Problem A.1

Newton's Method

Due Date: 4/5/2019
Folder: `Applied`
File Name: `A1_Newton_Name.py`
Points: 5 points

Learning Objectives

- Implement a given algorithm

Problem Background

Newton's method is an algorithm for approximating the solution to an algebraic equation of the form,

$$f(z) = 0.$$

The goal is to find a value of z the makes the function $f = 0$. This is done with Newton's method by an iterative process. We begin with an initial iterate z_0. This is a good guess to the solution (though sometimes the guess isn't very good at all if we have no information about the solution). We then attempt to "improve" the guess with the following equation

$$z_1 = z_0 - \frac{f(z_0)}{f'(z_0)}.$$

The idea being that z_1 is a better approximation. This is continued multiple times, using the iterative formula

$$z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}.$$

This iterative process is stopped when the iterations begin to get close together. So each iteration we test to see if $|z_{n+1} - z_n| < \text{TOL}$. Where TOL is some small number we choose, depending on how accurate we want our approximation. We may also stop the iterative process if we have done too many iterations, as this process can continue forever in some situations.

Program Criteria

Write a program that does the following:

- Define a variable `N = 100` to represent the maximum number of iterations to perform.
- Define a variable `TOL = 1e-4` to represent the tolerance at which we will stop the iterative process.
- Define a variable `z0` for the initial iterate of Newton's method
- Create a `lambda` function for $f(z)$ and another for $f'(z)$.
- Implement Newton's method. Make your iterations stop **either** when $|z_{n+1} - z_n| <$TOL, or when you have done `N` iterations. Store all the z_i iterations you calculate, as well as the spacing between them, that is $|z_{i+1} - z_i|$.
- Print out if the iterations stopped due to reaching tolerance, or stopped because you did the max number of iterations. The first means you converged, the second means you did not.
- Print out all iterations you calculated, along with the difference between them that you stored.

Deliverables

Place the following in a folder named `Applied` in your repository:

- A Python file `A1_Newton_Name.py` that satisfies the program criteria.
- A pdf file `A1_Newton_Name.pdf` created with Latex:
  - ⋆ Use your program to approximate all solutions to the equation

$$\frac{1}{100}[x^4+(e-2-\sqrt{2})x^3+(2\sqrt{2}-\sqrt{2}e-3-2e)x^2+(2\sqrt{2}e+3\sqrt{2}-3e)x+3\sqrt{2}e]=0.$$

    Write down all approximations to the solution
  - ⋆ Use your program to approximate two positive and two negative solutions to the equation,

$$\tan(x)-x-2=0.$$