Easy Statistical Analysis in PostgreSQL with PL/R

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Intro to PL/R

What is R?

 An open source language and environment for statistical computing and graphics. . .

What is PostgreSQL?

 PostgreSQL is a powerful, open source object-relational database system. It has more than 25 years of active development and a proven architecture that has earned it a strong reputation for reliability, data integrity, and correctness.

What is PL/R?

 R Procedural Language Handler for PostgreSQL. Enables user-defined SQL functions to be written in the R language. Actively developed since early 2003.

http://www.postgresql.org



Pros

- Leverage people's knowledge and skills
 - statistics/math, database, web are distinct specialties
- Leverage hardware
 - server better able to handle analysis of large datasets
- Processing/bandwidth efficiency
 - why send large datasets across the network?
- Consistency of analysis
 - ensure analysis done consistently once vetted
- Abstraction of complexity
 - keep system understandable and maintainable
- Leverage R
 - rich core functionality and huge ecosystem



Cons

- PostgreSQL user
 - Slower than standard SQL aggregates and PostgreSQL functions for simple cases
 - New language to learn
- R user
 - \bullet Debugging more challenging than working directly in R
 - Less flexible for ad hoc analysis
 - New language to learn

Creating PL/R Functions

A little different from standard R functions

```
func_name <- function(myarg1 [,myarg2...]) {
  function body referencing myarg1 [, myarg2 ...]
}</pre>
```

But similar to other PostgreSQL PLs

```
CREATE OR REPLACE FUNCTION func_name(arg-type1 [, arg-type2 ...])
RETURNS return-type AS $$
function body referencing arg1 [, arg2 ...]
$$ LANGUAGE 'plr';

CREATE OR REPLACE FUNCTION func_name(myarg1 arg-type1
[, myarg2 arg-type2 ...])
RETURNS return-type AS $$
function body referencing myarg1 [, myarg2 ...]
$$ LANGUAGE 'plr';
```

Example of Use

Highlighted Features

- RPostgreSQL Compatibility
- Custom SQL aggregates
- Window functions
- R object \Rightarrow bytea

RPostgreSQL Compatibility

- Allows prototyping using R, move to PL/R for production
- Queries performed in current database
- Driver/connection parameters ignored; dbDriver, dbConnect, dbDisconnect, and dbUnloadDriver are no-ops

RPostgreSQL Compatibility Example

PostgreSQL access from R

```
tsp_tour_length<-function() {</pre>
  require(TSP)
  require(fields)
  require(RPostgreSQL)
  drv <- dbDriver("PostgreSQL")</pre>
  conn <- dbConnect(drv, user="postgres", dbname="plr", host="localhost")</pre>
  sql.str <- "select id, st_x(location) as x, st_y(location) as y,
               location from stands"
  waypts <- dbGetQuery(conn, sql.str)</pre>
  dist.matrix <- rdist.earth(waypts[,2:3], R=3949.0)</pre>
  rtsp <- TSP(dist.matrix)</pre>
  soln <- solve_TSP(rtsp)</pre>
  dbDisconnect(conn)
  dbUnloadDriver(drv)
  return(attributes(soln)$tour_length)
}
```

RPostgreSQL Compatibility Example

Same function from PL/R

```
CREATE OR REPLACE FUNCTION tsp_tour_length() RETURNS float8 AS $$
  require(TSP)
  require(fields)
  require(RPostgreSQL)
  drv <- dbDriver("PostgreSQL")</pre>
  conn <- dbConnect(drv, user="postgres", dbname="plr", host="localhost")</pre>
  sql.str <- "select id, st_x(location) as x, st_y(location) as y,
               location from stands"
  waypts <- dbGetQuery(conn, sql.str)</pre>
  dist.matrix <- rdist.earth(waypts[,2:3], R=3949.0)</pre>
  rtsp <- TSP(dist.matrix)</pre>
  soln <- solve_TSP(rtsp)</pre>
  dbDisconnect(conn)
  dbUnloadDriver(drv)
  return(attributes(soln)$tour_length)
$$ LANGUAGE 'plr' STRICT:
```

RPostgreSQL Compatibility Example (cont.)

Output from R

```
tsp_tour_length()
[1] 2804.581
```

Same function from PL/R

```
SELECT tsp_tour_length();
tsp_tour_length
------
2804.58129355858
(1 row)
```

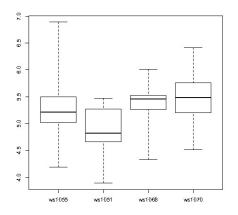
Aggregates

- Aggregates in PostgreSQL are extensible via SQL commands
- State transition function and possibly a final function are specified
- Initial condition for state function may also be specified

Aggregates Example

```
CREATE OR REPLACE FUNCTION r_quartile(ANYARRAY) RETURNS ANYARRAY AS $$
 quantile(arg1, probs = seq(0, 1, 0.25), names = FALSE)
$$ LANGUAGE 'plr':
CREATE AGGREGATE quartile (ANYELEMENT) (
 sfunc = array_append,
  stype = ANYARRAY,
 finalfunc = r_quantile,
  initcond = '{}'):
SELECT workstation, quartile(id_val) FROM sample_numeric_data
WHERE ia_id = 'G121XB8A' GROUP BY workstation;
 workstation |
                         quantile
            | {4.19,5.02,5.21,5.5,6.89}
 1055
 1051
             | {3.89,4.66,4.825,5.2675,5.47}
 1068
             | {4.33,5.2625,5.455,5.5275,6.01}
 1070
             {4.51,5.1975,5.485,5.7575,6.41}
(4 rows)
```

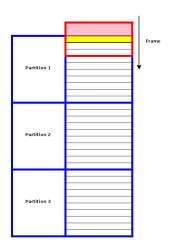
Aggregates Example - Quartile Boxplot Output



Window Functions

- Window Functions are available as of PostgreSQL 8.4
- Provide ability to calculate across sets of rows related to current row
- Similar to aggregate functions, but does not cause rows to become grouped
- Able to access more than just the current row of the query result

Window Functions



Window Function Example

```
CREATE TABLE test_data
  (fyear integer, firm float8, eps float8);
INSERT INTO test data
SELECT (b.f + 1) \% 10 + 2000 AS fyear.
       floor((b.f+1)/10) + 50 AS firm.
       f::float8/100 + random()/10 AS eps
FROM generate_series(-500,499,1) b(f);
-- find slope of the linear model regression line
CREATE OR REPLACE FUNCTION r_regr_slope(float8, float8)
RETURNS float8 AS $BODY$
  slope <- NA
  v <- farg1
  x <- farg2
  if (fnumrows==9) try (slope <- lm(y ~ x)$coefficients[2])
  return(slope)
$BODY$ LANGUAGE plr WINDOW;
```

Window Function Example

```
SELECT *, r_regr_slope(eps, lag_eps) OVER w AS slope_R
FROM (SELECT firm AS f, fyear AS fyr, eps,
  lag(eps) OVER (PARTITION BY firm ORDER BY firm, fyear) AS lag_eps
FROM test_data) AS a WHERE eps IS NOT NULL
WINDOW w AS (PARTITION BY firm ORDER BY firm, fyear ROWS 8 PRECEDING);
      fyr |
                 eps
                                    lag_eps
                                                        slope_r
     1991 l
            -4.99563754182309 I
     1992
            -4.96425441872329
                                 -4.99563754182309
     1993 I
            -4.96906093481928
                                 -4.96425441872329
     1994 I
            -4.92376988714561
                                 -4.96906093481928
     1995 I
            -4.95884547665715
                                 -4.92376988714561
     1996 I
            -4.93236254784279
                                 -4.95884547665715
     1997 I
            -4.90775520844385
                                 -4.93236254784279
     1998
            -4.92082695348188
                                 -4.90775520844385
     1999
            -4.84991340579465
                                 -4.92082695348188
                                                     0.691850614092383
    2000 I
            -4.86000917562284
                                 -4.84991340579465
                                                     0.700526929134053
```

Stock Data Example

- get Hi-Low-Close data from Yahoo for any stock symbol
- plot with Bollinger Bands and volume
- requires extra R packages from R:

```
install.packages(c('xts','Defaults','quantmod','cairoDevice','RGtk2'))
```

Stock Data Example

```
CREATE OR REPLACE FUNCTION plot_stock_data(sym text) RETURNS bytea AS $$
  library(quantmod)
  library(cairoDevice)
  library(RGtk2)
  pixmap <- gdkPixmapNew(w=500, h=500, depth=24)
  asCairoDevice(pixmap)
  getSymbols(c(sym))
  chartSeries(get(sym), name=sym, theme="white",
                 TA="addVo():addBBands():addCCI()")
  plot_pixbuf <- gdkPixbufGetFromDrawable(NULL, pixmap,</pre>
                 pixmap$getColormap(),0, 0, 0, 0, 500, 500)
  buffer <- gdkPixbufSaveToBufferv(plot_pixbuf, "jpeg",</pre>
                 character(0).character(0))$buffer
  return(buffer)
$$ LANGUAGE plr:
```

Stock Data Example

Need screen buffer on typical server:

```
Xvfb :1 -screen 0 1024x768x24
export DISPLAY=:1.0
```

Calling it from PHP for CYMI

Stock Data Example - Output



- Detecting Potential Fraud
 - Use Benford's law (also called first-digit law)
- Applies to data approximating geometric sequence
- Examples include, for example:
 - Sales figures
 - Census data
 - Medical claims
 - Expense reports
 - Energy savings

http://en.wikipedia.org/wiki/Benford's_law



- California Energy Efficiency Program Data
- Create and populate table with investment cost data

```
CREATE TABLE open_emv_cost(value float8, district int);
COPY open_emv_cost
FROM 'open-emv.cost.csv'
WITH delimiter ',';

http://open-emv.com/data
```

• Create and Benford's Law function

```
CREATE TYPE benford t AS (
  actual mean float8.
 n int,
 expected_mean float8,
 distorion float8.
 z float8
);
CREATE OR REPLACE FUNCTION benford(numarr float8[])
RETURNS benford t AS $$
 xcoll \leftarrow function(x) \{return ((10 * x) / (10 ^ (trunc(log10(x)))))\}
 numarr <- numarr [numarr >= 10]
 numarr <- xcoll(numarr)
 actual mean <- mean(numarr)
 n <- length(numarr)
 expected mean <- (90 / (n * (10 ^ (1/n) - 1)))
 distorion <- ((actual_mean - expected_mean) / expected_mean)
 z<-(distorion / sd(numarr))
 retval <- data.frame(actual mean.n.expected mean.distorion.z)
 return(retval)
$$ LANGUAGE plr;
```

Execute Benford's Law function

Data looks about right...

- Solve the famous Traveling Salesman Problem
 - Given list of location and distances, find a shortest possible tour that visits each location exactly once
- NP-hard problem in combinatorial optimization
- Applications include, for example:
 - Logistics
 - Land management
 - Semiconductor inspection
 - Geonome sequencing
 - Routing of SONET rings

 $\verb|http://en.wikipedia.org/wiki/Travelling_salesman_problem|\\$

http://www.tsp.gatech.edu/apps/index.html



Create and populate table with locations

```
CREATE TABLE stands (id serial primary key,
                     strata integer not null,
                     initage integer);
SELECT AddGeometryColumn('', 'stands', 'boundary', '4326', 'MULTIPOLYGON', 2):
CREATE INDEX "stands_boundary_gist" ON "stands" USING gist ("boundary" gist_geometry_ops);
SELECT AddGeometryColumn('', 'stands', 'location', '4326', 'POINT', 2);
CREATE INDEX "stands_location_gist" ON "stands" USING gist ("location" gist_geometry_ops);
INSERT INTO stands (id, strata, initage, boundary, location) VALUES
 (1.1.1.GeometryFromText('MULTIPOLYGON(((59.250000 65.000000.55.000000 65.000000.55.000000 51.750000.
 60.735294 53.470588, 62.875000 57.750000, 59.250000 65.000000 ))), 4326),
 GeometryFromText('POINT( 61.000000 59.000000 )', 4326 ))
.(2.2.1.GeometryFromText('MULTIPOLYGON(((67.000000 65.000000.59.250000 65.000000.62.875000 57.750000.
  67.000000 60.500000, 67.000000 65.000000 ))), 4326),
 GeometryFromText('POINT( 63.000000 60.000000 )', 4326 ))
,(3,3,1,GeometryFromText('MULTIPOLYGON(((67.045455 52.681818,60.735294 53.470588,55.000000 51.750000,
  55.000000 45.000000. 65.125000 45.000000. 67.045455 52.681818 ))), 4326).
 GeometryFromText('POINT( 64.000000 49.000000 )', 4326 ))
```

Create and populate table with locations

INSERT INTO stands (id, strata, initage, boundary, location) VALUES

```
(4,4,1,GeometryFromText('MULTIPDLYGON(((71.500000 53.500000,70.357143 53.785714,67.045455 52.681818, 65.125000 45.000000, 71.500000 45.000000, 71.500000 53.500000)))', 4326), GeometryFromText('POLINT( 68.000000 48.000000)', 4326))
,(5,5,1,GeometryFromText('MULTIPDLYGON(((69.750000 65.000000,67.000000 65.000000,67.000000 60.500000), 70.357143 53.785714, 71.500000 53.500000, 74.928571 54.642857, 69.750000 65.000000)))', 4326), GeometryFromText('POLINT( 71.000000 60.000000)', 4326))
,(6,6,1,GeometryFromText('MULTIPDLYGON(((80.000000 65.000000,69.750000 65.000000,74.928571 54.642857, 80.000000 55.423077, 80.000000 65.000000))', 4326))
,(7,7,1,GeometryFromText('POLINT( 73.000000 61.000000)', 4326))
,(7,7,1,GeometryFromText('MULTIPDLYGON(((80.000000) 55.423077,74.928571 54.642857,71.500000 53.500000, 71.500000 45.000000, 80.000000 45.000000, 80.000000 55.423077)))', 4326),
GeometryFromText('POLINT( 75.000000 48.000000) ', 4326))
,(8,8,1,GeometryFromText('MULTIPDLYGON(((67.000000 60.500000,62.875000 57.750000,60.735294 53.470588,67.045455 52.681818, 70.357143 53.785714, 67.000000 60.500000)))', 4326))
```

Create result data type and plr_modules

```
DROP TABLE IF EXISTS events CASCADE:
CREATE TABLE events
  segid int not null primary key, -- visit sequence #
 plotid int, -- original plot id
  bearing real, -- bearing to next wavpoint
 distance real, -- distance to next waypoint
 velocity real, -- velocity of travel, in nm/hr
 traveltime real. -- travel time to next event
 loitertime real. -- how long to hang out
  totaltraveldist real, -- cummulative distance
 totaltraveltime real -- cummulaative time
):
SELECT AddGeometryColumn('','events','location','4326','POINT',2);
CREATE INDEX "events_location_gist" ON "events" USING gist ("location" gist_geometry_ops);
CREATE TABLE plr_modules (
 modseg int4 primary key,
 moderc text
):
```

Create main PL/R function

```
CREATE OR REPLACE FUNCTION solve_tsp(makemap bool, mapname text) RETURNS SETOF events AS
$$
 require(TSP)
 require(fields)
  sql.str <- "select id, st_x(location) as x, st_y(location) as y, location from stands;"
 waypts <- pg.spi.exec(sql.str)
 dist.matrix <- rdist.earth(waypts[,2:3], R=3949.0)
 rtsp <- TSP(dist.matrix)
  soln <- solve_TSP(rtsp)
  tour <- as.vector(soln)
 pg.thrownotice( paste("tour.dist=", attributes(soln)$tour_length))
 route <- make.route(tour, wavpts, dist.matrix)
 if (makemap) {
    make.map(tour, waypts, mapname)
 return(route)
22
LANGUAGE 'plr' STRICT:
```

Install make.route() function

```
INSERT INTO plr_modules VALUES (0,
 $$ make.route <-function(tour, waypts, dist.matrix) {
    velocity <- 500.0
    starts <- tour[1:(length(tour))-1]
    stops <- tour[2:(length(tour))]
    dist.vect <- diag( as.matrix( dist.matrix )[starts.stops] )
    last.leg <- as.matrix( dist.matrix )[tour[length(tour)],tour[1]]</pre>
    dist.vect <- c(dist.vect, last.leg )
    delta.x <- diff( waypts[tour,]$x )
    delta.v <- diff( wavpts[tour.]$v )
    bearings <- atan( delta.x/delta.y ) * 180 / pi
    bearings <- c(bearings,0)
    for( i in 1:(length(tour)-1) ) {
      if( delta.x[i] > 0.0 && delta.y[i] > 0.0 ) bearings[i] <- bearings[i]
      if( delta.x[i] > 0.0 && delta.y[i] < 0.0 ) bearings[i] <- 180.0 + bearings[i]
      if( delta.x[i] < 0.0 && delta.v[i] > 0.0 ) bearings[i] <- 360.0 + bearings[i]
      if(delta.x[i] < 0.0 && delta.v[i] < 0.0 ) bearings[i] <- 180 + bearings[i]
    route <- data.frame(seg=1:length(tour), ptid=tour, bearing=bearings, dist.vect=dist.vect,
                        velocity=velocity, travel.time=dist.vect/velocity, loiter.time=0.5)
    route$total.travel.dist <- cumsum(route$dist.vect)
    route$total.travel.time <- cumsum(route$travel.time+route$loiter.time)
    route$location <- wavpts[tour.]$location
    return(route)}$$):
```

Install make.map() function

```
INSERT INTO plr_modules
 VALUES (1, $$
 make.map <-function(tour, waypts, mapname) {
    require(maps)
    jpeg(file=mapname, width = 480, height = 480, pointsize = 10, quality = 75)
    map('world2', xlim = c(20, 120), ylim=c(20,80))
    map.axes()
    grid()
    arrows(waypts[tour[1:(length(tour)-1)],]$x, waypts[tour[1:(length(tour)-1)],]$y,
           waypts[tour[2:(length(tour))],]$x, waypts[tour[2:(length(tour))],]$y,
           angle=10, lwd=1, length=.15, col="red")
    points( waypts$x, waypts$y, pch=3, cex=2)
    points( wavpts$x, wavpts$v, pch=20, cex=0.8 )
    text( waypts$x+2, waypts$y+2, as.character( waypts$id ), cex=0.8 )
    title( "TSP soln using PL/R" )
    dev.off()
 ጉ$$
);
```

(8 rows)

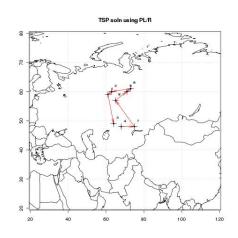
Run the TSP function

```
-- only needed if INSERT INTO plr_modules was in same session
SELECT reload plr modules():
SELECT seqid, plotid, bearing, distance, velocity, traveltime, loitertime, totaltraveldist
FROM solve_tsp(true, 'tsp.jpg');
NOTICE: tour.dist= 2804.58129355858
 seqid | plotid | bearing | distance | velocity | traveltime | loitertime | totaltraveldist
     1 I
              8 | 131.987 |
                             747.219 I
                                            500 I
                                                     1.49444
                                                                      0.5 I
                                                                                    747.219
     2 1
                      -90 I
                             322.719 L
                                            500 I
                                                    0.645437 I
                                                                      0.5 I
                                                                                    1069.94
     3 I
              4 | 284.036 | 195.219 |
                                            500 I
                                                    0.390438 |
                                                                      0.5 I
                                                                                    1265.16
              3 | 343.301 | 699.683 |
                                            500 I
                                                    1.39937 I
                                                                      0.5 L
                                                                                    1964.84
     5 I
              1 | 63.4349 | 98.2015 |
                                            500 I
                                                    0.196403 |
                                                                      0.5 I
                                                                                    2063.04
     6 I
              2 | 84.2894 | 345.957 |
                                            500 I
                                                    0.691915 I
                                                                      0.5 I
                                                                                       2409
              6 | 243.435 | 96.7281 |
                                            500 I
                                                    0.193456 I
                                                                      0.5 I
                                                                                    2505.73
     8 1
              5 I
                        0 1
                             298.855 I
                                            500 I
                                                  0.59771 l
                                                                      0.5 I
                                                                                    2804.58
```

• Run the TSP function (first row expanded)

```
١x
SELECT * FROM solve_tsp(true, 'tsp.jpg');
NOTICE: tour.dist= 2804.58129355858
-[ RECORD 1 ]---+----
segid
plotid
bearing
             I 104.036
distance
            I 195,219
velocity
            | 500
traveltime
            1 0.390438
loitertime
            1 0.5
totaltraveldist | 195.219
totaltraveltime | 0.890438
location
             I 0101000020E610000000000000005040000000000804840
-[ RECORD 2 ]---+----
Γ...1
```

Geospatial Example - Output



- Timeseries, waveform data
- Stored as array of floats recorded during seismic event at a constant sampling rate
- Available from online sources in individual file for each event
- Each file has about 16000 elements

Load 1000 seismic events

```
DROP TABLE IF EXISTS test_ts;
CREATE TABLE test_ts (dataid bigint NOT NULL PRIMARY KEY,
                      data double precision[]):
CREATE OR REPLACE FUNCTION load test(int) RETURNS text AS $$
 DECLARE.
         int:
    arr text:
    sql text;
 BEGIN
    arr := pg_read_file('array-data.csv', 0, 500000);
    FOR i TN 1..$1 LOOP
      sql := $i$INSERT INTO test_ts(dataid,data) VALUES ($i$ || i || $i$,'{$i$ || arr || $i$}')$i$;
     EXECUTE sal:
    END LOOP:
    RETURN 'OK';
 END:
$$ LANGUAGE plpgsql;
SELECT load test(1000):
 load test
-----
ΩK
(1 row)
Time: 37336.539 ms
```

• Load 1000 seismic events (alternate method)

```
DROP TABLE IF EXISTS test_ts_obj;
CREATE TABLE test_ts_obj (
    dataid serial PRIMARY KEY,
    data bytea
);

CREATE OR REPLACE FUNCTION make_r_object(fname text) RETURNS bytea AS $$
    myvar<-scan(fname,sep=",")
    return(myvar);
$$ LANGUAGE 'plr' IMMUTABLE;

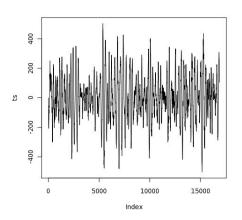
INSERT INTO test_ts_obj (data)
SELECT make_r_object('array-data.csv')
FROM generate_series(1,1000);

INSERT 0 1000
Time: 12166.137 ms
```

Plot the waveform

```
CREATE OR REPLACE FUNCTION plot ts(ts double precision[]) RETURNS bytea AS $$
 library(quantmod)
 library(cairoDevice)
 library(RGtk2)
 pixmap <- gdkPixmapNew(w=500, h=500, depth=24)
  asCairoDevice(pixmap)
 plot(ts,type="1")
 plot_pixbuf <- gdkPixbufGetFromDrawable(NULL, pixmap,
                                          pixmap$getColormap(),
                                          0, 0, 0, 0, 500, 500)
 buffer <- gdkPixbufSaveToBufferv(plot_pixbuf,
                                    "jpeg",
                                   character(0).
                                   character(0))$buffer
 return(buffer)
$$ LANGUAGE 'plr' IMMUTABLE:
SELECT plr_get_raw(plot_ts(data)) FROM test_ts WHERE dataid = 42;
```

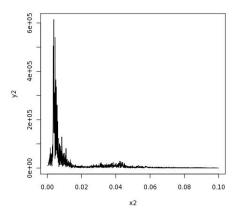
Seismic Data Example - Waveform Output



Analyze the waveform

```
CREATE OR REPLACE FUNCTION plot_fftps(ts bytea) RETURNS bytea AS $$
 library(quantmod)
 library(cairoDevice)
 library(RGtk2)
  fourier<-fft(ts)
 magnitude <- Mod (fourier)
 y2 <- magnitude[1:(length(magnitude)/10)]
 x2 <- 1:length(y2)/length(magnitude)
 mydf <- data.frame(x2,y2)
 pixmap <- gdkPixmapNew(w=500, h=500, depth=24)
  asCairoDevice(pixmap)
 plot(mvdf.tvpe="l")
 plot_pixbuf <- gdkPixbufGetFromDrawable(NULL, pixmap, pixmap$getColormap(),</pre>
                                                          0. 0. 0. 0. 500. 500)
 buffer <- gdkPixbufSaveToBufferv(plot_pixbuf, "jpeg", character(0),</pre>
                                                          character(0))$buffer
 return(buffer)
$$ LANGUAGE 'plr' IMMUTABLE:
SELECT plr_get_raw(plot_fftps(data)) FROM test_ts_obj WHERE dataid = 42;
```

Seismic Data Example - Waveform Analysis Output



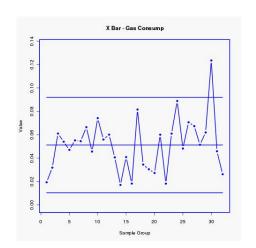
Statistical Process Control Example

Time: 21.986 ms

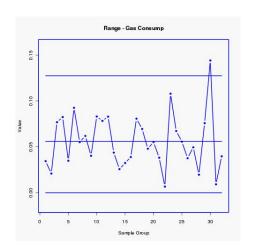
- Named controlChart R function loaded via plr_modules;
 about 120 lines of code
- controlchart() PL/R function; another 130 lines of code

```
http://www.joeconway.com/source_code/controlchart.sql
SELECT * FROM controlchart('G121XA34', 3, 0, array['/tmp/xbar.jpg','/tmp/r.jpg','/tmp/gma.jpg']);
SELECT * FROM controlchart('G121XA34', 3, 0, null) LIMIT 1;
-[ RECORD 1 ]-----
group_num | 1
xb
          I 0.0193605889310595
yhh.
          1 0 0512444187147061
xiic]
          1 0 0920736498010521
xlcl
         I 0.0104151876283601
          1 0.0344209665807481
r
rh
          1 0.0559304535429398
          I 0.127521434077903
rucl
rlcl
          1 0
gma
          I 0.0193605889310595
```

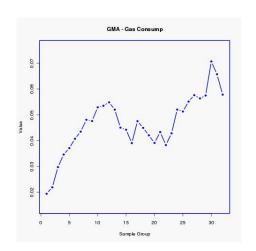
Statistical Process Control Example - X-Bar Chart Output



Statistical Process Control Example - R Chart Output



Statistical Process Control Example - GMA Chart Output



Questions?

Thank You! mail@joeconway.com