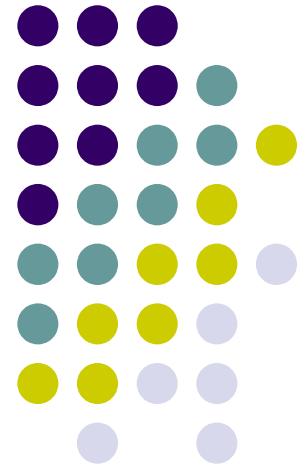


Chapter 10

Rotation of a Rigid Object about a Fixed Axis





Outline for W10,D3

Finish center of mass (Ch. 9)

Rotation of a rigid solid (Ch. 10)

θ , ω , and α

Relation between linear (s,v,a) and angular quantities

Torque

Homework

Ch. 10 P. 1,4-6,19-21,25,28-30,34,35,37,53,54,55,64,67,69

Do for Wed/Fri

Notes:

Lab this week is “2D Collisions”

See “NEW STUFF” for Ch. 10.

Ex) Find the COM of a non-uniform rod of length 1.0 m if its linear mass distribution is $\lambda(x)=3x+1$ kg/m, where $x=0$ at the origin.

- As before, rod is aligned with the x -axis, with one end on $(0,0)$, and $y_{\text{COM}} = z_{\text{COM}} = 0$.

Do integral using $\lambda = 3x+1$

$$x_{\text{COM}} = \frac{1}{M} \int_0^L x \lambda dx$$

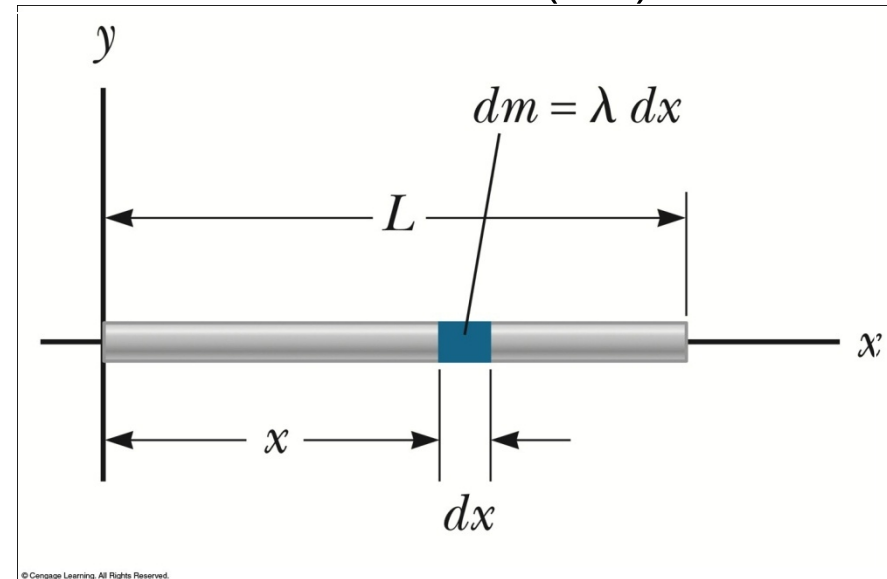
$$x_{\text{COM}} = \frac{1}{M} \int_0^{1.0} x (3x+1) dx$$

$$x_{\text{COM}} = \frac{1}{M} \left(x^3 + \frac{x^2}{2} \right)_0^{1.0}$$

So $x_{\text{com}} = 1.5/M$, but what is M ? M is the total mass.

$$M = \int_0^{1.0} \lambda dx = \int_0^{1.0} (3x+1) dx = \left(\frac{3x^2}{2} + x \right)_0^{1.0} = 5/2.$$

$$\text{So } x_{\text{com}} = (3/2)/(5/2) = 3/5 = 0.6 \text{ m}$$



Rigid Object

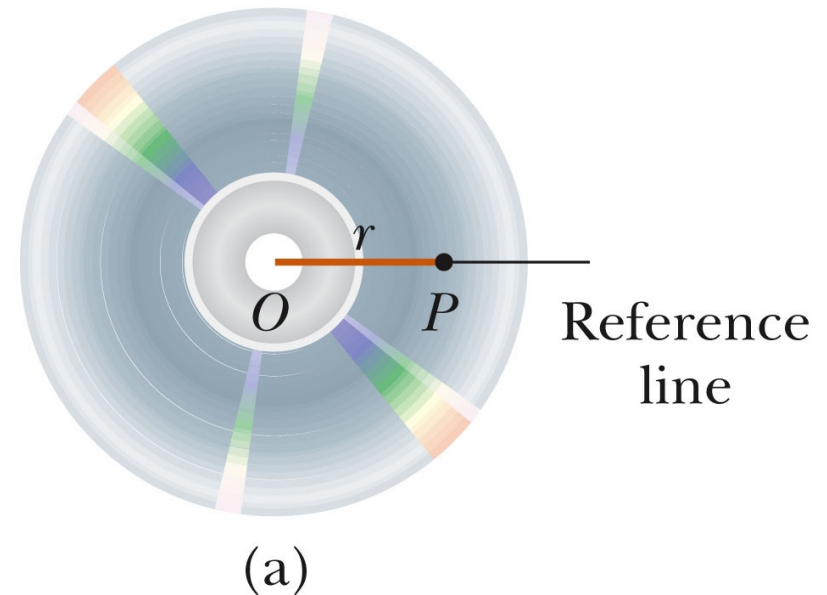


- A rigid object is one that is nondeformable
 - The relative locations of all particles making up the object remain constant
 - All real objects are deformable to some extent, but the rigid object model is very useful in many situations where the deformation is negligible
- This simplification allows analysis of the motion of an extended object



Angular Position

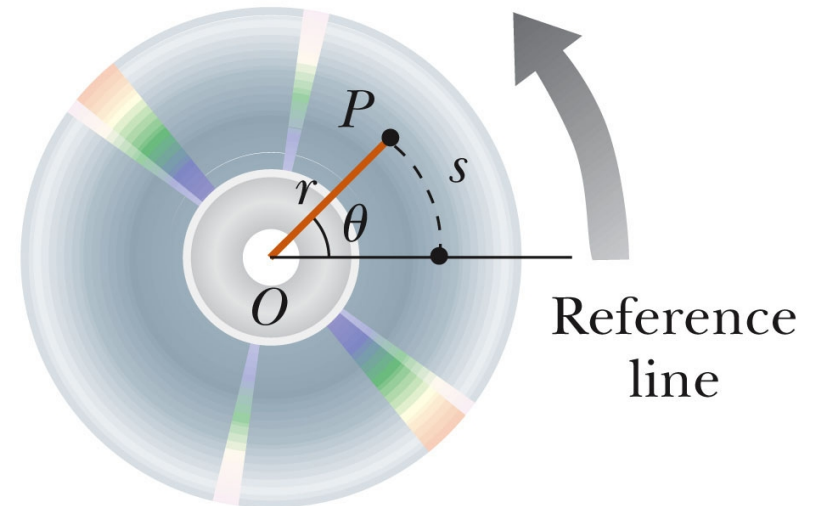
- Axis of rotation runs through the center of the disc, \perp the disk.
- Choose a fixed reference line
- Point P is at a fixed distance r from the origin



Angular Position, 2



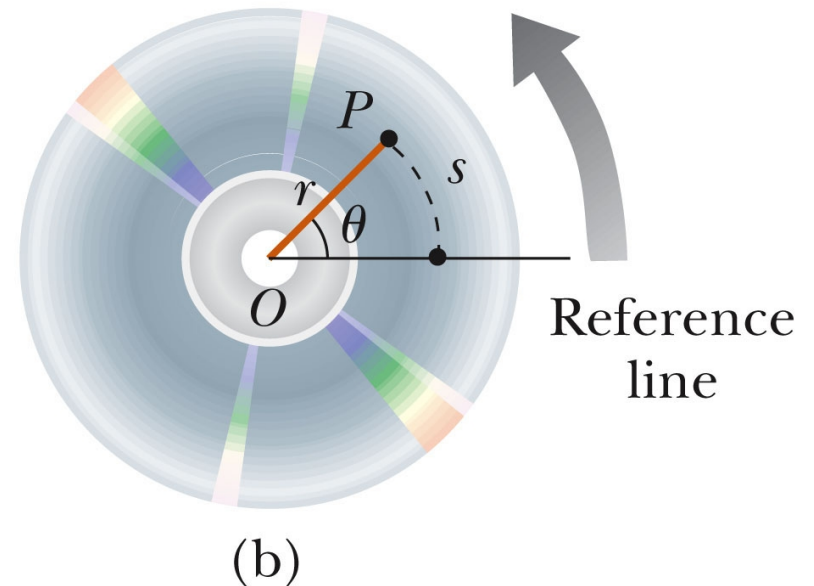
- Point P will rotate about the origin in a circle of radius r
- **Every** point on the disc undergoes circular motion about the center.
- Specify the position of point P in polar coordinates (r, θ) where θ is the measured counterclockwise from the reference line.

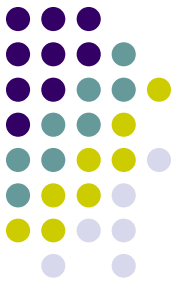


Angular Position, 3



- As the particle moves through θ , it moves through an arc length s .
- The arc length and r are related:
 - $s = \theta r$
 - where θ is in radians





The Radian

- This can also be expressed as

$$\theta = \frac{s}{r}$$

- θ is dimensionless, but is expressed in units of *radians* (rad).
- Ex) How many radians are subtended by an arc length of 6 inches if the radius of the arc is 3 in?
- Ex) How many radians are subtended by an arclength of 3 in if the radius is 3 in?
 - Try to estimate how many degrees this is!



Conversions

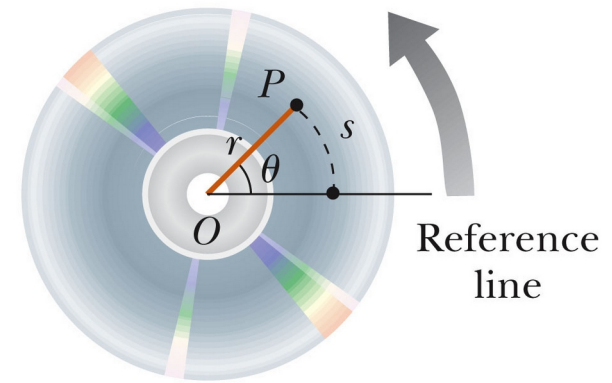
- Comparing degrees and radians

$$1 \text{ rad} = \frac{360^\circ}{2\pi} \simeq 57.3^\circ$$

- Converting from degrees to radians

$$\theta(\text{rad}) = \frac{\pi}{180^\circ} \theta(\text{degrees})$$

Angular Position, final



- So the *angular position* of a point P on an object is the angle θ , measured in radians or degrees.
- θ is the angle between a radial line running from the spin axis to P , and a reference line (usually the x-axis) also running through the spin axis.

DEMO: My CD has two points along the same radial line. How do their angular positions compare?

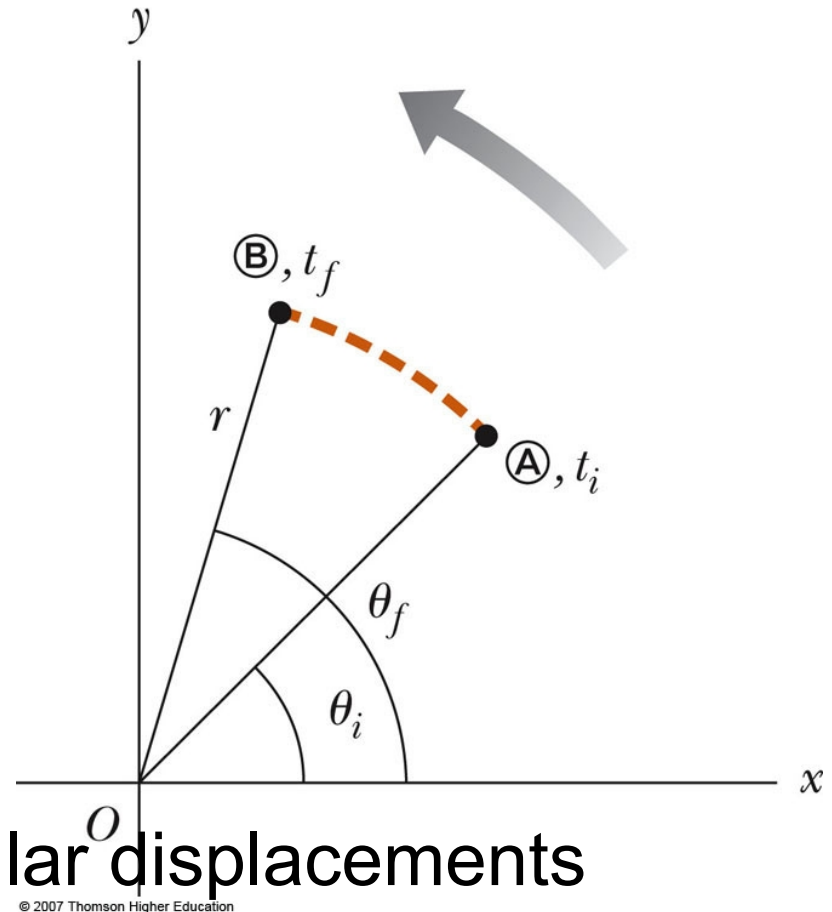


Angular Displacement

- The *angular displacement* is defined as the angle the object rotates through during some time interval

$$\Delta\theta = \theta_f - \theta_i$$

- This is the angle that the radial line of length r sweeps out.



DEMO: How do the angular displacements of the two dots on the CD compare?



Average Angular Speed

- The *average* angular speed, ω_{avg} , of a rotating rigid object is the ratio of the angular displacement to the time interval

$$\omega_{\text{avg}} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular Speed



- The *instantaneous* angular speed is defined as the limit of the average speed as the time interval approaches zero

$$\omega \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$



Angular Speed, final

- Units of angular speed are radians/sec
 - rad/s or s^{-1} since radians have no dimensions
- Angular speed will be positive if θ is increasing (counterclockwise)
- Angular speed will be negative if θ is decreasing (clockwise)

Average Angular Acceleration



- The average angular acceleration, α , of an object is defined as the ratio of the change in the angular speed to the time it takes for the object to undergo the change:

$$\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous Angular Acceleration



- The instantaneous angular acceleration is defined as the limit of the average angular acceleration as the time goes to 0

$$\alpha \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$



Angular Acceleration, final

- Units of angular acceleration are rad/s^2 or s^{-2} since radians have no dimensions
- Angular acceleration will be positive if an object rotating counterclockwise is speeding up
- Angular acceleration will also be positive if an object rotating clockwise is slowing down

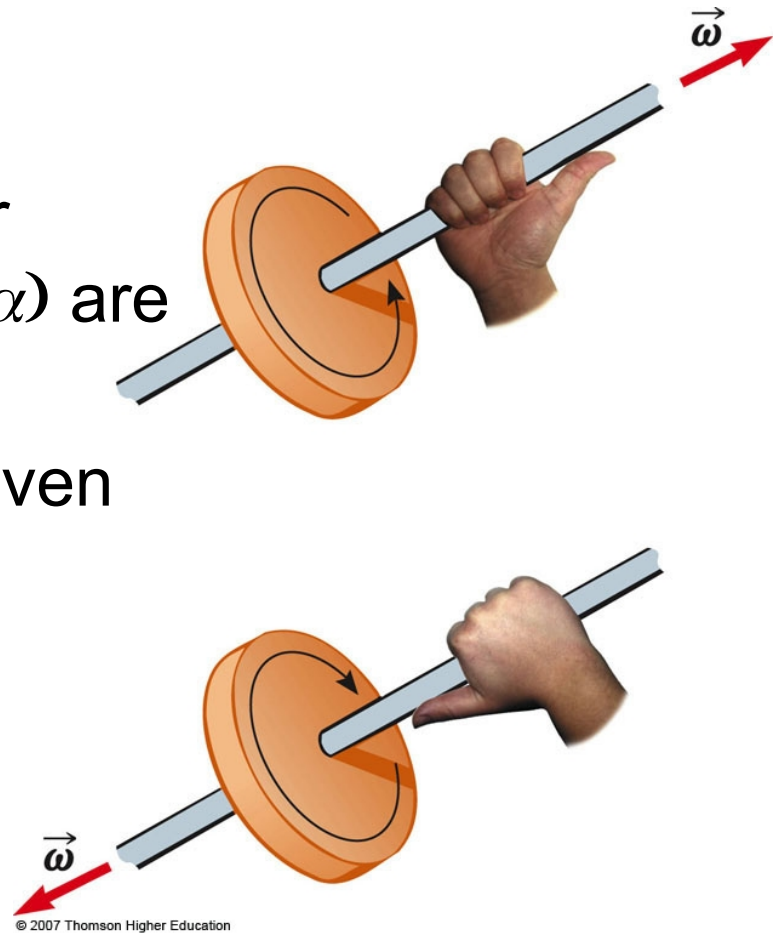


Angular Motion, mini-quiz

- T or F. The $\Delta\theta$, ω , and α are the same for every point on a rigid solid.
- T or F. The θ , $\Delta\theta$, ω , and α are the same for every point on a rigid solid.
- What is the ω_{avg} (in rad/sec) of a wheel that rotates 1 revolution in 2 seconds?
- If a CD spins up from 0 to 50 rad/s in 5 seconds, what is the α_{avg} ?

Directions, details

- Strictly speaking, the angular speed and acceleration (ω , α) are the magnitudes of vectors
- The directions are actually given by the right-hand rule.





Rotational Kinematics

- Under **constant angular acceleration**, we can describe the motion of the rigid object using a set of kinematic equations
 - These are similar to the kinematic equations for linear motion
 - The rotational equations have the same mathematical form as the linear equations
- The new model is a **rigid object under constant angular acceleration**
 - Analogous to the particle under constant acceleration model

Rotational Kinematic Equations



$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha (\theta_f - \theta_i)$$

$$\theta_f = \theta_i + \frac{1}{2} (\omega_i + \omega_f) t$$

all with constant α

Comparison Between Rotational and Linear Equations



TABLE 10.1

Kinematic Equations for Rotational and Translational Motion Under Constant Acceleration

Rotational Motion About a Fixed Axis	Translational Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$



Outline for W11,D1

Rotation of a rigid solid (Ch. 10)

Relation between (s, v_t, a_t) and (θ, ω, α)

Torque, $\tau = rF_{\perp}$

Rotational kinetic energy

Rotational inertia (or moment of inertia)

Homework

Ch. 10 P. 1,4-6,19-21,25,28-30,34,35,37,53,54,55,64,67,69

Do for Wed/Fri

Notes:

No lab this honors week.

No class Friday – activity instead.

See NEW “Exam-like” questions on Chs. 9-11.

Relationship Between Angular and Linear Quantities

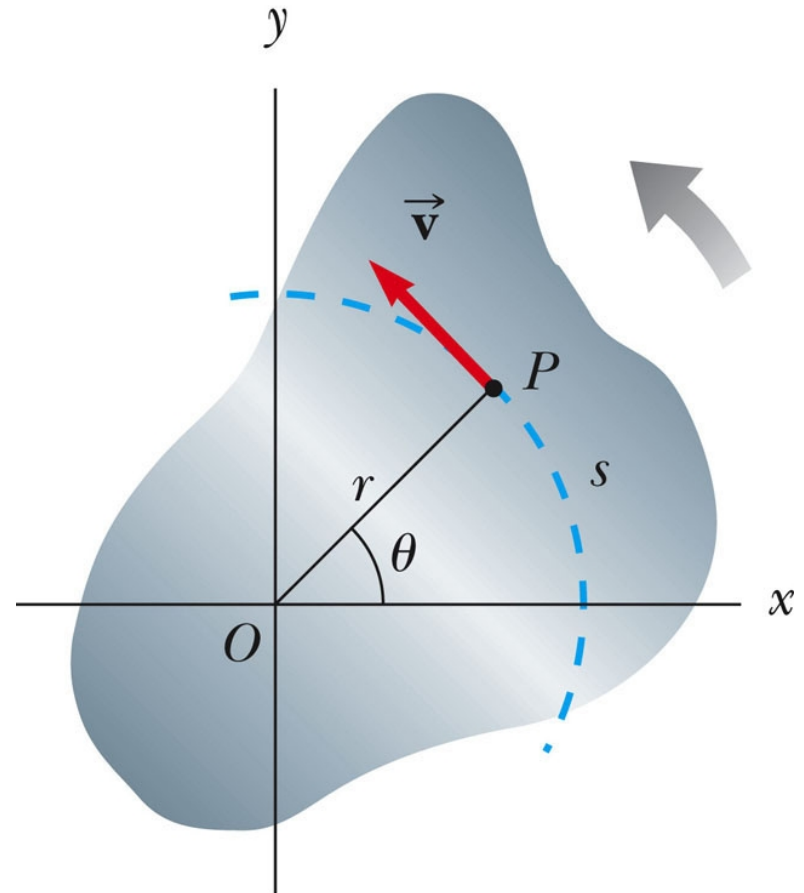


- Path length
$$s = \theta r$$
- Tangential speed
$$v_t = \omega r$$
- Tangential acceleration
$$a_t = \alpha r$$
- Centripetal acceleration
$$a_c = \omega^2 r$$
- Every point on the rotating object has the same angular motion
- Every point on the rotating object does **not** have the same linear motion

Speed Comparison

- The tangential velocity is a tangent to the circular path
- The magnitude of the velocity of point P is the tangential speed, v_t

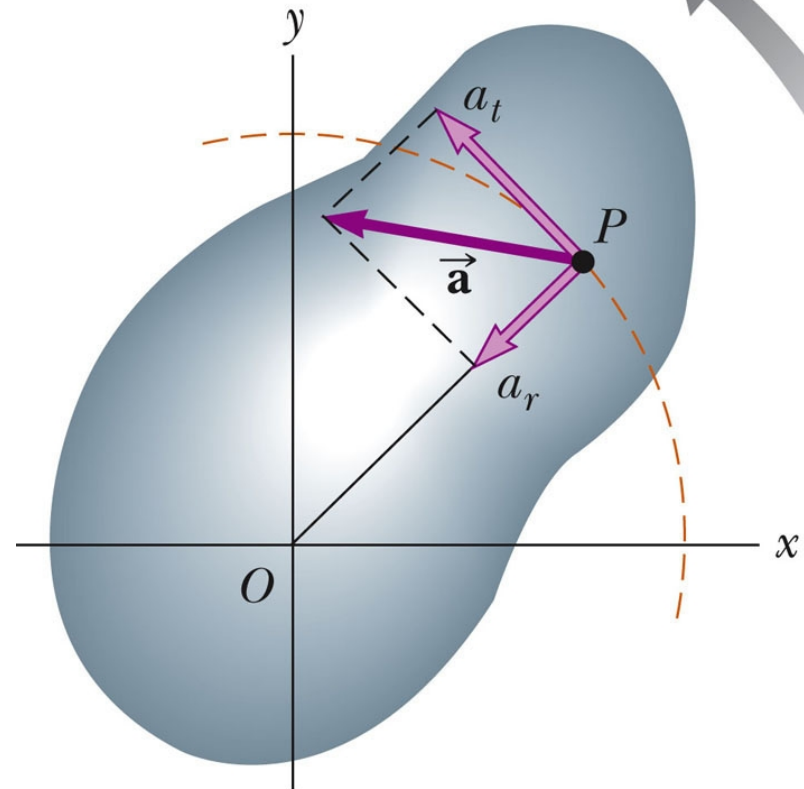
$$v_t = \frac{ds}{dt} = r \frac{d\theta}{dt} = r \omega$$



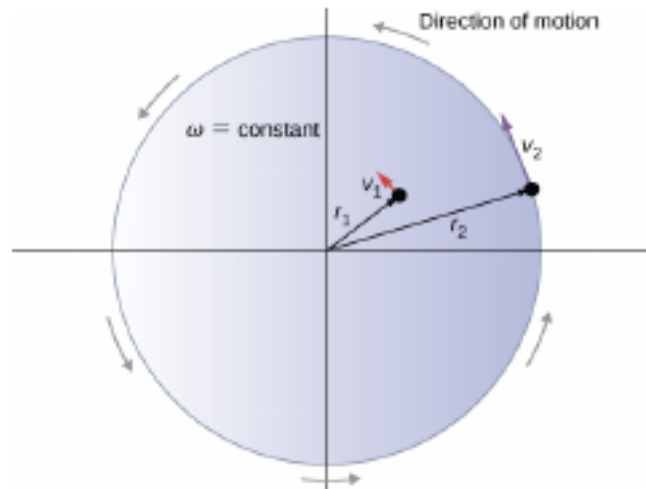
Acceleration Comparison

- The tangential acceleration is the derivative of the tangential speed

$$a_t = \frac{dv_t}{dt} = r \frac{d\omega}{dt} = r \alpha$$



Linear – angular relations. Examples.

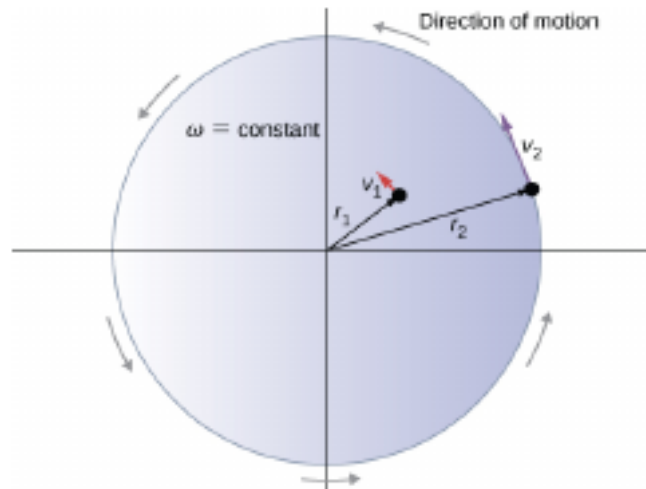


A solid, rotating disk.

In Figure 2, suppose the black dots are at $r_1 = 1.2$ cm and $r_2 = 3.6$ cm. Then answer these questions ...

11. If dot 1 has a tangential speed of $v_{t1} = 3$ cm/sec, what is the angular frequency of dot 1?
12. If dot 1 has a tangential speed of $v_{t1} = 3$ cm/sec, what is the angular frequency of dot 2?
13. If dot 1 has a tangential speed of $v_{t1} = 3$ cm/sec, what is the tangential speed of dot 2?
14. If dot 1 has a tangential speed of $v_{t1} = 3$ cm/sec, what is the centripetal acceleration of dot 2?

Linear – angular relations. Examples.



A solid, rotating disk.

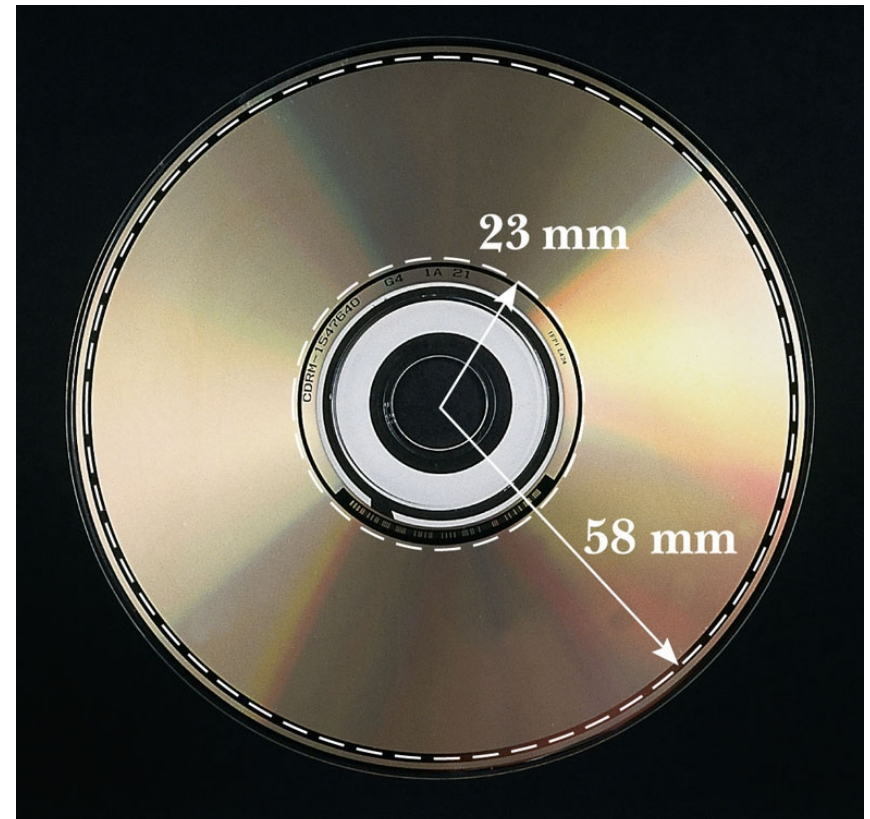
In Figure 2, suppose the black dots are at $r_1 = 1.2$ cm and $r_2 = 3.6$ cm. Then answer these questions ...

15. If dot 1 has a tangential speed of $v_{t1} = 3$ cm/sec, what is the centripetal acceleration of dot 1?
16. If a 0.002 kg bug is clinging on to the disk at dot 1, how much centripetal force must be exerted on the bug (by static friction)? (Recall $F_c = ma_c$.)
17. If a 0.002 kg bug is clinging on to the disk at dot 2, how much centripetal force must be exerted on the bug (by static friction)?



Rotational Motion Example

- For a compact disc player to read a CD, the angular speed must vary to keep the tangential speed constant ($v_t = \omega r$)
- At the inner sections, the angular speed is faster than at the outer sections



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Ex) Find v_t at $r=23\text{mm}$ if it spins at 500 RPM. $v_t=52.4 \cdot .023=1.20\text{m/s}$

Ex) Find v_t at $r=58\text{mm}$ if it spins at 200 RPM. $v_t=20.9 \cdot .058=1.21\text{m/s}$



Torque

- Torque, τ , is a force times a distance which changes the rotation rate of an object
 - Torque is a vector, but we will deal with its magnitude first. (Cross products appear in Ch. 11)
 - $\tau = F r \sin \phi = F d$
 - F is the force
 - ϕ is the angle the force makes with the line extending from the axis to the point of application of F .
 - d is the *moment arm* (or lever arm) of the force



Outline for W11,D2

Torque Example ($\tau = rF_{\perp}$)

Rotational kinetic energy

Rotational inertia (or moment of inertia)

Homework

Ch. 10 P. 1,4-6,19-21,25,28-30,34,35,37,53,54,55,64,67,69

Do for Wed/Fri

Ch. 11 P. 1,2,3,5,36,42,48 Do before Exam II (4/23 or 4/25)

Notes:

No lab this honors week.

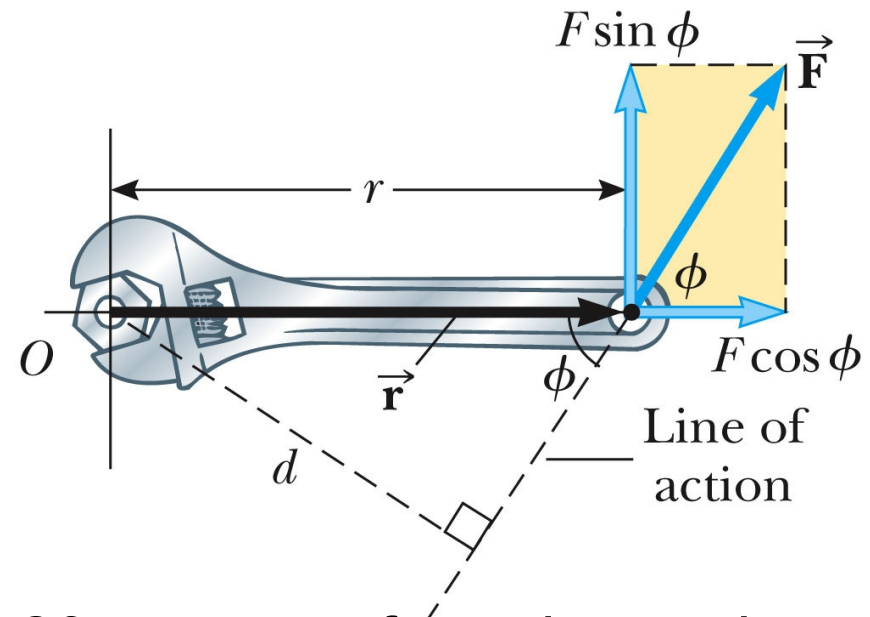
No class Friday – see email for activity instead.

See NEW lists of equations for Exam II and Ch. 11 links.

Torque, cont



- The moment arm, d , is the *perpendicular* distance from the axis of rotation to a line drawn along the direction of the force
 - $d = r \sin \phi$



Ex) A force of 10 N is applied 20 cm away from the nut it is tightening in a direction 60° away from the wrench arm. Find the torque.

Q: What if $\theta = 90^\circ$? Q: What if $r = 10$ cm and $\theta = 90^\circ$?



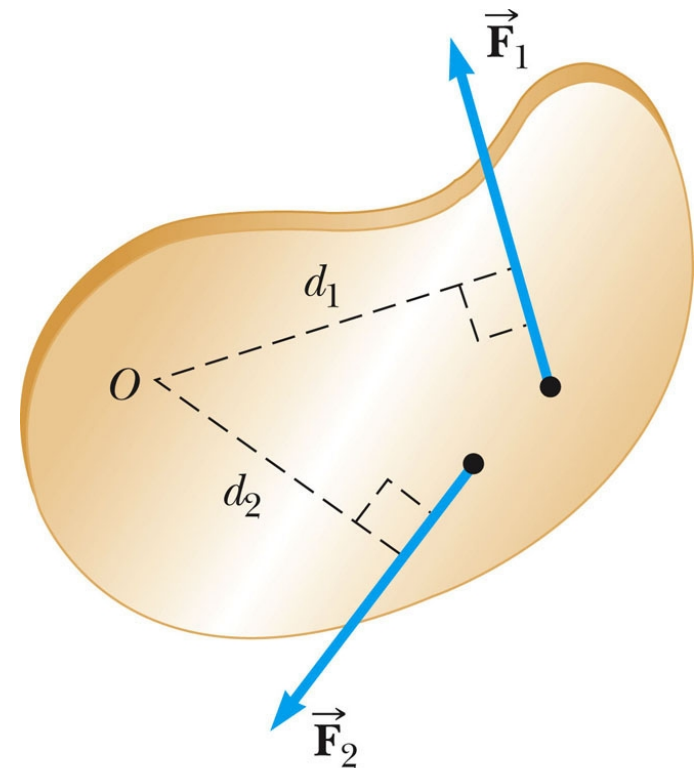
Torque, direction

- The horizontal component of the force ($F \cos \phi$) has no tendency to produce a rotation
- Torque has a direction
 - If the turning tendency of the force is counterclockwise (CCW), the torque will be positive
 - If the turning tendency is clockwise (CW), the torque will be negative



Net Torque

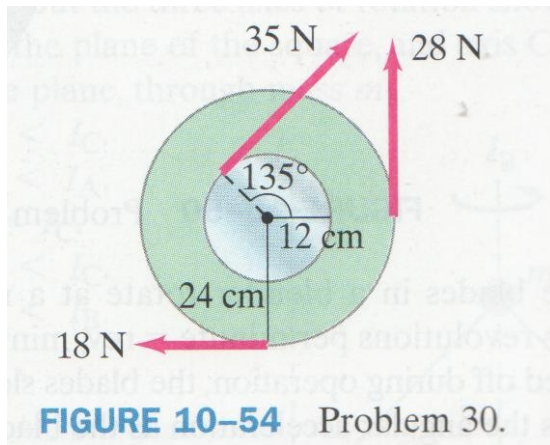
- The force \vec{F}_1 will tend to cause a counterclockwise rotation about O
- The force \vec{F}_2 will tend to cause a clockwise rotation about O
- $\Sigma\tau = \tau_1 + \tau_2 = F_1d_1 - F_2d_2$



Net Torque - Example



P. 30) Calculate the net torque about the axle of the wheel shown in Fig. 10-54. Assume that a friction torque of 0.60 Nm opposes the motion.



$$\tau_{net} = \Sigma \tau = \tau_1 + \tau_2 + \tau_3 + \tau_{fric}$$

$$\begin{aligned}\tau_{app} &= 28\text{N}(.24\text{m}) - 35\text{N}(.12\text{m}) - 18\text{N}(.24\text{m}) \\ &= 6.72 - 4.2 - 4.32 \\ &= -1.8 \quad (- \text{ implies CW})\end{aligned}$$

$$\text{Thus, } \tau_{fric} = 0.60 \text{ Nm} \quad (\text{CCW})$$

$$\text{and } \tau_{net} = -1.2 \text{ Nm}$$



Torque vs. Force

- Forces can cause a change in translational motion
 - Described by Newton's 2nd Law: $\mathbf{F}_{\text{net}} = m\mathbf{a}$
- Torques can cause a change in rotational motion
 - The Newton's 2nd law analog: $\tau_{\text{net}} = I \alpha$
 - Where I is rotational inertia



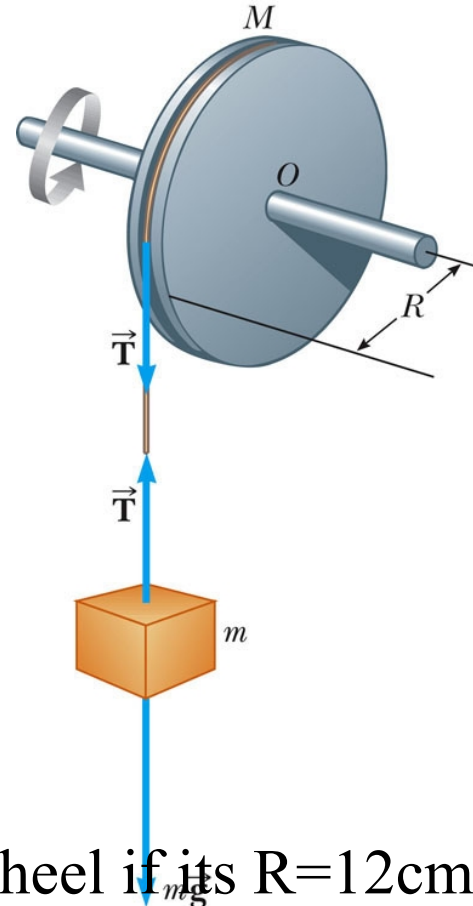
Torque Units

- The SI units of torque are N·m
 - Although torque is a force multiplied by a distance, it is very different from work and energy
 - The units for torque are reported in N·m and not changed to Joules

Torque and Angular Acceleration, Wheel Example



- Analyze:
- The wheel is rotating and so we apply $\Sigma \tau = I\alpha$
 - The tension supplies the tangential force
- The mass is moving in a straight line, so apply Newton's Second Law
 - $\Sigma F_y = ma_y = mg - T$



Ex) Find the angular acceleration of the wheel if its $R=12\text{cm}$ and its $I=0.05 \text{ kg m}^2$ and the hanging mass $m=2 \text{ kg}$. $\alpha=29.8 \text{ rad/s}^2$

Ex) Find the linear acceleration of the mass m . $a_y=3.58 \text{ m/s}^2$

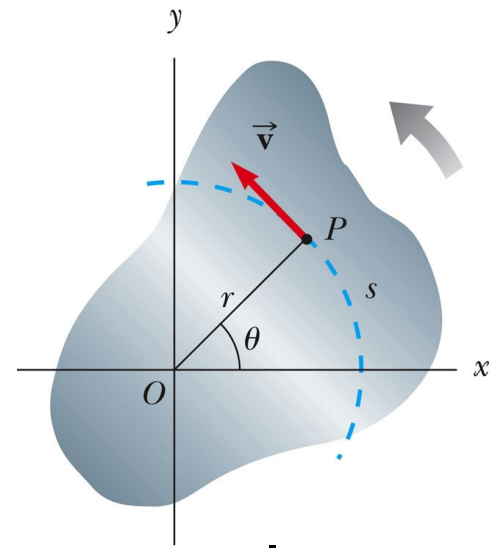
Torque and Angular Acceleration



See link “Torque and rotational kinematics example” for another worked example of $\tau = I\alpha$.

This one applies to a grinding wheel.

Rotational Kinetic Energy



- An object rotating about some axis with an angular speed, ω , has rotational kinetic energy. Lets derive $K_{\text{rot}} = \frac{1}{2} I \omega^2$
- Each particle, m_i , (like the one at P) has a kinetic energy of
 - $K_i = \frac{1}{2} m_i v_i^2$
- The v_i is a tangential velocity at P and can be replaced by $v_i = \omega_i r$



Rotational Kinetic Energy, cont

- The total rotational kinetic energy of the rigid object is the sum of the energies of all its particles

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

$$K_R = \frac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2 = \frac{1}{2} I \omega^2$$

- Where I is called the moment of inertia

Rotational Kinetic Energy, final



- There is an analogy between the kinetic energies associated with linear motion ($K = \frac{1}{2} m v^2$) and the kinetic energy associated with rotational motion ($K_R = \frac{1}{2} I \omega^2$)
- Rotational kinetic energy is not a new type of energy, the form is different because it is applied to a rotating object
- The units of rotational kinetic energy are Joules (J)



Rotational Kinetic Energy

Example) Find the total KE of a baseball (mass m , radius R) with a speed v and a spin ω .

Ans: $K_{\text{tot}} = K_{\text{rot}} + K_{\text{trans}}$



Moment of Inertia

- The definition of moment of inertia (for a collection of discrete masses) is

$$I = \sum_i r_i^2 m_i$$

- The dimensions of moment of inertia are ML^2 and its SI units are $kg \cdot m^2$

We can calculate the moment of inertia of an extended object by assuming it is divided into small volume elements, Δm_i , and taking the limit towards zero size: $\Delta m_i = dm$



Moment of Inertia, cont

- We can rewrite the expression for I in terms of Δm

$$I = \lim_{\Delta m_i \rightarrow 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$$

- With the small volume segment assumption,

$$I = \int \rho r^2 dV$$

- If ρ is constant, the integral can be evaluated with known geometry, otherwise its variation with position must be known



Notes on Various Densities

- Volumetric Mass Density \rightarrow mass per unit volume: $\rho = m / V$
- Surface Mass Density \rightarrow mass per unit thickness of a sheet of uniform thickness, t :
 $\sigma = \rho t$
- Linear Mass Density \rightarrow mass per unit length of a rod of uniform cross-sectional area: $\lambda = m / L = \rho A$

Moment of Inertia of a Uniform Rigid Rod



- The shaded area has a mass

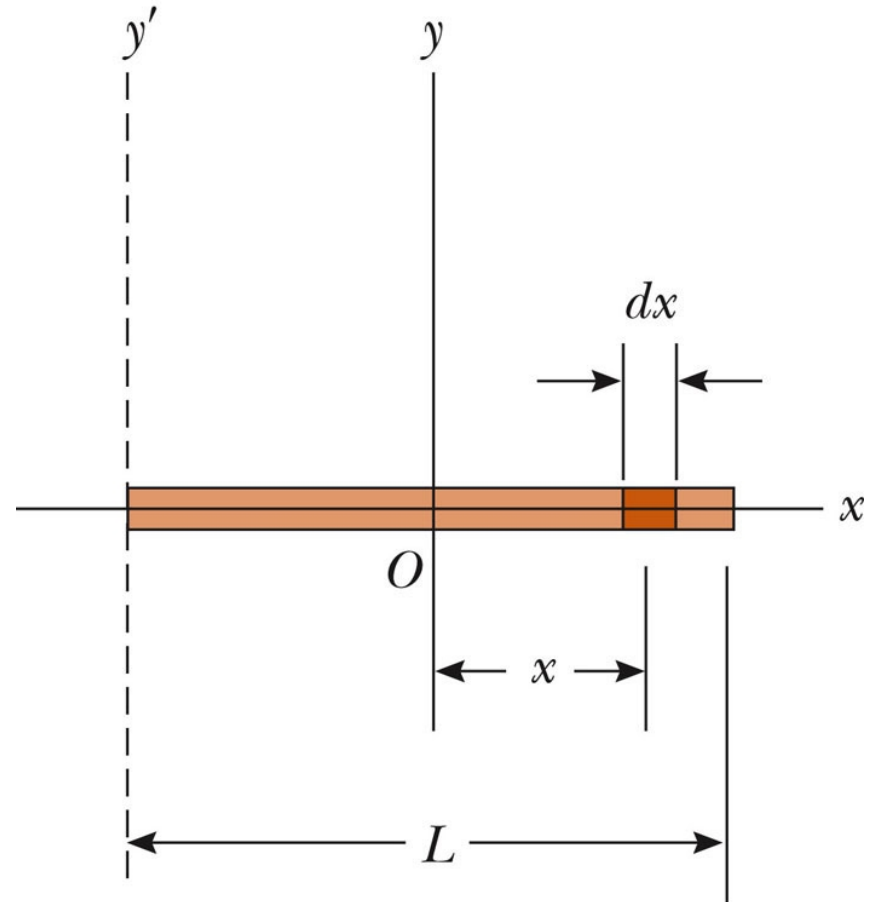
- $dm = \lambda dx$

- For a uniform rod, $\lambda = M/L$

- Then the moment of inertia is

$$I_y = \int r^2 dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx$$

$$I_y = \frac{1}{12} M L^2$$



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Q: What is $I_{y'}$ (relative to y')? Ans: $I_{y'} = \frac{4}{12} M L^2$

Moment of Inertia of a Uniform Solid Cylinder



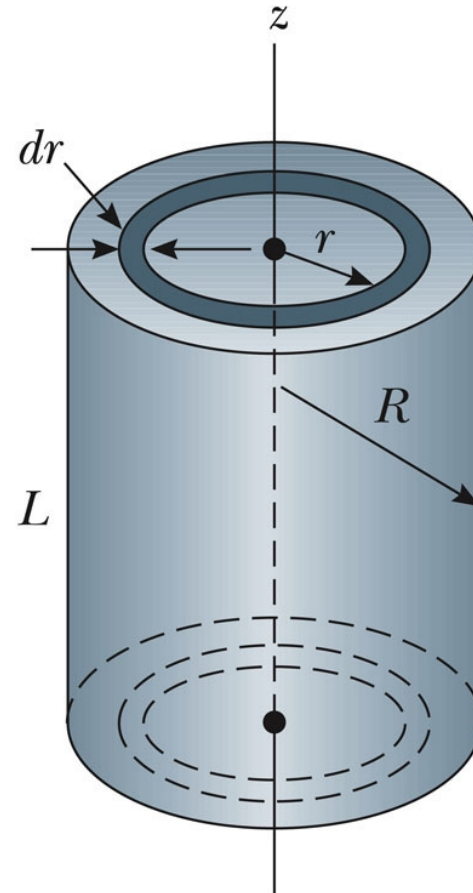
- Divide the cylinder into concentric shells with radius r , thickness dr and length L

- $dm = \rho dV = \rho 2\pi r L dr$

- Then for I

$$I_z = \int r^2 dm = \int r^2 (2\pi\rho L r dr)$$

$$I_z = \frac{1}{2} MR^2$$



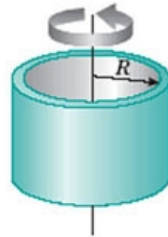
Moments of Inertia of Various Rigid Objects



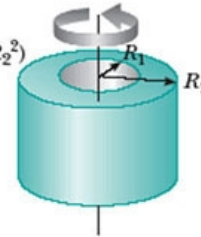
TABLE 10.2

Moments of Inertia of Homogeneous Rigid Objects with Different Geometries

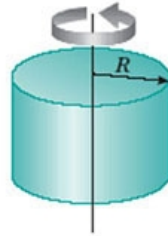
Hoop or thin
cylindrical shell
 $I_{\text{CM}} = MR^2$



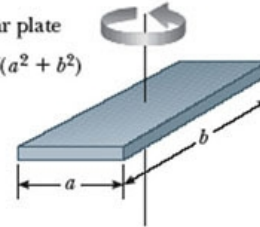
Hollow cylinder
 $I_{\text{CM}} = \frac{1}{2} M(R_1^2 + R_2^2)$



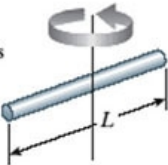
Solid cylinder
or disk
 $I_{\text{CM}} = \frac{1}{2} MR^2$



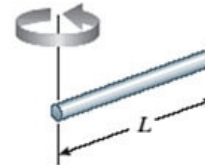
Rectangular plate
 $I_{\text{CM}} = \frac{1}{12} M(a^2 + b^2)$



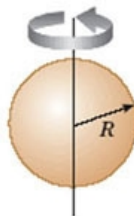
Long, thin rod
with rotation axis
through center
 $I_{\text{CM}} = \frac{1}{12} ML^2$



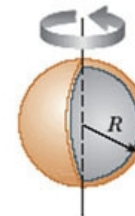
Long, thin
rod with
rotation axis
through end
 $I = \frac{1}{3} ML^2$



Solid sphere
 $I_{\text{CM}} = \frac{2}{5} MR^2$



Thin spherical
shell
 $I_{\text{CM}} = \frac{2}{3} MR^2$





Parallel-Axis Theorem

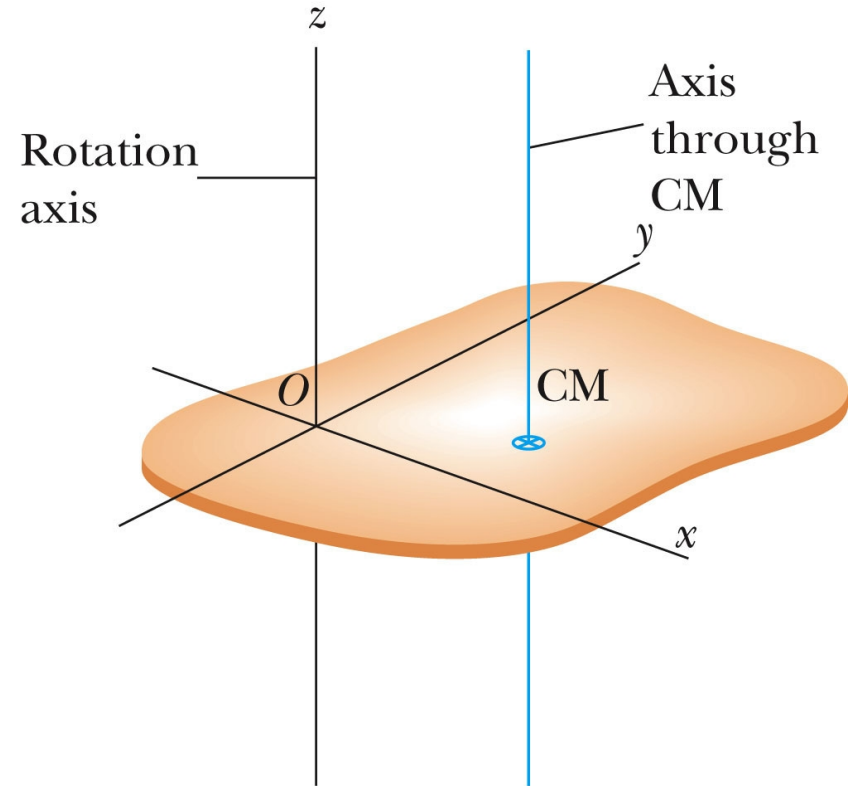
- In the previous examples, the axis of rotation coincided with the axis of symmetry of the object
- For an arbitrary axis, the parallel-axis theorem often simplifies calculations
- The theorem states $I = I_{\text{CM}} + MD^2$
 - I is about any axis parallel to the axis through the center of mass of the object
 - I_{CM} is about the axis through the center of mass
 - D is the distance from the center of mass axis to the arbitrary axis

Parallel-Axis Theorem

Example



- The axis of rotation goes through O
- The axis through the center of mass is shown
- The moment of inertia about the axis through O would be $I_O = I_{CM} + MD^2$



(b)

Moment of Inertia for a Rod Rotating Around One End



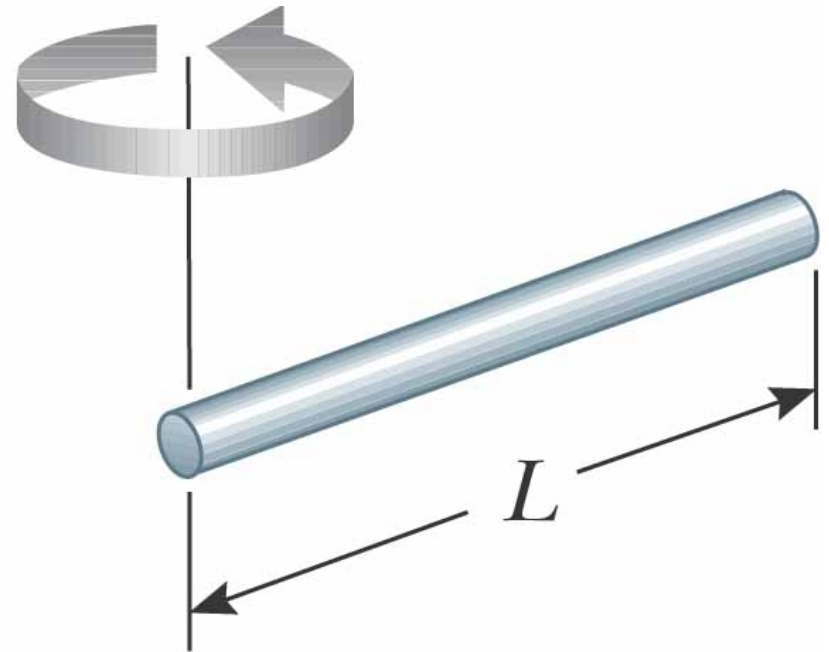
- The moment of inertia of the rod about its center is

$$I_{CM} = \frac{1}{12} ML^2$$

- D is $\frac{1}{2} L$
- Therefore,

$$I = I_{CM} + MD^2$$

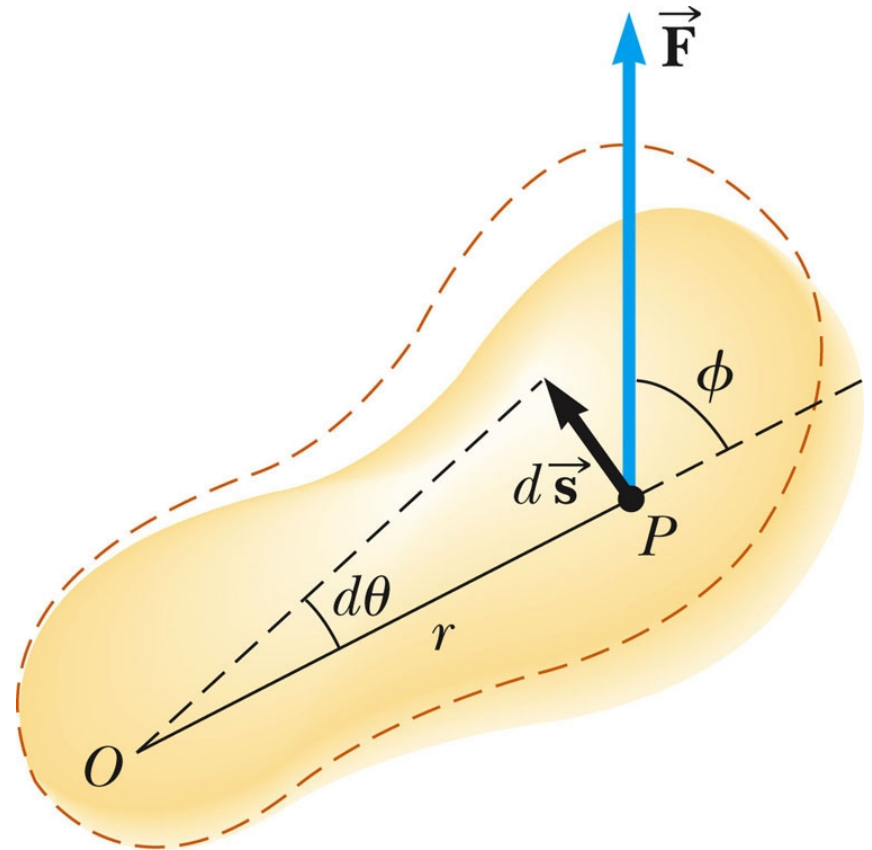
$$I = \frac{1}{12} ML^2 + M \left(\frac{L}{2} \right)^2 = \frac{1}{3} ML^2$$





Work in Rotational Motion

- Find the work done by \vec{F} on the object as it rotates through an infinitesimal distance $ds = r d\theta$
$$dW = \vec{F} \cdot d\vec{s}$$
$$= (F \sin \phi) r d\theta$$
- The radial component of the force does no work because it is perpendicular to the displacement



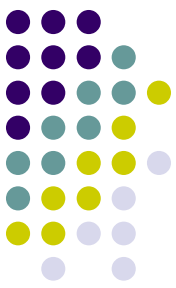


Power in Rotational Motion

- The rate at which work is being done in a time interval dt is

$$\text{Power} = \wp = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$

- This is analogous to $\wp = Fv$ in a linear system



Summary of Useful Equations

TABLE 10.3

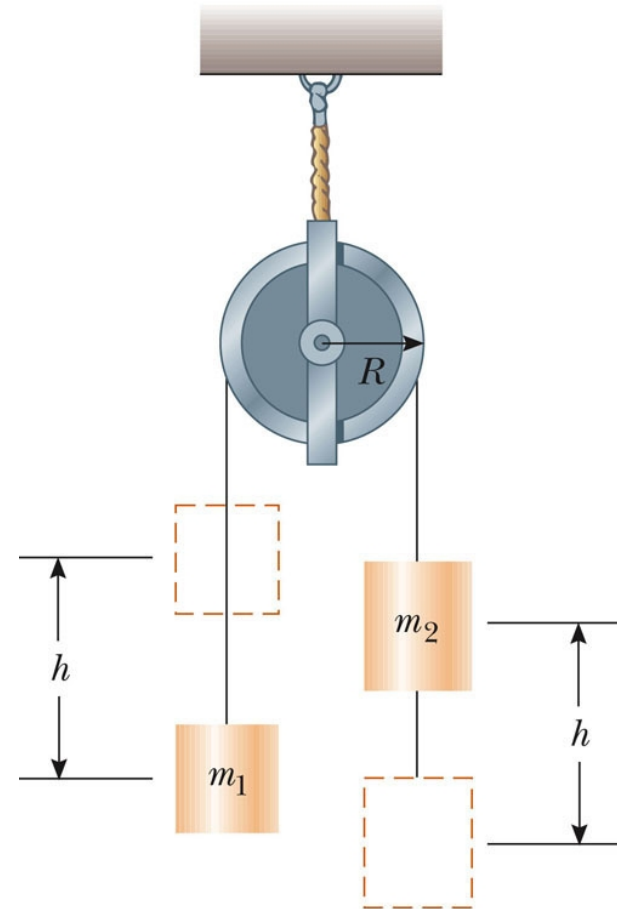
Useful Equations in Rotational and Translational Motion

Rotational Motion About a Fixed Axis	Translational Motion
Angular speed $\omega = d\theta/dt$	Translational speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Translational acceleration $a = dv/dt$
Net torque $\Sigma \tau = I\alpha$	Net force $\Sigma F = ma$
If $\alpha = \text{constant}$ $\begin{cases} \omega_f = \omega_i + \alpha t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If $a = \text{constant}$ $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau d\theta$	Work $W = \int_{x_i}^{x_f} F_x dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $\mathcal{P} = \tau\omega$	Power $\mathcal{P} = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma \tau = dL/dt$	Net force $\Sigma F = dp/dt$

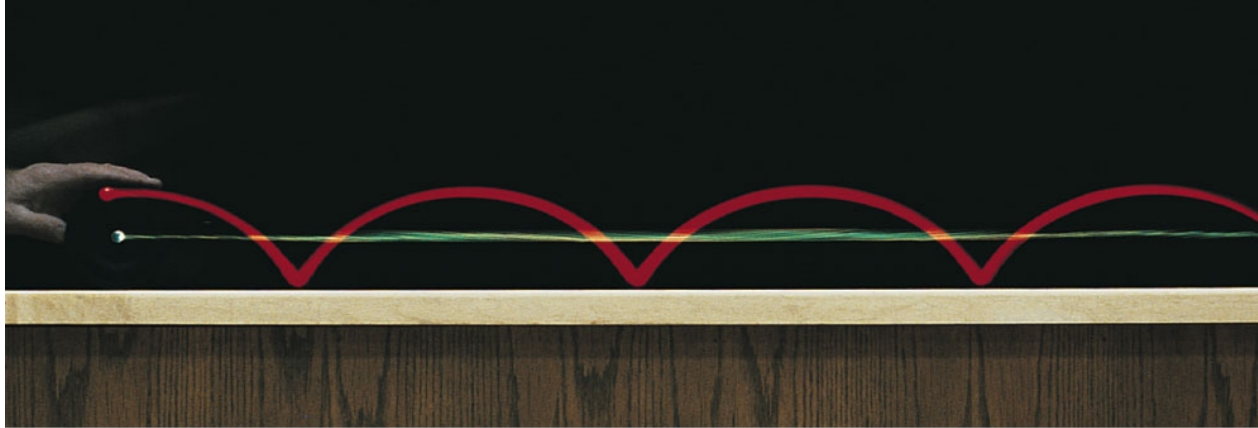
Energy in an Atwood Machine, Example



- The blocks undergo changes in translational kinetic energy and gravitational potential energy
- The pulley undergoes a change in rotational kinetic energy
- Use the active figure to change the masses and the pulley characteristics



Rolling Object



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- The red curve shows the path moved by a point on the rim of the object
 - This path is called a ***cycloid***
- The green line shows the path of the center of mass of the object



Pure Rolling Motion

- In pure rolling motion, an object rolls without slipping
- In such a case, there is a simple relationship between its rotational and translational motions

Rolling Object, Center of Mass

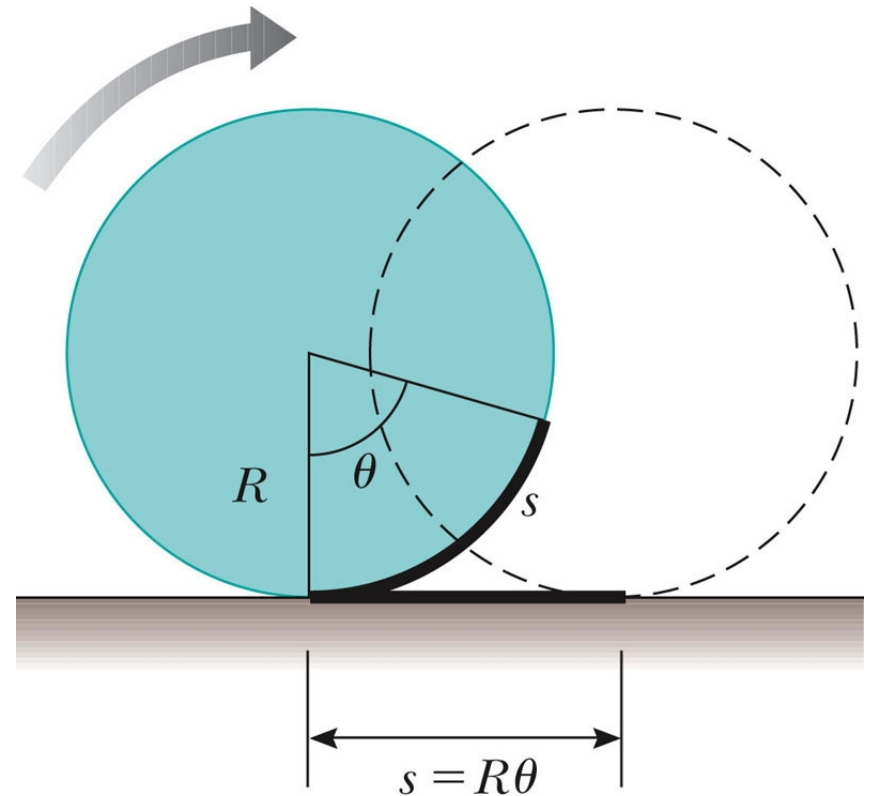


- The velocity of the center of mass is

$$v_{\text{CM}} = \frac{ds}{dt} = R \frac{d\theta}{dt} = R\omega$$

- The acceleration of the center of mass is

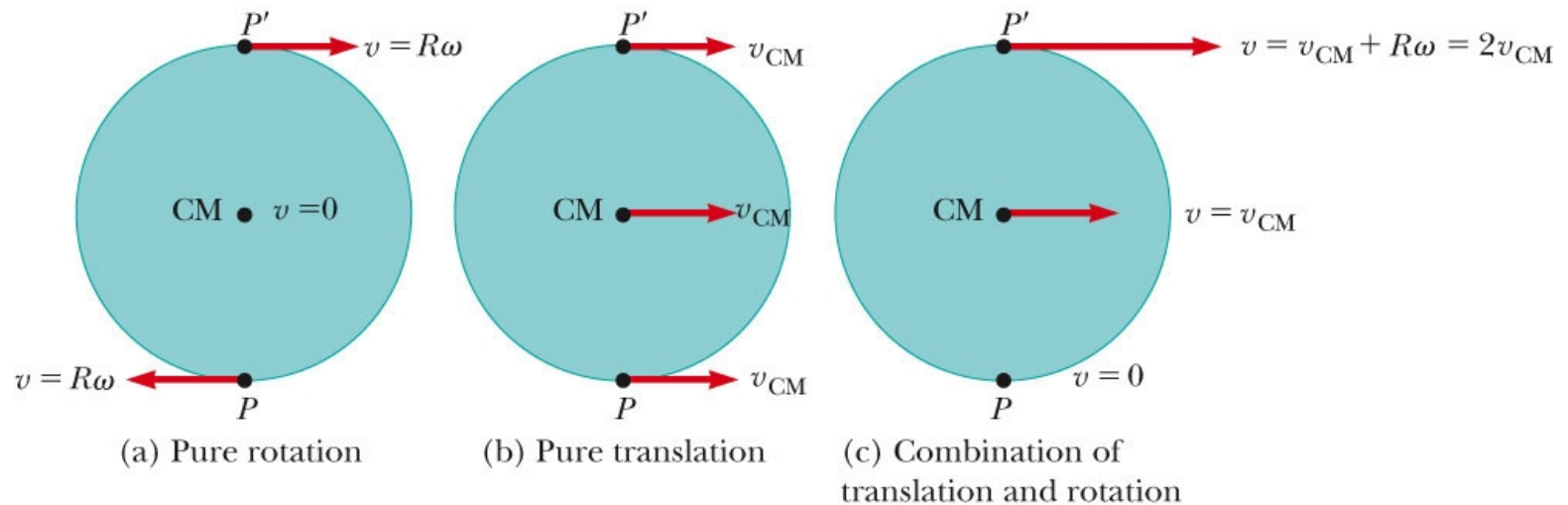
$$a_{\text{CM}} = \frac{dv_{\text{CM}}}{dt} = R \frac{d\omega}{dt} = R\alpha$$





Rolling Motion Cont.

- Rolling motion can be modeled as a combination of pure translational motion and pure rotational motion
- The contact point between the surface and the cylinder has a translational speed of zero (c)



Total Kinetic Energy of a Rolling Object

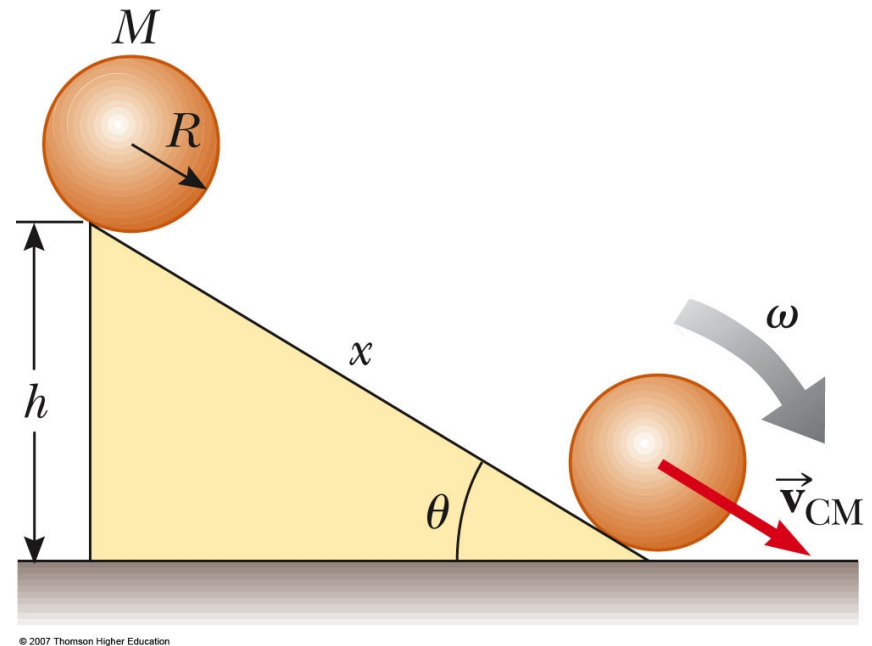


- The total kinetic energy of a rolling object is the sum of the translational energy of its center of mass and the rotational kinetic energy about its center of mass
 - $K = \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} M v_{CM}^2$
 - The $\frac{1}{2} I_{CM} \omega^2$ represents the rotational kinetic energy of the cylinder about its center of mass
 - The $\frac{1}{2} M v^2$ represents the translational kinetic energy of the cylinder about its center of mass

Total Kinetic Energy, Example



- Accelerated rolling motion is possible only if friction is present between the sphere and the incline
 - The friction produces the net torque required for rotation
 - No loss of mechanical energy occurs because the contact point is at rest relative to the surface at any instant
 - Use the active figure to vary the objects and compare their speeds at the bottom



Total Kinetic Energy, Example cont



- Apply Conservation of Mechanical Energy

- Let $U = 0$ at the bottom of the plane

- $K_f + U_f = K_i + U_i$

- $K_f = \frac{1}{2} (I_{CM} / R^2) v_{CM}^2 + \frac{1}{2} M v_{CM}^2 = \frac{1}{2} \left(\frac{I_{CM}}{R^2} + M \right) v_{CM}^2$

- $U_i = Mgh$

- $U_f = K_i = 0$

- Solving for v

$$v = \left[\frac{2gh}{1 + \left(\frac{I_{CM}}{MR^2} \right)} \right]^{\frac{1}{2}}$$

Sphere Rolling Down an Incline, Example



- **Conceptualize**

- A sphere is rolling down an incline

- **Categorize**

- Model the sphere and the Earth as an isolated system
 - No nonconservative forces are acting

- **Analyze**

- Use Conservation of Mechanical Energy to find v
 - See previous result

Sphere Rolling Down an Incline, Example cont



- **Analyze, cont**
 - Solve for the acceleration of the center of mass
- **Finalize**
 - Both the speed and the acceleration of the center of mass are independent of the mass and the radius of the sphere
- **Generalization**
 - *All homogeneous solid spheres experience the same speed and acceleration on a given incline*
 - Similar results could be obtained for other shapes