Problem N.3

Zero Divisors of $\mathbb{Z}_m$

Due Date: 3/25/2019
Folder: `NumberTheory`
File Name: `N3_ZeroDivisors_Name.py`
Points: 2 points

Learning Objectives

- Mix of programming skills
- Disprove a conjecture

Problem Background

Recall the basics of modular arithmetic and the group $\mathbb{Z}_m$. We know that the set of all residue classes is given by,

$$\mathbb{Z}_m = \{0, 1, 2, \ldots, (m-1)\}.$$

Addition, subtraction and multiplication on these elements is done similarly to regular integers, but using modular arithmetic.

For example, assume we are considering $\mathbb{Z}_7 = \{0, 1, 2, 3, 4, 5, 6\}$. In $\mathbb{Z}_7$ we have that 5+4=2, because $9 \equiv 2 (mod\ 7)$. Also, $2 \cdot 7 = 0$, because $2 \cdot 7 = 14$ and 14 is a multiple of 7, so it is zero in $\mathbb{Z}_7$.

Many things are similar between $\mathbb{Z}_m$ and the integers, but some things are different. One of those things involves zero divisors. A **zero divisor** is an element such that when you multiply it by a non-zero element you end up with zero. So for instance, about we saw that in $\mathbb{Z}_7$, when you multiple 2 and 7, they equal zero. Therefore, both 2 and 7 are zero divisors in the set $\mathbb{Z}_7$. The number of zero divisors and which elements are zero divisors will change depending on what the m is in $\mathbb{Z}_m$.

Program Criteria

Write a program that does the following:

- Create an input variable `m` that will represent which $\mathbb{Z}_m$ set we are working with.
- Determine which elements of $\mathbb{Z}_m$ are zero divisors.
- Print out all the zero divisors and how many zero divisors there are, with appropriate descriptive text.

Deliverables

Place the following in a folder named `NumberTheory` in your repository:

- A Python file `N3_ZeroDivisors_Name.py` that satisfies the program criteria.
- A pdf file `N3_ZeroDivisors_Name.pdf` describe a simple test for whether a particular element of $\mathbb{Z}_m$ is a zero divisor. This test will probably depend on m. This should not be a description of your program, but a simpler test that will describe all elements that are zero divisors.