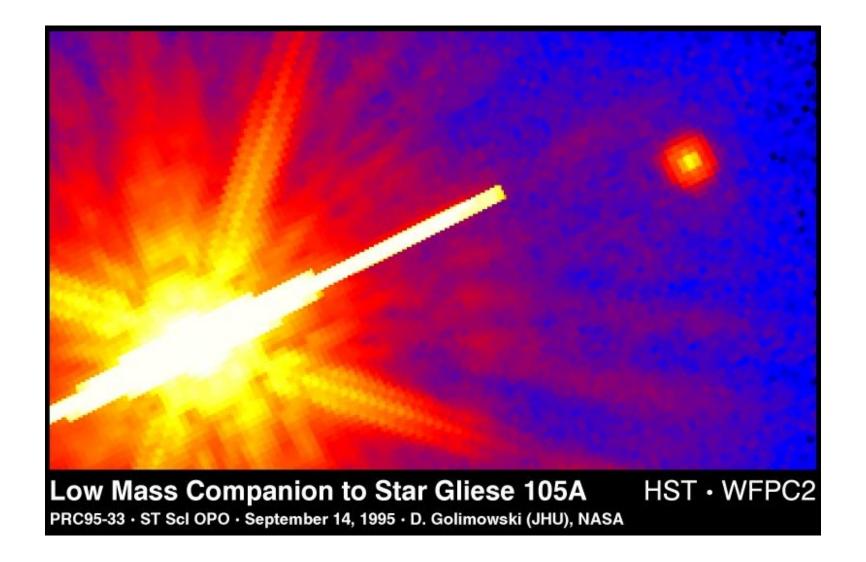
# Binary Star Systems

(Chapter 7)



## Binary Star Systems

- 1) Classification of binary star systems
  - 1) Optical Doubles
  - 2) Visual binary
  - 3) Astrometric Binary
  - 4) Eclipsing Binary (detached, semi-detached, unattached)
  - 5) Spectrum Binary
  - 6) Spectroscopic Binary
- 2) Stellar properties measured with binaries
  - 1) Mass
- 1) From Visual Binaries
- 2) Complications with Visual Binary method
- 2) Radii, Surface brightness, Surface temperature
- 3) Mass functions from Spectroscopic Binaries

- Multiple systems are more of a rule than an exception
  - At least half of all stars are multiple systems

#### Optical Double

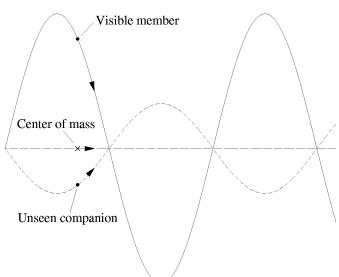
- Lie along the same line of sight and look as though they are companions
- They have no physical proximity to one another
- Not gravitationally bound
- Not a binary star system!

#### Visual binary

Sufficiently close to Earth and the stars are well enough separated that we can see the two stars individually (resolved) in a telescope and track their motion over a period of time

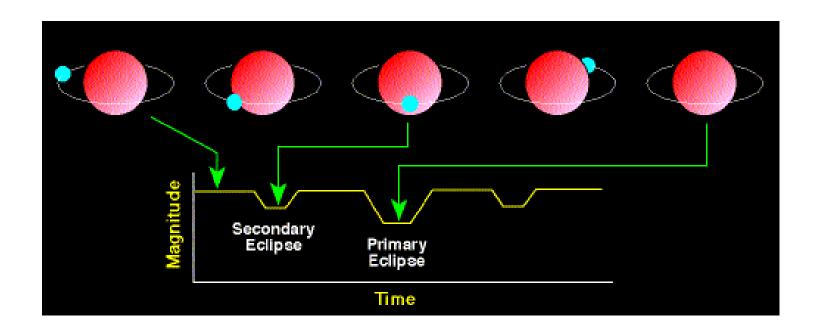
#### Astrometric binary

- Detect the presence of an unseen companion
   (faint, cool star) by its gravitational influence
   on the primary star
- Deviation in proper motion

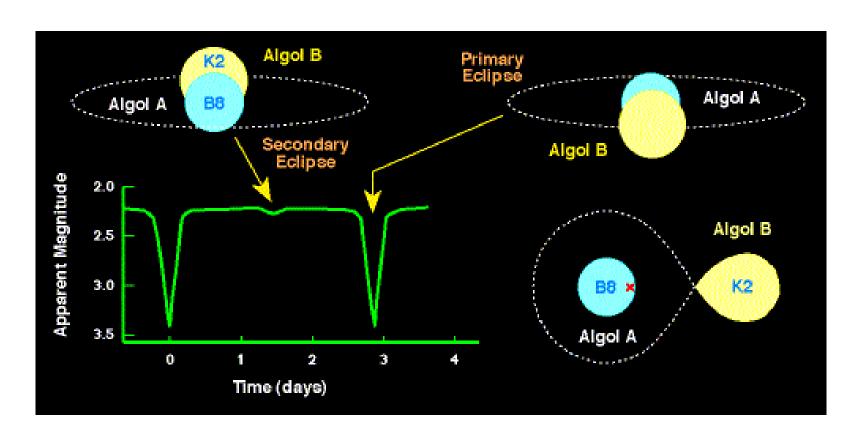


#### Eclipsing binary

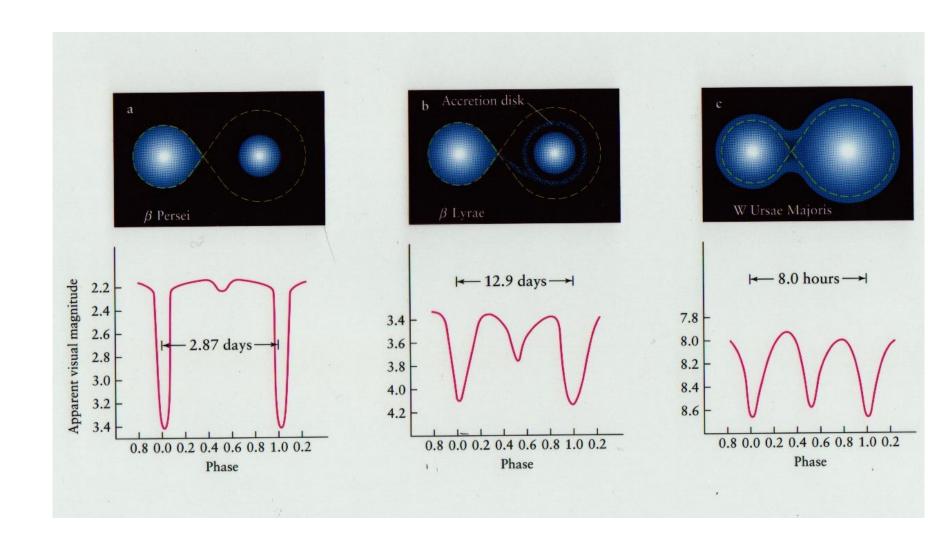
- Orbital planes along the line of sight (or nearly so)
- The system is usually not resolved in the telescope (not vis binary)
- Photoelectric or CCD photometry over time produces light curve
- Q: What can you tell from the shape of the eclipse dips?
- Q: How could you tell that the blue star moved left to right?



- Eclipsing binary Example Algol
  - Q: Why is the eclipse of the B8 star deeper than the eclipse of the K2 star?
    - Smaller blue star emits more light per unit surface area than the larger red star
  - Q: Why don't the minima have flat bottoms?
    - During eclipses, neither star gets completely covered up by the other.



Eclipsing binary Examples



#### Spectrum binary

- System with two superimposed, independent, discernable spectra

Normally, each star as a unique spectrum (spectral class). For example, a hot star has a spectrum rich in hydrogen lines



A cool star has thicker lines from metals, such as below

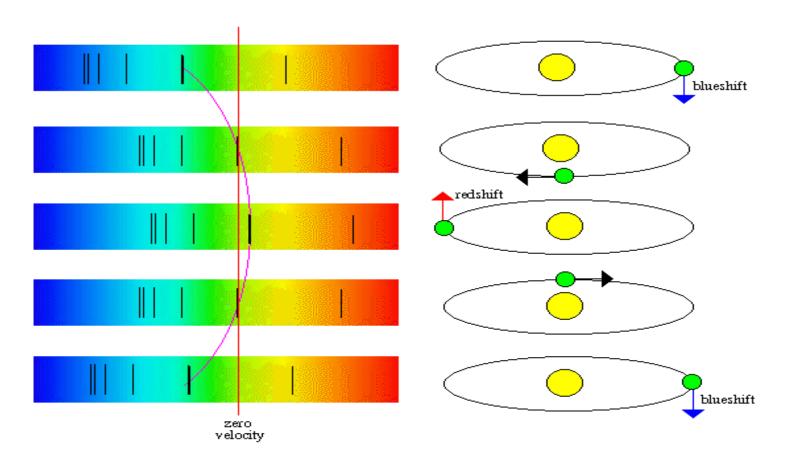


So, a spectrum binary is when you can not see two stars on the sky, but a spectrum of the object show two difference stellar classes, as below.

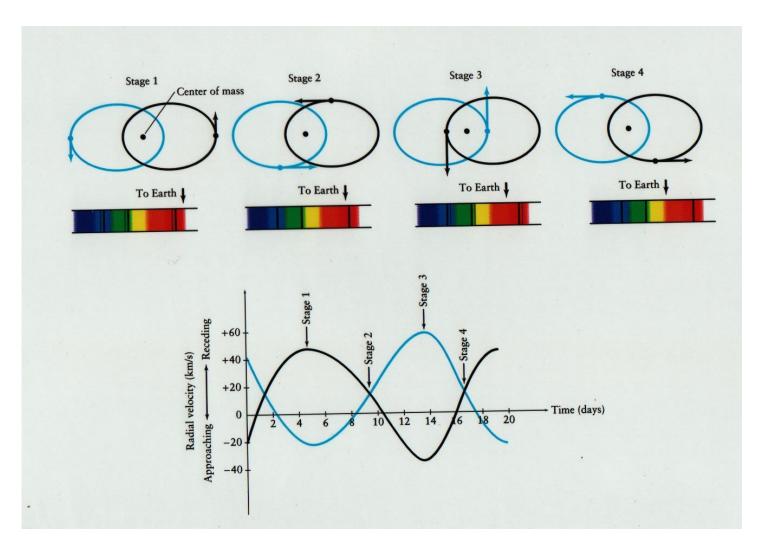


#### Spectroscopic Binary

- Pair cannot be resolved as a visual binary
- Measure relative velocities via the Doppler shift of their spectral lines
- Motion is usually seen in the lines of one star

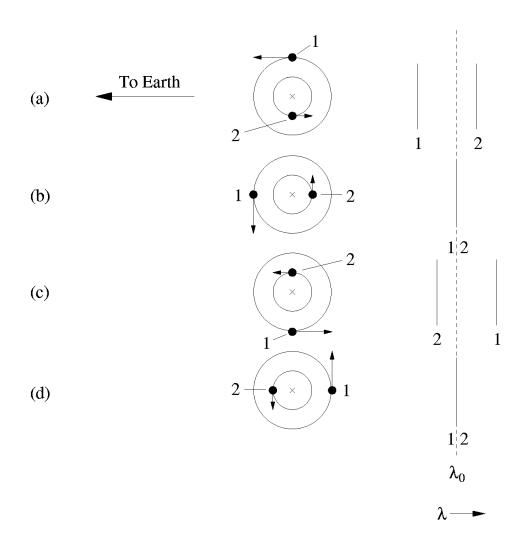


Spectroscopic binary



#### Spectroscopic Binary

Double-line spectroscopic binary



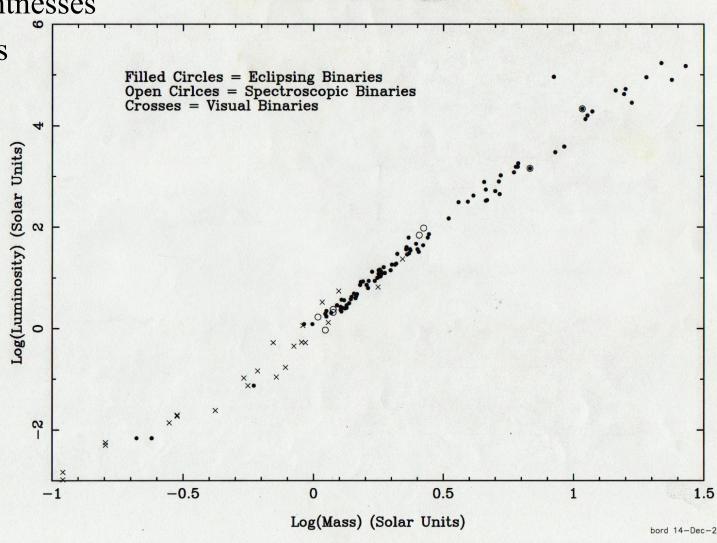
$$\frac{\Delta \lambda}{\lambda_0} = \frac{v}{c}$$

## Stellar Properties from binaries

Radii

Mass-Luminosity Relation Surface brightnesses **Temperatures** 

**MASSES** 

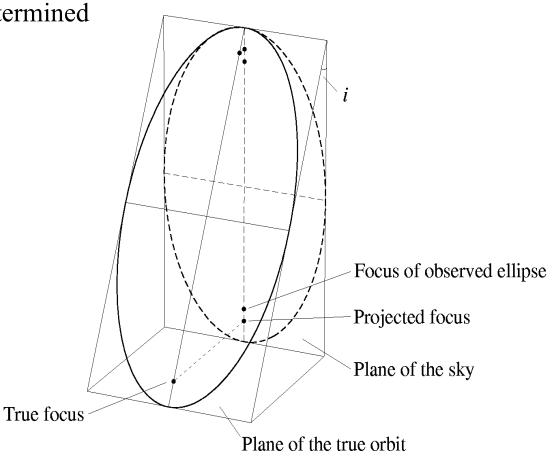


## Mass Determination Using Visual Binaries

- Angular separation greater than telescope resolution
- Observation of orbits yields ratio of stars' masses
- If distance is known

Individual masses can be determined

- angle of inclination, *i* 
  - Angle between the plane of the orbit and the plane of the sky



# Mass Determination Using Visual Binaries

- Let's assume  $i = 0^{\circ}$
- From the center of mass coordinates
  - Let's put CM at 0 and find ratio of massε

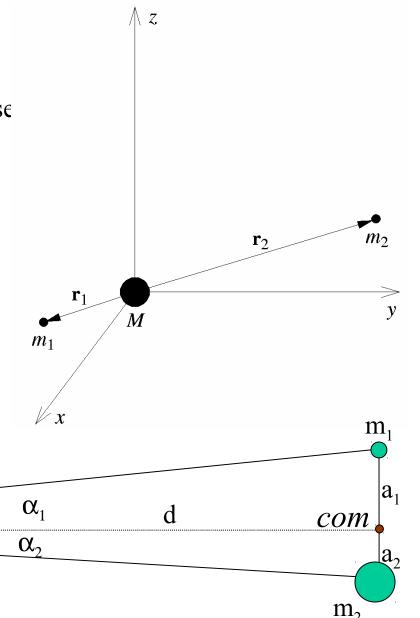
$$R = m_1 r_1 + m_2 r_2 = 0$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{a_2}{a_1}$$

• If the distance is known

$$\alpha_1 = \frac{a_1}{d}, \ \alpha_2 = \frac{a_2}{d}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}$$



# Mass Determination Using Visual Binaries

General Form of Kepler's third law

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

Where P is the period of orbit and a is the semi-major axis of the reduced mass

$$a=a_1+a_2$$

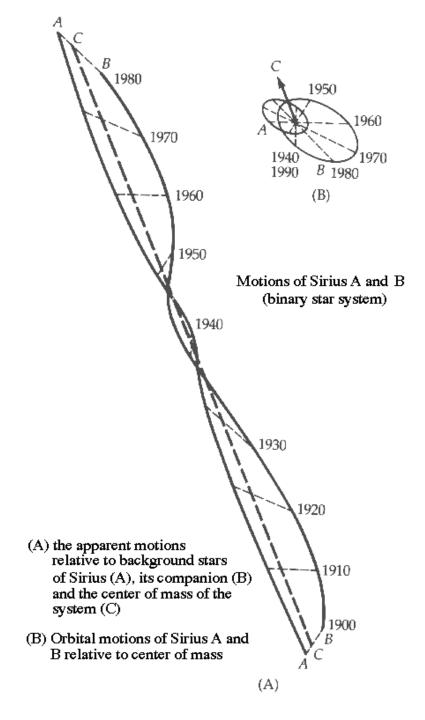
• Two equations and two unknowns, game over

$$\frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}$$

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

# Mass Determination... Other complications:

- Parallax
- Proper motion
  - Center of mass is at a constant velocity
  - Can be factored out



## Non-zero angle of Inclination

• The angles subtended by the semimajor axes will be reduced

$$\tilde{\alpha}_1 = \alpha_I \cos i$$

$$\tilde{\alpha}_2 = \alpha_2 \cos i$$

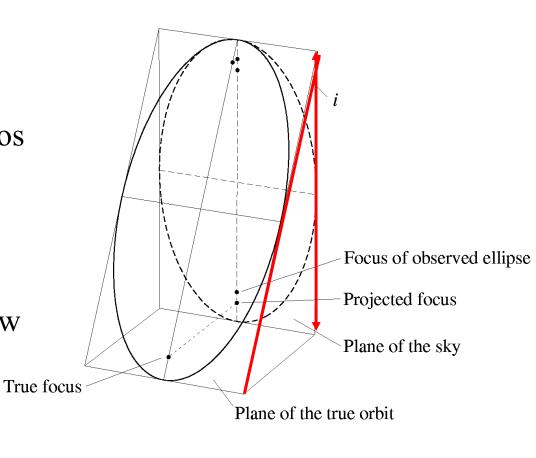
Doesn't matter for mass ratios

$$\frac{m_1}{m_2} = \frac{\tilde{\alpha}_2}{\tilde{\alpha}_1} = \frac{\alpha_2 \cos i}{\alpha_1 \cos i} = \frac{\tilde{\alpha}_2}{\tilde{\alpha}_1}$$

Does matter for Kepler's Law

$$P^{2} = \frac{4\pi^{2}}{G(m_{1} + m_{2})} a^{3}$$

$$\alpha = \frac{a}{d} \Rightarrow a = \alpha d$$



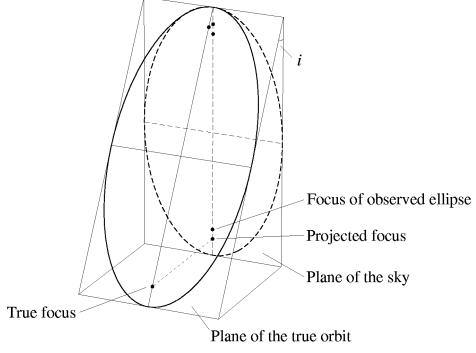
## Angle of Inclination

$$P^{2} = \frac{4\pi^{2}}{G\left(m_{1} + m_{2}\right)} (\alpha d)^{3}$$

$$\alpha = \frac{\tilde{\alpha}}{\cos i}$$

$$m_{1} + m_{2} = \frac{4\pi^{2}}{GP^{2}} \left(\frac{\tilde{\alpha}}{\cos i} d\right)^{3}$$

$$m_{1} + m_{2} = \frac{4\pi^{2}}{GP^{2}} (\tilde{\alpha} d)^{3} \left(\frac{1}{\cos i}\right)^{3}$$
True



- Angle of inclination must be determined
- Resolved by noting the *Projected focus* does not coincide with the *observed focus* 
  - Center of mass will be off
  - Inconsistencies will result

# Spectroscopic Binaries

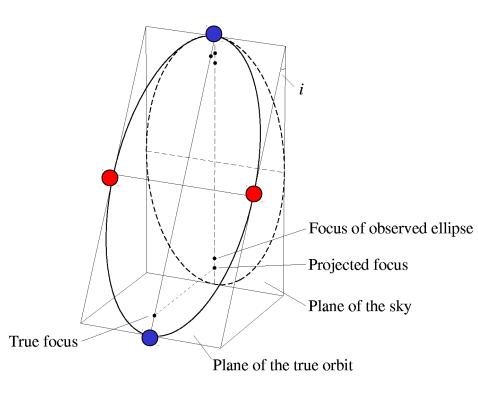
- Usually not a resolvable system
- Double line each star's spectra is seen
- Can determine:
  - Individual mass, radii, ratio of flux and temperature
- Angle of inclination important
  - Determines the radial velocities

$$v_{1r}^{\max} = v_I \sin i$$

$$v_{2r}^{\text{max}} = v_2 \sin i$$

**max** 

zero



# Spectroscopic Binaries for mass functions

- $v_{1r}$  and  $v_{2r}$  must both be measurable
  - Double-line spectroscopic binary vs. single-line spectroscopic binary
  - Comparable brightness or one star may be overwhelmed

$$\frac{m_{1}}{m_{2}} = \frac{v_{2r}}{v_{1r}}$$

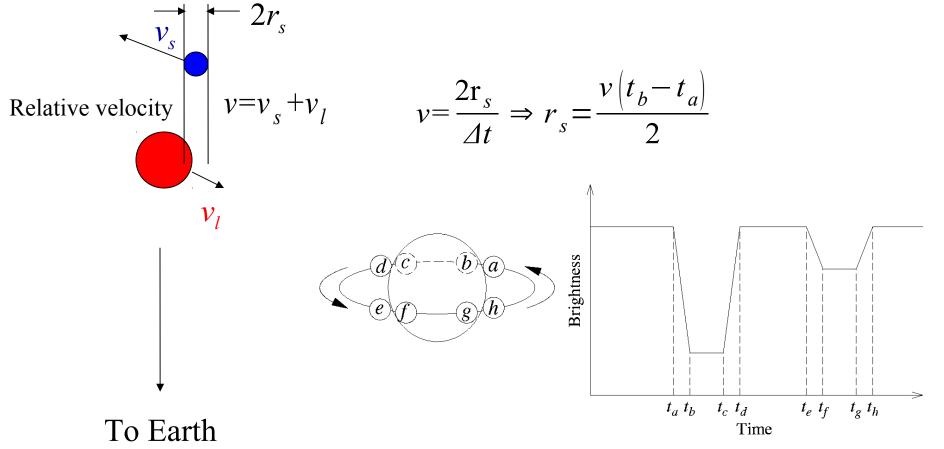
$$m_{1} + m_{2} = \frac{P}{2\pi G} \frac{(v_{1r} + v_{2r})^{3}}{\sin^{3} i} = \frac{Pv_{Ir}^{3}}{2\pi G} \frac{\left(1 + \frac{v_{2r}}{v_{1r}}\right)^{3}}{\sin^{3} i}$$

$$\frac{Pv_{Ir}^{3}}{2\pi G} \frac{\left(1 + \frac{m_{1}}{m_{2}}\right)^{3}}{\sin^{3} i} = \frac{Pv_{Ir}^{3}}{2\pi G} \frac{\left(\frac{m_{2}}{m_{2}} + \frac{m_{1}}{m_{2}}\right)^{3}}{\sin^{3} i} = \frac{Pv_{Ir}^{3}}{2\pi G} \frac{(m_{1} + m_{2})^{3}}{m_{2}^{3} \sin^{3} i}$$

Mass function 
$$\frac{m_2^3}{(m_1 + m_2)^2} \sin^3 i = \frac{P}{2\pi G} v_{1r}^3$$
 Depends on  $P$  and  $v_{Ir}$ 

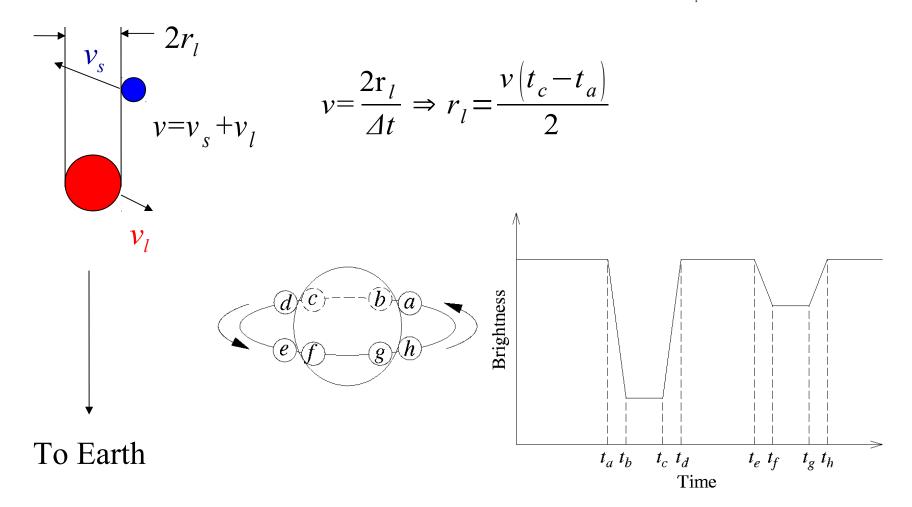
# Eclipsing, Spectroscopic Binaries for radii

- Estimation of radii
- Assume  $i \approx 90^{\circ}$
- Time duration from first contact  $(t_a)$  to minimum light  $(t_b)$  yields info about the radius of smaller star,  $\Delta t = t_b t_a$



# Eclipsing, Spectroscopic Binaries

- Time duration from first contact  $(t_a)$  to first exposure  $(t_c)$  yields info about radius of the larger star,  $\Delta t = t_c t_a$ 
  - This time the distance traveled in-between these events is  $r_1$



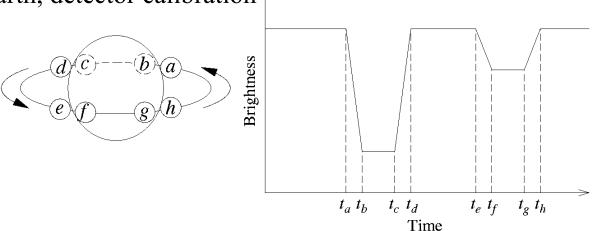
# **Eclipsing Binaries**

- Ratio of effective Temperatures
  - Obtained from the light curve
  - Radiative Surface Flux  $F_r = F_{surf} = \sigma T_e^4$
- Both stars visible with no obstructions
  - Amount of light detected

$$B_{o} = B_{l} + B_{s} = k (A_{l}F_{rl} + A_{s}F_{s}) = k (\pi r_{l}^{2}F_{rl} + \pi r_{s}^{2}F_{s})$$

k is a constant that depends on physical constants that will be the same for each star

*i.e.* distance from Earth, detector calibration \( \)



## **Eclipsing Binaries**

- Primary eclipse minima
  - Only the large star is visible

$$B_p = B_l = k\pi r_l^2 F_{rl}$$

- Secondary eclipse minima
  - Part of the large star is covered by the smaller, hotter star

$$B_{s} = B_{l} - B_{l}^{s} + B_{s} = k \left( \pi r_{l}^{2} F_{rl} - \pi r_{s}^{2} F_{rl} + \pi r_{s}^{2} F_{rs} \right)$$

Examining ratios which allow unknown factors to cancel

$$\begin{split} &\frac{B_{o} - B_{p}}{B_{o} - B_{s}} = \frac{\left(B_{l} + B_{s}\right) - \left(B_{l}\right)}{\left(B_{l} + B_{s}\right) - \left(B_{l} - B_{l}^{s} + B_{s}\right)} = \frac{B_{s}}{B_{l}^{s}} \\ &\frac{k\pi r_{s}^{2} F_{rs}}{k\pi r_{s}^{2} F_{rl}} = \frac{F_{rs}}{F_{rl}} = \left(\frac{T_{s}}{T_{l}}\right)^{4} \Rightarrow \frac{B_{o} - B_{p}}{B_{o} - B_{s}} = \left(\frac{T_{s}}{T_{l}}\right)^{4} \end{split}$$