Notes on Integration by Parts (IBP)

The continuous IBP formula is as follows:

$$\int_{\Omega} (\nabla \cdot \mathbf{u})\, q \, dx = - \int_{\Omega} \mathbf{u} \cdot \nabla q \, dx \tag{1}$$

Let

$$\mathbf{u}_h \in \mathcal{F}_h, \qquad q_h \in \mathcal{C}_h$$

be discrete fields, and define:

$$D : \mathcal{F}_h \to \mathcal{C}_h, \qquad G : \mathcal{C}_h \to \mathcal{F}_h$$

as mimetic *divergence* and *gradient* operators, respectively.

Then, the discrete analog of (1) is:

$$\langle D\mathbf{u}_h, q_h \rangle_Q = -\langle \mathbf{u}_h, G q_h \rangle_P, \qquad \forall\, \mathbf{u}_h, q_h. \tag{2}$$

where Q and P are diagonal positive definite matrices that define the discrete inner products for vector and scalar fields, respectively.

In contrast with the *Mimetic Finite Difference (MFD)* framework developed at Los Alamos, where one operator (typically the divergence) is taken as the primary operator and the other is derived from it through the *discrete duality principle*, in the approach of Corbino and Castillo both the divergence and the gradient are constructed independently. Although they are not derived from each other, their definitions are coupled by the requirement that they satisfy (2).