case_6.3

May 29, 2020

1 How much should properties be worth in Milwaukee, Wisconsin? (Part I)

[2]:
```
### Load relevant packages
import pandas                  as pd
import numpy                   as np
import matplotlib.pyplot       as plt
import seaborn                 as sns
import statsmodels.formula.api as smf
import statsmodels.api         as sm
import scipy

%matplotlib inline
plt.style.use('ggplot')
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters()
```

1.1 Introduction (5 mts)

Business Context. Real estate markets can sometimes be irrational, and buying a house can certainly be an emotional and highly psychological process. For example, the asking price can "anchor" the negotiations, and it can be very hard as a buyer to "forget" that initial number.

You are a property developer who frequently buys properties. It would be very useful to get a fair estimate of the price of a property before seeing the asking price, based on features like its size and location. Besides making you a more informed buyer, having a pricing model could have multiple uses, such as automatically detecting under-priced properties that come on the market, or estimating the value added to a property if it was extended, divided into apartments, or converted into offices.

Business Problem. Your task is to **build a model to predict property prices in the city of Milwaukee, Wisconsin**.

Analytical Context. The dataset consists of property sales (commercial and residential) in Milwaukee, Wisconsin from 2002 to 2018. Linear regression is a simple idea: At the end of the day, we are just fitting a line through data. But its simplicity is also its strength: coefficients are easily interpretable and it is straightforward to understand the underlying model. However, many things can go wrong during the process of fitting a linear model.

The case is structured as follows: you will 1) explore the data to pick up on some initial patterns; 2) analyze **residuals**, the difference between the actual values and their estimates, to diagnose an initial model; 3) perform variable transformations to partially deal with problems; and finally 4) look at how to deal with outliers to improve the model further.

1.2 Data exploration (25 mts)

Let's start by taking a look at the available features:

1. **PropType**: the property category ("Commercial", "Residential", "Lg Apartment", "Vacant Land", or "Condominium")
2. **Taxkey**: a unique identifier for the property
3. **Address**: the street address of the property
4. **CondoProject**: for condominiums, the name of the project
5. **District**: integer between 1 and 15 identifying the city district
6. **Nbhd**: integer identifying one of 591 neighborhoods
7. **Style**: information about the building architectural style, commerical use or type of building
8. **Extwall**: type of exterior wall (e.g. "Brick")
9. **Stories**: number of stories
10. **Year_Built**: the year the building was built
11. **Nr_of_rms**: number of rooms
12. **Fin_sqft**: finished square feet
13. **Units**: number of units (e.g. apartments) in the building
14. **Bdrms**: number of bedrooms
15. **Fbath**: number of full bathrooms
16. **Hbath**: number of half bathrooms
17. **Lotsize**: size of the lot in square feet
18. **Sale_date**: the date of the sale in YYYY-MM-DD format
19. **Sale_price**: sale price in US dollars

[3]:
```
data = pd.read_csv("2002-2018-property-sales-data.csv",
    dtype = { # indicate categorical variables
        "PropType": "category",
        "District": "category",
        "Extwall": "category",
        "Nbhd": "category",
        "Style": "category",
    },
    parse_dates=["Sale_date"], # the Sale_date column is parsed as a date
)
data.head()
```

[3]:
```
     PropType      Taxkey                 Address CondoProject District  \
0  Commercial  3230461110             2628 N 6TH ST          NaN        6
1  Commercial  3590192000        1363 N PROSPECT AV          NaN        3
```

```
2  Commercial  4161194000           617 S 94TH ST          NaN       10
3  Commercial  1719836000  3624 W SILVER SPRING DR          NaN        1
4  Commercial  3480290000         3830 W LISBON AV          NaN       15

   Nbhd                                             Style Extwall  Stories  \
0  6258                                 Commercial Exempt     NaN      2.0
1  6262                    Mansions With Commercial Usage     NaN      2.0
2  6272                                  Service Building     NaN      1.0
3  6218  Store Bldg - Multi Story (Store & Apt, Store & O     NaN      2.0
4  6254  Store Bldg - Multi Story (Store & Apt, Store & O     NaN      2.0

   Year_Built  Nr_of_rms  Fin_sqft  Units  Bdrms  Fbath  Hbath  Lotsize  \
0        1880          0      1840      1      0      0      0    12750
1        1876          0      6377      1      0      0      0    11840
2        1954          0      5022      1      0      0      0     9700
3        1955          0      6420      1      0      0      0     8792
4        1909          0      5956      1      0      0      0     4840

   Sale_date  Sale_price
0 2002-01-01       15900
1 2002-01-01      850000
2 2002-01-01      119000
3 2002-01-01      210000
4 2002-01-01       48500
```

[4]:
```
data.shape
data.isnull().sum()
```

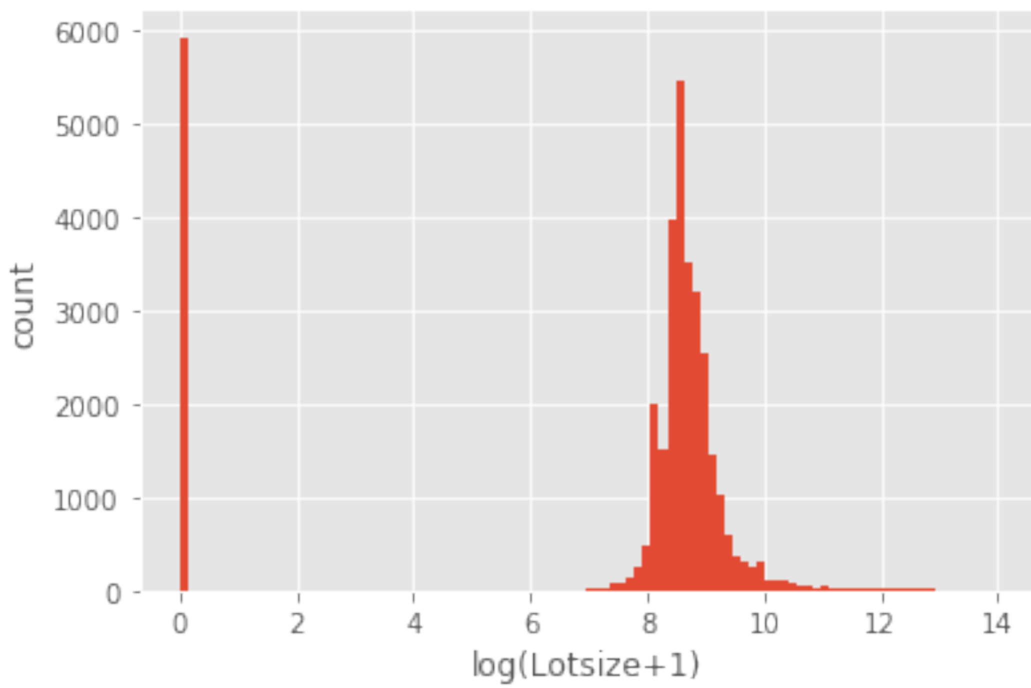
[4]:
```
PropType           3
Taxkey             0
Address            0
CondoProject   27867
District           0
Nbhd               0
Style              6
Extwall         9899
Stories            0
Year_Built         0
Nr_of_rms          0
Fin_sqft           0
Units              0
Bdrms              0
Fbath              0
Hbath              0
Lotsize            0
Sale_date          0
Sale_price         0
```

```
dtype: int64
```

Sometimes missing numbers are entered as zeros, which can mess up the analysis (see `Lotsize` below as an example). Before proceeding, we will remove rows with zeros in `Year_Built`, `Fin_sqft`, `Lotsize`, and `Sale_price`, as these are the numerical variables where erroneous zero values can skew the distribution:

[5]:
```
plt.hist(np.log(data["Lotsize"]+1), bins=100)
plt.xlabel("log(Lotsize+1)")
plt.ylabel("count")
```

[5]:
```
Text(0, 0.5, 'count')
```



For the sake of removing potential confounding factors from consideration, we will focus on residential properties only (commercial properties are subject to all sorts of economic and market forces that residential properties are not):

[6]:
```
def remove_unused_categories(data):
    """ The `remove_unused_categories` method in pandas
        removes categories from a Series if there are no
        elements of that category.

        This function is a convenience function that removes
        unused categories for all categorical columns
        of a data frame.
```

```
        The reason this is useful is that when we
        fit a linear regression, `statsmodels` will
        create a coefficient for every category in a column,
        and so unused categories pollute the results.
    """
    for cname in data:
        col = data[cname]
        if pd.api.types.is_categorical_dtype(col):
            data[cname] = col.cat.remove_unused_categories()
    return data

clean = np.where(
    (data["Sale_price"] > 0) &
    (data["Year_Built"] > 1800) &
    (data["Fin_sqft"] > 0) & # must have non-zero finished square feet
    (data["Lotsize"] > 0)  & # must have non-zero lot size
    (data["PropType"] == "Residential")
    )
data_clean = data.iloc[clean].copy()
remove_unused_categories(data_clean).head()
```

[6]:

```
         PropType      Taxkey           Address CondoProject District  Nbhd  \
10    Residential  3080013000      3033 N 35TH ST          NaN        7  2960
51    Residential  3190434000   1908 E WEBSTER PL          NaN        3  3170
67    Residential  3891722000       812 N 25TH ST          NaN        4  3040
116   Residential  3880628000       959 N 34TH ST          NaN        4  2300
134   Residential  3880406000     3209 W WELLS ST          NaN        4  2300

                   Style Extwall  Stories  Year_Built  Nr_of_rms  Fin_sqft  \
10                  AP 1   Frame      2.0        1913          0      3476
51   Rm or Rooming House   Frame      2.0        1897          0      1992
67   Rm or Rooming House   Frame      2.0        1907          0      2339
116                 AP 1   Frame      2.0        1890          0      2329
134              Mansion   Stone      2.5        1891          0      7450

     Units  Bdrms  Fbath  Hbath  Lotsize  Sale_date  Sale_price
10       4      9      1      0     5040 2002-02-01       42000
51       4      2      2      0     2880 2002-05-01      145000
67       6      0      1      0     3185 2002-06-01       30000
116      4      4      1      0     5781 2002-10-01       66500
134      2      7      6      0    15600 2002-11-01      150500
```

[7]:

```
data_clean.shape
```

[7]:

```
(24450, 19)
```

1.2.1 Exercise 1: (15 mts)

1.1 Write code to visualize the relationship between the logarithm of the sale price per square foot (`Fin_sqft`) and the following variables:

1. the number of units
2. the year the building was built
3. the city district
4. the logarithm of the finished square footage
5. the number of bedrooms
6. the sale date

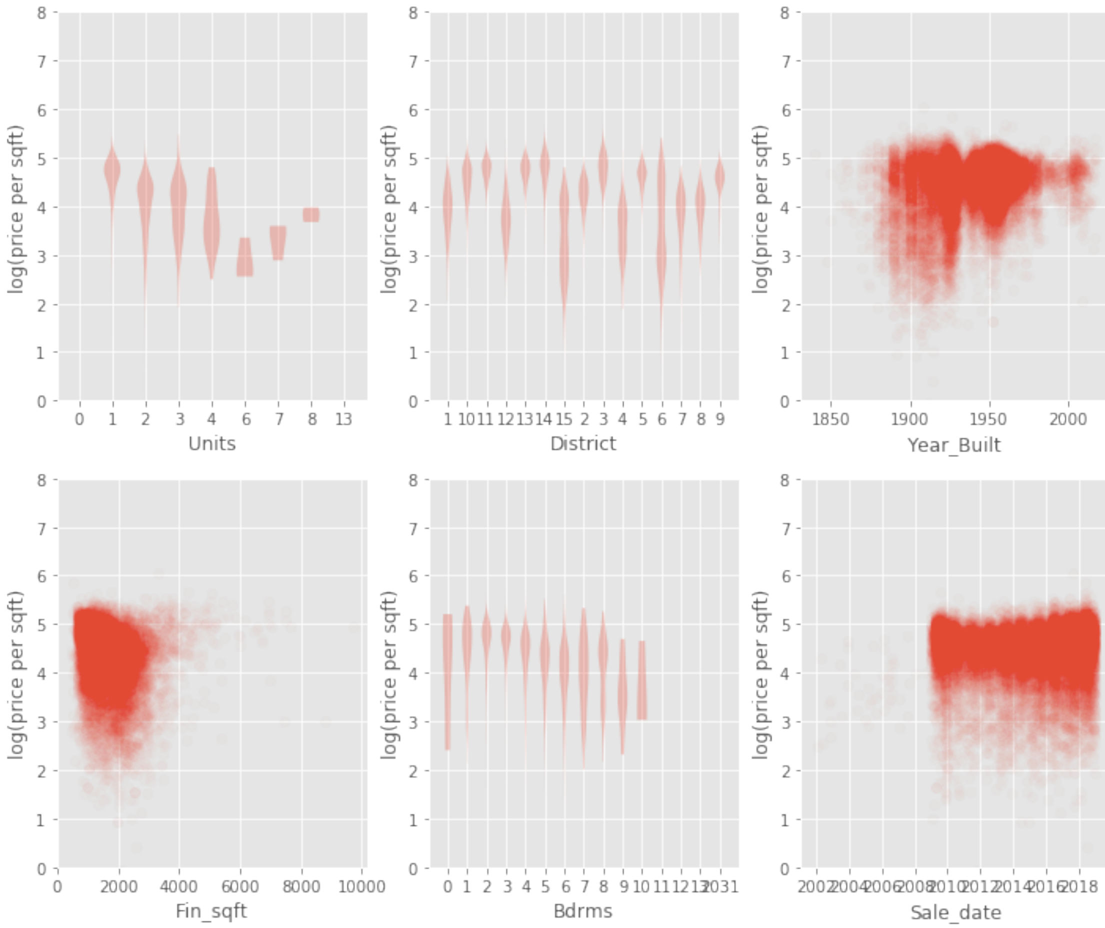
For categorical variables, it can be helpful to use **violin plots**, which first separates the data by category and then shows the distribution of the y variable as a smooth histogram for each category.

Answer. One possible solution is shown below:

[8]:
```
plt.figure(figsize=(12,10))
varstolook = ["Units", "District", "Year_Built", "Fin_sqft", "Bdrms",␣
 ↪"Sale_date"]
y = np.log(data_clean["Sale_price"] / data_clean["Fin_sqft"])
for i,feature in enumerate(varstolook):
    plt.subplot(2,3,i+1)
    colvalues = data_clean[feature]

    unique = sorted(set(colvalues.dropna().values))
    if len(unique) < 16:
        # categorical: let's make a violin plot
        plt.violinplot([y.values[colvalues == level] for level in unique],
                       positions=range(len(unique)),
                       showextrema=False,
                       )
        plt.xticks(range(len(unique)), labels=unique)
    else:
        # alpha=0.02 makes each data point almost completely
        # transparent, making it easier to see the distribution
        # of the data
        plt.scatter(colvalues, y, alpha=0.02, edgecolor=None)
    plt.xlabel(feature)
    plt.ylabel('log(price per sqft)')
    plt.ylim(0,8)
;
```

[8]:
```
''
```



1.2 What can you conclude from these exploratory plots? Select all that apply.

(a) Properties built in the last ten years tend to be more valuable (per square foot)

(b) Dividing a single-unit property into smaller units would tend to make it more valuable

(c) Many of the most valuable properties are in districts 3 and 14

(d) Having more bedrooms makes a property less valuable

Answer.

(a) False. Some of the most valuable properties (but also some of the *least* valuable properties are older. There is no clearly discernible trend.

(b) False. While this may be true, it cannot be seen in these plots alone. In fact the plots show that properties with more units are typically *less* valuable per square foot than single-unit properties. It may be reasonable to infer that the residents of Milwaukee have a marked preference for living in their own houses. But single-unit and multi-unit properties differ in other ways (for example they may be concentrated in different neighborhoods), which makes it impossible to draw solid conclusions from this plot alone.

(c) True. Indeed the upper tails of the violin plot for districts 3 and 14 extend further than the other districts. So those districts have some of the most valuable properties, at least on a per-square-foot basis. A quick look at a map of the districts shows that these are both near Lake Michigan and a short distance from the central business district (District 4).

(d) False. While this may appear to be true from the plot, it is reasonable to suspect that properties with many bedrooms are larger and have multiple units, which *confounds* the trend seen here.

1.2.2 Exercise 2: (5 mts)

Provide the code to fit a multiple linear regression of the sale price against the district, number of units, and finished square footage.

Answer. One possible solution is shown below:

[9]:
```
model_lin = smf.ols(formula = "Sale_price ~ District + Units+ Fin_sqft",
                    data = data_clean).fit()
model_lin.summary()
```

[9]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:             Sale_price   R-squared:                       0.675
Model:                            OLS   Adj. R-squared:                  0.674
Method:                 Least Squares   F-statistic:                     3165.
Date:                Wed, 13 Nov 2019   Prob (F-statistic):               0.00
Time:                        17:57:13   Log-Likelihood:            -2.9839e+05
No. Observations:               24450   AIC:                         5.968e+05
Df Residuals:                   24433   BIC:                         5.969e+05
Df Model:                          16
Covariance Type:            nonrobust
================================================================================
==
                     coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
--
Intercept       2.148e+04   1775.106     12.098      0.000      1.8e+04
2.5e+04
District[T.10]  5.036e+04   1784.909     28.212      0.000     4.69e+04
5.39e+04
District[T.11]  7.328e+04   1718.694     42.637      0.000     6.99e+04
7.66e+04
District[T.12] -3335.9809   2778.758     -1.201      0.230    -8782.516
2110.554
District[T.13]  7.137e+04   1779.286     40.111      0.000     6.79e+04
```

```
7.49e+04
District[T.14]  8.839e+04   1776.422     49.758      0.000    8.49e+04
9.19e+04
District[T.15] -4.969e+04   2628.207    -18.906      0.000   -5.48e+04
-4.45e+04
District[T.2]    1.94e+04   1992.257      9.738      0.000    1.55e+04
2.33e+04
District[T.3]   1.382e+05   2042.091     67.692      0.000    1.34e+05
1.42e+05
District[T.4]  -6.223e+04   4101.571    -15.173      0.000   -7.03e+04
-5.42e+04
District[T.5]   5.588e+04   1715.179     32.582      0.000    5.25e+04
5.92e+04
District[T.6]  -6433.0661   2411.944     -2.667      0.008   -1.12e+04
-1705.509
District[T.7]  -1.445e+04   2145.136     -6.735      0.000   -1.87e+04
-1.02e+04
District[T.8]   -384.3202   2291.476     -0.168      0.867   -4875.753
4107.112
District[T.9]   4.308e+04   2088.960     20.622      0.000     3.9e+04
4.72e+04
Units          -6.983e+04    834.735    -83.660      0.000   -7.15e+04
-6.82e+04
Fin_sqft          98.4410      0.640    153.885      0.000      97.187
99.695
==============================================================================
Omnibus:                    16670.121   Durbin-Watson:                   1.740
Prob(Omnibus):                  0.000   Jarque-Bera (JB):          1483125.778
Skew:                           2.504   Prob(JB):                         0.00
Kurtosis:                      40.825   Cond. No.                     3.26e+04
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 3.26e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
"""
```

1.3 Inspecting residuals to diagnose a fitted model (30 mts)

Linear regression is conceptually simple — after all, we are just fitting a line through our data — but a lot can go wrong. **Residual analysis** is an important tool for diagnosing many problems that can affect a linear regression. The "residuals" are the difference between the observations y and the corresponding fitted values $\hat{y}$; visually it is the distance between the fitted line and the data points.

Residual analysis allows us to test some of the theoretical assumptions that underpin linear regression. The short version is that linear regression works best when the residuals are identically normally distributed. You may have come across statements before claiming that "linear regression assumes the data are normally distributed", which is not entirely correct. Linear regression can still be a useful and powerful (and theoretically justified!) tool even if the data deviates from this assumption. That said, a distribution with fat tails is a particular problem for linear regression, because points "in the tails" that are far away from their fitted values can disproportionately affect the fitted coefficients and the predictions. Even a single such point, which could for example be caused by a misplaced decimal point when the raw data was manually entered into a spreadsheet by a distracted employee, can wreak havoc on a model. Such a data point is said to have high **leverage** on the model.

1.3.1 Exercise 3: (15 mts)

3.1 Obtain the residuals by comparing observations against fitted values (`model_lin.fittedvalues`). Check that you recover the same values as those stored in `model_lin.resid`.

Answer. Statistical software conveniently computes a lot of things for us. But when learning a new tool, it is helpful to check that we understand where these numbers come from. This is a small example of this principle, where we make sure we know how to compute residuals, even though they are also directly available as an attribute of the model object. Note that in turn, the fitted values could have been "manually" obtained as `model_lin.predict(data_clean)`.

[10]:
```
my_resid = data_clean.Sale_price - model_lin.fittedvalues
# check that the differences between my residuals and the
# precomputed ones are all zero (within numerical rounding error):
np.allclose(my_resid, model_lin.resid) # should return True
```

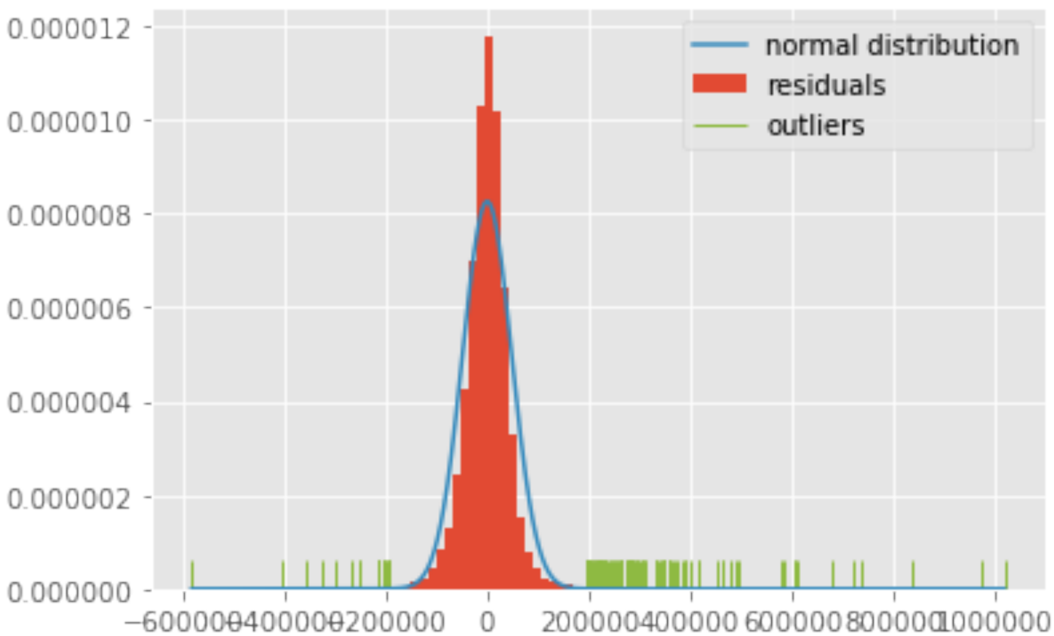
[10]:
```
True
```

3.2 Plot a histogram and a QQ plot of the residuals. What do you notice? (Hint: for the histogram, use the `rugplot()` function in the `seaborn` library, which is used to show residuals and can therefore visualize outliers.)

Answer. To compare the residuals histogram to a normal distribution, we use the `density=True` argument, which puts the histogram on the same scale as the normal distribution. We then use the `scipy.stats.norm.pdf` function to overlay the probability density function of the normal distribution. For the purpose of this rugplot, outliers are arbitrarily defined to be those residuals which are more than four standard deviations away from zero:

[11]:
```
plt.hist(model_lin.resid,
    density=True,     # the histogram integrates to 1
                      # (so it can be compared to the normal distribution)
    bins=100,         #  draw a histogram with 100 bins of equal width
    label="residuals" # label for legend
    )
```

```
# now plot the normal distribution for comparison
xx = np.linspace(model_lin.resid.min(), model_lin.resid.max(), num=1000)
plt.plot(xx, scipy.stats.norm.pdf(xx, loc=0.0, scale=np.sqrt(model_lin.scale)),
    label="normal distribution")
outliers = np.abs(model_lin.resid)>4*np.sqrt(model_lin.scale)
sns.rugplot(model_lin.resid[outliers],
            color="C5", # otherwise the rugplot has the same color as the␣
 ↪histogram
            label="outliers")
plt.legend(loc="upper right");
```



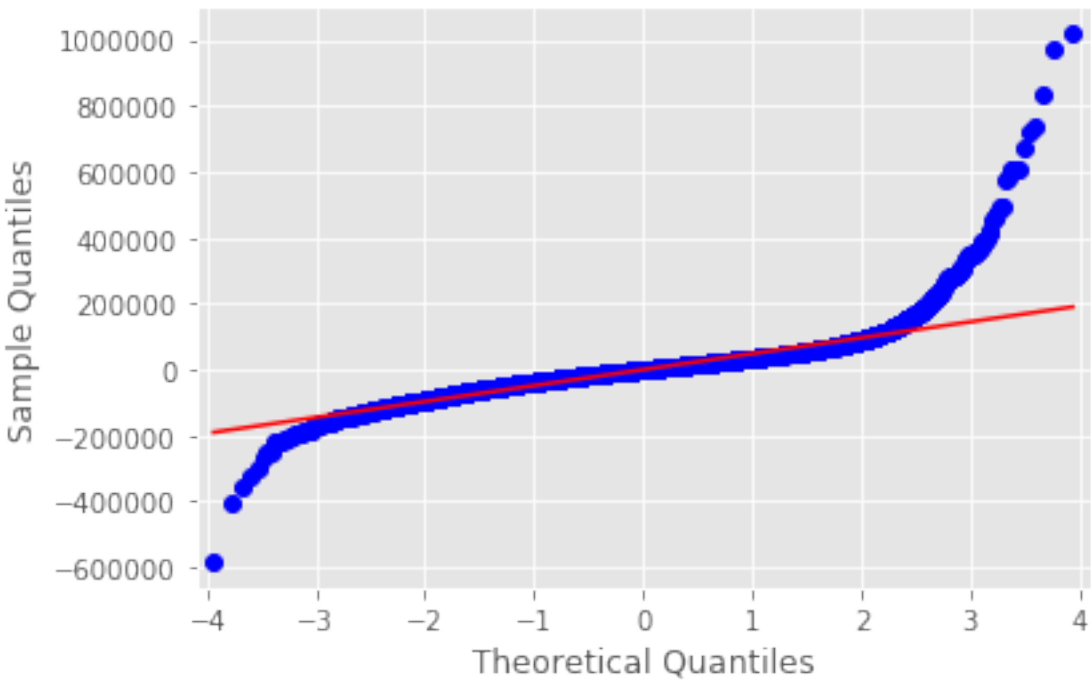
We immediately notice a few things:

1. The peak of the residuals is higher than the normal distribution
2. The bulk of the distribution of the residuals is narrower than the normal distribution
3. There are some large outliers on both sides
4. The residuals are not symmetrical; i.e. there are some particularly large outliers on the right side

NOTE: We are emphasizing visual diagnoses here, but there are metrics reported in the summary statistics table for the residuals that will tell the same story: the positive skew and very high kurtosis are causes for concern.

We also create the QQ plot of the residuals, which shows nonlinearity and is another strong warning sign:

[12]:
```
sm.qqplot(model_lin.resid, line="s");
```



Another troublesome situation that can be detected using residual analysis is **heteroscedasticity**, which means that the residuals have small variance for in some subsets of the data, and high variance in others. We saw this exactly early on in the previous case, with the data points "fanning out" around the line of best fit, and determined that this would cause problems. The opposite of heteroscedasticity is **homoscedasticity**, which is what we want to see in the data, and means that the residuals have similar variance across all subsets of the data.

1.3.2 Exercise 4: (10 mts)

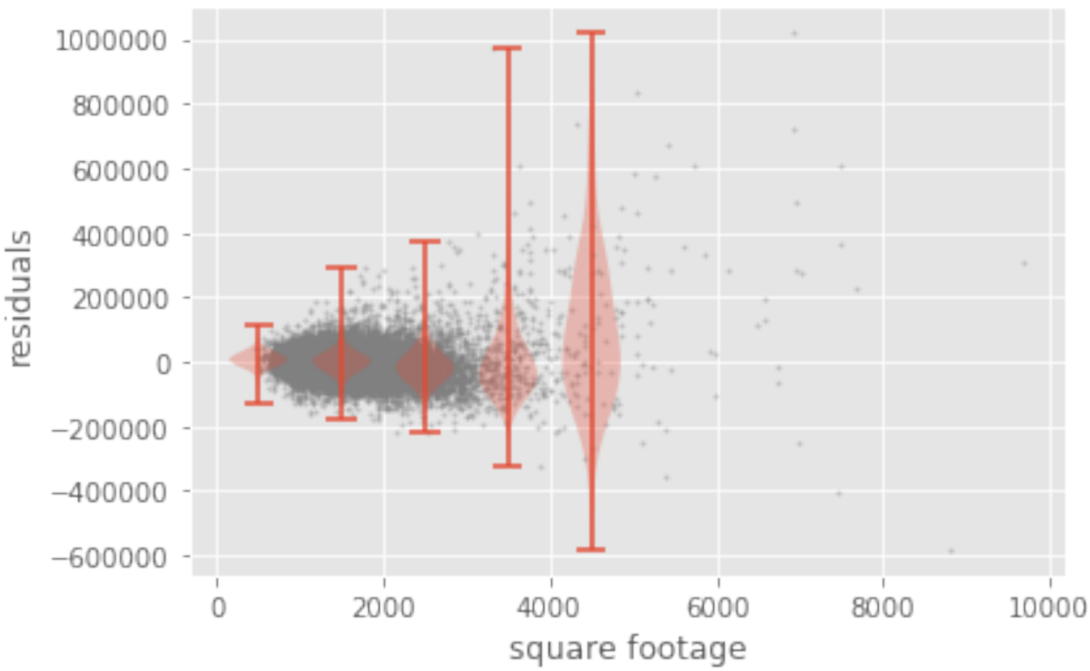
For the model above, are the residuals homoscedastic or heteroscedastic?

Answer. One possible solution is given below:

[13]:
```
Fin_sqft = data_clean["Fin_sqft"]
# scatter plot of the residuals:
plt.scatter(Fin_sqft, model_lin.resid, s=2, alpha=0.3, color="grey")
# violin plot of the same data, divided into 5 bins:
bins = np.array([0, 1000, 2000, 3000, 4000])
sqft_bin = np.digitize(Fin_sqft, bins)
plt.violinplot([model_lin.resid.values[sqft_bin == ibin] for ibin in␣
 ↪range(1,len(bins)+1)],
               positions=[500, 1500, 2500, 3500, 4500],
               widths=700,
               showextrema=True,
               )
```

```
plt.xlabel("square footage")
plt.ylabel("residuals")
```

[13]: Text(0, 0.5, 'residuals')



Answer. The violin plots show that the residuals get wider as the properties get larger, which demonstrates heteroscedasticity.

As a side note, heteroscedasticity is easy to show, but homoscedasticity is hard to demonstrate, because we have to show that *none* of the variables exhibit heteroscedasticity. In general, use your *intuition* to hunt for heteroscedasticity; here, it was perhaps foreseeable that small (and hence cheap) houses would have a more predictable sales price than large (expensive) houses.

1.4 Transforming variables to mitigate residuals issues (30 mts)

Now that we have seen that the residuals are problematic, we need to figure out what to do about them. One possible solution is to borrow the technique we learned in a previous case – taking a transformation of the response variable. Let's try making the response variable the logarithm of `Sale_price`. It is a natural assumption that the sale price should be proportional to the square footage of the property. Therefore, we will also take the logarithm of the square footage as a predictor variable in this model:

[14]:
```
model_log = smf.ols(formula = "np.log(Sale_price) ~ District + Units"
                              "+ np.log(Fin_sqft)",
                  data = data_clean).fit()
```

```
model_log.summary()
```

[14]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:     np.log(Sale_price)   R-squared:                       0.599
Model:                            OLS   Adj. R-squared:                  0.599
Method:                 Least Squares   F-statistic:                     2283.
Date:                Wed, 13 Nov 2019   Prob (F-statistic):               0.00
Time:                        17:58:57   Log-Likelihood:                -10020.
No. Observations:               24450   AIC:                         2.007e+04
Df Residuals:                   24433   BIC:                         2.021e+04
Df Model:                          16
Covariance Type:            nonrobust
================================================================================
====
                       coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
----
Intercept            5.2346      0.057     91.435      0.000       5.122
5.347
District[T.10]       0.6272      0.013     46.510      0.000       0.601
0.654
District[T.11]       0.8221      0.013     63.362      0.000       0.797
0.847
District[T.12]      -0.0038      0.021     -0.183      0.854      -0.045
0.037
District[T.13]       0.8019      0.013     59.694      0.000       0.776
0.828
District[T.14]       0.8701      0.013     64.855      0.000       0.844
0.896
District[T.15]      -0.4857      0.020    -24.462      0.000      -0.525
-0.447
District[T.2]        0.3132      0.015     20.827      0.000       0.284
0.343
District[T.3]        1.0223      0.015     66.953      0.000       0.992
1.052
District[T.4]       -0.1997      0.031     -6.464      0.000      -0.260
-0.139
District[T.5]        0.6896      0.013     53.242      0.000       0.664
0.715
District[T.6]       -0.1841      0.018    -10.103      0.000      -0.220
-0.148
District[T.7]       -0.0692      0.016     -4.272      0.000      -0.101
-0.037
```

```
District[T.8]         0.1567      0.017      9.050      0.000       0.123
0.191
District[T.9]         0.5799      0.016     36.764      0.000       0.549
0.611
Units                -0.3523      0.006    -55.450      0.000      -0.365
-0.340
np.log(Fin_sqft)      0.8687      0.008    104.173      0.000       0.852
0.885
==============================================================================
Omnibus:                     8475.447   Durbin-Watson:                   1.741
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           120216.866
Skew:                          -1.272   Prob(JB):                         0.00
Kurtosis:                      13.561   Cond. No.                         186.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

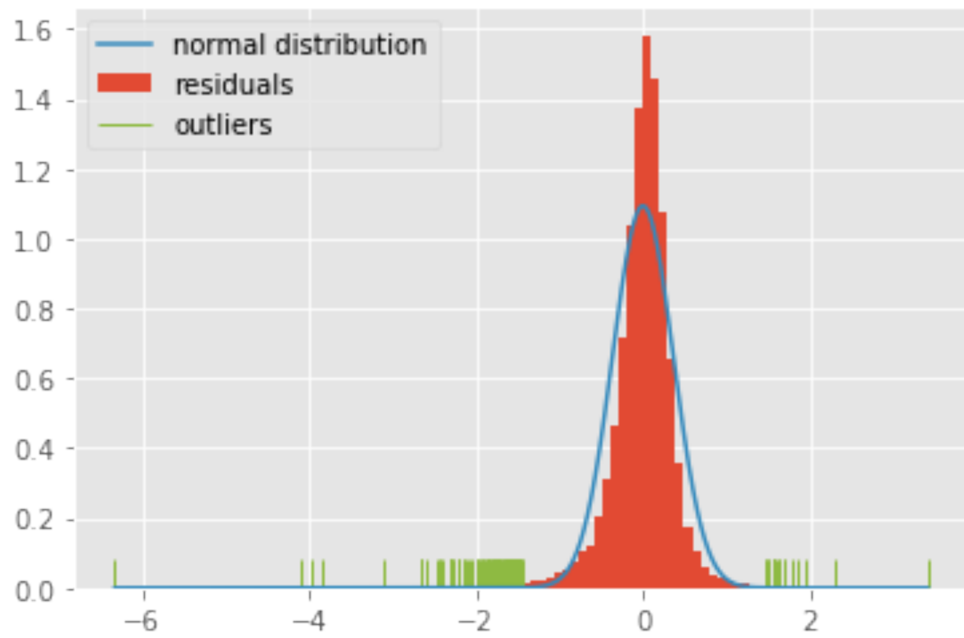
1.4.1 Exercise 5:

5.1 Plot a histogram of the residuals, similar to what we did in Exercise 3. Did the log transformation fix the skewness and outlier issues that we noticed earlier?

Answer. One possible solution is shown below:

[15]:
```python
plt.hist(model_log.resid,
    density=True,      # the histogram integrates to 1
                       # (so it can be compared to the normal distribution)
    bins=100,          #  draw a histogram with 100 bins of equal width
    label="residuals" # label for legend
    )
# now plot the normal distribution for comparison
xx = np.linspace(model_log.resid.min(), model_log.resid.max(), num=10000)
plt.plot(xx, scipy.stats.norm.pdf(xx, loc=0.0, scale=np.sqrt(model_log.scale)),
    label="normal distribution")
sns.rugplot(model_log.resid[np.abs(model_log.resid)>4*np.sqrt(model_log.scale)],
            color="C5", # otherwise the color was the same as the histogram
            label="outliers")
plt.legend(loc="upper left")
;
```

[15]:
```
''
```



Answer. Visually, it is not obvious that the residuals have improved. However, looking at the details paints a different picture. There are no fewer outliers, but many are now below the peak rather than above. Low outliers are easier to explain away as products of known external factors than high outliers; e.g. if the owner sells their house to a friend or relative, or the property is in particularly bad condition, then it could be sold at a severely reduced rate.

The residuals are also less skewed, and the tails of the distribution are less fat. This can be deduced from the `Skew` and `Kurtosis` metrics at the bottom of the summaries of the models (reproduced below for convenience):

[16]:
```
model_lin.summary().tables[-1]
```

[16]: <class 'statsmodels.iolib.table.SimpleTable'>

[17]:
```
model_log.summary().tables[-1]
```

[17]: <class 'statsmodels.iolib.table.SimpleTable'>

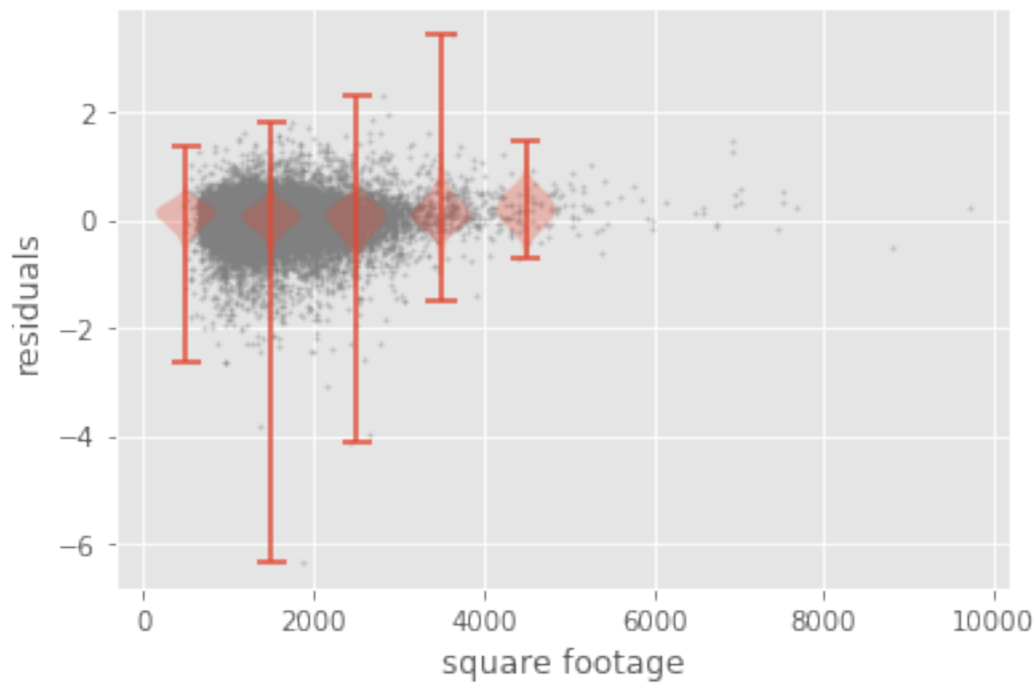
5.2 Did the heteroscedasticity issue improve from Exercise 4?

Answer. One possible solution is shown below:

[18]:
```
Fin_sqft = data_clean["Fin_sqft"]
plt.scatter(Fin_sqft, model_log.resid, s=2, alpha=0.3, color="grey")
bins = np.array([0, 1000, 2000, 3000, 4000])
sqft_bin = np.digitize(Fin_sqft, bins)
```

```
plt.violinplot([model_log.resid.values[sqft_bin == ibin] for ibin in␣
↪range(1,len(bins)+1)],
               positions=[500, 1500, 2500, 3500, 4500],
               widths=700,
               showextrema=True,
               )
plt.xlabel("square footage")
plt.ylabel("residuals")
;
```

[18]: ''



The distribution of the residuals as a function of square footage has stabilized, which supports the argument that the logarithmic scale is more natural to model property prices.

1.4.2 Exercise 6: (5 mts)

6.1 How do we interpret the `Units` coefficient?

Answer. All else being equal, an additional unit decreases the log-price of the property by 0.35, which corresponds to a slightly less than 30% reduction in price (this is calculated as $1 - e^{-0.35}$).

6.2 What story does the `np.log(Fin_sqft)` coefficient tell us in this model?

$c \cdot \log(x)$

$e^c \cdot e^{\log(x)} = x \cdot e^c$

$c<1$

(a) The price of a property is proportional to its square footage

(b) The price of a property goes down with square footage

(c) The price of a property goes up logarithmically with its square footage

(d) The price of a property goes up exponentially with its square footage

(e) The price of a property goes up with square footage, but there are diminishing returns

Answer. (e). The fitted coefficient is 0.87, which is positive but less than 1. In light of the fact that we have taken the logarithm of `Fin_sqft`, this implies *sublinear* growth; i.e. the price of a property goes up with square footage, but there are diminishing returns. Do you find this intuitive or surprising?

1.5 Dealing with outliers (30 mts)

Our analysis of residuals shows a number of outliers. As mentioned above, low outliers are not entirely unexpected; e.g. when people sell a property to friends or relatives, or if the property is in disrepair, the sale price could be significantly lower than expected by the model. Nonetheless, outliers can destabilize a model and significantly reduce their predictive ability despite only representing a fringe subset of the overall data.

1.5.1 Question: (5 mts)

What should an analyst do with outliers?

It is tempting to simply remove data points with large residuals; however, this viewpoint suffers from hindsight bias. Outliers with high residuals cannot be determined before the model is created, which means that they cannot be removed in advance.

Rather, the starting point should always be to inspect the initial raw data for any data points with unusually small or large values for certain features, and understand why they are different. If there is something clearly wrong with those data (like a misplaced decimal point), or if the reason for these small or large values can be effectively explained via an external factor that is not captured by the data itself, then this justifies removing them. Otherwise, it is best to keep these data points in mind throughout the modeling process, and deal with them during the modeling process itself.

1.5.2 Exercise 7: (10 mts)

Print the characteristics of the property with the greatest absolute residual (i.e. the worst outlier in the dataset). For this property, what is the fitted (predicted) and actual sale price?

Find all other sales of the same property in the dataset. Can you see an explanation for these extreme outliers?

Answer: The worst outlier is a house that our model predicted should sell for about \$56,000, and instead was sold for \$100. This same house was sold 8 months earlier for \$189,000, which confirms that \$100 is not the real market price. It could, for example, be a transaction between relatives.

[19]:
```python
iworst = np.abs(model_log.resid.values).argmax()
print("predicted sale price: ${:.0f}".format(
        np.exp(model_log.fittedvalues.iloc[iworst])))
data_clean.iloc[iworst] # select the worst row
```

```
predicted sale price: $56174
```

[19]:
```
PropType                 Residential
Taxkey                    3515031000
Address                1938 N 19TH ST
CondoProject                     NaN
District                          15
Nbhd                            3000
Style                       Colonial
Extwall             Aluminum / Vinyl
Stories                            2
Year_Built                      2010
Nr_of_rms                          0
Fin_sqft                        1860
Units                              1
Bdrms                              4
Fbath                              3
Hbath                              1
Lotsize                         5681
Sale_date        2010-10-01 00:00:00
Sale_price                       100
Name: 6484, dtype: object
```

[20]:
```python
# show all sales for this property
data_clean.loc[data_clean.Taxkey == data_clean.iloc[iworst].
 ↪Taxkey,["Sale_date","Sale_price"]]
```

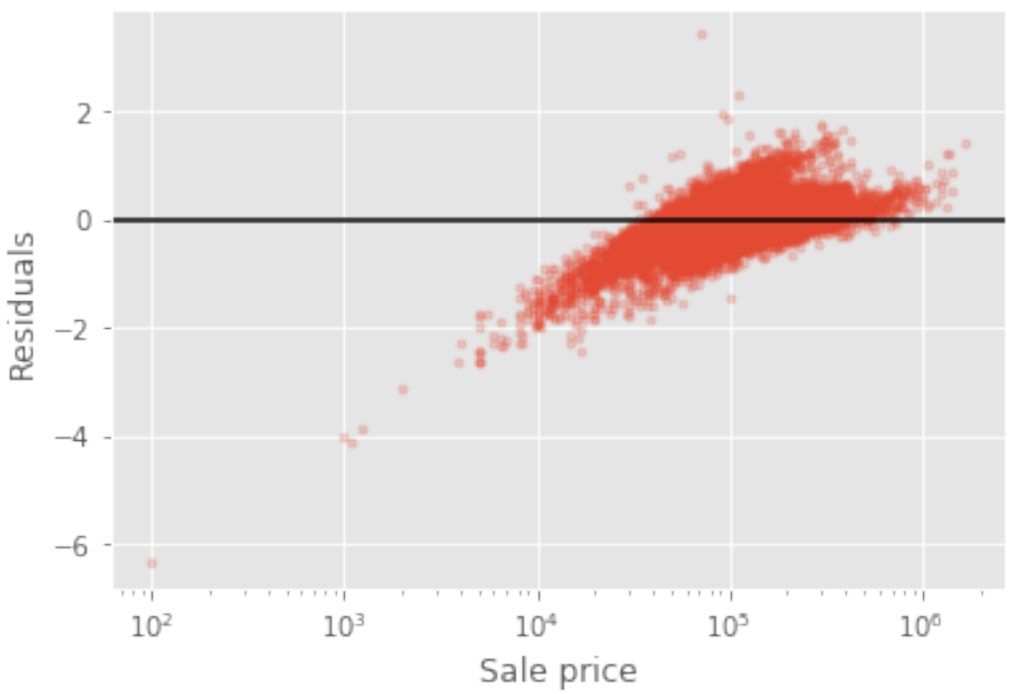
[20]:

	Sale_date	Sale_price
4990	2010-02-01	189000
6484	2010-10-01	100

The scatter plot below shows the residuals vs. the sale price for all properties in the data. The cluster in the bottom left corner shows properties that sold at a much lower price than expected by the model, which can be explained as transactions below market rate between friends or relatives. This would justify removing all transactions below about $2,000 from the dataset before re-running the analysis:

[21]:
```python
plt.semilogx(data_clean["Sale_price"], model_log.resid, ".", alpha=0.2)
plt.ylabel("Residuals")
plt.xlabel("Sale price")
plt.axhline(0, color="black")
```

[21]:
```
<matplotlib.lines.Line2D at 0x11eb28d68>
```



We can quantify the effect of these outliers by removing them and re-running the regression, and seeing how the fitted coefficients change.

1.5.3 Exercise 8: (15 mts)

8.1 Re-run the same model one a reduced dataset which removes all outliers with absolute residuals greater than 1.5.

1. What percentage of the data is removed?
2. What is the value of the fitted `Units` coefficient before and after removing outliers?
3. Is the difference significant?
4. What coefficients move by more than two standard errors after removing outliers?

Answer. One possible solution is given below:

[22]:
```
# Removing outliers with residuals greater than 1.5
# results in a noticably different `Units` coefficient,
# even though only 0.5% of observations are removed
outliers = np.abs(model_log.resid) < 1.5
data_noout = data_clean.loc[outliers, :]
# refit a model with the reduced data
model_noout = smf.ols(formula = "np.log(Sale_price) ~ District + Units"
                                "+ np.log(Fin_sqft)",
                      data = data_noout).fit()
# inspect the `Units` coefficient
```

```
print("{:.2f}% of data removed".format((1-data_noout.size/data_clean.size)*100))
print("`Units` coefficient in full model:          {:.3f}".format(model_log.
 ↪params["Units"]))
print("`Units` coefficient with excluded outliers: {:.3f}".format(model_noout.
 ↪params["Units"]))
```

```
0.56% of data removed
`Units` coefficient in full model:          -0.352
`Units` coefficient with excluded outliers: -0.382
```

[23]:
```
# how many SE's does the coefficient move by:
move = (model_log.params-model_noout.params)/model_log.bse
move[np.abs(move)>2] # coefficients that move by more than 2 SE's
```

[23]:
```
District[T.15]   -2.044336
District[T.6]    -3.645634
Units             4.660283
dtype: float64
```

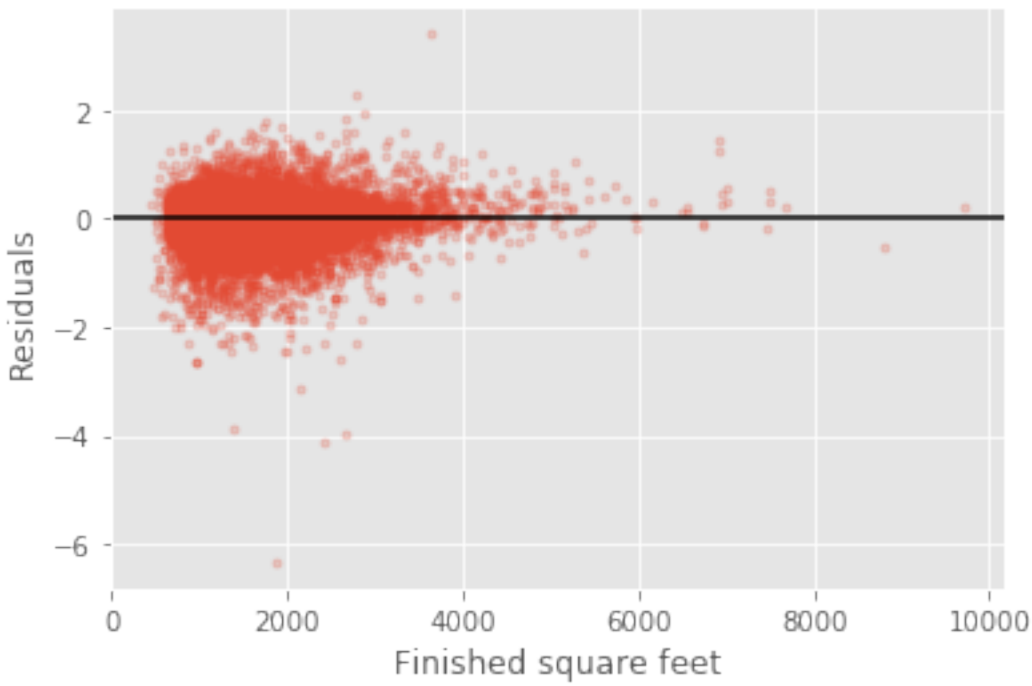
This procedure only removes around 0.5% of the data, but the `Units` coefficient changes significantly from -0.352 to -0.382. This may not seem like much, but the standard error of this coefficient was 0.006, so the estimate has shifted by almost *five standard errors*, which is indeed a significant change.

8.2 Check whether particularly small or large properties also tend to result in outliers.

Answer. No. As one can see from the residuals plot below, there isn't a notable cluster of properties anywhere far away from the zero line:

[24]:
```
plt.plot(data_clean["Fin_sqft"], model_log.resid, ".", alpha=0.2)
plt.ylabel("Residuals")
plt.xlabel("Finished square feet")
plt.axhline(0, color="black")
```

[24]:
```
<matplotlib.lines.Line2D at 0x11d45cd30>
```



The bulge in the center of the scatterplot of residuals above may look like heteroscedasticity. However, this effect can appear simply because a lot of properties are between 1,000 to 2,000 square feet, which causes the scatter plot to "bulge out" in the middle. You can construct a violin plot of the same data to dispel any doubts about this claim.

1.6 Conclusions (5 mts)

We used linear regression to predict property sale price in Milwaukee, Wisconsin, and illustrated some potential issues with linear regression. When some of the assumptions of linear regression are severly violated, like normality of the residuals and homoescedasticity, the fitted model can be destabilized. As an example, we saw how a small number of outliers can shift a fitted coefficient by several standard errors.

1.7 Takeaways (5 mts)

To fix some of the issues detected in the analysis of residuals, transforming the outcome variable can be helpful. Here, modeling the logarithm of the sale price reduced the skewness of the residuals, reduced the severity of outliers (the thickness of the tails of the residuals), and largely eliminated heteroscedasticity as a function of the size of the property. Other datasets may invite different transformations. You should combine intuition and experimentation to find the right transformation.