Job Scheduling

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Ordonnancement

Content of the Lecture

- What is job scheduling?
- Single machine problems and results
- Makespan problems on parallel machines
- Utilization problems on parallel machines
- Completion time problems on parallel machines
- Exemplary workload problem

Examples of Job Scheduling

- Processor scheduling
  - Jobs are executed on a CPU in a multitasking operating system.
  - Users submit jobs to web servers and receive results after some time.
  - Users submit batch computing jobs to a parallel processor.

- Bandwidth scheduling
  - Users call other persons and need bandwidth for some period of time.
- Airport gate scheduling
  - Airlines require gates for their flights at an airport.
- Repair crew scheduling
  - Customer request the repair of their devices.

Job Properties

- Independent jobs
  - No known precedence constraints
    - Difference to task scheduling
- Atomic jobs
  - No job stages
    - Difference to job shop scheduling
- Batch jobs
  - No deadlines or due dates
    - Difference to deadline scheduling

p_j	processing time of job j	
r_j	release date of job j	earliest starting time
w_j	weight of job j	importance of the job
m_j	size of job j	parallelism of the job

Machine Environments

- 1: single machine
  - Many job scheduling problems are easy.
- **P_m: m parallel identical machines**
  - Every job requires the same processing time on each machine.
  - Use of machine eligibility constraints M_j if job j can only be executed on a subset of machines
    - Airport gate scheduling: wide and narrow body airplanes
- Q_m: m uniformly related machines
  - The machines have different speeds v_i that are valid for all jobs.
  - In deterministic scheduling, results for P_m and Q_m are related.
  - In online scheduling, there are significant differences between P_m and Q_m.
- R_m: m unrelated machines
  - Each job has a different processing time on each machine.

Restrictions and Constraints

- Release dates r_j
- Parallelism m_j
  - Fixed parallelism: m_j machines must be available during the whole processing of the job.
  - Malleable jobs: The number of allocated machines can change before or during the processing of the job.
- Preemption
  - The processing of a job can be interrupted and continued on another machine.
  - Gang scheduling: The processing of a job must be continued on the same machines.
- Machine eligibility constraints M_j
- Breakdown of machines
  - m(t): time dependent availability

rarely discussed in the literature

Objective Functions

- Completion time of job j: C_j
- Owner oriented:
  - Makespan: $C_{max} = \max(C_1, \ldots, C_n)$
    - completion time of the last job in the system
  - Utilization U_t: Average ratio of busy machines to all machines in the interval (0,t] for some time t.
- User oriented:
  - Total completion time: $\Sigma\, C_j$
  - Total weighted completion time: $\Sigma\, w_j C_j$
  - Total weighted waiting time: $\Sigma\, w_j (C_j - p_j - r_j) = \Sigma\, w_j C_j - \underbrace{\Sigma\, w_j (p_j + r_j)}_{\text{const.}}$
  - Total weighted flow time: $\Sigma\, w_j (C_j - r_j) = \Sigma\, w_j C_j - \underbrace{\Sigma\, w_j r_j}_{\text{const.}}$
- Regular objective functions:
  - non decreasing in $C_1, \ldots, C_n$

Workload Classification

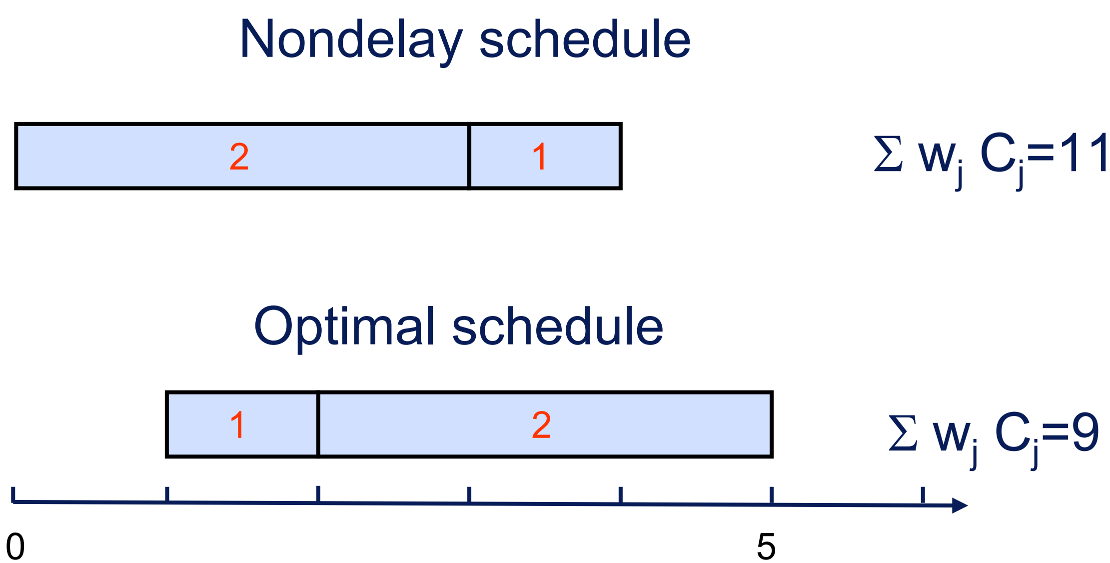
- Deterministic scheduling problems
  - All problem parameters are available at time 0.
  - Optimal algorithms,
  - Simple individual approximation algorithms
  - Polynomial time approximation schemes
- Online scheduling problems
  - Parameters of job j are unknown until r_j (submission over time).
  - p_j is unknown C_j (nonclairvoyant scheduling).
  - Competitive analysis
- Stochastic scheduling
  - Known distribution of job parameters
  - Randomized algorithms
- Workload based scheduling
  - An algorithm is parameterized to achieve a good solution for a given workload.

Nondelay (Greedy) Schedule

- No machine is kept idle while a job is waiting for processing.
  An optimal schedule need not be nondelay!

Example: $1 \mid \mid \Sigma\, w_j C_j$

jobs	1	2
p_j	1	3
r_j	1	0
w_j	2	1



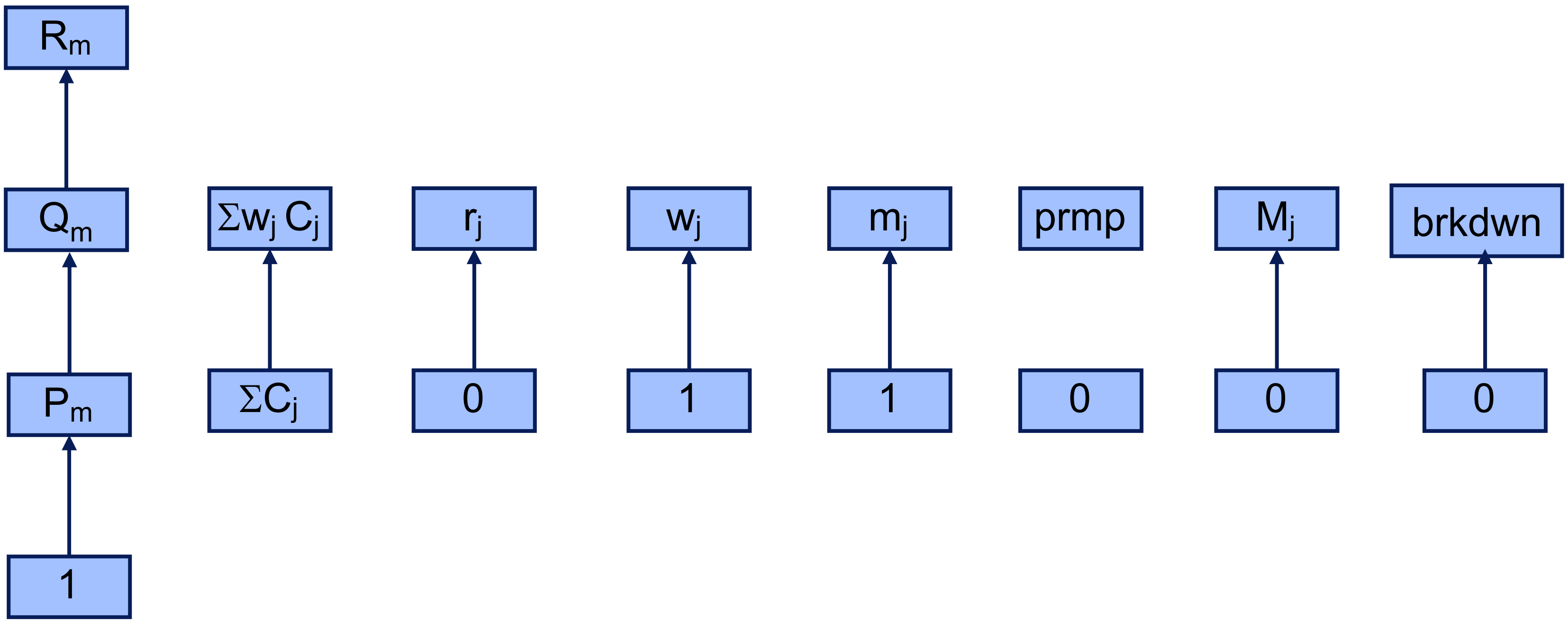
Complexity Hierarchy

Some problems are special cases of other problems:

Notation: $\alpha_1 \mid \beta_1 \mid \gamma_1 \;\propto$ (reduces to) $\alpha_2 \mid \beta_2 \mid \gamma_2$

Examples:

$$1 \mid\mid \Sigma C_j \propto 1 \mid\mid \Sigma w_j C_j \propto P_m \mid\mid \Sigma w_j C_j \propto P_m \mid m_j \mid \Sigma w_j C_j$$



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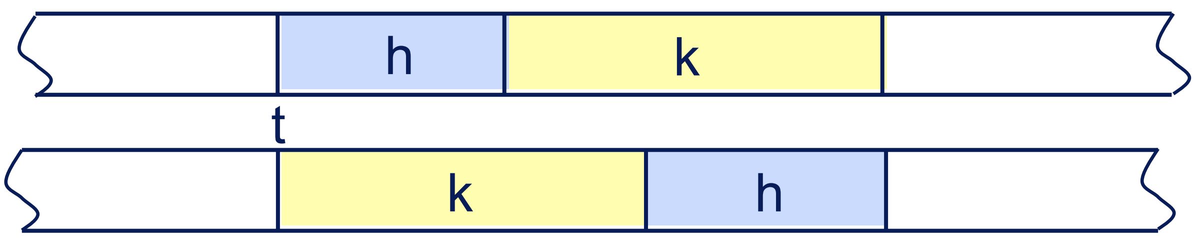
$1 \,||\, \Sigma\, w_j C_j$

- $1 \,||\, \Sigma\, w_j C_j$ is easy and can be solved by sorting all jobs in decreasing Smith order w_j/p_j (weighted shortest processing time first (WSPT) rule, Smith, 1956).
  - Nondelay schedule
  - Proof by contradiction and localization:

    If the WSPT rule is violated then it is violated by a pair of neighboring task h and k.

S_1: $\Sigma\, w_j C_j = \ldots + w_h(t+p_h) + w_k(t + p_h + p_k)$



S_1-S_2:

$w_k\, p_h - w_h\, p_k > 0$

$w_k/p_k > w_h/p_h$

S_2: $\Sigma\, w_j C_j = \ldots + w_k(t+p_k) + w_h(t + p_k + p_h)$

Other Single Machine Problems

- Every nondelay schedule has
  - optimal makespan and
  - optimal utilization for any interval starting at time 0.
- WSPT requires knowledge of the processing times
  - No direct application to nonclairvoyant scheduling
- $1 \mid prmp \mid \Sigma\, C_j$ is easy.
  - The online nonclairvoyant version (Round Robin) has a competitive factor of 2-2/(n+1) (Motwani, Phillips, Torng,1994).
- $1 \mid r_j\, ,prmp \mid \Sigma\, C_j$ is easy.
  - The online, clairvoyant version is easy.
- $1 \mid r_j \mid \Sigma\, C_j$ is strongly NP hard.
- $1 \mid r_j\, ,prmp \mid \Sigma\, w_j\, C_j$ is strongly NP hard.
  - The WSRPT (remaining processing time) rule is not optimal.

Optimal versus Approximation

- $1 \mid r_j, prmp \mid \Sigma\, w_j (C_j - r_j)$ and $1 \mid r_j, prmp \mid \Sigma\, w_j C_j$
  - Same optimal solution
  - Larger approximation factor for $1 \mid r_j, prmp \mid \Sigma\, w_j (C_j - r_j)$.
  - No constant approximation factor for the total flowtime objective (Kellerer, Tautenhahn, Wöginger, 1999)

$$\frac{\sum w_j \cdot (C_j(S) - r_j)}{\sum w_j \cdot (C_j(OPT) - r_j)} =$$

$$= \frac{\frac{\sum w_j \cdot C_j(S)}{\sum w_j \cdot C_j(OPT)} \sum w_j \cdot (C_j(OPT) - r_j) + \left(\frac{\sum w_j \cdot C_j(S)}{\sum w_j \cdot C_j(OPT)} - 1 \right) \sum w_j \cdot r_j}{\sum w_j \cdot (C_j(OPT) - r_j)} =$$

$$= \frac{\sum w_j \cdot C_j(S)}{\sum w_j \cdot C_j(OPT)} + \left(\frac{\sum w_j \cdot C_j(S)}{\sum w_j \cdot C_j(OPT)} - 1 \right) \cdot \frac{\sum w_j \cdot r_j}{\sum w_j \cdot (C_j(OPT) - r_j)}$$

Approximation Algorithms

- $1 \mid r_j \mid \Sigma\, C_j$
  - Approximation factor e/(e-1)=1.58 (Chekuri, Motwani, Natarajan, Stein, 2001)
  - Clairvoyant online scheduling: competitive factor 2 (Hoogeveen, Vestjens,1996)
- $1 \mid r_j \mid \Sigma\, w_j C_j$
  - Approximation factor 1.6853 (Goemans, Queyranne, Schulz, Skutella, Wang, 2002)
  - Clairvoyant online scheduling: competitive factor 2 (Anderson, Potts, 2004)
- $1 \mid r_j\ ,prmp \mid \Sigma\, w_j\, C_j$
  - Approximation factor 1.3333,
  - Randomized online algorithm with the competitive factor 1.3333
  - WSPT online algorithm with competitive factor 2 (all results: Schulz, Skutella, 2002)

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P_m and Makespan with m_j=1

- A scheduling problem for parallel machines consists of 2 steps:
  - Allocation of jobs to machines
  - Generating a sequence of the jobs on a machine
- A minimal makespan represents a balanced load on the machines if no single job dominates the schedule.

$$C_{max}(OPT) \geq \max\left\{ \max\{p_j\}, \frac{1}{m} \cdot \sum p_j \right\}$$

- Preemption may improve a schedule even if all jobs are released at the same time.

$$C_{max}(OPT) = \max\left\{ \max\{p_j\}, \frac{1}{m} \cdot \sum p_j \right\}$$

- Optimal schedules for parallel identical machines are nondelay.

$P_m \,||\, C_{max}$

- $P_m \,||\, C_{max}$ is strongly NP-hard (Garey, Johnson 1979).
- Approximation algorithm: Longest processing time first (LPT) rule (Graham, 1966)
  - Whenever a machine is free, the longest job among those not yet processed is put on this machine.
  - Tight approximation factor: $\frac{C_{max}(LPT)}{C_{max}(OPT)} \leq \frac{4}{3} - \frac{1}{3m}$
  - The optimal schedule $C_{max}(OPT)$ is not necessarily known but a simple lower bound can be used:

$$C_{max}(OPT) \geq \frac{1}{m}\sum_{j=1}^{n} p_j$$

LPT Proof (1)

- If the claim is not true, then there is a counterexample with the smallest number n of jobs.
- The shortest job n in this counterexample is the last job to start processing (LPT) and the last job to finish processing.
  - ➔ If n is not the last job to finish processing then deletion of n does not change C_{max} (LPT) while C_{max} (OPT) cannot increase.
  - ➔ A counter example with n – 1 jobs
- Under LPT, job n starts at time $C_{max}(LPT)-p_n$.
  - ➔ In time interval $[0, C_{max}(LPT) - p_n]$, all machines are busy.

$$C_{max}(LPT) - p_n \leq \frac{1}{m}\sum_{j=1}^{n-1} p_j$$

LPT Proof (2)

$$C_{max}(LPT) \le p_n + \frac{1}{m}\sum_{j=1}^{n-1} p_j = p_n\left(1-\frac{1}{m}\right) + \frac{1}{m}\sum_{j=1}^{n} p_j$$

$$\frac{4}{3}-\frac{1}{3m} < \frac{C_{max}(LPT)}{C_{max}(OPT)} \le \frac{p_n\left(1-\frac{1}{m}\right)}{C_{max}(OPT)} + \frac{\frac{1}{m}\sum_{j=1}^{n} p_j}{C_{max}(OPT)} \le \frac{p_n(1-1/m)}{C_{max}(OPT)} + 1$$

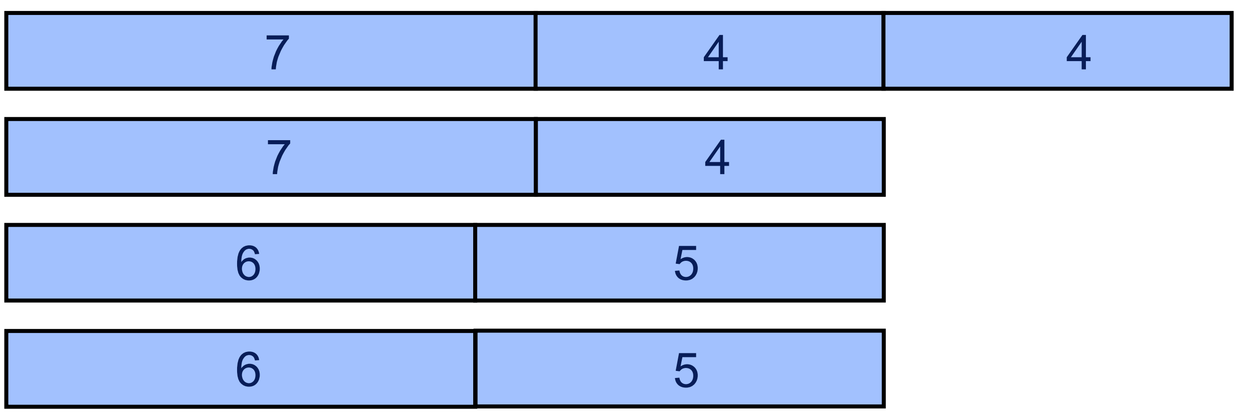
$$C_{max}(OPT) < 3p_n$$

At most two jobs are scheduled on each machine.
For such a problem, LPT is optimal.

A Worst Case Example for LPT

jobs	1	2	3	4	5	6	7	8	9
p_j	7	7	6	6	5	5	4	4	4

- 4 parallel machines: $P4||C_{max}$
- $C_{max}(OPT) = 12 = 7+5 = 6+6 = 4+4+4$
- $C_{max}(LPT) = 15 = 11+4 = (4/3 - 1/(3 \cdot 4)) \cdot 12$



List Scheduling

- LPT requires knowledge of the processing times.
  - ➜ No direct application to nonclairvoyant scheduling
- Arbitrary nondelay schedule (List Scheduling, Graham, 1966)
  - ➜ Tight approximation factor: $\dfrac{C_{max}(\text{LIST})}{C_{max}(\text{OPT})} \le 2 - \dfrac{1}{m}$

$C_{max}(\text{LIST})=11$

$C_{max}(\text{OPT})=6$

Online Transformation

Let A be an algorithm for a job scheduling problem without release dates and with

$$\frac{C_{max}(A)}{C_{max}(OPT)} \leq k$$

Then there is an algorithm A' for the corresponding online job scheduling problem with

$$\frac{C_{max}(A')}{C_{max}(OPT)} \leq 2k$$

(Shmoys, Wein, Williamson, 1995)

Transformation Proof

- S_0: Jobs available at time $0=F_{-1}=F_{-2}$
- $F_0=C_{max}(A,S_0)$
- S_{i+1}: Jobs released in $(F_{i-1},F_i]$
- $F_i=C_{max}(A,S_i)$ such that no job from S_i starts before F_{i-1}.
- Assume that all jobs in S_i are released at time F_{i-2}
  - $C_{max}(OPT)$ cannot increase while $C_{max}(A')$ remains unchanged.
- Proof

$$F_{i-2}+F_i-F_{i-1}\le k\cdot C_{max}(A,S_i)=k\cdot C_{max}(A')$$

$$F_{i-1}-F_{i-2}\le F_{i-3}+F_{i-1}-F_{i-2}\le k\cdot C_{max}(A,S_{i-1})<k\cdot C_{max}(A')$$

$$F_i<2k\cdot C_{max}(A')$$

List Scheduling Extensions

- The List scheduling bound 2-1/m also applies to $P_m|r_j|C_{max}$ (Hall, Shmoys, 1989).
- Online extension of List scheduling to parallel jobs:
  - No machine is kept idle while there is at least one job waiting and there are enough machines idle to start this job (nondelay).
- The List scheduling bound 2-1/m also applies to $P_m|m_j|C_{max}$ (Feldmann, Sgall, Teng, 1994).
- The List scheduling bound 2-1/m also applies to $P_m|m_j,r_j|C_{max}$ (Naroska, Schwiegelshohn, 2002).
  - 2-1/m is a competitive factor for the corresponding online nonclairvoyant scheduling problem.
  - Proof by induction on the number of different release dates

$P_m \mid m_j \mid C_{max}$ Proof

- The bound holds if during the whole schedule there is no interval with at least m/2 idle machines.

$$C_{max}(OPT) \geq \frac{1}{m}\sum m_j \cdot p_j \geq \frac{m+1}{2m} \cdot C_{max}(S) \geq$$
$$\frac{m}{2m-1} \cdot C_{max}(S) = \frac{1}{2-\frac{1}{m}} \cdot C_{max}(S)$$

- The sum of machines used in any two intervals is larger than m unless the jobs executed in one interval are a subset of the jobs executed in the other interval.

$$C_{max}(S) \leq \max\left\{\left(2-\frac{1}{m}\right) \cdot \sum m_j \cdot p_j, \left(2-\frac{1}{m}\right) \cdot \max\{p_j\}\right\}$$

Makespan with Preemptions

- P_m |prmp| C_{max} is easy.
  - Transformation of a nonpreemptive single machine schedule in a preemptive parallel schedule (McNaughton, 1959)
    - The single machine schedule is split into at most m schedules of length C_{max}(OPT).
    - Each schedule is executed on a different machine.
    - There are at most m-1 preemptions.
- P_m |r_j, prmp| C_{max} is easy.
  - Longest remaining processing time algorithm.
  - Clairvoyant online scheduling
    - Competitive factor 1 for allocation as late as possible.
    - Competitive factor e/(e-1)=1.58 for allocation of machine slots at submission time (Chen, van Vliet, Wöginger, 1995)
  - Nonclairvoyant online scheduling: same competitive factor 2-1/m as for the nonpreemptive case (Shmoys, Wein Williamson, 1995)

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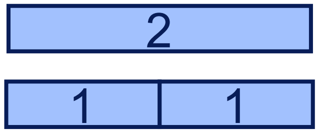
Utilization

- Utilization U_t is closely related to the makespan C_{max} if $t=C_{max}$.
  - In online job scheduling problems, there is no last submitted job.
  - U_t with t being the actual time is better suited than the makespan objective.
- $P_m \mid r_j \mid U_t$
  - Nonclairvoyant online scheduling: tight competitive factor for any nondelay schedule 1.3333 (Hussein, Schwiegelshohn, 2006)
  - Proof by induction on the different release dates.

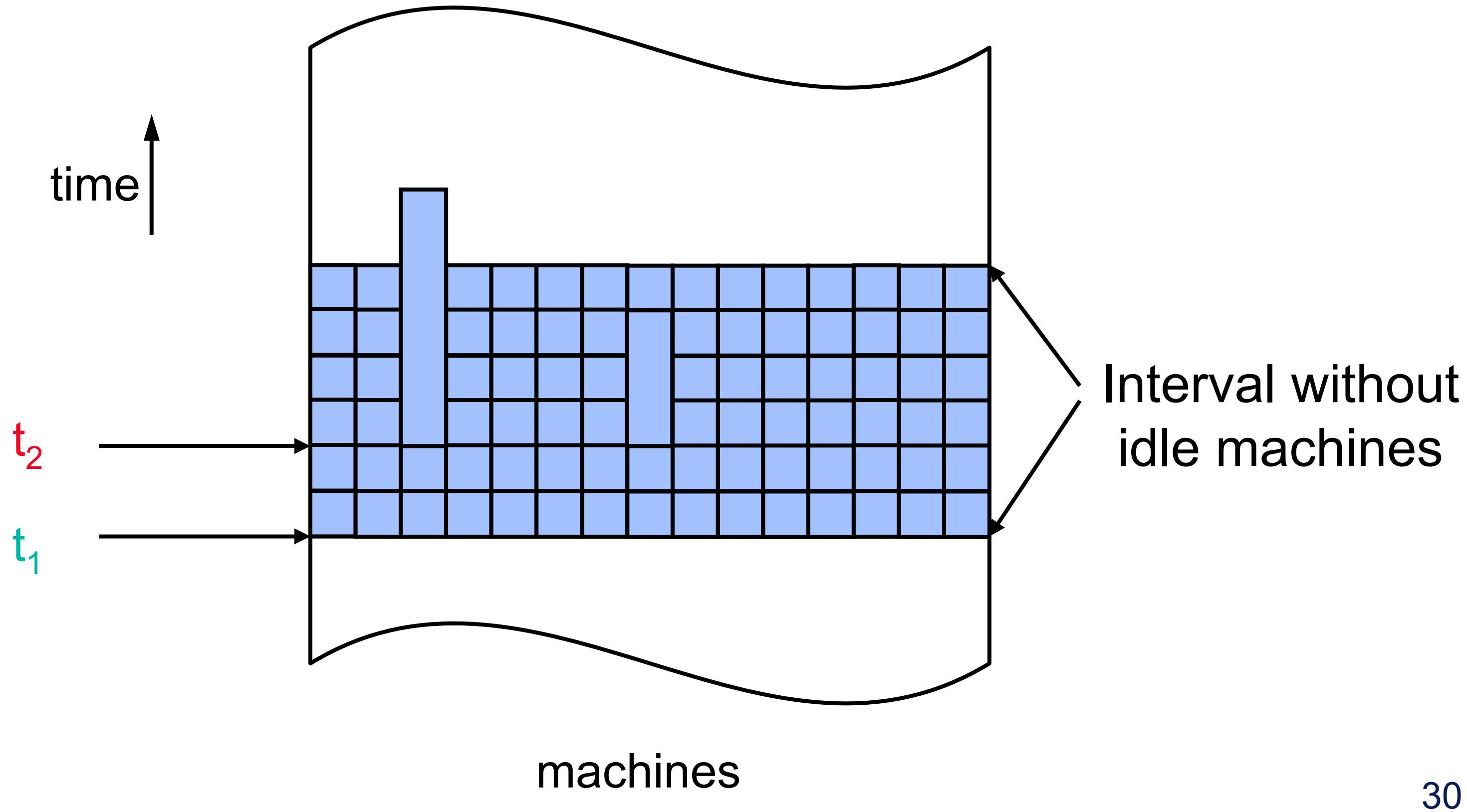
$U_2(LIST)=0.75$



$U_2(OPT)=1$

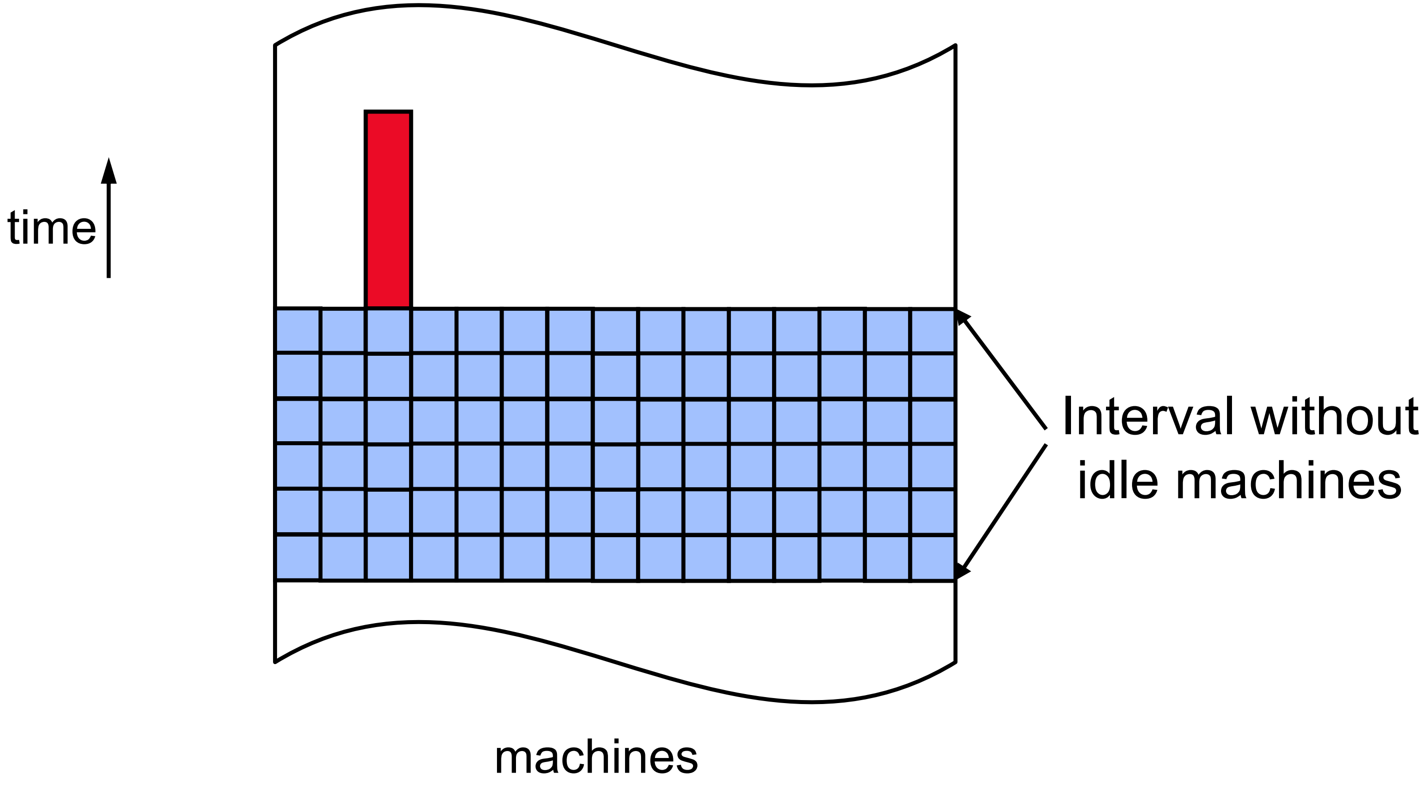
Utilization Proof (1)

- Transformation of the job system
  - Reduction of the release dates



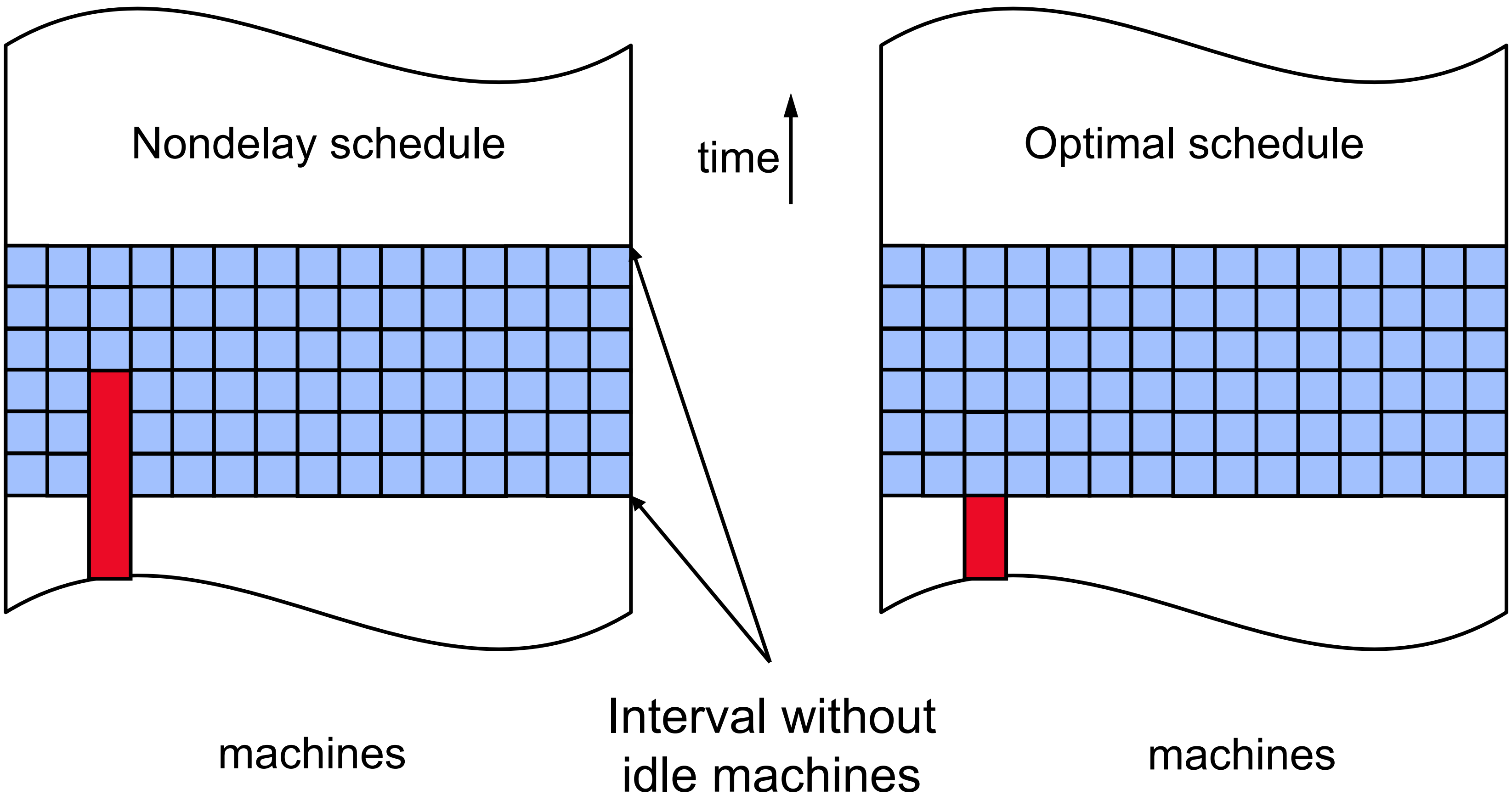
Utilization Proof (2)

- Transformation of the job system
  - Splitting of jobs
  - The system only contains short and long jobs.
    - All long jobs start at the end of an interval.



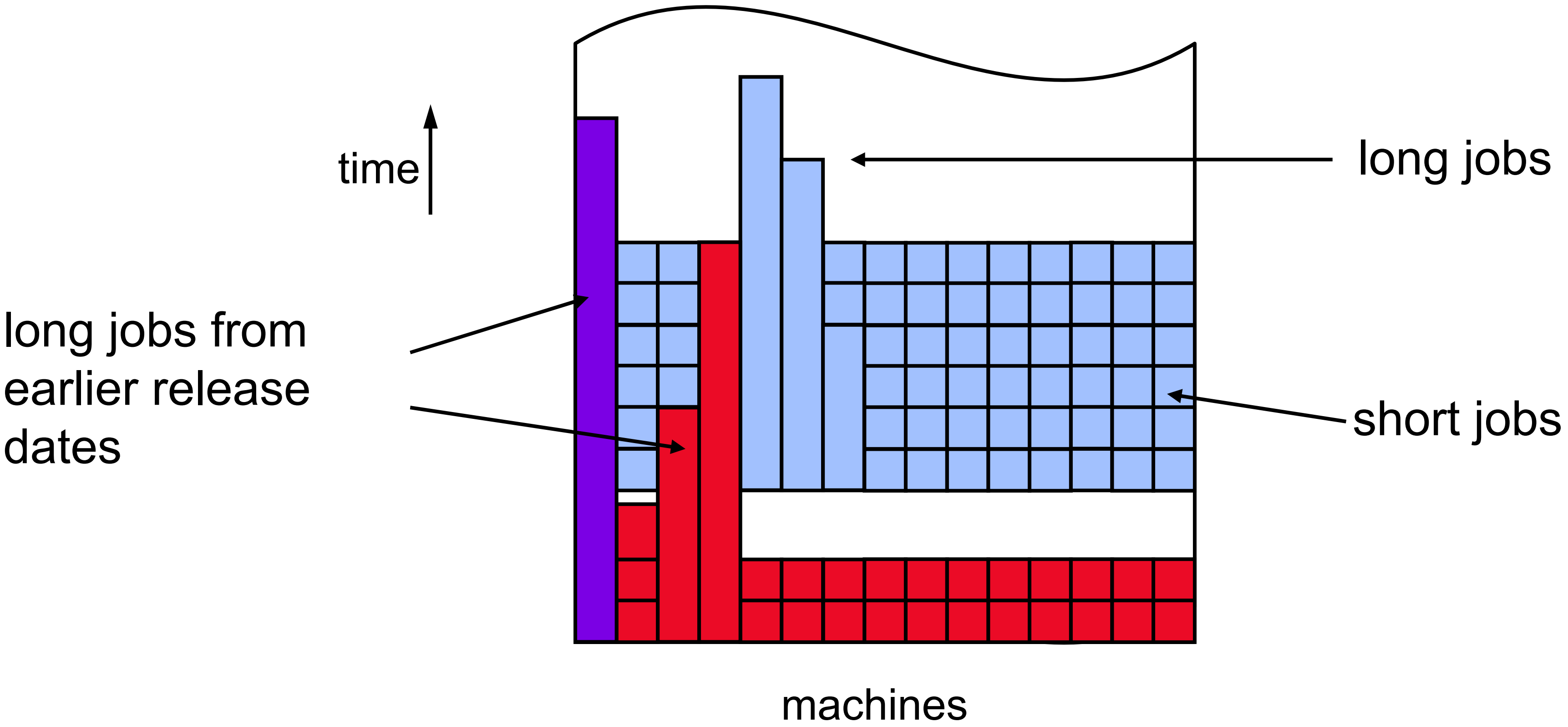
Utilization Proof (3)

- Transformation of the job system
  - Modification of jobs with earlier release dates

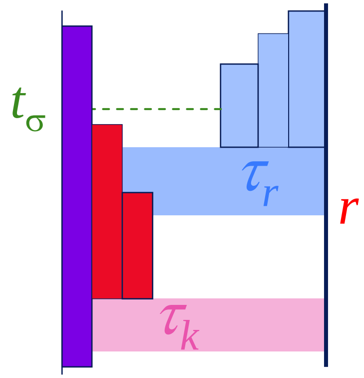


Utilization Proof (4)

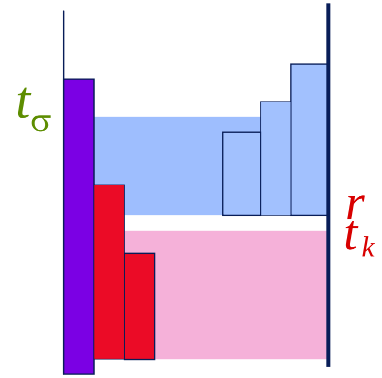
If all long jobs of a transformed job system start at their release date, then the utilization is maximal for all t and the equal priority completion time is minimal.



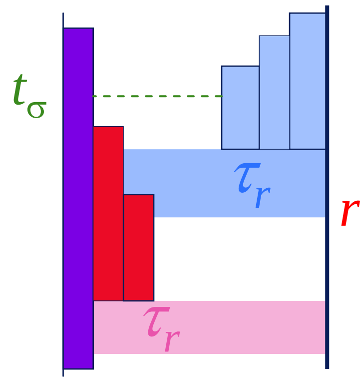
Utilization Proof (5)



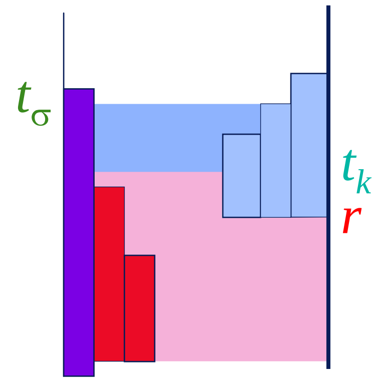
Nondelay schedule S



Optimal schedule



Nondelay schedule S

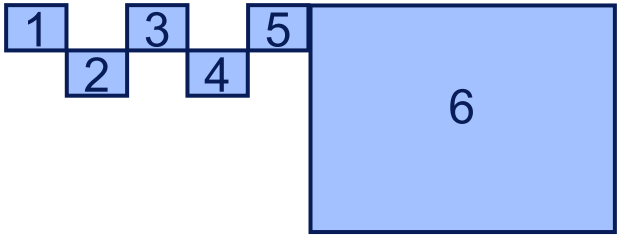


Optimal schedule

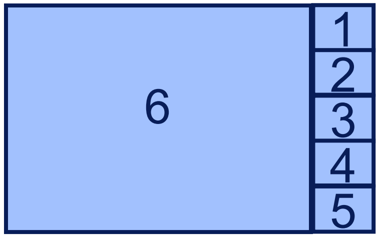
$P_m \mid r_j, m_j \mid U_t$

- Parallel jobs may cause intermediate idle time even if all jobs are released at time 0.
- Nonclairvoyant online scheduling:
  - Competitive factor → m in the worst case
  - Competitive factor → 2 if the actual time >> max{p_j}

Jobs	1	2	3	4	5	6
p_j	$1+\varepsilon$	$1+\varepsilon$	$1+\varepsilon$	$1+\varepsilon$	1	5
r_j	0	1	2	3	4	0
m_j	1	1	1	1	1	5



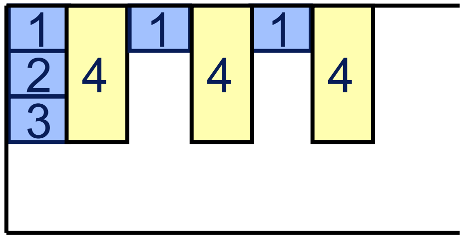
$U_5(\text{LIST})=0.2+0.16\varepsilon$



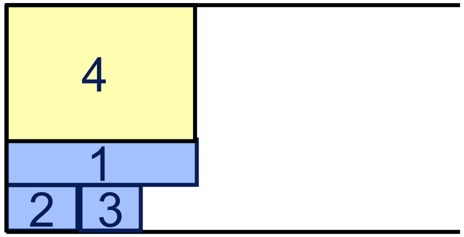
$U_5(\text{OPT})=1$

$P_m \mid r_j, m_j, prmp \mid U_t$

- Here, preemption of parallel jobs is based on gang scheduling.
  - All allocated machines concurrently start, interrupt, resume, and complete the execution of a parallel job.
  - There is no migration or change of parallelism.
- Nonclairvoyant online scheduling: competitive factor 4 (Schwiegelshohn, Yahyapour, 2000)



$U_3(A)=7/15$



$U_3(OPT)=14/15$

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$P_m \,||\, \Sigma C_j$

- $P_m \,||\, \Sigma C_j$ is easy.
  - Shortest processing time (SPT) (Conway, Maxwell, Miller, 1967)
  - Single machine proof:
    - $\Sigma\, C_j = n\, p_{(1)} + (n-1)\, p_{(2)} + \ldots\, 2\, p_{(n-1)} + p_{(n)}$
    - $p_{(1)} \le p_{(2)} \le p_{(3)} \le \ldots.. \le p_{(n-1)} \le p_{(n)}$ must hold for an optimal schedule.
  - Parallel identical machines proof:
    - Dummy jobs with processing time 0 are added until n is a multiple of m.
    - The sum of the completion time has n additive terms with one coefficient each:
      m coefficients with value n/m
      m coefficients with value n/m – 1
      :
      m coefficients with value 1
    - If there is one coefficient h>n/m then there must be a coefficient k<n/m.
    - Then we replace h with k+1 and obtain a smaller ΣC_j .
- P_m |prmp| ΣC_j is easy (Shortest remaining processing time).

$P_m \mid\mid \Sigma w_j C_j$

- $P_m \mid\mid \Sigma w_j C_j$ is strongly NP-hard.
  - The WSPT algorithm has a tight approximation factor of 1.207 (Kawaguchi, Kyan, 1986)
  - It is sufficient to consider instances where all jobs have the same ratio w_j/p_j.
  - Proof by induction on the number of different ratios.
    - *J* is the set of all jobs with the largest ratio in an instance I.
    - The weights of all jobs in *J* are multiplied by a positive factor $\varepsilon < 1$ such that those jobs now have the second largest ratio.
    - This produces instance I'.
    - The WSPT order is still valid.
    - The WSPT schedule remains unchanged.
    - The optimal schedule may change.

Different Ratios

- Induction Proof
  - $\Sigma w_j C_j(\text{WSPT},I') \le \lambda \cdot \Sigma w_j C_j(\text{OPT},I')$ (induction assumption)
  - x: contribution of all jobs in *J* to $\Sigma w_j C_j(\text{WSPT},I)$
  - y: contribution of all jobs not in *J* to $\Sigma w_j C_j(\text{WSPT},I)$
  - x': contribution of all jobs in *J* to $\Sigma w_j C_j(\text{OPT},I)$
  - y': contribution of all jobs not in *J* to $\Sigma w_j C_j(\text{OPT},I)$
  - $x \le \lambda \cdot x'$ (induction assumption)
  - $\Sigma w_j C_j(\text{WSPT},I) = x+y$ and $\Sigma w_j C_j(\text{WSPT},I') = \varepsilon \cdot x+y$,
  - $\Sigma w_j C_j(\text{OPT},I) = x'+y'$ and $\Sigma w_j C_j(\text{OPT},I') \le \varepsilon \cdot x'+y'$
  - $y \le \lambda \cdot y' \rightarrow \Sigma w_j C_j(\text{WSPT},I) \le \lambda \cdot \Sigma w_j C_j(\text{OPT},I)$
  - $y > \lambda \cdot y' \rightarrow \lambda \cdot x'y > x \cdot \lambda \cdot y' \rightarrow x'/y' > x/y \rightarrow x'y-xy' > 0 \rightarrow x'y-xy' > \varepsilon(x'y-xy')$
  - $\Sigma w_j C_j(\text{WSPT},I') \cdot \Sigma w_j C_j(\text{OPT},I) = (\varepsilon \cdot x+y)(x'+y') > (\varepsilon \cdot x'+y')(x+y) \ge \Sigma w_j C_j(\text{OPT},I') \cdot \Sigma w_j C_j(\text{WSPT},I)$
  - $\Sigma w_j C_j(\text{WSPT},I) \le \lambda \cdot \Sigma w_j C_j(\text{OPT},I)$
- Assumption: $w_j = p_j$ holds for all jobs j.

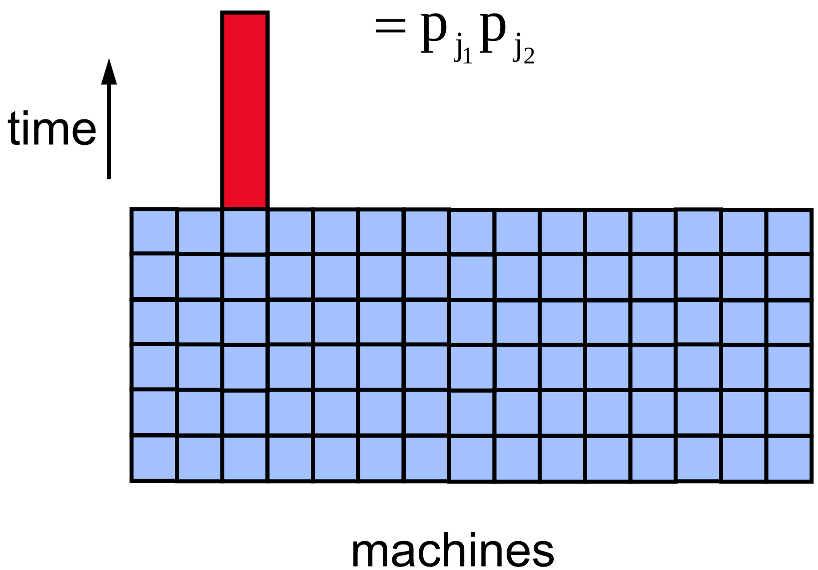
WSPT Proof (1)

- Transformation of the job system
  - Splitting of job j into jobs j_1 and j_2.
  - The system only contains short and long jobs.
    - All long jobs start at the end of busy interval in the list schedule.

$$\sum w_j C_j(S') - \sum w_j C_j(S) = p_j C_j(S') - p_{j_1} C_{j_1}(S) - p_{j_2} C_{j_2}(S) =$$

$$= (p_{j_1} + p_{j_2}) \cdot C_j(S') - p_{j_1} \cdot (C_j(S') - p_{j_2}) - p_{j_2} C_j(S') =$$

$$= p_{j_1} p_{j_2}$$



$$\frac{\sum w_j C_j(S)}{\sum w_j C_j(OPT)} \geq \frac{\sum w_j C_j(S') - p_{j_1} p_{j_2}}{\sum w_j C_j(OPT') - p_{j_1} p_{j_2}} \geq$$

$$\geq \frac{\sum w_j C_j(S')}{\sum w_j C_j(OPT')}$$

WSPT Proof (2)

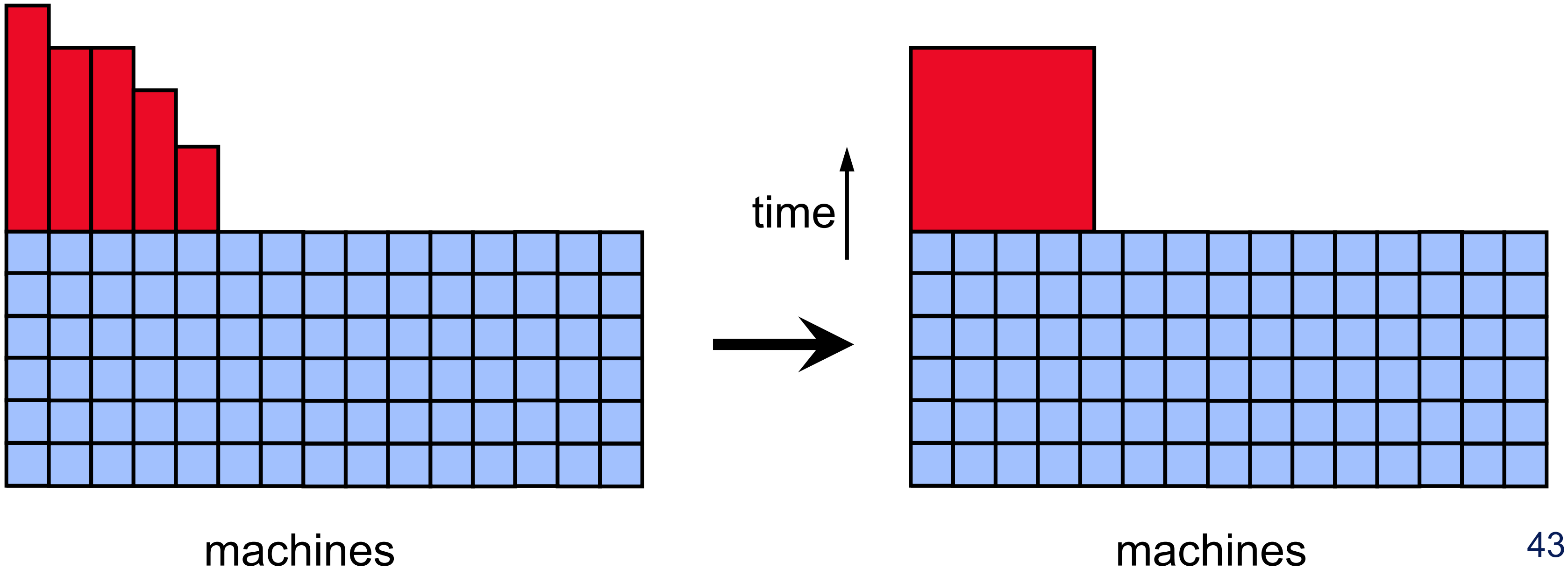
- Single machine without intermediate idle time
  - $w_j = p_j$ holds for all jobs.
  - $\sum w_j C_j(S) = \sum w_j C_j(OPT) = 0.5((\sum p_j)^2 + \sum p_j^2)$
  - Proof by induction on the number of jobs

$$\sum w_j C_j(S) = \frac{1}{2}\left(\left(\sum p_j\right)^2 + \sum p_j^{\ 2}\right) + p_{j'}\left(p_{j'} + \sum p_j\right) =$$

$$= \frac{1}{2}\left(\left(\sum p_j\right)^2 + 2p_{j'}\sum p_j + p_{j'}^{\ 2}\right) + \frac{1}{2}\left(\sum p_j^{\ 2} + p_{j'}^{\ 2}\right)$$

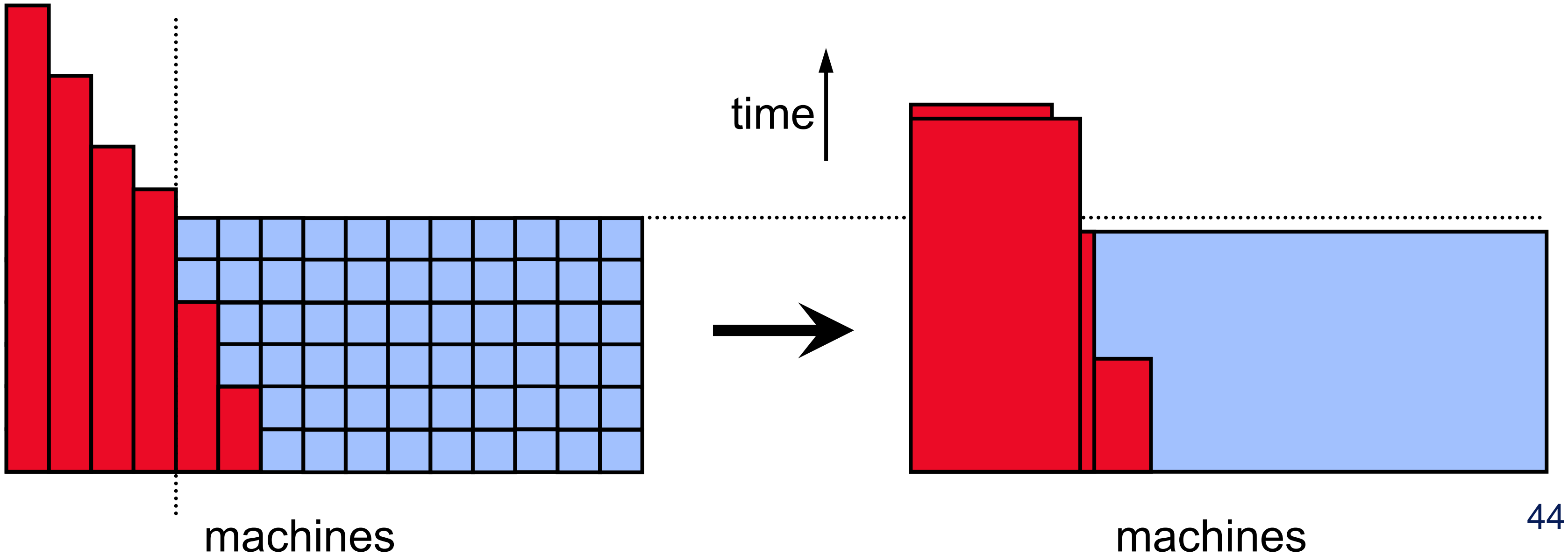
WSPT Proof (3)

- Equalization of the long jobs
  - Assumption of a continuous model (fraction of machines)
  - k long jobs with different processing times are transformed into n(k) jobs with the same processing time p(k) such that $\sum p_j = n(k) \cdot p(k)$ and $\sum p_j^2 = n(k) \cdot (p(k))^2$ hold.
  - $p(k) = \sum p_j^2 / \sum p_j$ and $n(k) = (\sum p_j)^2 / \sum p_j^2$
  - Then we have $k \geq n(k)$ for reasons of convexity.



WSPT Proof (4)

- Modification of the job system
  - Partitioning of the long jobs into two groups
  - Equalization of the both groups separately
  - The maximum completion time of the small jobs decreases due to the large rectangle.
  - The jobs of the small rectangle are rearranged.
  - New equalization of the large rectangle
  - Determination of the size of the large rectangle



Release Dates

- $P_m \,|r_j|\, \Sigma C_j$
  - Approximation factor 2
  - Clairvoyant, randomized online scheduling: competitive factor 2
- $P_m \,|r_j,prmp|\, \Sigma C_j$
  - Approximation factor 2
  - Clairvoyant, randomized online scheduling: competitive factor 2
- $P_m \,|r_j|\, \Sigma w_j C_j$
  - Approximation factor 2
  - Clairvoyant, randomized online scheduling: competitive factor 2
- $P_m \,|r_j,prmp|\, \Sigma w_j C_j$
  - Approximation factor 2
  - Clairvoyant, randomized online scheduling: competitive factor 2 (all results Schulz, Skutella, 2002)

Parallel Jobs

- $P_m \,|m_j, prmp|\, \Sigma w_j C_j$
  - Use of gang scheduling without any task migration
  - Approximation factor 2.37 (Schwiegelshohn, 2004)
- $P_m \,|m_j, prmp|\, \Sigma C_j$
  - Nonclairvoyant approximation factor 2-2/(n+1) if all jobs are malleable with linear speedup (Deng, Gu, Brecht, Lu, 2000).
- $P_m \,|m_j|\, \Sigma w_j C_j$
  - Approximation factor 7.11 (Schwiegelshohn, 2004)
  - Approximation factor 2 if $m_j \leq 0.5m$ holds for all jobs (Turek et al., 1994)
- $P_m \,|m_j|\, \Sigma C_j$
  - Approximation factor 2 if the jobs are malleable without superlinear speedup (Turek et al., 1994)

Online Problems

- $P_m \,|m_j, r_j, prmp|\, \Sigma w_j C_j$
  - Nonclairvoyant online scheduling with gang scheduling and $w_j = m_j \cdot p_j$: competitive factor 3.562 (Schwiegelshohn, Yahyapour, 2000)
    - $w_j = m_j \cdot p_j$ guarantees that no job is preferred over another job regardless of its resource consumption as all jobs have the same (extended) Smith ratio.
    - All jobs are started in order of their arrival (FCFS).
    - Any job started after a job j can increase the flow time $C_j - r_j$ by at most a factor of 2
  - Clairvoyant online scheduling with malleable jobs and linear speedup:
    - Competitive factor $12+\varepsilon$ for a deterministic algorithm
    - Competitive factor 8.67 for a randomized algorithm (both results Chakrabarti et al.,1996)

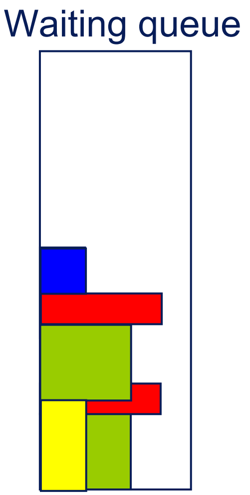
Content of the Lecture

- What is job scheduling?
- Single machine problems and results
- Makespan problems on parallel machines
- Utilization problems on parallel machines
- Completion time problems on parallel machines
- Exemplary workload problem

MPP Problem

- Machine model
  - *Massively parallel processor (MPP)*: *m* parallel identical machines
- Job model
  - Multiple independent users
  - Nonclairvoyant (unknown processing time p_j) with estimates
  - Online (submission over time r_j)
  - Fixed degree of parallelism m_j during the whole processing
  - No preemption
- Objective
  - Machine utilization
  - Average weighted response time (*AWRT*): $p_j \cdot m_j \cdot (C_j - r_j)$
  - Based on user groups

Algorithmic Approach



- Reordering of the waiting queue
  - Parameters of jobs in the waiting queue
  - Actual time
  - Scheduling situations: weekdays daytime (8am – 6pm), weekdays nighttime (6pm – 8am), weekends
- Selected sorting criteria
- Selected objective
  - Consideration of 2 user groups: $10\ AWRT_1 + 4\ AWRT_2$
- Parameter training with Evolution Strategies
  - Recorded workloads and simulations
  - Workload scaling for comparison

Workloads and User Groups

User Group	1	2	3	4	5
RC_u/RC	> 8%	2 – 8 %	1 – 2 %	0.1 – 1 %	< 0.1 %

User group definition

Identifier	CTC	KTH	LANL	SDSC 00	SDSC 95	SDSC 96
Machine	SP2	SP2	CM-5	SP2	SP2	SP2
Period	06/26/96 – 05/31/97	09/23/96 – 08/29/97	04/10/94 – 09/24/96	04/28/98 – 04/30/00	12/29/94 – 12/30/95	12/27/95 – 12/31/96
Processors (m)	1024	1024	1024	1024	1024	1024
Jobs (n)	136471	167375	201378	310745	131762	66185

Workload scaling

Sorting Criteria

$$f_1(Job) = \sum_{i=1}^{|Groups|} w_i \cdot \left(K_i + a \cdot \frac{waitTime}{requestedTime} + b \cdot \frac{requestedTime}{processors} \right)$$

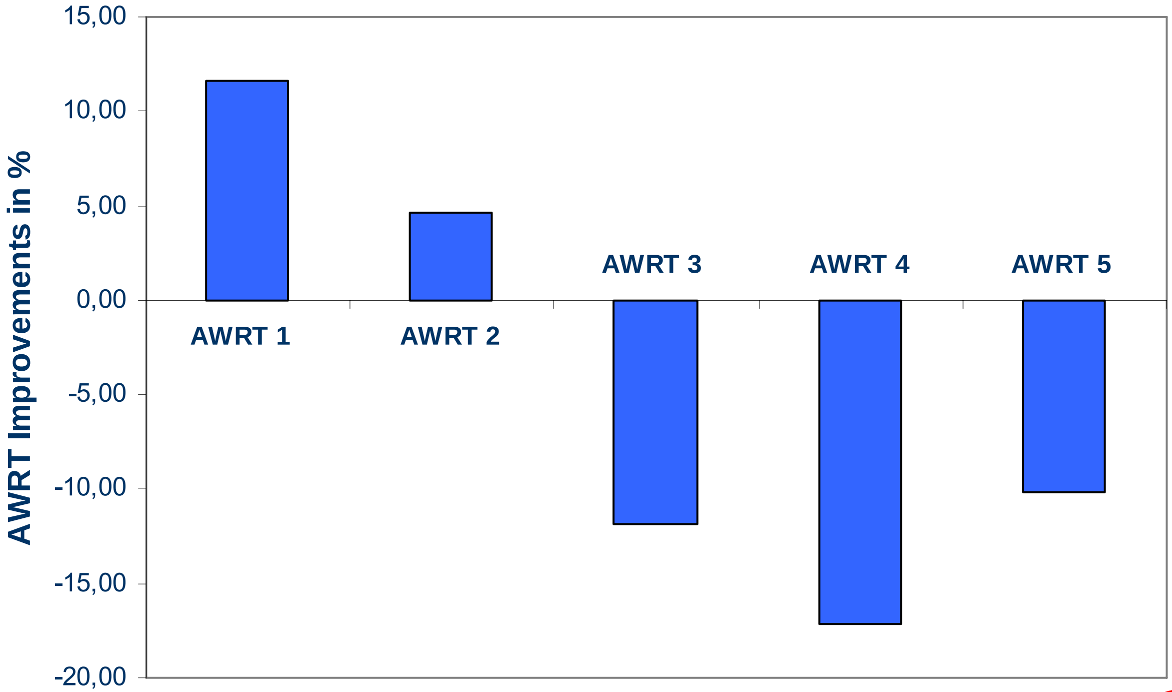
$$f_2(Job) = \sum_{i=1}^{|Groups|} w_i \cdot \left(K_i + a \cdot waitTime + b \cdot \frac{requestedTime}{processors} \right)$$

$$f_3(Job) = \sum_{i=1}^{|Groups|} w_i \cdot \left(K_i + a \cdot \frac{waitTime}{requestedTime \cdot processors} \right)$$

$$f_4(Job) = \sum_{i=1}^{|Groups|} w_i \cdot \left(K_i + a \cdot waitTime + b \cdot requestedTime \cdot processors \right)$$

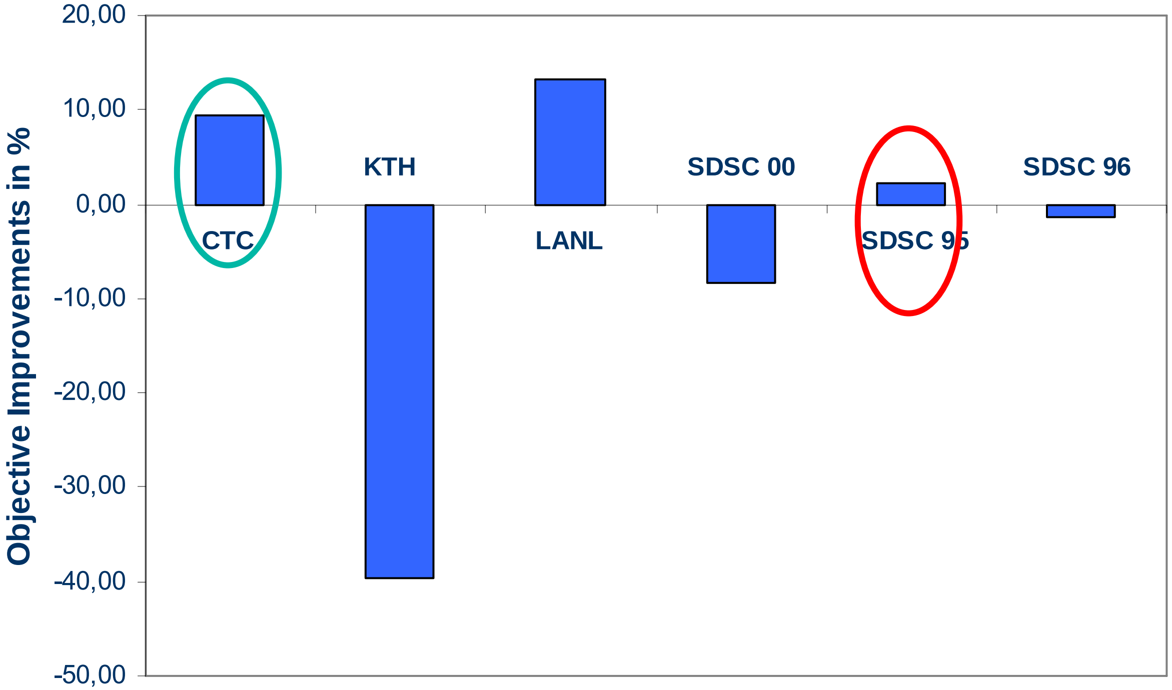
Training of parameters w_i, K_i, a, b with Evolution Strategies

CTC Training and CTC Workload



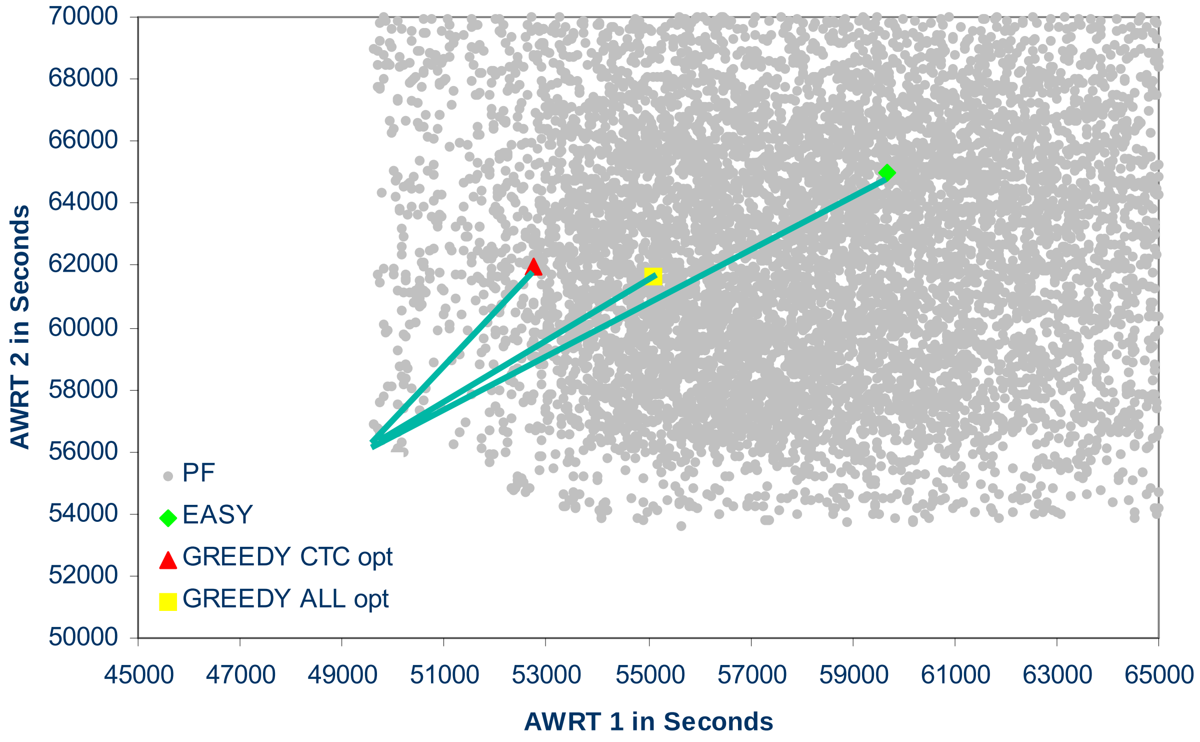
Method	AWRT 1	AWRT 2	AWRT 3	AWRT 4	AWRT 5	UTIL
GREEDY	52755.80 s	61947.65 s	56275.18 s	54017.23 s	35085.84 s	66.99 %
EASY	59681.28 s	64976.07 s	50317.47 s	46120.02 s	31855.68 s	66.99 %

CTC Training and All Workloads

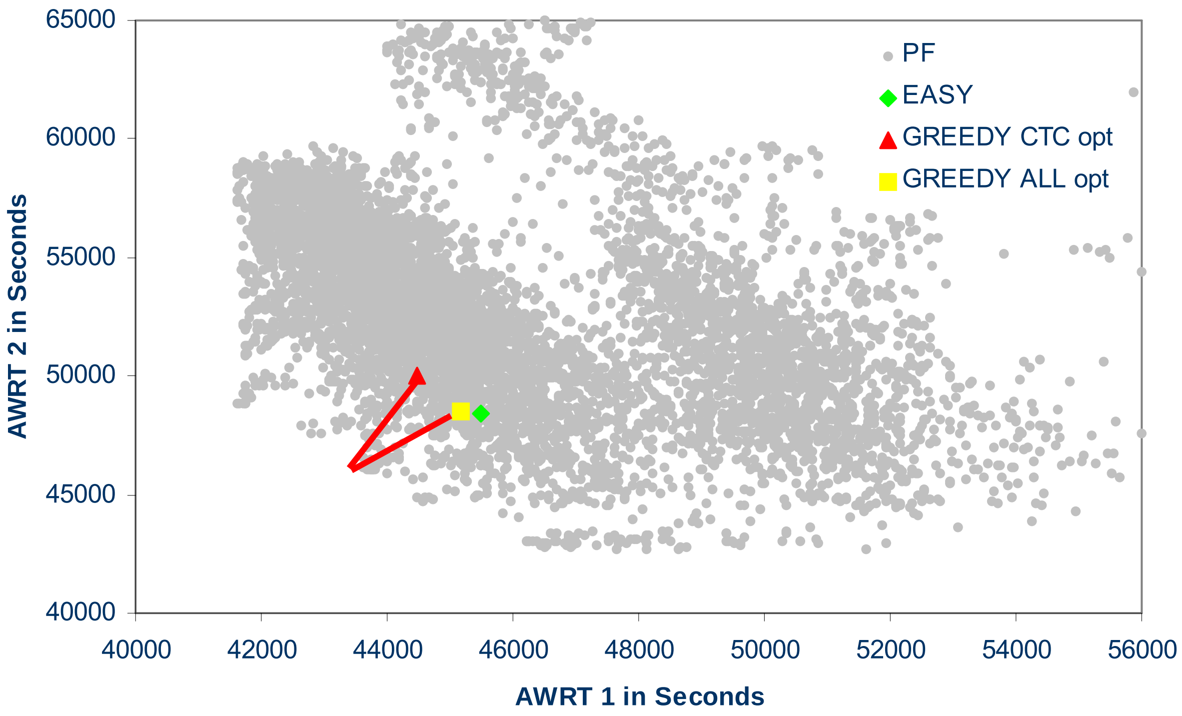


- Some workloads are similar (CTC, LANL).
- Some workloads are significantly different (CTC, KTH).

Results in CTC Paretofront



Results in SDSC 95 Paretofront



Conclusion

- Most deterministic job scheduling problems are NP hard.
  - Approximation algorithms
    - Polynomial time approximation schemes
    - Simple algorithms
- Complete problem knowledge is rare in practice.
  - Online algorithms
    - Competitive analysis
  - Stochastic scheduling
    - Randomized algorithms
- Challenges
  - Partial information
    - Recorded workloads
    - User estimates
  - Scheduling objectives and constraints