The Classification of Stellar Spectra
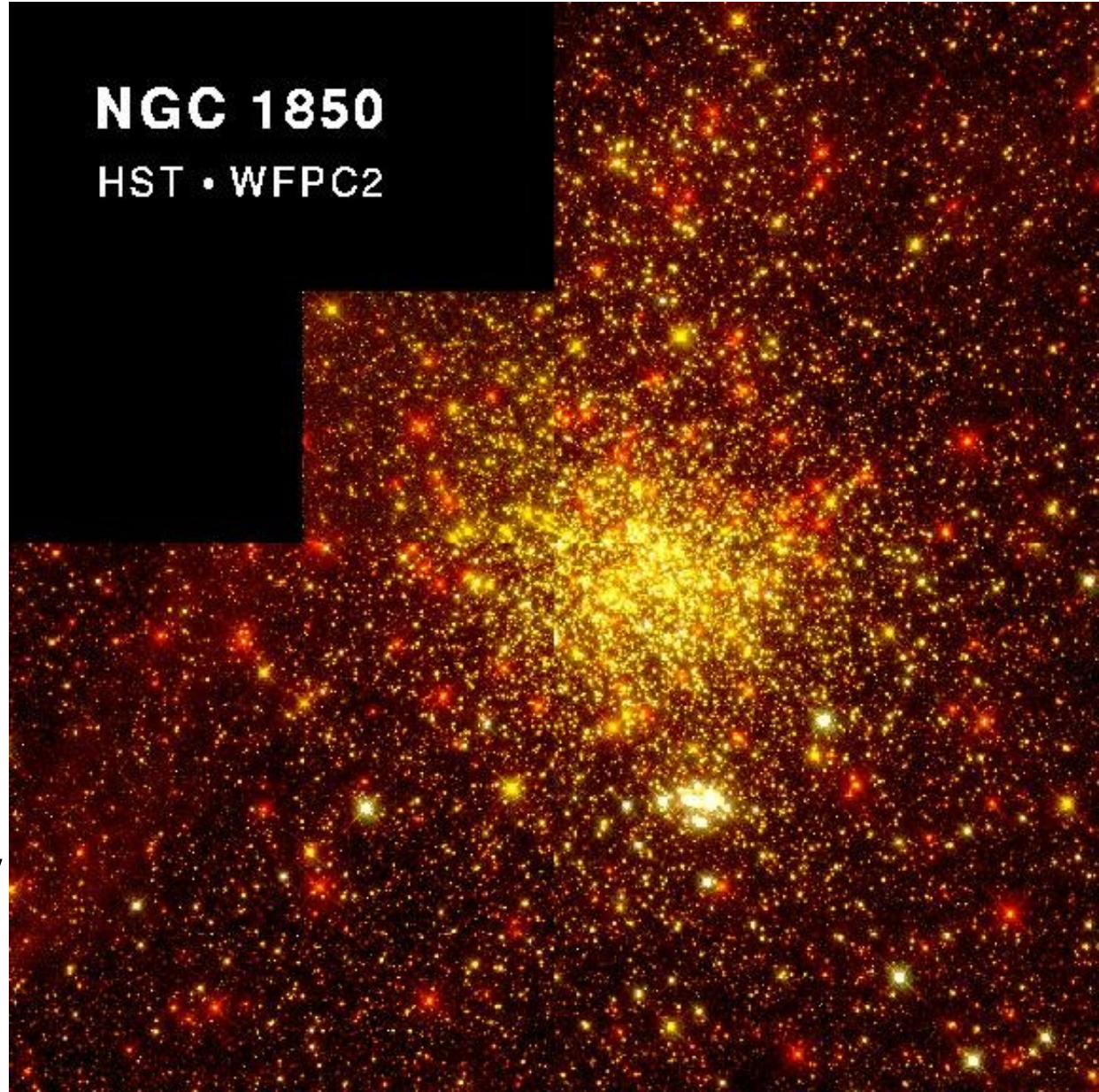
Chapter 8

Star Clusters in the Large Magellanic Cloud

NGC 1850
HST • WFPC2

http://www.seds.org/hst/ NGC1850.html

The Classification of Stellar Spectra

- Classification scheme developed before the physics
- Parameters that could be used to classify stars
  - Apparant brightness (bad idea)
  - Luminosity (Intrinsic brightness)
  - Temperature (Color)
  - Spectra (absorption lines)
  - Mass (only for binaries)

- The Henry Draper Catalogue
  - Contained >100,000 spectral classifications from A.J. Cannon and others from Harvard
  - Used OBAFGKM

The Harvard "Computers" of the Harvard College Observatory

http://cannon.sfsu.edu/%7Egmarcy/cswa/history/pick.html

The Classification of Stellar Spectra

- Originally organized by strength of H Balmer lines (A,B,...).
- Atomic physics allowed connection to temperature to be made.
- Spectral Type:

Hotter ← O B A F G K M L T → Cooler

Violet   Blue   Yellow   Red
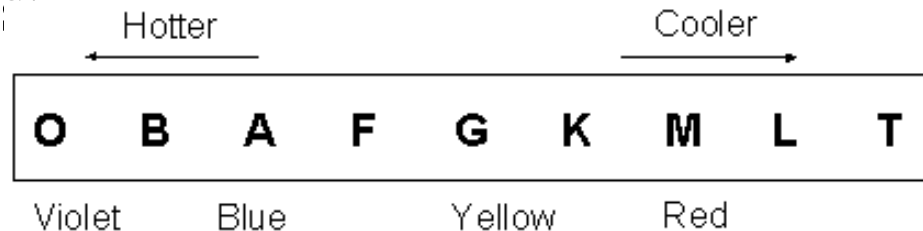
Early type → late type

L and T are more modern additions – Brown Dwarfs. R N S also used after M.

- – Subdivisions in tenths: 0 → 9 (early → late, hot → cool) within a Spectral Type). E.g., A0 is hotter than A5.
- – The Sun is a G2 – an early G-type star
  - G – yellow star (continuum peak in green/yellow)
    - – H lines weak
    - – Ca II (singly ionized) lines continue becoming stronger
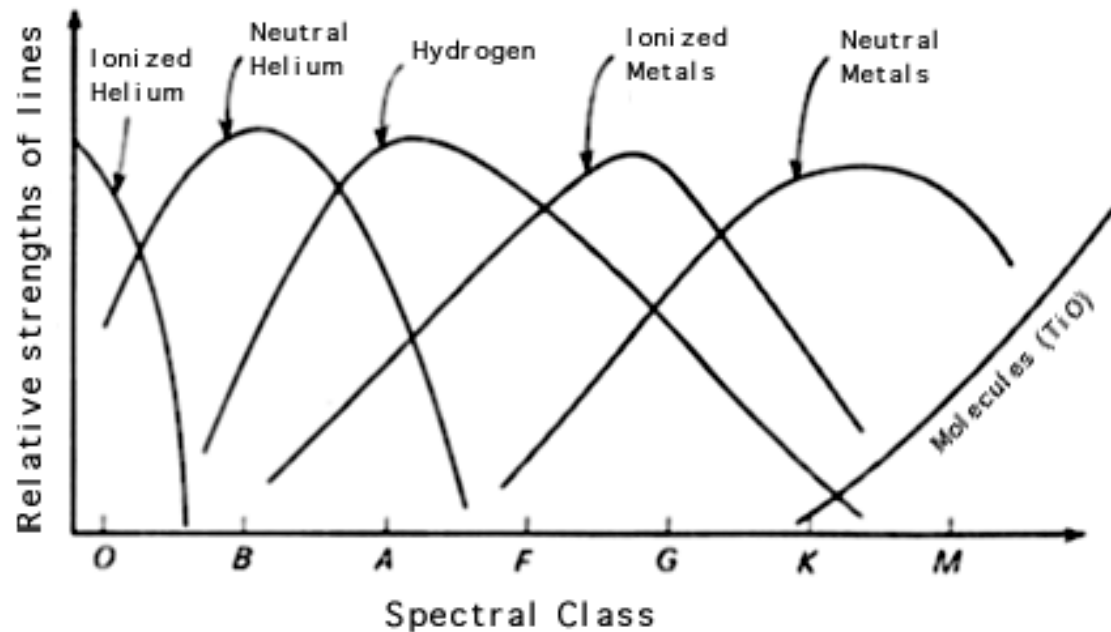   - – Fe I, other *neutrals* metal lines become stronger

O to G example
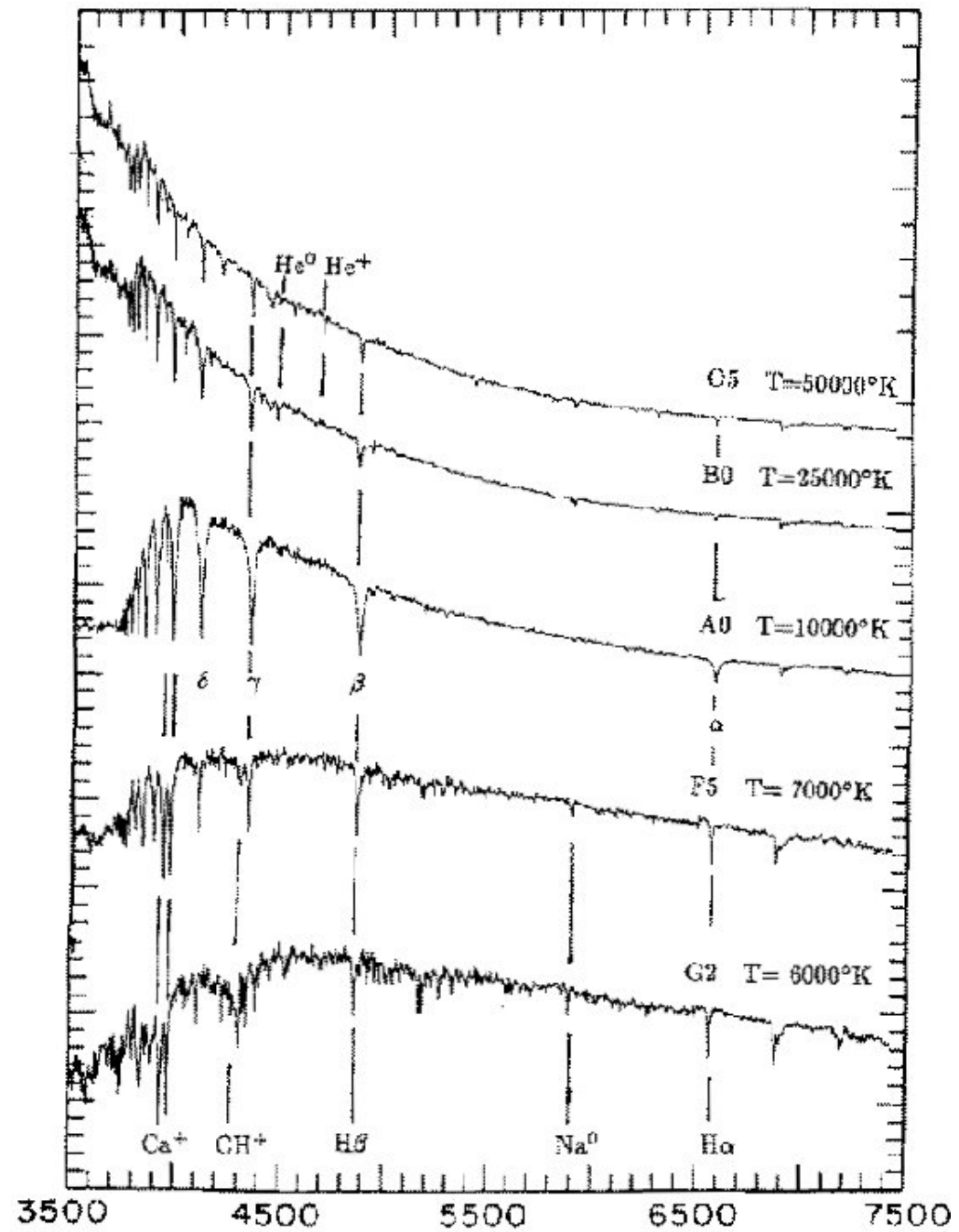
G to M example
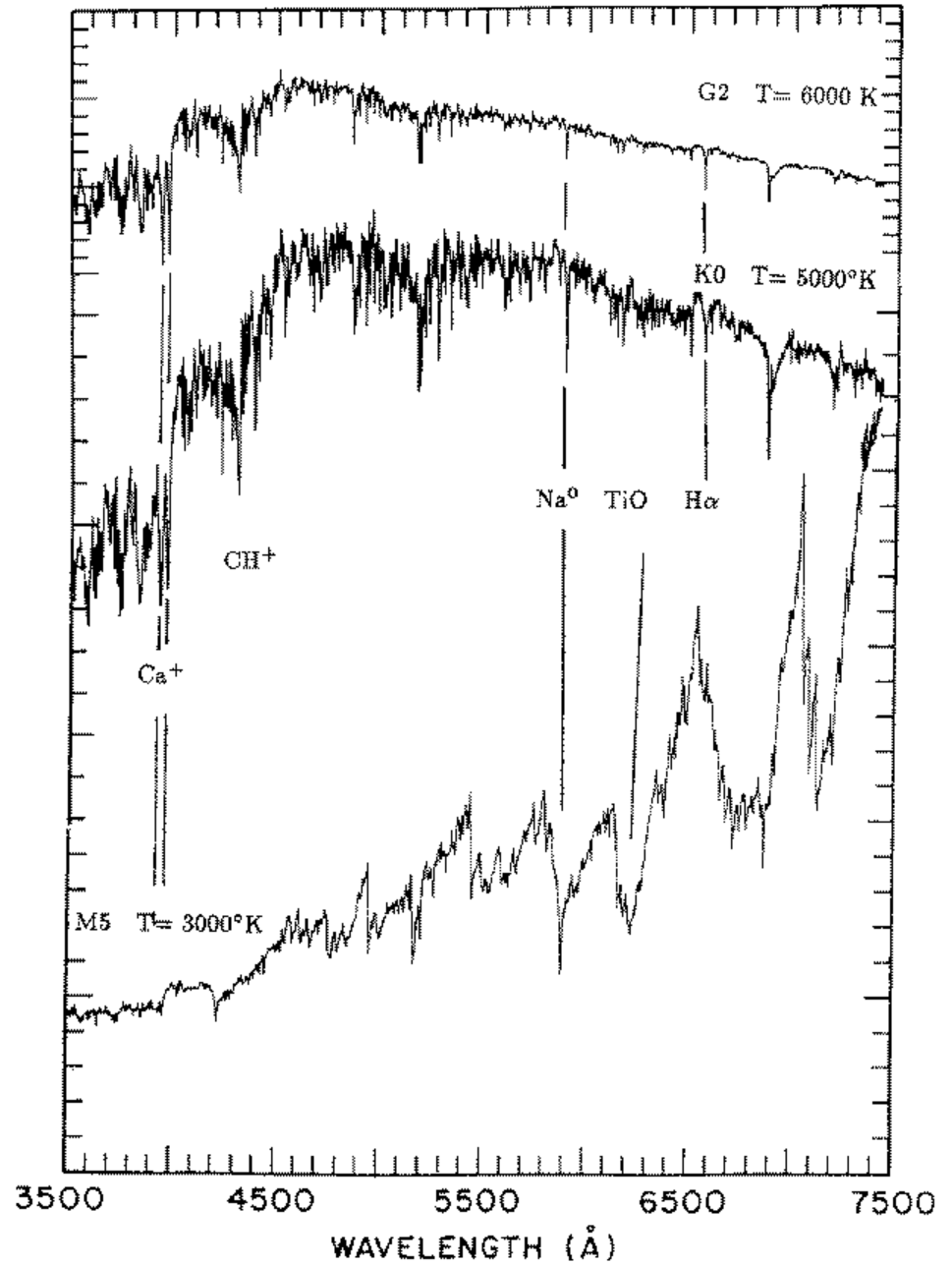
The Formation of Spectral Lines

- Question: What causes the differences in the observed spectra??
    - [Absorption by intervening material. Earth's atmos., ISM.]
    - Composition
    - Temperature
    - Surface gravity / pressure

- Answer:
    - Temperature is the main factor

The Formation of Spectral Lines

- Big Question of Ch.8: Why are the H balmer lines strongest for A stars, which seem to have T_surf = 10,000K?

- To find answer:
    - Need Ch.5's info about the Bohr atom … energy levels.
    - Need Kirchoff's laws → our gas is the upper "atmosphere" of the star.
    - Need statistical mechanics – study of large numbers of particles that can occupy different states
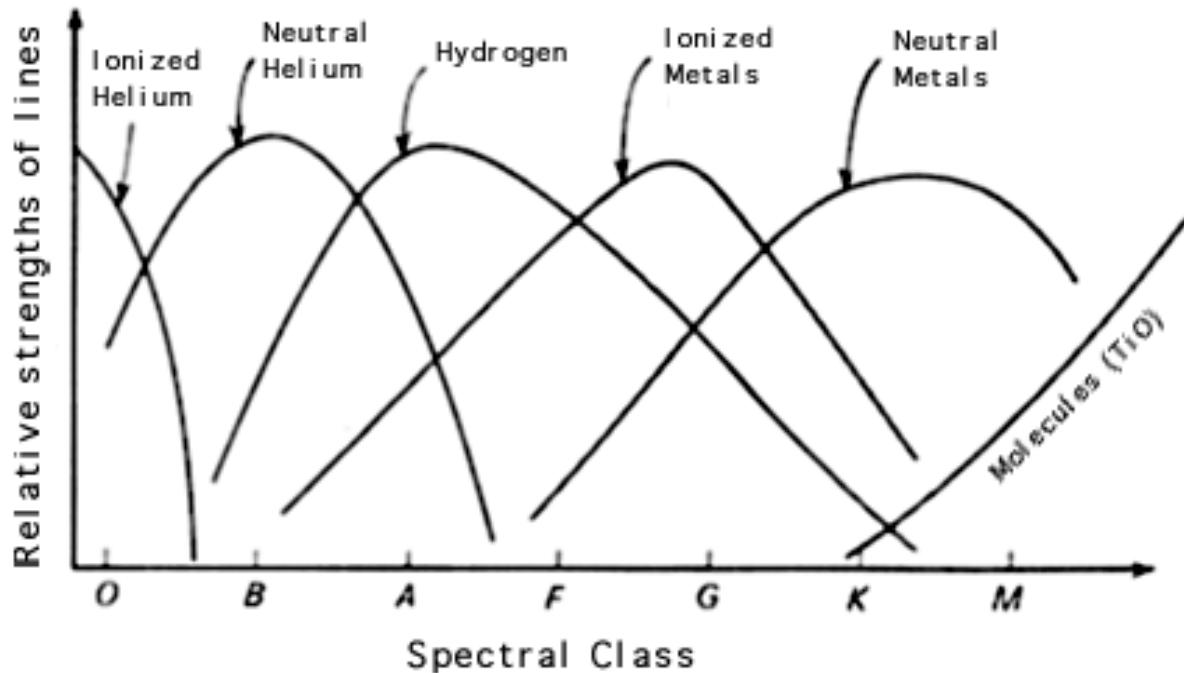
The Formation of Spectral Lines

- Distribution of electrons in different atomic orbitals depends on temperature

- Electrons can jump up in energy by absorption of a photon OR collision with a particle! So KE of surrounding particles important.

- What is the probability of finding an electron in a particular orbital?
  - Answer with Statistical Mechanics…
  - Maxwell-Boltzmann (velocity) Distribution
    - Assumes thermal equilibrium
    - Number of gas particles per unit volume have a speed between v and v+dv

$$n_v \, dv = n \left(\frac{m}{2\pi \, kT} \right)^{\frac{3}{2}} e^{-\frac{1}{2} mv^2 / kT} \, 4\pi \, v^2 \, dv$$

Maxwell-Boltzmann Distribution

$$n_v\,dv = n\left(\frac{m}{2\pi\,kT}\right)^{\frac{3}{2}} e^{-\frac{1}{2}mv^2/kT}\; 4\pi\,v^2\,dv$$

- Most probable speed

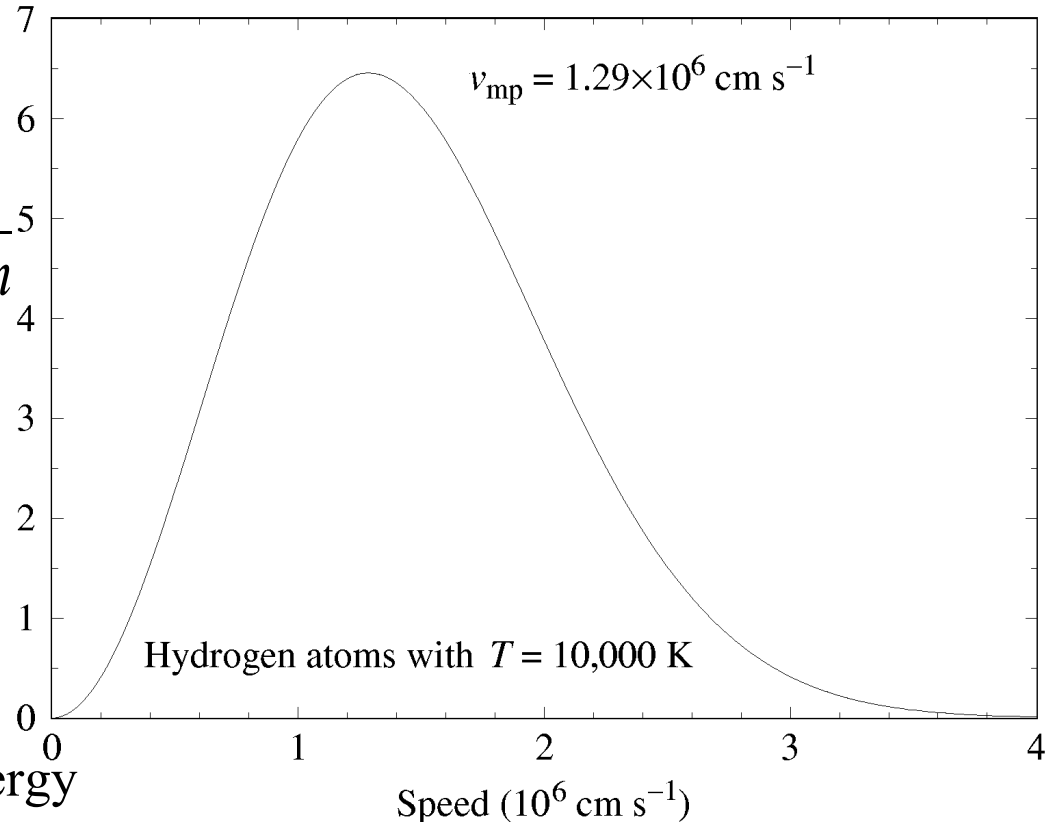$$v_{mp} = \sqrt{\frac{2\,kT}{m}} = 1.4\,\sqrt{kT/m}$$

- Root-mean-square

$$v_{rms} = \sqrt{\frac{3\,kT}{m}} = 1.73\,\sqrt{kT/m}$$

- Average

$$v_{avg} = \sqrt{\frac{8\,kT}{\pi m}} = 1.6\,\sqrt{kT/m}$$

- Collisional energy causes
  a distribution of electrons
  among the atomic orbitals
  (Kinetic Energy → Potential Energy

$v_{mp} = 1.29\times10^6$ cm s^{-1}

Hydrogen atoms with $T = 10{,}000$ K

n_v/n (10^{-7} s cm^{-1})

Speed (10^6 cm s^{-1})

Boltzmann Factor

- The higher the energy of a state, the less likely it will be occupied

$$P_a \propto e^{\frac{-E_a}{kT}}$$

  - For the Maxwell-Boltzmann distribution, the energy is Kinetic Energy

$$P_v \propto e^{-\frac{1}{2}mv^2/kT}$$

- The "kT" term is associated with the thermal energy of the "gas" as a whole

- Ratio of Probabilities for two different states (and energies)

$$\frac{P_b}{P_a} = \frac{e^{\frac{-E_b}{kT}}}{e^{\frac{-E_a}{kT}}} = e^{\frac{-(E_b - E_a)}{kT}}$$

Degeneracy Factor

- An energy (eigenvalue) is associated with each set of quantum numbers (eigenstate or eigenfunction)

- *Degenerate States* have different quantum numbers but the same energy

- Modify the Boltzmann factor

$$P_a \propto g_a e^{\frac{-E_a}{kT}}$$

   - The probability of being in any of the g_a degenerate states with energy E_a

     - g_a is the _degeneracy_ or _statistical weight_ of state a

- Ratio of probabilities between states with two different energies

$$\frac{P_b}{P_a} = \frac{g_b}{g_a} e^{\frac{-(E_b - E_a)}{kT}}$$

Degeneracy Factor

- Details of quantum mechanics determines the energies and quantum numbers…

- Visit the following site on the next page and browse…

- Quantum numbers for Hydrogen $\{n, l, m_l, m_s\}$

  - Table 8.2

State	n Principal quantum number n	l Orbital quantum number	m_l Magnetic quantum number	m_s Spin quantum number	Maximum number of electrons
1s	1	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2
2s	2	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2
2p	2	1	−1,0,+1	$+\frac{1}{2}, -\frac{1}{2}$	6
3s	3	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2
3p	3	1	−1,0,+1	$+\frac{1}{2}, -\frac{1}{2}$	6
3d	3	2	−2,−1,0,1,2	$+\frac{1}{2}, -\frac{1}{2}$	10

2s + 2p $\Rightarrow$ 8

3s + 3p + 3d $\Rightarrow$ 18

$= 2n^2$

Boltzmann Equation

- Number of atoms in a particular state a

$$N_a = NP_a$$

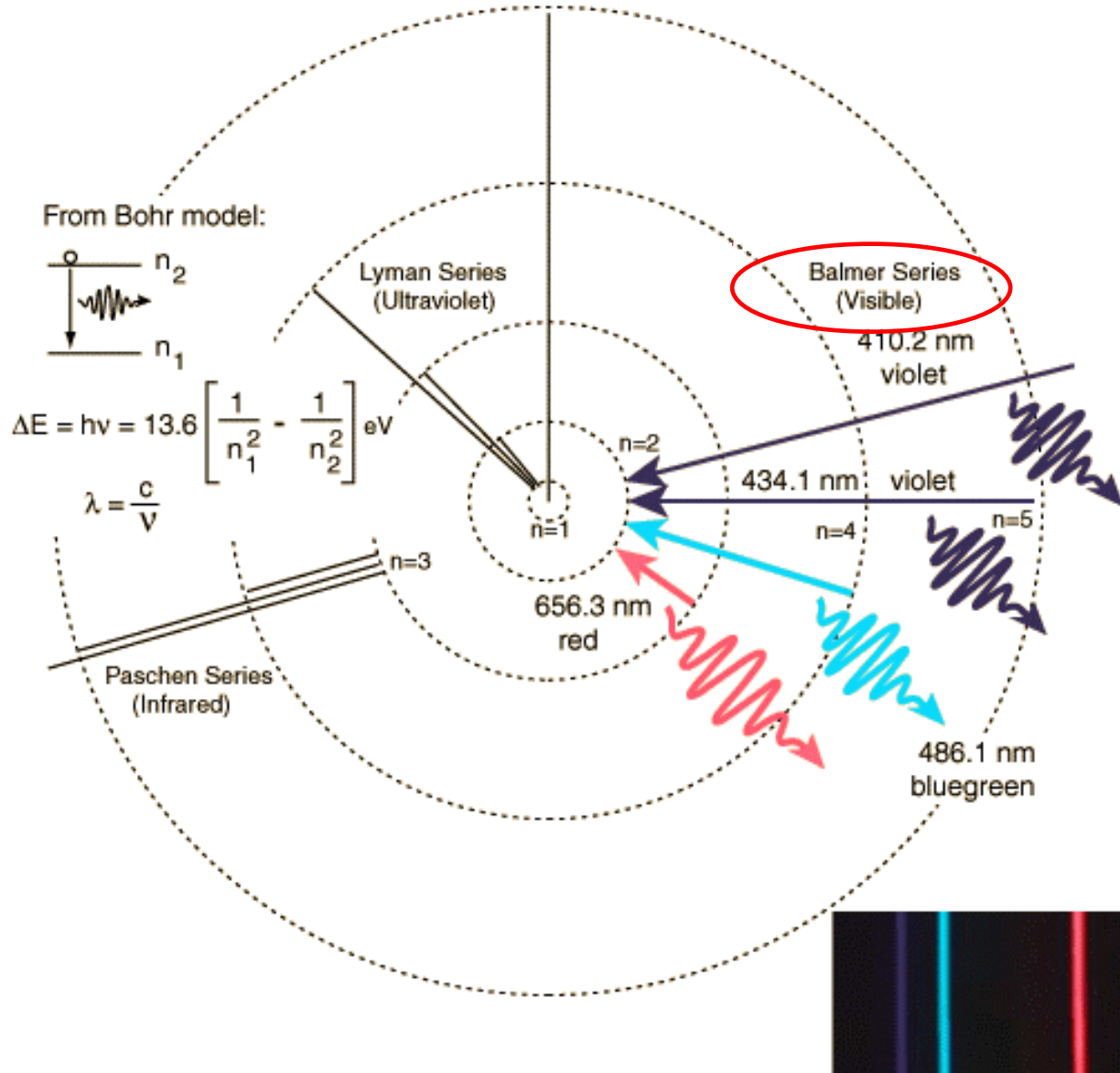N = total number of atoms

N_a = number of atoms in state a

P_a = probability of being in state a

$$\Rightarrow \frac{N_b}{N_a} = \frac{g_b}{g_a} e^{\frac{-\left(E_b - E_a\right)}{kT}}$$

Hydrogen Atom Examples

Hydrogen Atom

- Balmer series absorption spectra is an upward transition from $n = 2$

- Observation: this series has a peak absorption spectrum at ~9520 K.

From Bohr model:

$$\Delta E = h\nu = 13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] eV$$

$$\lambda = \frac{c}{\nu}$$

Lyman Series
(Ultraviolet)

Balmer Series
(Visible)

410.2 nm
violet

434.1 nm   violet

656.3 nm
red

$n=2$

$n=1$

$n=3$

$n=4$

$n=5$

486.1 nm
bluegreen

Paschen Series
(Infrared)

Hydrogen Atom Populations

- We just saw that not many Hydrogen atoms are in the *n=1* state at 9520 K!

  - Shouldn't the intensity keep growing as the temperature increases since there is a higher probability for an H atom to be in the *n=2* state?!?!
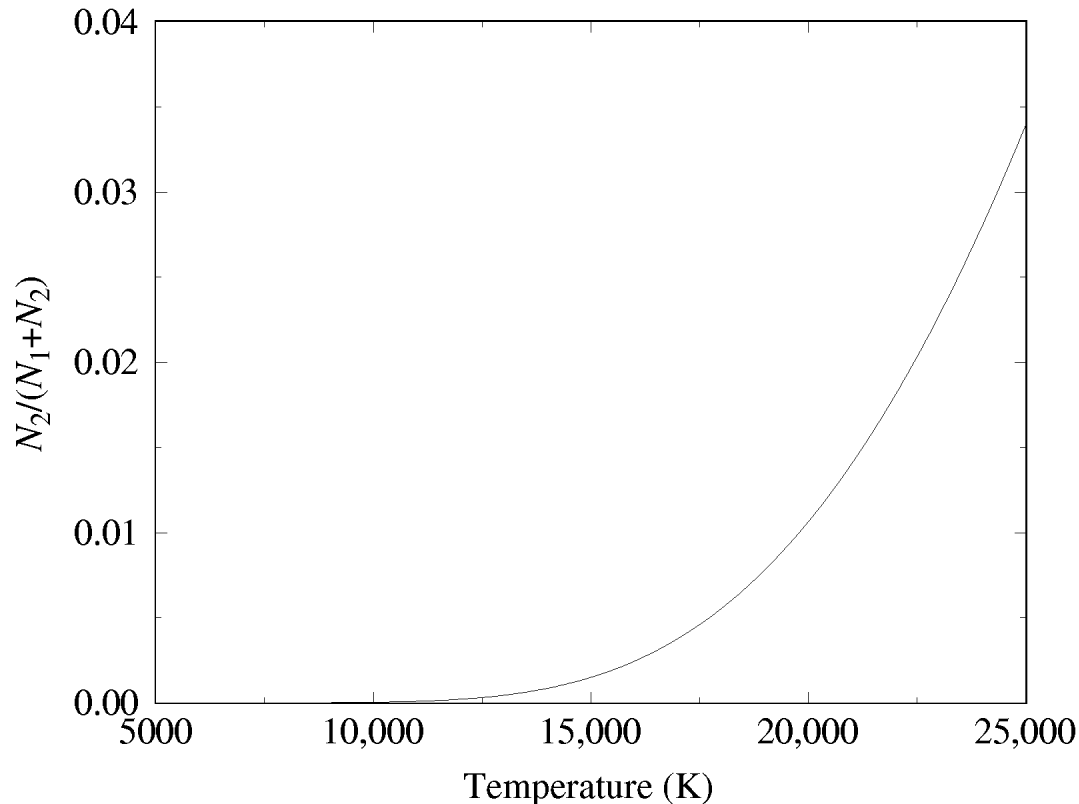
Partition Function

- We also have to figure in all states that have a significant population
- For one state we have: $P_1 \propto g_1 e^{\frac{-E_1}{kT}}$

- Ratio between two states: $\dfrac{P_2}{P_1} = \dfrac{g_2 e^{\frac{-E_2}{kT}}}{g_1 e^{\frac{-E_1}{kT}}} = \dfrac{g_2}{g_1} e^{\frac{-(E_2 - E_1)}{kT}}$

- Ratio of state *2* to *all* other states with reference to the ground state:

$$\frac{P_2}{P_{all}} = \frac{g_b e^{\frac{-(E_2 - E_1)}{kT}}}{g_1 e^{\frac{-(E_1 - E_1)}{kT}} + g_2 e^{\frac{-(E_2 - E_1)}{kT}} + g_3 e^{\frac{-(E_3 - E_1)}{kT}} + \cdots} = \frac{g_2 e^{\frac{-(E_2 - E_1)}{kT}}}{Z}$$

Partition Function

- This tell us how many states are accessible or available at a given temperature (thermal energy)

$$Z = g_1 e^{\frac{-(E_1 - E_1)}{kT}} + g_2 e^{\frac{-(E_2 - E_1)}{kT}} + g_3 e^{\frac{-(E_3 - E_1)}{kT}} + \cdots$$

$$= g_1 + \sum_i g_i e^{\frac{-(E_i - E_1)}{kT}}$$

- The higher the temperature, the more states that are available
- At zero K, everything will be in the ground state
  - Bose-Einstein Condensates

Partition Function and Atoms

- We also have to handle ionization!
- Nomenclature: H I – neutral hydrogen

    H II – singly ionized hydrogen

    He I – neutral Helium

    He II – singly ionized Helium

    He III – doubly ionized Helium

- Ionization Energy for H I to H II

$$\chi_I = 13.6\, eV$$

  – Rather than $N_2/N_1 \rightarrow \infty$, the atom will ionize before this happens

Saha Equation

- Determines the ratio of numbers of ionized atoms
- Need distinct partition functions since energy levels of atoms are different for different ionization stages
  - Z_i is the initial stage of ionization
  - Z_{i+1} is the final stage of ionization
- Ratio of the number of atoms in each of these stages

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi\, m_e\, kT}{h^2}\right)^{\frac{3}{2}} e^{-\chi_i\, kT}$$

  - n_e is the electron density (an ideal gas of electrons)
    - Electron pressure $\quad P_e = n_e\, kT$
    - Electrons recombine with H II to give H I

Ionized Hydrogen Atoms

- Fraction of hydrogen atoms that are ionized
- If we have H II, we can't have the Balmer series!
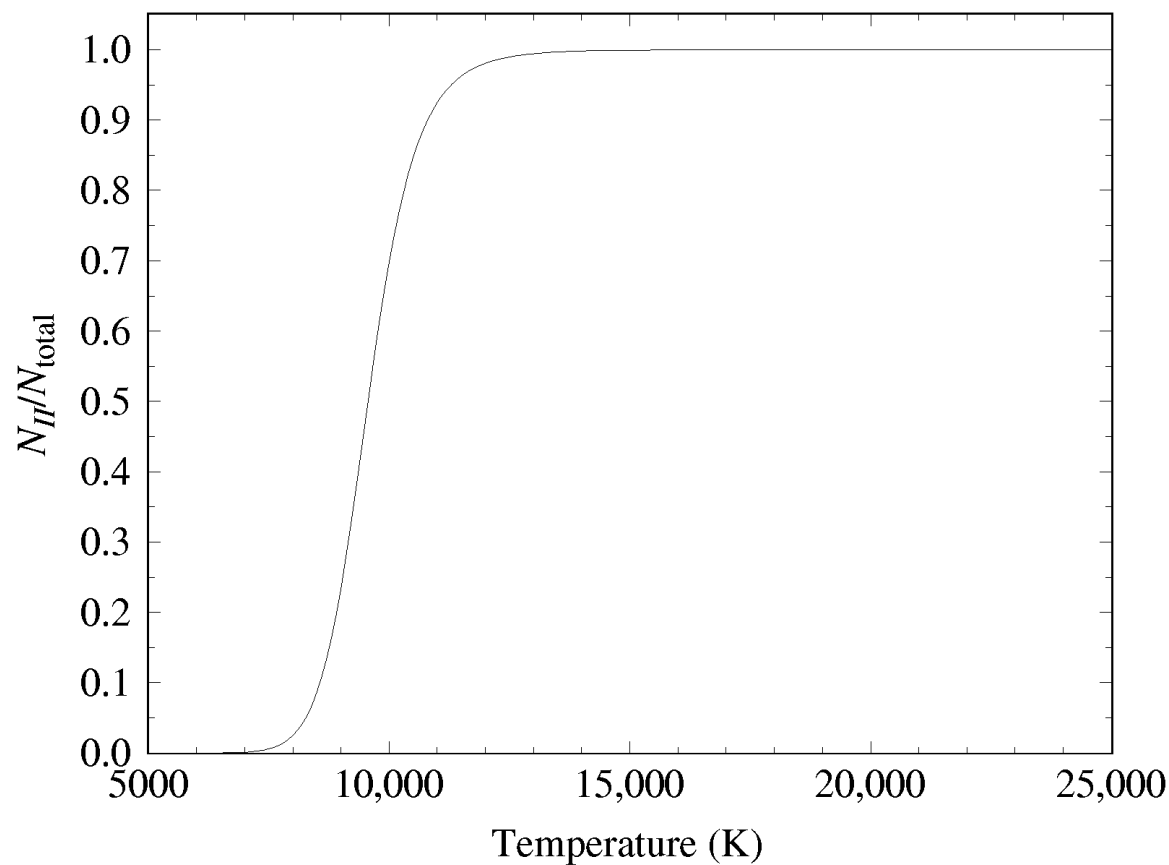
H I $n = 2$ population

- Includes the Boltzmann factor, partition function and ionization
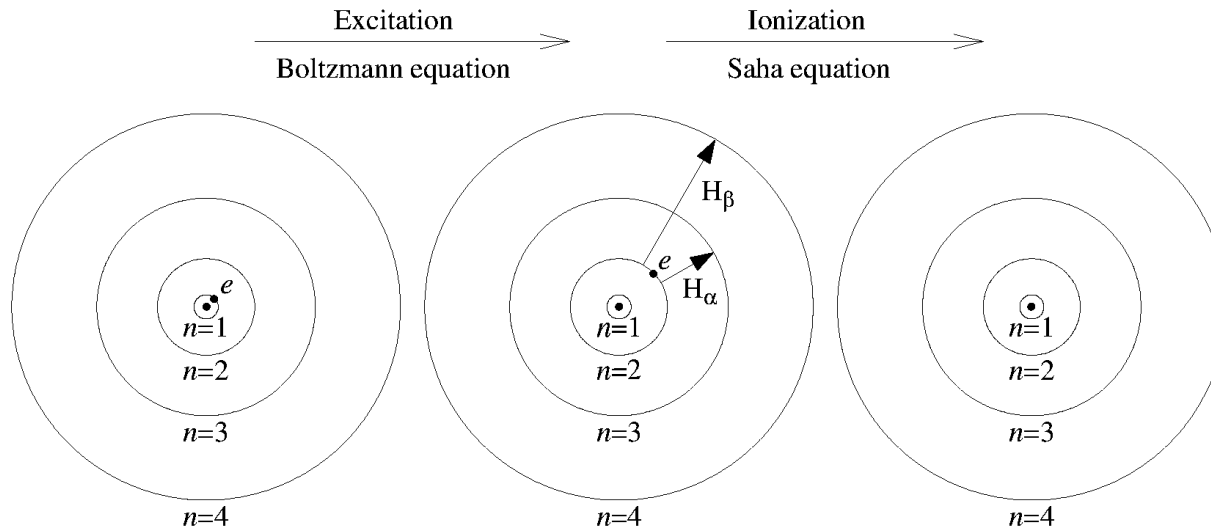- Population peak at 9520 K, in agreement with observation of the Balmer series
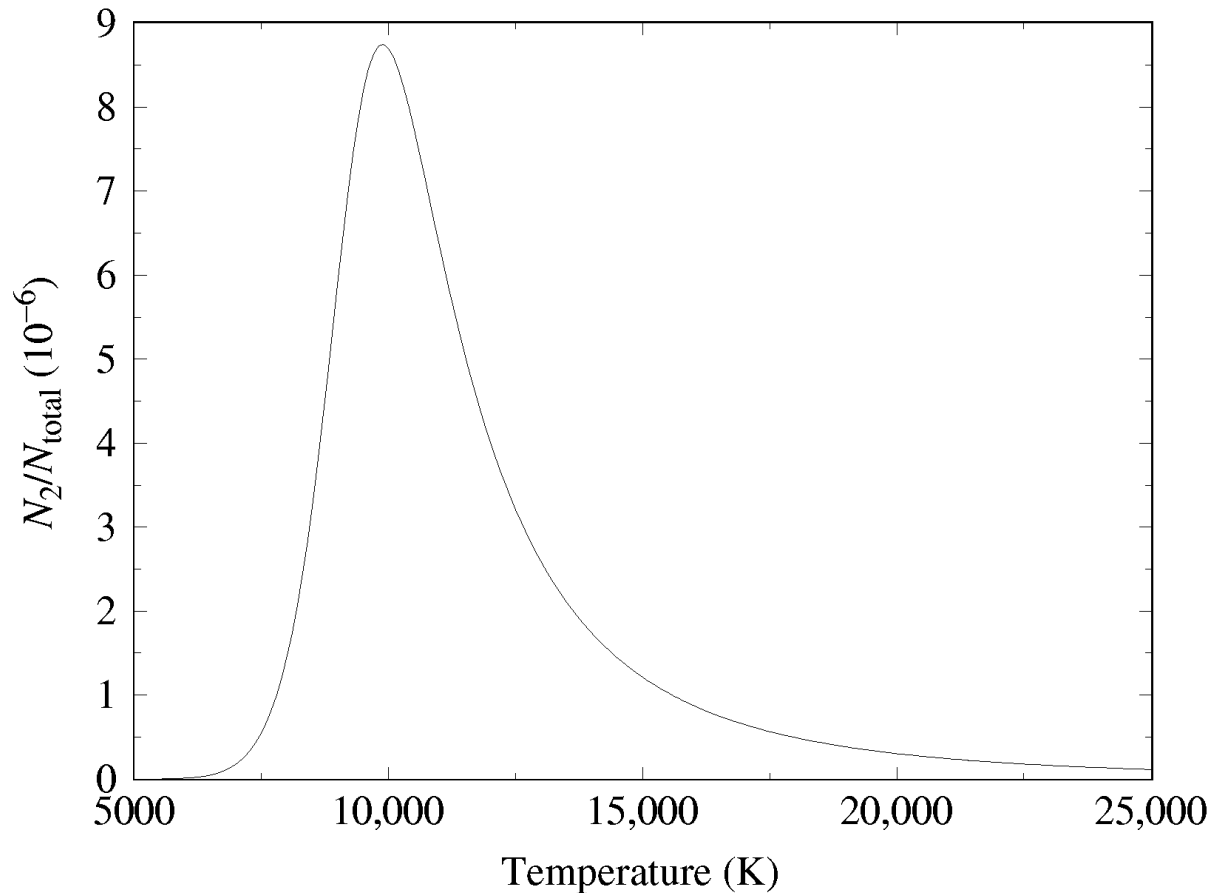
H I $n = 2$ population

$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_I + N_2}\right)\left(\frac{N_I}{N_I + N_{II}}\right)$$

Fraction of non-ionized hydrogen Atoms in the $n = 2$ state

Fraction of non-ionized hydrogen atoms

Example 8.3

- Degree of ionization in a stellar atmosphere of pure hydrogen for the temperature range of 5000-25000 K $\dfrac{N_{II}}{N_{Total}}$

- Given electron pressure $P_e = 200 \dfrac{dyne}{cm^2} = 20 \, N/m^2$

- Saha equation

$$\frac{N_{II}}{N_I} = \frac{2kTZ_{II}}{P_e Z_I} \left(\frac{2\pi \, m_e \, kT}{h^2} \right)^{\frac{3}{2}} e^{-\chi_i \, kT}$$

- Must determine the partition functions
  - Hydrogen ion is a proton, so $Z_{II} = 1$
  - Neutral hydrogen over this temp range

  $\Delta E = E_2 - E_1 = 10.2 \text{ eV}$

  $\Delta E \gg kT$, then $\quad e^{-\Delta E / kT} \ll 1$

  $\Rightarrow Z_I = g_1 + \displaystyle\sum_i g_i \, e^{\frac{-(E_i - E_1)}{kT}} \quad g_1 = 2$

$T := 5000 \text{K}$

$k \cdot T = 0.43 \text{ eV}$
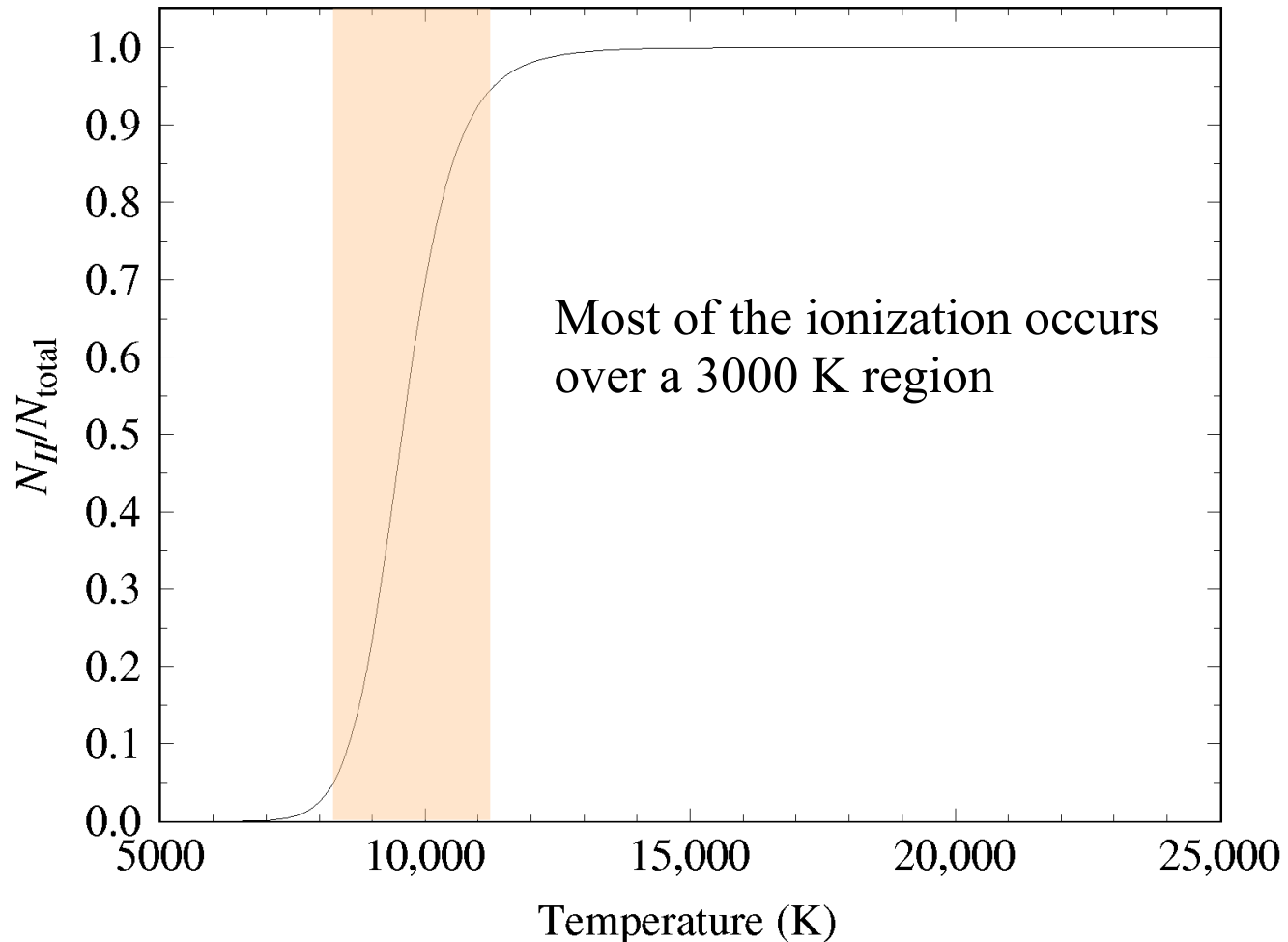
$T := 25000 \text{K}$

$k \cdot T = 2.15 \text{ eV}$

Example 8.3

- Degree of Ionization

$$\frac{N_{II}}{N_I} = \frac{2kT(1)}{P_e(2)}\left(\frac{2\pi\, m_e\, kT}{h^2}\right)^{\frac{3}{2}} e^{-\chi_i kT}$$

$$\frac{N_{II}}{N_I + N_{II}} =$$

$$\frac{N_{II}/N_I}{1 + N_{II}/N_I}$$

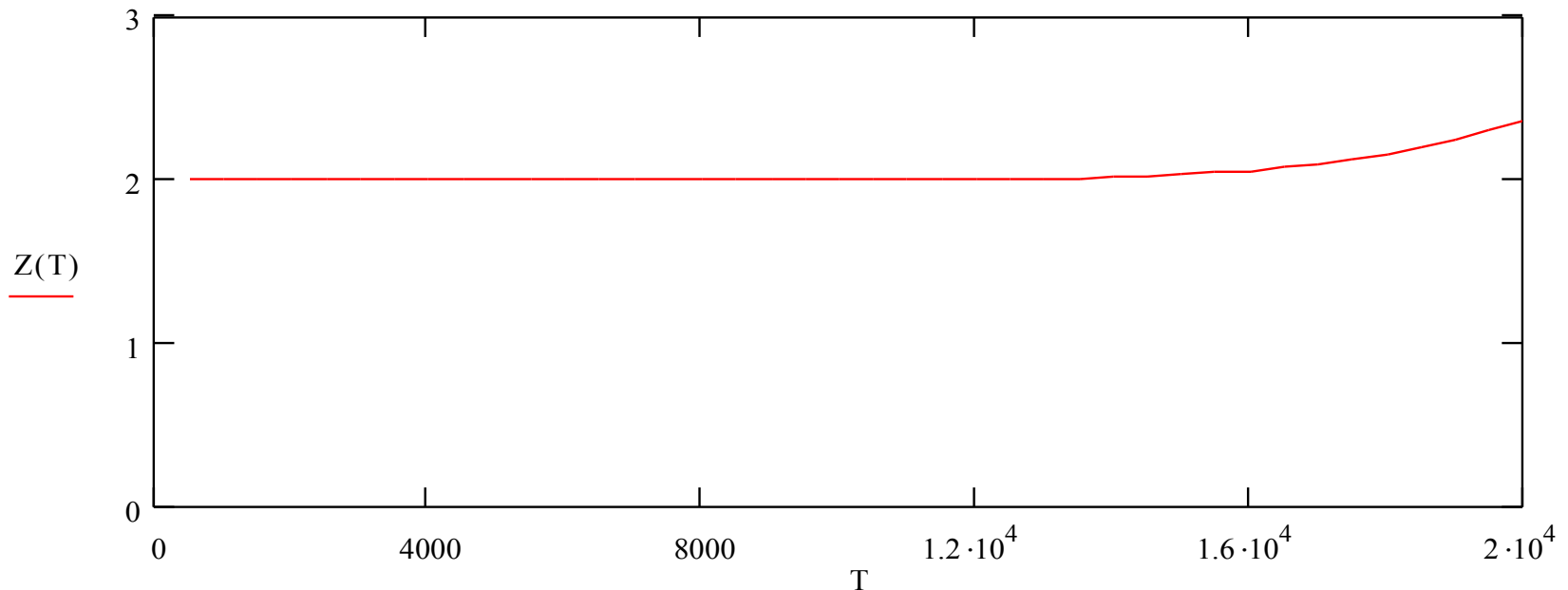Most of the ionization occurs over a 3000 K region

Problem 8.7

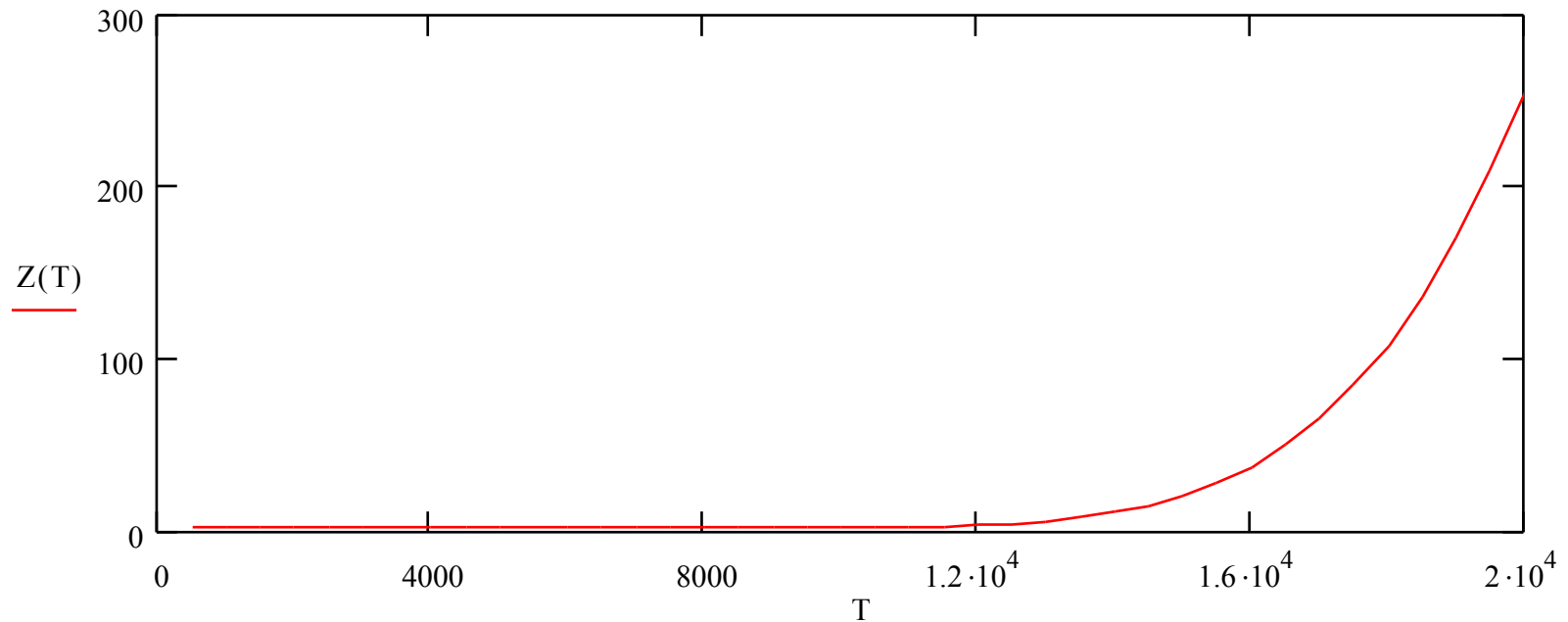- Evaluate the first three terms of the partition function for 10000K

Partition Function: Counting the first ten states... Energy: $E(n) := \dfrac{-13.6\text{eV}}{n^2}$ Degeneracy: $g(n) := 2 \cdot n^2$

$$f_B(n, T) := \exp\left[\frac{-(E(n) - E(1))}{k \cdot T}\right] \qquad Z(T) := \sum_{n=1}^{10} \left(g(n) \cdot f_B(n, T)\right) \qquad T := 0, 500 .. 20000$$

$Z(6000\text{K}) = 2.0000$ $Z(10000\text{K}) = 2.0002$ $Z(15000\text{K}) = 2.0292$

Problem 8.8

- The partition function diverges at $n \rightarrow \infty$
 - Why do we ignore large n?

Partition Function: Counting the first 100 states...   <u>Energy:</u> $E(n) := \dfrac{-13.6\text{eV}}{n^2}$ <u>Degeneracy:</u> $g(n) := 2 \cdot n^2$

$$f_B(n,T) := \exp\left[\frac{-(E(n) - E(1))}{k \cdot T}\right] \qquad Z(T) := \sum_{n=1}^{100} \left(g(n) \cdot f_B(n,T)\right) \qquad T := 0, 500 .. 20000$$

$$Z(6000K) = 2.0000 \qquad Z(10000K) = 2.0952 \qquad Z(15000K) = 20.2988$$

Problem 8.8

Partition Function: Counting the first 1000 states... <u>Energy:</u> $E(n) := \dfrac{-13.6\text{eV}}{n^2}$ <u>Degeneracy:</u> $g(n) := 2 \cdot n^2$

$$f_B(n, T) := \exp\left[\frac{-(E(n) - E(1))}{k \cdot T}\right] \qquad Z(T) := \sum_{n=1}^{1000} \left(g(n) \cdot f_B(n, T)\right) \qquad T := 0, 500 .. 20000$$
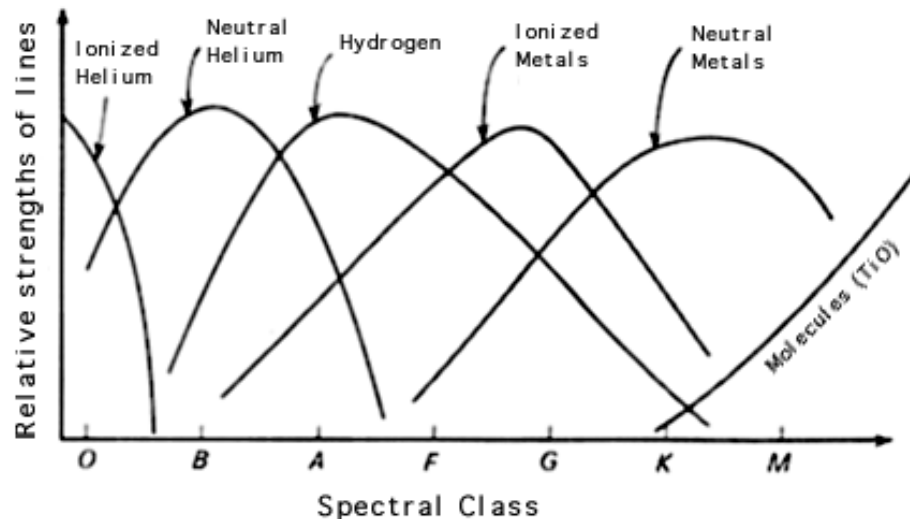
$Z(6000\text{K}) = 2.0025 \qquad Z(10000\text{K}) = 95.4311 \qquad Z(15000\text{K}) = 1.7998 \times 10^4$
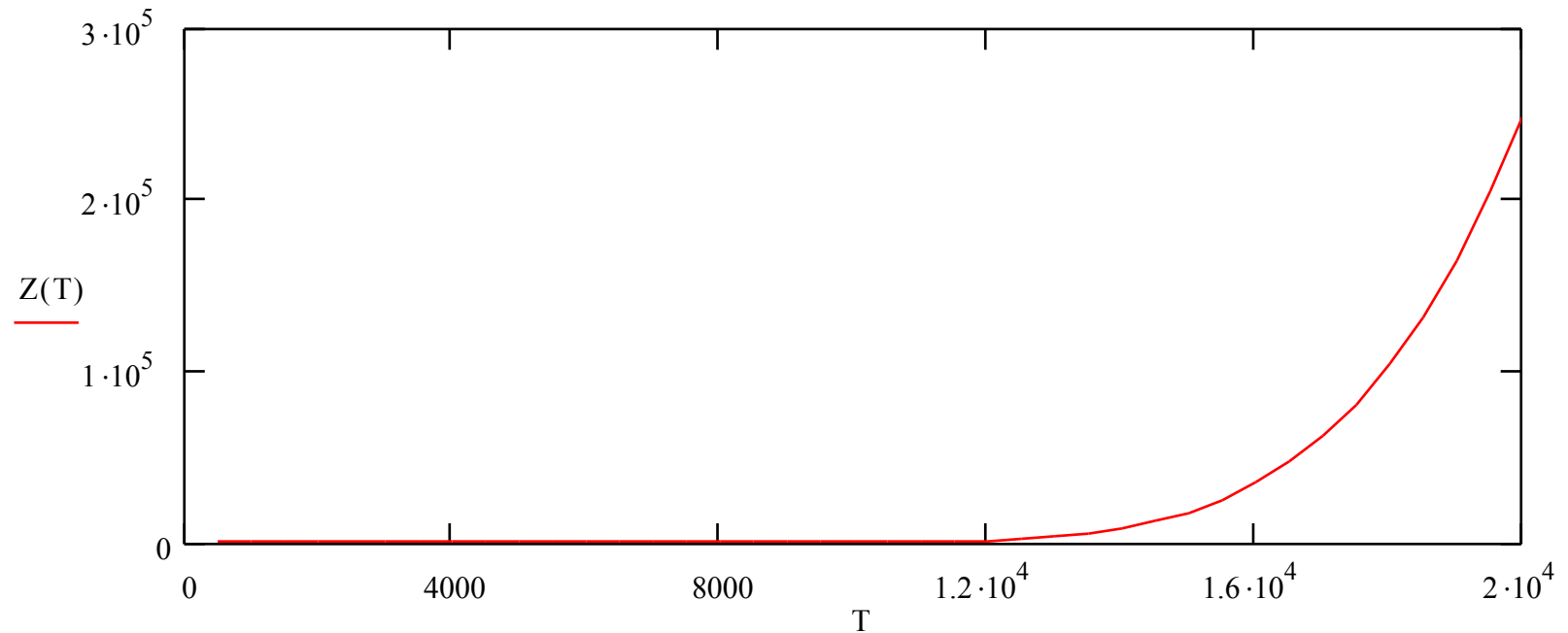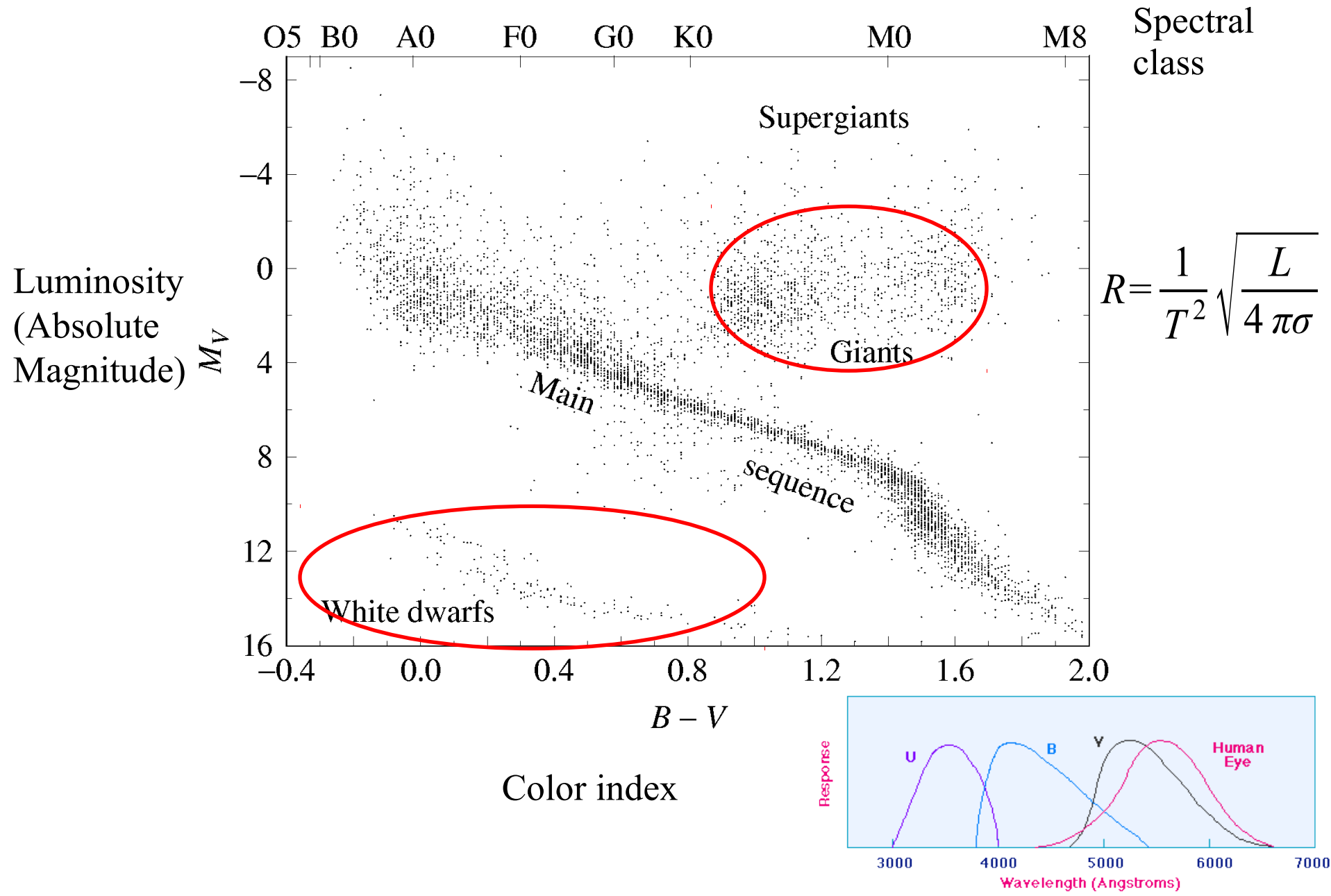

- Ionization
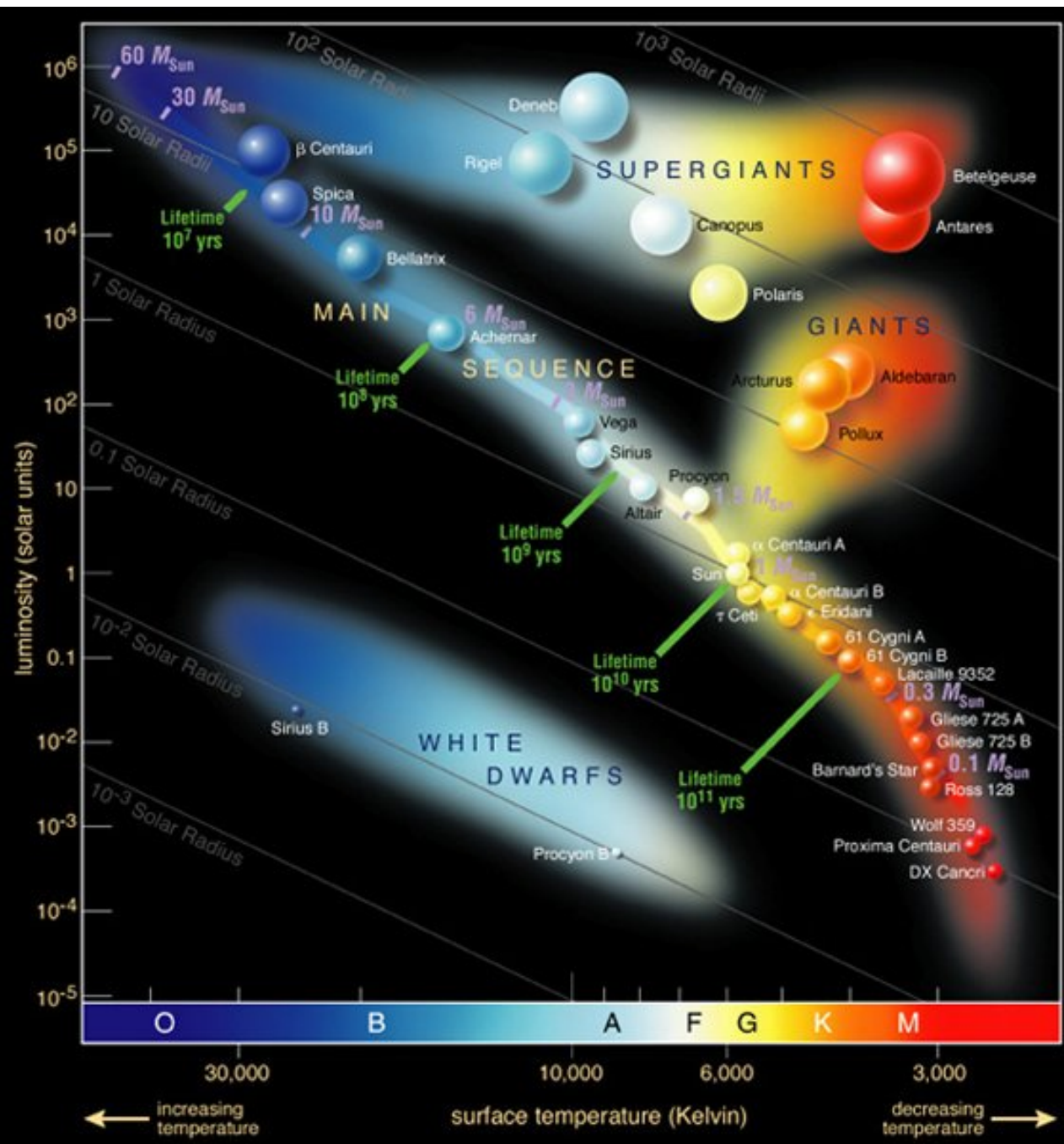- Unphysical orbital size $\quad r_n = a_o n^2$

Example 8.4

- Surface of the Sun has 500,000 hydrogen atoms per calcium atom, but calcium absorption lines are much stronger than the Balmer series lines.

- The Boltzmann and Saha equations reveal that there are 400× more Ca atoms in the ground electronic state than in the *n=2* hydrogen state.

- Calcium is not more abundant

- Differences are due to sensitive temperature dependence

Hertzsprung-Russell (H-R) Diagram

Spectral class

O5 B0 A0 F0 G0 K0 M0 M8

Luminosity (Absolute Magnitude) M_V

Supergiants

Giants

Main sequence

White dwarfs

$B - V$

Color index

$$R = \frac{1}{T^2} \sqrt{\frac{L}{4\pi\sigma}}$$

A colorful H–R Diagram

Betelgeuse

Antares

Sun (1 pixel)
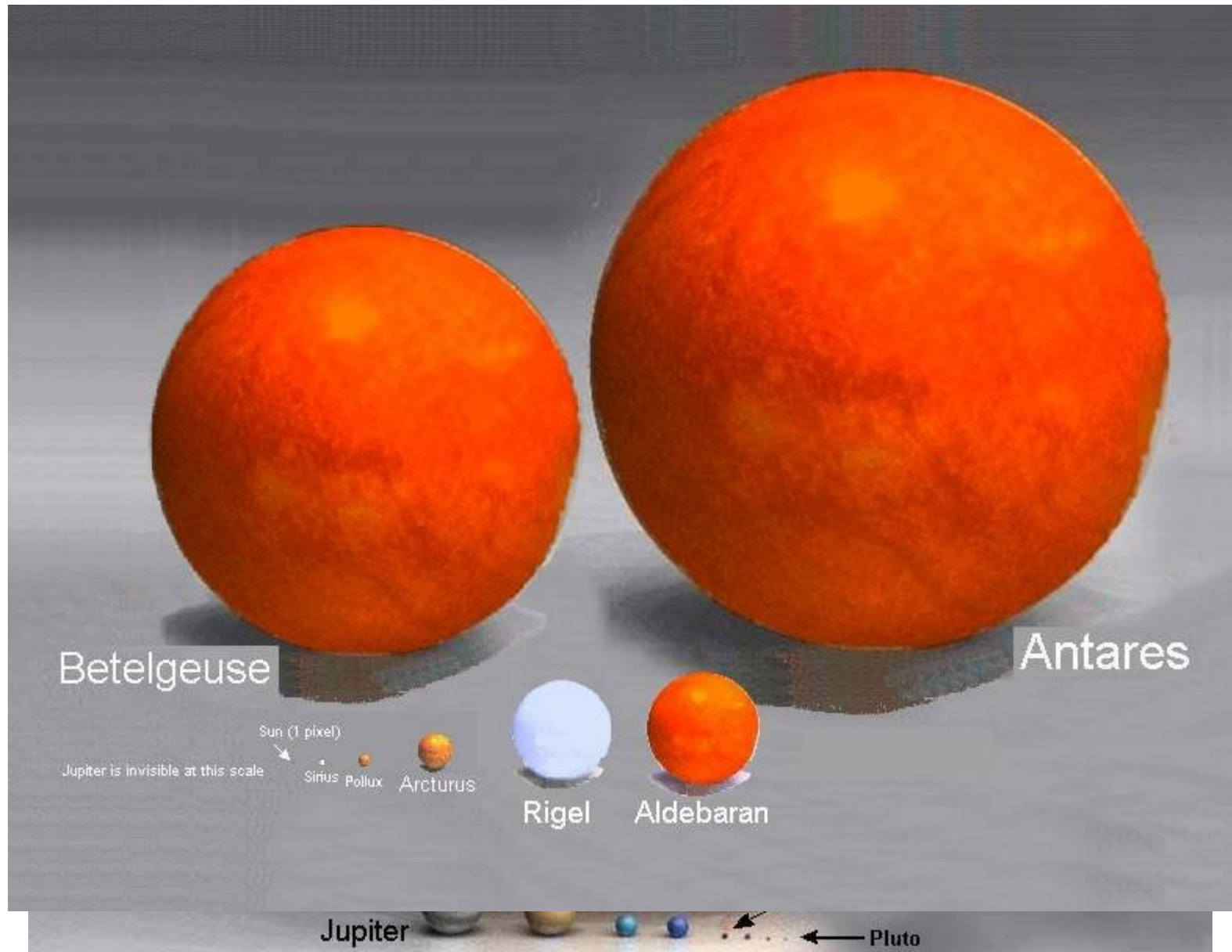
Jupiter is invisible at this scale

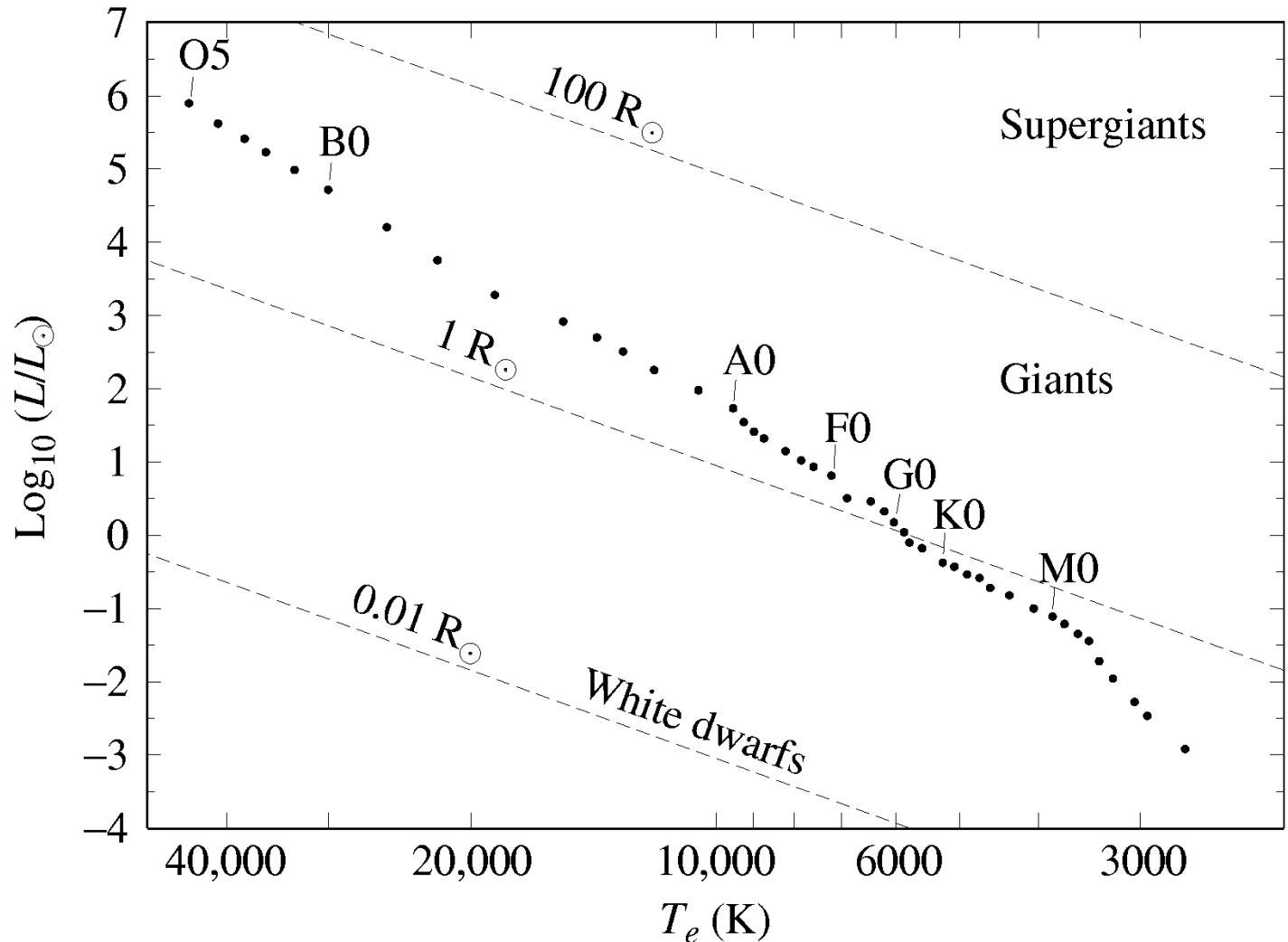Sirius Pollux
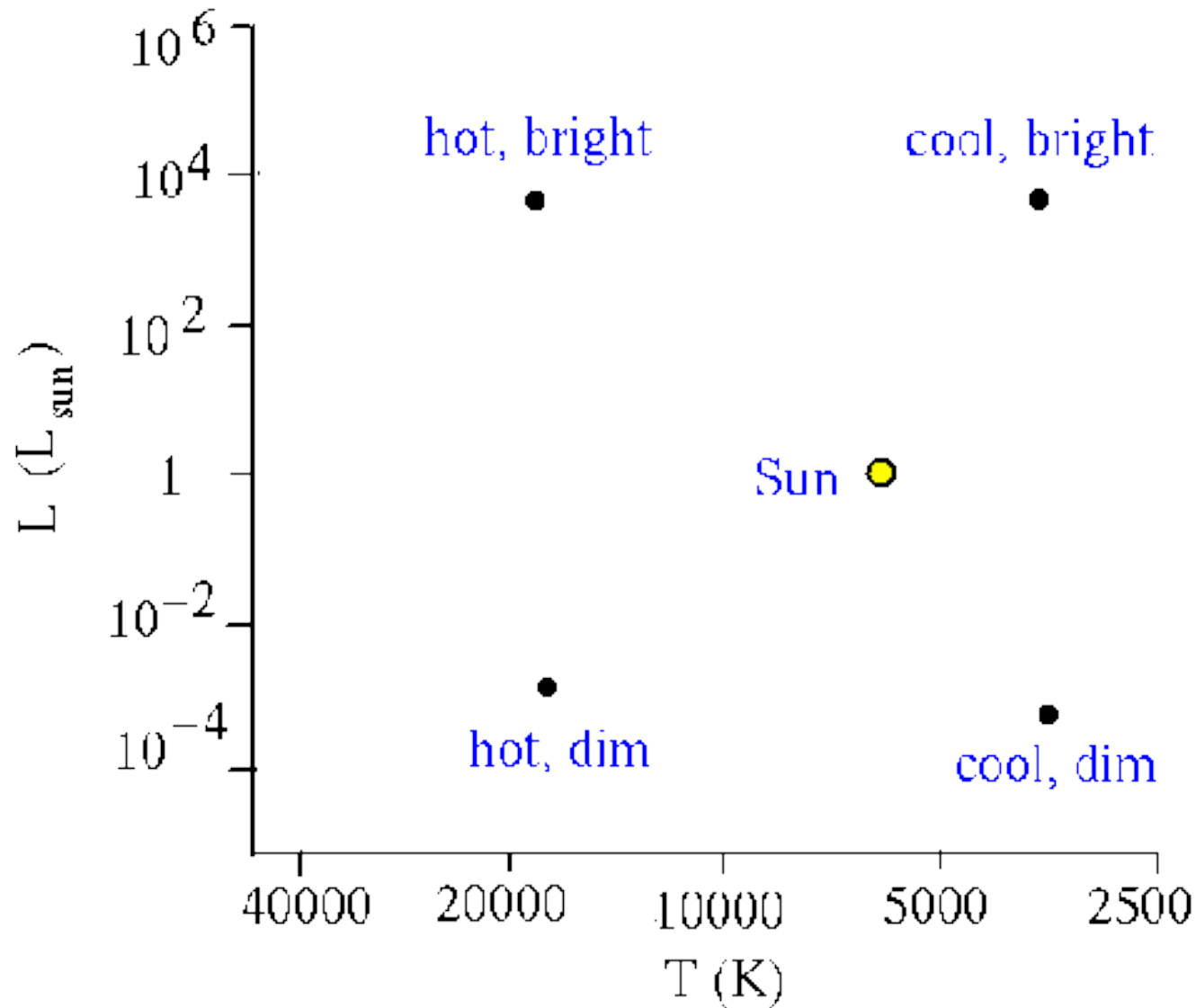
Arcturus

Rigel

Aldebaran

Jupiter
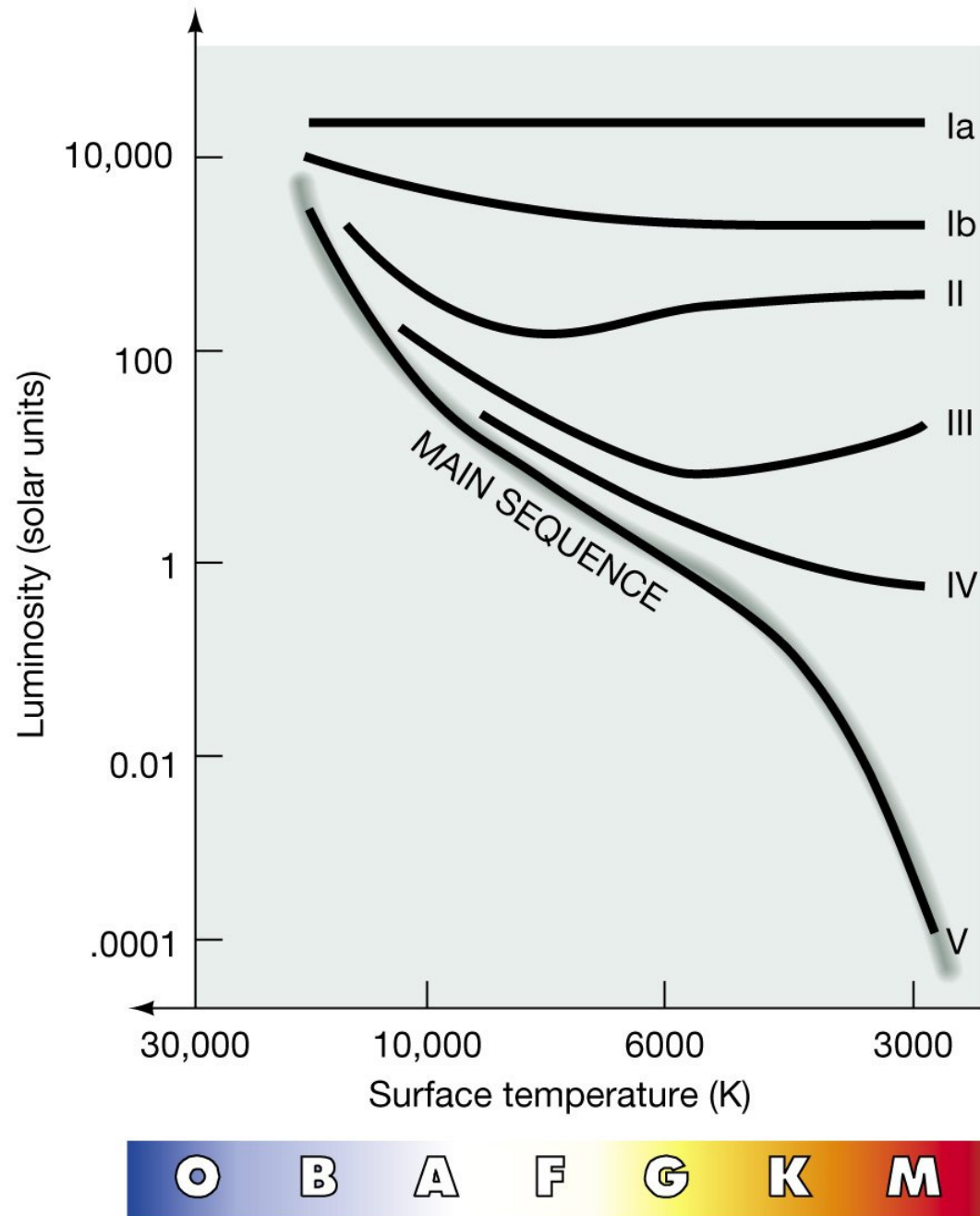
Pluto

Hertzsprung-Russell (H-R) Diagram

- Star Radius

$$R = \frac{1}{T^2} \sqrt{\frac{L}{4\pi\sigma}}$$

Hertzsprung-Russell (H-R) Diagram

Luminosity Classes

Ia

Ib

II

III

MAIN SEQUENCE

IV

V

Luminosity (solar units)

10,000

100

1

0.01

.0001

Surface temperature (K)

30,000   10,000   6000   3000

O   B   A   F   G   K   M

Spectral classification

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Stellar Luminosity Classes

Class	Description
Ia	Bright supergiants
Ib	Supergiants
II	Bright giants
III	Giants
IV	Subgiants
V	Main-sequence stars and dwarfs

TABLE 17.3 Stellar Luminosity Classes

Some define VI and wd (or D)

Luminosity classes can be discerned by line widths.

A3 I   Comparison of Spectral Lines for Supergiant & Main Sequence Stars

← Thinner lines →
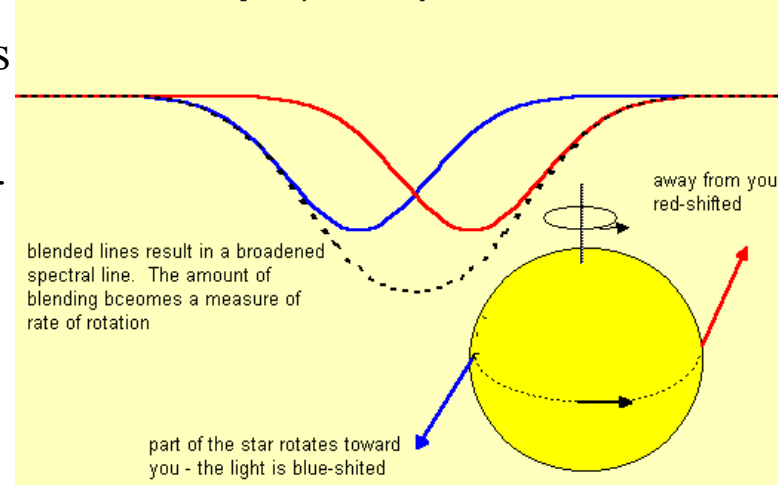
A3 V

Pressure broadening of lines
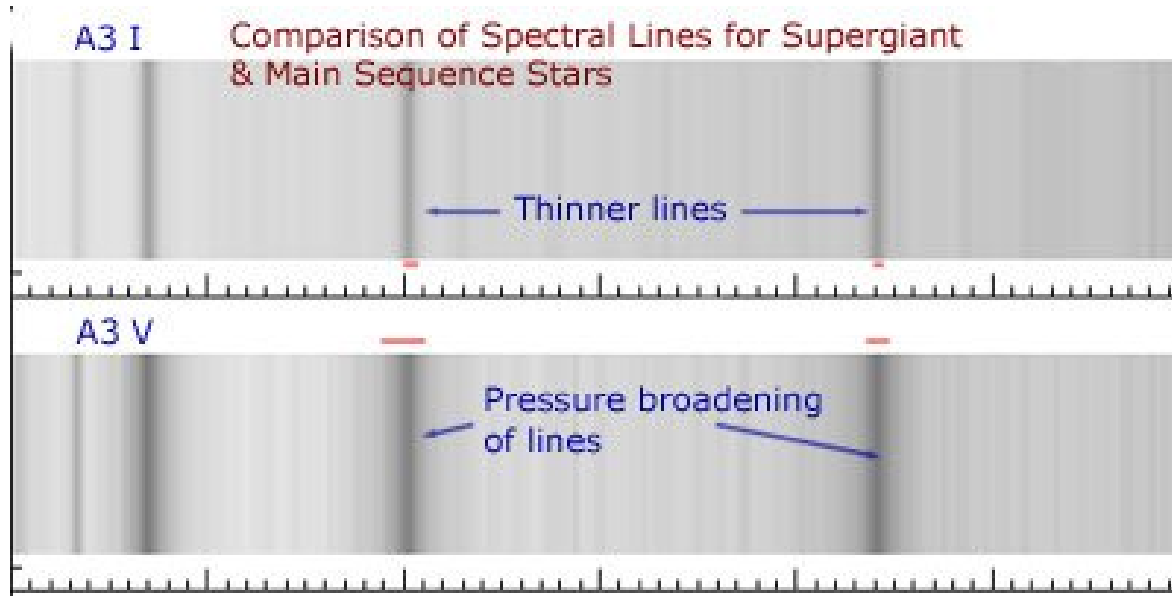
Courtesy ANU.

Pressure Broadening is greater for the smaller, denser star. The giant star has a more tenuous atmosphere amd will produce narrower spectral lines
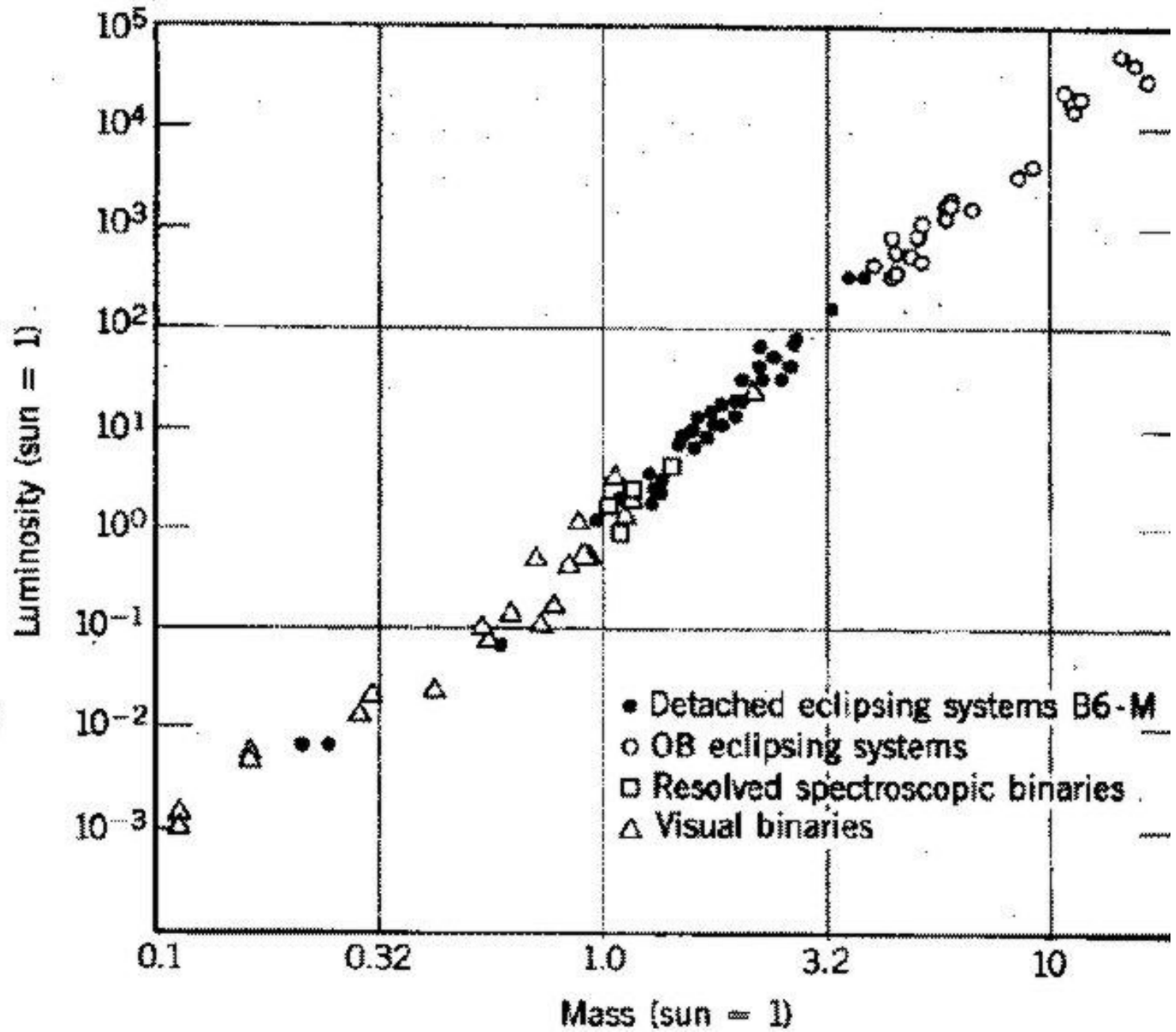
giant star - more tenuous atmosphere

dwarf star: denser atmosphere
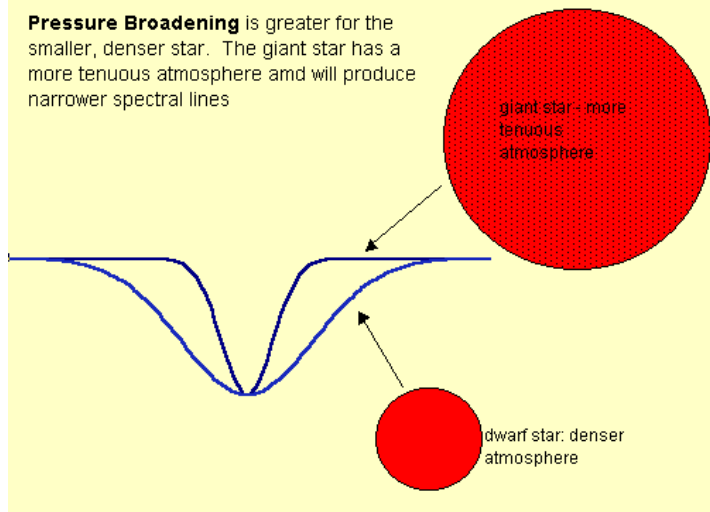
However,
Other processes
Can
Broaden lines...

Rotational Broadening: a way of measuring stellar rotation

blended lines result in a broadened spectral line. The amount of blending bceomes a measure of rate of rotation

away from you red-shifted

part of the star rotates toward you - the light is blue-shited

Mass-Luminosity Relation from Binary Systems

The mass – luminosity relation for stars, as determined from binary systems, in which the individual masses can be found.
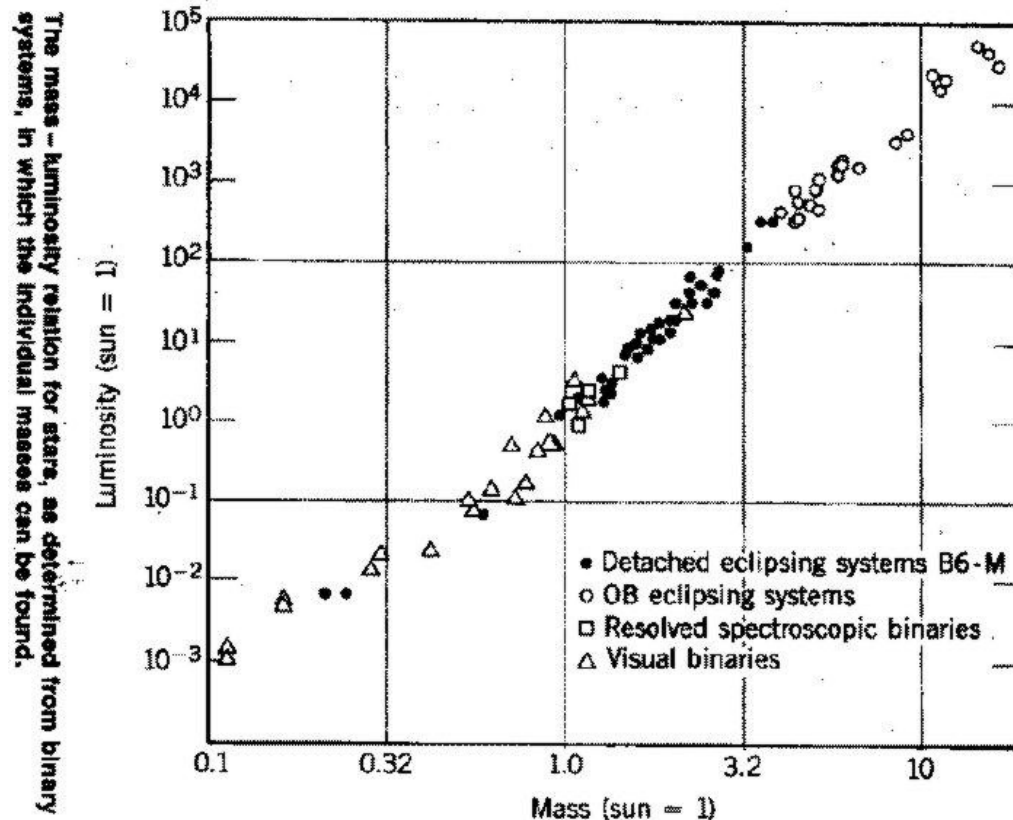
Legend:
- • Detached eclipsing systems B6-M
- ○ OB eclipsing systems
- □ Resolved spectroscopic binaries
- △ Visual binaries

Y-axis: Luminosity (sun = 1)
X-axis: Mass (sun = 1)

Mass-Luminosity Relation

- Early theories had "early" O-type (bright, hot, massive) stars evolving to "old" M-type stars (dim, cool, less massive)
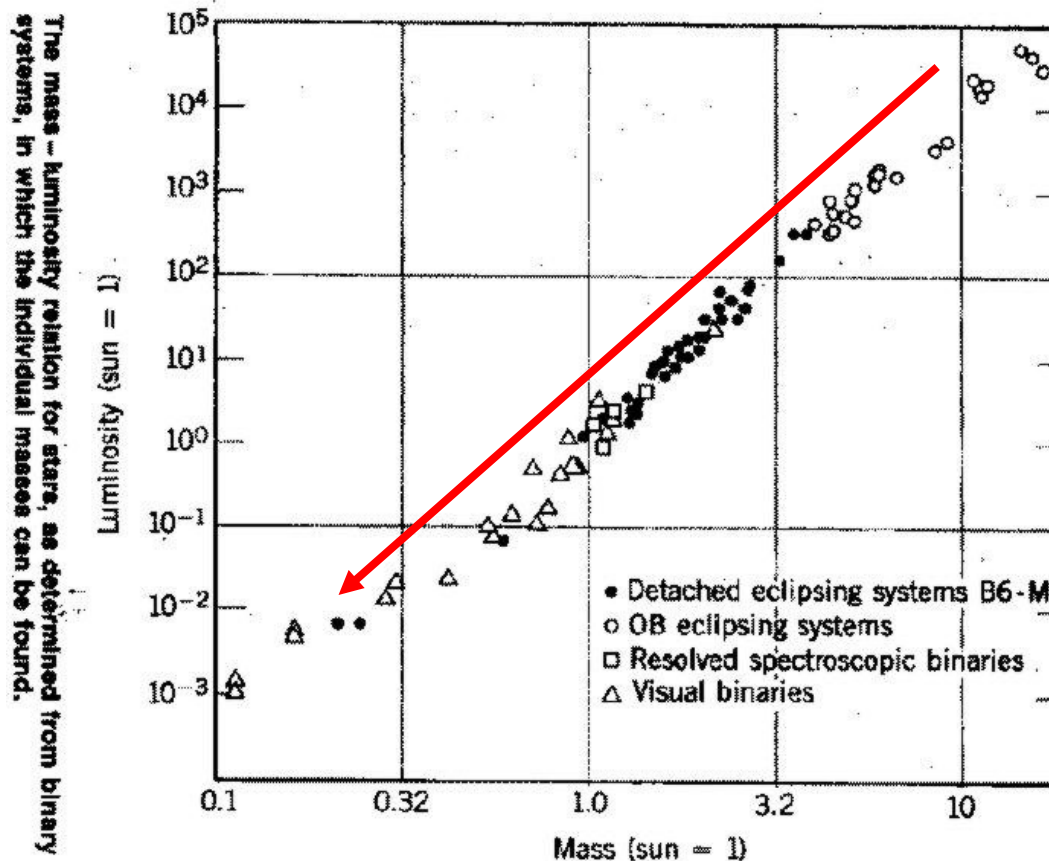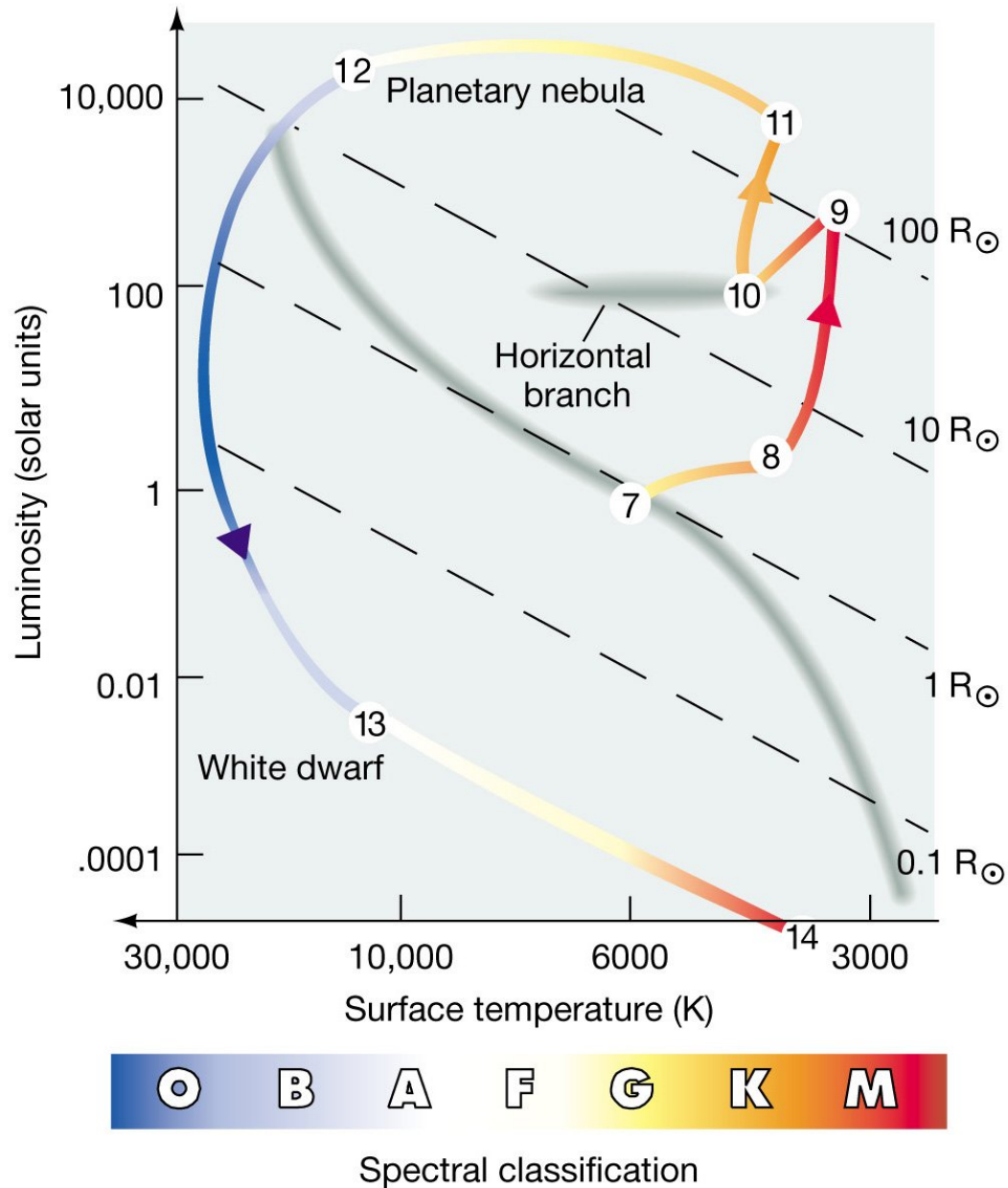
Mass-Luminosity Relation

- Luminosity (power output) comes from nuclear fusion at the core of the stars.
- L increases dramatically with M: $L \sim M^{3.5}$ (from M-L relation)
- From this, we can derive a lifetime for stars on the Main Seq.:
  - Lifetime = Fuel/(Rate of burning fuel)
  - Lifetime = M/L
  - Lifetime = $M/M^{3.5} = M^{-2.5}$
- Actually, $L \sim M^4$ for M>1, so
- Lifetime $\sim M^{-3}$

The mass – luminosity relation for stars, as determined from binary systems, in which the individual masses can be found.

- ● Detached eclipsing systems B6-M
- ○ OB eclipsing systems
- □ Resolved spectroscopic binaries
- △ Visual binaries

Luminosity (sun = 1)

Mass (sun = 1)

Evolution of 1 M$_\odot$ star.

Evolutionary tracks.

Figure 17-17
Stellar Distances

~10,000 pc

OBAFGKM

Spectroscopic parallax

~200 pc

Stellar parallax

~1 A.U.

Distance

Radar ranging
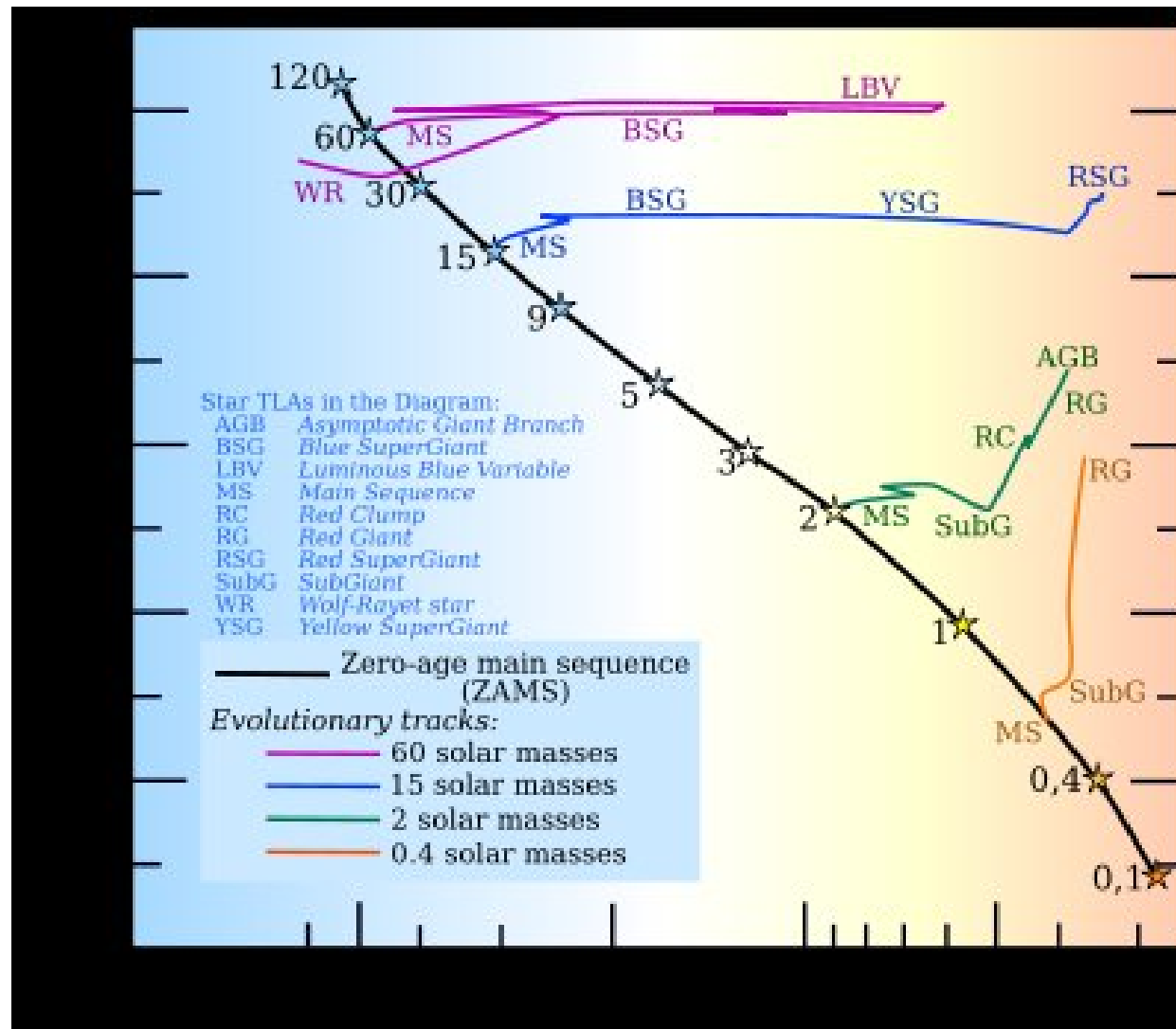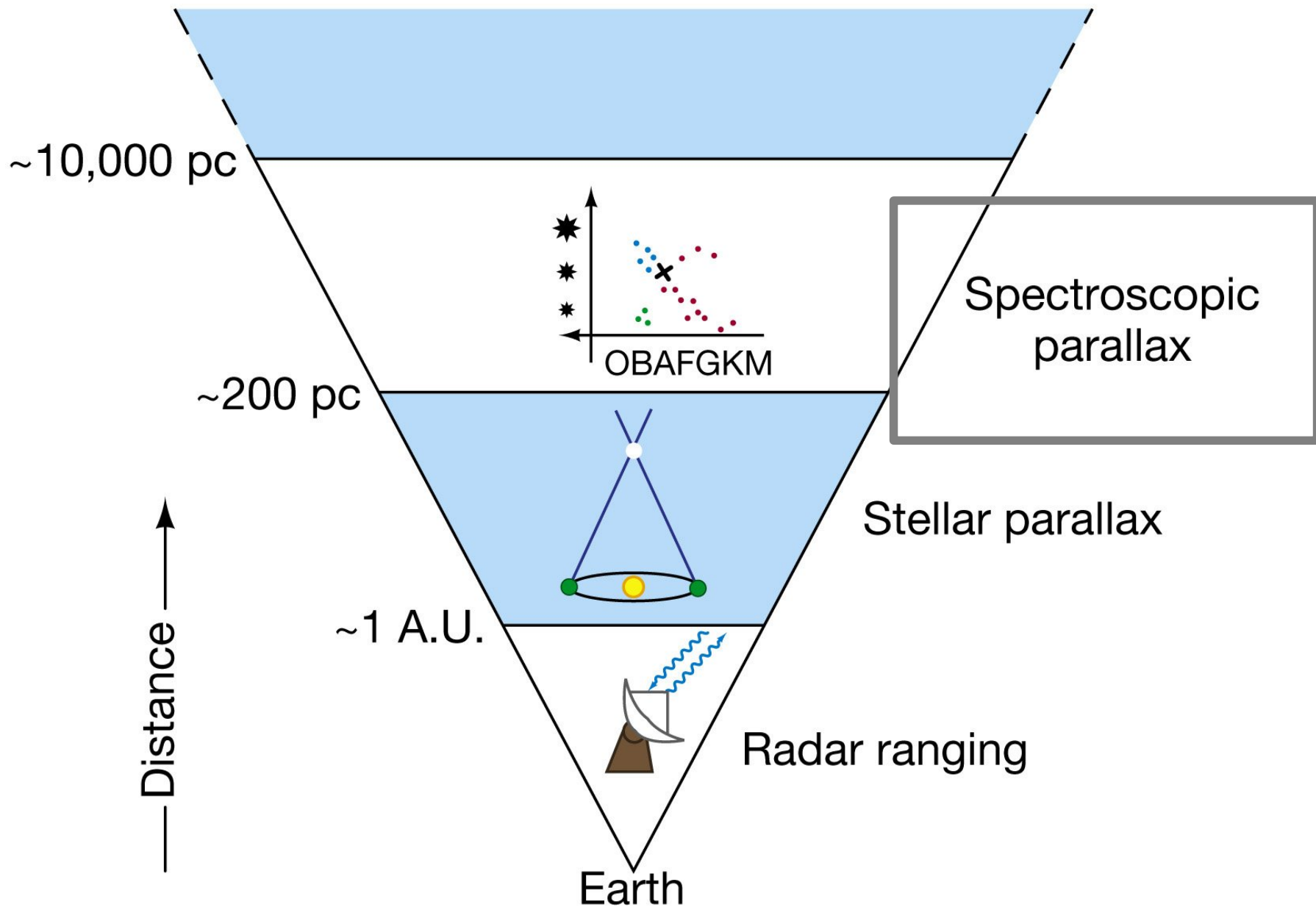
Earth

Spectroscopic "Parallax"

- Method to determine a stars distance
  - Determine the spectral class and luminosity class.
  - Read the absolute magnitude from the H-R diagram
  - Compare to apparent magnitude to determine distance
  - Accurate to a factor of ± 1 magnitude
    - $10^{1/5} = 1.6$

Stellar and Spectroscopic Parallax

Stellar Parallax works out to 200pc (ground), 1000 pc (Hipparcos)
Spectroscopic Parallax works for stars for which a good spectrum
 can be observed (about 8 kpc), but ...
• **Not precise for individual stars, especially giants**
• **Entire clusters of stars works better! ("main-sequence fitting")**
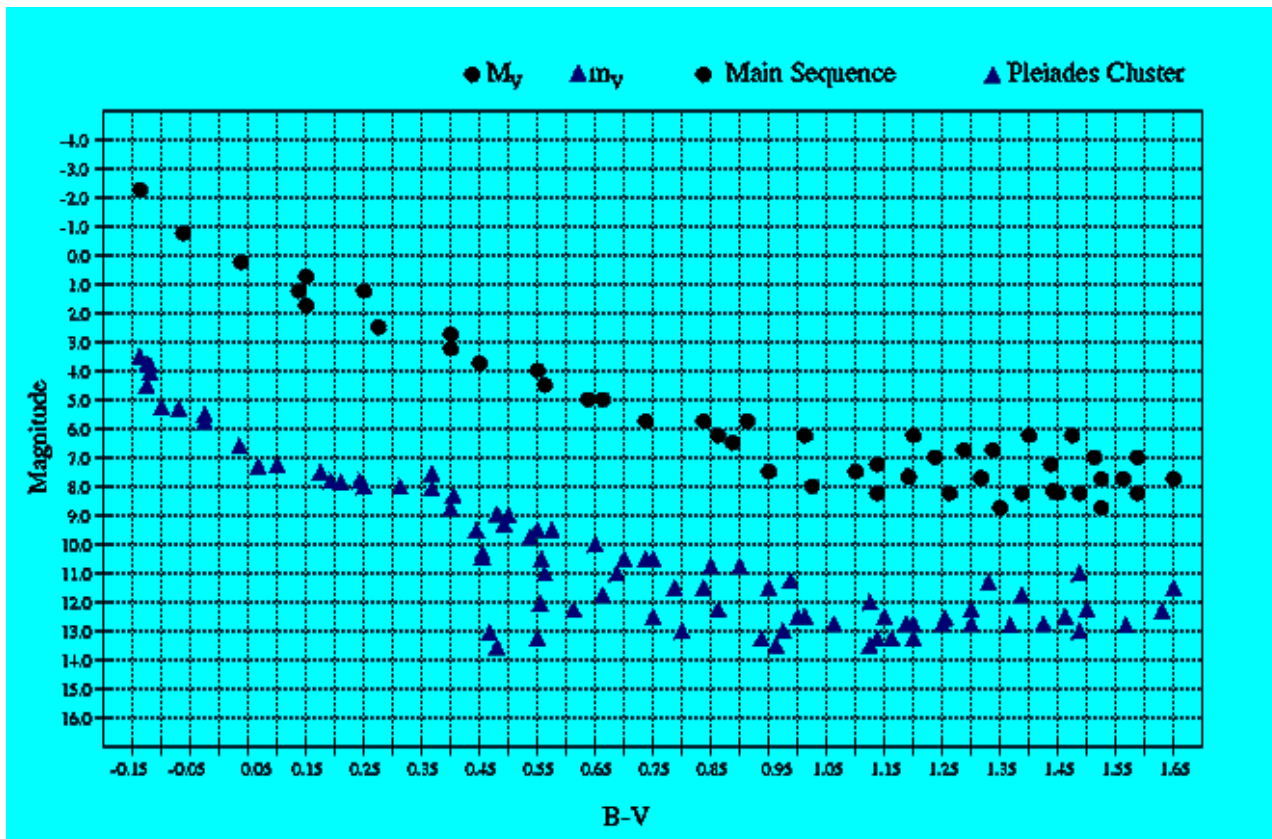
$$m-M=5\log(d/10)$$

Spec Parallax assumes, for example, that all A0V stars have the same M.
That makes A0V stars "standard candles".