ECE 4705L **EXPERIMENT 2**

RESPONSE OF FILTERS TO PULSE TRAIN[1]

INTRODUCTION

This week, the objective of this experiment is to become familiar with the lowpass, highpass and bandpass filters and their effects on the pulse train in the time domain and in the frequency domain. These various types of filters can be made to behave in certain ways based on their specific 3dB cutoff frequency. A filter's 3dB cutoff frequency is the frequency at which any signal above the established cutoff is filtered out; it can also be thought of as the frequency where the magnitude drops by 3dB on the magnitude vs frequency graph. By combining a resitor and capacitor and, depending on the orientation of these components, we can create different orders for each filter, where each order is related to the amount of resistor/capacitor pairs present. A 1^{st} order filter, for either typer of filter, is commprised of a single resitor and capaicitor in combination. For this lab, we focused on the Krohn-Hite Filter which is a fourth order (k=4) Butterworth filter. The equations and graphs for the three filters's magnitude are shown below:

Low-Pass Filter: $L(f) = \dfrac{1}{\sqrt{1+\left(\dfrac{f}{f_c}\right)^{2k}}}$, where f_c is the 3dB cutoff frequency.

High-Pass Filter: $H(f) = \dfrac{1}{\sqrt{1+\left(\dfrac{f_c}{f}\right)^{2k}}}$, where f_c is the 3dB cutoff frequency.

Band-Pass Filter: $L(f) = \dfrac{1}{\sqrt{1+\left(\dfrac{f}{f_{ch}}\right)^{2k}}} \cdot \dfrac{1}{\sqrt{1+\left(\dfrac{f_{cl}}{f}\right)^{2k}}}$, where f_{ch} and f_{cl} are the 3dB high and low frequency cutoff frequencies, respectively.

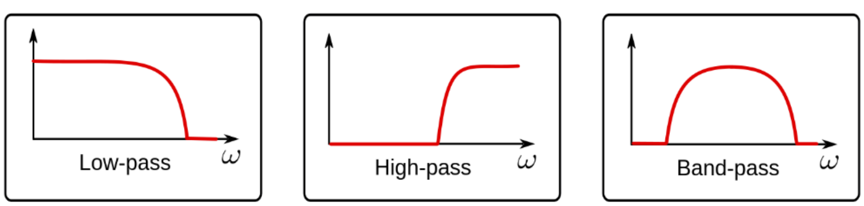

Fig. 1 – Generic Magintude vs Frequency Graphs of Lowpass, Highpass, and Bandpass Filters

As seen in Fig. 1, the lowpass filter passes all low frequencies of a given signal, the highpass filter passes only frequencies above the cutoff, and the bandpass filter passes frequencies between a low and high cutoff frequency. We will now show the behaviors of these different filters, and what happens to the amplitude spectrums of a signal when the filters are applied at the given 3dB cutoff frequencies.

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[1] Based on a lab from Dr. James Kang

LAB

The MATLAB graphs below depict the lowpass, highpass, and bandpass frequency conditions for the simulated Krohn-Hite Butterworth filters. The graphs are shown in gain vs frequency, with the frequency in log-scale hertz (Hz) to show all pertinent frequency values.

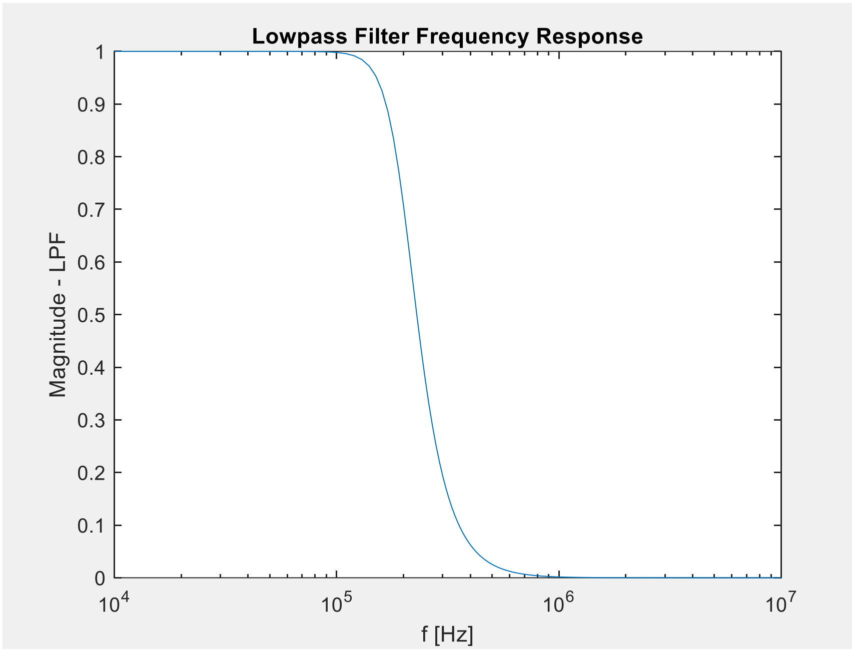

Fig. 1 – Lowpass Filter Log-Scale Frequency Response

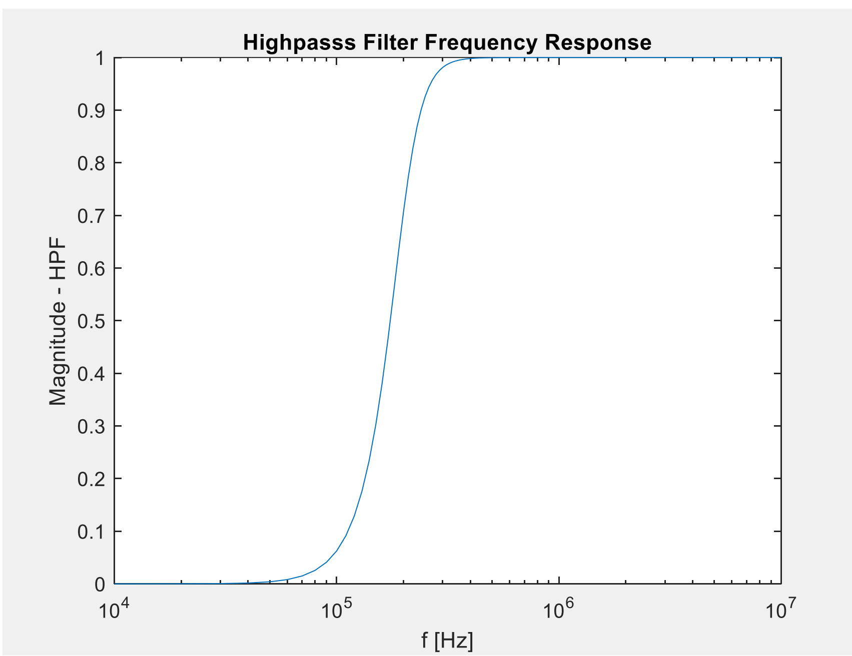

Fig. 2 – Highpass Filter Log-Scale Frequency Response

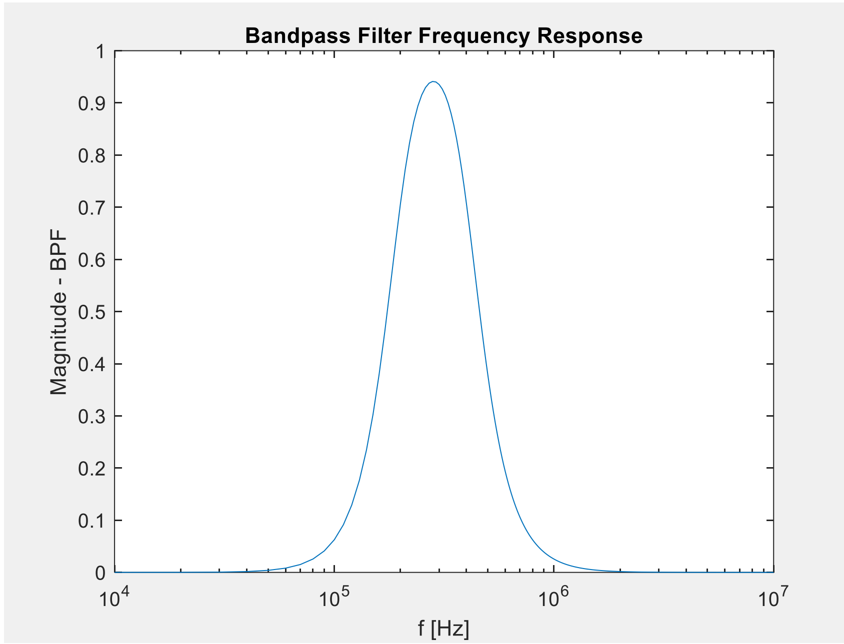

Fig. 3 – Bandpass Filter Log-Scale Frequency Response

From the graphs, we can more or less predict how the amplitude spectrum of the filtered signal will react to the filters. Given a rectangular pulse train with amplitude 100mV, period 25µs (freq=40kHz), and duty cycle 1/5, we can apply each filter condition to the signal and observe the amplitude spectrum output using MATLAB. For the lowpass and highpass filter, a 3dB cutoff frequency of 200kHz was implemented, and for the bandpass filter, the low cutoff was also 200kHz and the high cutoff was set to 400kHz. Given these conditions, we can see the effects of each filter on the rectangular pulse train. First, however, we will view the amplitude spectrum of the unfiltered signal to which we will compare the filtered signals.

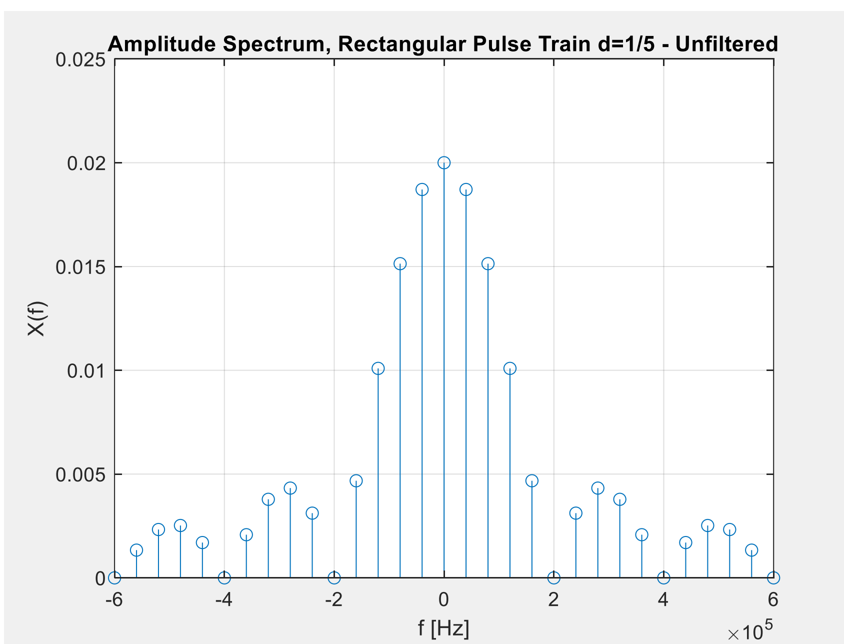

Fig. 4 – Unfiltered Amplitude Spectrum of Rectangular Pulse with Duty Cycle = 1/5

As we already know, the double-sided amplitude spectrum of a rectangular pulse train with duty cycle 1/5 shows the amplitude of the signal at distinct frequencies, and a null at every fifth point on the spectrum. Now let us apply the filters and examine the resulting spectra of the lowpass, highpass, and bandpass filter outputs. It is expected that the spectrum of the lowpass filtered signal will pass all magnitudes under the +/-200kHz cutoff frequency and drastically reduces the magnitudes at frequencies above the cutoff.

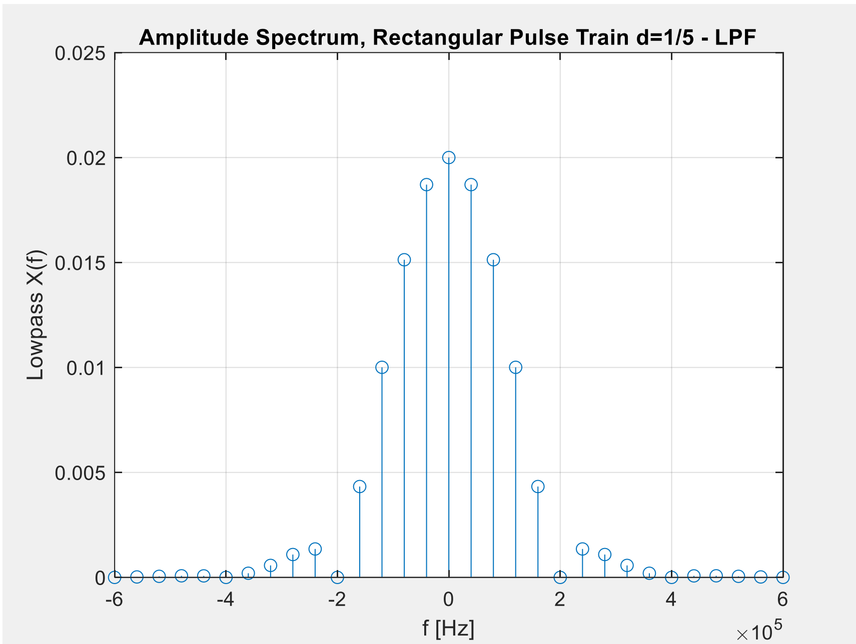

Fig. 5 – Lowpass Filtered Amplitude Spectrum of Rectangular Pulse with Duty Cycle = 1/5

With the lowpass filtered signal having been simulated, we confirm that the filter is functioning properly because all values above +200kHz and below -200kHz are practically reduced to zero. Next let us look at the highpass filterd signal. For this filter, we expect the output amplitude spectrum to be opposite that of the lowpass filter, i.e. magnitude values at less than +200kHz and greater than -200kHz will be filtered out.

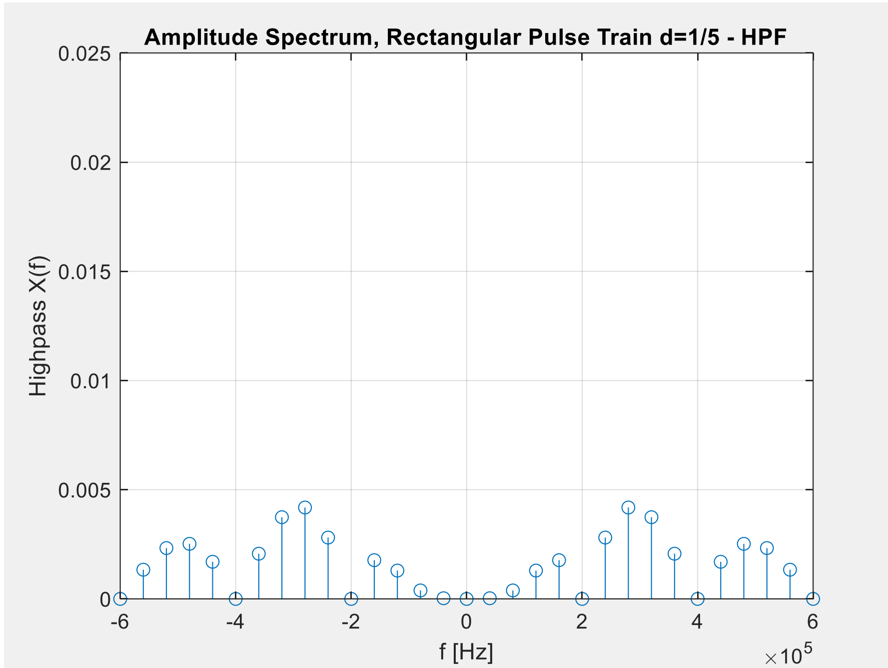

Fig. 6 – Highpass Filtered Amplitude Spectrum of Rectangular Pulse with Duty Cycle = 1/5

Comparing with the unfiltered signal, the highpass filter output amplitude spectrum falls in line with the predicted behavior of a highpass filter given the filter conditions. An interesting observation to take note of is that, at the edges of the cutoff frequency, some of the signal passes through, resulting in magnitudes slightly greater than zero at frequencies within the cutoff. Finally the bandpass filter will be simulated to examine the behavior of the filter compared to the rest. For this filter, we would expect only the magnitudes within the low and high cutoff to pass.

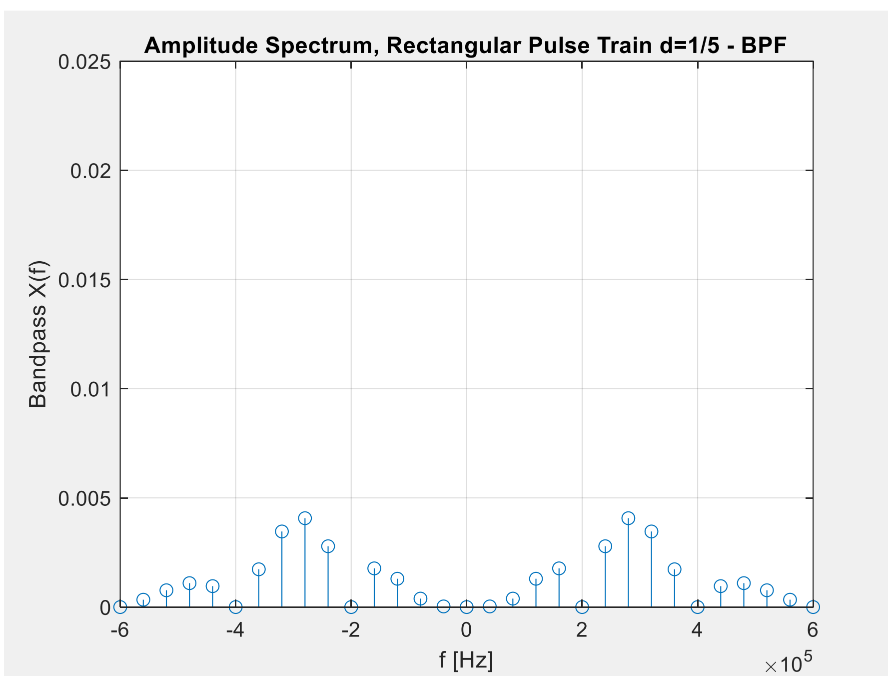

Fig. 7 – Bandpass Filtered Amplitude Spectrum of Rectangular Pulse with Duty Cycle = 1/5

CONCLUSION

In this lab, we were tasked with examining the effects of lowpass, highpass, and bandpass filters on a pulse train. After performing the lowpass, highpass, and bandpass filter simulations through MATLAB, we were able to view and analyze the behavior of each filter. More specifically, we analyzed what would happen to the amplitude spectrum of a 1/5 duty cycle rectangular pulse train when passed through each filter. We became familiar with the conditions required of each filter and, with the simulations, we were able to also familiarize ourselves with how the filters affect a given signal. All our expected results matched with the actual results, with lowpass filtering out all values above the cutoff, highpass filtering out all values below the cutoff, and bandpass filtering out values less than the low cutoff and greater than the high cutoff. The resulting MATLAB graphs also confirmed our predictions and provided a clearer understanding of the behavior of the different filters.

Appendix A

I. MATLAB code:

```
% Part 1
clear all; clc;
f = 1000*linspace(0,10000,1000);
k = 4; fc = 200e3;
fcl = 200e3; fch = 400e3;
lowpass = 1./sqrt(1+((f/fc).^(2*k)));
highpass = 1./sqrt(1+((fc./f).^(2*k)));
bandpass = (1./sqrt(1+((f/fch).^(2*k)))).*(1./sqrt(1+((fcl./f).^(2*k))));

figure(1)
semilogx(f,lowpass);
xlabel('f [Hz]'); ylabel('Magnitude - LPF');
title('Lowpass Filter Frequency Response')

figure(2)
semilogx(f,highpass);
xlabel('f [Hz]'); ylabel('Magnitude - HPF');
title('Highpasss Filter Frequency Response')

figure(3)
semilogx(f,bandpass);
xlabel('f [Hz]'); ylabel('Magnitude - BPF');
title('Bandpass Filter Frequency Response')


% Part 2
n=-15:15;
h=100e-3; T0=25e-6; f0=40e3;
f=n*f0;
d=1/5;
k=4; fc=200e3; fcl=200e3; fch=400e3;

% Rectangular Pulse Train - unfiltered
Xn=h*d*sinc(n*d);
Xf=abs(Xn);

figure(4)
stem(f,Xf)
title('Amplitude Plot, Rectangular Pulse Train d=1/5 - Unfiltered');
xlabel ('f [Hz]'); ylabel('X(f)'); grid on;


% Rectangular Pulse Train - LPF
L=1./sqrt((1+((f./fc).^(2*k))));
XnL=Xf.*L;

figure(5)
stem(f,XnL)
title('Amplitude Plot, Rectangular Pulse Train d=1/5 - LPF');
xlabel ('f [Hz]'); ylabel('X(f)'); grid on;
```

```
% Rectangular Pulse Train - HPF
H=1./sqrt(1+((fc./f).^(2*k)));
XnH=Xf.*H;

figure(6)
stem(f,XnH)
title('Amplitude Plot, Rectangular Pulse Train d=1/5 - HPF');
xlabel ('f [Hz]'); ylabel('X(f)'); grid on;
axis([-6e5 6e5 0 0.025]);


% Rectangular Pulse Train - BPF
B=(1./sqrt(1+((f/fch).^(2*k)))).*(1./sqrt(1+((fcl./f).^(2*k))));
XnB=Xf.*B;

figure(7)
stem(f,XnB)
title('Amplitude Plot, Rectangular Pulse Train d=1/5 - BPF');
xlabel ('f [Hz]'); ylabel('X(f)'); grid on;
axis([-6e5 6e5 0 0.025]);
```