Do there exist significant differences between the balances of my various customers' cohorts?

Introduction

Business Context. You are leading a business analytics unit in a bank and have been asked to support the marketing unit to conduct a customer segmentation analysis. You are provided with a dataset comprising a sample of customers, their bank account balances, and some demographic information about them. Different populations across the country have different income levels and may have different spending profiles. Your marketing team wants to know if there are significant differences in the bank balances of different subsegments of your customer base, so that they can design targeted products for different groups.

Business problem. The marketing department wants you to determine: **"Do there exist statistically significant differences in the bank balances of your customer segments (by age, job, education, marital status, etc.)?"**

Analytical Context. The dataset that we will use in this case was retrieved from Kaggle (https://www.kaggle.com/skverma875/bank-marketing-dataset). In this case, we will take the first step towards transitioning from **exploratory data analysis** to **confirmatory analysis**. We will: (1) learn a formal framework for hypothesis testing; (2) learn about p - values; (3) generate a hypothesis from exploratory data analysis; and finally (4) analyze the results of a hypothesis test.

In [2]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as sm
from statsmodels.formula.api import ols
```

```python
import statsmodels
from scipy import stats
from pingouin import pairwise_ttests #this is for performing the pairwi
se tests
```

Data Exploration

The data includes the account balance of 45,211 customers, along with several other characteristics:

In [3]:
```python
bank = pd.read_csv("bank-full.csv")
bank.shape
```

Out[3]: (45211, 17)

In [4]:
```python
bank.head()
```

Out[4]:

	age	job	marital	education	default	balance	housing	loan	contact	day	month
0	58	management	married	tertiary	no	2143	yes	no	unknown	5	may
1	44	technician	single	secondary	no	29	yes	no	unknown	5	may
2	33	entrepreneur	married	secondary	no	2	yes	yes	unknown	5	may
3	47	blue-collar	married	unknown	no	1506	yes	no	unknown	5	may
4	33	unknown	single	unknown	no	1	no	no	unknown	5	may

The relevant features we will use in this case are:

- **balance:** bank balance; key variable of interest
- **job:** the title of the job (“management”, “technician”, etc)
- **marital:** marital status ("single", "married" or "divorced")
- **education:** different levels of education ("primary","secondary","tertiary","unknown")

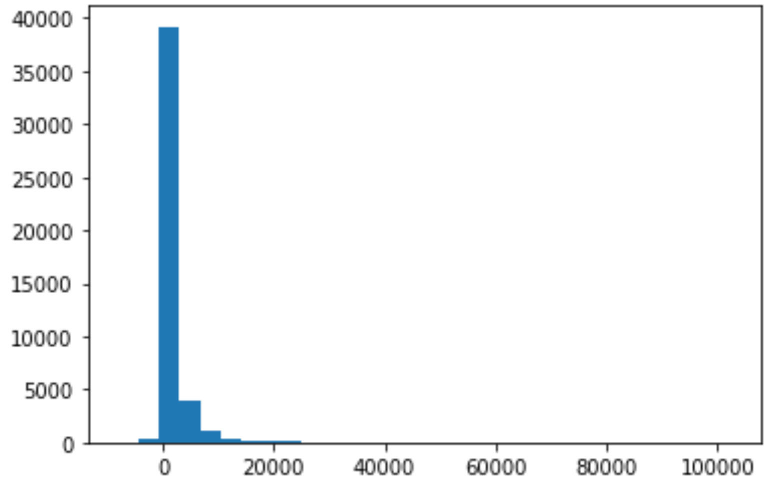
- **default:** the customer defaulted ("yes" or "no")
- **loan:** the customer took out a loan ("yes" or "no")

In [5]:
```
bank.describe()
```

Out[5]:

	age	balance	day	duration	campaign	pdays	
count	45211.000000	45211.000000	45211.000000	45211.000000	45211.000000	45211.000000	452
mean	40.936210	1362.272058	15.806419	258.163080	2.763841	40.197828	
std	10.618762	3044.765829	8.322476	257.527812	3.098021	100.128746	
min	18.000000	-8019.000000	1.000000	0.000000	1.000000	-1.000000	
25%	33.000000	72.000000	8.000000	103.000000	1.000000	-1.000000	
50%	39.000000	448.000000	16.000000	180.000000	2.000000	-1.000000	
75%	48.000000	1428.000000	21.000000	319.000000	3.000000	-1.000000	
max	95.000000	102127.000000	31.000000	4918.000000	63.000000	871.000000	27

In [6]:
```
plt.hist(bank['balance'],bins = 30);
```



Exercise 1:

From the tables and histogram above, we see a huge range in the bank balance of your customers. Why do you think this is?

Answer: There are many subgroups in our population. For example, there are people with varying degrees of education, job profiles, etc. Thus we will expect these subgroups to have different bank balances.

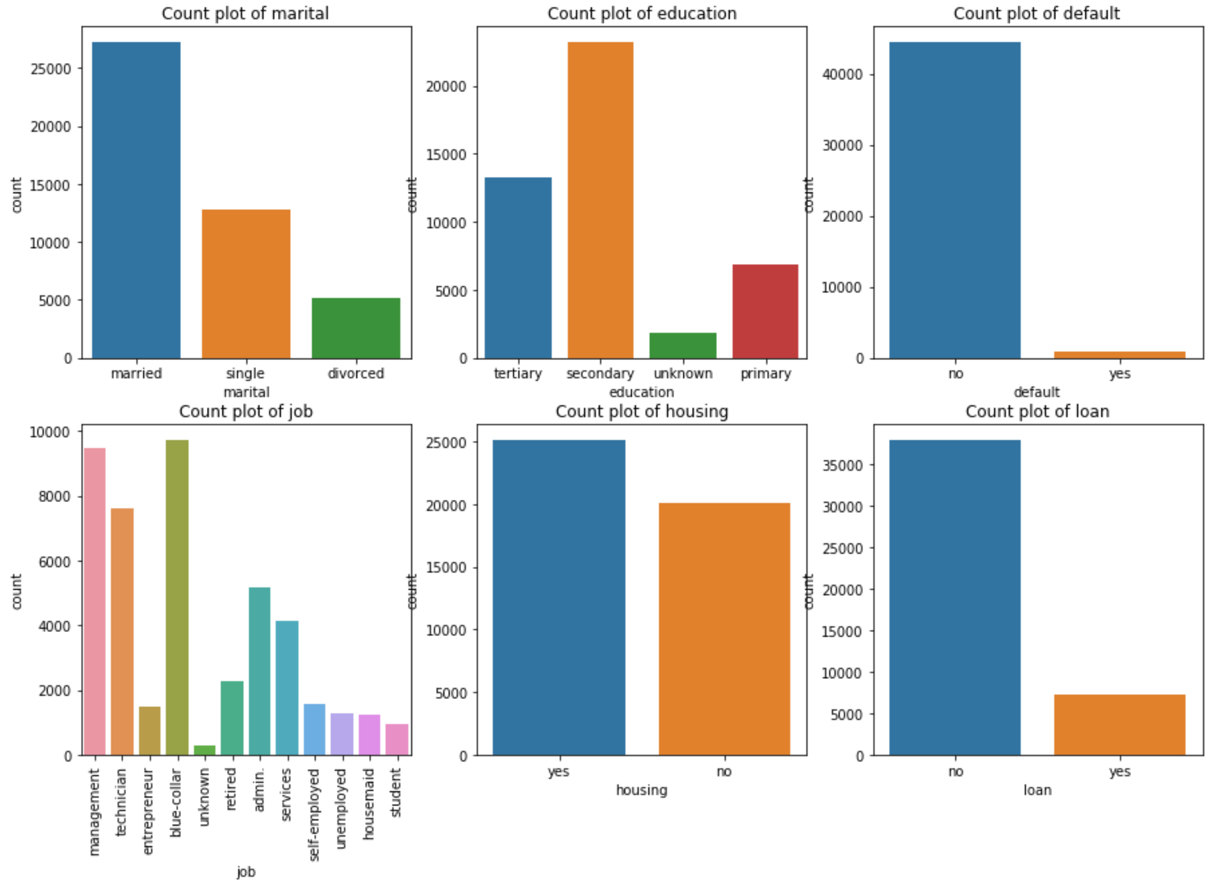
Let us understand our data further by plotting the variables.

Exercise 2:

Consider the variables `job`, `marital`, `education`, `default`, `housing`, `loan`. For each variable, write code to plot the number of customers in each category of that variable, using the command `sns.countplot()`.

Answer. One possible solution is given below:

In [7]:
```
plt.figure(figsize=(15,10))
vars_to_look = ['marital','education','default','job','housing','loan']
for i, var in enumerate(vars_to_look):
    plt.subplot(2,3,i+1)
    if i ==3:
        plt.xticks(rotation = 90)
    sns.countplot(bank[var])
    plt.title("Count plot of " + var)
```



Exercise 3:

Last year the average balance for the entire population of the bank's customers was $1341.12. How much higher is the sample average balance this year? Is this difference significant?

Answer.

In [8]:
```
bank['balance'].mean()
```

Out[8]:
```
1362.2720576850766
```

This is a difference of $1362.27 - \$1341.12 = $22.15. The bank management thinks that the consumer behavior might have changed slightly, and that, on average, customers are keeping more money in their bank account.

However, this difference might be due to statistical variation because of sampling variability. We would need to conduct a more rigorous test to determine if this difference is actually significant after factoring in such variability.

Analytical framework for hypothesis testing

The procedure that we use to help us decide whether a difference between the mean μ of a population and a reference value μ_0 is **statistically significant** is called **hypothesis testing**. In our context, μ is the average bank balance of the customers this year and μ_0 is their average bank balance last year.

The first step is defining the null hypothesis (often indicated as H_0). The null hypothesis **always** corresponds to the hypothesis of no change; that is, the status quo is still valid. Formally, this is written as: $H_0 : \mu = \mu_0$.

In our case, H_0 would be equivalent in hypothesizing that the average balance for the entire customer population μ is the same as the average balance last year $\mu_0 = \$1341.12$. We wish to test if H_0 is wrong; that is, if μ is different from $\mu_0 = \$1341.12$.

Question:

We said before that the null hypothesis **always** corresponds to the hypothesis of no change; that is, the status quo is still valid. Why is this? Why didn't we set up the null hypothesis to be $H_0 : \mu > \mu_0$?

Answer: This is just convention; no other reason.

Alternative Hypothesis

In opposition to the null hypothesis, we define an alternative hypothesis (often indicated with H_1 or H_a) to challenge the status quo. We can have three different ways to define an alternative hypothesis:

1. $H_a : \mu \neq \mu_0$ (two-sided test)
2. $H_a : \mu > \mu_0$ (one-sided test)
3. $H_a : \mu < \mu_0$ (one-sided test)

The statistical test will help us decide if there is enough evidence to reject the null hypothesis in favor of an alternative.

Conducting a hypothesis test

Returning to our case, suppose we wish to perform a statistical test to assess the hypothesis of management:

$$H_0 : \mu = 1341.12$$
$$H_a : \mu \neq 1341.12$$

There are two possible outcomes for this test: (1) We conclude H_0 is false, and say we **reject** H_0. In this case we will conclude that there is statistical evidence for the alternative H_a and that the bank balance of customers this year is indeed different from 1341.12 USD. Or (2) we **fail to reject** H_0. In this case, we conclude that there is not enough statistical evidence to say for sure that H_0 is false. **Notice that in the second case we cannot say that the original hypothesis is true.** (In fact, there is no test out there that will tell you that a hypothesis is true. Why do you think that is?)

The following command lets us run this test:

In [9]:
```
stats.ttest_1samp(bank['balance'], popmean=1341.122)
```

Out[9]:
```
Ttest_1sampResult(statistic=1.4769973489267905, pvalue=0.13968331332845
219)
```

Reading the output of a statistical test: p - values

Statistical tests report a p **- value**. This is the key quantity that we will use to determine if the outcome of the test was significant. Let's introduce another quantity α which we will call the significance level; this will be explained later. For now, set $\alpha = 0.05$.

We can have two outcomes:

1. If this probability is smaller than our significance level ($p < \alpha$) we reject H_0 and we claim that the observed difference is "statistically significant".
2. If this probability is greater than our significance level ($p > \alpha$) we have to retain H_0 and we claim that the observed difference is not statistically significant.

Question:

What do you conclude from the output? Do we reject H_0 or retain it?

Answer. Since the p - value (0.13968) is larger than $\alpha = 0.05$ we have to retain the null hypothesis.

Question:

What change do we have to make if the alternative was one-sided?

Answer. Suppose that our alternative was one-sided: $H_a : \mu > \mu_0$ (one-sided test) or $H_a : \mu < \mu_0$ (one-sided test). In this case, just run the two-sided test and then divide the p - value by half.

In [10]:
```
T,p = stats.ttest_1samp(bank['balance'], popmean=1341.122)
p_value = p/2
```

Exercise 4:

We retained H_0 above. Are we 100% sure this decision is correct? Why or why not?

Answer: Since statistical hypothesis testing procedures aim to infer about an unknown population parameter using the information contained in a sample, it will not always lead us to the correct decision. However, as you will learn later, the p - value gives us a good sense of the degree of confidence we can have in our conclusion being correct. For most applications, being at least 95% confident will be sufficient.

Errors due to a wrong conclusion from a hypothesis test

There are two ways that a test can lead us to an incorrect decision:

1. When H_0 is true and we reject it. This is called **Type 1 Error**. It corresponds to obtaining a **false positive**.
2. When H_0 is false and we do not reject it. This is called **Type 2 Error**. It corresponds to having a **false negative**.

	H_0 **is true**	H_0 **is False**
Reject H_0	Type I error	Correct Decision (True Positive)
Fail to Reject H_0	Correct Decision (True negative)	Type II error

In general, we cannot control both the Type I and Type II error. So the type of an error we control depends on the situation.

Exercise 5:

Discuss the following two scenarios with your teammates:

1. A patient is getting a diagnostic test for finding out if they are infected with HIV virus. What is the null hypothesis here? What is more serious here: making a Type I error (false positive) or Type II error (false negative)? Why?
2. You are getting a lot of spam emails, so you are writing a spam filter algorithm to detect whether an email is spam or not. What is the null hypothesis here? What is more serious here: making a Type I error (false positive) or Type II error (false negative)? Why?

Answer.

1. H_0: the person does not have HIV. Making a type II error is serious here as this could lead to a severely shortened life expectancy if they do not receive the proper treatments they need.
2. H_0: the email is not spam. Making a type I error is more serious here. Type I error means that you falsely conclude a non-spam email to be spam. This could hurt you since you may miss an important email from your boss.

Controlling the Type I error: significance level

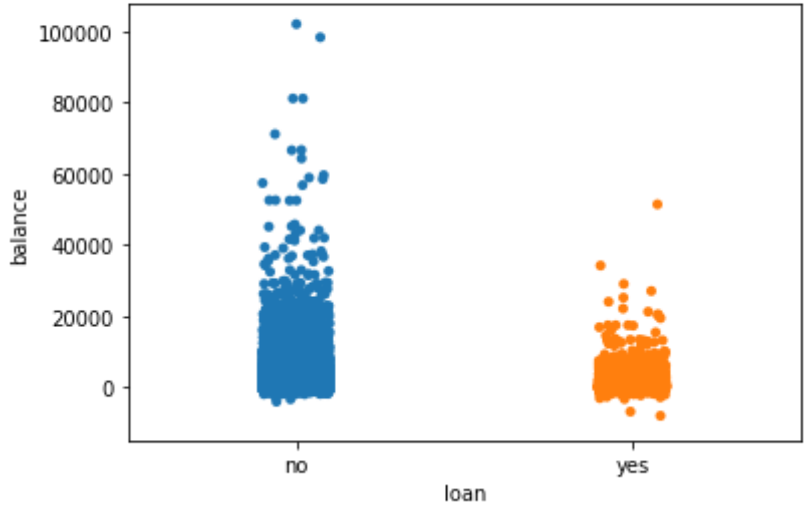
It is standard practice to minimize the probability of making a Type I error. The probability, usually denoted by α, is the significance level we introduced above. Typically we choose our significance level α to be 5%. Thus, if we reject H_0 with $\alpha = 0.05$, then there is only a 5% chance that the conclusion we make is a false positive. Lowering the α value (say to 1%) will decrease the probability of making a false positive conclusion.

Of course, as we discussed before, because we control α, we cannot control the Type II error we make.

Finding cohorts within your customer base

The goal of your team is to determine whether there are differences among different groups of customers in terms of bank balances. We start by looking at balances for two cohorts: those who took out a loan and those who didn't:

In [11]:
```
ax = sns.stripplot(x="loan", y="balance", data=bank)
plt.ylabel('balance')
plt.show()
```



Since the distribution of the data has heavy tails, it is hard to tell much from the strip plot whether there is a difference between the two subgroup means. We continue our data analysis by looking at summary statistics for each of the two groups:

In [12]:
```
bank[bank.loan=="yes"].balance.describe()
```

Out[12]:
```
count     7244.000000
mean       774.309912
std       1908.283253
min      -8019.000000
25%          2.000000
50%        258.000000
75%        864.250000
max      51439.000000
Name: balance, dtype: float64
```

In [13]:
```
bank[bank.loan=="no"].balance.describe()
```

Out[13]:
```
count     37967.000000
mean       1474.453631
std        3204.088951
min       -4057.000000
25%          94.000000
50%         496.000000
75%        1558.000000
max      102127.000000
Name: balance, dtype: float64
```

Are the means of the loan and no loan groups significantly different? (5 mts)

We would like to test statistically whether the two group means are different from each other; that is, whether the difference between the mean balance in the groups with a loan (μ_1) is different than the mean balance in the group with no loan (μ_2). The testing procedure that we described can be used also to answer this question:

$$H_0 : \mu_1 = \mu_2$$
$$H_a : \mu_1 \neq \mu_2$$

We get the following output:

In [14]:
```
statistic, pvalue = stats.ttest_ind(bank[bank.loan=="yes"].balance, ban
k[bank.loan=="no"].balance, equal_var=False)
statistic, pvalue
```

Out[14]:
```
(-25.18086057755715, 2.7640564777544156e-137)
```

Question:

What would you conclude from the above test?

Answer: The p - value is very small (p = 2.76 e-1376); hence, we reject the null hypothesis that the two groups have the same mean.

Differences by education:

In our search to determine what factors are important in customer segmentation, we can identify in this large dataset if education plays a role in customer behavior. The variable `education` has 4 categories:

In [15]:
```
bank["education"].unique()
```

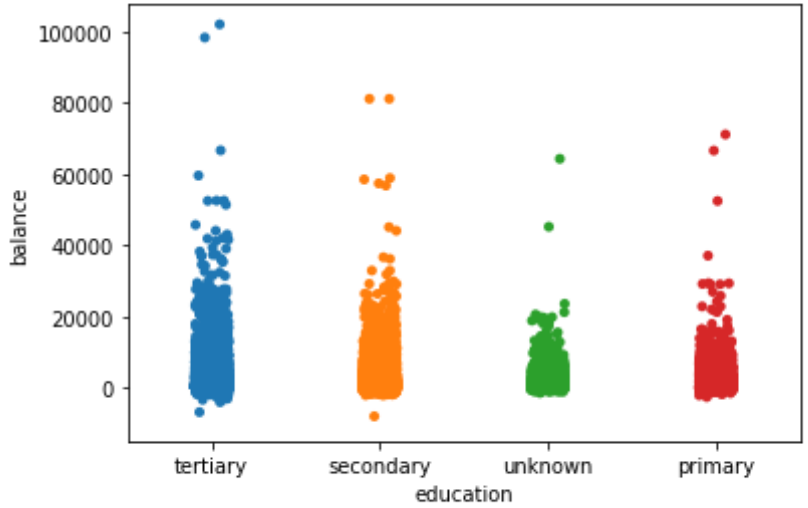
Out[15]:
```
array(['tertiary', 'secondary', 'unknown', 'primary'], dtype=object)
```

Exercise 6:

Write code to visualize the balance by education via a strip plot (i.e. grouped 1D scatterplot). From your visual exploration, do you think there is a difference across the groups?

Answer. One possible solution is given below:

In [16]:
```
ax = sns.stripplot(x="education", y="balance", data=bank)
plt.ylabel('balance')
plt.show()
```



Answer. The standard visual representation in this case does not provide too much insight; thus, we need an alternative method to investigate further.

Do bank balances differ significantly across education cohorts?

Our new hypothesis will be:

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$
$$H_a : \text{At least one of the means } \mu_j \text{ is different from the others.}$$

To test this hypothesis we need an extension of the capabilities of the t - test (which can test only two groups at the same time). This test is called **Analysis of Variance (ANOVA)**.

In [17]:
```
mod = ols('balance ~ education', data=bank).fit()
aov_table = sm.stats.anova_lm(mod, typ=2)
aov_table
```

Out[17]:

	sum_sq	df	F	PR(>F)

	sum_sq	df	F	PR(>F)
education	3.220417e+09	3.0	116.682074	2.849538e-75
Residual	4.159034e+11	45207.0	NaN	NaN

As we can see, looking at the p - value that accompanies the F - statistics, we obtain a strong rejection of the null hypothesis, leading us to conclude that education groups have some differences in their means. The ANOVA test does not tell us which pair of groups have means that are different from each other. To investigate these differences further, we are going first to report the descriptive statistics by group, and then display the group means in a bar chart:

In [18]:
```
bank[bank.education=="primary"].balance.describe()
```

Out[18]:
```
count     6851.000000
mean      1250.949934
std       2690.743991
min      -2604.000000
25%         61.000000
50%        403.000000
75%       1390.000000
max      71188.000000
Name: balance, dtype: float64
```

In [19]:
```
bank[bank.education=="secondary"].balance.describe()
```

Out[19]:
```
count    23202.000000
mean      1154.880786
std       2558.256739
min      -8019.000000
25%         55.000000
50%        392.000000
75%       1234.000000
max      81204.000000
Name: balance, dtype: float64
```

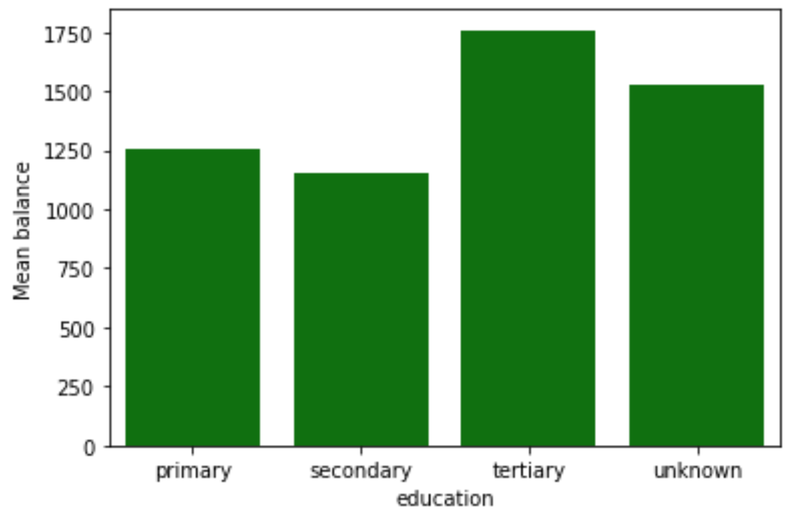
In [20]:
```
bank[bank.education=="tertiary"].balance.describe()
```

Out[20]:
```
count     13301.000000
mean       1758.416435
std        3839.088305
min       -6847.000000
25%         104.000000
50%         577.000000
75%        1804.000000
max      102127.000000
Name: balance, dtype: float64
```

In [21]:
```
bank[bank.education=="unknown"].balance.describe()
```

Out[21]:
```
count     1857.000000
mean      1526.754443
std       3152.228273
min      -1445.000000
25%        106.000000
50%        568.000000
75%       1699.000000
max      64343.000000
Name: balance, dtype: float64
```

In [22]:
```
mean_balance_education=bank.groupby(by="education").balance.mean()
sns.barplot(x=mean_balance_education.index, y=mean_balance_education.va
lues, color="green")
plt.ylabel("Mean balance")
plt.show()
```



Qualitatively there seems to be a big difference between tertiary and lower education levels, but the difference between primary and secondary education groups is more difficult to assess. We want to test if there is a significant difference in each pairwise comparison. To test this sequence of hypotheses we can use a variant of the t - test, called a **pairwise t - test**.

In [23]:
```
gb = bank.groupby(['education'])
group_names = bank["education"].unique()
print("\t\t\tstatistic\t\tpvalue")
for i in range(len(group_names)):
  for j in range(i+1, len(group_names)):
    group1 = gb[["balance"]].get_group(group_names[i])
    group2 = gb[["balance"]].get_group(group_names[j])
    stat, pvalue = stats.ttest_ind(group1, group2, equal_var = False)
    print(group_names[i] + " vs. " + group_names[j] + "\t" + str(stat[0
]) + "\t" + str(pvalue[0]))
```

```
                        statistic               pvalue
tertiary vs. secondary  16.18717929985782       1.4525474790099814e-58
tertiary vs. unknown    2.8825321831203117      0.003976238237673688
tertiary vs. primary    10.906634331433306      1.3023845080432282e-27
secondary vs. unknown   -4.95482328681934       7.83111710943088e-07
```

```
secondary vs. primary   -2.6255174227477864     0.0086639693661419
unknown vs. primary     3.445496944400963       0.0005789323657180873
```

The summary output above signals that there is a significant difference between each pairwise comparison.

Exercise 7:

There are six comparisons in total that have to be conducted because the variable education has 4 levels. Discuss with a partner if, in your opinion, performing multiple tests simultaneously can increase the risk of erroneous inferences. What will increase: Type I error or Type II error?

Answer. Yes, the more comparisons we run simultaneously, the higher the risk of observing some "false positives" or making a Type I error. Basically, if your torture the data enough, it will confess.

Multiple comparisons: the Bonferroni correction

There are many ways to correct this issue, known as the "multiple comparisons" problem. One of the most classical methods is the Bonferroni correction. This method divides the significance level α by the number of multiple comparisons being performed (6 in this case). So if our $\alpha = 0.05$ we will reject the null hypothesis only if the p - value is less than 0.00833.

Notice that the p - value for the comparison between "primary" and "secondary" education is 0.02126, and therefore higher than the adjusted confidence level. After the correction we cannot reject the null hypothesis that this pair has significantly different means. Several software packages directly adjust the p - values for the increased probability of observing a false positive. If we perform this method for `education`, we get the following output:

In [24]:
```
pairwise_results = pairwise_ttests(dv='balance', between = ['education'
], padjust='bonf', data=bank)
pairwise_results
```

Out[24]:

	Contrast	A	B	Paired	Parametric	T	dof	Tail	p-unc
0	education	tertiary	secondary	False	True	16.187179	20183.403282	two-sided	1.452547e-58
1	education	tertiary	unknown	False	True	2.882532	2688.204879	two-sided	3.976238e-03
2	education	tertiary	primary	False	True	10.906634	18353.418018	two-sided	1.302385e-27
3	education	secondary	unknown	False	True	-4.954823	2056.380989	two-sided	7.831117e-07
4	education	secondary	primary	False	True	-2.625517	10768.241876	two-sided	8.663969e-03
5	education	unknown	primary	False	True	3.445497	2633.681528	two-sided	5.789324e-04

In the above table, `p-unc` stands for the uncorrected p - value, and `p-adjust` stands for the corrected p - value. The adjusted p - value for "secondary vs. primary" is 0.052 (greater than 0.05), confiriming our calculation. We thus reject all of the null hypotheses except for the "secondary vs. primary" case.

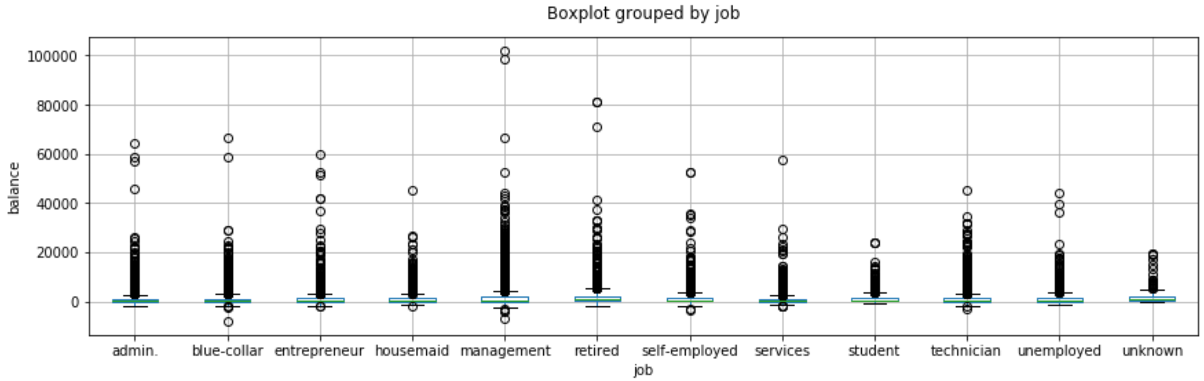
Exercise 8:

Perform an exploratory data analysis of the bank balance by job title. Write code to answer the following two questions:

1. Are the group means the same? Write a hypothesis for this, clearly stating what H_0 is.
2. Do a pairwise hypothesis test for comparing the different group means, applying the Bonferroni correction.
3. Do you find any of the results of hypothesis tests of customers in management vs. other groups surprising?

Answer. One possible solution is given below:

In [25]:
```
boxplot = bank.boxplot(column=['balance'], by="job",figsize=(14,4))
boxplot.axes.set_title("")
plt.ylabel('balance')
plt.show()
```



In [26]:
```
plt.figure(figsize=(14,4))
ax = sns.stripplot(x="job", y="balance", data=bank)
plt.ylabel('balance')
plt.show()
```

In [27]:
```
mod = ols('balance ~ job', data=bank).fit()
aov_table = sm.stats.anova_lm(mod, typ=2)
```

```
aov_table
```

Out[27]:

	sum_sq	df	F	PR(>F)
job	4.341414e+09	11.0	43.007783	5.709430e-94
Residual	4.147824e+11	45199.0	NaN	NaN

In [28]:
```
pairwise_job_results = pairwise_ttests(dv='balance', between = ['job'],
 padjust='bonf', data=bank)
pairwise_job_results[pairwise_job_results['A']=='management']
```

Out[28]:

	Contrast	A	B	Paired	Parametric	T	dof	Tail	
0	job	management	technician	False	True	10.429971	16520.443985	two-sided	2.169
1	job	management	entrepreneur	False	True	2.111899	1902.921967	two-sided	3.482
2	job	management	blue-collar	False	True	15.083784	15180.499966	two-sided	4.851
3	job	management	unknown	False	True	-0.048726	316.659804	two-sided	9.611
4	job	management	retired	False	True	-2.196665	3131.535808	two-sided	2.811
5	job	management	admin.	False	True	11.667427	13855.652407	two-sided	2.619
6	job	management	services	False	True	14.825903	12787.547265	two-sided	2.556
7	job	management	self-employed	False	True	1.148353	2184.520136	two-sided	2.509
8	job	management	unemployed	False	True	2.530687	1875.469495	two-sided	1.146
9	job	management	housemaid	False	True	3.973197	1818.298260	two-sided	7.368

	Contrast	A	B	Paired	Parametric	T	dof	Tail	
10	job	management	student	False	True	4.225002	1439.557060	two-sided	2.539

The account balance of unemployed customers and management customers is not significantly different, and this is surprising!

Conclusions

After doing exploratory data analysis, we formally introduced hypothesis tests. We saw that education level definitely affects bank balance; customers with a tertiary education seem to have a statistically significant difference in bank balance compared to the rest of the population. However, after adjusting for multiple testing, there does not seem to be a statistically significant difference between customers with a primary and secondary education. We also saw quite a few statistically significant differences in the bank balances of customers with different job profiles.

Takeaways

Customer segmentation requires analyzing differences among different subgroups of customers. Observed differences do not necessarily correspond to statistically significant differences. Hypothesis testing can help us identify differences that are too extreme to happen at random.

However, hypothesis testing is not just a rote application of rules; we have to be mindful about the multiple comparisons issue. This is where the "monkeys on a typewriter" problem comes from. A Bonferroni correction goes a long way towards mitigating the problem of observing too many false positive results while doing a large number of hypothesis tests.

In [ ]: