P231 Week 1: measurements

Goals of Week 1:

- Learn about base and derived units
- Learn dimensions and dimensional analysis
- Understand the need for errors and significant figures
- Learn how to propagate errors in +, -, ×, and ÷
- Learn definitions of average velocity, speed, instantaneous velocity, etc.

Units

Base Units

Mechanical

Quantity	MKS unit	cgs unit
mass	kg (kilogram)	g
length	m (meter)	cm
time	s (second)	s

Other

Quantity	MKS unit
temperature	K (Kelvin)
current	A (amps)
amount of matter	mol (mole)
luminous intensity	cd (candela)

Derived Units

miles/hour
km/s
mol/liter
kg m/s^2
etc.
etc.
etc.

Unit systems

System	L	M	T
SI, or MKS	m	kg	s
cgs	cm	g	s
US Customery	ft (foot)	slug	s

Making convenient units with prefixes

TABLE 1.2 Multiples and Prefixes for Metric Units*

Multiple†	Prefix (and Abbreviation)	Pronunciation	Multiple†	Prefix (and Abbreviation)	Pronunciation
10^{24}	yotta- (Y)	yot′ta (*a* as in *a*bout)	10^{-1}	deci- (d)	des′i (as in *deci*mal)
10^{21}	zetta- (Z)	zet′ta (*a* as in *a*bout)	10^{-2}	centi- (c)	sen′ti (as in *senti*mental)
10^{18}	exa- (E)	ex′a (*a* as in *a*bout)	10^{-3}	milli- (m)	mil′li (as in *mili*tary)
10^{15}	peta- (P)	pet′a (as in *peta*l)	10^{-6}	micro- (μ)	mi′kro (as in *micro*phone)
10^{12}	tera- (T)	ter′a (as in *terra*ce)	10^{-9}	nano- (n)	nan′oh (*an* as in *ann*ual)
10^{9}	giga- (G)	ji′ga (*ji* as in *ji*ggle, *a* as in *a*bout)	10^{-12}	pico- (p)	pe′ko (*peek-oh*)
10^{6}	mega- (M)	meg′a (as in *mega*phone)	10^{-15}	femto- (f)	fem′toe (*fem* as in *fem*inine)
10^{3}	kilo- (k)	kil′o (as in *kilo*watt)	10^{-18}	atto- (a)	at′toe (as in an*atom*y)
10^{2}	hecto- (h)	hek′to (*heck-toe*)	10^{-21}	zepto- (z)	zep′toe (as in *zep*pelin)
10	deka- (da)	dek′a (*deck* plus *a* as in *a*bout)	10^{-24}	yocto- (y)	yock′toe (as in *sock*)

*For example, 1 gram (g) multiplied by 1000 (10^3) is 1 kilogram (kg); 1 gram multiplied by 1/1000 (10^{-3}) is 1 milligram (mg).

†The most commonly used prefixes are printed in color. Note that the abbreviations for the multiples 10^6 and greater are capitalized, whereas the abbreviations for the smaller multiples are lowercased.

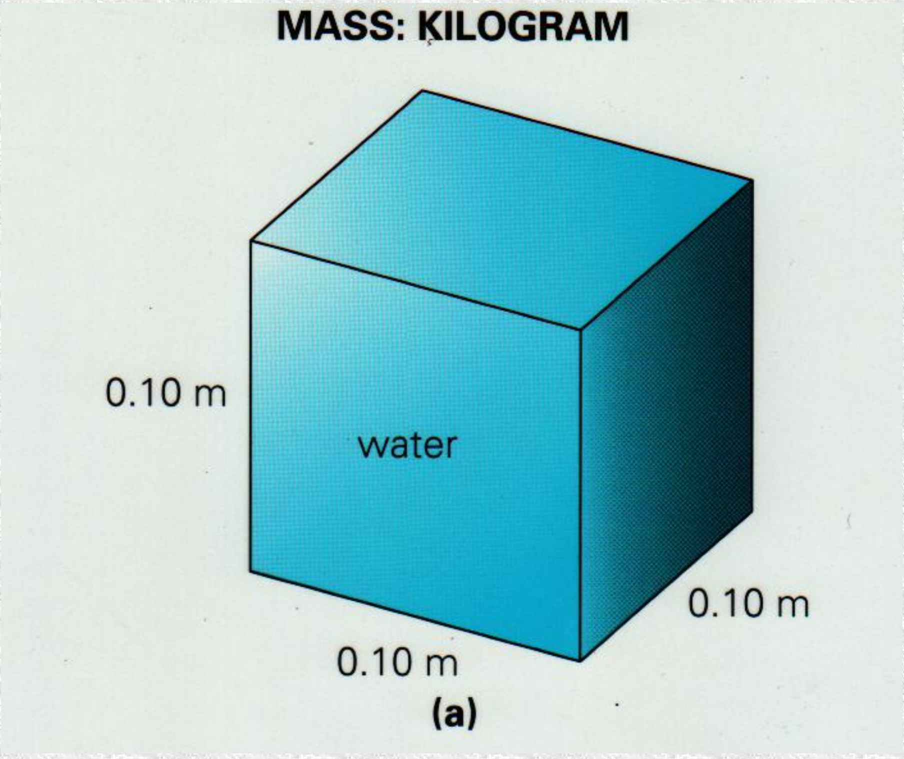
Unit Standards

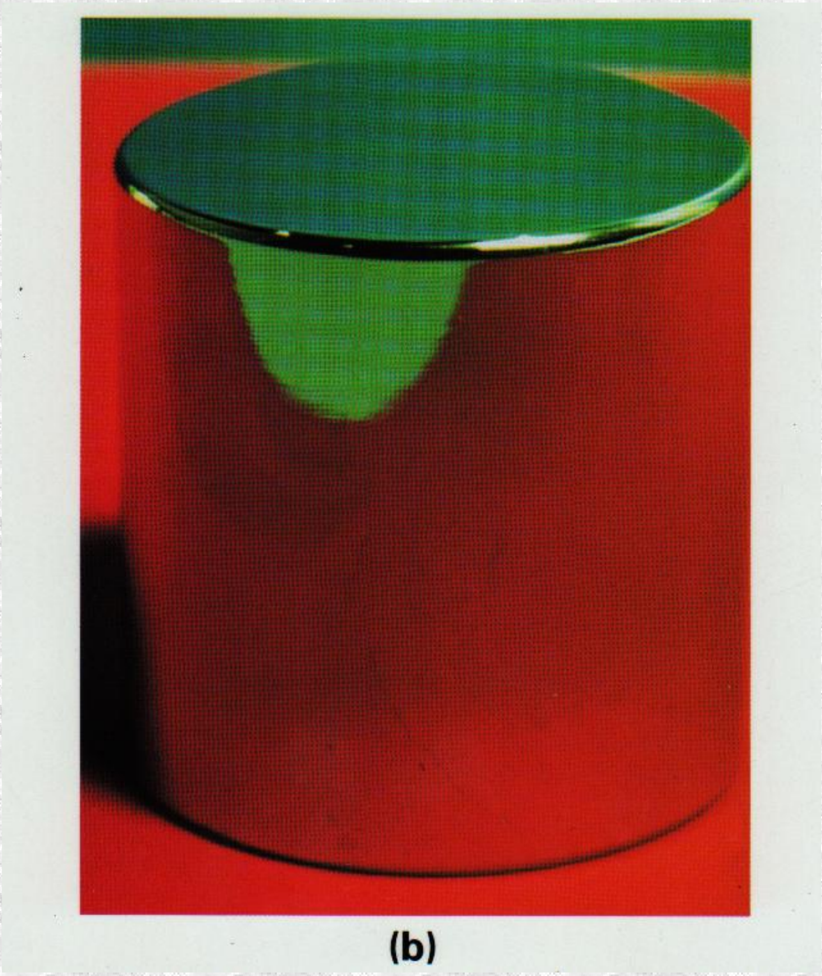
<u>Standard</u>: a real-life object or thing which defines a unit.

Why do we need standards?
Communication!

- **between scientists discussing experimental results**
- **between international businessmen selling goods "by the gallon" or "by the pound"**
- **between Earth and alien life (some day?)**

Unit Standards
Mass

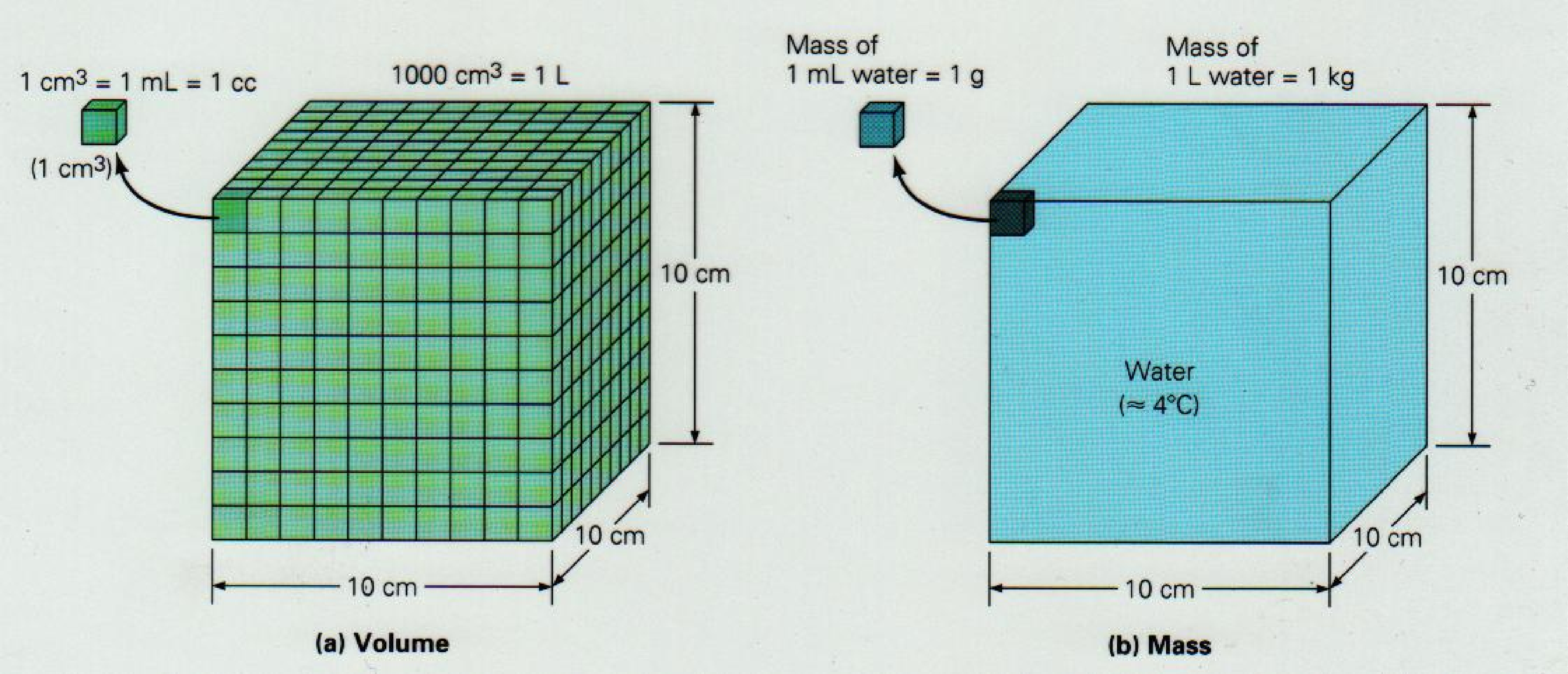




5/20/19 – the kg is based on exactly
Specifying Planck's constant!

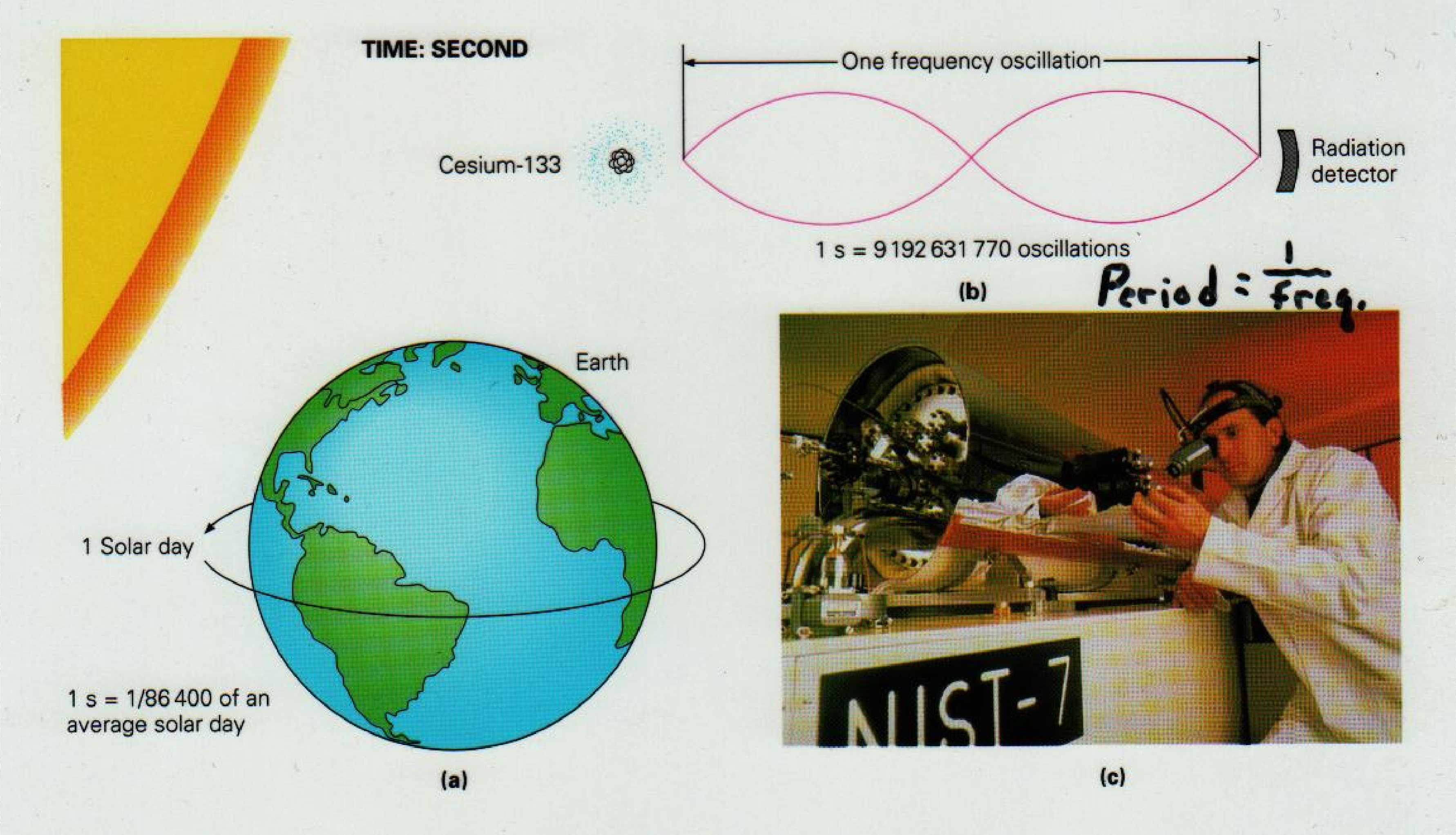
PtIr cylinder in Sevres, France

Unit Standards
Volume based on mass of H_2O

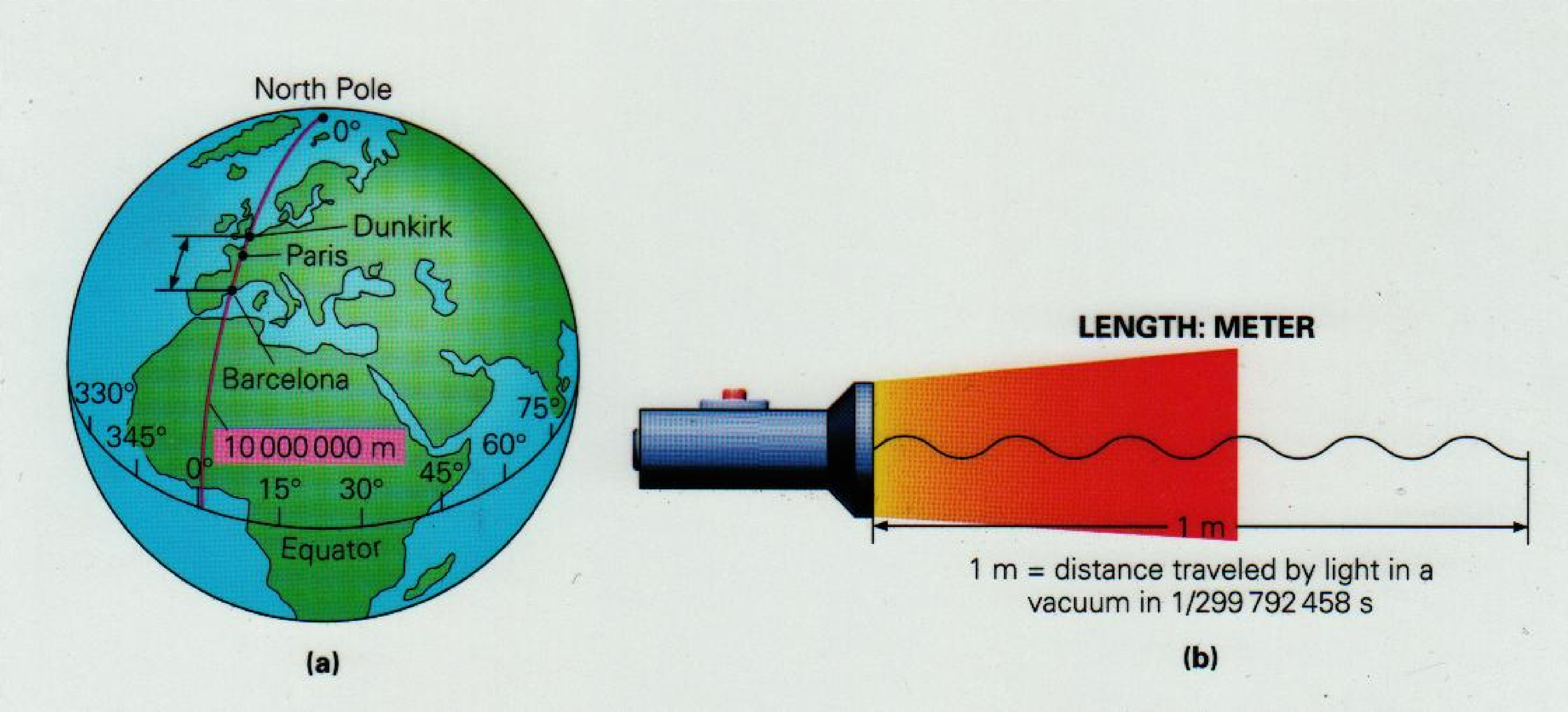


(1mL=1g is strictly true at T = 4 °C.)

Unit Standards
Time



Unit Standards
Length



The meter is now based on the speed of light in a vacuum.

Dimensions

<u>dimension</u>: "the manifoldness with which the fundamental units of time, length, and mass are involved in determining the units of other physical quantities."
<u>dimension</u>: the physical nature of a quantity

[x] = "the dimensions of x"

For mechanical base units …

Mechanical

Quantity	**Dimension**
mass	M
length	L
time	T

For some derived units …

[miles/hour] =	L/T
[km/s] =	L/T
[knot] =	L/T
[L (liter)] =	L^3
[kg m/s^2] =	ML/T^2
[density]=	M/L^3

Use of brackets: "[x]=" means "the dimensions of x are ..."

Dimensional Analysis

- *a way to figure out if an equation is (dimensionally) correct*
- *allows you to decide which equation to use.*

Ex. 1) Is this equation dimensionally correct?

$$ma = ½^{\dagger}mv^2$$

where m=mass, v=speed (L/T), a=acceleration (L/T^2)

Soln: $[ma]=ML/T^2$ and $[½\ mv^2]=ML^2/T^2$
since $ML/T^2 \neq ML^2/T^2$ the equation cannot be correct.

Ex. 2) Is this equation dimensionally correct?

$$y = at^2$$

where y=position (L), t=time, a=acceleration=L/T^2

Soln: $[y]=L$, $[at^2]=L/T^2 * T^2=L$.
since L = L, the equation is dimensionally correct.
However, the equation is still wrong! How?

† 1/2 is a dimensionless constant

Dimensional Analysis (cont)

Ex. 3) How long does it take to drive 20 miles (to Lima) at a constant 60 mph?

Soln: Let v=speed (L/T), d=distance (L) and t=time (T).

Possible (linear) equations: t=v*d, t=v/d, t=d/v

Check dimensions: L^2/T $1/T$ T

so: t=d/v = 20/60 = <u>1/3 hr or 20 minutes.</u>

Measurements

measurement: the act or result of measuring

Example: use a plastic ruler to measure a shoe's length
to be $L=12.0\pm0.1$ inches.
Example: use a Vernier scale to measure the same shoe length
to be $L=12.13\pm0.04$ inches.
Notice:
- A measurement consists of a *number*, an *error* (or uncertainty, or tolerance), and a *unit*. 3 things!
- The number of significant digits shown is related to the error.
- The caliper is more *precise* than the ruler.
- We did not determine which measurement device is more *accurate*.

Measurements

Accuracy and precision

i. <u>accuracy:</u> how close the measurement is to some accepted "true" value

ii. <u>precision:</u> how repeatable the measurement is with a given instrument and technique

Measurements -accuracy and precision

Example: two bathroom scales.
Step on and off them repeatedly in a consistent way.

digital scale	analog (yellow) scale
155.1 lbs	150. lbs
155.0	148
155.1	149
155.2	149
155.3	151

Q: Which scale has the greater “spread” in values?
Q: Which scale is more precise?
Q: Which scale is most accurate?

You go to the doctors office and they tell you 149.2 lbs.

Q: Which scale is most accurate?
Q: Which scale is more precise?

Measurements

Significant figures or (significant digits)
-- a way of suggesting precision.

significant figure: any digit of a number that is known with some certainty. The least significant digit (LSD) is the rightmost signif-igant digit and it is least certain.
Count the “sig figs”:

Examples:

1) 4,567,000 4
2) 4.567 0 5
3) 4,567,$\bar{0}$00 6
4) 4,567,000. 7
5) 0.03450 4
6) 30.003 5

Notes:

1. The digit left of a decimal point is significant for numbers greater than 1. (Ex. 4 & 5)
2. Errors should have 1 significant figure.
3. For homework after week 1, answers with 3 - 4 significant digits are ok.
4. The weights from the yellow scale should not be quoted to more than the 1's place.

Which digit is the LSD for each of the above?
Which place is occupied by the LSD in the above?

Measurements

Error (uncertainty, tolerance)
-- the best way to quantify precision.

How do you determine the error on a measurement?

a) From the number of significant figures?
Not good. There is NO universally accepted rule for deriving errors from significant digits.
Ex.) engineers say 32.4 means 32.4+-0.05

b) By looking at the smallest "tickmarks" on your instrument.
Error ("tolerance") is usually ½ of the tickmark spacing.

c) By considering how difficult it is to use the instrument.
Ex. using a stopwatch.

<u>d) By repeating the measurement many times and finding the spread of measurements. ("standard deviation") BEST!</u>

Mistake,
not error.

Measurements

Errors types of errors

random errors, instrumental errors, tolerance
- related to the precision of the measurement

systematic errors
- related to the accuracy of the measurement
- an effect that shifts all measurements in the same direction.

Ex) You use the previous yellow scale to weigh yourself.
It's zeropoint can be adjusted!

Ex) You are measuring the volume of an air-filled ball.
Answer will change depending on the pressure
and temperature inside and outside of the balloon.

Ex) You are measuring a length with a ruler.
* parallax * worn down ends * non-perpendicularity
* cheep rulers have bad tickmarks * lengths change w/T

Measurements

Errors ways of mathematically expressing errors

absolute errors

-- 155 +- 8 lbs has an absolute error of 8 lbs

fractional errors

-- 155 +- 8 lbs has a fractional error of 0.052

percentage errors

-- 155 +- 8 lbs has a percentage error of 5.2%

"Propagation"

Measurements

Error Propagation

How do you figure out the error for a number that was calculated from several measurements? (Append. B.8)

I. If only significant figures are shown:

a) Addition and subtraction: the final answer should have its LSD in the same place as the least precise input measurement

Ex) 5800 m + 121 m = 5900 m

Ex) 612800 s + 2011.5 s = 614,800 s

Ex) 220. - 115 = 105

b) Multiplication and division: the final answer should have the same number of sig figs as the input number with the fewest sig. figs.

Ex) 2000 x 15.143 = 30,000

Ex) 382,500 x 11. = 4,200,000 (not 4,207,500)

Ex) 520 / 3 = 200 (not 173.3)

Measurements

Error Propagation - cont.

II. If errors are explicitly shown

a) Addition and subtraction:

1) simple way: add error

Ex) 580.±2 m + 121 ±3 m = 701.±5

(This is an overestimate.)

2) correct way: add errors "in quadrature"

Ex) 580.±2 m + 121+-3m = 701.±e

where e= $\sqrt{(2)^2+(3)^2}=\sqrt{13}=3.61$

b) Multiplication and division:

1) simple way: "adding the fractional errors"

Ex) Hwk Problems 30-34 ...

2) correct way: add fractional errors in quadrature.

(We will use the Simple way instead.)

Note: the LSD of the answer must match the LSD of the error!

Note: the number of sig figs in the final answer does not have to be the same as the least precise input number, ala prev slide.

Measurements

Errors and statistics

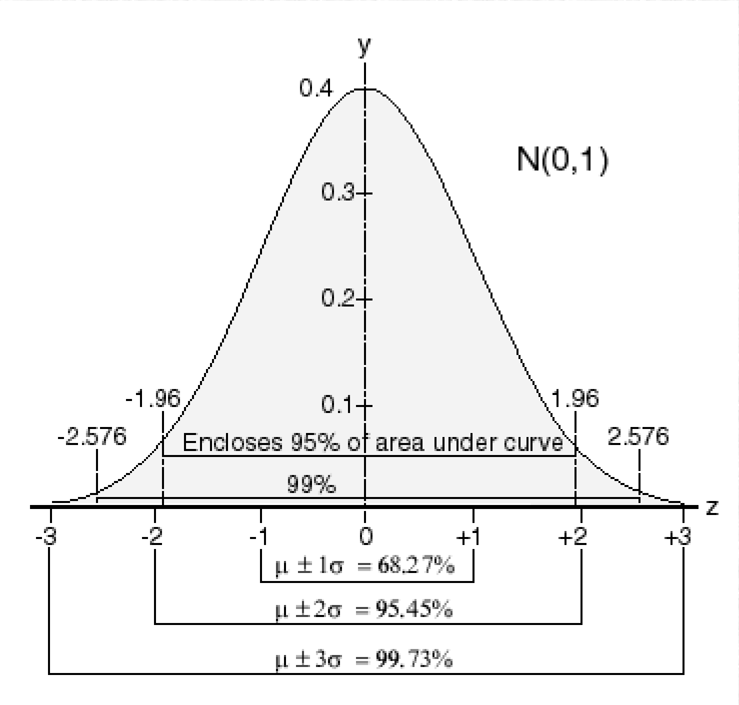
Mean $\mu = \frac{\sum x_i}{N}$

Standard Deviation $\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{(N-1)}}$

Standard Deviation of the mean

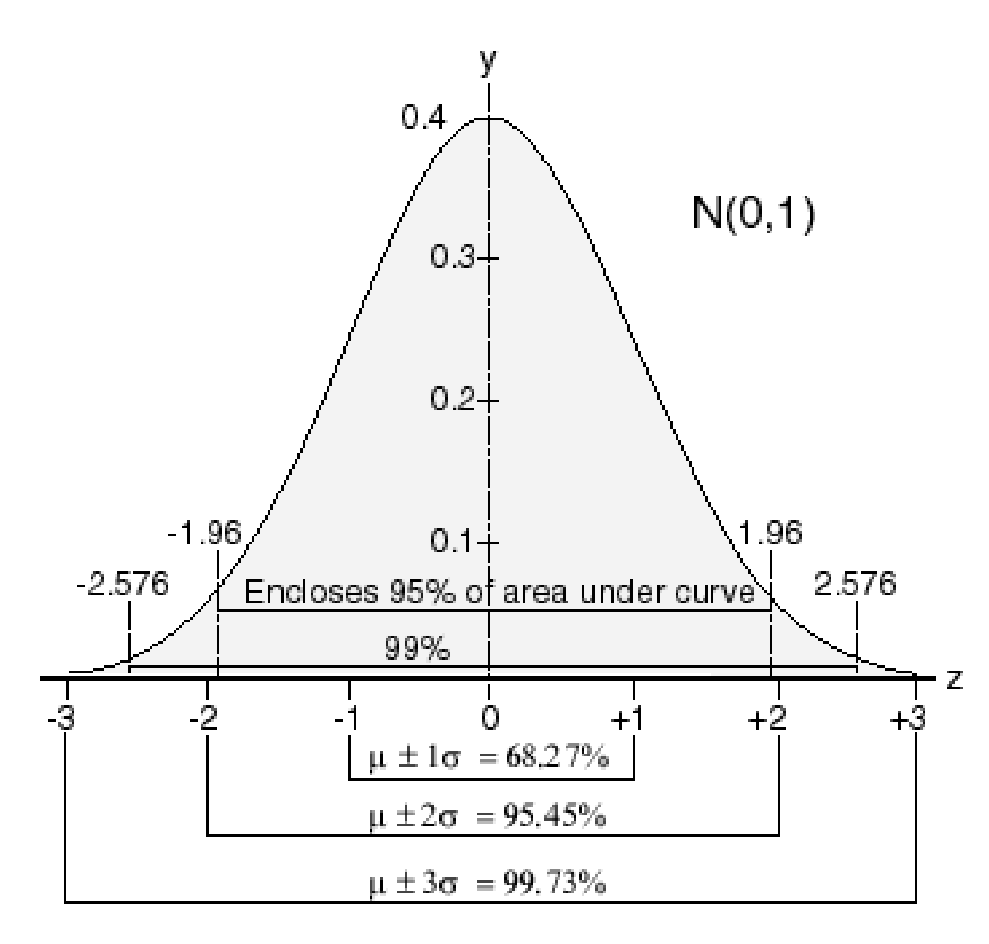
$$\sigma_\mu = \frac{\sigma}{\sqrt{N}}$$

Normal or Gaussian distribution



Measurements

Errors and statistics



IMPORTANT CONCEPT:
The Gaussian distribution can be interpreted as a <u>probability</u> distribution.

Ex) You measure a mean of 10000 weights to be 70.0 lbs with a standard deviation of σ = 10.0 lbs. If the weights are normally distributed, what is the probability that a single, new measurement will have a value greater than 90 lbs?

90-70 = 20 lbs

20 lbs = 2*10 = 2*σ

Area under curve between z=2*σ and z=+∞ is (100%-95.45%)/2 =2.275%=Ans.

Ex) What is probability that a single new measurement will be 50 or lower?

Ans=2.275%

Ex) What is the probability that a single new measurement will be within 70+-10 lbs? Ans=68.27%.