Physics 231

<u>Outline for Day 1</u>
Attendance and a list of units
Discuss syllabus
Units & Measurements

P231 Week 1: measurements

Goals of Week 1:

- Learn about base and derived units
- Learn dimensions and dimensional analysis
- Understand measurements and their errors
- Learn how to propagate errors
- Understand how μ, σ, and σ_{μ} are related to measurements and errors

Units

Base Units

Mechanical

Quantity	**MKS unit**	**cgs unit**
mass	kg (kilogram)	g
length	m (meter)	cm
time	s (second)	s

Other

Quantity	**MKS unit**
temperature	K (Kelvin)
current	A (amps)
amount of matter	mol (mole)
luminous intensity	cd (candela)

Derived Units

miles/hour
km/s
mol/liter
kg m/s^2
etc.
etc.
etc.

Dimensions

dimension: "the manifoldness with which the fundamental units of time, length, and mass are involved in determining the units of other physical quantities."
dimension: a measureable quantity

For mechanical base units …

Mechanical

Quantity	**Dimension**
mass	M
length	L
time	T

For some derived units …

[miles/hours] =	L/T
[km/s] =	L/T
[knot] =	L/T
[L (liter)] =	L^3
[kg m/s²] =	ML/T^2

Dimensional Analysis

- *a way to figure out if an equation is correct*
- *allows you to decide which equation to use.*

Ex. 1) Is this equation dimensionally correct?

$$ma = \tfrac{1}{2}^{\dagger} mv^2$$

where m=mass, v=speed, a=acceleration=L/T^2

Soln: $[ma]=ML/T^2$ and $[\tfrac{1}{2} mv^2]=ML^2/T^2$
since $ML/T^2 \neq ML^2/T^2$ the equation cannot be correct.

Ex. 2) Is this equation dimensionally correct?

$$y = at^2$$

where y=position (L), t=time, a=acceleration=L/T^2

Soln: $[y]=L$, $[at^2]=L/T^2 * T^2=L$.
since L = L, the equation is dimensionally correct.
However, the equation is still wrong! How?

† 1/2 is a dimension less constant

Dimensional Analysis (cont)

Ex. 3) How long does it take to drive 20 miles (to Lima) at a constant 60 mph?

Soln: Let v=speed (L/T), d=distance (L) and t=time (T).

Possible (linear) equations: t=v*d, t=v/d, t=d/v

check dimensions: L^2/T $1/T$ T

so: t=d/v = 20/60 = 1/3 hr or 20 minutes.

Measurements

measurement: the act or result of measuring

Example: use a plastic ruler to measure a shoe's length
to be $L=12.0\pm0.1$ inches.
Example: use a Vernier caliper to measure the same shoe length
to be $L=12.13\pm0.04$ inches.
Notice:
•A measurement consists of a *number*, an *error* (or uncertainty, or tolerance), and a *unit*. 3 things!
•The number of significant digits shown is related to the error.
•The caliper is more *precise* than the ruler.
•It is uncertain which measurement device is more *accurate*.

Measurements

Accuracy and precision
i. <u>accuracy:</u> how close the a measurement is to some accepted "true" value
ii. <u>precision:</u> how repeatable the measurement is with a given
instrument and technique

Measurements -accuracy and precision

Example: two bathroom scales.
Step on and off them repeatedly in a consistent way.

digital scale	analog (yellow) scale
155.1 lbs	150. lbs
155.0	148
155.1	149
155.2	149
155.3	151

Q: Which scale is more precise?
Q: Which scale has the greater “spread” in values?
Q: Which scale is most accurate?

You go to the doctors office and they tell you 157.2 lbs.

Q: Which scale is most accurate?
Q: Which scale is more precise?

Measurements

Significant figures or (significant digits)
-- a way of suggesting precision.

significant figure: any digit of a number that is known with certainty; any digit of a number beginning with the leftmost non-zero digit and ending with the rightmost non-zero digit (or a zero considered to be the exact value.

Examples:

1) 4,567,000 4
2) 4.567 0 5
3) 4,567,000 6
4) 4,567,000. 7
5) 0.03450 4
6) 30.003 5

Notes:

1. The number left of a decimal point is significant.
2. errors should have 1 significant figure.
3. For homework after week 1, give answers to 3 or 4 significant digits.
4. The *least significant digit* (LSD) is the rightmost significant digit.
5. the weights from the yellow scale should not be quoted to more than the 1's place.

Which digit is the LSD in each of the above?
Which place is occupied by the LSD in the above?

Measurements

Errors (uncertainty, tolerance)
-- the best way to quantify precision.

How do you determine an error on a measurement?

a) From the number of significant figures?
Not good. There is NO universally accepted rule for deriving errors from significant digits.
Ex.) engineers say 32.4 means 32.4+-0.05

b) By looking at the smallest "tickmarks" on your instrument.
Tolerance is usually ½ of the tickmark spacing.

c) By considering how difficult it is to use the instrument.
Ex. using a stopwatch.

d) By repeating the measurement many times and finding the spread of measurements. ("standard deviation") BEST!

Mistake,
not error.

Measurements

Errors types of errors

<u>*random, independent errors*</u>

- related to the <u>precision</u> of the measurement
- the type determined in the previous slide
- depends on the number of measurements

<u>*systematic errors*</u>

- related to the <u>accuracy</u> of the measurement
- an effect that shifts all measurements away from the true value.
  Ex) the zeropoint was set wrong on the yellow scale. If the zero point was increase by 5 lbs, it would give accurate readings
- use your best understanding and judgement of the measurement process to estimate.
  Ex) you are measuring the volume of a balloon.
  You understand that the answer will depend on the pressure and temperature inside and outside of the balloon.

Measurements

Errors ways of mathematically expressing errors

absolute errors

-- 155 +- 8 lbs has an absolute error of 8 lbs

fractional errors

-- 155 +- 8 lbs has a fractional error of 0.052

percentage errors

-- 155 +- 8 lbs has a percentage error of 5.2%

"Propagation"

Measurements

Error Propagation

How do you figure out the error for a number that was calculated from numbers with errors?

I. If only significant figure are shown:

a) Addition and subtraction: the final answer should have its LSD in the same place as the least precise input measurement

Ex) 5800 m + 121 m = 5900 m

Ex) 612800 s + 2011.5 s = 614,800 s

Ex) 220. - 115 = 105

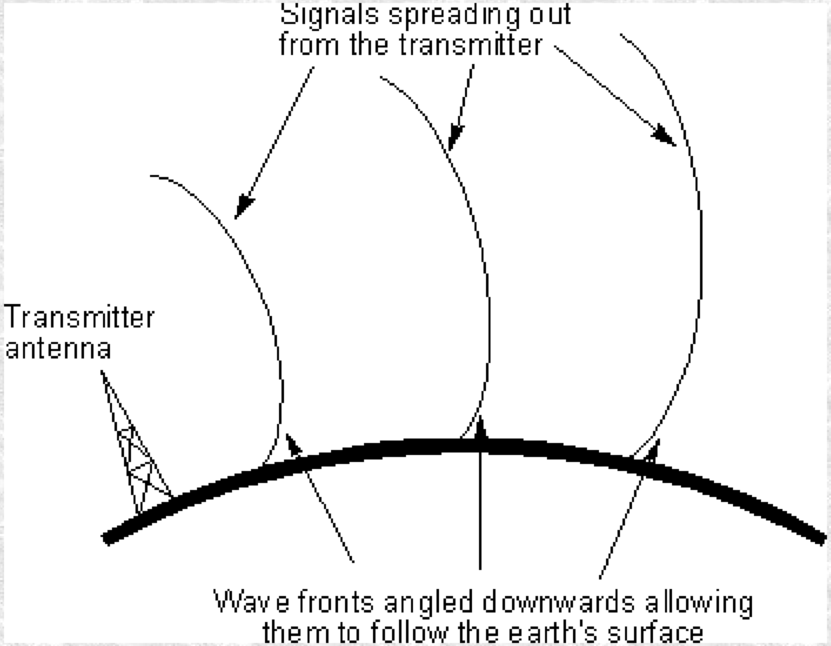
b) Multiplication and division: the final answer should have the same number of sig figs as the input number with the fewest sig. figs.

Ex) 2000 x 15.143 = 30,000

Ex) 382,500 x 11. = 4,200,000 (not 4,207,500)

Ex) 520 / 3 = 200 (not 173.3)

Measurements



Error Propagation - cont.

II. If errors are explicitly shown

a) Addition and subtraction:

1) simple way: add error

Ex) 580.±2 m + 121 ±3 m = 701.±5

(This is an overestimate.)

2) correct way: add errors "in quadrature"

Ex) 580.±2 m + 121+-3m = 701.±e

where e= $\sqrt{(2)^2+(3)^2}=\sqrt{13}=3.61$

b) Multiplication and division:

1) simple way: "adding the percentages"

Ex) Hwk Problems 2-5 ...

2) correct way: add fractional errors in quadrature.

(We will use the Simple way instead.)

Note: the LSD of the answer must match the LSD of the error!

Note: the number of sig figs in the final answer does not have to be the same as the least precise input number, ala prev slide.

Measurements

Errors and statistics

Mean $\mu = \frac{\sum x_i}{N}$

Standard Deviation $\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{(N-1)}}$

Standard Deviation of the mean

$$\sigma_\mu = \frac{\sigma}{\sqrt{N}}$$

Normal or Gaussian distribution