We are aiming to numerically calculate a Fourier integral using the FFT method.

$$f(x) = \int_R F(k)e^{2\pi ikx}dk$$

$$f(x) = \sum_{-\infty}^{\infty} F(k)e^{2\pi ikx}\Delta_k$$

$$\approx \sum_{-k_{max}}^{k_{max}} F(k)e^{2\pi ikx}\Delta_k$$

$$= \sum_{m=-M}^{M-1} F(m\Delta_k)e^{2\pi im\Delta_k x}\Delta_k \text{ where } k_{max} = M\Delta_k$$

$$= \Delta_k e^{-2\pi iM\Delta_k x} \sum_{m=0}^{2M-1} F((m-M)\Delta_k)e^{2\pi im\Delta_k x}$$

This form looks similar to that of the Inverse Discrete Fourier Transform, shown below:

$$a_n = \frac{1}{N}\sum_{m=0}^{M-1} A_k e^{2\pi i\frac{nm}{M}} \text{ for } n = 0...M-1$$

Let us discretize x to see further similarities.

$$\text{Let } x \to x_n = \Delta_x n \text{ for } n = -M...M-1$$

$$f(x_n) = \Delta_k e^{-2\pi ik_{max}\Delta_x n} \sum_{m=0}^{2M-1} F((m-M)\Delta_k)e^{2\pi imn\Delta_x\Delta_k} \text{ for } n = -M...M-1$$

$$f(x_n) = \Delta_k e^{-2\pi ik_{max}\Delta_x(n-M)} \sum_{m=0}^{2M-1} F((m-M)\Delta_k)e^{2\pi imn\Delta_x\Delta_k}e^{-2\pi imM\Delta_x\Delta_k} \text{ for } n = 0...2M-1$$

We can recover the Inverse DFT form if the following condition holds:

$$\Delta_x\Delta_k = \frac{1}{M} \to \Delta_x = \frac{1}{M\Delta_k} = \frac{1}{k_{max}}$$

$$\text{Therefore } x_{max} = \Delta_x M = \frac{1}{\Delta_k} = \frac{M}{k_{max}}$$

Since Δ_x scales with k_{max}, the only way to obtain higher resolution is to increase the cutoff point in k-space.