Chapter 9

Linear Momentum and Collisions

<u>Outline for W9,D2</u>

Power, P=dE/dt, P=F•v

Momentum, **p**=m**v**

Conservation of momentum

Relation to force

Homework

Ch. 9 P. 1,2,4-6,8,9,18,21,23,28,29,37,38,47,54,55 Do by Mon

Notes:

Practice quiz, exam-like Qs, etc. for Ch. 9 are under “NEW STUFF”.

Next exam after Ch. 11 (4/16 or 4/21).

Power (Ch. 8)

Power is the time rate of energy change.

The ***instantaneous power*** is defined as

$$P \equiv \frac{dE}{dt}$$

Average power is defined as

$$P_{avg} \equiv \frac{\Delta E}{\Delta t}$$

- The SI unit of power is called the watt: 1 watt = 1 joule / second = 1 kg · m^2 / s^3
- A unit of power in the US Customary system is horsepower: 1 hp = 746 W
- Units of power can also be used to express units of energy:

  1 kWh = (1000 W)(3600 s) = 3.6 x10^6 J

The ΔE in P_{avg} could be an energy lost or gained, or an energy produced or consumed, and it could involve any type of energy: work (W), heat (Q), ΔU, ΔK, etc..

A common example in problems is: $$P_{avg} = \frac{W}{\Delta t}$$

Ex) (P. 56) How long will it take a 1750-W motor to lift a 335 kg piano to a sixth floor window 18.0 m above?

Soln: $P_{avg} = \Delta U_g / \Delta t$ with $\Delta U_g = mg\Delta y = (335)(9.8)(18.0) = 59090$ J

(OR $P_{avg} = W_{mot} / \Delta t$ with $W_{mot} = mg\Delta y$.) So $\Delta t = mg\Delta y / P_{avg}$

$$\Delta t = (58090 \text{ J})/(1750 \text{J/s}) = 33.8 \text{ sec}$$

Ex) (P. 57) An 85-kg football player traveling 5.0 m/s is stopped in 1.0 sec by a tackler. (a) What is the original kinetic energy of the player?
(b) What is the average power required to stop him?

Soln: (a) $K = 1/2\ m\ v^2$. (b) $P_{avg} = \Delta K / \Delta t$

Power as force times velocity

Since differential work dW=**F** ·d**r**, the instantaneous power associated with work can be written:

$$P = \frac{dW}{dt} = \frac{\vec{F}\cdot d\,\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$

The **v** is the velocity of the point of application of the force doing the work.

Ex) (P. 58) If a car generates 18 hp when traveling at a steady 95 km/hr, what must be the average force exerted on the car due to friction and air resistance?

Soln: P = Fv → F = P/v.
Convert P=18hp (746 W/hp) = 13428 W
v=95 km/hr (1000m/km)(1hr/3600s)=26.4 m/s. F=13428/26.4=509 N

Linear Momentum

The **linear momentum** of an object of mass *m* moving with a velocity $\boldsymbol{v}$ is defined to be the product of the mass and velocity: $\mathbf{p} \equiv m\mathbf{v}$

- Linear momentum is a vector quantity.
- Its direction is the same as the direction of the velocity.

The dimensions of momentum are ML/T.

The SI units of momentum are kg · m / s.

Momentum can be expressed in component form:

- $p_x = m\, v_x$ $\quad$ $p_y = m\, v_y$ $\quad$ $p_z = m\, v_z$

Linear Momentum is also conserved in a closed system:

$$\mathbf{p}_{tot} = \mathbf{p}_{tot}'$$

Where $\mathbf{p}_{tot}$ is the total momentum before some interaction within the system, and $\mathbf{p}_{tot}'$ is the total momentum after the collision.

Why do we need momentum? Consider this problem...

A 60 kg archer stands on frictionless ice and fires a 0.5 kg arrow at 80 m/s. What is the archer's velocity after firing the arrow?

Approaches:

- Kinematics – no
  - No information about acceleration.
- Newton's Second Law – no
  - No information about *F* or *a*
- Energy conservation – no
  - No information about work or energy
- Momentum conservation – yes

Why do we need momentum? Consider this problem...

A 60 kg archer stands on frictionless ice and fires a 0.5 kg arrow at 80 m/s. What is the archer's velocity after firing the arrow?

Momentum conservation:

$\mathbf{p}_{initial} = m_{arch}\mathbf{v}_{arch} + m_{arr}\,\mathbf{v}_{arr} = 0 + 0 = 0$

$\mathbf{p}_{final} = m_{arch}\mathbf{v}_{arch} + m_{arr}\,\mathbf{v}_{arr} = (60\text{kg})\mathbf{v}_{arch} + (0.5\text{kg})(80\text{ m/s }\hat{\imath})$

$\mathbf{p}_{initial} = \mathbf{p}_{final}$

$0 = (60\text{ kg})\,\mathbf{v}_{arch} + (40\text{ kg m/s})\,\hat{\imath}$

$60\text{ kg }\mathbf{v}_{arch} = -40\text{ kg m/s }\hat{\imath}$

$\mathbf{v}_{arch} = -0.67\text{ m/s }\hat{\imath}$

Momentum and Kinetic Energy

Momentum and kinetic energy both involve mass and velocity.

There are major differences between them:

- Kinetic energy is a scalar and momentum is a vector.
- Kinetic energy is proportional to v^2, momentum is proportional to v.

Ex) If $m_1=4m_2$ and $K_1=K_2$, what is p_1/p_2 ?

(p_1 is the magnitude of $\mathbf{p}_1$)

Ans: $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$ so

$4m_2v_1^2 = m_2 v_2^2$

$v_1^2/v_2^2 = \frac{1}{4}$

$v_1/v_2 = \frac{1}{2}$

So $p_1/p_2 = m_1v_1/m_2v_2$

$p_1/p_2 = (4)(1/2)$

$= 2$

The more massive object has smaller v (if K's are equal).
The more massive object has larger p (if K's are equal).

The rate of change of Momentum is *Force*

Newton's Second Law can be used to relate the momentum of a particle to the resultant force acting on it.

$$\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}} = m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt}$$

with constant mass.

Thus, $\mathbf{F}_{net} = d\mathbf{p}/dt$

(The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.)

- This is the form in which Newton presented the Second Law.
- It is a more general form than the one we used previously.
- This form also allows for mass changes.

Integral of Force gives change in momentum

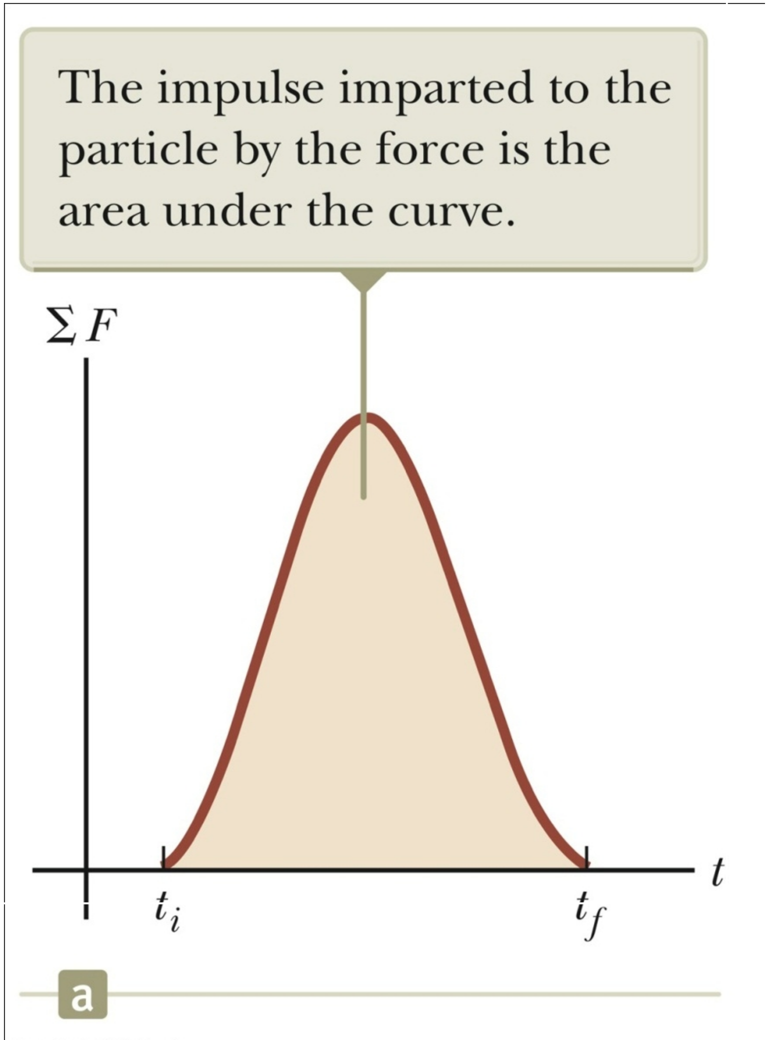
Change of momentum is called *impulse*.

I = Δ**p**

The magnitude of the impulse is equal to the area under the force-time curve.

- The force may vary with time.

Thus, $$\Delta \vec{p} = \int_{t_1}^{t_2} \vec{F}_{net}\, dt$$

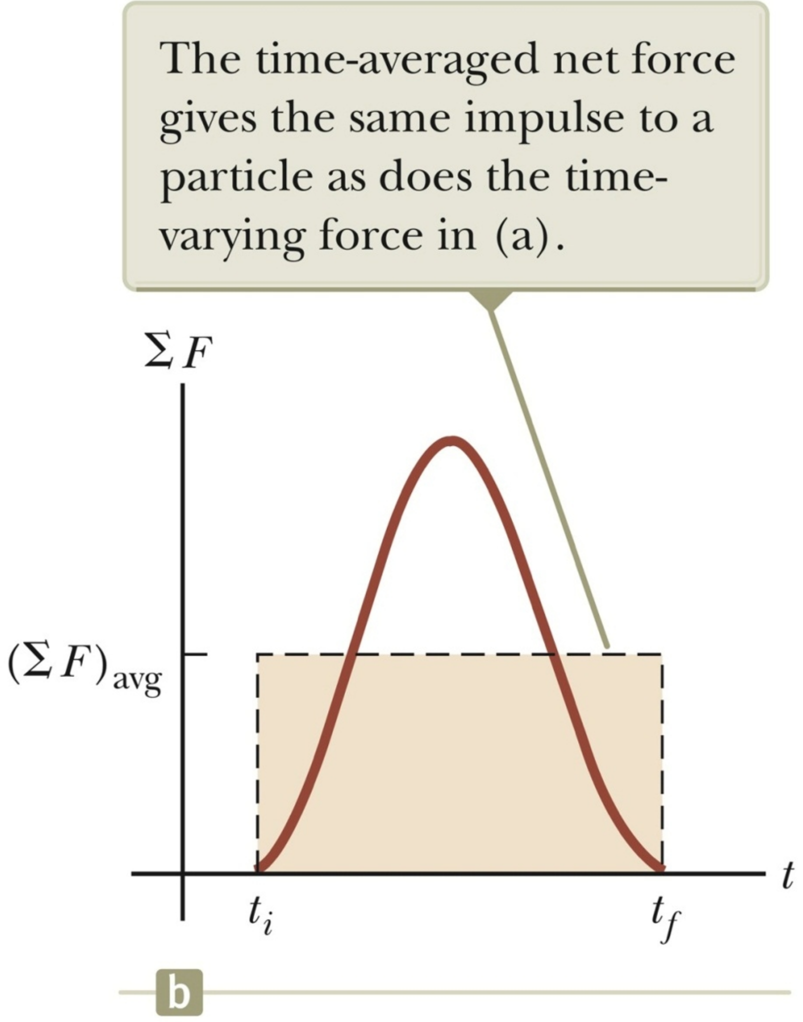


Change of momentum or Impulse

The impulse can also be found by using the time averaged force.

$$\mathbf{I} = \sum \mathbf{F}\Delta t$$

This would give the same impulse as the time-varying force does.



Outline for W9,D3

Momentum, **p**=m**v**

Relation to force **F**=d**p**/dt, $\Delta\vec{p}=\int_{t_1}^{t_2}\vec{F}_{net}\,dt$

1D Conservation of momentum

Types of collisions

Homework

Ch. 9 P. 1,2,4-6,8,9,18,21,23,28,29,37,38,47,54,55 Do by Mon

Notes:

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Next exam after Ch. 11 (4/16 or 4/21).

Examples of the relations between F and p

Ex) (P. 5) Find **F**(t) if **p** = $4.8t^2\,\hat{\imath} - 8\,j - 9.4t\,k$.

Ex) (P. 8) Find $\Delta\mathbf{p}$ between t=1.0 and t=2.0 sec if $\mathbf{F}=26i-12t^2j$.

Derive conservation of momentum from Newton's 3rd law!

Consider a collision between 2 compressible balls, m_1 and m_2.

During every instant of contact, $\mathbf{F}_{1by2}=-\mathbf{F}_{2by1}$ (by Newton's 3rd).

So $m_1 d\mathbf{v}_1/dt = -m_2 d\mathbf{v}_2/dt$ But if m's are constant, this means

$d(m_1\mathbf{v}_1)/dt = -d(m_2\mathbf{v}_2)/dt$

Or $d\mathbf{p}_1/dt + d\mathbf{p}_2/dt = 0$ which is $d(\mathbf{p}_1+\mathbf{p}_2)/dt = 0$ or $\Delta\mathbf{p}_{tot}=0$

Conservation of Linear Momentum

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

For a 2-body system: $$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}\,'_1 + m_2\vec{v}\,'_2$$

For an N-body system: $$\sum_{i=1}^{N} m_i\vec{v}_i = \sum_{i=1}^{N} m_i\vec{v}\,'_i$$

Or: $$\vec{p}_{tot} = \vec{p}\,'_{tot}$$

For a 2-body 1D collision: $$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}$$

For a 2-body 2D collision, include this equation with the above:

$$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}$$

Conservation of Linear Momentum

$$m_1\vec{v}_1+m_2\vec{v}_2=m_1\vec{v}\,'_1+m_2\vec{v}\,'_2$$

Example: (P. 6) A child in a boat throws a 5.3 kg package out horizontally with a speed of 10 m/s. Calculate the velocity of boat immediately after, assuming it was initially at rest. The mass of the child is 24 kg and the mass of the boat is 42 kg.

Soln: Apply cons of momentum to a 2-mass system.

Let m_1=m_child + m_boat = 66 kg (the child is fixed w.r.to the boat)

and let m_2=m_package = 5.3 kg

So $$m_1\vec{v}_1+m_2\vec{v}_2=m_1\vec{v}\,'_1+m_2\vec{v}\,'_2$$

becomes $$0 + 0 = 66\,\mathbf{v}_1' + 5.3\,\mathbf{v}_2'$$

$$0 = 66\,\mathbf{v}_1' + 5.3\,(10\text{ m/s})$$

$$-53\text{ kg m/s} = 66\text{ kg}\,\mathbf{v}_1'$$

$$-0.8\text{ m/s }\hat{\imath} = \mathbf{v}_1'$$

Example: see “Scan of old Ch. 9 notes”, 2^{nd} page.

3 Types of Collisions

For ALL types of collisions, momentum is conserved!!

Elastic collision: kinetic energy is also conserved.

- Elastic collisions occur on a microscopic level between molecules.
- In large-scale collisions, only approximately elastic collisions actually occur.
- Special equations for the 1-D, head-on case: $(v_A - v_B) = -(v_A' - v_B')$
  - $$v'_B = v_A\left(\frac{2m_A}{m_A + m_B}\right) + v_B\left(\frac{m_B - m_A}{m_A + m_B}\right) \qquad v'_A = v_A\left(\frac{m_A - m_B}{m_A + m_B}\right) + v_B\left(\frac{2m_B}{m_A + m_B}\right)$$
  -
  - See Prob. 34 in Giancoli

Inelastic collision: kinetic energy is not conserved.

Perfectly inelastic *collision:* kinetic energy is lost AND the objects stick together

- Special equation for 2-body case: $m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}\,'$

Elastic Collisions, cont.

Example of some special cases of elastic collisions:

- $m_1 = m_2$ The particles exchange velocities
- When a very heavy particle collides head-on with a very light one initially at rest, the heavy particle continues in motion unaltered and the light particle rebounds with a speed of about twice the initial speed of the heavy particle.
- When a very light particle collides head-on with a very heavy particle initially at rest, the light particle has its velocity reversed and the heavy particle remains approximately at rest.

Demonstrate head-on collisions with different mass ratios: 9.11.swf

Demonstrate glancing collisions with equal mass ratios: 9.11.swf

What is the angle between the final velocities?

Demonstrate glancing collisions with arbitrary masses: 9.11.swf

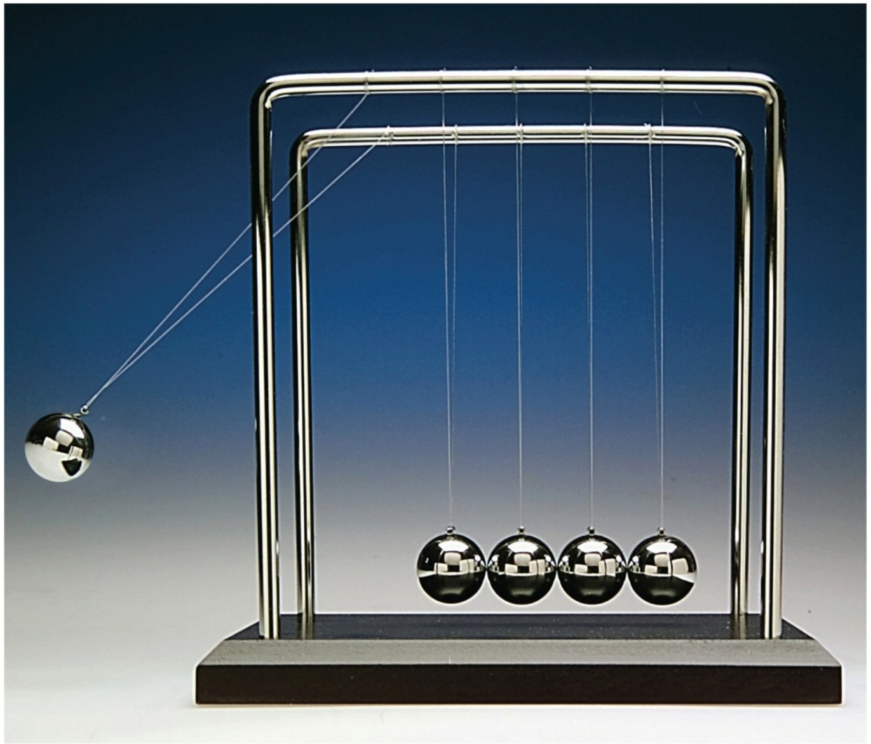
Example: Stress Reliever

Conceptualize

- Imagine one ball coming in from the left and two balls exiting from the right.
- Is this possible?

Categorize

- Categorize the system as isolated in terms of both momentum and energy.
- Elastic collisions



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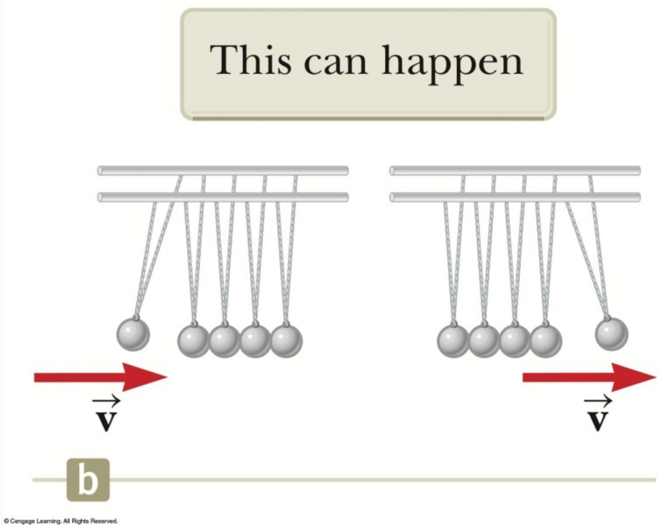
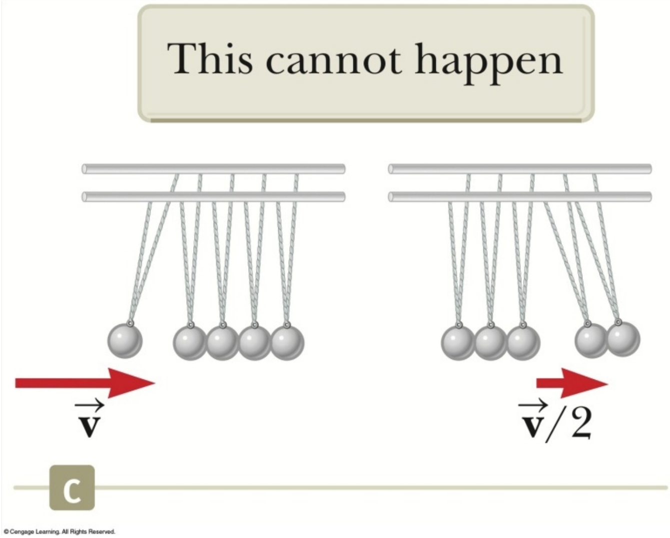
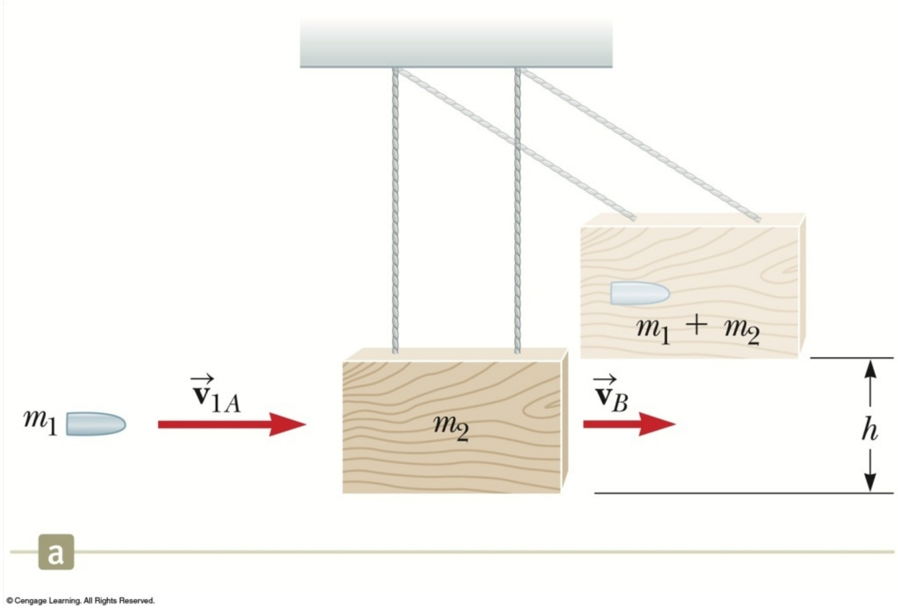
Example: Stress Reliever, final





Perfectly Inelastic Collision Example – Ballistic Pendulum



Conceptualize

- Observe diagram
- The projectile enters the pendulum, which swings up to some height where it momentarily stops.

Categorize

- Isolated system in terms of momentum for the projectile and block.
- Perfectly inelastic collision – the bullet is embedded in the block of wood.
- Momentum equation will have two unknowns
- Use conservation of energy from the pendulum to find the velocity just after the collision.
- Then you can find the speed of the bullet.

Ballistic Pendulum, cont.

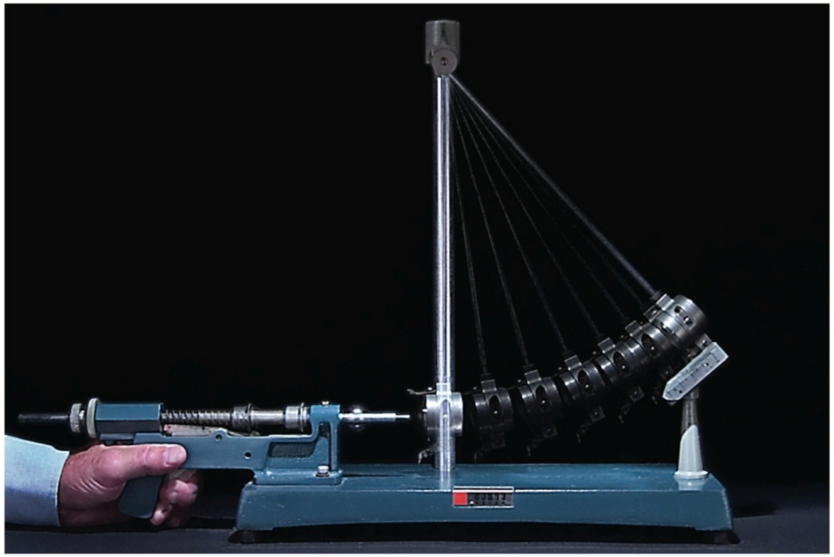
A multi-flash photograph of a ballistic pendulum.

Analyze

- Equations for the momentum and conservation of energy with kinetic and gravitational potential energies.
- Solve resulting system of equations.

Finalize

- Note different systems involved and different analysis models used.
- Some energy was transferred during the perfectly inelastic collision.



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Two-Dimensional Collisions

The momentum is conserved in all directions.

Use subscripts for

- Identifying the object
- Indicating initial or final values
- The velocity components

If the collision is elastic, use conservation of kinetic energy as a second equation.

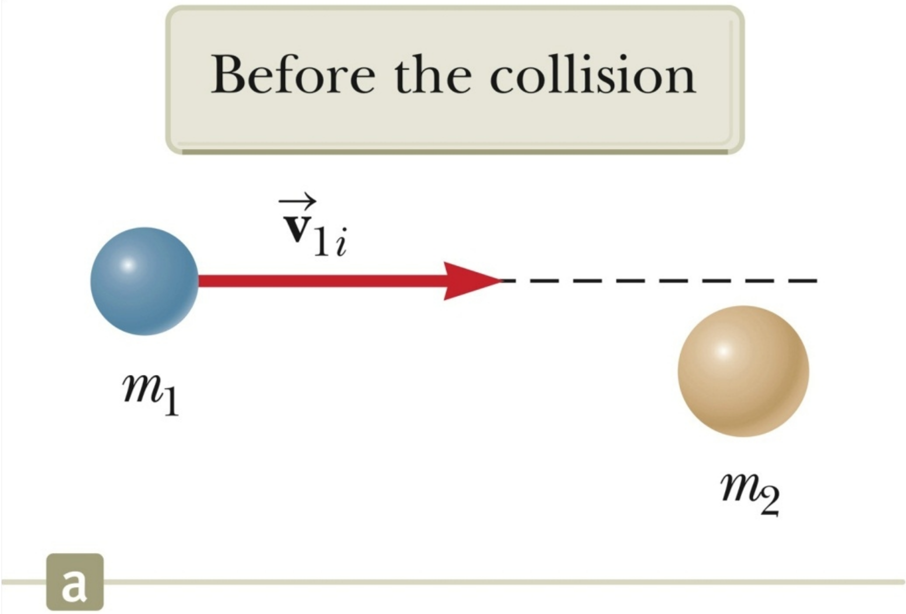
- Remember, the simpler equation can only be used for one-dimensional situations.

Two-Dimensional Collision, example

Particle 1 is moving at velocity $\mathbf{v}_{1i}$ and particle 2 is at rest.

In the *x*-direction, the initial momentum is $m_1 v_{1i}$.

In the *y*-direction, the initial momentum is 0.



Two-Dimensional Collision, example cont.

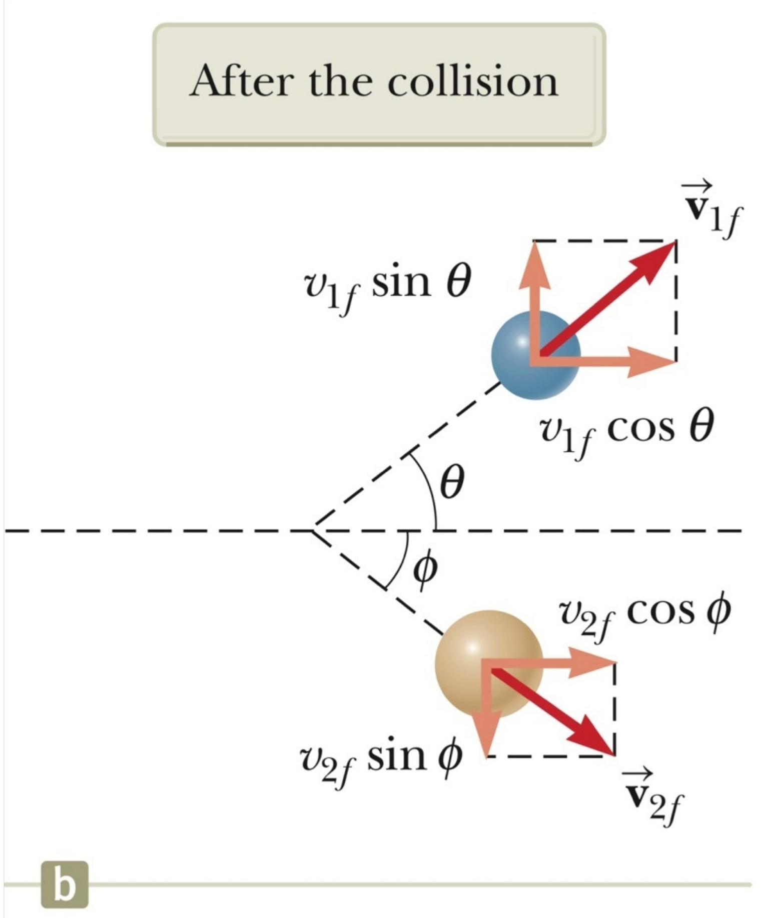
After the collision, the momentum in the x-direction is $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$.

After the collision, the momentum in the y-direction is $m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$.

- The negative sign is due to the component of the velocity being downward.

If the collision is elastic, apply the kinetic energy equation.

This is an example of a *glancing collision.*



Problem-Solving Strategies – Two-Dimensional Collisions

Conceptualize

- Imagine the collision.
- Predict approximate directions the particles will move after the collision.
- Set up a coordinate system and define your velocities with respect to that system.
  - It is usually convenient to have the *x*-axis coincide with one of the initial velocities.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.

Categorize

- Is the system isolated?
- If so, categorize the collision as elastic, inelastic or perfectly inelastic.

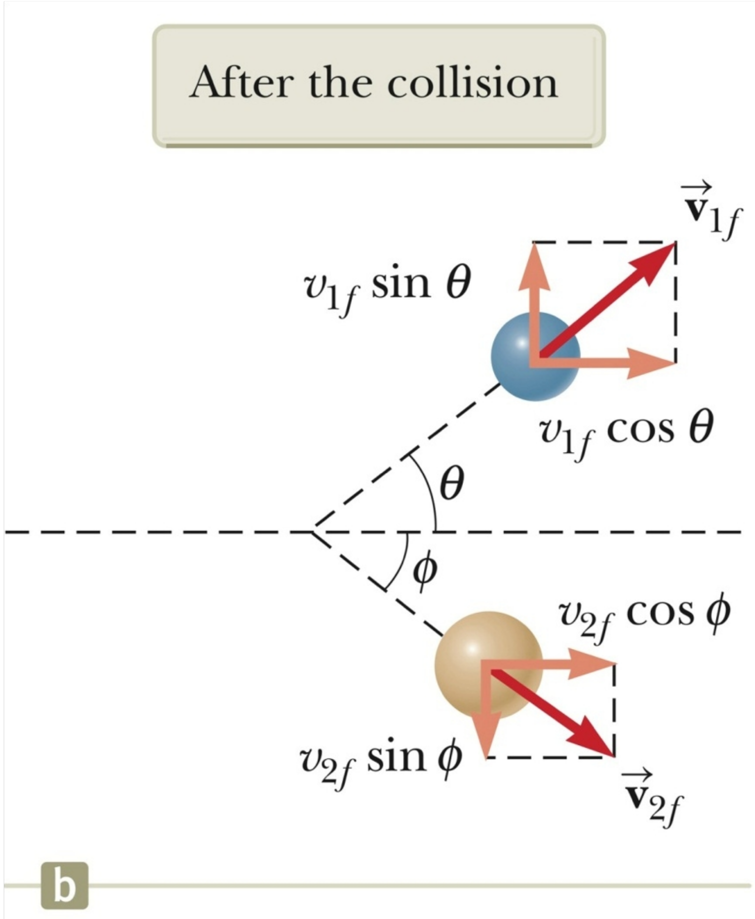
Two-Dimensional Collision Example

Conceptualize

- See picture
- Choose East to be the positive x-direction and North to be the positive y-direction.

Categorize

- Ignore friction
- Model the vehicles as particles.
- Model the system as isolated in terms of momentum.
- The collision is perfectly inelastic.
  - The vehicles stick together.



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The Center of Mass

There is a special point in a system or object, called the ***center of mass***, that moves as if all of the mass of the system is concentrated at that point.

The system will move as if an external force were applied to a single particle of mass *M* located at the center of mass.

- *M* is the total mass of the system.

This behavior is independent of other motion, such as rotation or vibration, or deformation of the system.

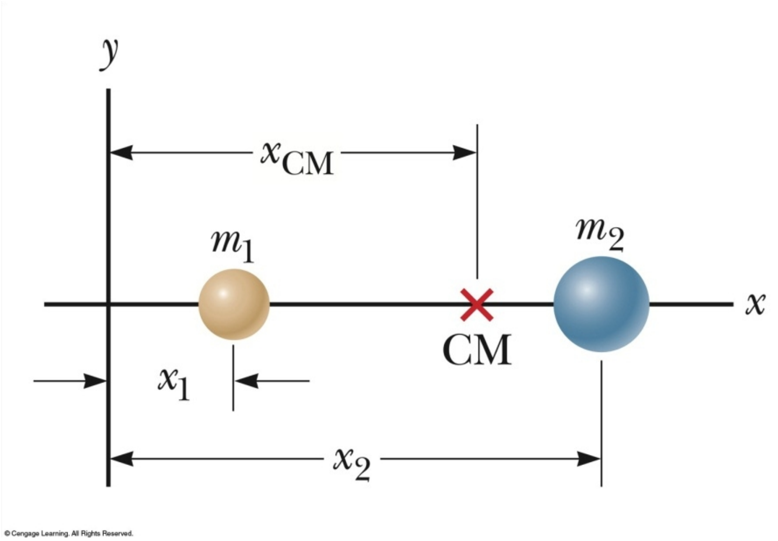
- This is the particle model.

Center of Mass, Coordinates

The coordinates of the center of mass are

$$x_{CM} = \frac{\sum_i m_i x_i}{M} \qquad y_{CM} = \frac{\sum_i m_i y_i}{M}$$

$$z_{CM} = \frac{\sum_i m_i z_i}{M}$$



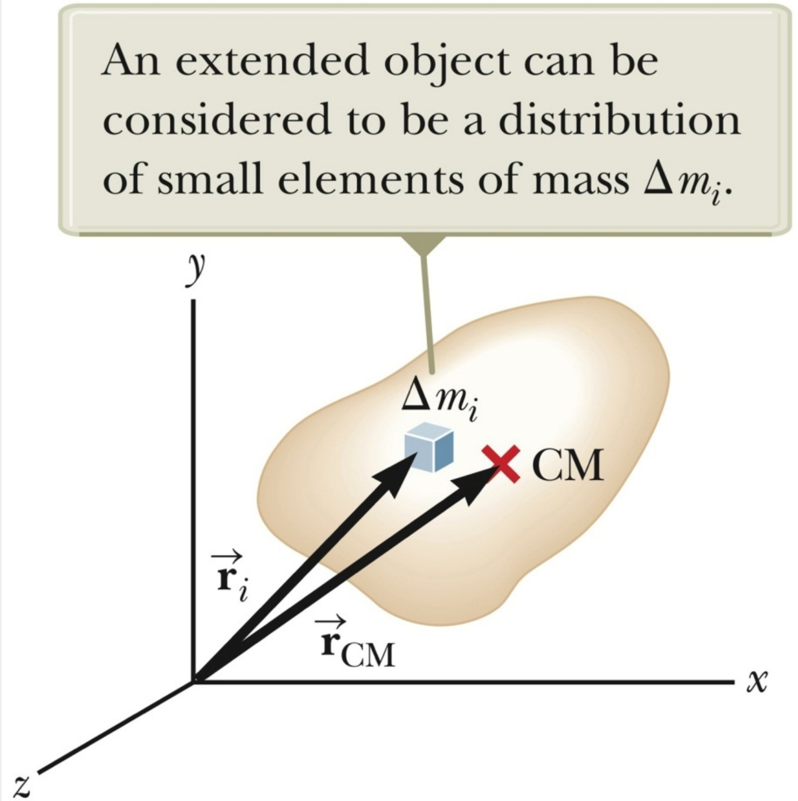
- M is the total mass of the system.
  - Use the active figure to observe effect of different masses and positions.

Center of Mass, Extended Object

Similar analysis can be done for an extended object.

Consider the extended object as a system containing a large number of small mass elements.

Since separation between the elements is very small, it can be considered to have a constant mass distribution.



Center of Mass, position

The center of mass in three dimensions can be located by its position vector, $\mathbf{r}_{CM}$.

- For a system of particles,

$$\vec{\mathbf{r}}_{CM} = \frac{1}{M}\sum_i m_i \vec{\mathbf{r}}_i$$

  - $\mathbf{r}_i$ is the position of the i^{th} particle, defined by

$$\mathbf{r}_i = x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}$$

- For an extended object,

$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\, dm$$

Center of Mass, Symmetric Object

The center of mass of any symmetric object of uniform density lies on an axis of symmetry and on any plane of symmetry.

Center of Gravity

Each small mass element of an extended object is acted upon by the gravitational force.

The net effect of all these forces is equivalent to the effect of a single force $M\mathbf{g}$ acting through a point called the **center of gravity.**

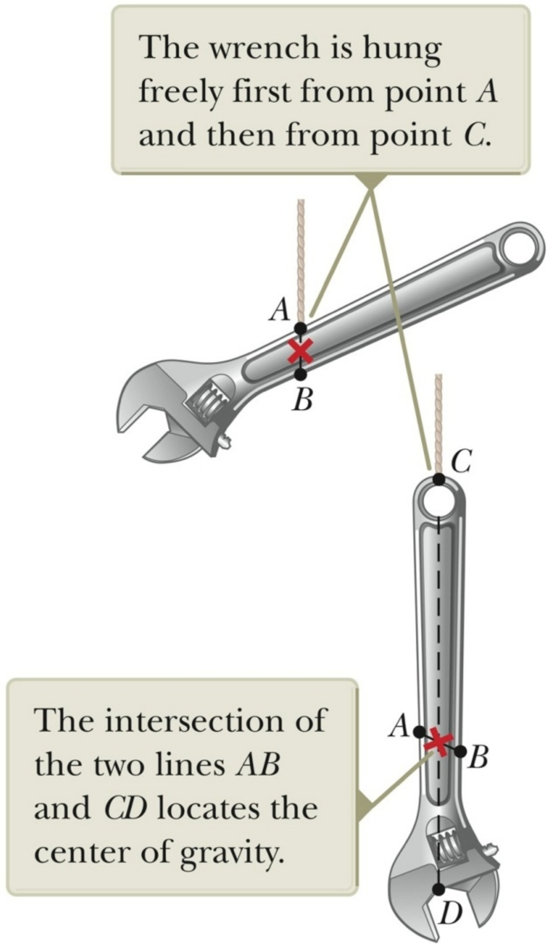
- If $\mathbf{g}$ is constant over the mass distribution, the center of gravity coincides with the center of mass.

Finding Center of Gravity, Irregularly Shaped Object

Suspend the object from one point.

Then, suspend from another point.

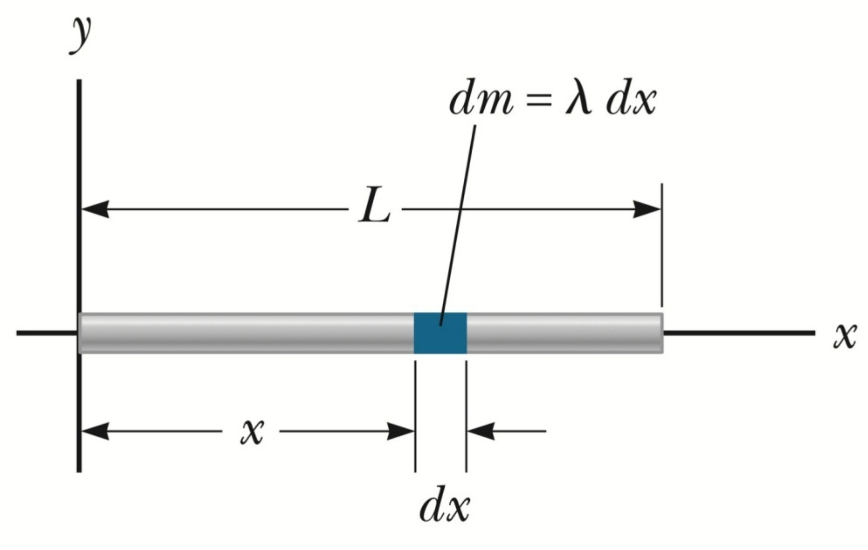
The intersection of the resulting lines is the center of gravity and half way through the thickness of the wrench.



Center of Mass, Rod

Conceptualize

- Find the center of mass of a rod of mass *M* and length *L.*
- The location is on the *x*-axis (or $y_{CM} = z_{CM} = 0$)



Categorize

- Analysis problem

Analyze

- Use equation for x_{cm}
- $x_{CM} = L / 2$

Motion of a System of Particles

Assume the total mass, M, of the system remains constant.

We can describe the motion of the system in terms of the velocity and acceleration of the center of mass of the system.

We can also describe the momentum of the system and Newton’s Second Law for the system.

Velocity and Momentum of a System of Particles

The velocity of the center of mass of a system of particles is

$$\vec{\mathbf{v}}_{\text{CM}} = \frac{d\mathbf{r}_{\text{CM}}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{v}}_i$$

The momentum can be expressed as

$$M\mathbf{v}_{\text{CM}} = \sum_i m_i \mathbf{v}_i = \sum_i \mathbf{p}_i = \mathbf{p}_{\text{tot}}$$

The total linear momentum of the system equals the total mass multiplied by the velocity of the center of mass.

Acceleration and Force in a System of Particles

The acceleration of the center of mass can be found by differentiating the velocity with respect to time.

$$\vec{\mathbf{a}}_{CM} = \frac{d\mathbf{v}_{CM}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{a}}_i$$

The acceleration can be related to a force.

$$M\,\mathbf{a}_{CM} = \sum_i \mathbf{F}_i$$

If we sum over all the internal force vectors, they cancel in pairs and the net force on the system is caused only by the external forces.

Newton's Second Law for a System of Particles

Since the only forces are external, the net external force equals the total mass of the system multiplied by the acceleration of the center of mass:

$$\sum \mathbf{F}_{ext} = M\mathbf{a}_{CM}$$

The center of mass of a system of particles of combined mass *M* moves like an equivalent particle of mass *M* would move under the influence of the net external force on the system.

Impulse and Momentum of a System of Particles

The impulse imparted to the system by external forces is

$$\int \sum \mathbf{F}_{ext} dt = M \int d\mathbf{v}_{CM} \rightarrow \Delta \mathbf{p}_{tot} = \mathbf{I}$$

The total linear momentum of a system of particles is conserved if no net external force is acting on the system.

$$M\mathbf{v}_{CM} = \mathbf{p}_{tot} = constant \quad when \sum \mathbf{F}_{ext} = 0$$

For an isolated system of particles, both the total momentum and the velocity of the center of mass are constant in time.

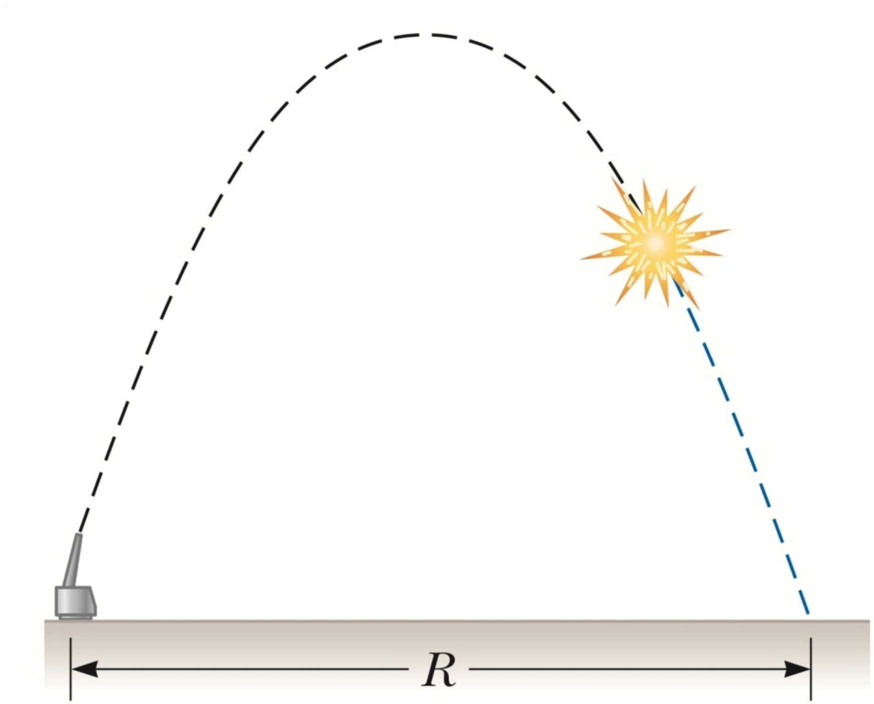
- This is a generalization of the isolated system (momentum) model for a many-particle system.

Motion of the Center of Mass, Example

A projectile is fired into the air and suddenly explodes.

With no explosion, the projectile would follow the dotted line.

After the explosion, the center of mass of the fragments still follows the dotted line, the same parabolic path the projectile would have followed with no explosion.



Deformable Systems

To analyze the motion of a deformable system, use Conservation of Energy and the Impulse-Momentum Theorem.

$$\Delta E_{system} = \sum T \rightarrow \Delta K + \Delta U = 0$$

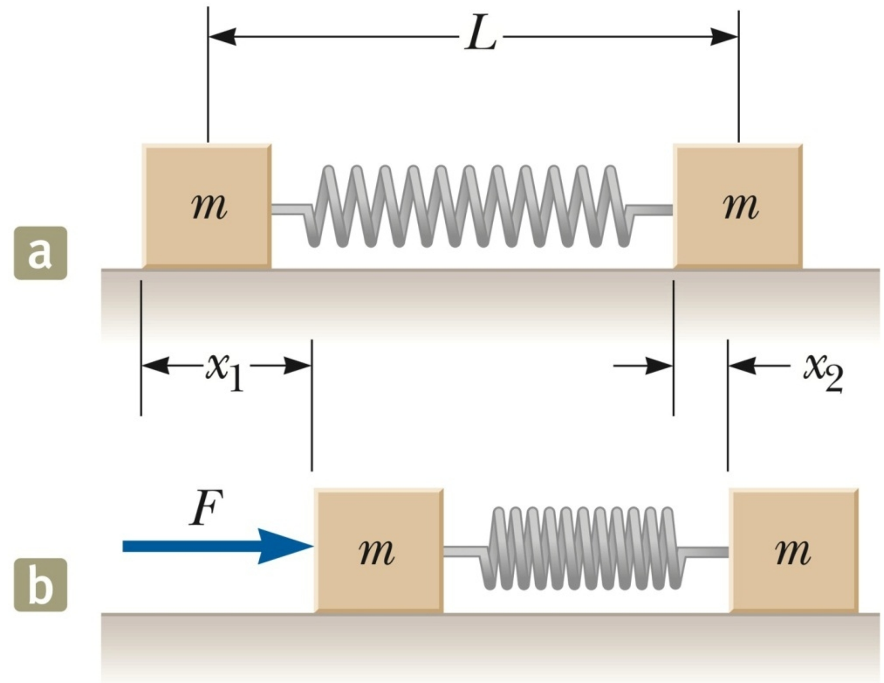
$$\Delta \vec{\mathbf{p}}_{tot} = \vec{\mathbf{I}} \rightarrow m\Delta \vec{\mathbf{v}} = \int \vec{\mathbf{F}}_{ext}\, dt$$

- If the force is constant, the integral can be easily evaluated.

Deformable System (Spring) Example

Conceptualize

- See figure
- Push on left block, it moves to right, spring compresses.
- At any given time, the blocks are generally moving with different velocities.
- After the force is removed, the blocks oscillate back and forth with respect to the center of mass.



Spring Example, cont.

Categorize

- Non isolated system in terms of momentum and energy.
  - Work is being done on it by the applied force.
- It is a deformable system.
- The applied force is constant, so the acceleration of the center of mass is constant.
- Model as a particle under constant acceleration.

Analyze

- Apply impulse-momentum
- Solve for v_{cm}

Spring Example, final

Analyze, cont.

- Find energies

Finalize

- Answers do not depend on spring length, spring constant, or time interval.

Rocket Propulsion

When ordinary vehicles are propelled, the driving force for the motion is friction.

- The car is modeled as an non-isolated system in terms of momentum.
- An impulse is applied to the car from the roadway, and the result is a change in the momentum of the car.

The operation of a rocket depends upon the law of conservation of linear momentum as applied to an isolated system, where the system is the rocket plus its ejected fuel.

As the rocket moves in free space, its linear momentum changes when some of its mass is ejected in the form of exhaust gases.

- Because the gases are given momentum when they are ejected out of the engine, the rocket receives a compensating momentum in the opposite direction.
- In free space, the center of mass of the system moves uniformly.

Rocket Propulsion, 2

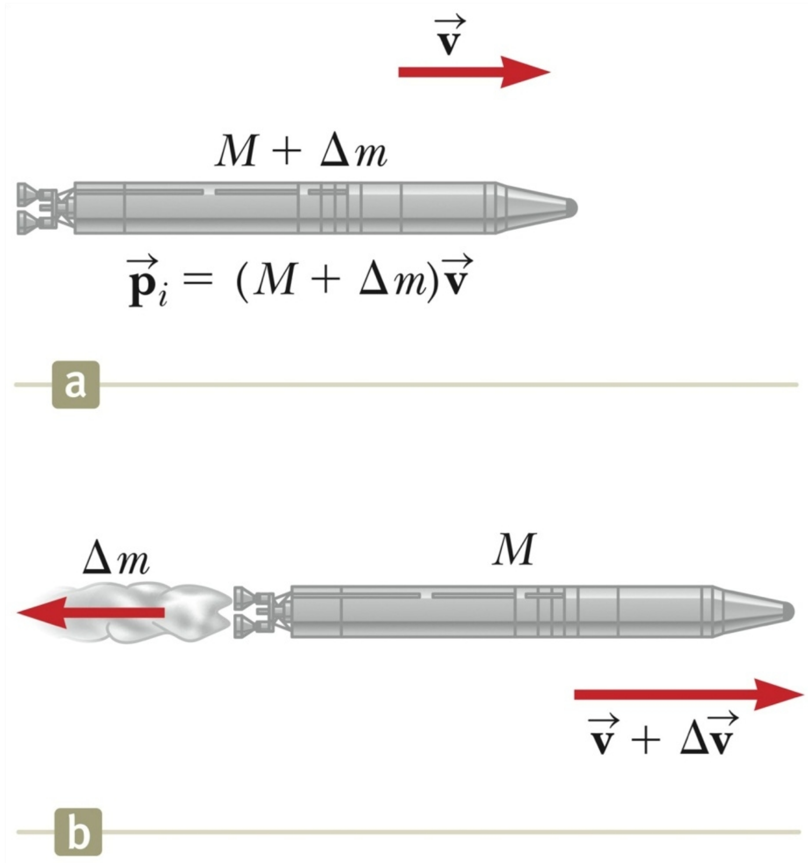
The initial mass of the rocket plus all its fuel is $M + \Delta m$ at time t_i and speed v.

The initial momentum of the system is

$$\mathbf{p}_i = (M + \Delta m)\mathbf{v}$$

At some time $t + \Delta t$, the rocket's mass has been reduced to M and an amount of fuel, Δm has been ejected.

The rocket's speed has increased by Δv.



Rocket Propulsion, 3

The basic equation for rocket propulsion is

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$$

The increase in rocket speed is proportional to the speed of the escape gases (v_e).

- So, the exhaust speed should be very high.

The increase in rocket speed is also proportional to the natural log of the ratio M_i/M_f.

- So, the ratio should be as high as possible, meaning the mass of the rocket should be as small as possible and it should carry as much fuel as possible.

Thrust

The thrust on the rocket is the force exerted on it by the ejected exhaust gases.

$$thrust = M\frac{dv}{dt} = \left| v_e \frac{dM}{dt} \right|$$

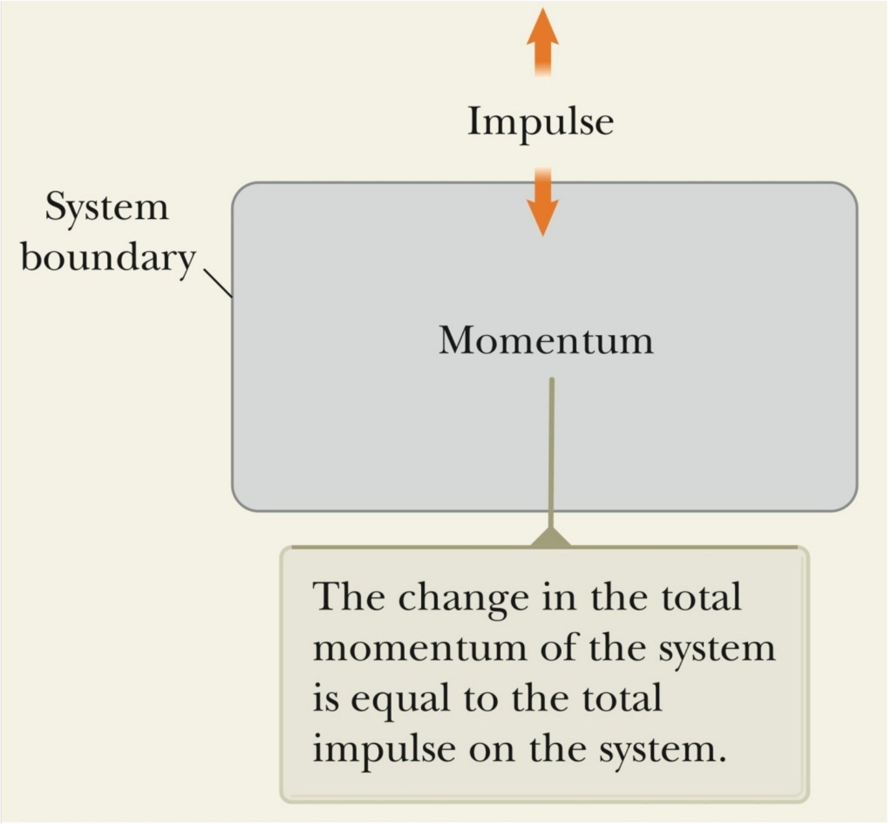
The thrust increases as the exhaust speed increases.

The thrust increases as the rate of change of mass increases.

- The rate of change of the mass is called the **burn rate.**

Problem Solving Summary – Non-isolated System

If a system interacts with its environment in the sense that there is an external force on the system, use the impulse-momentum theorem.

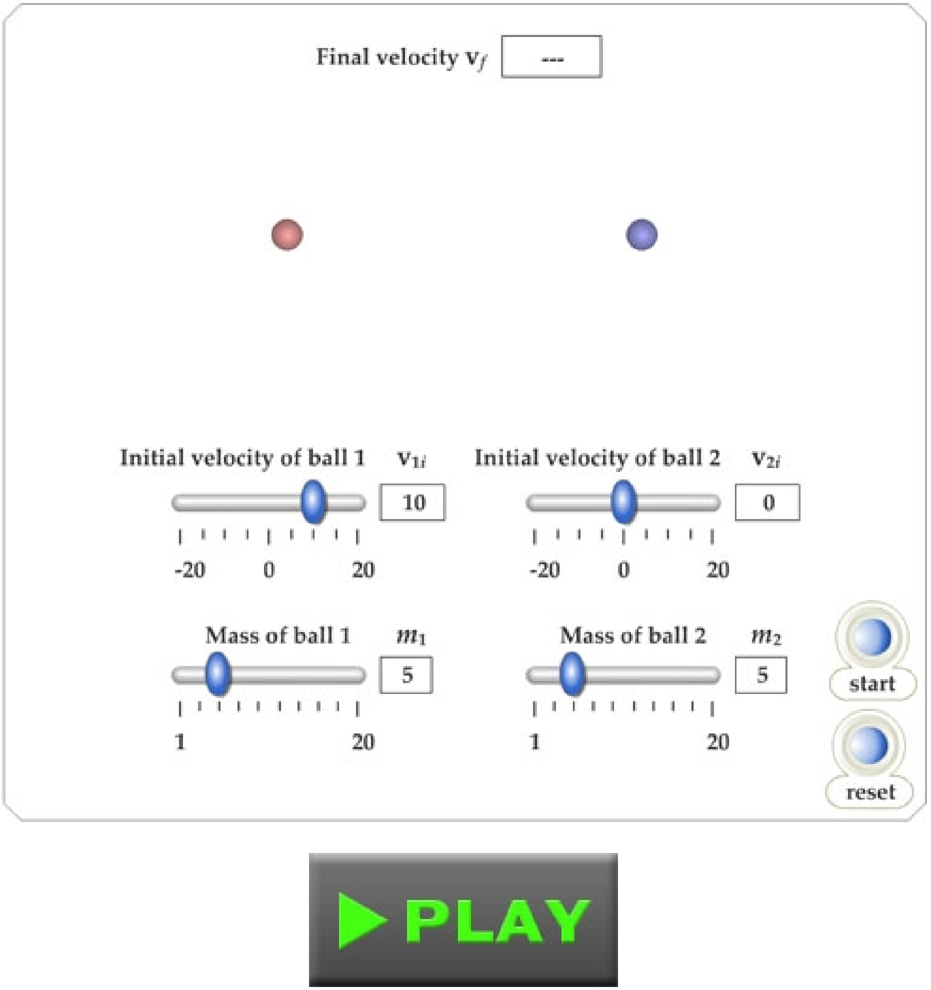


Problem Solving Summary – Isolated System

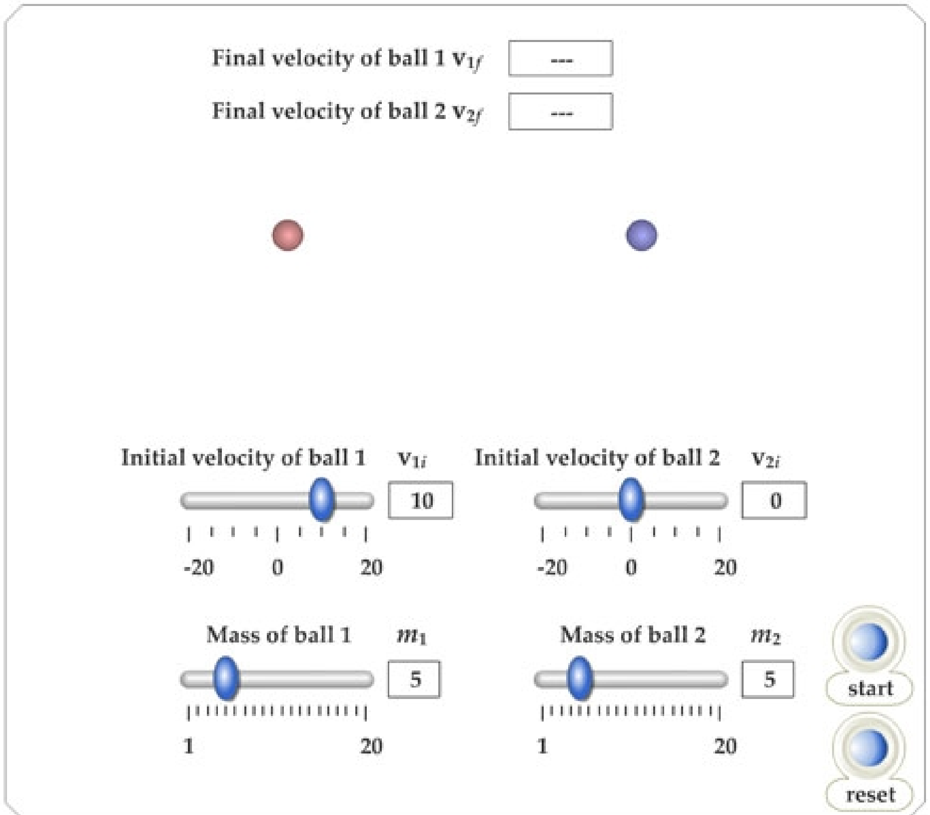
If there are no external forces, the principle of conservation of linear momentum indicates that the total momentum of an isolated system is conserved regardless of the nature of the forces between the members of the system.

The system may be isolated in terms of momentum but non-isolated in terms of energy.

9.6 Perfectly Inelastic Collisions



9.7 Perfectly Elastic Collsions



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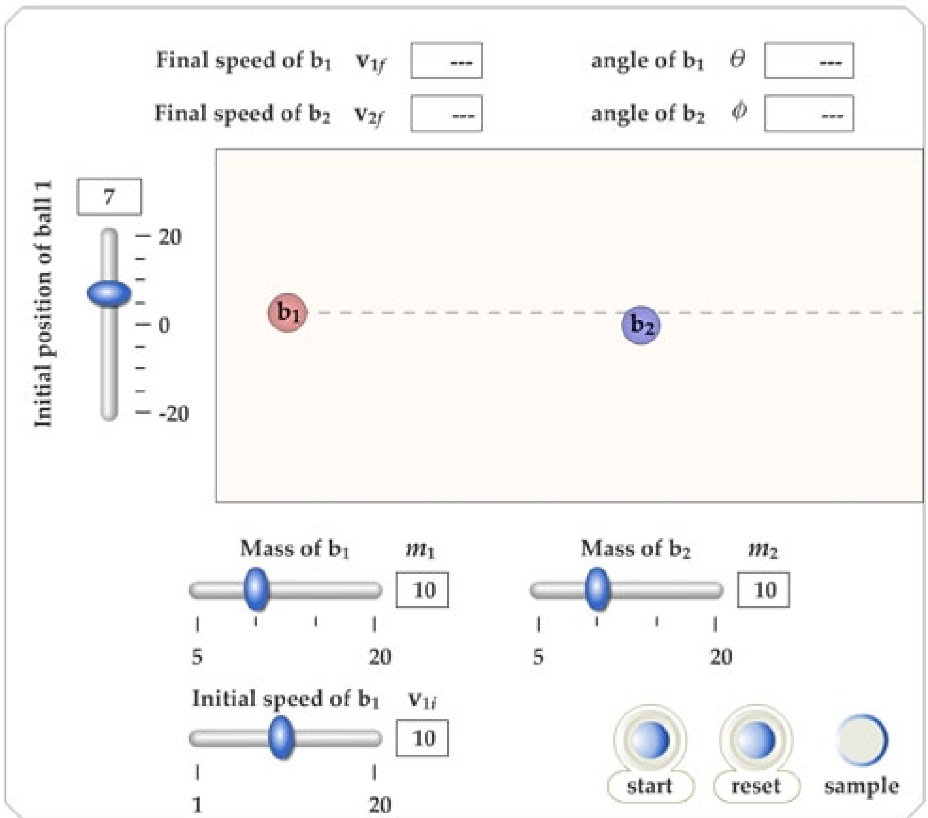
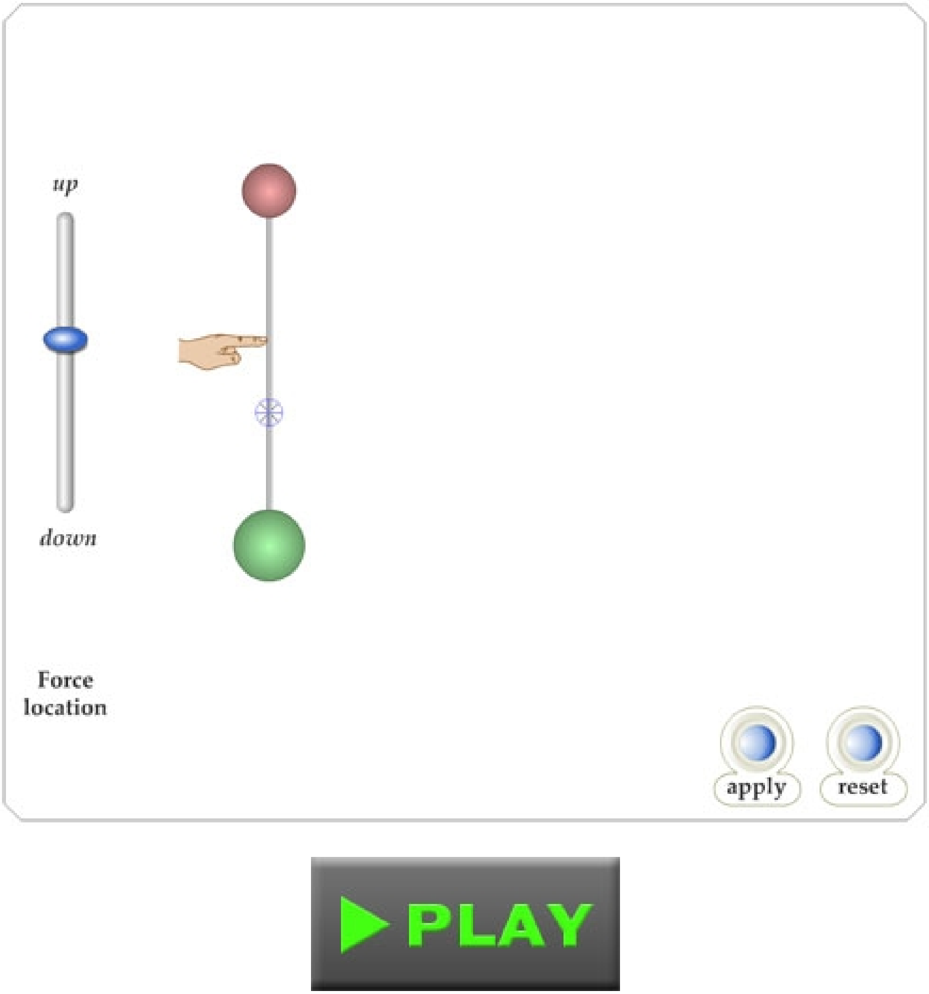
9.11 Collisions in Two Dimensions



9.13 The Center of Mass



9.14 Calculating the Center of Mass