Chapter 9

Linear Momentum and Collisions

Momentum Analysis Models

Force and acceleration are related by Newton's second law.

When force and acceleration vary by time, the situation can be very complicated.

The techniques developed in this chapter will enable you to understand and analyze these situations in a simple way.

Will develop momentum versions of analysis models for isolated and non-isolated systems

These models are especially useful for treating problems that involve collisions and for analyzing rocket propulsion.

Thought Experiment

An archer stands on frictionless ice and fires an arrow. What is the archer's velocity after firing the arrow?

- Motion models such as a particle under constant acceleration cannot be used.
  - No information about the acceleration of the arrow
- Model of a particle under constant force cannot be used.
  - No information about forces involved
- Energy models cannot be used.
  - No information about the work or the energy (energies) involved

A new quantity is needed – linear momentum.

Linear Momentum

The **linear momentum** of a particle or an object that can be modeled as a particle of mass *m* moving with a velocity $\vec{\mathbf{v}}$ is defined to be the product of the mass and velocity:

- $\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$
  - The terms momentum and linear momentum will be used interchangeably in the text.

Linear momentum is a vector quantity.

- Its direction is the same as the direction of the velocity.

The dimensions of momentum are ML/T.

The SI units of momentum are kg · m / s.

Momentum can be expressed in component form:

- $p_x = m\, v_x \qquad p_y = m\, v_y \qquad p_z = m\, v_z$

Momentum and Kinetic Energy

Momentum and kinetic energy both involve mass and velocity.

There are major differences between them:

- Kinetic energy is a scalar and momentum is a vector.
- Kinetic energy can be transformed to other types of energy.
  - There is only one type of linear momentum, so there are no similar transformations.

Analysis models based on momentum are separate from those based on energy.

This difference allows an independent tool to use in solving problems.

Newton's Second Law and Momentum

Newton's Second Law can be used to relate the momentum of a particle to the resultant force acting on it.

$$\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} = \frac{d\left(m\vec{\mathbf{v}}\right)}{dt} = \frac{d\vec{\mathbf{p}}}{dt}$$

with constant mass

The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.

- This is the form in which Newton presented the Second Law.
- It is a more general form than the one we used previously.
- This form also allows for mass changes.

Conservation of Linear Momentum

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

- The momentum of the system is conserved, not necessarily the momentum of an individual particle.
  - Avoid applying conservation of momentum to a single particle.
- This also tells us that the total momentum of an isolated system equals its initial momentum.

Conservation of Momentum, 2

Conservation of momentum can be expressed mathematically in various ways:

- $\vec{p}_{total} = \vec{p}_1 + \vec{p}_2 = \text{constant}$
- $\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$
- This is the mathematical statement of a new analysis model, the isolated system (momentum).

In component form, the total momenta in each direction are independently conserved.

- $p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$ $\quad p_{1iy} + p_{2iy} = p_{1fy} + p_{2fy}$ $\quad p_{1iz} + p_{2iz} = p_{1fz} + p_{2fz}$

Conservation of momentum can be applied to systems with any number of particles.

The momentum version of the isolated system model states *whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.*

Forces and Conservation of Momentum

In conservation of momentum, there is no statement concerning the types of forces acting on the particles of the system.

The forces are not specified as conservative or non-conservative.

There is no indication if the forces are constant or not.

The only requirement is that the forces must be internal to the system.

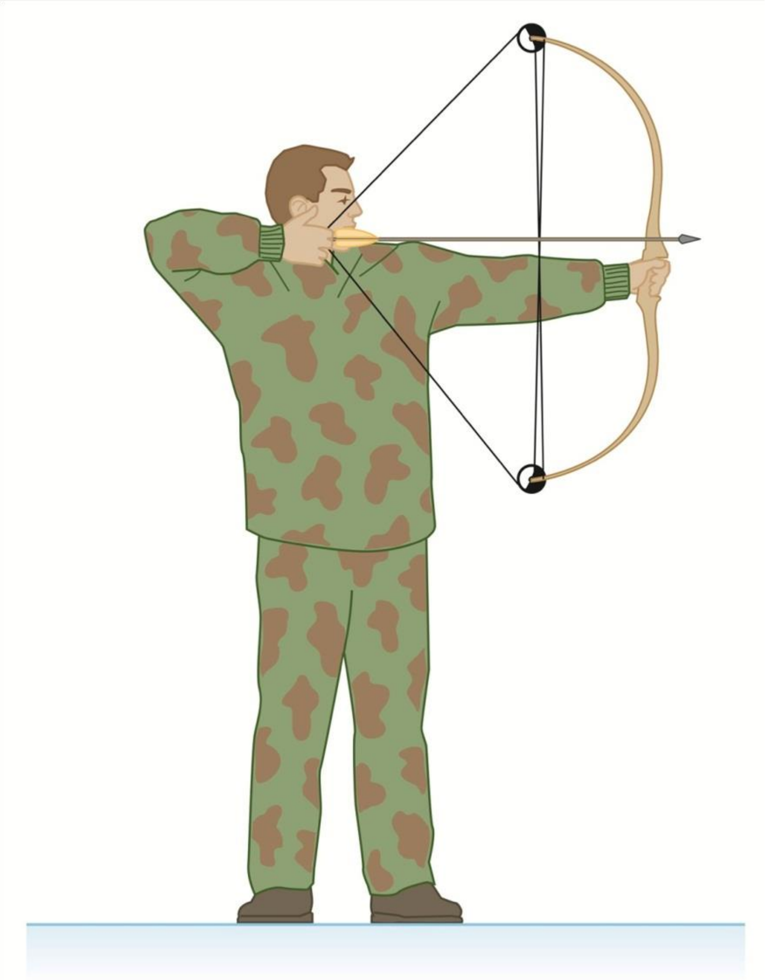
- This gives a hint about the power of this new model.

Conservation of Momentum, Archer Example Revisited

The archer is standing on a frictionless surface (ice).

Approaches:

- Motion – no
  - No information about velocities, etc.
- Newton's Second Law – no
  - No information about *F* or *a*
- Energy approach – no
  - No information about work or energy
- Momentum – yes



Archer Example, 2

Conceptualize

- The arrow is fired one way and the archer recoils in the opposite direction.

Categorize

- Momentum
  - Let the system be the archer with bow (particle 1) and the arrow (particle 2).
  - It is not an isolated system in the *y*-direction because the gravitational force and the normal force act on it.
  - There are no external forces in the *x*-direction, so it is isolated in terms of momentum in the *x*-direction.
    - Apply the isolated system (momentum) model in terms of momentum components in the *x*-direction.

Archer Example, 3

Analyze, cont.

- Total momentum before releasing the arrow is 0
- The total momentum after releasing the arrow is

$$\vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} = 0 \rightarrow m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

Finalize

- The final velocity of the archer is negative.
  - Indicates he moves in a direction opposite the arrow
  - Archer has much higher mass than arrow, so velocity is much lower

Notes

- The problem seems very simple, but could not be solved using previous analysis models.
- Using the new momentum model made the solution quite simple.

Impulse and Momentum

The momentum of a system changes if a net force from the environment acts on the system.

For momentum considerations, a system is non-isolated if a net force acts on the system for a time interval.

From Newton's Second Law, $\vec{\mathbf{F}} = \dfrac{d\vec{\mathbf{p}}}{dt}$

Solving for $d\vec{\mathbf{p}}$ gives $d\vec{\mathbf{p}} = \sum \vec{\mathbf{F}} dt$

Integrating to find the change in momentum over some time interval.

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \int_{t_i}^{t_f} \vec{\mathbf{F}} dt = \vec{\mathbf{I}}$$

The integral is called the *impulse,* $\vec{\mathbf{I}}$, of the force acting on an object over Δt.

Impulse-Momentum Theorem

This equation expresses the **impulse-momentum theorem**: The change in the momentum of a particle is equal to the impulse of the new force acting on the particle.

- $\Delta\vec{\mathbf{p}} = \vec{\mathbf{I}}$
- This is equivalent to Newton's Second Law.
- This is identical in form to the conservation of energy equation.
- This is the most general statement of the principle of conservation of momentum and is called the conservation of momentum equation.
  - This form applies to non-isolated systems.
- This is the mathematical statement of the **non-isolated system (momentum) model.**

More About Impulse

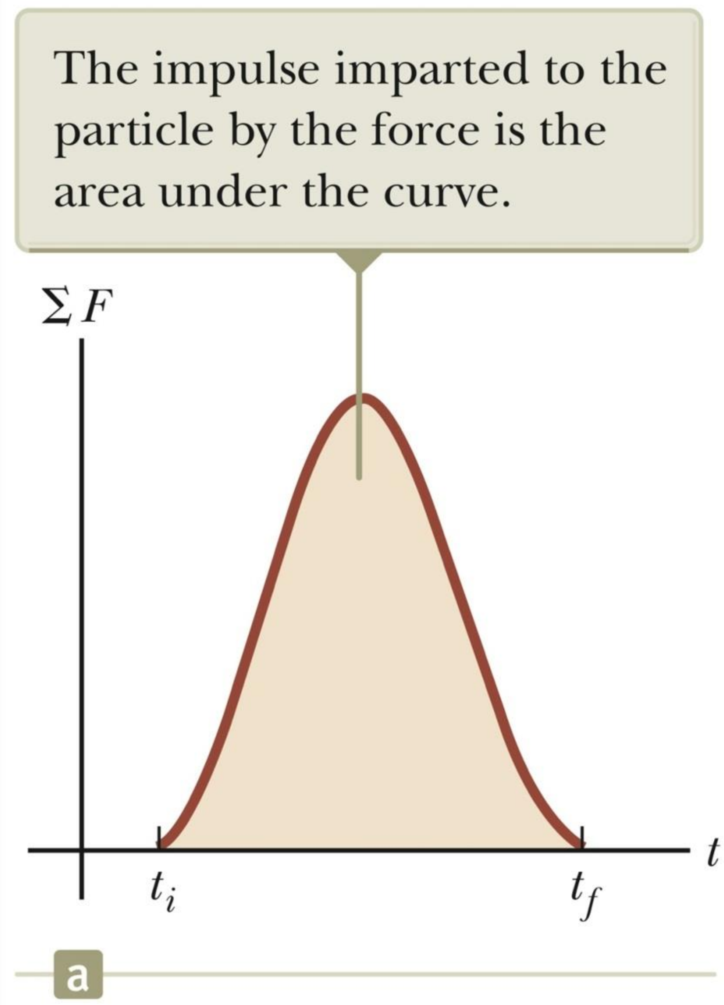
Impulse is a vector quantity.

The magnitude of the impulse is equal to the area under the force-time curve.

- The force may vary with time.

Dimensions of impulse are M L / T

Impulse is not a property of the particle, but a measure of the change in momentum of the particle.

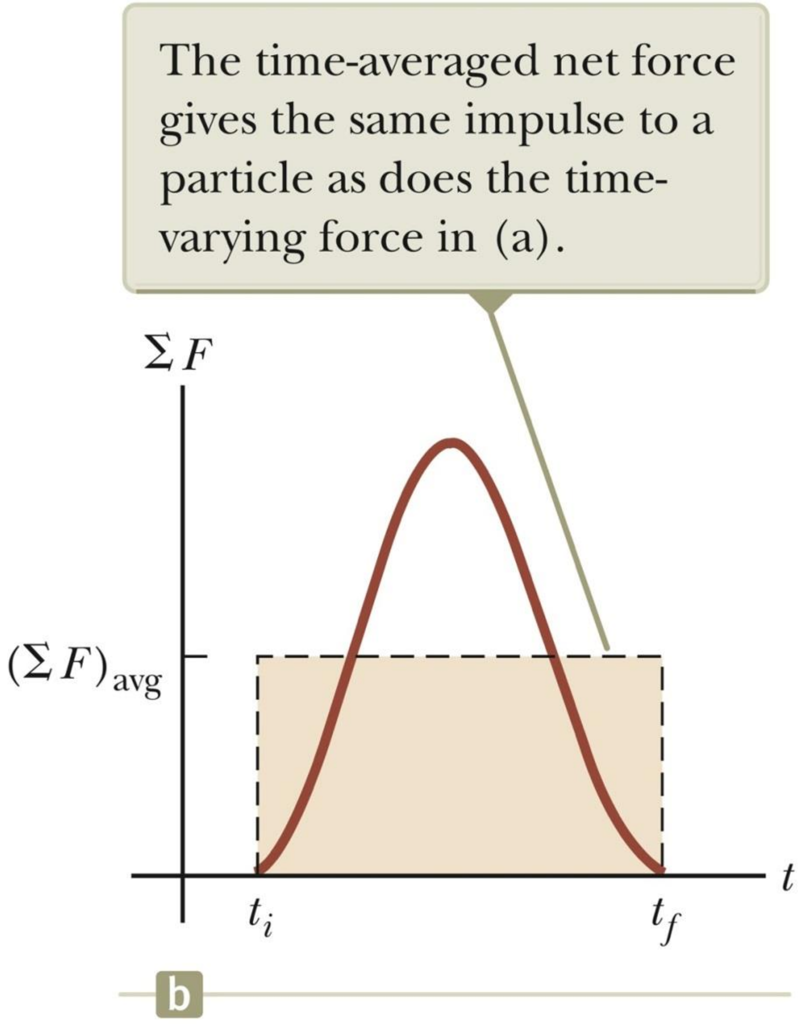


Impulse, Final

The impulse can also be found by using the time averaged force.

$$\vec{\mathbf{I}} = \sum \vec{\mathbf{F}} \Delta t$$

This would give the same impulse as the time-varying force does.



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Impulse Approximation

In many cases, one force acting on a particle acts for a short time, but is much greater than any other force present.

When using the Impulse Approximation, we will assume this is true.

- Especially useful in analyzing collisions

The force will be called the *impulsive force.*

The particle is assumed to move very little during the collision.

$\vec{\mathbf{p}}_i$ *and* $\vec{\mathbf{p}}_f$ represent the momenta *immediately* before and after the collision.

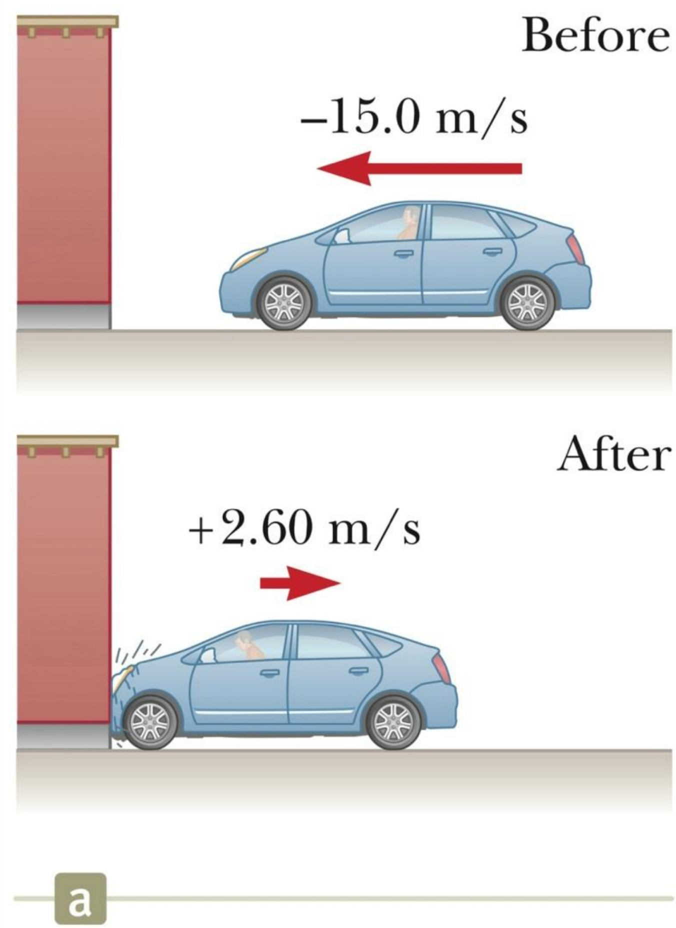
Impulse-Momentum: Crash Test Example

Conceptualize

- The collision time is short.
- We can image the car being brought to rest very rapidly and then moving back in the opposite direction with a reduced speed.

Categorize

- Assume net force exerted on the car by wall and friction with the ground is large compared with other forces.
- Gravitational and normal forces are perpendicular and so do not effect the horizontal momentum.



Crash Test Example, 2

Categorize, cont.

- Can apply impulse approximation
- The car's change in momentum is due to an impulse from the environment.
- Therefore, the non-isolated system (momentum) model can be applied.

Analyze

- The momenta before and after the collision between the car and the wall can be determined .
- Find
  - Initial momentum
  - Final momentum
  - Impulse
  - Average force

Crash Test Example, 3

Finalize

- The net force is a combination of the normal force on the car from the wall and nay friction force between the tires and the ground as the front of the car crumples.
- Check signs on velocities to be sure they are reasonable

Collisions – Characteristics

The term **collision** represents an event during which two particles come close to each other and interact by means of forces.

- May involve physical contact, but must be generalized to include cases with interaction without physical contact

The interaction forces are assumed to be much greater than any external forces present.

- This means the impulse approximation can be used.

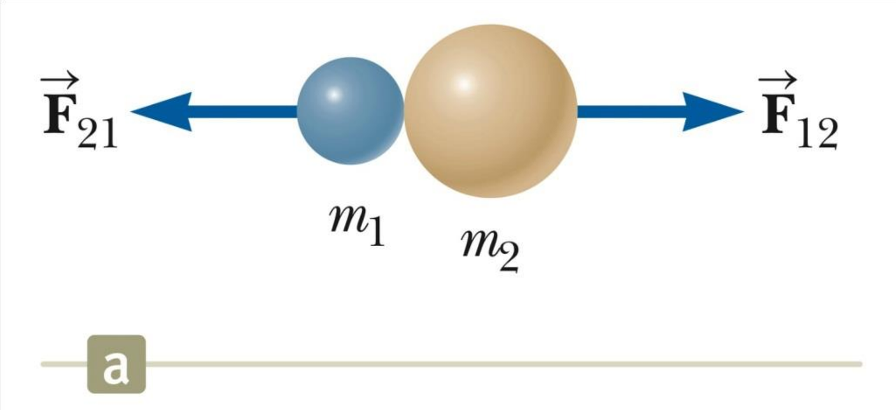
Collisions – Example 1

Collisions may be the result of direct contact.

The impulsive forces may vary in time in complicated ways.

- This force is internal to the system.
- Observe the variations in the active figure.

Momentum is conserved.

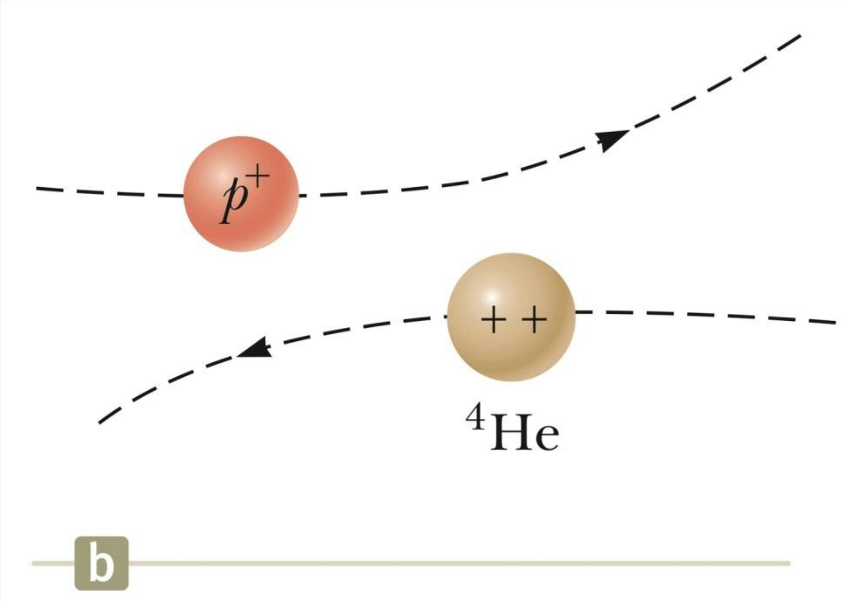


Collisions – Example 2

The collision need not include physical contact between the objects.

There are still forces between the particles.

This type of collision can be analyzed in the same way as those that include physical contact.



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Types of Collisions

In an ***elastic*** collision, momentum and kinetic energy are conserved.

- Perfectly elastic collisions occur on a microscopic level.
- In macroscopic collisions, only approximately elastic collisions actually occur.
  - Generally some energy is lost to deformation, sound, etc.
- These collisions are described by the isolated system model for both energy and momentum.
  - There must be no transformation of kinetic energy into other types of energy within the system.

In an ***inelastic*** collision, kinetic energy is not conserved, although momentum is still conserved.

- If the objects stick together after the collision, it is a ***perfectly inelastic*** collision.

Collisions, cont.

In an inelastic collision, some kinetic energy is lost, but the objects do not stick together.

Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types .

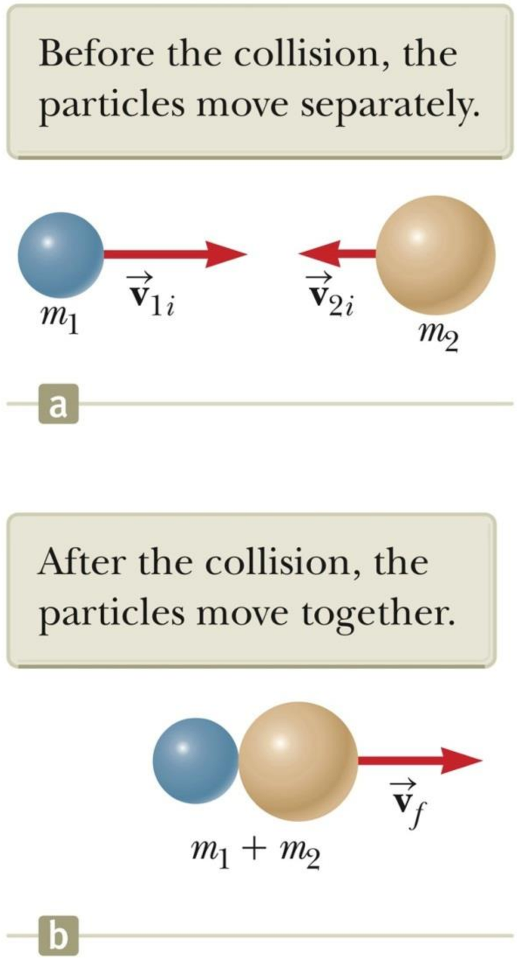
Momentum is conserved in all collisions

Perfectly Inelastic Collisions

Momentum of an isolated system is conserved in any collision, so the total momentum before the collision is equal to the total momentum of the composite system after the collision.

Since the objects stick together, they share the same velocity after the collision.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = (m_1 + m_2)\vec{\mathbf{v}}_f$$



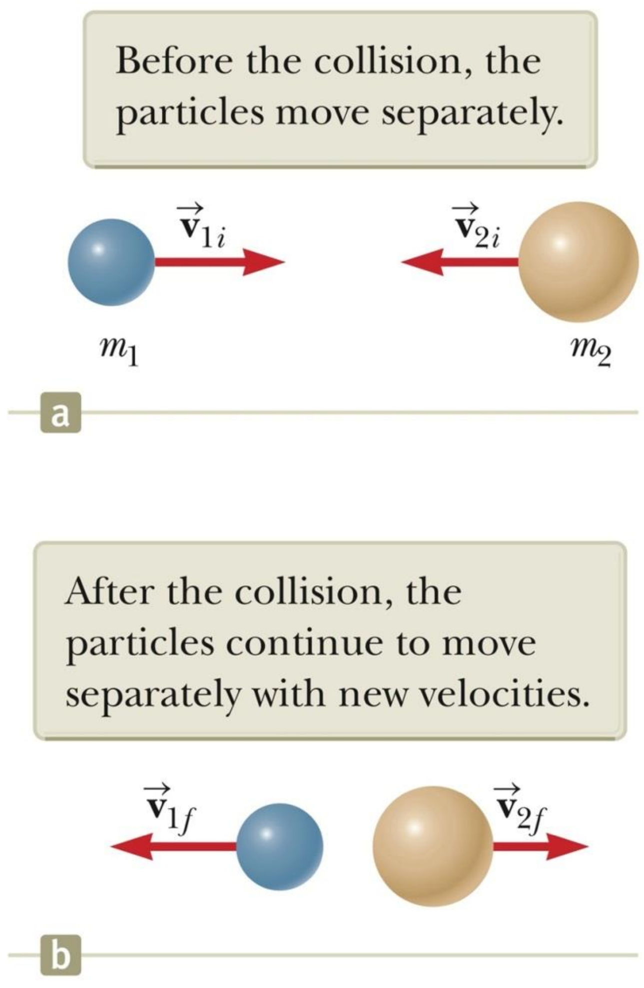
Elastic Collisions

Both momentum and kinetic energy are conserved.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$\frac{1}{2}m_1\mathbf{v}_{1i}^2 + \frac{1}{2}m_2\mathbf{v}_{2i}^2 = \frac{1}{2}m_1\mathbf{v}_{1f}^2 + \frac{1}{2}m_2\mathbf{v}_{2f}^2$$

Typically, there are two unknowns to solve for and so you need two equations.



Elastic Collisions, cont.

The kinetic energy equation can be difficult to use.

With some algebraic manipulation, a different equation can be used.

$v_{1i} - v_{2i} = v_{1f} + v_{2f}$

This equation, along with conservation of momentum, can be used to solve for the two unknowns.

- It can only be used with a one-dimensional, elastic collision between two objects.
- Using this equation eliminates the need for using an equation with quadratic terms (from the kinetic energy equation).

Remember to use the appropriate signs for all velocities.

Elastic Collisions, final

Example of some special cases:

- $m_1 = m_2$ – the particles exchange velocities
- When a very heavy particle collides head-on with a very light one initially at rest, the heavy particle continues in motion unaltered and the light particle rebounds with a speed of about twice the initial speed of the heavy particle.
- When a very light particle collides head-on with a very heavy particle initially at rest, the light particle has its velocity reversed and the heavy particle remains approximately at rest.

Problem-Solving Strategy: One-Dimensional Collisions

Conceptualize

- Image the collision occurring in your mind.
- Draw simple diagrams of the particles before and after the collision.
- Include appropriate velocity vectors.

Categorize

- Is the system of particles isolated?
- Is the collision elastic, inelastic or perfectly inelastic?

Problem-Solving Strategy: One-Dimensional Collisions

Analyze

- Set up the mathematical representation of the problem.
- Solve for the unknown(s).

Finalize

- Check to see if the answers are consistent with the mental and pictorial representations.
- Check to be sure your results are realistic.

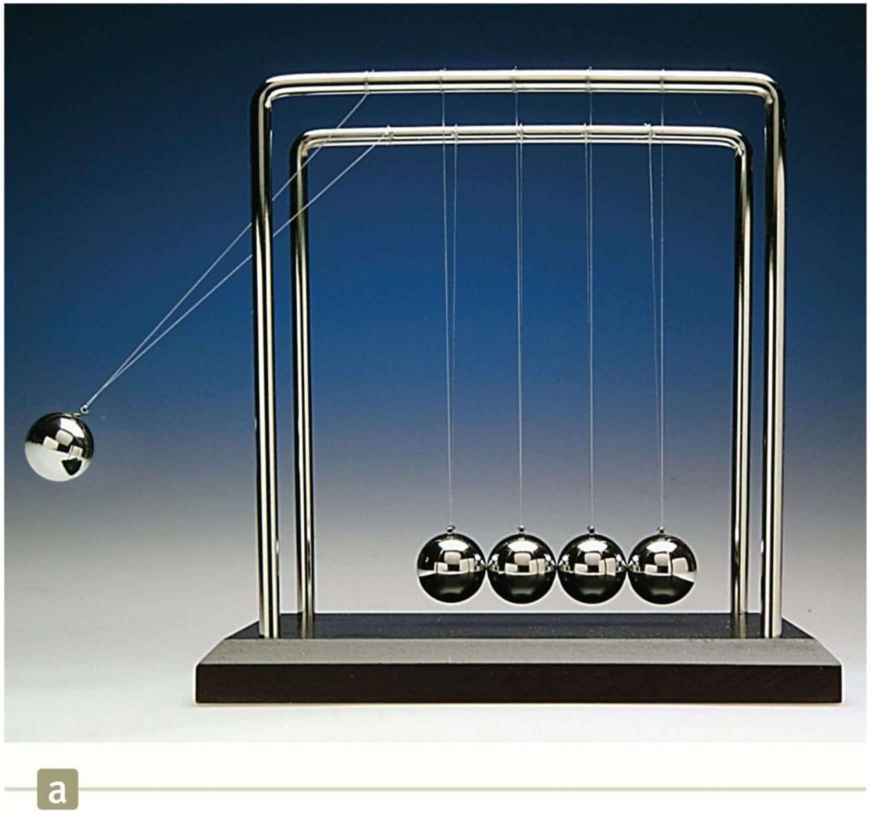
Example: Stress Reliever

Conceptualize

- Imagine one ball coming in from the left and two balls exiting from the right.
- Is this possible?

Categorize

- Due to shortness of time, the impulse approximation can be used.
- Categorize the system as isolated in terms of both momentum and energy.
- Elastic collisions



Example: Stress Reliever, cont.

Analyze

- Check to see if momentum is conserved.
  - It is
- Check to see if kinetic energy is conserved.
  - It is not
  - Therefore, the collision couldn’t be elastic.

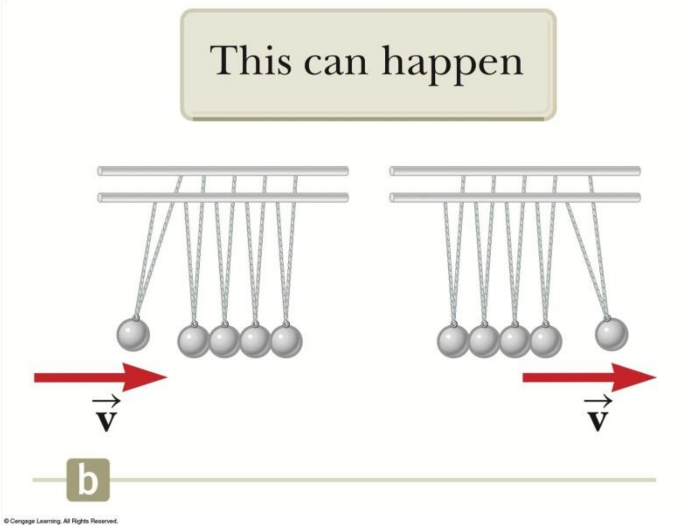
Finalize

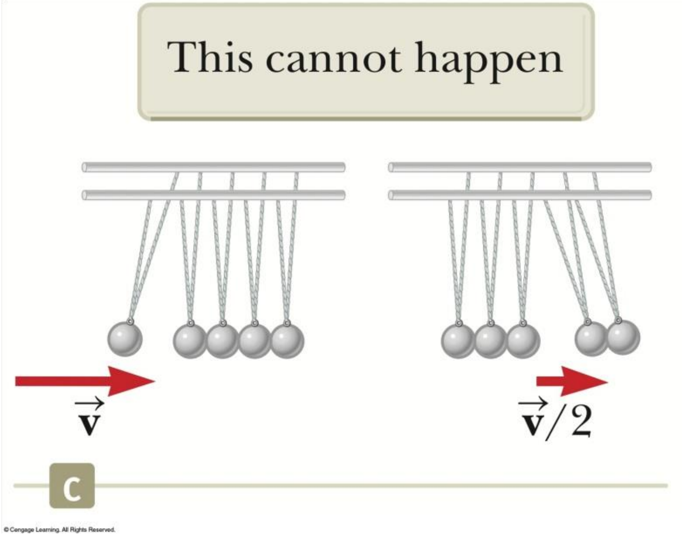
- Having two balls exit was not possible if only one ball is released.

Example: Stress Reliever, final

For a collision to actually occur, both momentum and kinetic energy must be conserved.

- One way to do so is with equal numbers of balls released and exiting.
- Another way is to have some of the balls taped together so they move as one object.





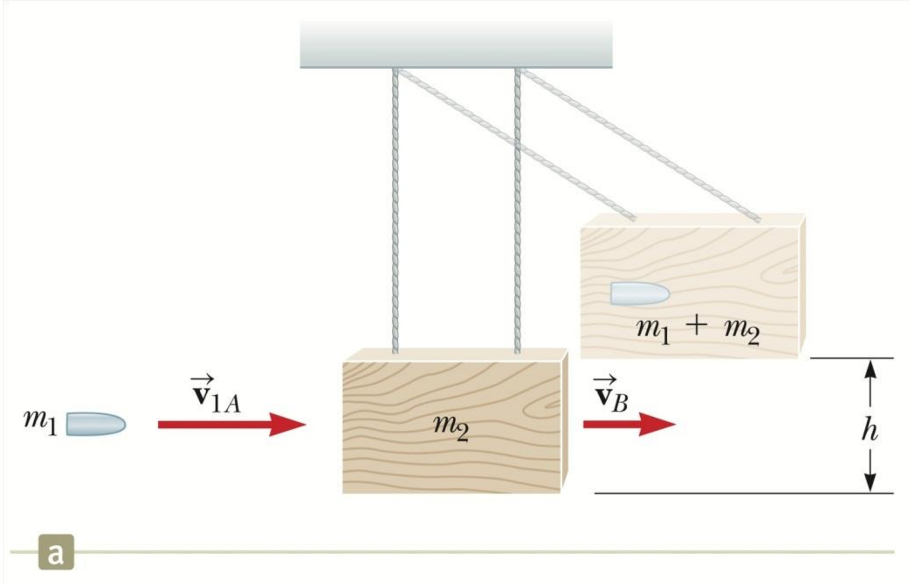
Collision Example – Ballistic Pendulum

Conceptualize

- Observe diagram
- The projectile enters the pendulum, which swings up to some height where it momentarily stops.

Categorize

- Isolated system in terms of momentum for the projectile and block.
- Perfectly inelastic collision – the bullet is embedded in the block of wood.
- Momentum equation will have two unknowns
- Use conservation of energy from the pendulum to find the velocity just after the collision.
- Then you can find the speed of the bullet.



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Ballistic Pendulum, cont.

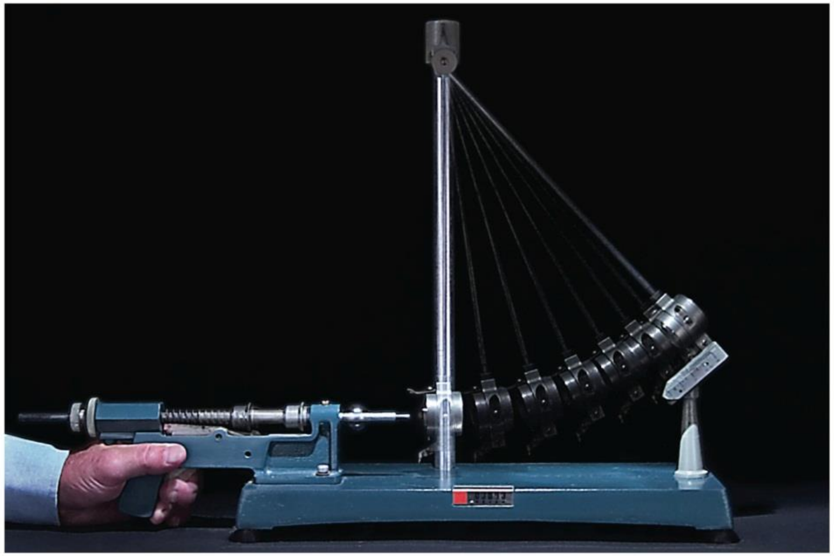
A multi-flash photograph of a ballistic pendulum.

Analyze

- Equations for the momentum and conservation of energy with kinetic and gravitational potential energies.
- Solve resulting system of equations.

Finalize

- Note different systems involved and different analysis models used.
- Some energy was transferred during the perfectly inelastic collision.



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Two-Dimensional Collisions

The momentum is conserved in all directions.

Use subscripts for

- Identifying the object
- Indicating initial or final values
- The velocity components

If the collision is elastic, use conservation of kinetic energy as a second equation.

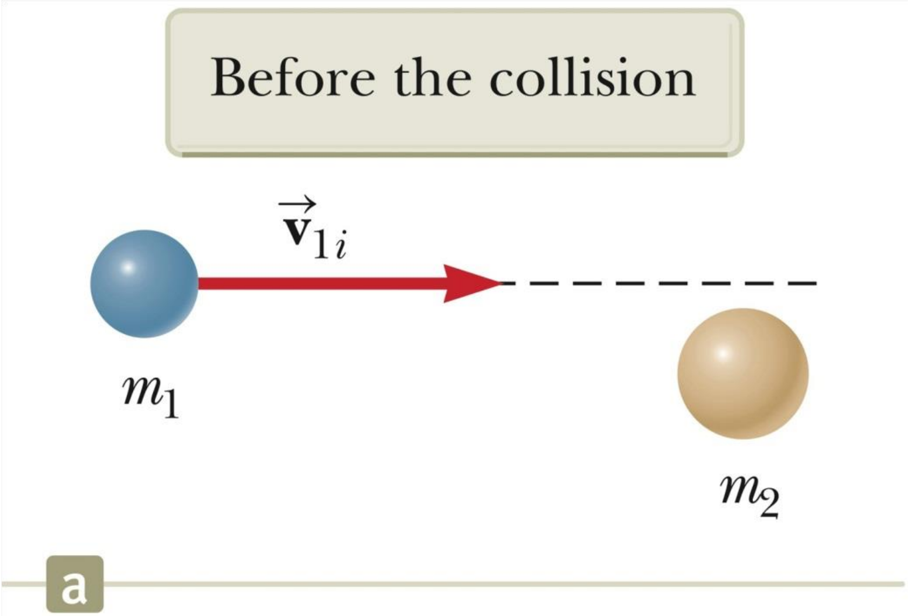
- Remember, the simpler equation can only be used for one-dimensional situations.

Two-Dimensional Collision, example

Particle 1 is moving at velocity $\vec{\mathbf{v}}_{1i}$ and particle 2 is at rest.

In the *x*-direction, the initial momentum is $m_1 v_{1i.}$

In the *y*-direction, the initial momentum is 0.



Two-Dimensional Collision, example cont.

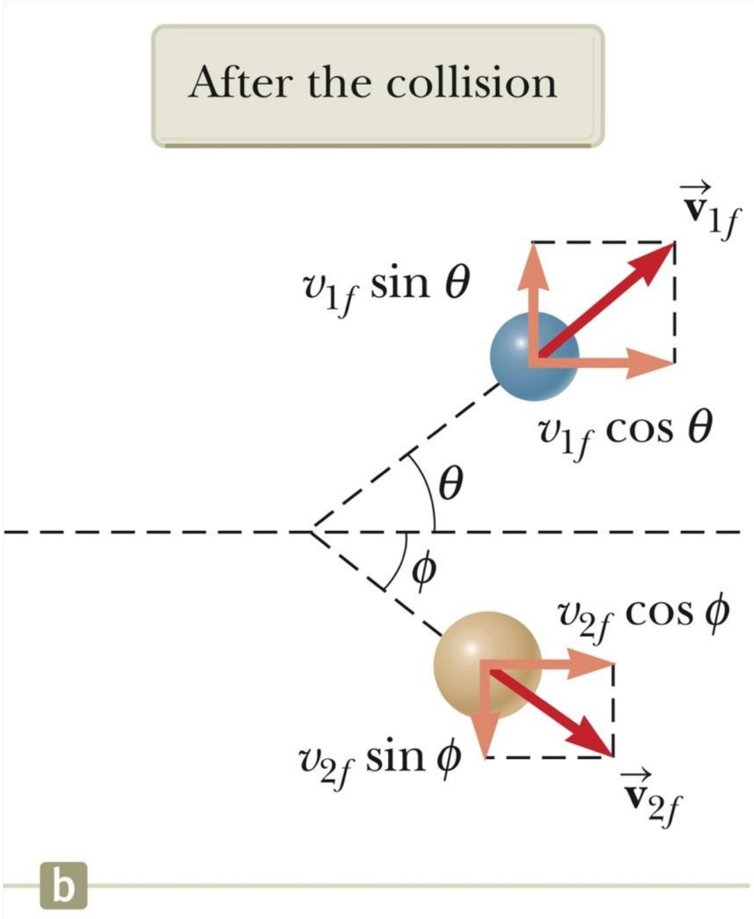
After the collision, the momentum in the x-direction is $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$.

After the collision, the momentum in the y-direction is $m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$.

- The negative sign is due to the component of the velocity being downward.

If the collision is elastic, apply the kinetic energy equation.

This is an example of a *glancing collision.*



Problem-Solving Strategies – Two-Dimensional Collisions

Conceptualize

- Imagine the collision.
- Predict approximate directions the particles will move after the collision.
- Set up a coordinate system and define your velocities with respect to that system.
  - It is usually convenient to have the *x*-axis coincide with one of the initial velocities.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.

Categorize

- Is the system isolated?
- If so, categorize the collision as elastic, inelastic or perfectly inelastic.

Problem-Solving Strategies – Two-Dimensional Collisions, 2

Analyze

- Write expressions for the x- and y-components of the momentum of each object before and after the collision.
  - Remember to include the appropriate signs for the components of the velocity vectors.
- Write expressions for the total momentum of the system in the x-direction before and after the collision and equate the two. Repeat for the total momentum in the y-direction.
- If the collision is inelastic, kinetic energy of the system is not conserved, and additional information is probably needed.
- If the collision is perfectly inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknowns.

Problem-Solving Strategies – Two-Dimensional Collisions, 3

Analyze, cont.

- If the collision is elastic, the kinetic energy of the system is conserved.
  - Equate the total kinetic energy before the collision to the total kinetic energy after the collision to obtain more information on the relationship between the velocities.

Finalize

- Check to see if your answers are consistent with the mental and pictorial representations.
- Check to be sure your results are realistic.

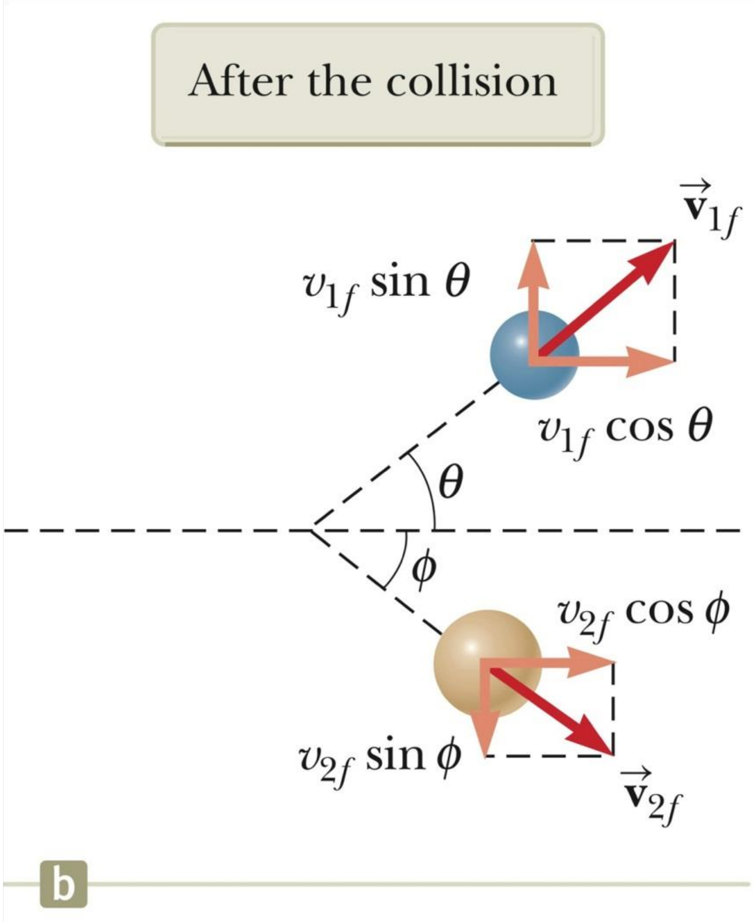
Two-Dimensional Collision Example

Conceptualize

- See picture
- Choose East to be the positive x-direction and North to be the positive y-direction.

Categorize

- Ignore friction
- Model the vehicles as particles.
- Model the system as isolated in terms of momentum.
- The collision is perfectly inelastic.
  - The vehicles stick together.



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Two-Dimensional Collision Example, cont.

Analyze

- Before the collision, the car has the total momentum in the x-direction and the truck has the total momentum in the y-direction.
- After the collision, both have x- and y-components.
- Write expressions for initial and final momenta in both directions.
  - Evaluate any expressions with no unknowns.
- Solve for unknowns

Finalize

- Check to be sure the results are reasonable.

The Center of Mass

There is a special point in a system or object, called the ***center of mass***, that moves as if all of the mass of the system is concentrated at that point.

The system will move as if an external force were applied to a single particle of mass *M* located at the center of mass.

- *M* is the total mass of the system.

This behavior is independent of other motion, such as rotation or vibration, or deformation of the system.

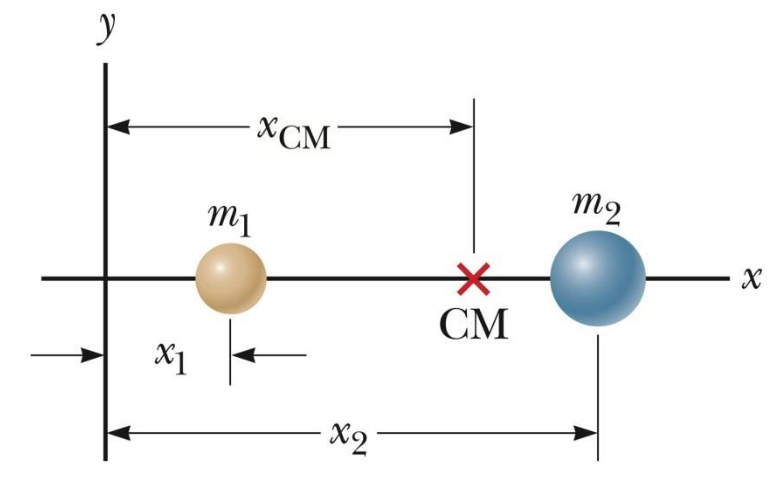
- This is the particle model.

Center of Mass, Coordinates

The coordinates of the center of mass are

$$x_{CM} = \frac{\sum_i m_i x_i}{M} \qquad y_{CM} = \frac{\sum_i m_i y_i}{M}$$

$$z_{CM} = \frac{\sum_i m_i z_i}{M}$$



- M is the total mass of the system.
  - Use the active figure to observe effect of different masses and positions.

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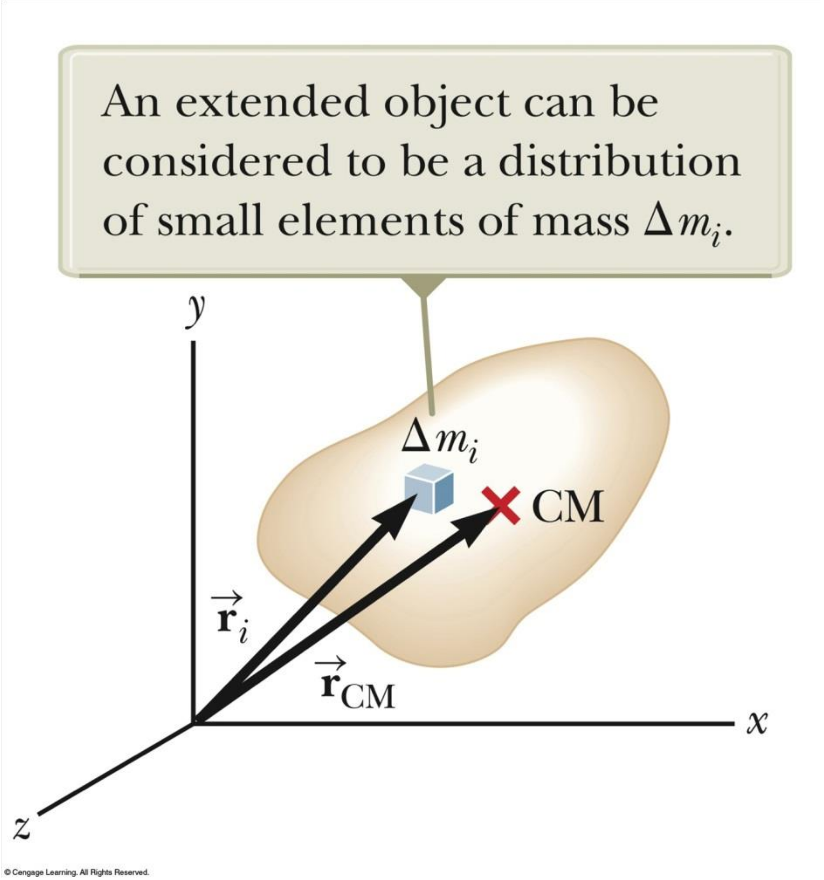
Center of Mass, Extended Object

Similar analysis can be done for an extended object.

Consider the extended object as a system containing a large number of small mass elements.

Since separation between the elements is very small, it can be considered to have a constant mass distribution.



Center of Mass, position

The center of mass in three dimensions can be located by its position vector, $\vec{\mathbf{r}}_{CM}$.

- For a system of particles,

$$\vec{\mathbf{r}}_{CM} = \frac{1}{M}\sum_i m_i \vec{\mathbf{r}}_i$$

  - $\vec{\mathbf{r}}_i$ is the position of the i^{th} particle, defined by

$$\mathbf{r}_i = x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}$$

- For an extended object,

$$\vec{\mathbf{r}}_{\text{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\, dm$$

Center of Mass, Symmetric Object

The center of mass of any symmetric object of uniform density lies on an axis of symmetry and on any plane of symmetry.

Center of Gravity

Each small mass element of an extended object is acted upon by the gravitational force.

The net effect of all these forces is equivalent to the effect of a single force $M\vec{\mathbf{g}}$ acting through a point called the **center of gravity.**

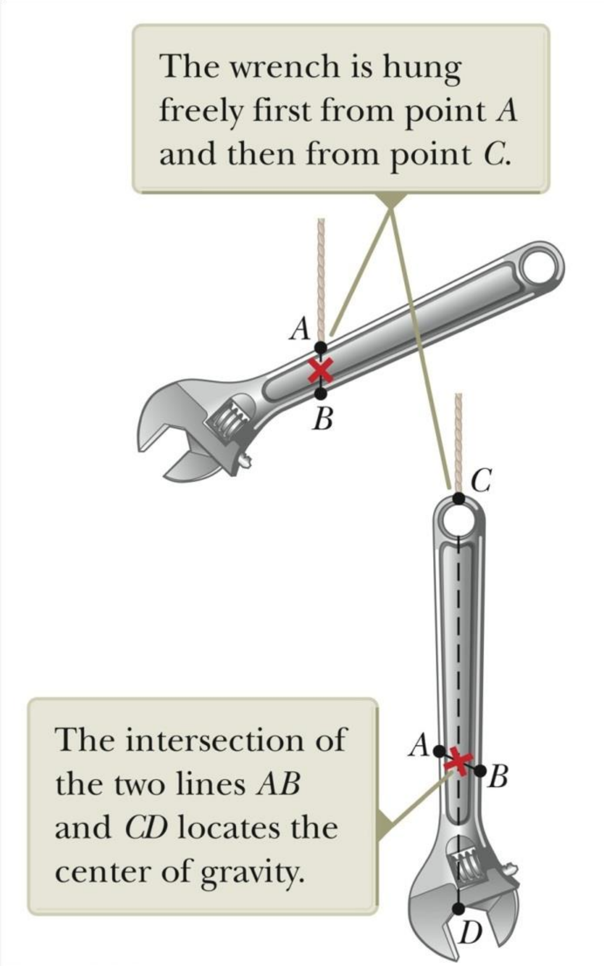
- If $\vec{\mathbf{g}}$ is constant over the mass distribution, the center of gravity coincides with the center of mass.

Finding Center of Gravity, Irregularly Shaped Object

Suspend the object from one point.

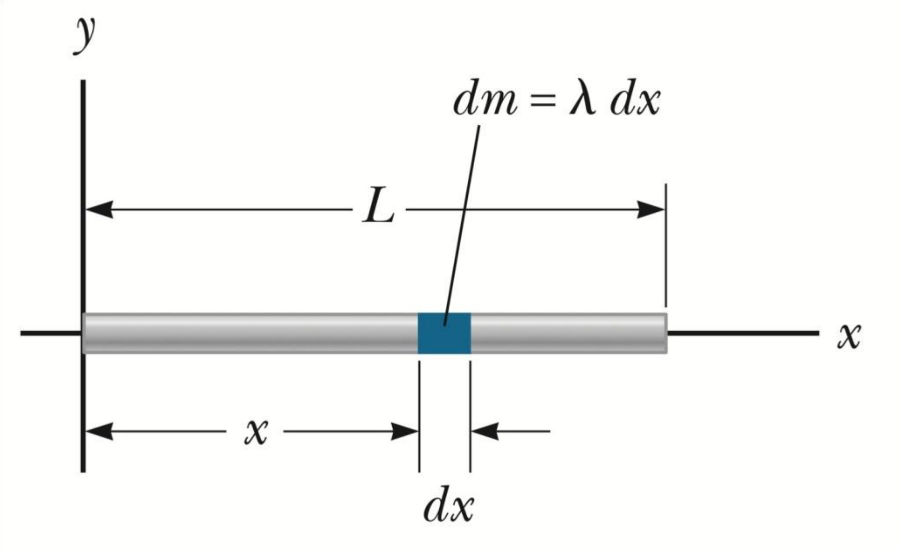
Then, suspend from another point.

The intersection of the resulting lines is the center of gravity and half way through the thickness of the wrench.



Conceptualize

- Find the center of mass of a rod of mass *M* and length *L.*
- The location is on the *x*-axis (or $y_{CM} = z_{CM} = 0$)



Categorize

- Analysis problem

Analyze

- Use equation for x_{cm}
- $x_{CM} = L / 2$

Motion of a System of Particles

Assume the total mass, M, of the system remains constant.

We can describe the motion of the system in terms of the velocity and acceleration of the center of mass of the system.

We can also describe the momentum of the system and Newton’s Second Law for the system.

Velocity and Momentum of a System of Particles

The velocity of the center of mass of a system of particles is

$$\vec{\mathbf{v}}_{\text{CM}} = \frac{d\vec{\mathbf{r}}_{\text{CM}}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{v}}_i$$

The momentum can be expressed as

$$M\,\vec{\mathbf{v}}_{\text{CM}} = \sum_i m_i \vec{\mathbf{v}}_i = \sum_i \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_{\text{tot}}$$

The total linear momentum of the system equals the total mass multiplied by the velocity of the center of mass.

Acceleration and Force in a System of Particles

The acceleration of the center of mass can be found by differentiating the velocity with respect to time.

$$\vec{\mathbf{a}}_{\text{CM}} = \frac{d\vec{\mathbf{v}}_{\text{CM}}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{a}}_i$$

The acceleration can be related to a force.

$$M\vec{\mathbf{a}}_{\text{CM}} = \sum_i \vec{\mathbf{F}}_i$$

If we sum over all the internal force vectors, they cancel in pairs and the net force on the system is caused only by the external forces.

Newton's Second Law for a System of Particles

Since the only forces are external, the net external force equals the total mass of the system multiplied by the acceleration of the center of mass:

$$\sum \vec{\mathbf{F}}_{ext} = M\vec{\mathbf{a}}_{CM}$$

The center of mass of a system of particles of combined mass *M* moves like an equivalent particle of mass *M* would move under the influence of the net external force on the system.

Impulse and Momentum of a System of Particles

The impulse imparted to the system by external forces is

$$\int \sum \vec{\mathbf{F}}_{ext} dt = M \int d\vec{\mathbf{v}}_{CM} \rightarrow \Delta \vec{\mathbf{p}}_{tot} = \vec{\mathbf{I}}$$

The total linear momentum of a system of particles is conserved if no net external force is acting on the system.

$$M \vec{\mathbf{v}}_{CM} = \vec{\mathbf{p}}_{tot} = constant \quad when \sum \vec{\mathbf{F}}_{ext} = 0$$

For an isolated system of particles, both the total momentum and the velocity of the center of mass are constant in time.

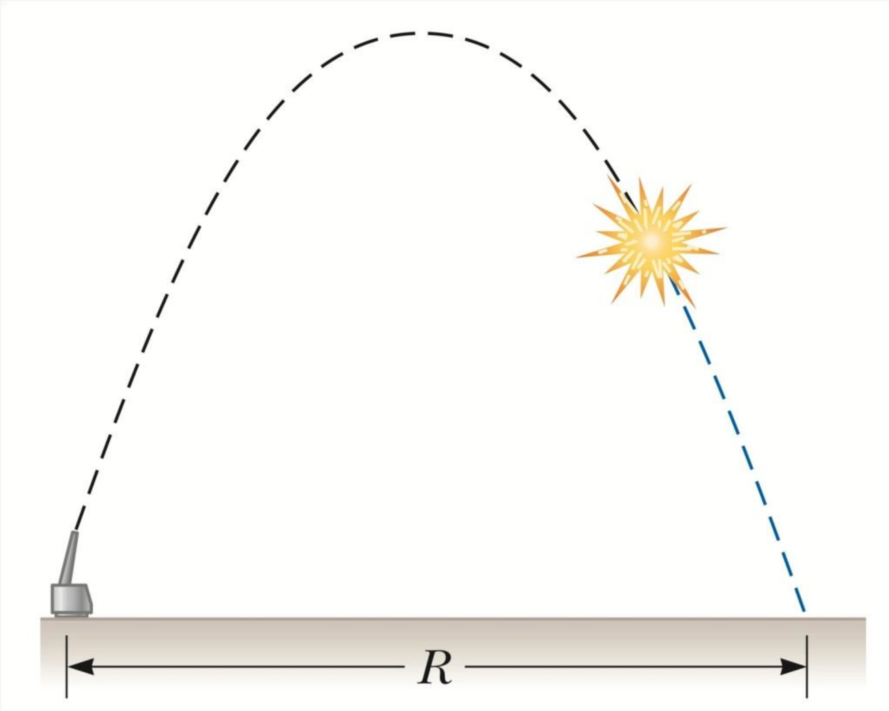
- This is a generalization of the isolated system (momentum) model for a many-particle system.

Motion of the Center of Mass, Example

A projectile is fired into the air and suddenly explodes.

With no explosion, the projectile would follow the dotted line.

After the explosion, the center of mass of the fragments still follows the dotted line, the same parabolic path the projectile would have followed with no explosion.



Deformable Systems

To analyze the motion of a deformable system, use Conservation of Energy and the Impulse-Momentum Theorem.

$$\Delta E_{system} = \sum T \rightarrow \Delta K + \Delta U = 0$$

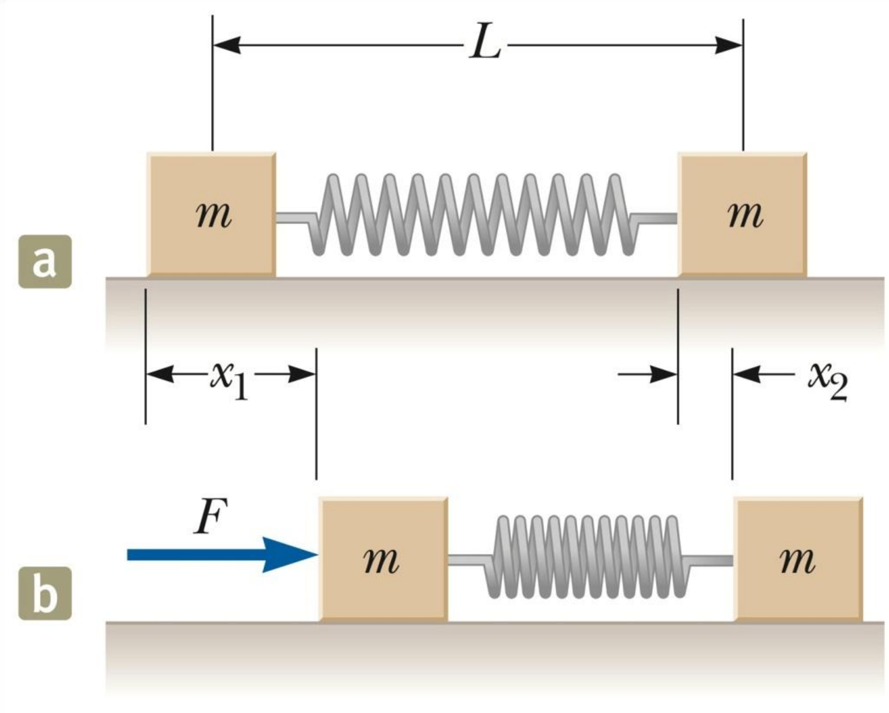
$$\Delta \vec{\mathbf{p}}_{tot} = \vec{\mathbf{I}} \rightarrow m\Delta \vec{\mathbf{v}} = \int \vec{\mathbf{F}}_{ext} dt$$

- If the force is constant, the integral can be easily evaluated.

Deformable System (Spring) Example

Conceptualize

- See figure
- Push on left block, it moves to right, spring compresses.
- At any given time, the blocks are generally moving with different velocities.
- After the force is removed, the blocks oscillate back and forth with respect to the center of mass.



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Spring Example, cont.

Categorize

- Non isolated system in terms of momentum and energy.
  - Work is being done on it by the applied force.
- It is a deformable system.
- The applied force is constant, so the acceleration of the center of mass is constant.
- Model as a particle under constant acceleration.

Analyze

- Apply impulse-momentum
- Solve for v_{cm}

Spring Example, final

Analyze, cont.

- Find energies

Finalize

- Answers do not depend on spring length, spring constant, or time interval.

Rocket Propulsion

When ordinary vehicles are propelled, the driving force for the motion is friction.

- The car is modeled as an non-isolated system in terms of momentum.
- An impulse is applied to the car from the roadway, and the result is a change in the momentum of the car.

The operation of a rocket depends upon the law of conservation of linear momentum as applied to an isolated system, where the system is the rocket plus its ejected fuel.

As the rocket moves in free space, its linear momentum changes when some of its mass is ejected in the form of exhaust gases.

- Because the gases are given momentum when they are ejected out of the engine, the rocket receives a compensating momentum in the opposite direction.
- In free space, the center of mass of the system moves uniformly.

Rocket Propulsion, 2

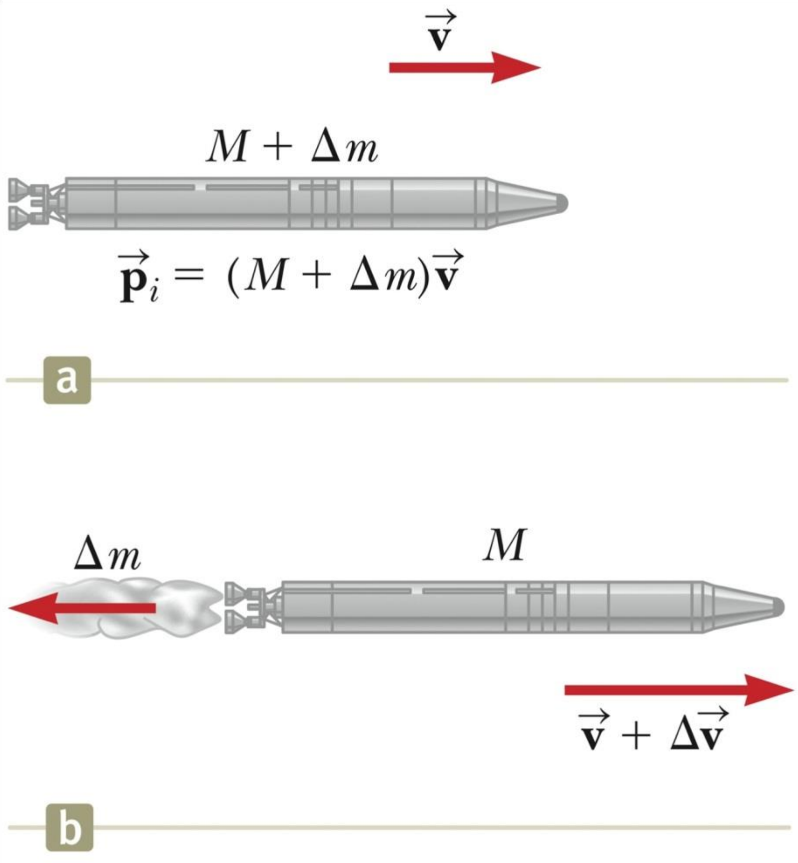
The initial mass of the rocket plus all its fuel is $M + \Delta m$ at time t_i and speed v.

The initial momentum of the system is

$$\vec{\mathbf{p}}_i = (M + \Delta m)\vec{\mathbf{v}}$$

At some time $t + \Delta t$, the rocket's mass has been reduced to M and an amount of fuel, Δm has been ejected.

The rocket's speed has increased by Δv.



Rocket Propulsion, 3

The basic equation for rocket propulsion is

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$$

The increase in rocket speed is proportional to the speed of the escape gases (v_e).

- So, the exhaust speed should be very high.

The increase in rocket speed is also proportional to the natural log of the ratio M_i/M_f.

- So, the ratio should be as high as possible, meaning the mass of the rocket should be as small as possible and it should carry as much fuel as possible.

Thrust

The thrust on the rocket is the force exerted on it by the ejected exhaust gases.

$$thrust = M\frac{dv}{dt} = \left| v_e \frac{dM}{dt} \right|$$

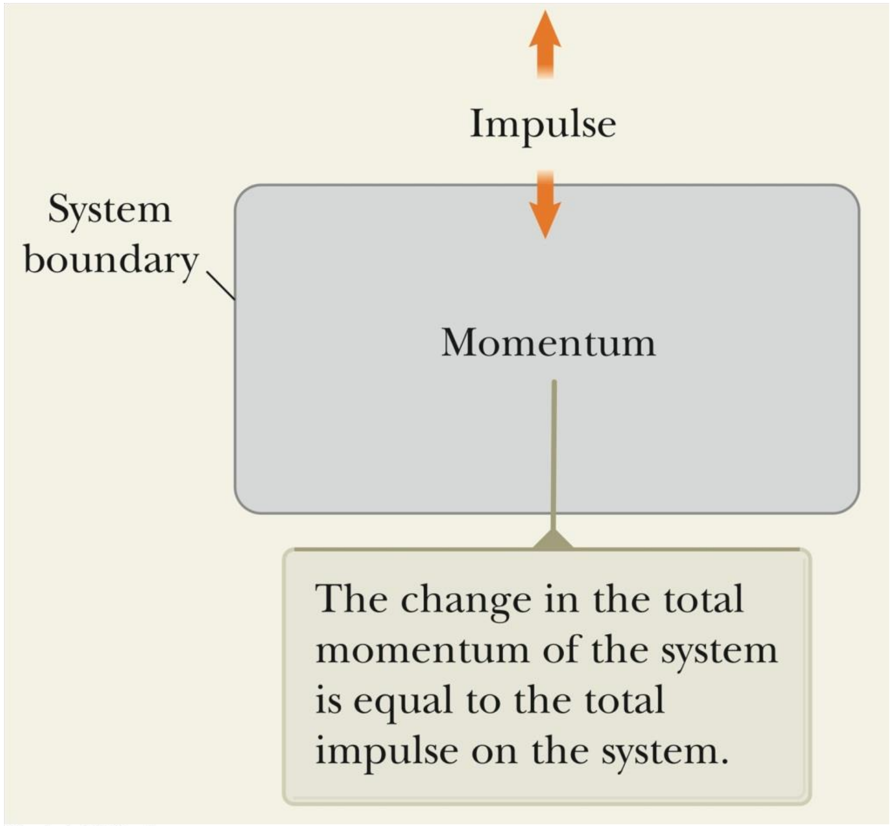
The thrust increases as the exhaust speed increases.

The thrust increases as the rate of change of mass increases.

- The rate of change of the mass is called the **burn rate.**

Problem Solving Summary – Non-isolated System

If a system interacts with its environment in the sense that there is an external force on the system, use the impulse-momentum theorem.

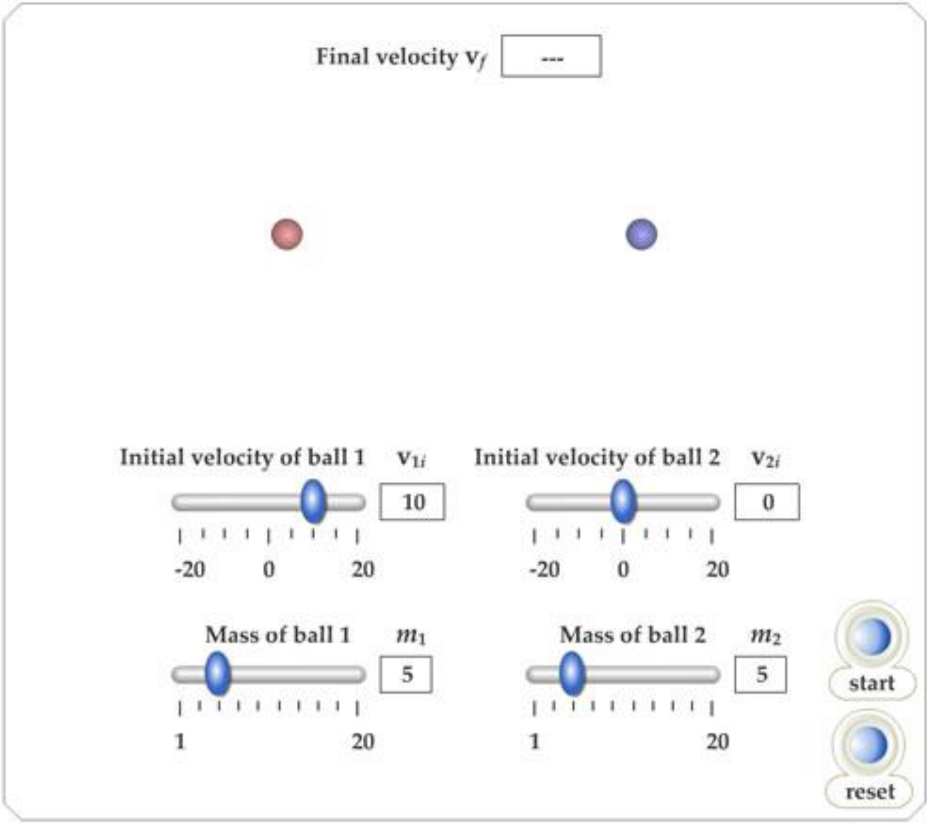


Problem Solving Summary – Isolated System

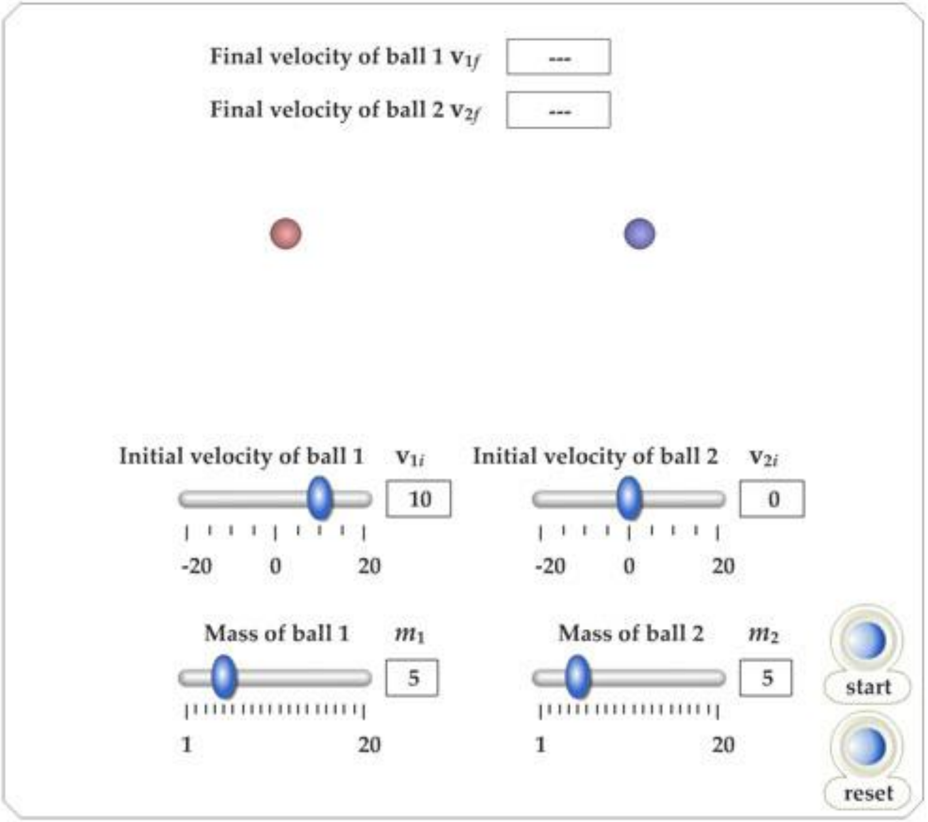
If there are no external forces, the principle of conservation of linear momentum indicates that the total momentum of an isolated system is conserved regardless of the nature of the forces between the members of the system.

The system may be isolated in terms of momentum but non-isolated in terms of energy.

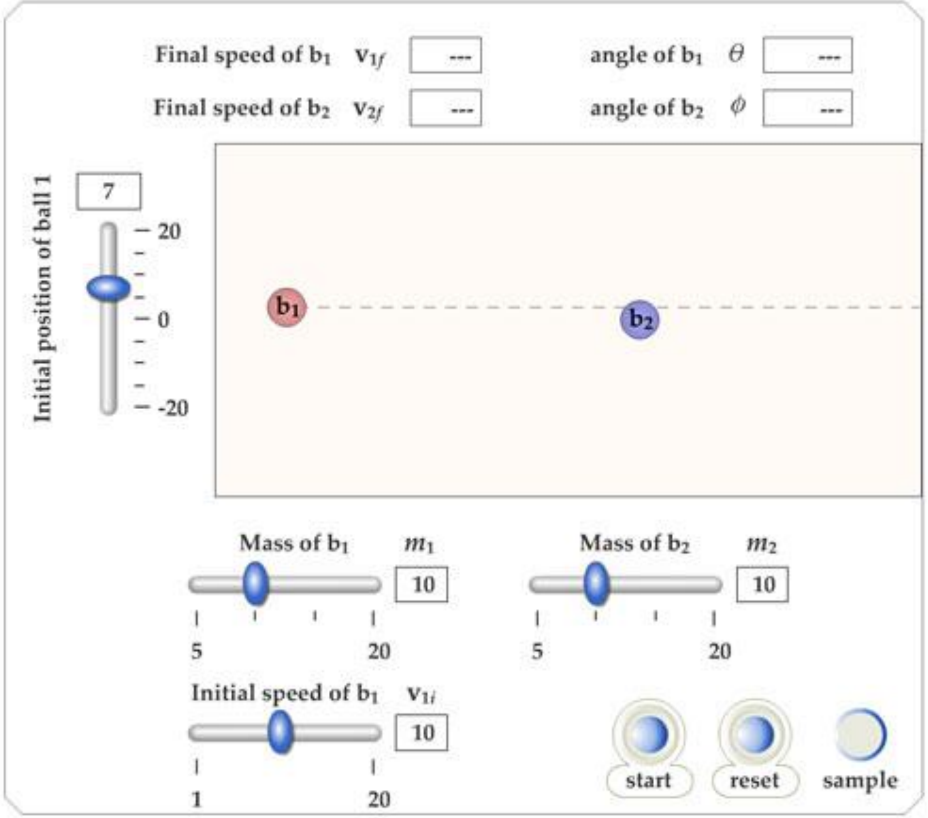
9.6 Perfectly Inelastic Collisions



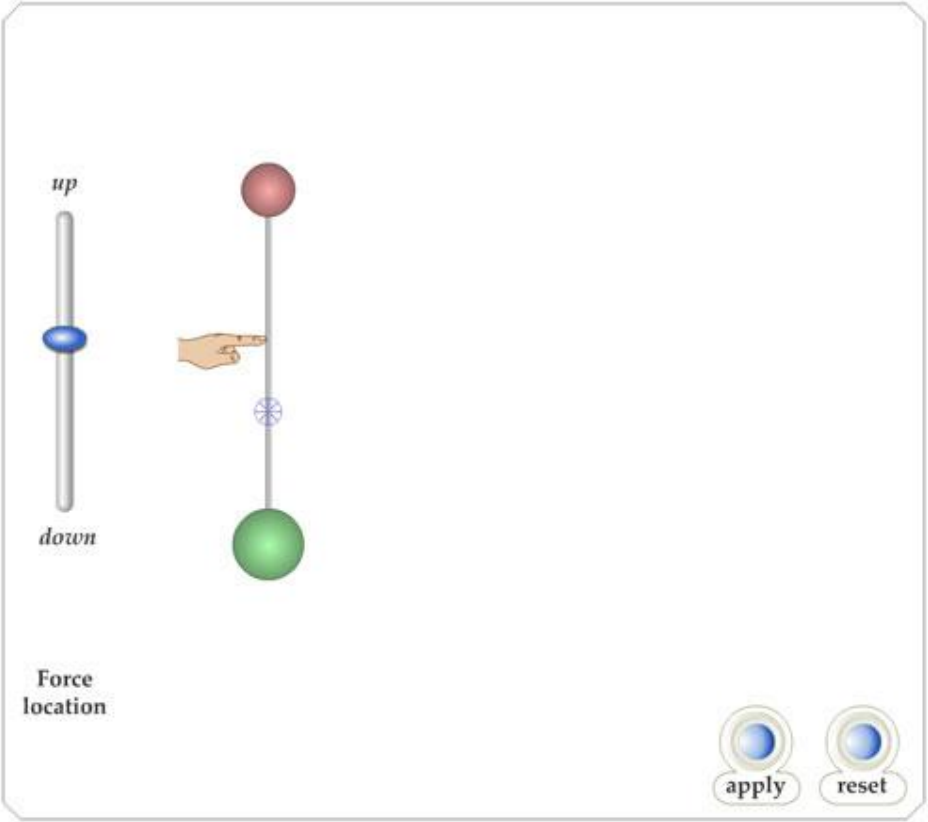
9.7 Perfectly Elastic Collsions



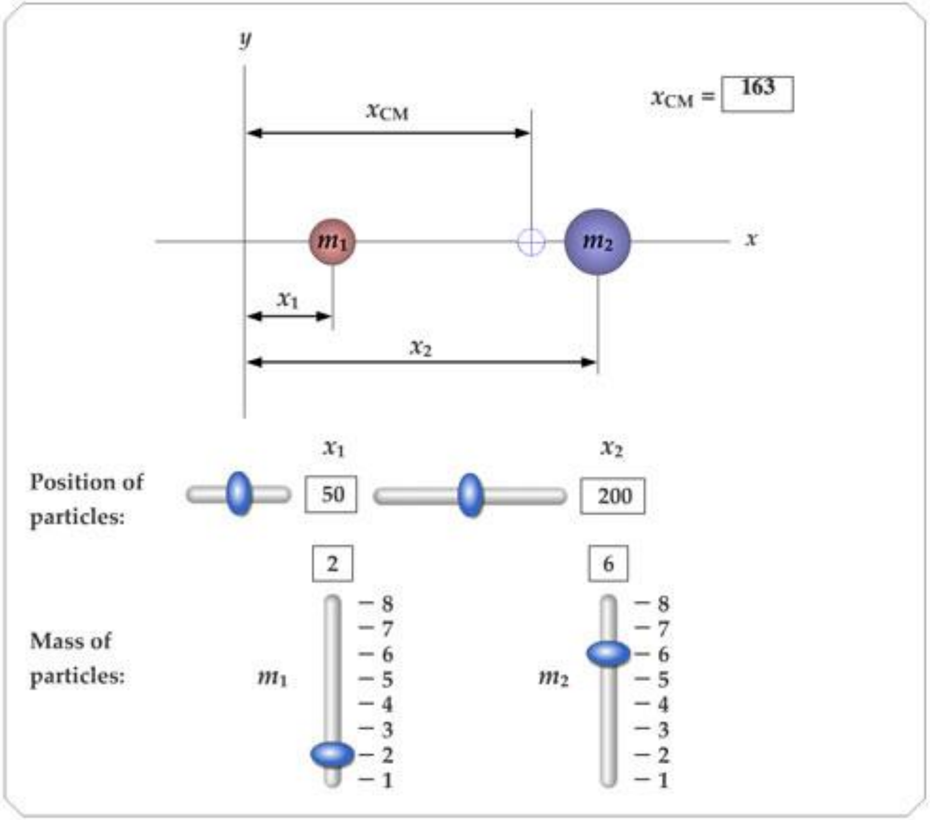
9.11 Collisions in Two Dimensions



9.13 The Center of Mass



9.14 Calculating the Center of Mass



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