Problem I.5

Fibonacci Sequence

Due Date: 2/13/2019
Folder: `IntroToProgramming`
File Name: `FibSeq_LastName.py`

Learning Objectives

- Programming Skills ⋆ For loops
- Use numerical data to determine new identity

Problem Background

The Fibonacci sequence has been around for millenia, but has been in the western world since about the 1200s. It is defined recursively in the following way. The first two terms are given as

$$F_0 = 0, F_1 = 1,$$

adn the following terms in the sequence are determined by the recursive relationship,

$$F_n = F_{n-1} + F_{n-2}.$$

The Fibonacci sequence has been seen in multiple places in nature, more that would be expected from such a simple to define sequence. Some examples are music, optics, rabbit population (one of the earliest observations), botany, and computer search algorithms. If you are interested, you can start looking for more information on the Wikipedia page.

In addition to its appearance in nature, the Fibonacci sequence has many interesting mathematical properties. One such property is another recurence relation between its terms, called Cassini's identity,

$$F_n^2 - F_{n-1}F_{n+1} = (-1)^{n-1}.$$

A generalization of Cassini's identity is Catalan's identity,

$$F_n^2 - F_{n+r}F_{n-r} = (-1)^{n-r}F_r^2,$$

for

Programming Reminders

- Syntax for a `for` loop: `for ii in range(N):`
- To create an empty list use `my_list = []`

Program Criteria

Write a program that does the following:

- Create an input variables `F0` and `F1` for the first two terms of your sequence.
- Create an input variable `N` for the number of terms to generate in your sequence.
- Generate the first N terms for the sequence using the Fibonacci recursion relation

  $$F_n = F_{n-1} + F_{n-2},$$

  but using the initial terms `F0` and `F1`. **Be sure to store the sequence in a list, not an** `np.array`.
- Write code to check whether the sequence you generate satisfies Catalan's identity.
- Print out the last 10 terms in the sequence you generate, with an appropriate description.

Deliverables

For this project, you will be testing whether Catalan's identity is satisfied when the two initial terms of the sequence are changed. If you find the identity is not satisfied, can you generate a similar identity that depends on the initial terms?

Place the following in a folder named `IntroToProgramming` in your repository:

- A Python file `FibSeq_LastName.py` that satisfies the program criteria.
- A Latex document `FibSeq_LastName.pdf` with the following information:
  - ⋆ Explain briefly what you did to determine if Catalan's identity holds when the initial terms of the sequence change.
  - ⋆ If it does not hold, state how you think it changes when the initial terms change.
  - ⋆ If you were able to determine a similar identity, state the identity and how it depends on the initial terms.