Chapter 7

Energy of a System

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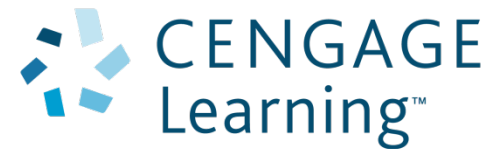
Outline for Week 7,D1

Exam I return

Work by a constant force $\quad W = F \, \Delta \, r \cos \theta$

Scalar dot products, **A•B = C**

Homework

Ch. 7 Read 7.1-7.4

    P. 1,2,5,8,12,18,20,22,25,37,39,41,45,55,
       56,59,60,65

    MiscQ: 1-13 (odd)   Do for next Mon

Notes:

   "NEW STUFF": added chapter7.pdf, two practice quizzes, and Ch 7 hwk key.

Outline for Week 7,D2

What is energy?

Work by a constant force $\quad W = F\,\Delta\,r\cos\theta$

Scalar dot products, **A•B** = C

Work by a variable force $\quad W = \int \vec{F}\cdot d\,\vec{r}$

Homework

Ch. 7 Read 7.1-7.4

P. 1,2,5,8,12,18,20,22,25,37,39,41,45,55,
56,59,60,65

MiscQ: 1-13 (odd)   Do for next Mon (after break)

Notes:

"NEW STUFF": updated chapter7.pdf.

Midterms will be based on Exam 1.

What is Energy?

Some problems are very difficult to solve with Newton's Laws alone, but the *Law of Conservation of Energy* makes them easier.

Energy: in physics, a quantity measured in Joules (SI), ergs, calories, BTU, N-m, ft-lbs, or kW-hrs. It's dimensions are ML^2/T^2. It is a scalar, NOT a vector.

Energy: "the ability to do work"   (Energy is not easily defined!)

Types of energy:

Energies stored in a system: thermal (or internal), kinetic (energy of motion), potential (energy of position: gravitational, elastic, nuclear, chemical)

Energies transferred into a system: **work** (W) and heat (Q). (Others: T_{ET} T_{MT} T_{ER})

It is wrong to say "the system contains a lot of work". Instead you can say "work was done on the system". You <u>can</u> say "the system contains a lot of kinetic and potential energy" or "work increased the kinetic energy of the system".

Work

The **work**, *W*, done on a system by a *constant force* is given by:

$$W = F \, \Delta r \cos \theta$$

where *F* is the magnitude of the force, Δr is the magnitude of the displacement of the point of application of the force, and θ is the angle between the force and the displacement vectors.

- The meaning of the term *work* is distinctly different in physics than in everyday meaning.

- Work is <u>not</u> a vector, even though it involves two vectors, **F** and Δr.

- Work can be + or – because *cos θ* can be + or -.

- Two geometric interpretations:

    - F cos *θ* is the component of F parallel to Δr, (i.e., $W = F_{\parallel} \Delta r$)

    - Δr cos *θ* is the component of Δr parallel to F. (i.e., $W = F \Delta r_{\parallel}$)

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Work, cont.

$W = F \Delta r \cos \theta$

- The displacement, $\Delta \mathbf{r}$, is that of the <u>point of application</u> of the force.

- A force does no work on the object if the force does not move through a displacement.

- The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application.

F → **F** Force without motion
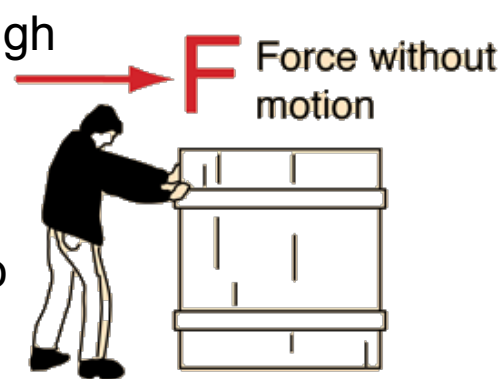
When a force is exerted on an object which does not move, no work is done on the object.

↑F Force perpendicular to the motion
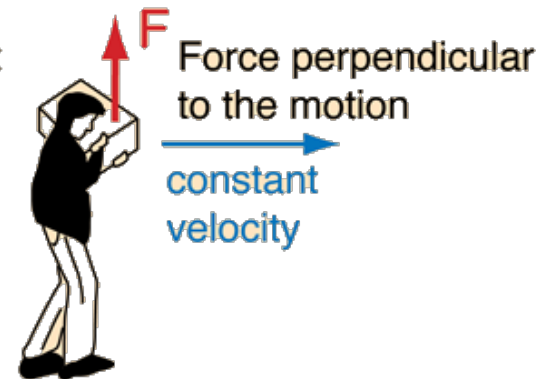
constant velocity

When an object is carried at constant velocity by a force which acts at right angles to the motion, no work is done on the object.

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Work, cont.

$W = F \, \Delta r \cos \theta$

Q1: What work is done by F =10 N if θ=0 degrees and Δr=2 m?

Q2: What work is done by F=10 N if θ=60 degrees and Δr=2 m?

Q3: What work is done by F=10 N if θ=90 degrees and Δr=2 m?

Q4: The box weighs 20 N. What is the work done by the normal force on the box when θ=0 degrees and Δr=2 m?

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Q5: The box weighs 20 N. What is the work done by the force of gravity on the box when θ=0 degrees and Δr=2 m?

Q6: What is the work done by the force of friction on the 20N box if μ_k = 0.5 and when θ=0 degrees and Δr=2 m?

Section 7.2
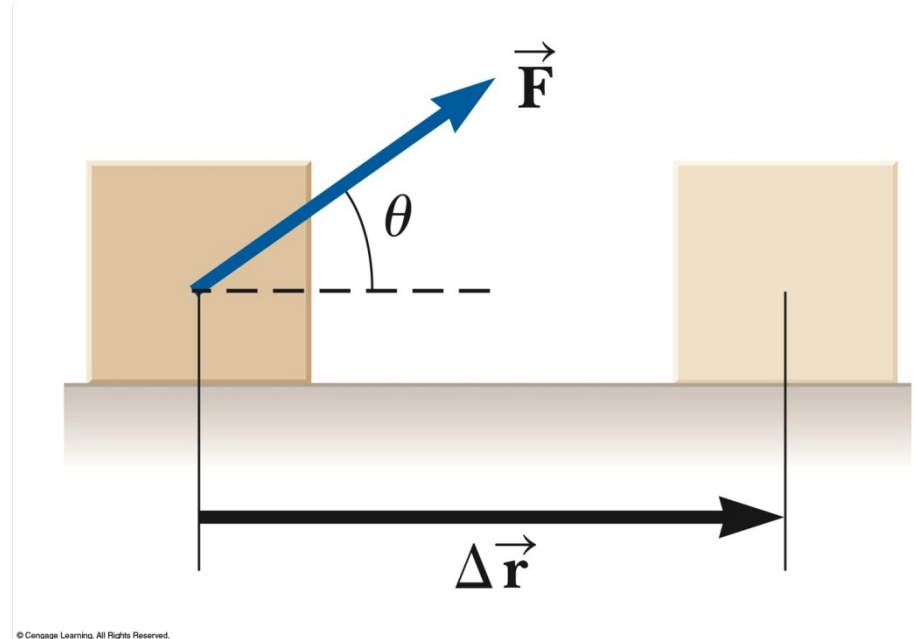
Work, cont.

$W = F \, \Delta r \cos \theta$

Q7: What work is done by **F** =20 N î if Δ**r**=2 m î?

Q8: What work is done by **F**=-20 N î if Δ**r**=2 m i? How could this happen?

Q9: What work is done by **F**=20 î + 15 ĵ N if Δ**r**=3 m î?

Q10: What work is done by **F**=20 î + 0 ĵ N if Δ**r**=3 m î – 2m ĵ ?
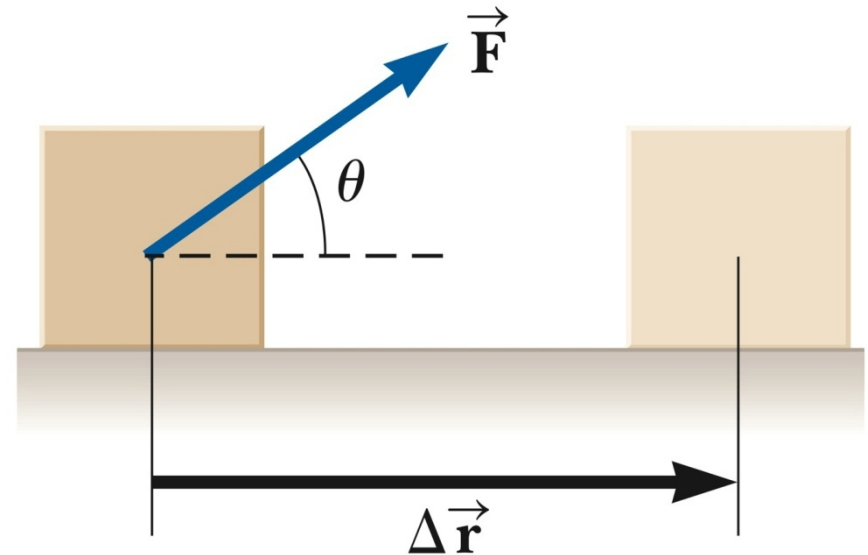
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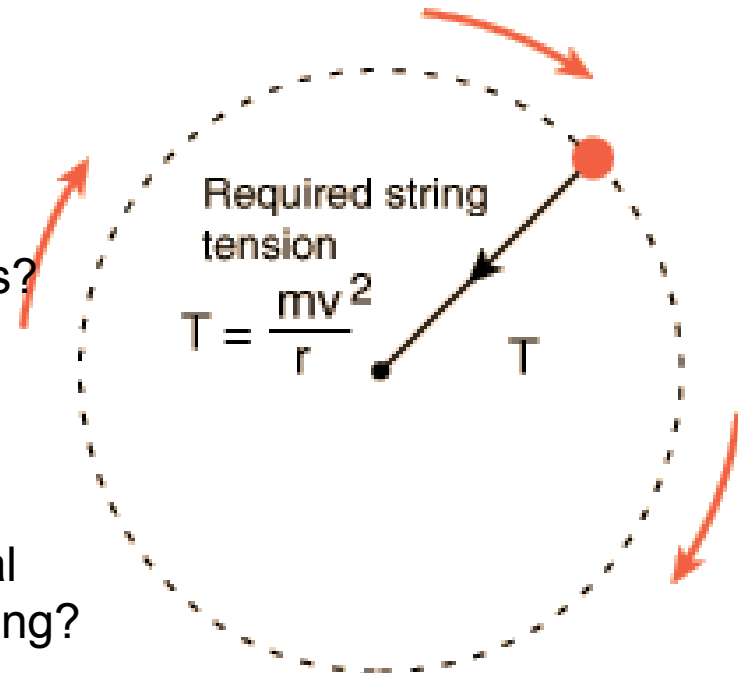# Work, cont.

What is the work done by the
tension force in 1 revolution of the mass?

What is the work by the tension force
in ½ revolution of the mass?

What is the work done by the centripetal
force for any angle swept out by the string?

What is the work done by the net force on
the mass for any angle swept out by the string?

Required string
tension

$$T = \frac{mv^2}{r}$$

T

Displacement in the Work Equation

The displacement is that of the point of application of the force.

If the force is applied to a rigid object that can be modeled as a particle, the displacement is the same as that of the particle.

For a deformable system, the displacement of the object generally is not the same as the displacement associated with the forces applied.

Work by friction is also hard to explain because there is no single point of application. But it can ultimately be modeled by $W = f_k \, \Delta r \cos \theta$ and is usually negative because θ=180.

Outline for Week 7,D3

Scalar dot products, **A•B** = C

Work by a variable force $\qquad W = \int \vec{F} \cdot d\,\vec{r}$

Homework

Ch. 7 Read 7.1-7.4

     P. 1,2,5,8,12,18,20,22,25,37,39,41,45,55,
       56,59,60,65

     MiscQ: 1-13 (odd)   Do for next Mon (after break)

Notes:

   "NEW STUFF": added exam-like questions for Ch.7, updated chapter7.pdf.

    Midterms will be based on Exam 1.

Work (cont.)

The normal force and the gravitational force do no work on the object.
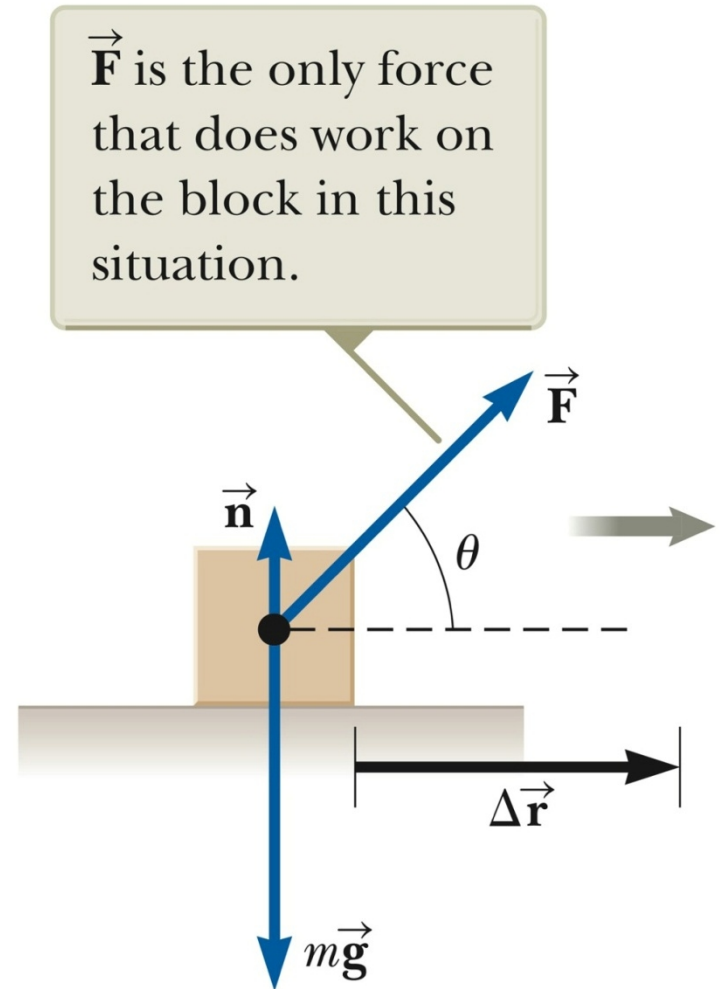
- $\cos\theta = \cos 90° = 0$

The force **F** is the only force that does work on the object.

Example with acceleration.

Q: How much work is done pushing a 46 kg crate to the right with **a**=2.5 m/s²î over 10.3 m across a floor with μ_k=0.4?

Sol: First find F_{app}, then W=$F_{app}\Delta r$
F_{net}=m**a** = F_{app} + **f**$_k$

… W = 3042 J.

F is the only force that does work on the block in this situation.

$\vec{F}$

$\vec{n}$

θ

$\Delta\vec{r}$

$m\vec{g}$

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Work Is An Energy Transfer

If the work done on a system is positive, energy is transferred into the system.

If the work done on the system is negative, energy is transferred out of the system.

Example system: a mass on a string (a pendulum).

Energy that can be stored: gravitational potential energy and kinetic energy

Q: How can you use work to transfer energy in?

Q: How can you use work to transfer energy out?
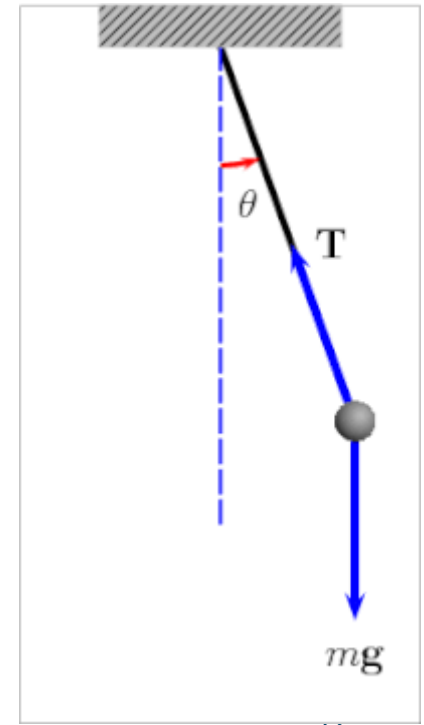
Scalar (Dot) Product of Two Vectors

The scalar product of two vectors is written as $\vec{A} \cdot \vec{B}$.

- It is also called the dot product.

  $\vec{A} \cdot \vec{B} = AB \cos\theta$

- θ is the angle *between A and B*

Applied to work, this means

$$W = F\Delta r \cos\theta = \vec{F} \cdot \Delta\vec{r}$$

(for the case of constant forces)

$\vec{B}$

θ

$\vec{A} \cdot \vec{B} = AB \cos\theta$

$B \cos\theta$

$\vec{A}$
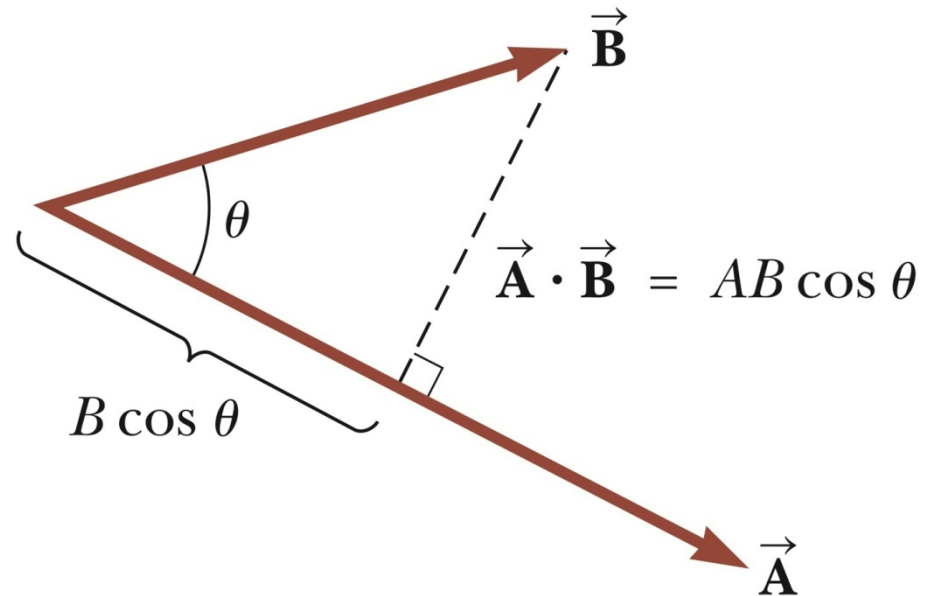
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Scalar Product, cont

The scalar product is commutative.

- $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

The scalar product obeys the distributive law of multiplication.

- $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

Dot Products of Unit Vectors

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$
$$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$$

Using component form with vectors:
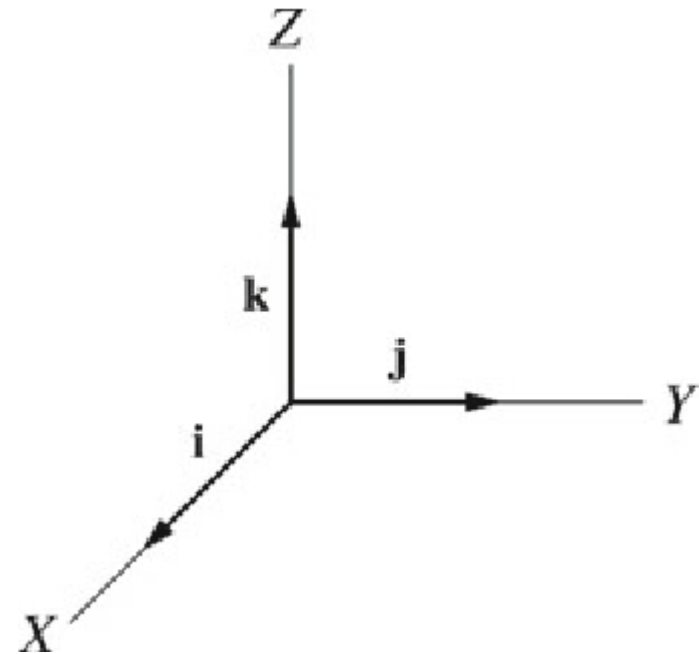
$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$
$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$
$$\vec{A} \cdot \vec{B} = AxBx + AyBy + AzBz$$

In the special case where

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

Dot Products - example

$$\vec{A} \cdot \vec{B} = AxBx + AyBy + AzBz$$

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

Ex) (P. 20) Find the angle between **A**=5.8i-3.5j-6.2k
and   **B**=8.2i+4.3j-7.0k

$$\vec{A} \cdot \vec{B} = 5.8 \cdot 8.2 + (-3.5) \cdot 4.3 + (-6.2) \cdot (-7.0)$$

A·B = 75.91
Find |**A**| = 9.145   and |**B**|=11.61
Then |**A**||**B**| cos θ = 75.91
cos θ = 75.91/(9.145*11.61) → θ = 44.4 ° (fixed)

Work Done by a Varying Force

$$W = \int \vec{F} \cdot d\vec{r}$$

To use W = F Δ r cos θ, the force must be constant, so the equation cannot be used to calculate the work done by a varying force.

Assume that during a very small displacement, Δx, *F* is constant.

For that displacement, $W \sim F \Delta x$

For all of the intervals,

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$

The total work done for the displacement from x_i to x_f is approximately equal to the sum of the areas of all the rectangles.

Area $= F_x \Delta x$

F_x

F_x

x_i

x_f

x

Δx

a

Section 7.4
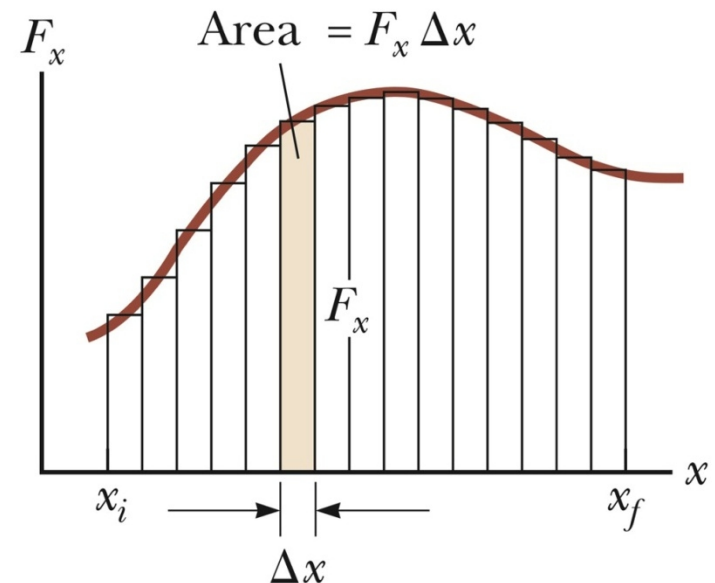
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Work Done by a Varying Force, cont.

Let the size of the small displacements approach zero .

Since

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

Or, more generally,

$$W = \int \vec{F} \cdot d\vec{r}$$

The work done is equal to the area under the curve between x_i and $x_{f.}$

The work done by the component F_x of the varying force as the particle moves from x_i to x_f is *exactly* equal to the area under the curve.

b

Section 7.4
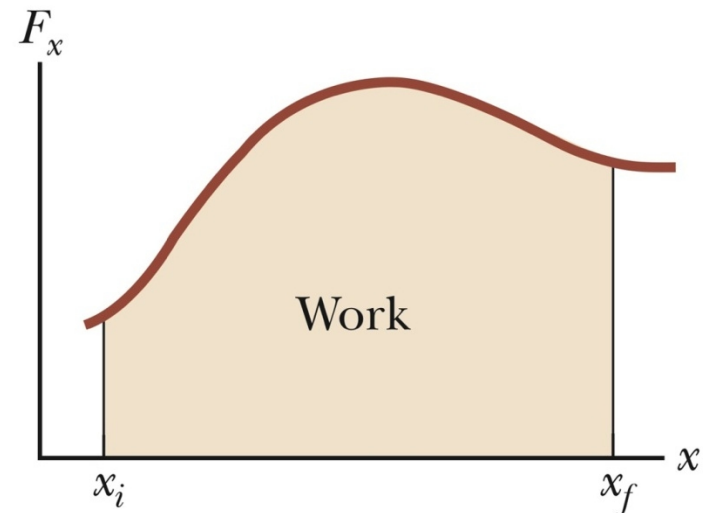
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Work Done by a Varying Force, cont.

$$W = \int \vec{F} \cdot d\vec{r}$$

Example) P. 7.41. Find the work done on a particle moving according to this graph …

a) between x=0 to x=10 m
Ans: 2800 J

b) between x=0 to x=15 m
Ans: 2100 J

Work Done by a Varying Force, cont.

$$W = \int \vec{F} \cdot d\vec{r}$$

Example) P. 7.45. $F(x) = \dfrac{A}{\sqrt{x}}$

With A = 3.00 Nm$^{1/2}$ what is the work done by this force when the object moves from x_i=0 to x_f=1.0?

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Work Done By Multiple Forces

If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is the work done by the net force.

$$\sum W = W_{ext} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx$$

In the general case of a net force whose magnitude and direction may vary.

$$\sum W = W_{ext} = \int_{x_i}^{x_f} \left(\sum \mathbf{F} \right) d\vec{\mathbf{r}}$$

The subscript "ext" indicates the work is done by an *external* agent on the system.

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Work Done by Multiple Forces, cont.

If the system cannot be modeled as a particle, then the total work is equal to the algebraic sum of the work done by the individual forces.

$$\sum W = W_{\text{ext}} = \sum_{forces} \left(\mathbf{F} \cdot d\mathbf{r} \right)$$

- Remember work is a scalar, so this is the sum of a bunch of scalars.

Outline for Week 8,D1

Work by the spring force $W = -\dfrac{1}{2}k\left(x_f^2 - x_i^2\right)$

The work-KE principle $W_{ext} = \Delta K$

Homework

Ch. 7 Read 7.1-7.4

    P. 1,2,5,8,12,18,20,22,25,37,39,41,45,55,
        56,59,60,65 MiscQ: 1-13 (odd)   Do for today

Ch. 8 Read 8.1-8.6,8.8-8.9   Do P. 1, 3, 4, 6, 9,12,13,17,19,28,
        29,30,55,56,58,61,73 for next Monday

Notes: Lab on "Cons. Of Energy"

    "NEW STUFF": updating chapter7.pdf.

    Midterms are based on Exam 1.

Last time: Work Done by a Varying Force

Can't use $W = F\Delta r \cos\theta = \vec{F}\cdot\Delta\vec{r}$

But you can add up a bunch of **F**·Δ**r** terms to approximate the work. Or, do an integral for the exact right amount:

$$\lim_{\Delta x\to 0}\sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

Or, more generally,

$$W = \int \vec{F}\cdot d\vec{r}$$

The work done is equal to the area under the curve between x_i and x_f.

The work done by the component F_x of the varying force as the particle moves from x_i to x_f is *exactly* equal to the area under the curve.

F_x

Work

x_i x_f x

b
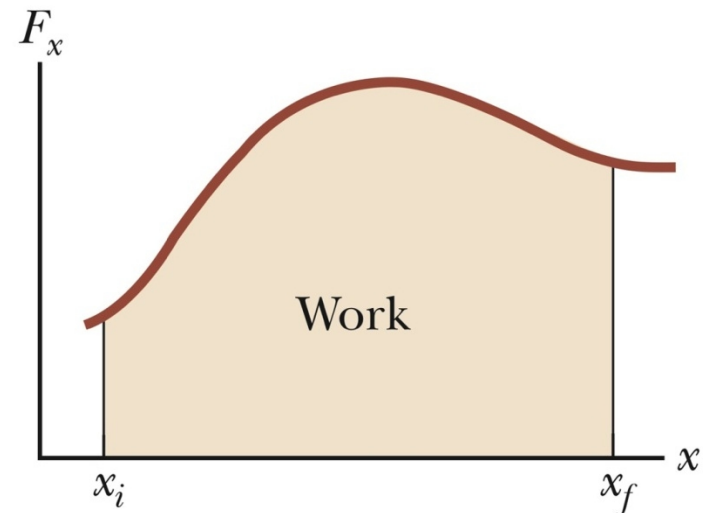
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The Spring Force   is a variable force!

$x = 0$ $\vec{F}_s$

When x is positive (stretched spring), the spring force is directed to the left.
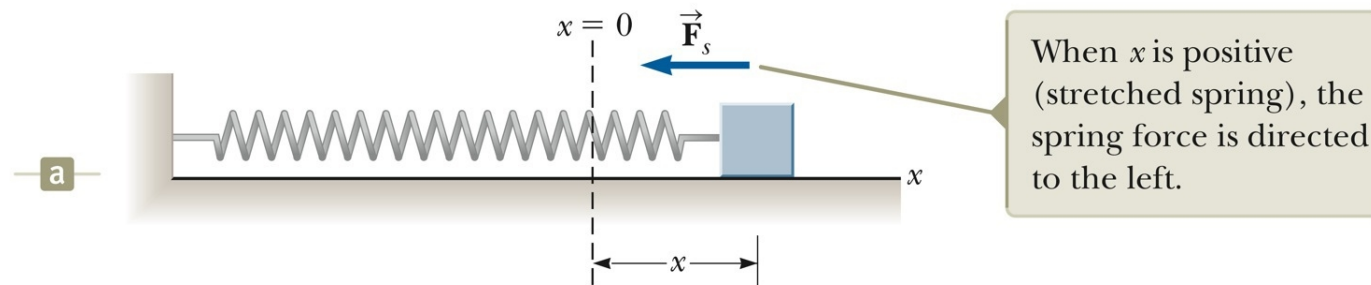
In order to calculate the work done by the spring force, we need to know how the force varies with position, which is given by ...

Hooke's Law:

$$F_s = -kx$$

F_s is the spring force,
k is the spring constant (N/m), and
x is the distance of [the block] from the equilibrium position.

The "-" indicates that the force is in the opposition direction of the displacement from equilibrium.

The block is usually on a horizontal, frictionless surface.

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Work Done By A Spring

Observe the motion of the block with various values of the spring constant.

See 7.9.swf

When x is positive (stretched spring), the spring force is directed to the left.

When x is zero (natural length of the spring), the spring force is zero.

When x is negative (compressed spring), the spring force is directed to the right.

The work done by the spring force on the block as it moves from $-x_{max}$ to 0 is the area of the shaded triangle, $\frac{1}{2}kx_{max}^2$.

Hooke's Law, cont.

When x is positive (stretched spring), the spring force is directed to the left.
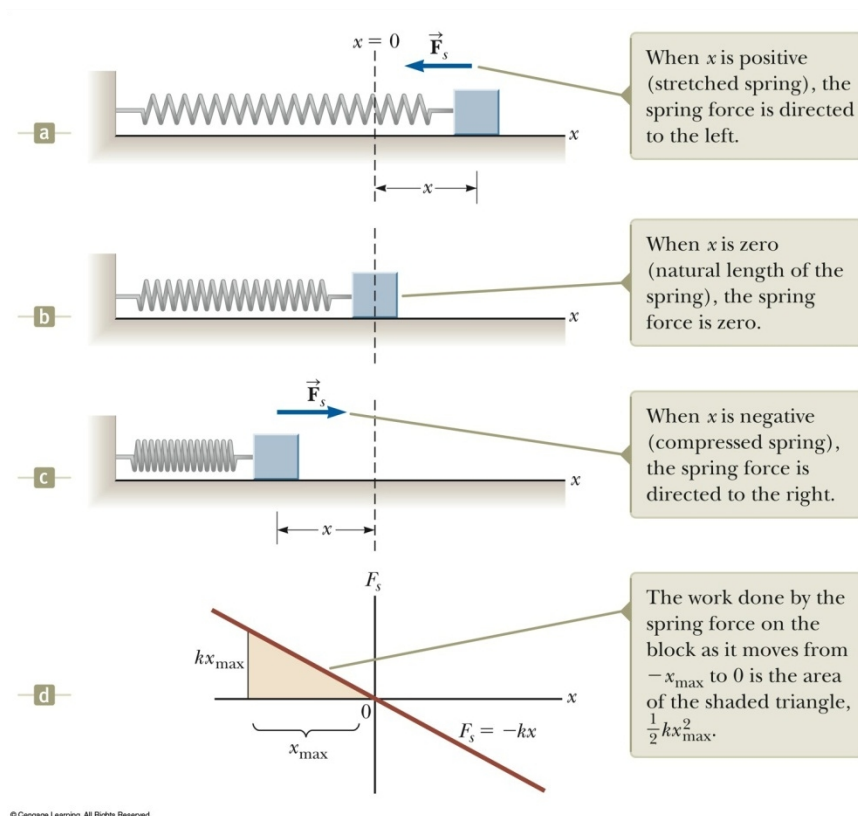
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The vector form of Hooke's Law is

$$\mathbf{F}_s = F_x\hat{\mathbf{i}} = -kx\hat{\mathbf{i}}$$

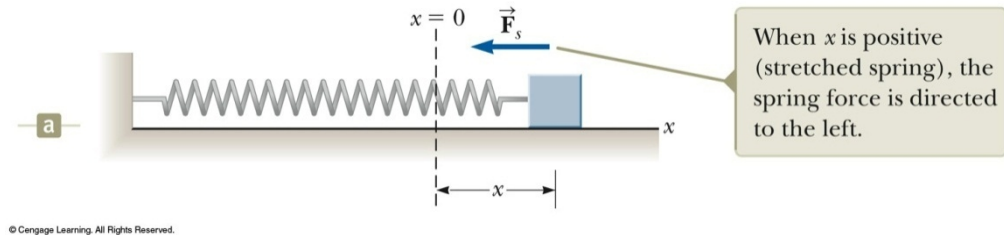When x is positive (spring is stretched), F is negative

When x is 0 (at the equilibrium position), F is 0

When x is negative (spring is compressed), F is positive
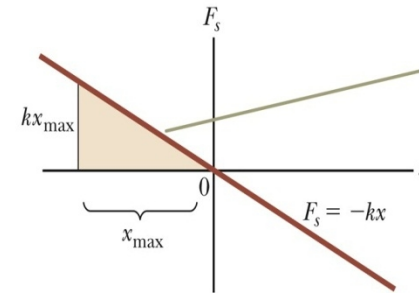
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Work Done by a Spring

Identify the block as the system.

Ex) Calculate the work by the spring as the block moves from $x_i = -x_{max}$ to $x_f = 0$.

$$W_s = \int_{x_i}^{x_f} \vec{F}_s \cdot d\vec{r} = \int \left(-kx\,\hat{i}\right)\cdot\left(dx\,\hat{i}\right)$$

$$W_s = \int_{-x_{max}}^{0} -kx\,dx = \frac{1}{2}(-k)\left(0^2 - (-x_{max})^2\right) = \frac{1}{2}kx_{max}^2$$

The work done by the spring force on the block as it moves from $-x_{max}$ to 0 is the area of the shaded triangle, $\frac{1}{2}kx_{max}^2$.

F_s

kx_{max}

$F_s = -kx$

x_{max}

x

0

$-d-$

Notice that this is the area of the shaded triangle!

Q: What is the work done by the spring if the block moves from 0 to x_{max}?

Q: What is the work done by the spring if the block moves from $-x_{max}$ to x_{max} ?

Q: What is the work done by the spring if the block moves from X_i=-0.5 to x_f=0.3 m if k=300 N/m?

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Work Done by a Spring, cont.

Assume the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$.

The work done by the spring on the block is

$$W_s = \int_{x_i}^{x_f} (-kx)\, dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

- Or $W_s = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$
- If the motion ends where it begins, W = 0

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Spring with an Applied Force

Suppose an external agent, F_{app}, like someone's hand, stretches the spring.

The applied force is equal and opposite to the spring force (if <u>block's speed constant</u>).

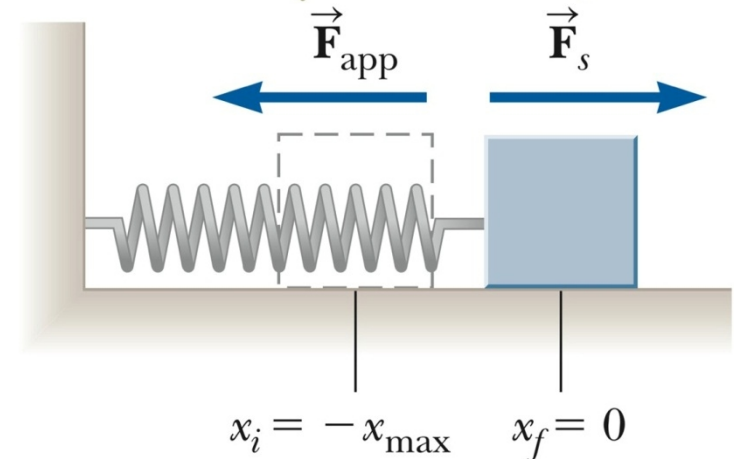$$\mathbf{F}_{app} = F_{app}\hat{\mathbf{i}} = -\mathbf{F}_s = -\left(-kx\hat{\mathbf{i}}\right) = kx\hat{\mathbf{i}}$$

Work done by F_{app} as the block moves from $-x_{max}$ to $x = 0$ is equal to

$-\frac{1}{2}kx^2_{max}$

For any displacement, the work done by the applied force (at const speed) is

$$W_{app} = \int_{x_i}^{x_f} (kx)\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

If the process of moving the block is carried out very slowly, then $\vec{\mathbf{F}}_{app}$ is equal in magnitude and opposite in direction to $\vec{\mathbf{F}}_s$ at all times.

$\vec{\mathbf{F}}_{app}$ $\vec{\mathbf{F}}_s$

$x_i = -x_{max}$ $x_f = 0$

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Spring with an Applied Force

For any displacement, the work done by the applied force is

$$W_{app} = \int_{x_i}^{x_f} (kx)\, dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

Q: What is the work done by a hand pushing the block at <u>constant speed</u> from $x_i=0$ to $x_f=0.5$ m if k=300 N/m?

Ans: ½(300)(0.5)² = 37.5 J

Q: What is the net work done by a hand pushing the spring-block at constant speed from x_i=-0.5 m to x_f=0.5 m if k=300 N/m?

Ans: ½(300)(0.5²-0.5²) = 0 J

If the process of moving the block is carried out ~~very slowly,~~ at const. speed then $\vec{F}_{app}$ is equal in magnitude and opposite in direction to $\vec{F}_s$ at all times.

$\vec{F}_{app}$ $\vec{F}_s$

$x_i = -x_{max}$ $x_f = 0$

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Spring with an Applied Force

For any displacement, the work done by the applied force is

$$W_{app} = \int_{x_i}^{x_f} (kx)\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$
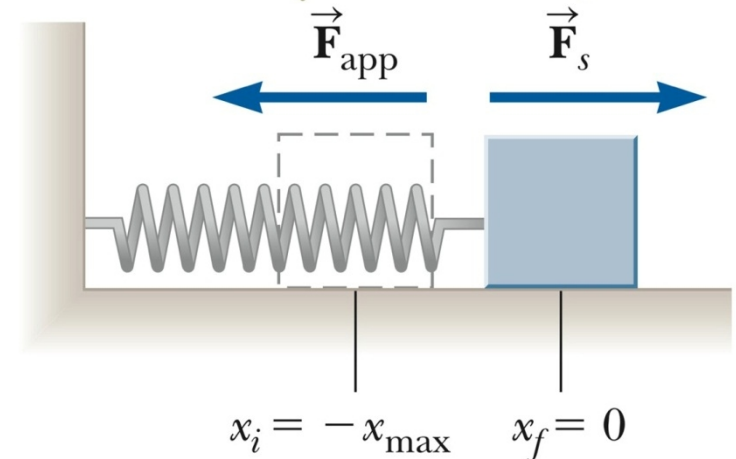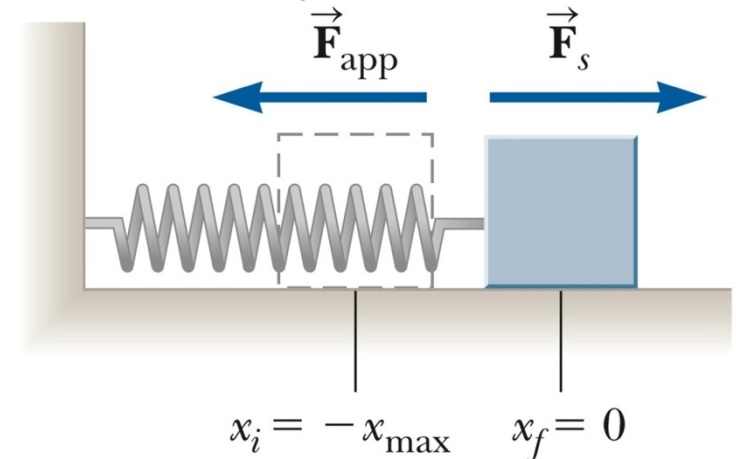
Q: What is the work done by a hand pushing the block at <u>constant speed</u> from $x_i=0$ to $x_f=0.5$ m if k=300 N/m?

 Ans: ½(300)(0.5)² = 37.5 J

Q: What is the net work done by a hand pushing the spring-block at constant speed from $x_i=-0.5$ m to $x_f=0.5$ m if k=300 N/m?

 Ans: ½(300)(0.5²-0.5²) = 0 J

If the process of moving the block is carried out ~~very slowly,~~ at const. speed then $\vec{\mathbf{F}}_{app}$ is equal in magnitude and opposite in direction to $\vec{\mathbf{F}}_s$ at all times.

$\vec{\mathbf{F}}_{app}$ $\vec{\mathbf{F}}_s$

$x_i = -x_{max}$ $x_f = 0$

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Outline for Week 8,D2

Finish work-KE principle $W_{ext} = \Delta K$

Conservative and non-conservative forces
Potential energy (gravity, elastic, general)

Homework

Ch. 8 Read 8.1-8.6,8.8-8.9   Do P. 1, 3, 4, 6, 9,12,13,17,19,28, 29,30,55,56,58,61,73 for next Monday

Notes:

Lab on "Cons. Of Energy"
"NEW STUFF": Added Practice Quiz on Ch. 8.
Updated chapter7.pdf again.

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Kinetic Energy

One possible result of work acting as an influence on a system is that the system changes its speed.

The system could possess *kinetic energy.*

Kinetic Energy is the energy of a particle due to its motion.

- *$K = \frac{1}{2} mv^2$*
    - *K* is the kinetic energy
    - *m* is the mass of the particle
    - *v* is the speed of the particle

A change in kinetic energy is one possible result of doing work to transfer energy into (or out of) a system.
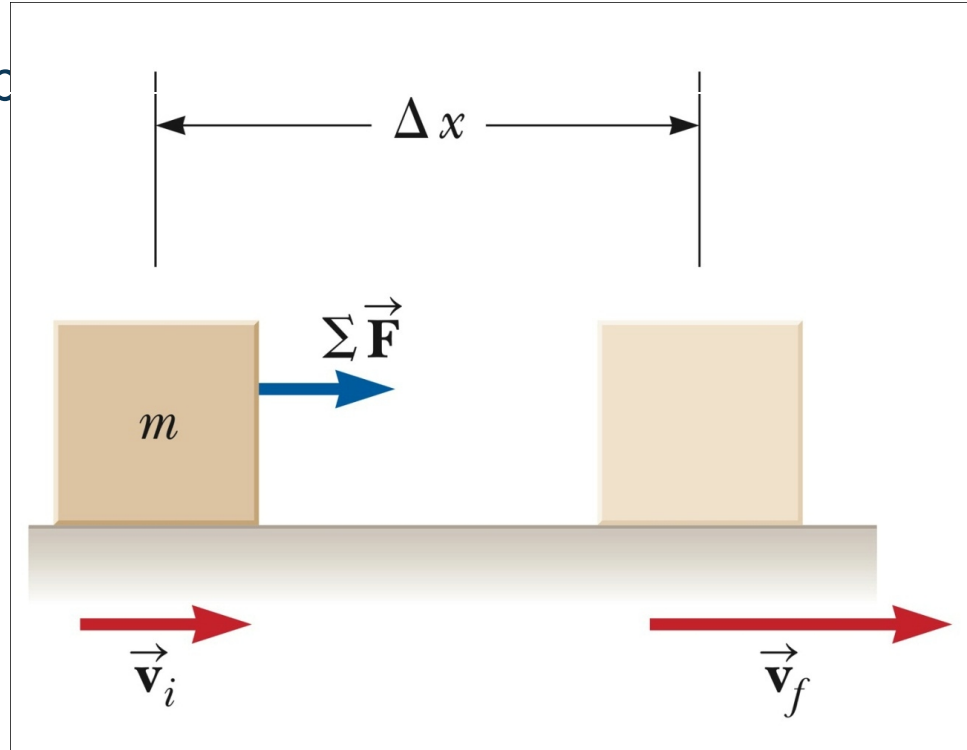
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Work - Kinetic Energy princip...

Derivation:

$$W_{ext} = \int_{x_i}^{x_f} \sum F \, dx = \int_{x_i}^{x_f} ma \, dx$$

$$W_{ext} = \int_{v_i}^{v_f} mv \, dv$$

$$W_{ext} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{ext} = K_f - K_i = \Delta K$$

Ex) Find speed of frictionless block (m=0.5 kg) at x=0 if it is attached to a spring with k=300 N/m and released from rest at x=0.3 m.

Ans: W_{ext}=W_s=-½k(0-0.3²)=½ m (v_f^2-0) … v_f = 7.35 m/s

Ex) (P. 60) An 85 g arrow is fired from a bow whose string exerts an average force of 105 N over a distance of 75 cm. What is the speed of the arrow as it leaves the bow?

Ans: W_{ext}=1/2 m (v_f^2-0) with W_{ext}=F_{avg}Δx=105(.75m). v_f = 43 m/s

Work-Kinetic Energy Theorem

The Work-Kinetic Energy Theorem states $W_{ext} = K_f - K_i = \Delta K$

When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system.

- The speed of the system increases if the work done on it is positive.

- The speed of the system decreases if the net work is negative.

- Also valid for changes in rotational speed

The work-kinetic energy theorem is not valid if other changes (besides its speed) occur in the system or if there are other interactions with the environment besides work.

Ex) You exert a 20 N upward force on an object that weighs 10 N. The force is applied over a distance of 0.3 m and the object starts at rest. What is the change in KE of the object? Ans: the work is also changing the gravitational potential energy of the object, so $W_{hand} \neq \Delta KE$. Instead, $W_{hand} = \Delta KE + \Delta PE$, or $W_{hand+grav} = \Delta KE$. Using the latter: **20N(0.3)-10N(0.3)=ΔKE=3J**.

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Work-Kinetic Energy Theorem – Example

Find the final velocity of the block.

The normal and gravitational forces do no work since they are perpendicular to the direction of the displacement.
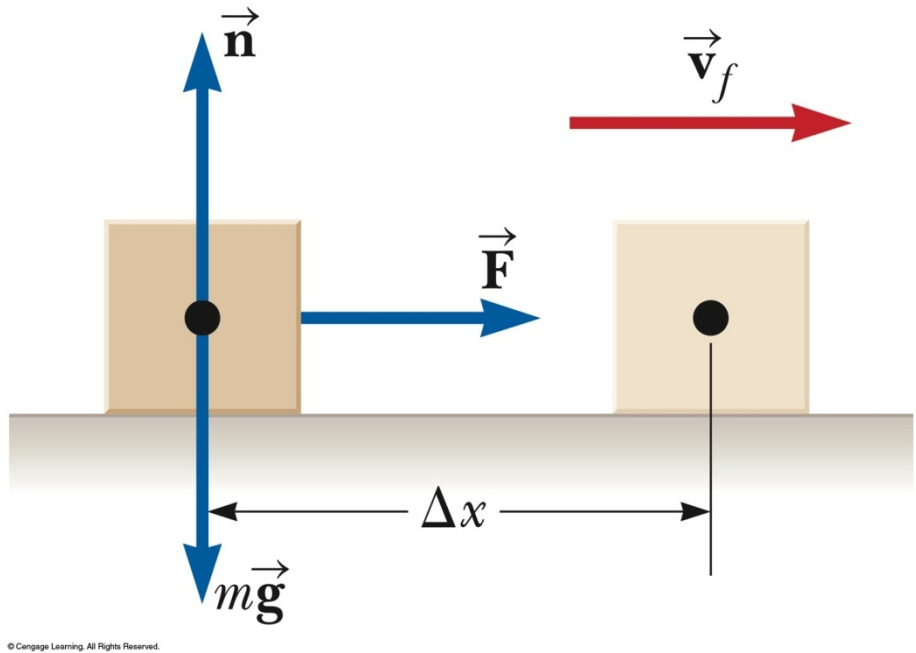
$W_{ext} = \Delta K = \frac{1}{2}mv_f^2 - 0$

$F\Delta x = \frac{1}{2}mv_f^2 \rightarrow v_f = (2F\Delta x/m)^{\frac{1}{2}}$

The answer for v_f could be checked using the kinematic equations …

Substitute $a = F_{net}/m$ into

$v_f^2 - v_i^2 = 2a(\Delta x)$, again giving $v_f = (2F\Delta x/m)^{\frac{1}{2}}$

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What if friction, f_k, acted in the opposite direction of the applied force?
Ans: you modify the Work-KE Theorem to be

$W_{ext} = \Delta K + f_k\Delta x$.   This gives $v_f = (2[F-f_k]\Delta x/m)^{\frac{1}{2}}$

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Conservative Forces

The work done by a conservative force on a particle moving between any two points is *independent of the path* taken by the particle.

The work done by a conservative force on a particle moving through any *closed* path is zero.

- A closed path is one in which the beginning and ending points are the same.

Examples of conservative forces:

- Gravity (DEMO: 7.15.swf)

- Spring force

Examples of non-conservative forces:
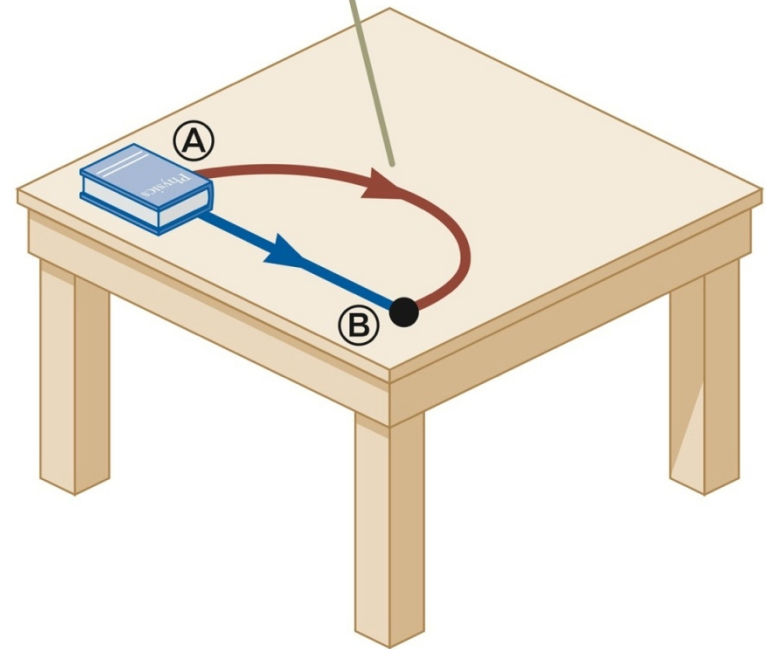
- Friction

- Air resistance

- An applied force which counters friction.

Non-conservative Forces, cont.

The work done against friction is greater along the brown path than along the blue path.

Because the work done depends on the path, friction is a non-conservative force.

The work done in moving the book is greater along the brown path than along the blue path.

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Conservative Forces (cont.)
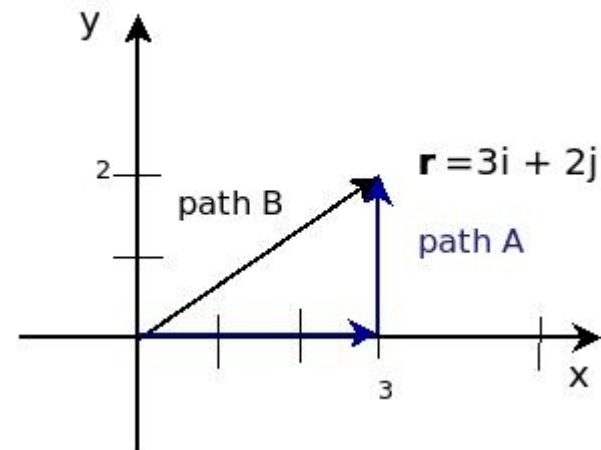
Ex) Show that the work done by gravity on mass m moved from (0,0) to (3,2) is the same for 2 straight-line paths.

Path A: (0,0) to (3,0) to (3,2)

Path B: (0,0) to (3,2)

$$W_A = \int_{(0,0)}^{(3,0)} \vec{F}_g \cdot d\vec{r} + \int_{(3,0)}^{(3,2)} \vec{F}_g \cdot d\vec{r}$$

$$W_A = \int_{(0,0)}^{(3,0)} -mg\,\hat{j} \cdot dx\,\hat{i} + \int_{(3,0)}^{(3,2)} -mg\,\hat{j} \cdot dy\,\hat{j}$$

Conservative Forces, cont

We can associate a potential energy for a system with any conservative force acting between members of the system.

- This can be done only for conservative forces.

- In general: $W_{int} = -\Delta U$

  - W_{int} is used as a reminder that the work is done by one member of the system on another member and is internal to the system.

- Positive work done by an outside agent on a system causes an increase in the potential energy of the system.

- Work done on a component of a system by a conservative force internal to an isolated system causes a decrease in the potential energy of the system.

Example: make equations for a falling apple when the system is the Earth+apple And when the system is just the apple.

Non-conservative Forces

A non-conservative force does not satisfy the conditions of conservative forces.

Non-conservative forces acting in a system cause a *change* in the **mechanical energy** of the system.

$E_{mech} = K + U$

- K includes the kinetic energy of all moving members of the system.

- U includes all types of potential energy in the system.

Conservation of mechanical energy:

$\Delta E_{mech} = \Delta K + \Delta U = 0$ for a closed system with no non-conservative forces

$\Delta E_{mech} = \Delta K + \Delta U < 0$ for a closed system with non-conservative forces

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Conservative Forces and Potential Energy

Define a potential energy function, *U*, such that the work done by a conservative force equals the decrease in the potential energy of the system.

The work done by such a force, *F*, is

$$W_{int} = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

•ΔU is negative when *F* and *x* are in the same direction

~

Ex) Find the potential energy function for a spring.

Ex) Find the potential energy function for gravity.

Conservative Forces and Potential Energy

The conservative force is related to the potential energy function through.

$$F_x = -\frac{dU}{dx}$$

The *x* component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to *x*.

- Can be extended to three dimensions

Conservative Forces and Potential Energy – Check

Look at the case of a stretched spring:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

- This is Hooke's Law and confirms the equation for $U_s = \frac{1}{2}kx^2$

U is an important function because a conservative force can be derived from it.

Potential Energy

Potential energy is energy determined by the configuration of a system in which the components of the system interact by forces.

- The forces are internal to the system.

- Can be associated with only specific types of forces acting between members of a system

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Gravitational Potential Energy

The system is the Earth and the book.

$\mathbf{F}_{app}$ is external, $\mathbf{F}_{g}$ is internal.

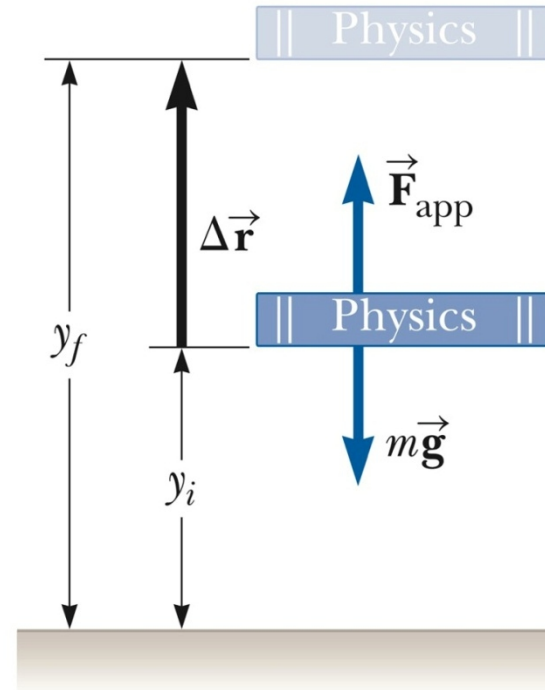$\mathbf{F}_{app}$ does work on the book, lifting it at constant speed through a vertical displacement.

$$\Delta\mathbf{r} = (y_f - y_i)\,\hat{\mathbf{j}}$$

The work done on the system by the hand must appear as an increase in the energy of the system.

The energy storage mechanism is called *potential energy.*

The work done by the agent on the book–Earth system is $mgy_f - mgy_i$.

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Gravitational Potential Energy, cont

Assume the book in fig. 7.15 is allowed to fall at a constant speed.

There is no change in kinetic energy since the book starts and ends at the same speed.

Gravitational potential energy is the energy associated with an object at a given location above the surface of the Earth.

$$W_{ext} = \left(\mathbf{F}_{app} \right) \cdot \Delta \mathbf{r}$$

$$W_{ext} = (mg\hat{\mathbf{j}}) \cdot \left[(y_f - y_i)\hat{\mathbf{j}} \right]$$

$$W_{ext} = mgy_f - mgy_i$$

Gravitational Potential Energy, final

The quantity *mgy* is identified as the gravitational potential energy, U_g.

- $U_g = mgy$

Units are joules (J)

It is a scalar

Work may change the gravitational potential energy of the system.

- $W_{ext} = \Delta U_g$

Potential energy is always associated with a system of two or more interacting objects.

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Gravitational Potential Energy, Problem Solving

The gravitational potential energy depends only on the vertical height of the object above Earth's surface.

In solving problems, you must choose a reference configuration for which the gravitational potential energy is set equal to some reference value, normally zero.

- The choice is arbitrary because you normally need the difference in potential energy, which is independent of the choice of reference configuration.

Often having the object on the surface of the Earth is a convenient zero gravitational potential energy configuration.

The problem may suggest a convenient configuration to use.

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Elastic Potential Energy

Elastic Potential Energy is associated with a spring.

The force the spring exerts (on a block, for example) is $F_s = -kx$

The work done by an external applied force on a spring-block system is

- $W = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$

- The work is equal to the difference between the initial and final values of an expression related to the configuration of the system.
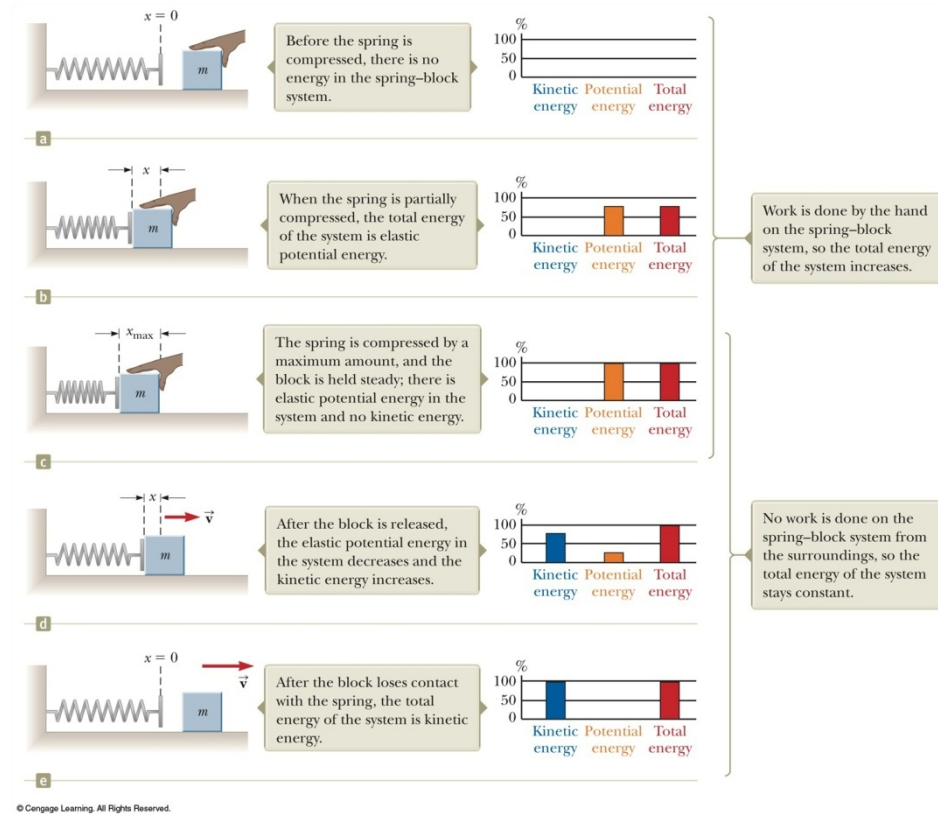
Elastic Potential Energy, cont.

This expression is the elastic potential energy:

$$U_s = \tfrac{1}{2} kx^2$$

The elastic potential energy can be thought of as the energy stored in the deformed spring.

The stored potential energy can be converted into kinetic energy.

Observe the effects of different amounts of compression of the spring.

Before the spring is compressed, there is no energy in the spring–block system.

When the spring is partially compressed, the total energy of the system is elastic potential energy.

Work is done by the hand on the spring–block system, so the total energy of the system increases.

The spring is compressed by a maximum amount, and the block is held steady; there is elastic potential energy in the system and no kinetic energy.

After the block is released, the elastic potential energy in the system decreases and the kinetic energy increases.

No work is done on the spring–block system from the surroundings, so the total energy of the system stays constant.

After the block loses contact with the spring, the total energy of the system is kinetic energy.

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Elastic Potential Energy, final

The elastic potential energy stored in a spring is zero whenever the spring is not deformed (U = 0 when x = 0).
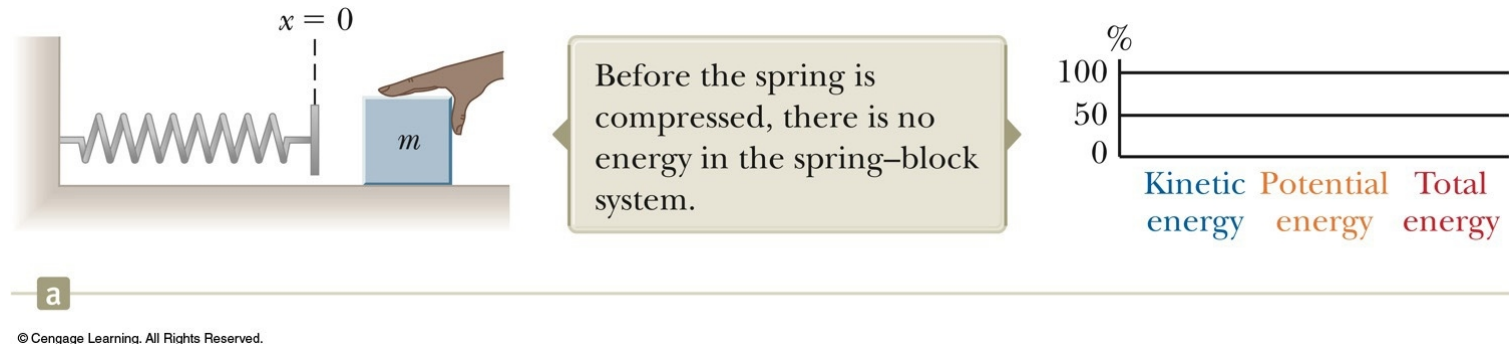
- The energy is stored in the spring only when the spring is stretched or compressed.

The elastic potential energy is a maximum when the spring has reached its maximum extension or compression.

The elastic potential energy is always positive.

- x^2 will always be positive.

Energy Bar Chart Example

$x = 0$

Before the spring is compressed, there is no energy in the spring–block system.

%
100
50
0

Kinetic   Potential   Total
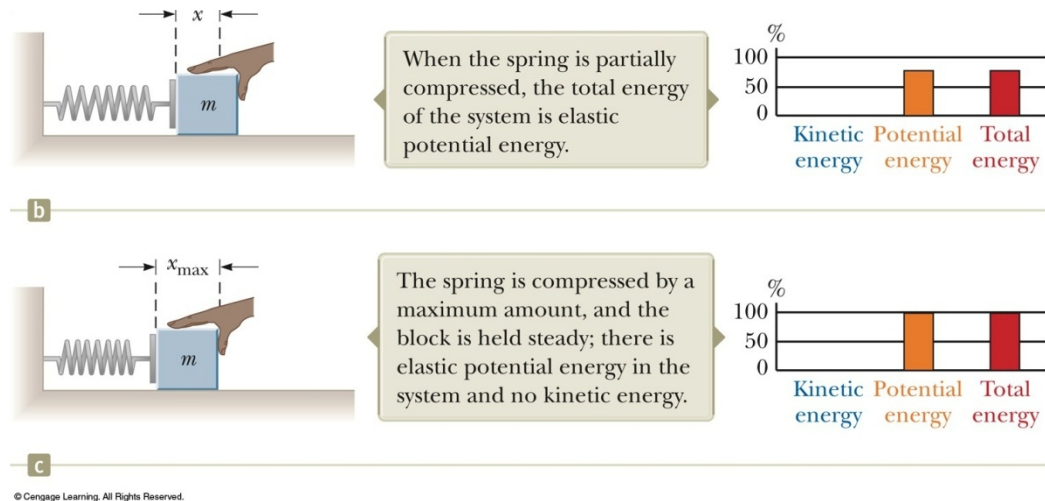energy   energy   energy

a

An energy bar chart is an important graphical representation of information related to the energy of a system.

- The vertical axis represents the amount of energy of a given type in the system.

- The horizontal axis shows the types of energy in the system.

In a, there is no energy.

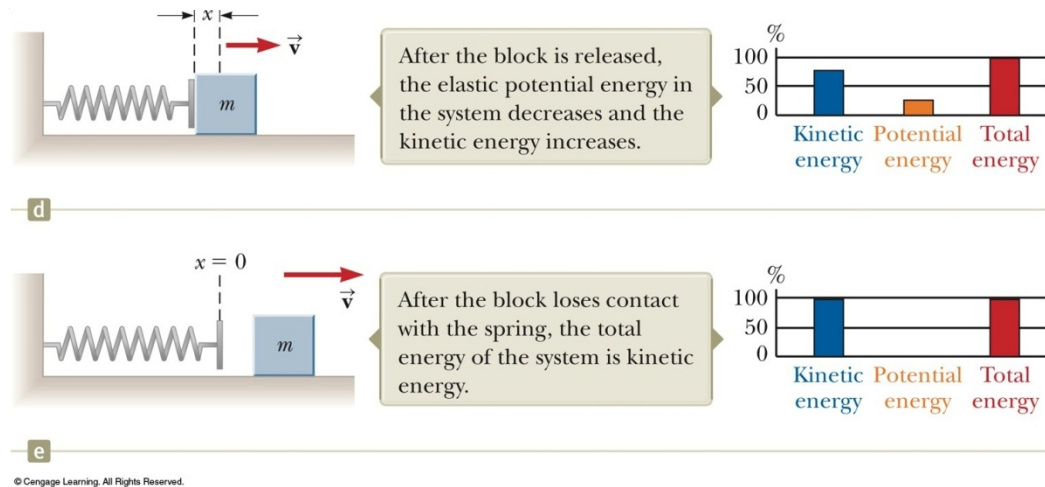- The spring is relaxed, the block is not moving

Energy Bar Chart Example, cont.

When the spring is partially compressed, the total energy of the system is elastic potential energy.

The spring is compressed by a maximum amount, and the block is held steady; there is elastic potential energy in the system and no kinetic energy.

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Between b and c, the hand has done work on the system.

- The spring is compressed.

- There is elastic potential energy in the system.

- There is no kinetic energy since the block is held steady.

Energy Bar Chart Example, final

After the block is released, the elastic potential energy in the system decreases and the kinetic energy increases.

After the block loses contact with the spring, the total energy of the system is kinetic energy.

In d, the block has been released and is moving to the right while still in contact with the spring.

- The elastic potential energy of the system decreases while the kinetic energy increases.

In e, the spring has returned to its relaxed length and the system contains only kinetic energy associated with the moving block.
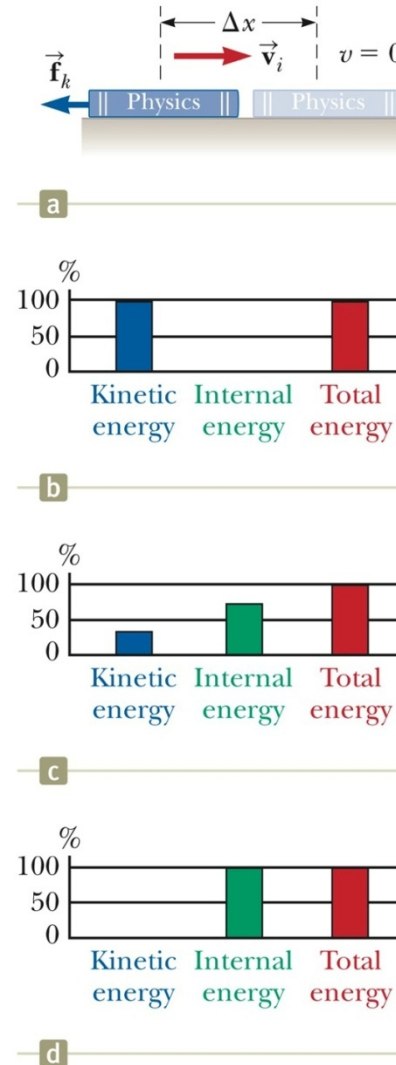
Internal Energy

The energy associated with an object's temperature is called its *internal energy*, $E_{int.}$

In this example, the surface is the system.

The friction does work and increases the internal energy of the surface.

When the book stops, all of its kinetic energy has been transformed to internal energy.

The total energy remains the same.

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Conservative Forces

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The work done by a conservative force on a particle moving through any closed path is zero.

- A closed path is one in which the beginning and ending points are the same.

Examples of conservative forces:

- Gravity

- Spring force

Conservative Forces, cont

We can associate a potential energy for a system with any conservative force acting between members of the system.

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Non-conservative Forces

A non-conservative force does not satisfy the conditions of conservative forces.

Non-conservative forces acting in a system cause a *change* in the mechanical energy of the system.

$E_{mech} = K + U$

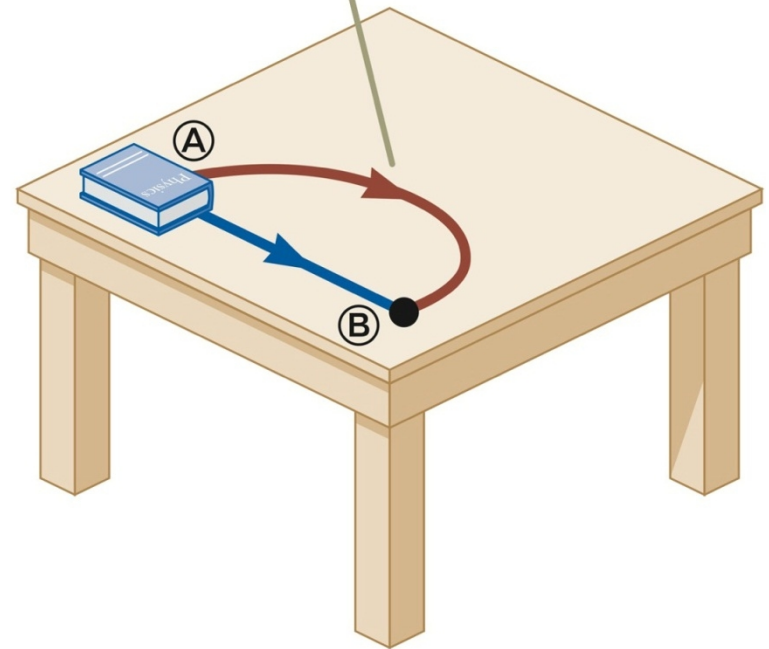- K includes the kinetic energy of all moving members of the system.

- U includes all types of potential energy in the system.

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Non-conservative Forces, cont.

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Because the work done depends on the path, friction is a non-conservative force.

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Conservative Forces and Potential Energy

Define a potential energy function, U, such that the work done by a conservative force equals the decrease in the potential energy of the system.

The work done by such a force, F, is

$$W_{int} = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

- ΔU is negative when F and x are in the same direction

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Conservative Forces and Potential Energy

The conservative force is related to the potential energy function through.

$$F_x = -\frac{dU}{dx}$$

The *x* component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to *x*.

- Can be extended to three dimensions
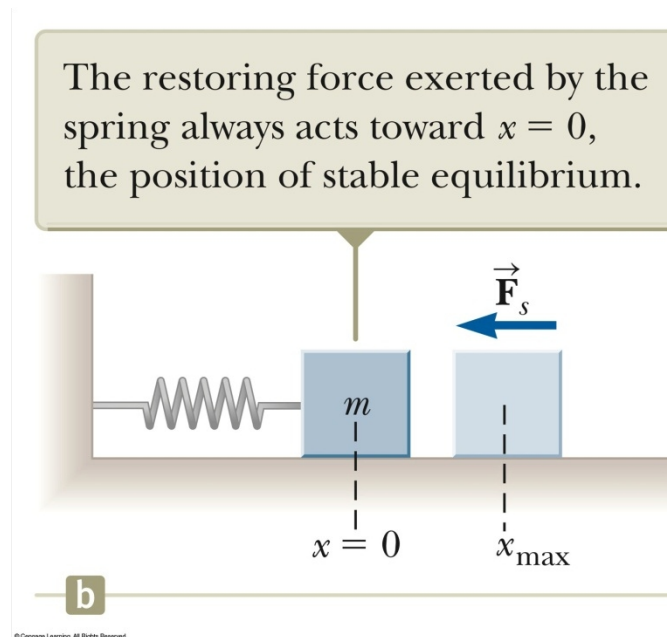
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Conservative Forces and Potential Energy – Check

Look at the case of a deformed spring:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

- This is Hooke's Law and confirms the equation for U

U is an important function because a conservative force can be derived from it.

Energy Diagrams and Equilibrium

The restoring force exerted by the spring always acts toward $x = 0$, the position of stable equilibrium.

$\vec{F}_s$

m

$x = 0$ x_{max}

b

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Motion in a system can be observed in terms of a graph of its position and energy.

In a spring-mass system example, the block oscillates between the turning points, $x = \pm x_{max.}$

The block will always accelerate back toward $x = 0$.

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Energy Diagrams and Stable Equilibrium

The $x = 0$ position is one of **stable equilibrium.**

▪ Any movement away from this position results in a force directed back toward x = 0.

Configurations of stable equilibrium correspond to those for which $U(x)$ is a minimum.

$x = x_{max}$ and $x = -x_{max}$ are called the turning points.

$$U_s = \frac{1}{2}kx^2$$
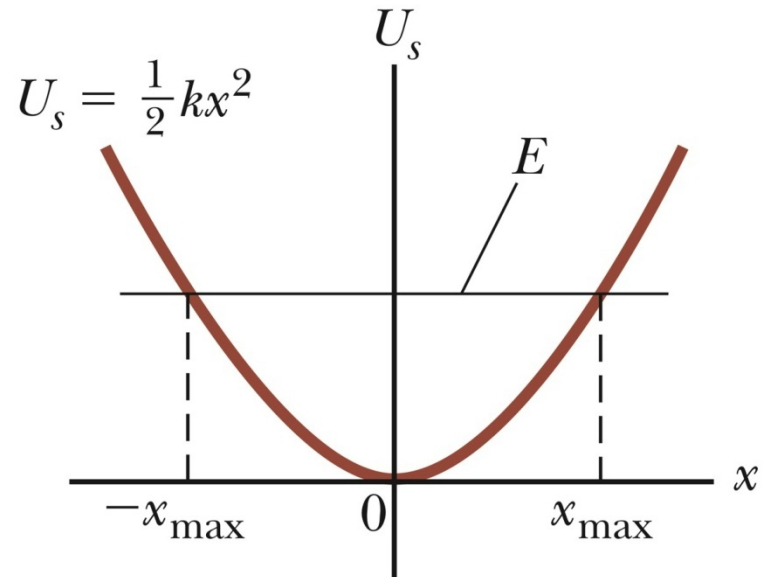
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Energy Diagrams and Unstable Equilibrium

$F_x = 0$ at $x = 0$, so the particle is in equilibrium.

For any other value of x, the particle moves away from the equilibrium position.

This is an example of **unstable equilibrium.**

Configurations of unstable equilibrium correspond to those for which $U(x)$ is a maximum.

Positive slope $x < 0$

Negative slope $x > 0$
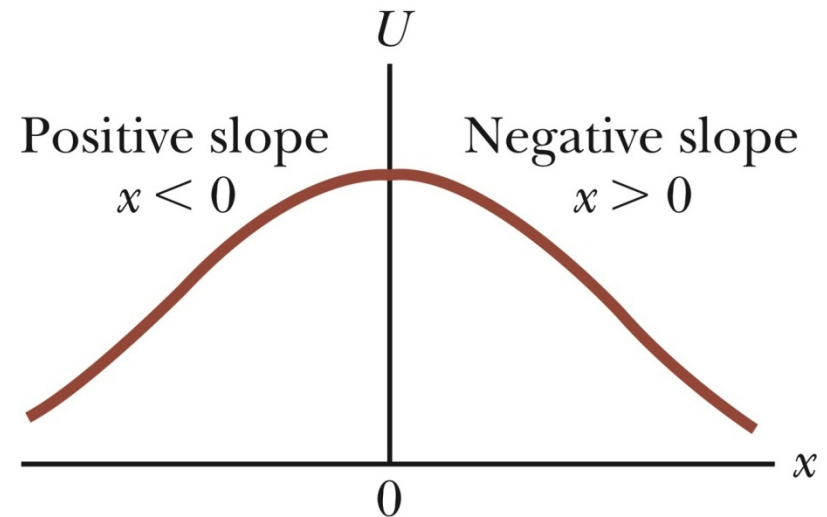
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Neutral Equilibrium

Neutral equilibrium occurs in a configuration when *U* is constant over some region.

A small displacement from a position in this region will produce neither restoring nor disrupting forces.

Potential Energy in Molecules

There is potential energy associated with the force between two neutral atoms in a molecule which can be modeled by the *Lennard-Jones* function.
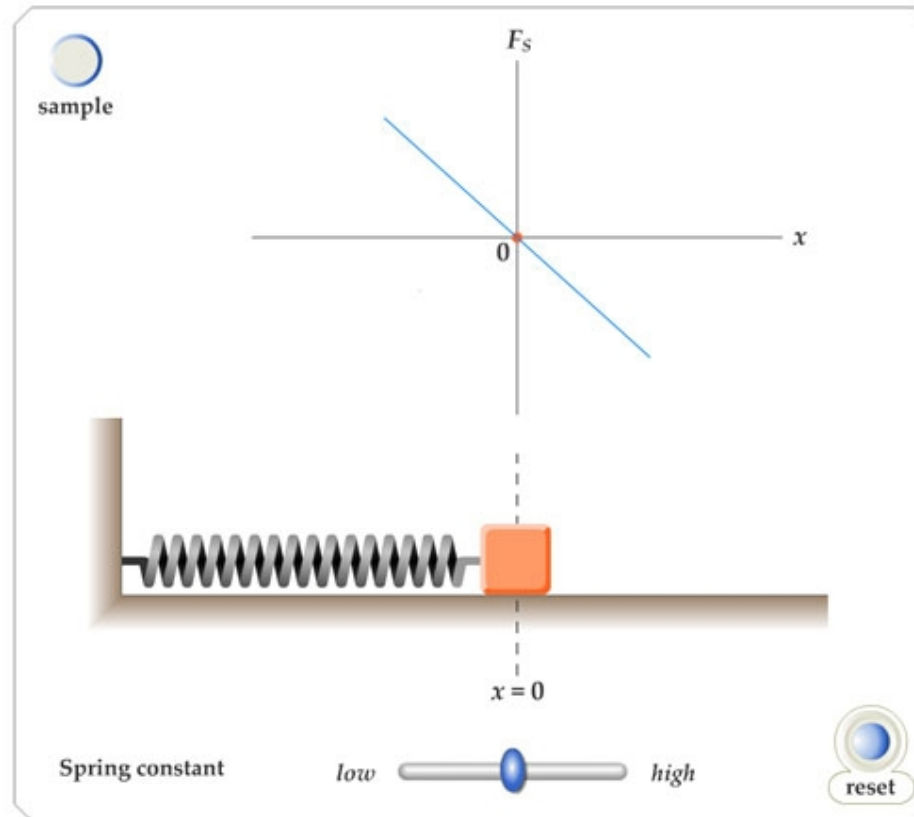
$$U(x) = 4 \in \left[\left(\frac{\sigma}{x} \right)^{12} - \left(\frac{\sigma}{x} \right)^{6} \right]$$

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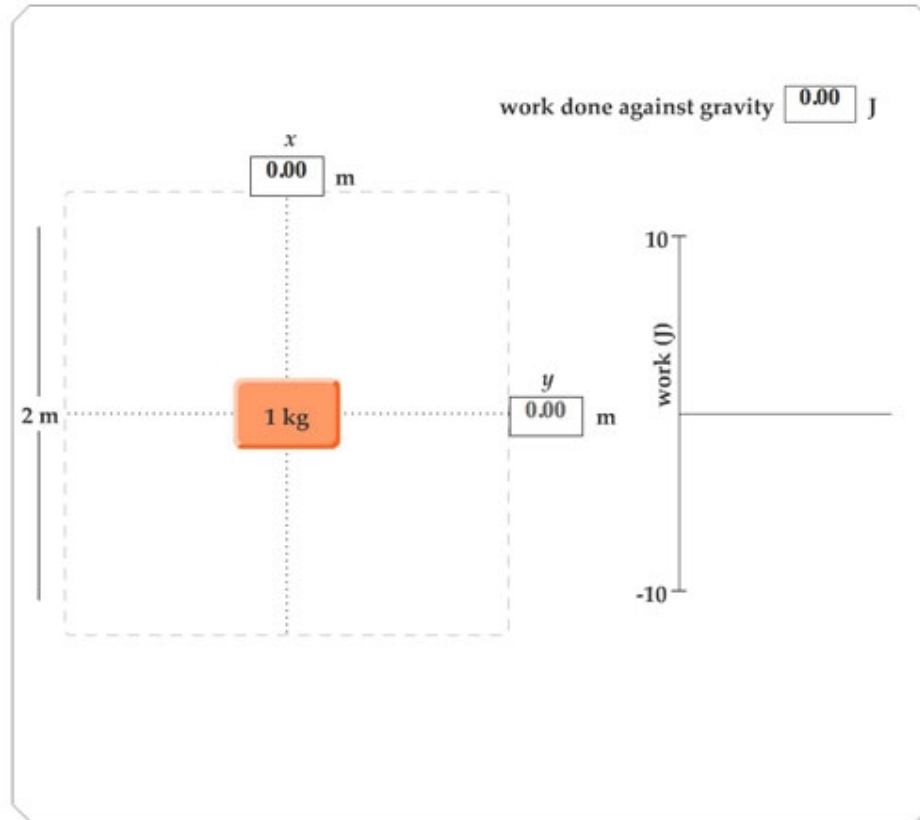Find the minimum of the function (take the derivative and set it equal to 0) to find the separation for stable equilibrium.

The graph of the Lennard-Jones function shows the most likely separation between the atoms in the molecule (at minimum energy).
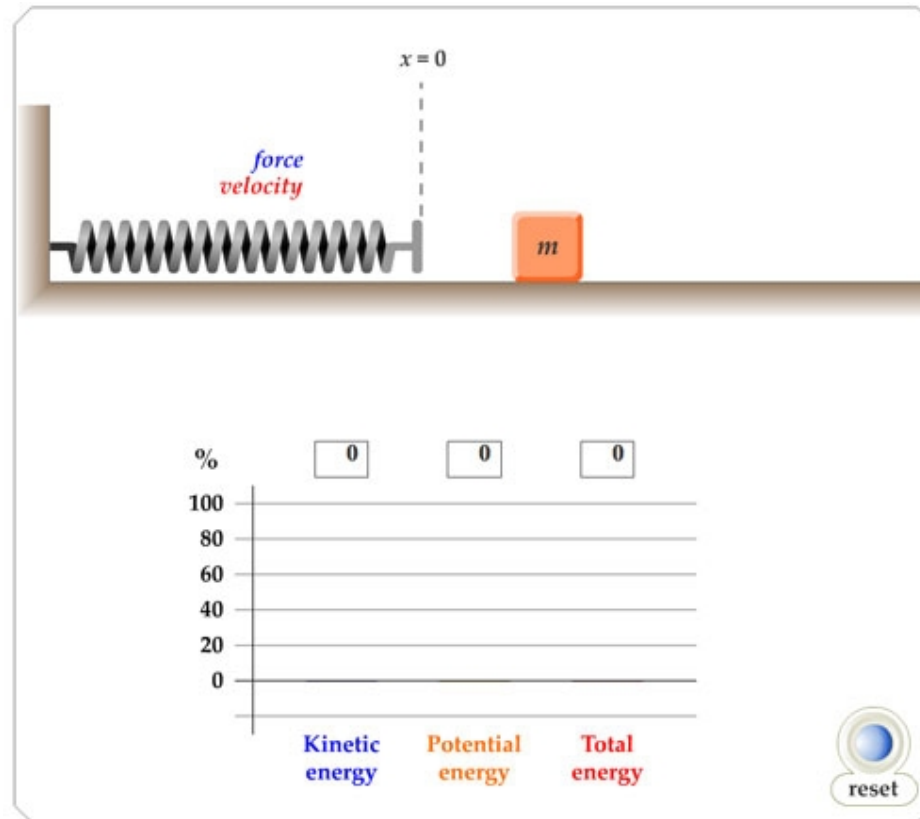
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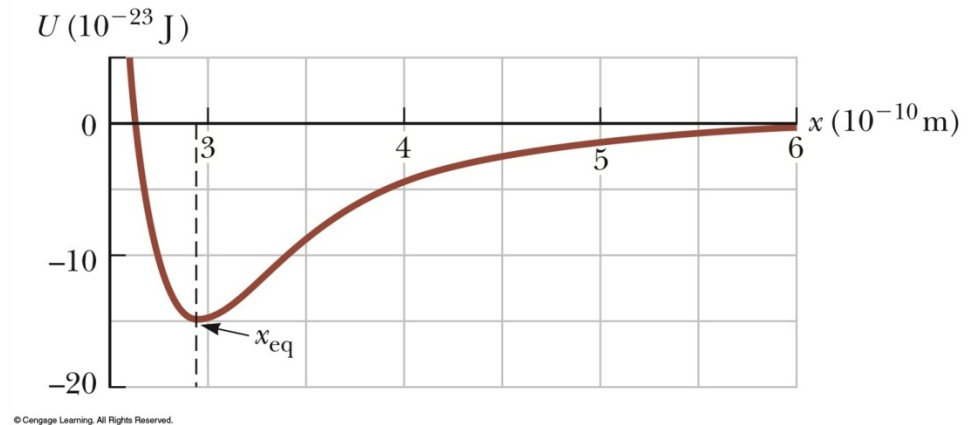
7.9 Hooke's Law and Restoring Forces

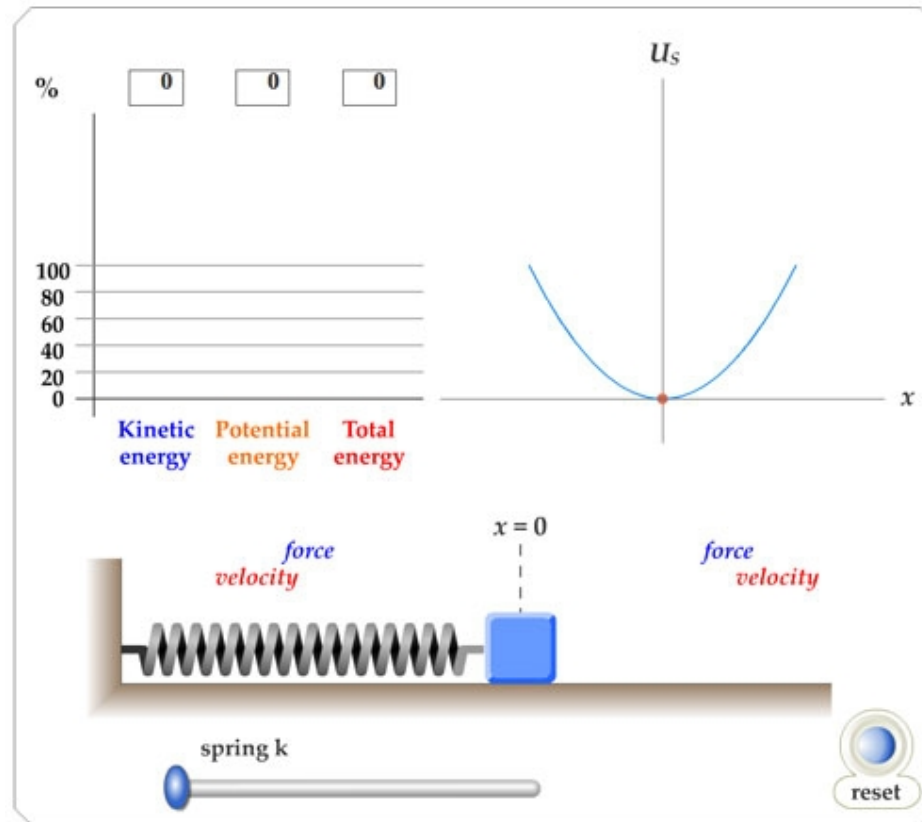# 7.15 Work Done By External Agent on System of the Book

7.16 Conservation Mechanical Energy in Block-Spring System

7.18 Work When Kinetic Friction is Present

7.20 An Oscillating Block-Spring System

7.20b Conservation of Mechanical Energy for a Pendulum