Exercise

Compute the probability that the Markov Model in page 17 (Chapter 5 slides) generates the string "bcba".

b	c	b	a
1	1	1	2
2	2	2	3

b	c	b	a
1	1	1	
	2	2	3

, and state 2 cannot repeat itself.

Valid sequence of states: **1 1 1 3**, **1 1 2 3**, **1 2 1 3**

P(bcba|M)=P(bcba,1113F)+P(bcba,1123F)+P(bcba,1213F)=
P(bcba|1113F)*P(1113F)+ P(bcba|1123F)*P(1123F)+ P(bcba|1213F)*P(1213F)=0.0073386

In this resolution, we have first to calculate all the possible sequences of states that generate the string. We might leave out some sequence of states …

Forward algorithm

The forward algorithm is used to compute P(y|M) without need to enumerate explicitly the sequence of states that derive the string 'y'.

Instead of calculating a priori the sequence of states, the Forward algorithm performs a series of recursive calculations to compute P(y|M) and also obtains the sequence of states that derive the string 'y'.

Basic concept

α(q,t): where 'q' is a state and 't' is a sequence of 't' symbols; e.g. **α(1,b), α(1,bc), α(2,bca),….**

For example, for string y $\equiv$ bcaa

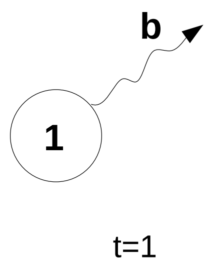
states \ sequence of symbols	t=1 (b) b	t=2 (c) bc	t=3 (a) bca	t=4 (a) bcaa
1	α(1,b)	α(1,bc)	α(1,bca)	α(1,bcaa)
2	α(2,b)	α(2,bc)	α(2,bca)	α(2,bcaa)
3	α(3,b)	α(3,bc)	α(3,bca)	α(3,bcaa)

α**(q,t)** is the probability that a Markov model (M) reaches state 'q' at instant 't' and emits the string $y_1, y_2, \ldots, y_t$

More specifically, let's assume we have three states (1,2,3):

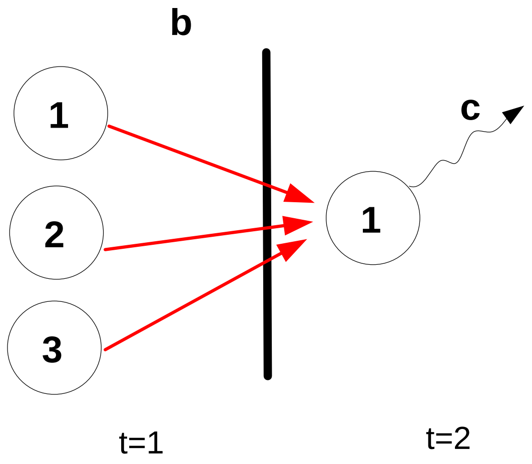
t=1

α**(1,b):** probability that M reaches state 1 at t=1 and emits symbol 'b': α**(1,b)=**P(1)P(b|1)



t>1

α**(1,bc):** probability that M reaches state 1 at t=2 and emits symbol 'c' having emitted before symbol 'b' with any other state. The emission before is recursively computed.



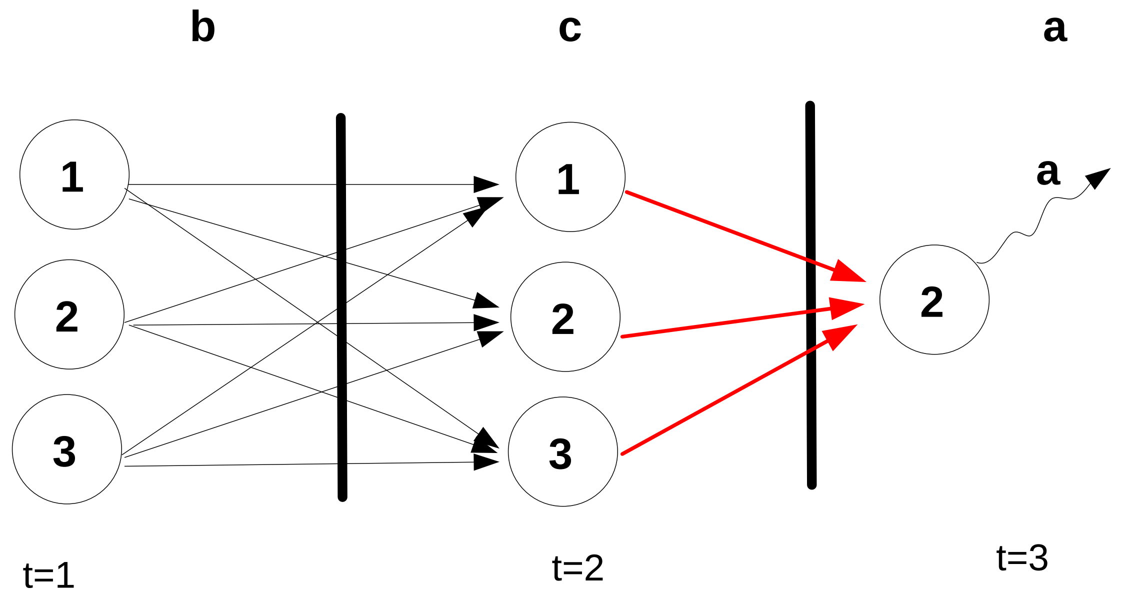
probability of reaching state 1 at t=2 having emitted before “b”

$$\alpha(1,bc)=P(c|1)*(\boxed{\alpha(1,b)*P(1|1)+\alpha(2,b)*P(1|2)+\alpha(3,b)*P(1|3)})$$

α(1,bc):

1. probability that state 1 emits symbol c **(P(c|1)) ***
2. probability of reaching state 1 having emitted before symbol b; i.e., the probability that when state 1 is reached, the symbol ‘b’ has already been emitted **(α(1,b)*P(1|1)+ α(2,b)*P(1|2)+ α(3,b)*P(1|3))**

α(2,bca): probability that M reaches state 2 at t=3 and emits symbol ‘a’ having emitted before symbols ‘bc’ with any other states.



$$\alpha(2,bca)=P(a|2)*\boxed{(\alpha(1,bc)*P(2|1)+\alpha(2,bc)*P(2|2)+\alpha(3,bc)*P(2|3)}\,)$$

probability of reaching state 2 at t=3 having emitted before “bc”

$$P(\mathbf{y}|\mathbf{M}) = \sum_{q\in Q} \boldsymbol{\alpha}(\mathbf{q},|\mathbf{y}|)\mathbf{P}(\mathbf{F}|\mathbf{q})$$

$$\boldsymbol{P}(\mathbf{bcba}|\mathbf{M}) = \sum_{\boldsymbol{q} \in \boldsymbol{Q}} \boldsymbol{\alpha}(\mathbf{q}, |\mathbf{bcba}|)\mathbf{P}(\mathbf{F}|\mathbf{q}) = \sum_{\boldsymbol{q'} \in \boldsymbol{Q}} \boldsymbol{\alpha}(\mathbf{q}', |\mathbf{bcb}|)\ \mathbf{P}(\mathbf{a}|\mathbf{q})\mathbf{P}(\mathbf{q}|\mathbf{q}')\mathbf{P}(\mathbf{F}|\mathbf{q}) = \cdots$$

Forward

	b **t=1**	**c** **t=2**	**b** **t=3**	**a** **t=4**	
1	$\alpha(1,b) = P(b\|1)P(1) = B_{1b}\pi_1 = \mathbf{0.3}$	$\alpha(1,bc) = P(c\|1)\sum_q P(1\|q)\alpha(q,b) = P(c\|1)\big(P(1\|1)\,\alpha(1,b) + P(1\|2)\,\alpha(2,b) + P(1\|3)\,\alpha(3,b)\big) = \mathbf{0.042}$	$\alpha(1,bcb) = P(b\|1)\sum_q P(1\|q)\alpha(q,bc) = P(b\|1)\big(P(1\|1)\,\alpha(1,bc) + P(1\|2)\,\alpha(2,bc) + P(1\|3)\,\alpha(3,bc)\big) = \mathbf{0.00297}$	$\alpha(1,bcba) = P(a\|1)\sum_q P(1\|q)\alpha(q,bcb) = P(a\|1)\big(P(1\|1)\,\alpha(1,bcb) + P(1\|2)\,\alpha(2,bcb) + P(1\|3)\alpha(3,bcb)\big) = 0$	
2	$\alpha(2,b) = P(b\|2)P(2) = B_{2b}\pi_2 = 0$	$\alpha(2,bc) = P(c\|2)\sum_q P(2\|q)\alpha(q,b) = P(c\|2)(P(2\|1)\alpha(1,b) + P(2\|2)\alpha(2,b) + P(2\|3)\alpha(3,b)) = \mathbf{0.015}$	$\alpha(2,bcb) = P(b\|2)\sum_q P(2\|q)\alpha(q,bc) = P(b\|2)\big(P(2\|1)\,\alpha(1,bc) + P(2\|2)\,\alpha(2,bc) + P(2\|3)\,\alpha(3,bc)\big) = \mathbf{0.0126}$	$\alpha(2,bcba) = P(a\|2)\sum_q P(2\|q)\alpha(q,bcb) = P(a\|2)\big(P(2\|1)\,\alpha(1,bcb) + P(2\|2)\,\alpha(2,bcb) + P(2\|3)\,\alpha(3,bcb)\big) = 0.0004455$	
3	$\alpha(3,b) = P(b\|3)P(3) = B_{3b}\pi_3 = 0$	$\alpha(3,bc) = P(c\|3)\sum_q P(3\|q)\alpha(q,b) = P(c\|3)(P(3\|1)\alpha(1,b) + P(3\|2)\alpha(2,b) + P(3\|3)\alpha(3,b)) = 0$	$\alpha(3,bcb) = P(b\|3)\sum_q P(3\|q)\alpha(q,bc) = P(b\|3)\big(P(3\|1)\,\alpha(1,bc) + P(3\|2)\,\alpha(2,bc) + P(3\|3)\,\alpha(3,bc)\big) = 0$	$\alpha(3,bcba) = P(a\|3)\sum_q P(3\|q)\alpha(q,bcb) = P(a\|3)\big(P(3\|1)\,\alpha(1,bcb) + P(3\|2)\,\alpha(2,bcb) + P(3\|3)\alpha(3,bcb)\big) = \mathbf{0.012231}$	
F					$P(bcba\|M) = \sum_q \alpha(q,\mathrm{bcba})A_{qF} = \sum_q \alpha(q,\mathrm{bcba})P(F\|q) = \alpha(1,\mathrm{bcba})P(F\|1) * \alpha(2,\mathrm{bcba})P(F\|2) * \boldsymbol{\alpha(3,\mathrm{bcba})P(F\|3)}$ $\mathbf{0.0073386}$

Viterbi

V	b t=1	c t=2	b t=3	a t=4	
1	$V(1,b) =$ $P(b\|1)P(1) =$ $B_{1b}\pi_1 = \mathbf{0.3}$	$V(1,bc) =$ $P(c\|1)\max_q P(1\|q)V(q,b) =$ $P(c\|1)P(1\|1)V(1,b) =$ $\mathbf{0.042}$	$V(1,bcb) =$ $P(b\|1)\max_q P(1\|q)V(q,bc) =$ $P(b\|1)P(1\|1)V(1,bc) =$ 0.0025	$V(1,bcba) =$ $P(a\|1)\max_q P(1\|q)V(q,bcb) =$ $P(a\|1)P(1\|2)V(2,bcb) =$ 0	
2	$V(2,b) =$ $P(b\|2)P(2) =$ $B_{2b}\pi_2 = 0$	$V(2,bc) =$ $P(c\|2)\max_q P(2\|q)V(q,b) =$ $P(c\|2)P(2\|1)V(1,b) =$ 0.015	$V(2,bcb) =$ $P(b\|2)\max_q P(2\|q)V(q,bc) =$ $P(b\|2)P(2\|1)V(1,bc) =$ $\mathbf{0.0126}$	$V(2,bcba) =$ $P(a\|2)\max_q P(2\|q)V(q,bcb) =$ $P(a\|2)P(2\|1)V(1,bcb) =$ 0.0004	
3	$V(3,b) =$ $P(b\|3)P(3) =$ $B_{3b}\pi_3 = 0$	$V(3,bc) =$ $P(c\|3)\max_q P(3\|q)V(q,b) =$ $P(c\|3)P(3\|1)V(1,b) =$ 0	$V(3,bcb) =$ $P(b\|3)\max_q P(3\|q)V(q,bc) =$ $P(b\|3)P(3\|2)V(2,bc) =$ 0	$V(3,bcba) =$ $P(a\|3)\max_q P(3\|q)V(q,bcb) =$ $P(a\|3)P(3\|2)V(2,bcb) =$ $\mathbf{0.0113}$	
F					$P(bcba\|M) =$ $\max_q V(q,bcba)A_{qF} =$ $max_q V(q,bcba)P(F\|q) =$ $\boldsymbol{max_q V(3,bcba)P(F\|3)} =$ $0.0113 * 0.6 =$ $\mathbf{0.00678}$

The most probable sequence of states for generating string “bcba” is {1123F}