

# Design and Development of a Low-cost Inverted Pendulum for Control Education

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**Abstract**—This paper describes the design and development of a low-cost inverted pendulum device for purposes of control education. The device is based on a modular construction in the form of assembly kit. The individual parts can be made of laser cut fiberboard or any appropriate material. The pendulum uses a stepper motor as the actuator for cart movement and rotary encoders for sensing the angle of rod and position of a cart. The main electronics used for control of motor and sensors reading is a micro-controller board with Atmel ATMega2560 8-bit MCU. The paper also describes the principles of operation of the device, along with the communication and external control interface written in MATLAB. The MATLAB command line interface contains a set of simple functions for signal acquisition and control of the main actuator in terms of position, velocity and acceleration. Students can use these to incorporate the device into their own operation algorithms and control scenarios. For educational purposes, the paper also deals with the mathematical modeling of the system and its simplifications that can be applied in the case of stepper motor usage.

## I. INTRODUCTION

A quality of control education highly depends on an engineering equipment available to students. It allows them to understand the fundamental principles of systems' behavior, and thus, to select appropriate approaches to control design. Still, in these days, many institutions build their control education on the use of computer simulations. Even this approach is valid, and in some context contributing to the understanding of physical systems, it cannot provide deeper insight into the real control problems.

At the Institute of Information Engineering, Automation and Mathematics (IAM), we try to provide our students with as much practical experimentation as possible. Therefore, some training systems have been developed and deployed into curricula in the past years. These are e.g. 2-dimensional plotting device, set of robotic vehicles, ball-and-beam system, and intelligent room. To ensure that students does not acquire their practical skills only as users of devices, but also as their developers, we encourage them to participate in the development of engineering equipment. Some of the systems such as robotic cars [1] have been developed with the help of students, and other like 2D plotter [2] have been created solely by students. We believe that the best systems for control design practicing are those of challenging nature. Interesting dynamical properties of a system are for example a natural instability, nonlinearity, fast dynamics, and various uncertainties. Therefore, we focused our attention on the development

of a mechanical system that exhibits mentioned properties and simultaneously operates on very basic principles of physics that students can understand even without deeper knowledge of mechanics.

One of the often used systems in control education is inverted pendulum. This is due to the fact that it exhibits interesting dynamical properties and this system is relatively comprehensible even for people with basic knowledge of mechanics and motion. Inverted pendulum, in its various forms, has been used in control education for many decades [3]. Even the pendulum is often in the education presented in its rawest form, its balancing nature is useful for many control applications that share the same concept. These are all self-balancing robots and one particular type known as a Segway [4]. Interesting applications of inverted-pendulum-based robotics can be found all over the literature. From general purpose scientific and educational robots [5], [6], through medical applications [7], to personal transportation and entertainment [8], the mechanics of inverted pendulum are present in a large class of physical systems. Therefore, it is essential for educators to provide students with a real control engineering equipment of such nature.

World of academia provides various low-cost solutions to the development of inverted pendulum systems for control education [9], [10]. The majority of these solutions are based on a typical setup that uses balanced rod attached on a cart that is controlled by a linear actuator (motor-driven belt or lead screw). Other solutions employ different physical principles of operation, such as electromagnetic actuation [11].

In this paper, we describe a design and development of an inverted pendulum for control education. The pendulum has been developed by one of the master's degree students as the part of a semestral project. Even the device is an *ad hoc* solution, it has been designed to provide several key features. The pendulum can be produced and provided as an assembly kit. For other educators interested in acquiring their copy, the device can be duplicated from the vector-based design sheet. Additionally, the pendulum is re-configurable in terms of parameters. Several parameters that influence the dynamic behavior can be changed or adjusted. Moreover, the device is provided with the control software and communication interface for MATLAB/Simulink. One of the main benefits of the device is its price. Using the low-cost parts the overall price (depending on the selected components) can be pushed below

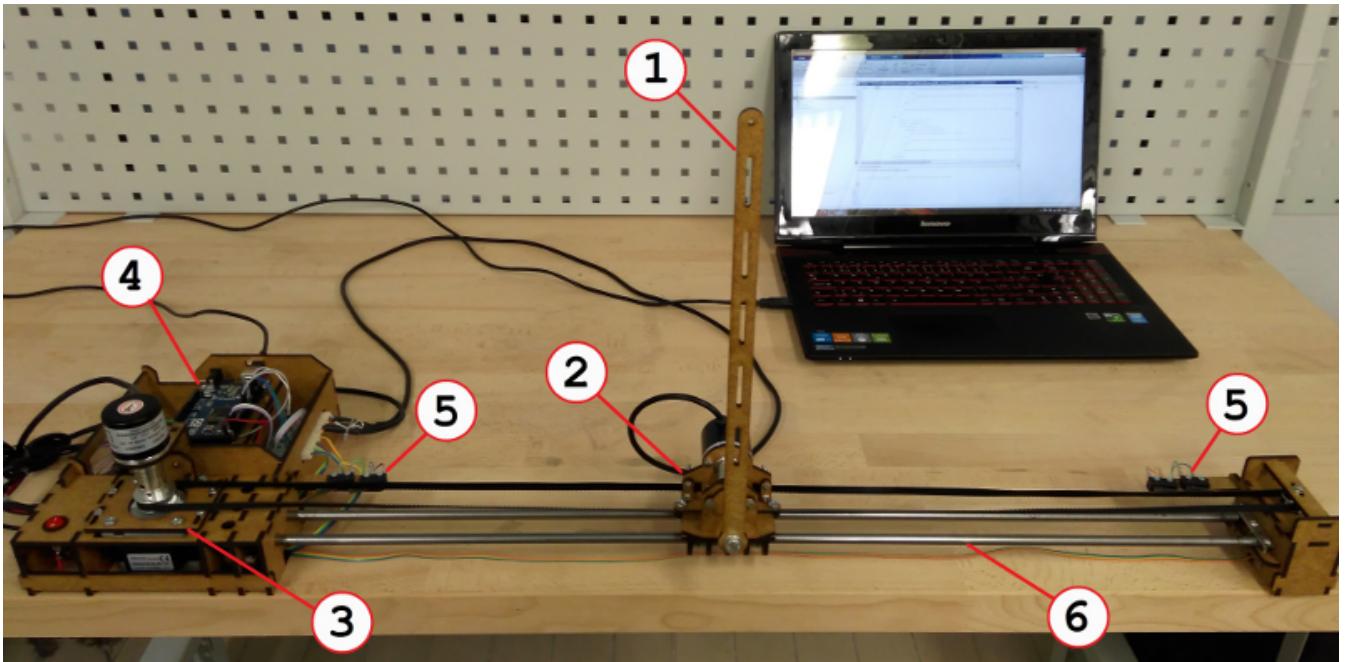


Fig. 1. Inverted pendulum with highlighted parts: 1 - rod of the pendulum, 2 - cart with encoder, 3 - motor with encoder, 4 - motor driver and micro-controller board, 5 - limit switches, 6 - cart leading rods.

the 200EUR, which makes the device much cheaper than similar devices from professional manufacturers of control education equipment.

## II. DEVICE DESCRIPTION

### A. Mechanical Construction

Construction of a pendulum (Fig. 1) is based on modular assembly kit and consists of roughly 50 individual parts. The main frame of the device was created via a computer design, and it is represented in vector graphics. This allows manufacturing the whole main frame components from a single laser-cut plate. The construction can be made of any material, such as metal, plastic, acrylic glass, wood, etc. The first instance of the device was manufactured from medium-density fibreboard (MDF). This material makes the frame of pendulum lightweight and robust at the same. The only parts that are not made of main construction material are the leading rods for pendulum cart, rotary encoders, switches, motor, and electronics. These can be chosen by a particular developer (or assembler) to fit the needs of an application. Since the modular nature of device provides possibility to interchange the actuators and sensors, educators can adjust or significantly change the parameters of the system. This includes the range of cart movement, axial/rotary friction, maximum force applied to the cart, encoder precision and accuracy, mass of pendulum, and position of pendulum's center of gravity.

Dimensional parameters of the device are as follows. The pendulum is 95cm in length, 10cm in width, and 14cm in height. These are dimensions of the main part. Controller

box that contains motor driver and micro-controller board can be attached to the side of the pendulum and its dimensions are 18/14/10cm. The overall length of cart-leading rods is 72cm and usable operating range (from left to right limit switch) is 60cm. This is also the maximum traveling distance of pendulum's cart that can be used before the actuator is turned off for security reasons. The length of the pendulum's balancing rod is 30cm and its center of gravity is in the middle by default. Rod is perforated, and it allows further weight distribution by attaching metal screws. This allows the adjustment of rod's/pendulum's parameters. Additionally two exchangeable rods of 35cm and 40cm are available.

The construction of the device is designed in such a way that it is fully reproducible. Anyone who is interested in assembling the own device can ask developers for sketches and manufacture the parts. The vector sketch of an assembly kit is shown in Fig. 2.

### B. Motor

The main actuator of the device is a rubber belt attached to the motor. It allows controlling the balance of the pendulum by applying an axial force to cart. Both, the velocity and acceleration can be used as control variables.

A current version of pendulum device is driven by a bipolar hybrid stepper motor Trinamic QSH618-45-28-110. This motor provides enough power and precision to move the cart fluently. Stepper motor is controlled by time-based switching (energizing) of two sets of motor coils, that represent the north and south poles of a stator. The rotor is made of permanent magnet. Some of the most important parameters of used motor are shown in Table I.

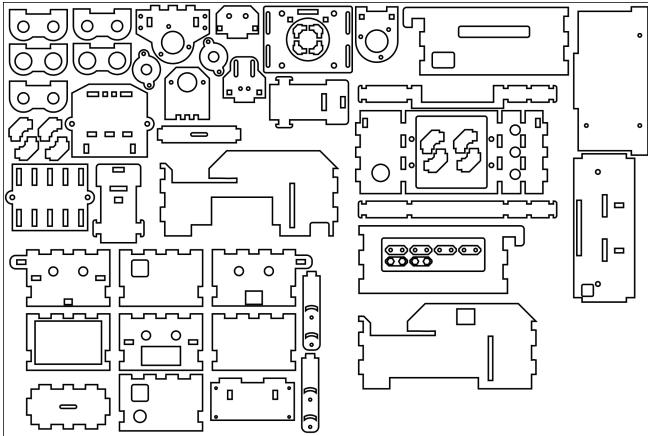


Fig. 2. Vector-defined parts of pendulum's main frame

TABLE I  
MOTOR TECHNICAL DATA

Specification	Value	Unit
Dimensions	60 × 60 × 45	mm
Weight (Mass)	0.6	kg
Connection wires	4	N°
Rated voltage	2.1	V
Max. voltage	75	V
Rated current	2.8	A
Holding torque	1.1	N m
Step angle	1.8	°
Step angle accuracy	5	%
Max. axial/radial force	15/75	N

### C. Motor Driver

The stepping of the motor is controlled via a driver. The driver used in the developed device is a low-cost variation of popular TB6600, namely a model HY-DIV268N-5A (Fig. 3) with a price tag of approximately 35EUR.

The driver is configurable by a set of DIP switches that can set up the nominal/peak current for coils and step resolution up to 16 divisions of one nominal step. Stepper motor



Fig. 3. Stepper motor driver HY-DIV268N-5A

separately energizes the coils A and B to perform predefined step-size movements based on input control signal. Binary signal *PUL*+/− is in charge of stepping motion, where one pulse performs a predefined portion of the step, and another binary signal *DIR*+/− controls the direction of motor's rotation. Additionally, the driver contains input signal *ENA*+/− for disconnecting the main power source from coil circuits. This feature allows to stop the motor's movement at any moment or to disable the holding torque.

### D. Sensors

Pendulum device uses two set of sensors to determine the position of cart and pendulum (Fig. 4). One incremental rotary encoder is attached to pendulum itself to provide feedback about the angle and the second one is mounted on a shaft of the motor to get an absolute reading of position, velocity, and acceleration of cart. LPD3806-400BM-G5-24C encoders used on a device fall into the category of low-cost sensors with an approximate cost of 25EUR per unit. With a reasonable resolution of 400 pulses per revolution, they provide enough precision for an intended application. Moreover, the photo-electric encoders are more suitable for this kind of application than for example potentiometers, because their readings do not subject to electrical noise.

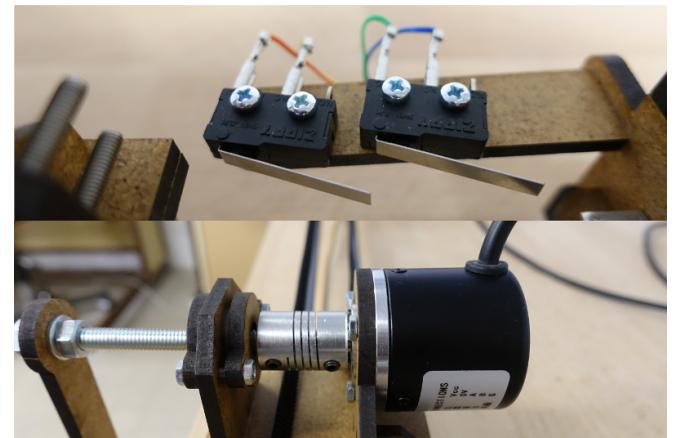


Fig. 4. Sensors used on device: top - limit switches, bottom - incremental rotary encoder

Additionally, four contact switches (in the groups of two) are located on each side of cart's leading rods. The first limit switch on each side is used as a homing reference during the initiation phase or as a soft turn-off switch during the operation of the pendulum. These two switches are connected to the main micro-controller and their function can be reprogrammed. The second two switches are connected directly to *ENA*+/− signals of the motor driver. This provides a secure method for disconnecting the motor from a power source in the case of an abnormal shift of the cart, and thus, to avoid damage to cart, rubber belt or main frame.

### E. Controller

A micro-controller board primarily controls the inverted pendulum (Fig. 5). The board is equipped with a high-performance, low-power consumption 8-bit AVR micro-controller unit (MCU) ATMega2560. This MCU operates at 16 MHz clock frequency, has 256 KB of flash memory and 8 KB of SRAM. The board provides 54 digital I/O pins, 16 pins with 10-bit ADCs (analog inputs), six independent timers (two 8-bit and four 16-bit), and six pins with external interrupt capabilities.

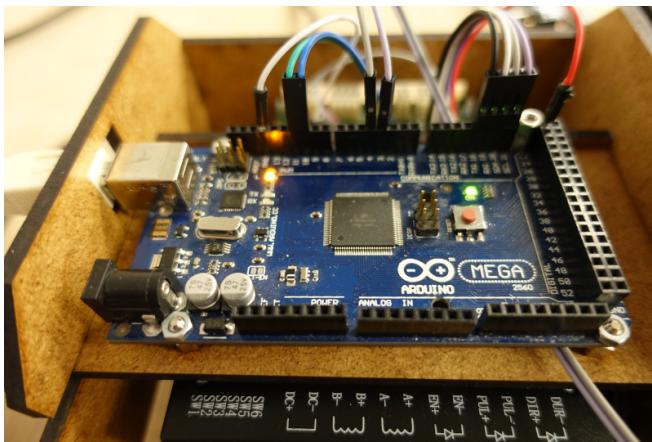


Fig. 5. Micro-controller board with integrated 8-bit MCU Atmel ATMega2560

## III. OPERATION OF DEVICE

### A. Main Program

The operation of inverted pendulum device is based on a program executed by the main controller Mega2560. This program is written in C/C++, and it periodically performs two main tasks. Firstly it reads the position of pendulums rod and cart via attached encoders, and secondly, it issues commands for stepper motor driver to move the cart. Encoders are read on a hardware level, using external interrupt pins of Mega2560. Each encoder issues two pulse signals that combined carry the information about the angular movement and direction. To avoid the loss of information, an amount of pulses from encoders are stored in counter registers during each reading cycle of the main program. These reading are then used to calculate the actual angle/angular velocity for pendulum and position/velocity for the cart. Actual states of the system are stored in the main program until the following reading rewrites them. The stepper motor is controlled via the variable-frequency square signal issued by single output pin to perform the stepping motion, and two logic signals for direction and power supply control.

The main program uses different sampling period for process read/update and communication. This allows the user to program the controller directly into Mega2560. In such a case the experimenter does not require an external computer to control the device. The second scenario is to use the Mega2560

only as the open loop controller and employ the serial communication interface to control the device via computer (using e.g. MATLAB). Using such scenario, the Mega2560 is still in charge of controlling the main actuator and performing data acquisition, but it does not contain the main closed-loop controller. The control computation is performed in an external computational environment.

### B. Communication

The serial interface serves the communication between the Mega2560 and computer with predefined baud rate 115200. The information is exchanged in the binary form. The transferred messages use a simple structure to allow fast communication speeds. The command message (sent from a computer to Mega2560) is 6 bytes long. The first byte represents the command type as a decimal number (unsigned integer from 0 to 255). Following 4 bytes can be used as optional parameters. For example, the motor control command uses all 4 bytes and encodes a single-precision floating point number in IEEE 754 format. The last byte of the message is used as a message terminator. A current version of the program uses a line feed character (0x0A). List of currently available commands is shown in Table II.

The communication used by the developed interface is of synchronous nature (on-demand mechanism). Each command message sent through a serial interface is immediately followed by a response message. The response can be of variable length, but it always contains at least 2 bytes. The first byte contains a decimal number of original command that invoked the action. The last byte is the terminator 0x0A. Additionally, commands 5-9 also return the values of sensor readings. These are represented by 9 bytes, containing floating point values of 2 encoder readings in angular units and 1 byte with positions of limit switches.

### C. MATLAB Interface

Students can use and control the device from MATLAB. The only requirements are the environment itself and class called Pendulum. The control of experiment via the command line is straightforward. The user creates an object that represents the device by creating an instance of a class and defining a communication port of the device.

```
% Instance of Pendulum  
my_pendulum = Pendulum('COM33');  
% establishes communication  
my_pendulum.connect();
```

Using this command, a MATLAB creates the my\_pendulum variable that contains all properties and methods required for control of pendulum. The user can read the process variables by simply calling data acquisition methods.

```
% Reading of pendulum's angle  
angle = my_pendulum.getAngle();  
  
% Reading of cart's position  
cart_pos = my_pendulum.getCartPos();
```

TABLE II  
SERIAL COMMANDS FOR PENDULUM DEVICE

Command type	Byte 1	Byte 2-5	Byte 6	Description
Initialization	DEC 1	HEX 0x00000000	HEX 0x0A	resets internal variables to initial values
Actuator ON	DEC 2	HEX 0x00000000	HEX 0x0A	energizes motor coils (applies holding torque)
Actuator OFF	DEC 3	HEX 0x00000000	HEX 0x0A	de-energizes motor coils (turns off holding torque)
Homing	DEC 4	HEX 0x00000000	HEX 0x0A	moves cart to position of left limit switch
Move cart abs.	DEC 5	FLOAT X	HEX 0x0A	moves cart to $X$ mm from left limit switch
Move cart rel.	DEC 6	FLOAT X	HEX 0x0A	moves cart to $X$ mm from its last position
Move cart vel.	DEC 7	FLOAT V	HEX 0x0A	moves cart with defined velocity $V$
Move cart acc.	DEC 8	FLOAT A	HEX 0x0A	moves cart with defined acceleration $A$
Get sensors	DEC 9	HEX 0x00000000	HEX 0x0A	performs no action and returns sensor readings

```
% Reading all sensors at once
sensors = my_pendulum.getSensors();

% Extracting angle and switch position
angle = sensors.angle;
switch1 = sensors.switch(1);
```

All data acquisition command in MATLAB take the last acquired values from device. These are store in class instance and periodically updated by timer-based function. The update frequency can be set by changing `updateFreq` property using one of the following commands.

```
% setting upd. freq. to 50Hz
my_pendulum.setUpdateFreq(50);
% the same in sampling time
my_pendulum.setSamplingTime(0.02);
```

Actuator can be controlled directly via a change of position, velocity, or acceleration of the cart.

```
% move cart absolutely to x = 250mm
my_pendulum.moveAbs(250);
% move cart relatively by 10mm
my_pendulum.moveRel(10);
% move cart with velocity 100mm/s
my_pendulum.setVel(100);
% move cart with acceleration 20mm/s^2
my_pendulum.setAcc(20);
```

Each movement command also accepts negative values. These represent the movement in the opposite direction. Additionally, the units of length can be changed from default millimeters to meters by command `my_pendulum.setLengthUnit('m')`. Immediate stop of pendulum's motion can be triggered via command `my_pendulum.stop()`. After the experiment is over, the session is terminated using `my_pendulum.close()`.

Using these commands, students can compose their control scenarios using either M-files or provided a Simulink model of the device.

#### IV. MATHEMATICAL MODEL

Students that will work with the device require a model of its natural behavior. In fact, learning to derive the mathematical description of mechanical systems is an excellent introduction to system's modeling. Since these systems are very comprehensible, students can better understand the mechanics and overall "what is going on" principle than for other classes

of systems. Mathematical modeling of an inverted pendulum is well-known procedure [12]–[14]. It can be derived using standard force balance equations based on Newton's laws of motion [12] or, alternatively, by using the Lagrange's equations [13]. Both approaches yield the same result.

The motion of the pendulum's base (cart) can be expressed as a 2nd order differential equation (1).

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta} \cos \theta - ml\dot{\theta}^2 \sin \theta = F \quad (1)$$

This equation defines the forces acting on a cart, where  $x$  is the position of the cart,  $M$  is a mass of the cart,  $m$  is the mass of a pendulum,  $l$  is a length of the pendulum,  $b$  is a friction coefficient of the cart movement,  $\theta$  is an angle of the pendulum's rod, and  $F$  is a force acting on the cart. The second governing equation (2) defines the motion of pendulum's rod

$$(I + ml^2)\ddot{\theta} + mgl \sin \theta = -ml\ddot{x} \cos \theta, \quad (2)$$

where  $I = \frac{ml^2}{3}$  is a mass moment of inertia of a uniform rod rotating about one end.

If a stepper motor is used as the main actuator, the model can be significantly simplified. Assuming that a movement command for a motor is directly translated to the change of cart's position and that the holding torque of the motor is high enough to overcome the forces of pendulum acting backward on the cart, the equation (1) can be neglected. In such a setup, control input for rod balancing is not a force  $F$ , but velocity  $v$  or acceleration  $\dot{v}$  of cart respectively. These assumptions reduce the model to the single equation

$$(I + ml^2)\ddot{\theta} + mgl \sin \theta = -ml\dot{v} \cos \theta. \quad (3)$$

Since the model (3) is nonlinear in  $\theta$ , it must be linearized to obtain form suitable for linear control design methods.

As it is evident from the general behavior of inverted pendulum, it has two natural equilibrium points, both in the vertical position of pendulum's rod. The interesting equilibrium point for control applications is the upward position of the pendulum because it is the unstable point. For this point  $\theta = \pi$  a small deviation of pendulum's position  $\phi$  reflects to the change of angle as  $\theta = \pi + \phi$ . Then using simple approximations

$$\ddot{\theta} = \ddot{\phi} \quad (4)$$

$$\cos \theta = \cos (\pi + \phi) \approx -1 \quad (5)$$

$$\sin \theta = \sin (\pi + \phi) \approx -\phi, \quad (6)$$

the model can be linearized in the point of its unstable equilibrium, resulting into the following form.

$$(I + ml^2)\ddot{\phi} - mgl\phi = -ml\dot{\phi} \quad (7)$$

Using this linear model, students can derive transfer function or state-space representation in order to analyze dynamical behavior and to design an appropriate controller.

## V. CONCLUSIONS

The developed device of inverted pendulum provides some interesting features especially suitable for educational purposes. The design of the device allows other educators to manufacture and assemble their own copies of the pendulum. This is because it uses modular construction and other parts that can be easily acquired on the market. Additionally, the overall cost of the device is much lower than similar devices from industrial control equipment manufacturers. One of the benefits is also the possibility to control the device via the popular computational software MATLAB that is used in the majority of educational institutions in the field of automatic control. The inverted pendulum is a very comprehensible physical system that allows students to deal with the simple motion control that is an essential base for many other systems in robotics.

Future work will be focused on the development of communication interface via the computer network. This will allow the educators to incorporate the device into the remote labs and share them with other people in the academic area.

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