ECE 4705 Lab
Experiment 3 – Sampling Theory
Julio Ortiz Guzman
ECE4705L_03

ECE 4705L **EXPERIMENT 3**

SAMPLING THEORY[1]

INTRODUCTION

This week the objective of the experiment is to become familiar with sampling and reconstruction of a signal. The purpose behind sampling is to be able to take a continuous-time signal and translate it to a discrete-time signal that can be exactly copied and recreated. Sampling works by recording the value of the signal measured at certain intervals in time, i.e. certain frequencies. We can sample at higher frequency rates to receive more data values per cycle within a given time period. To perfectly recreate the sampled signal, sampling must ideally be done at a frequency that is two times greater than the maximum frequency of the sampled signal, and this is known as the Nyquist Rate. The equation of the sampled signal, g(t), can be obtained by multiplying the equation of the input signal, x(t) with the sampling function, $\delta T_s(t)$.

$$g(t) = x(t) \cdot \delta_{T_s}(t)$$

$$g(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Or

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s)$$

Using this equation, we can get the instantaneous value of the signal to be recreated. By sampling at at least the Nyquist rate within the same time period, we can perfectly recreate the input signal. There are two forms of sampling theory, ideal sampling and practical sampling. With ideal sampling, a signal is sampled using a continuous impulse train at the Nyquist rate, and these impulses have nearly zero width due to them occuring instantaneously. The following diagram provides a general demostration of ideal sampling in the time domain.

[1] Based on a lab from Dr. James Kang

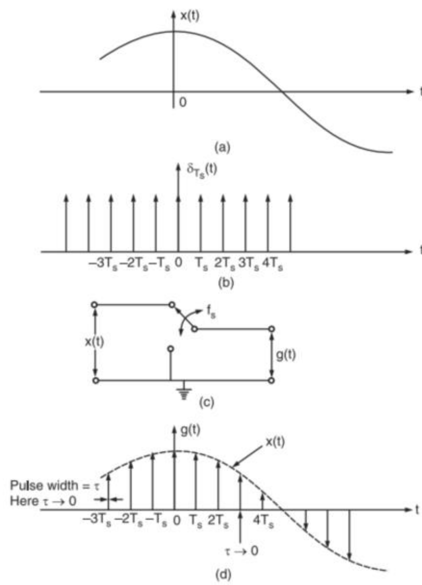

Fig. 1 – Input Signal, Sampling Function, and Ideally Sampled Signal in the Time Domain

However, in practice, ideal sampling is not possible due to the near zero width of the impulses of the sampling function that the power in the instantaneously sampled pulse is negligible. Thus, this method is not suitable for transmission purpose. Practical sampling, on the other hand, utilizes an impulse train with impulses of width τ allowing for it to be used practically, as the name implies. Due to the nature of this type of sampling, sampling at greater than the Nyquist Rate is required to obtain the same level of accuracy in the reconstructed signal. The diagram below shows practical sampling in the time domain.

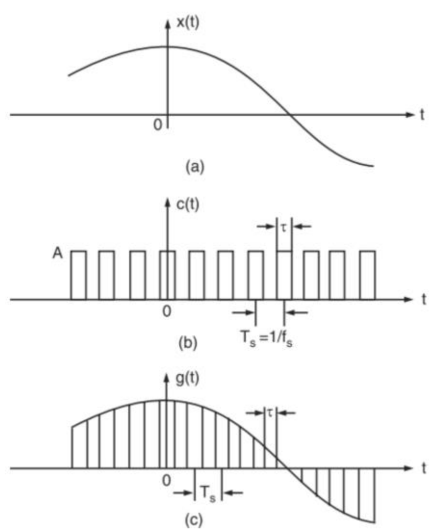

Fig. 2 – Input Signal, Sampling Function, and Practically Sampled Signal in the Time Domain

For this lab, we will be using a periodic rectangular pulse train with period $1/f_s$ = 20 usec, duty cycle ¼, and pulse height, h=1 volt to perform practical sampling on the given signals.

LAB

For this lab, we worked with two input signals and attempted to sample the signals utilizing the two types of sampling we learned about in the introduction. Through MATLAB, we were able to set up a function for the equation of the sample signal and graph the amplitude spectrum of that signal. Let us first start by looking at the signal $x(t)=2\cos(2p\times10000t)$.

1a) Ideal Sampling for $x(t)=2\cos(2p\times10000t)$

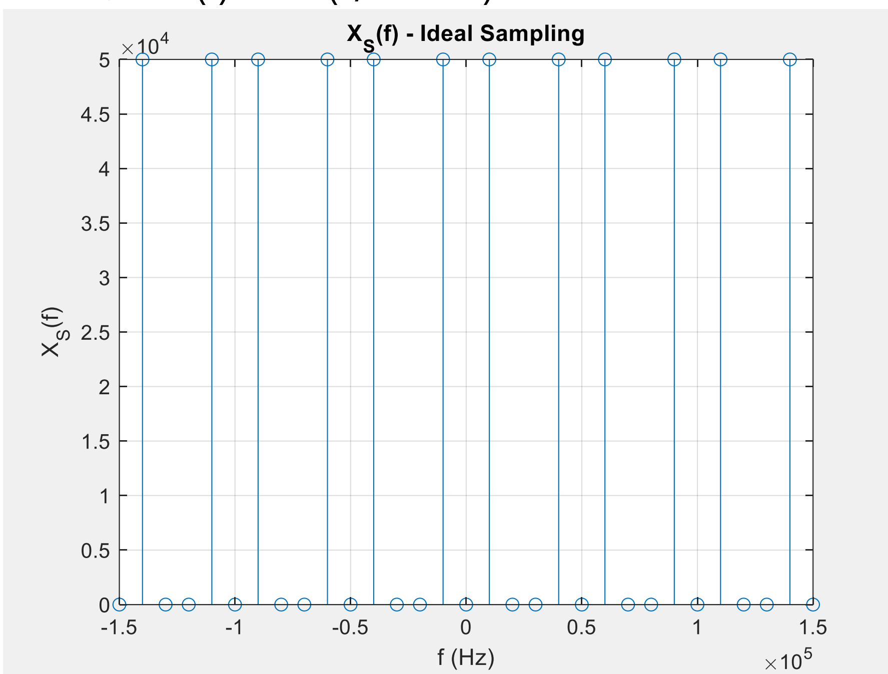

Fig. 3 – Two-Sided Amplitude Spectrum for $x(t)=2\cos(2p\times10000t)$ with Ideal Sampling

1b) Practical Sampling for $x(t)=2\cos(2p\times10000t)$

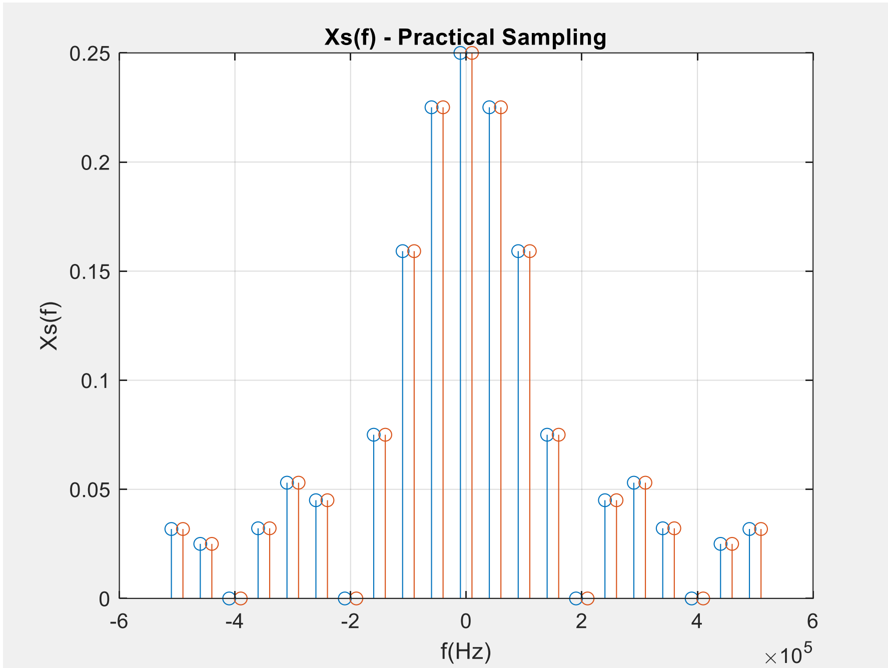

Fig. 4 – Two-Sided Amplitude Spectrum for $x(t) = 2\cos(2\pi \times 10000t)$ with Practical Sampling

It is important to note how the reconstructed signal matches very closesly to the signal that it is sampling in shape and magnitude. This is to be expected due to the equation for the input signal being multiplied by the sampling signal, which in the lab has an amplitude of 1 volt.

2a) Ideal Sampling for $x(t) = 0.2\cos(2\pi \times 2000t) + 0.6\cos(2\pi \times 10000t)$

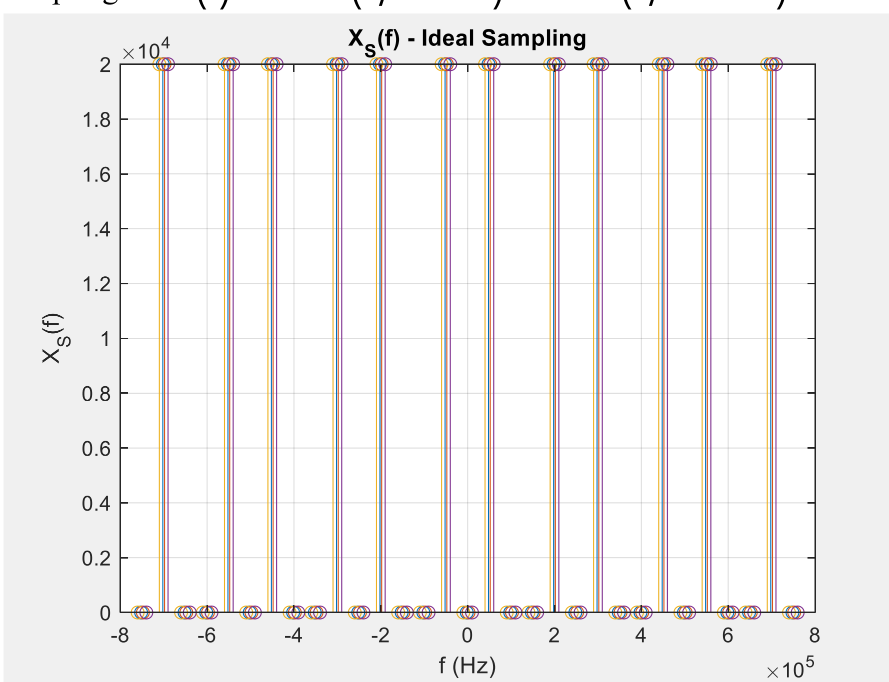

Fig. 5 – Two-Sided Amplitude Spectrum for $x(t) = 0.2\cos(2\pi \times 2000t) + 0.6\cos(2\pi \times 10000t)$ with Ideal Sampling

2b) Practical Sampling for $x(t) = 0.2\cos(2p \times 2000t) + 0.6\cos(2p \times 10000t)$

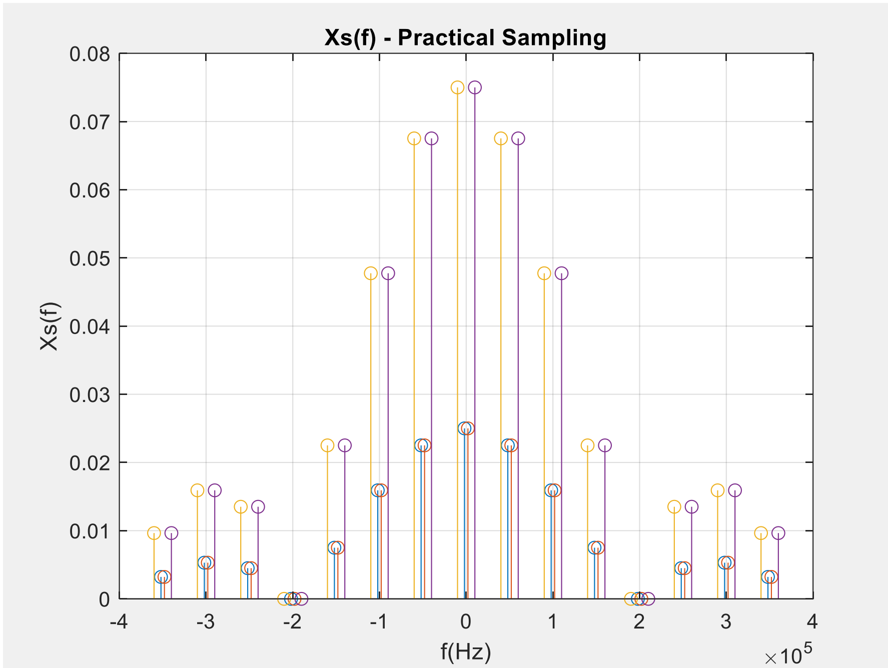

Fig. 6 – Two-Sided Amplitude Spectrum for $x(t) = 0.2\cos(2p \times 2000t) + 0.6\cos(2p \times 10000t)$ with Practical Sampling

Again we see a similar pattern where the reconstructed sampled signal is essentially a copy of the amplitude spectrum of the sampling signal. Due to the voltage of the sampling signal for practical sampling being exactly equal to one, the amplitude values of the pulse trains do not experience much change. The resulting amplitude spectrums were reminiscent of those of the pulse trains that were examined in earlier labs.

CONCLUSION

In this lab, we became famliar with sampling theory and the two different kinds of sampling methods, Ideal and Practical. By using MATLAB software, we were able to simulate each kind of sampling and present the two sided amplitude spectrums after sampling the given signals with the two kinds of sampling. With ideal sampling, an impulse train of 50kHz frequency was used to sample the signals, and when we used practical sampling, we sampled the given signal using a rectangular pulse train with duty cycle ¼. The results show that with ideal sampling, much more values are recorded leading to a better reconstruction of the original signal.

Appendix A

I. MATLAB code:

```
% Part 1a
clear;clc;
fm=10e3; A=2;
fs=50e3; Ts=1/fs;
N=15; k=-N:N;
f=k*fm;

X=A/2*(dirac(f-fm)+dirac(f+fm));
Xs = 0.0;
for n=k
    Xs=Xs+(fs*A/2.*(dirac(f-fm-fs.*n)+(dirac(f+fm-fs.*n))));
    jdx=Xs==inf;
    Xs(jdx)=fs*A/2;
end

T1a = table(k',Xs','VariableNames',{'k','Xs(f)'})

figure(1)
stem(f,Xs);
title('X_S(f) - Ideal Sampling');
xlabel ('f (Hz)'); ylabel('X_S(f)');
grid on;


% Part 1b
N=10; k=-N:N;
fs=50e3; Ts1=1/fs;
f=k*fm;
d=1/4; A=1;

Xn=A*d*sinc(k*d);
Xn1=abs(Xn);
T1b = table(k',Xn1','VariableNames',{'Harmonic Number n','Xn1'})

figure(2)
stem(fs*k-fm,Xn1);hold on;
stem(fs*k+fm,Xn1); hold off;
title('Xs(f) - Practical Sampling');
xlabel ('f(Hz)'); ylabel('Xs(f)'); grid on;


% Part 2a
N=15; k=-N:N;
fs=50e3;
fm1=2e3; fm2=10e3;
f1=k*fm1; f2=k*fm2;
A1=0.2; A2=0.6;

X1=A1/2*(dirac(f1-fm1)+dirac(f1+fm1)).*(A2/2*(dirac(f2-fm2)+dirac(f2+fm2)));
X_s = 0.0;
for n=k
```

```
    X_s=X_s+(fs*A1/2.*(dirac(f1-fm1-fs.*n)+(dirac(f1+fm1-
fs.*n))))+(fs*A2/2.*(dirac(f2-fm2-fs.*n)+(dirac(f2+fm2-fs.*n))));
    jdx=X_s==inf;
    X_s(jdx)=fs*A1/2+fs*A2/2;
end

T2a = table(k',X_s','VariableNames',{'k','X_s(f)'})

figure(3)
stem(fs*k-fm1,X_s); hold on;
stem(fs*k+fm1,X_s);
stem(fs*k-fm2,X_s);
stem(fs*k+fm2,X_s); hold off;
title('X_S(f) - Ideal Sampling');
xlabel ('f (Hz)'); ylabel('X_S(f)');
grid on;

% Part 2b
N=7; k=-N:N;
fs=50e3;
fm1=2e3; fm2=10e3;
f1=k*fm1; f2=k*fm2;
A1=0.2; A2=0.6;
d=1/4; h=1;

Xk=h*d*sinc(k.*d);

Xs1=A1/2.*abs(Xk);
Xs2=A2/2.*abs(Xk);
T2b = table(k',Xk',Xs1',Xs2','VariableNames',{'k','Xk','|Xs1|','|Xs2|'})

figure(4)
stem(fs*k-fm1,Xs1); hold on;
stem(fs*k+fm1,Xs1);
stem(fs*k-fm2,Xs2);
stem(fs*k+fm2,Xs2); hold off;

title('Xs(f) - Practical Sampling');
xlabel ('f(Hz)'); ylabel('Xs(f)'); grid on;
```