Homework 4
Computer Science
B351 Spring 2017
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April 4, 2017

All the work herein is mine.

Homework Questions

1. Convert the following logical sentences to clausal form:

   (a) $\exists\ y\ p(y) \vee [\exists\ y\ (q(y) \rightarrow (\exists\ x\ (p(x) \vee\ q(x, y, C)))]$
   $\exists\ y\ p(y) \vee [\exists\ y\ (\neg q(y) \vee (\exists\ x\ (p(x) \vee\ q(x, y, C)))]$
   $\exists\ y\ p(y) \vee [\exists\ z\ (\neg q(z) \vee (\exists\ x\ (p(x) \vee\ q(x, z, C)))]$
   $\exists\ y\ \exists\ z\ p(y) \vee (\neg q(z) \vee (\exists\ x\ (p(x) \vee\ q(x, z, C)))$
   $\exists\ y\ \exists\ z\ \exists\ x\ p(y) \vee (\neg q(z) \vee ((p(x) \vee\ q(x, z, C)))$
   $\exists\ z\ \exists\ x\ p(f()) \vee (\neg q(z) \vee ((p(x) \vee\ q(x, z, C)))$
   $\exists\ x\ p(f()) \vee (\neg q(g()) \vee ((p(x) \vee\ q(x, g(), C)))$
   $p(f()) \vee (\neg q(g()) \vee ((p(h()) \vee\ q(h(), g(), C)))$
   $p(f()), (\neg q(g()), ((p(h()),\ q(h(), g(), C)))$
   $[p[f[]], [\neg, q, [g, []]], [p, [h, []]],\ [q[h[], g[], C]]]$

   (b) $\forall x \forall y \forall x\ d(x, y) \wedge d(y, z) \rightarrow d(x, z)$

   (c) $(P \vee Q) \wedge (\neg P \rightarrow (Q \vee R))$
   $(P \vee Q) \wedge (\neg\neg P \vee (Q \vee R))$
   $(P \vee Q) \wedge (P \vee (Q \vee R))$
   $[\wedge, [\vee, P, Q], [\vee, P, [\vee, Q, R]]]$

2. Let $\mathcal{U} = \{1, 2, 3\}$, $p = \{1, 3\}$, $m = \{(1, 1), (2, 1), (3, 2)\}$

   (a) Determine $\models \forall\ x\ \exists\ y\ m(x, y)$
   for x = 1
   (1,1)
   for x = 2 (2,1)
   for x = 3 (3,2)
   Therefore $\models \forall\ x\ \exists\ y\ m(x, y)$ is valid.

   (b) Determine $\models \forall\ y\ \exists\ x\ m(x, y)$
   for y = 1
   (1,1),(2,1)
   for y = 2
   (3,2)
   for y = 3

{}
therefore $\models \forall\ y\ \exists\ x\ m(x, y)$ is not valid.

(c) Determine $\models \forall\ x\ \exists\ x\ m(x, x)$
for x = 1
(1,1)
for x = 2
{}
Therefore $\models \forall\ x\ \exists\ x\ m(x, x)$ is not valid.

(d) Determine $\models \exists\ x\ \forall\ y\ m(x, y)$
for x = 1
(1,1)
for x = 2
(2,1)
for x = 3
(3,2)
Because none of these x values holds all there y values, $\models \exists\ x\ \forall\ y\ m(x, y)$ is not valid.

(e) Determine $\models \exists\ x\ \forall\ y\ m(y, x)$
y = 1
(1,1)
y = 2
(2,1)
y = 3
(3,2)
Therefore $\models \exists\ x\ \forall\ y\ m(y, x)$ is valid.

(f) Determine $\models \exists\ x\ \forall\ x\ m(x, x)$
for x = 1
(1,1)
Therefore $\models \exists\ x\ \forall\ x\ m(x, x)$ is valid.

3. You've decided to add a new quantifier: M that takes one variable. The syntax is $M\ x\ f(x)$ for some sentence f. The meaning of $M\ x\ f(x)$ is that the number of times $\sigma(u, x)f(x)$ is true, where $\sigma(u, x)$ is substituting a value from the domain $u \in \mathcal{U}$ is at least 1.5 times more than when it is false. We can assume $\mathcal{U}$ is finite too. Use the model in the previous problem.
Let U = 1,2,3, p = 1,3, m = (1,1),(2,1),(3,2)

   (a) Determine $\models Mx\ p(x)$
   (b) Determine $\models \forall\ x\ My\ m(y, x) \rightarrow p(x)$

4. Ursala, Kaiser, and Shilah are dogs. We know the following:

   (a) Ursala is silver.
   (b) Shilah is gray and loves Kaiser.
   (c) Kaiser is either gray or silver (but not both) and loves Ursala.

   What does this sentence mean? $\exists x \exists y(gray(x) \wedge silver(y) \wedge loves(x, y)$. Use resolution refutation to prove this.

5. Consider a robot that works in a mine – it has to push some objects and not push others depending on a colored tag that is either green or red. Here are the facts:

   - If pushable objects are green, the non-pushable are red.
   - All objects are either green or red.
   - If there is a non-pushable object, the all pushable objects are green.

- Object 1, a cart, is pushable.
- Object 2, a pile of ore, is not pushable.

Assume you're trying to prove that there is a red object.

- Rewrite the statements into FOL (formal) and show their robotic equivalent (Python).
  - If pushable objects are green, the non-pushable are red.
    $(\exists x\ green(x) \wedge pushable(x)) \models (\exists y\ red(y) \wedge \neg pushable(y))$
    $[\models, [\exists, x, [\wedge, [green, [x]], [pushable, [x]]]][\exists, y, [\wedge, [red, [x]], [\neg, [pushable, [x]]]]$
  - All objects are either green or red.
    $\forall x\ green(x) \vee red(x)$
  - If there is a non-pushable object, the all pushable objects are green.
  - Object 1, a cart, is pushable.
  - Object 2, a pile of ore, is not pushable.
- Convert to clausal form.
- Use refutation to prove the there is a red object, by working *only* on the robotic equivalent. Clearly in indicate the process.

6. Assume $\mathcal{U} = \{Alex, Bob, Cathy\}$, $M(x)$ means x is a mechanic, $N(x)$ means x works at NASA, $W(x, y)$ means x worked with y, $I(x, y, z)$ means x introduced y to z. Write constants A, B, C to mean $Alex, Bob, Cathy$, respectively. Write the following in FOL:

   - Cathy is a mechanic. Example: $M(C)$
   - Bob is not a mechanic.
     $\neg M(B)$
   - Either Alex is a mechanic or Bob is, but I know Cathy works at NASA.
     $(M(A) \vee M(B)) \wedge M(C)$
   - Bob introduced Alex to Cathy, since Cathy works at NASA.
     $N(C) \models I(B, A, C)$
   - Someone is a mechanic, but everyone works at NASA. $\exists x \forall y M(x) \wedge N(y)$
   - Bob introduced himself to Cathy.
     $I(B, B, C)$
   - Nobody has been introduced to Alex.
     $\forall x \forall y \neg I(x, y, A)$
   - If someone introduced Bob to Alex, then Bob isn't a mechanic.
     $\exists x I(x, B, A) \models \neg M(B)$
   - Nobody works with anyone here.
     $\forall x \forall y \neg W(x, y)$
   - Somebody works with Cathy, but it's not a mechanic, because Cathy works at NASA.
     $\exists x W(x, C) \wedge \neg M(x) \models N(C)$