

Problem F.5

Newton Fractals

Due Date: 5/3/2019

Folder: FinalProject

File Name: F5_Newton_Name.py

Points: 20 points

Problem Background

Newton's method is a way of solving an equation

$$f(z) = 0.$$

It is an iterative method. We start it with an initial "guess" to the solution, z_0 . We then find a better approximation, z_1 of the solution through the following equation,

$$z_1 = z_0 - \frac{f(z_0)}{f'(z_0)},$$

where $f'(z)$ is the derivative of $f(z)$. We repeat this process multiple times with the same formula,

$$z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)},$$

until eventually we converge and the iterations start to get close together,

$$|z_{n+1} - z_n| < \text{TOL},$$

where TOL is some small tolerance defining how close we want the iterations to get before we stop.

The final approximation to $f(z) = 0$ depends on where we start, z_0 . If we choose a different starting point, we might converge to a different solution, as $f(z) = 0$ might have multiple solutions. One thing to note is that the solutions might be complex, and so the initial starting points z_0 can also be complex.

A Newton fractal is a colored complex plane that denotes pictorially which solution of $f(z) = 0$ each initial point z_0 converges to. A number of these fractals can be seen at [Newton Fractal](#). As an example consider the equation

$$f(z) = z^3 - 1.$$

This has three solutions, $z = 1$, $z = -e^{i\pi/3}$, $z = e^{2i\pi/3}$. We label each of these solutions with a color. Then, using Newton's method, say we start with an initial iterate $z_0 = 1 + i$. Using this initial guess, Newton's method converges to 1. Therefore, we would color a dot on the complex plane at (1,1) red. However, if we start at $z_0 = -1 - i$, this converges to $-e^{i\pi/3}$ and so we would color a dot on the complex plane at (-1,-1) blue. This would be done for all initial iterates in the complex plane, and all those colored dots would create a Newton fractal.

In this project, you will create a program to generate a Newton Fractal for any function $f(z)$. Then you can play around with the function and see what different Newton fractals are generated. In addition, some pictures incorporate the number of iterations it takes to converge in the image. This can also be incorporated into your code to create and even more interesting picture.

Program Criteria

Write a program that does the following:

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Deliverables

Place the following in a folder named **FinalProject** in your repository:

- A Python file `F5_Newton_Name.py` that satisfies the program criteria.