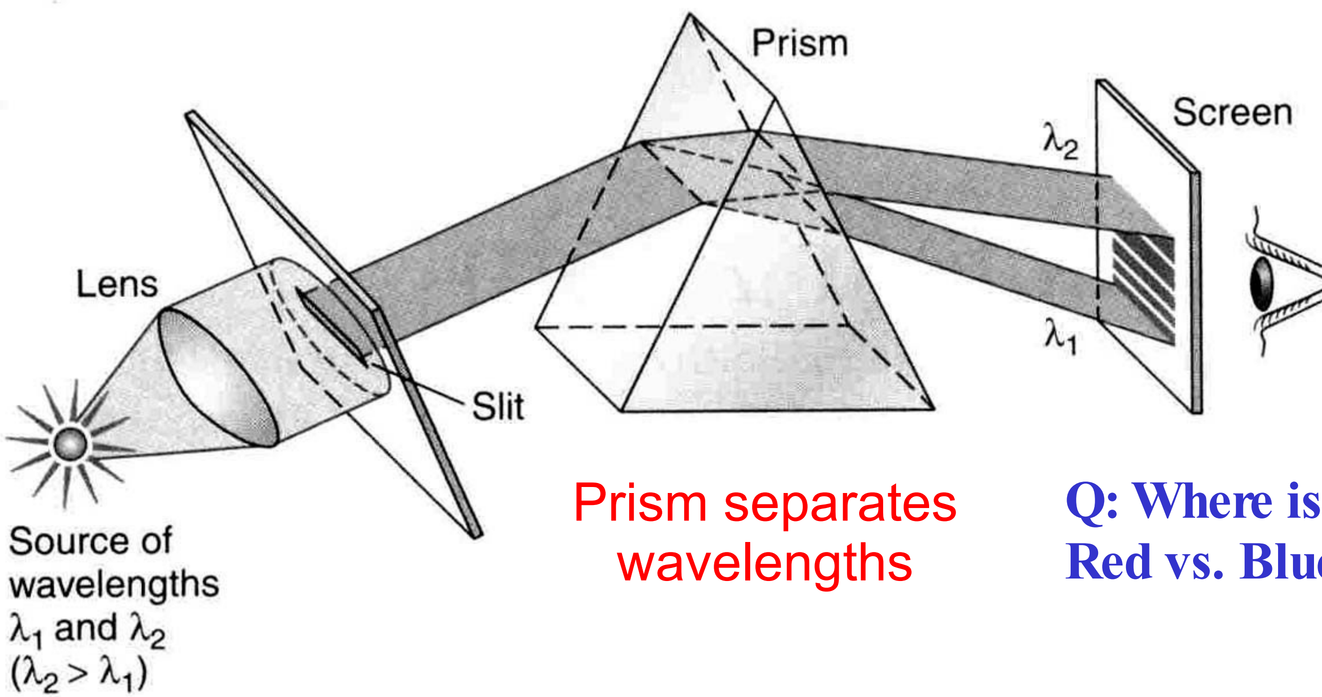
The Interaction of Light and Matter

Outline

- **(1) Motivation: Why spectral lines?**
  - **the Birth of Spectroscopy**
  - **Kirchoff's Laws**
- (2) **Photons – the particle nature of light**
  - **Blackbody radiation (Planck introduces quantum of light)**
  - **Photoelectric Effect**
  - **Compton Scattering**
- **(3) The Bohr Model of the Atom**
  - **a theory to describe spectral lines,**
- **(4) Quantum Mechanics and the Wave-Particle Duality (SKIP on ExamI)**
  - **De Broglie wavelength**
  - **Schrodinger's probability waves.**

Spectroscopy - history

- **Trogg (50 million BC) – rainbow**
- **Newton (1642-1727) – decomposes light into spectrum and back again**
- **W. Herschel (1800) – discovers infrared**
- **J. W. Ritter (1801) – discovers ultraviolet**
- **W. Wollaston (1802) – discovers absorption lines in solar spectrum**



Prism separates wavelengths

Q: Where is Red vs. Blue?

Spectroscopy - history

- **J. Herschel, Wheatstone, Alter, Talbot and Angstrom studied spectra of terrestrial things (flames, arcs and sparks) ~1810**
- **Joseph Fraunhofer**
  - **Cataloged ~475 dark lines of the solar spectrum by 1814**
  - **Identifies sodium in the Sun from flame spectra in the lab!**
  - **Looks at other stars (connects telescope to spectroscope)**
- **Foucault (1848) – sees absorption lines in sodium flame with bright arc behind it.**

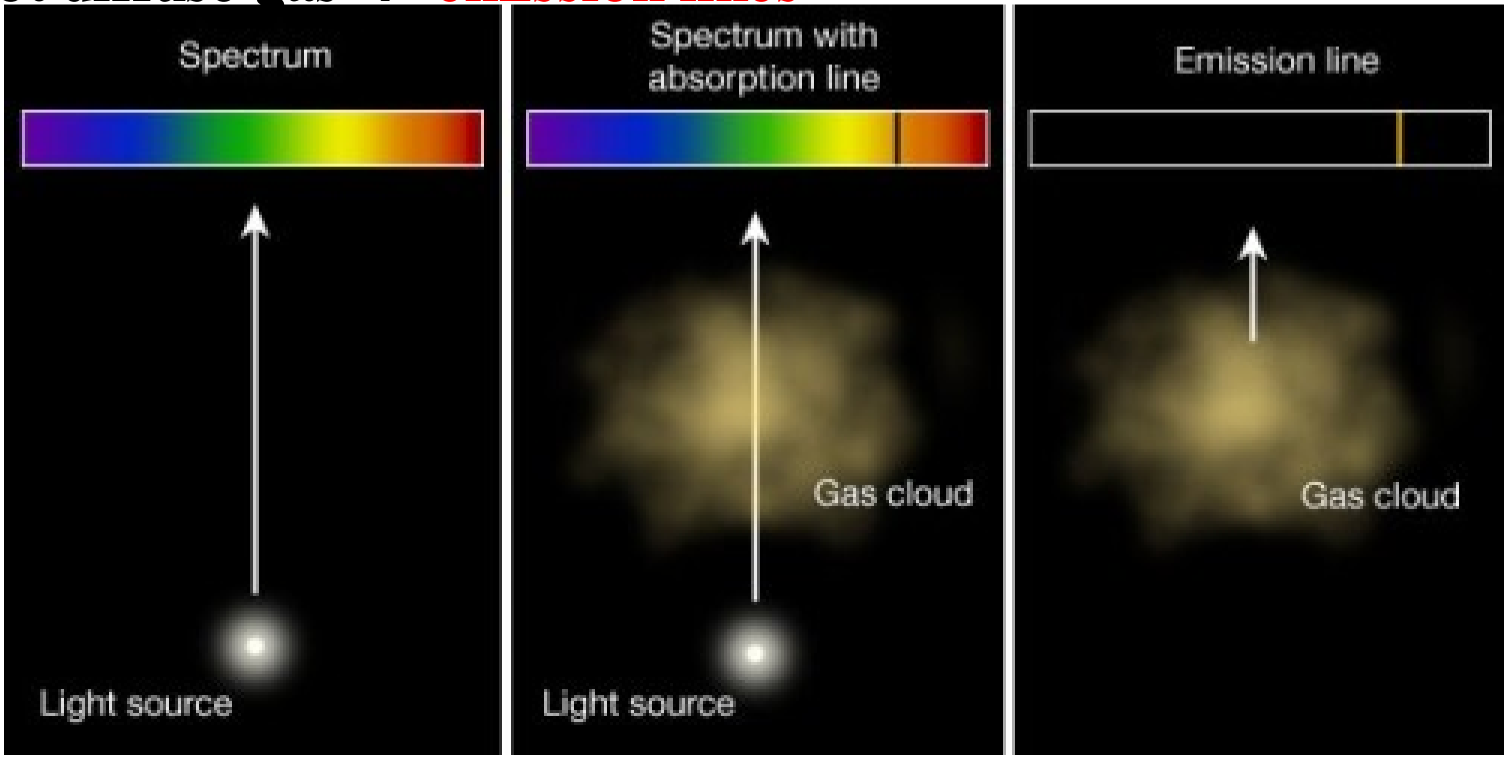
There was no accepted explanation for the absorption lines. *New physics* needed*!*

Kirchhoff's laws (1859):

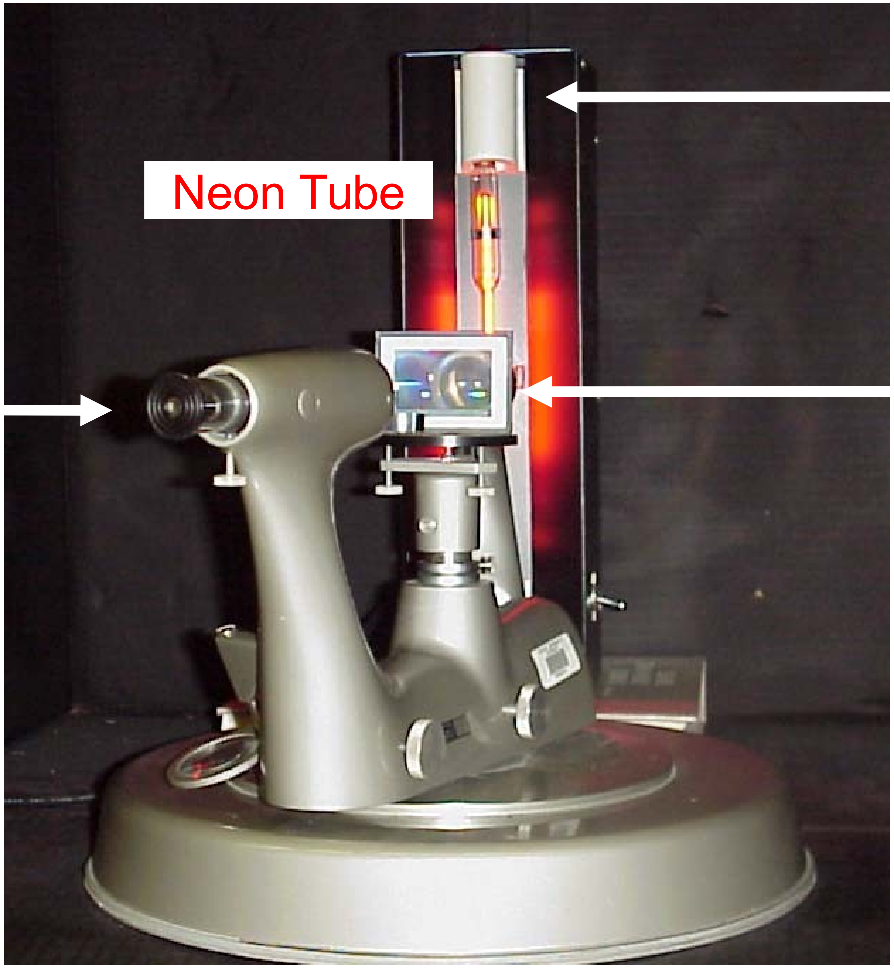
Kirchhoff worked with Bunsen on flame spectra

Developed a prism spectroscope

- **Hot solid or dense gas, → continuous spectrum (eg Blackbody)**
- **Cool diffuse gas in front of a blackbody → absorption lines**
- **Hot diffuse gas → emission lines**



Spectroscope for typical atomic physics lab



High Voltage Supply

Diffraction Grating

Eyepiece

$$d\,\mathbf{sin}\,\theta = n\lambda$$

Doppler shift (see also Ch. 4)

- **Spectral lines allow for the measurement of radial velocities**
- **At low velocities, $v_r << c$**
  - **Classical Doppler effect**
    - ***Radial velocity*, v_r**
    - ***Heliocentric correction* for Earth's motion, up to 29.8 km/s, depending on direction.**

$$\frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rst}} = \frac{\Delta\lambda}{\lambda_{rest}} = \frac{v_r}{c}$$

$$\Delta\lambda = \frac{v_r}{c}\lambda_{rest}$$

$$v_r = c\frac{\Delta\lambda}{\lambda_{rest}} = \mathbf{-14}\,\frac{\text{km}}{\textbf{sec}}$$

- **Example: H_α is 6562.80 Å**
  - **Vega is measured to be 6562.50 Å**
  - **Coupled with the *proper motion***
    - **Can determine total velocity**

$$v_\theta = r\mu = \mathbf{13}\,\frac{\text{km}}{\text{s}}$$

$$v = \sqrt{v_r^2 + v_\vartheta^2} = \mathbf{19}\,\frac{\text{km}}{\text{s}}$$

Doppler shift

- **Since most galaxies are moving away, astronomers call the Doppler shift a *redshift*, *z*.**

$$z = \frac{\Delta\lambda}{\lambda_{rest}}$$

- **At high velocities, v_r <~ c**
  - **Relativistic redshift parameter (Ch. 4):**

$$z = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} - 1$$

  - **Example: Prob. 4.8.**
    (should get: $v_r = 0.9337c$)

Particle/Wave Duality - Part 1

PART 1

- ***Electrons*** as discrete ***Particles***
  - **Measurement of** e/m **(CRT) and** e **(oil-drop expt.)**
- ***Photons*** as discrete ***Particles***
  - Blackbody Radiation**: Temp. Relations & Spectral Distribution**
  - Photoelectric **Effect: Photon "kicks out" Electron**
  - Compton **Effect: Photon "scatters" off Electron**

PART 2

- ***Wave Behavior:*** Diffraction and Interference
- ***Photons*** as ***Waves***: $\lambda = hc / E$
  - **X-ray Diffraction (Bragg's Law)**
- ***Electrons*** as ***Waves***: $\lambda = h / p$
  - **Low-Energy Electron Diffraction (LEED)**

Photons: Quantized Energy Particle

- Light comes in discrete energy "packets" called photons

Energy of Single Photon

$$E = h\nu = \frac{hc}{\lambda}$$

From Relativity:

$$E^2 = (pc)^2 + (mc^2)^2$$

Rest mass

For a Photon (m = 0):

$$E^2 = (pc)^2 + 0 \Rightarrow \boxed{E = pc}$$

Momentum of Single Photon

$$p = \frac{E}{c} = \frac{hc}{\lambda c} = \frac{h}{\lambda}$$

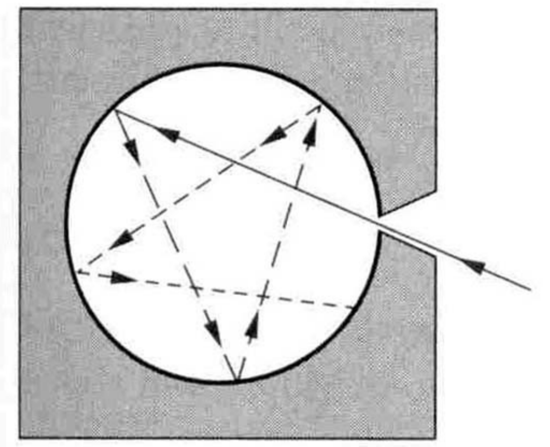
Blackbody Radiation: First clues to quantization

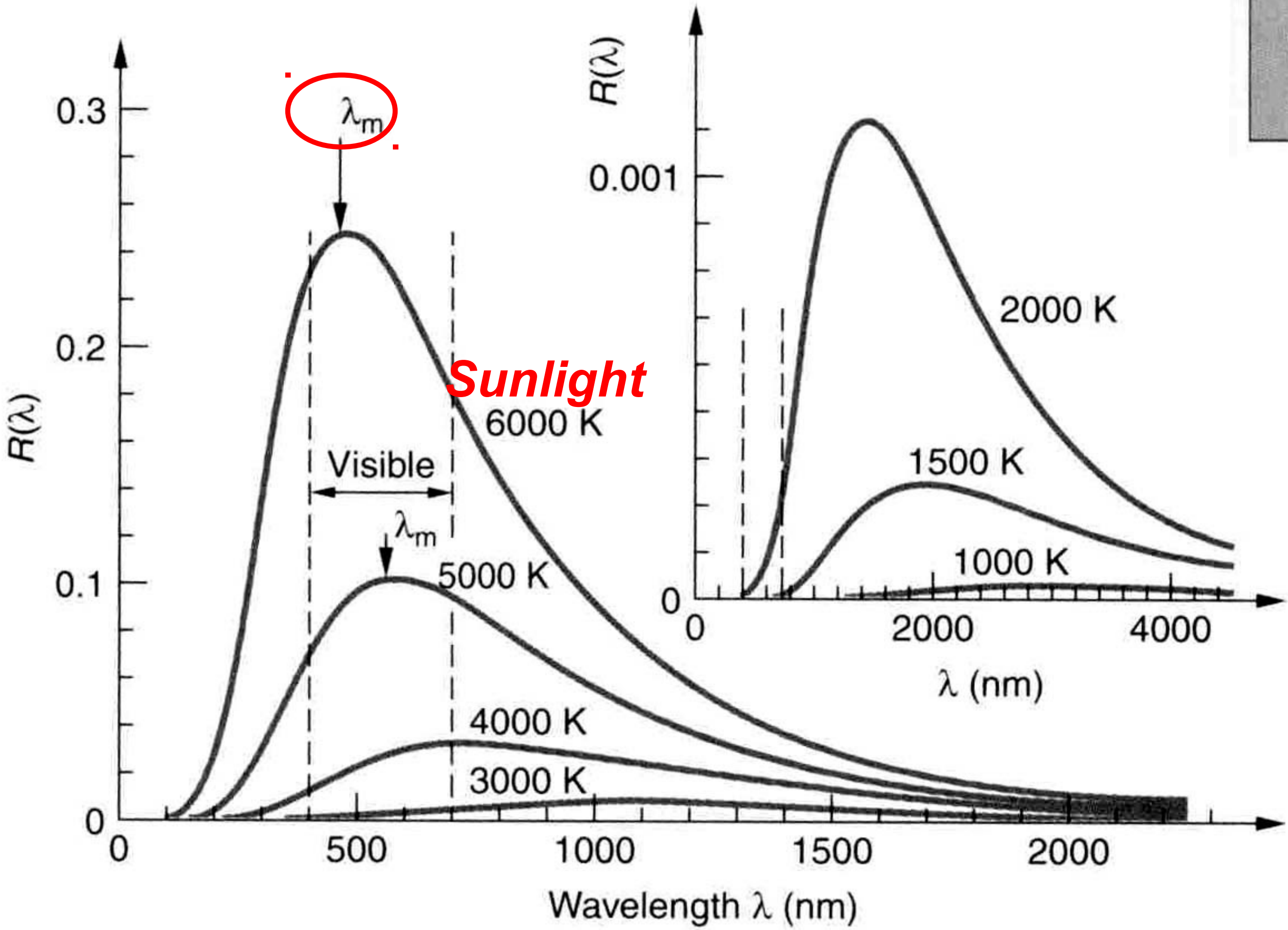
Recall Wien's Law:

$$\lambda_{max} = \frac{0.029}{T}\,\text{cm}\cdot\text{K}$$

and the Stefan-Boltzmann Law:

$$F = \sigma T^4$$





Spectral Distribution depends only on Temperature

Blackbody Radiation: Rayleigh-Jeans Equation

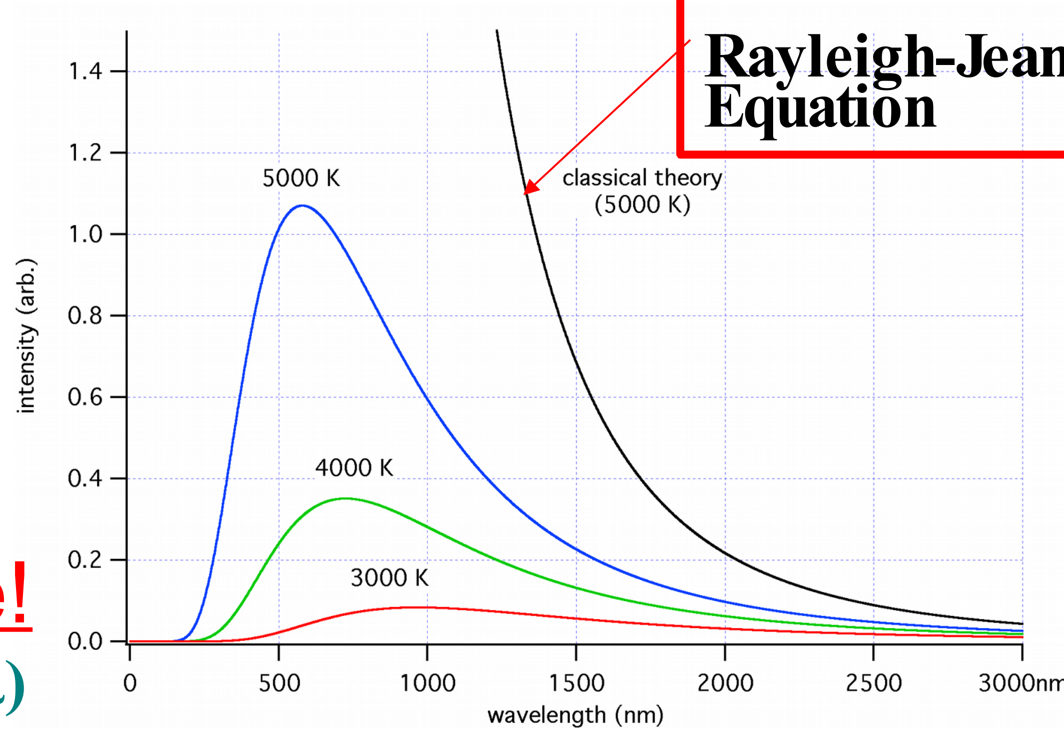
Classical physics led to a prediction for the spectrum of cavity (blackbody) radiation whereby

$$B_\lambda(T) = \frac{2ckT}{\lambda^4}$$

This was derived by assuming each mode of oscillation in the cavity would have an energy $E_{avg} = kT$ (k = 1.38 x 10^{-23} J/K is Boltzmann's constant)

The number of modes per wavelength interval increased as lambda decreased leading to excess energy at small λ s.

Rayleigh-Jeans Equation



UV Catastrophe!

(B explodes for small λ)

Spectral Blackbody: Planck's Law

- Planck's Law was found empirically (trial and error!)
- Quantize the E&M radiation so that the minimum energy for light at a given wavelength is:

$$E_\nu = h\nu = hc/\lambda$$

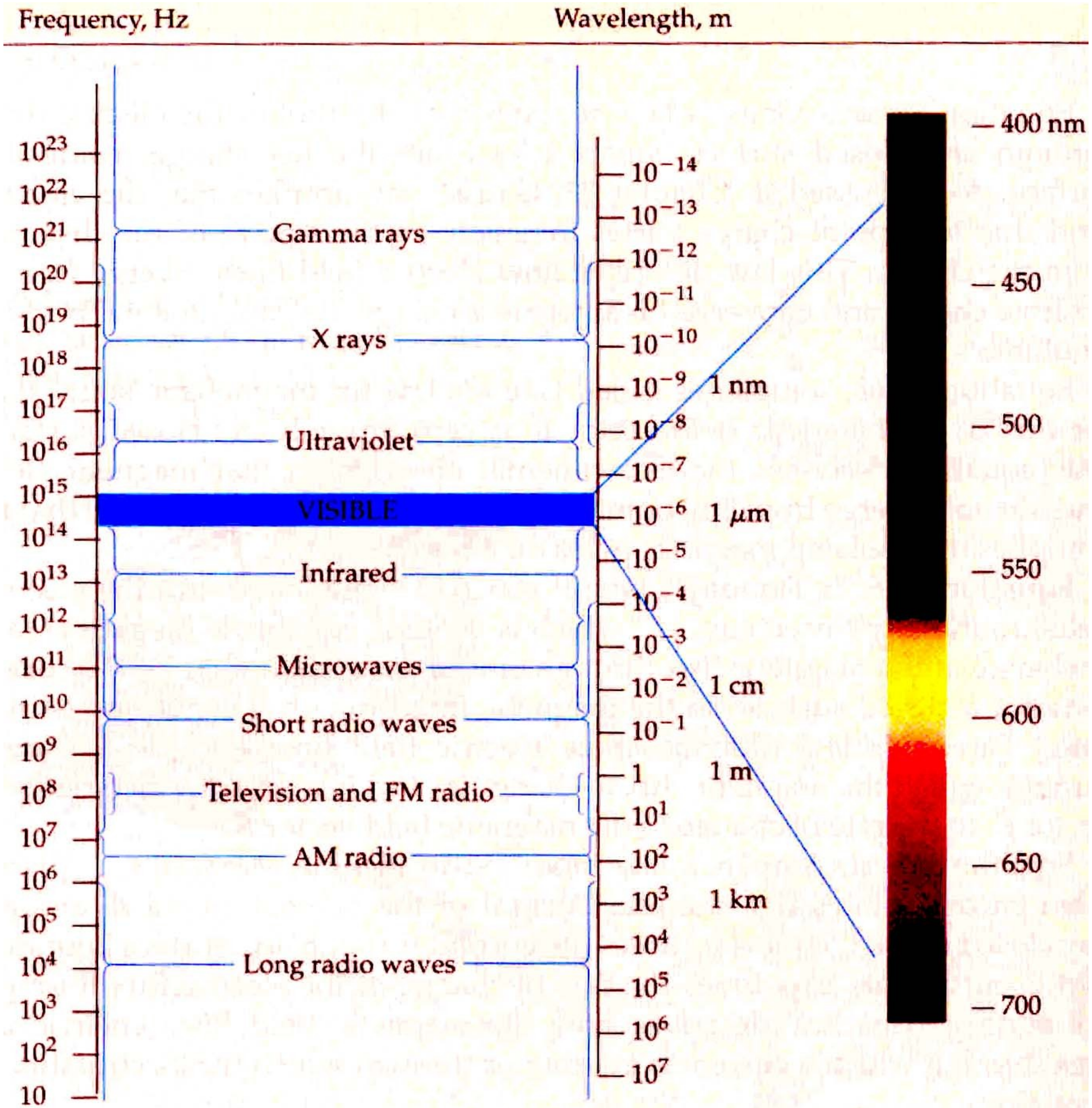
where h = Planck's Constant = 6.266 x 10^{-27} erg·s.

Then $E_\nu = nh\nu$, n= **0, 1, 2, 3**

can be used in replacing the classical *kT* expression for the average energy in a mode.

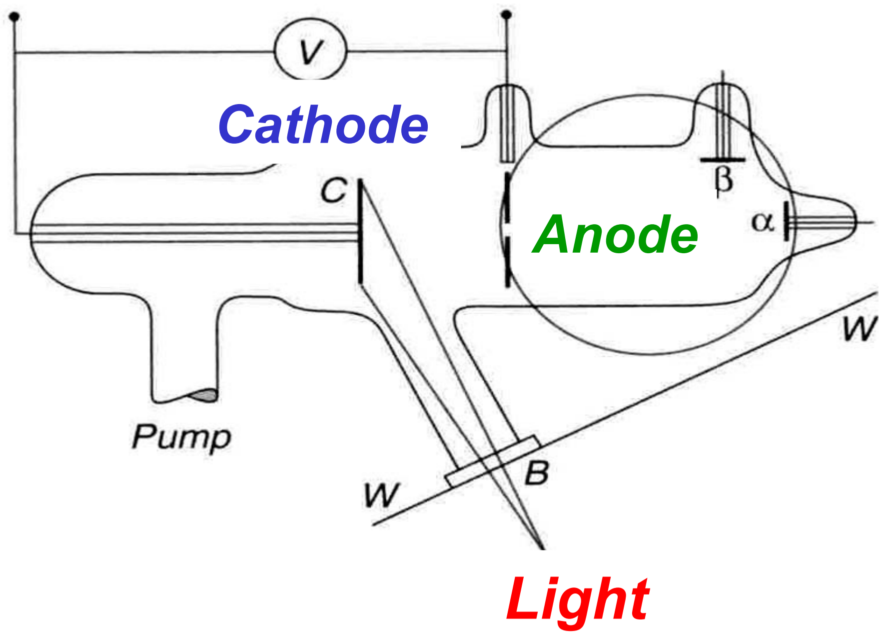
Now the entire hot object may not have enough energy to emit one photon of light at very small wavelengths, so *n=0*, and the UV catastrophe can be avoided.

Photons: Electromagnetic Spectrum



Photoelectric Effect: "Particle Behavior" of Photon

- **Shows quantum nature of light (Theory by Einstein & Expt. by Millikan).**
- Photons **hit metal cathode and instantaneously eject** electrons **(requires minimum energy = work function).**
- **Electrons travel from cathode to anode against** retarding voltage V_R
- **Electrons collected as** "photoelectric" current **at anode.**
- **Photocurrent becomes zero when retarding voltage V_R equals the** stopping voltage V_{stop}, **i.e.** $eV_{stop} = K_e$



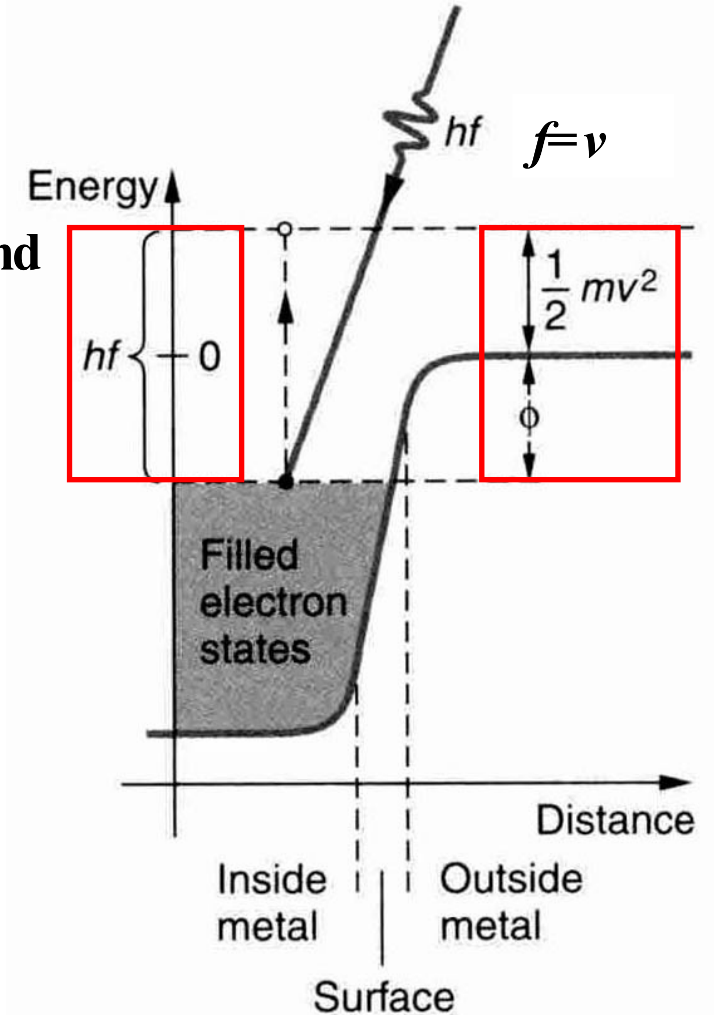
Photoelectric Effect - equation

- PHOTON IN ⇒ ELECTRON OUT
  - e– kinetic energy = Total photon energy – e– ejection energy

$$K_{max} = h\nu - \varphi$$

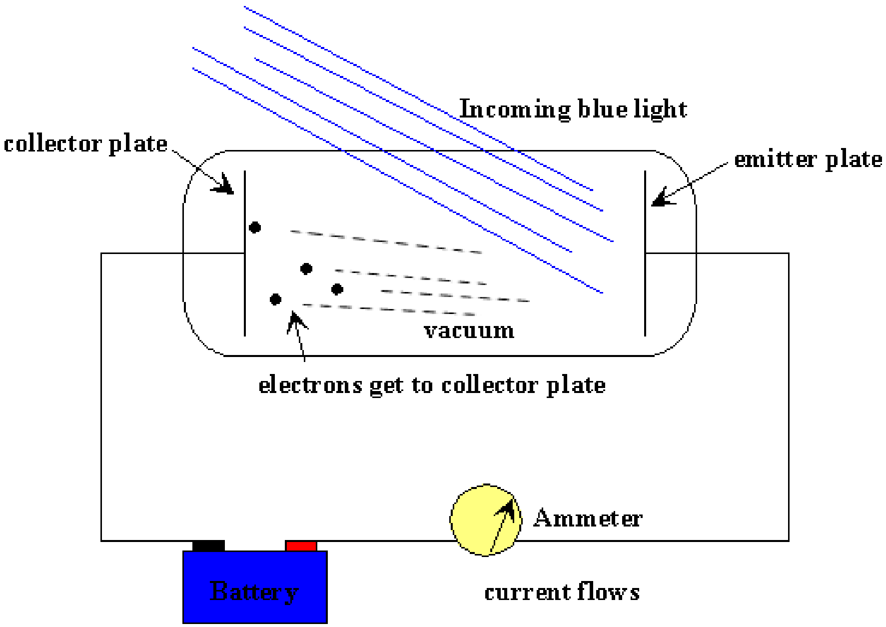
  - where *hν* = photon energy, ϕ = work function, and K_{max} = kinetic energy
  - $K_{max} = eV_{stop}$ = stopping energy
- Special Case: No kinetic energy ($V_o = 0$)
  - Minimum frequency *ν* to eject electron

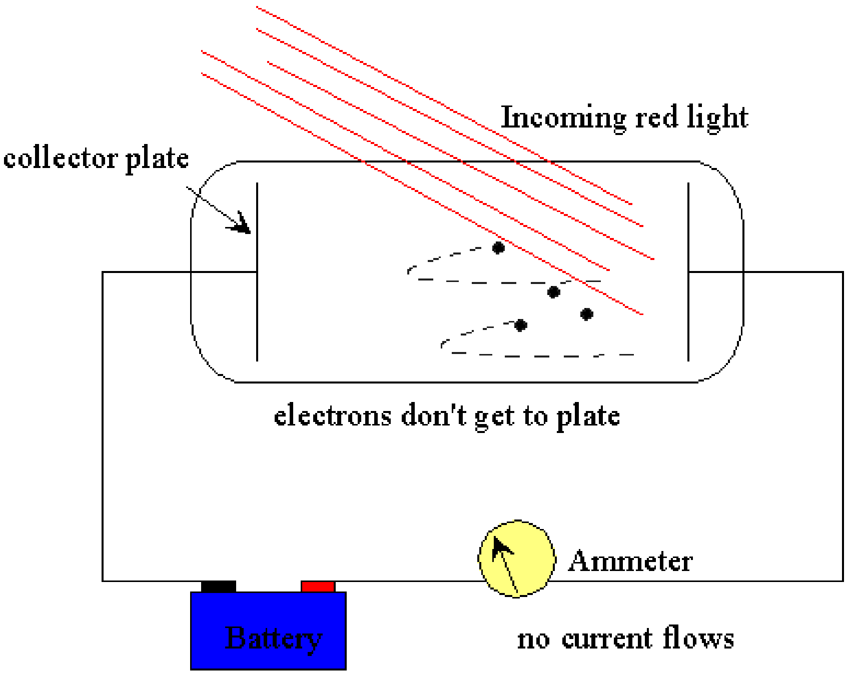
$$h\nu_{min} = \varphi$$



Photoelectric Effect

- In order to make electrons reach the collector plate, the light has to be “blue enough”; the intensity doesn't matter if light is red!

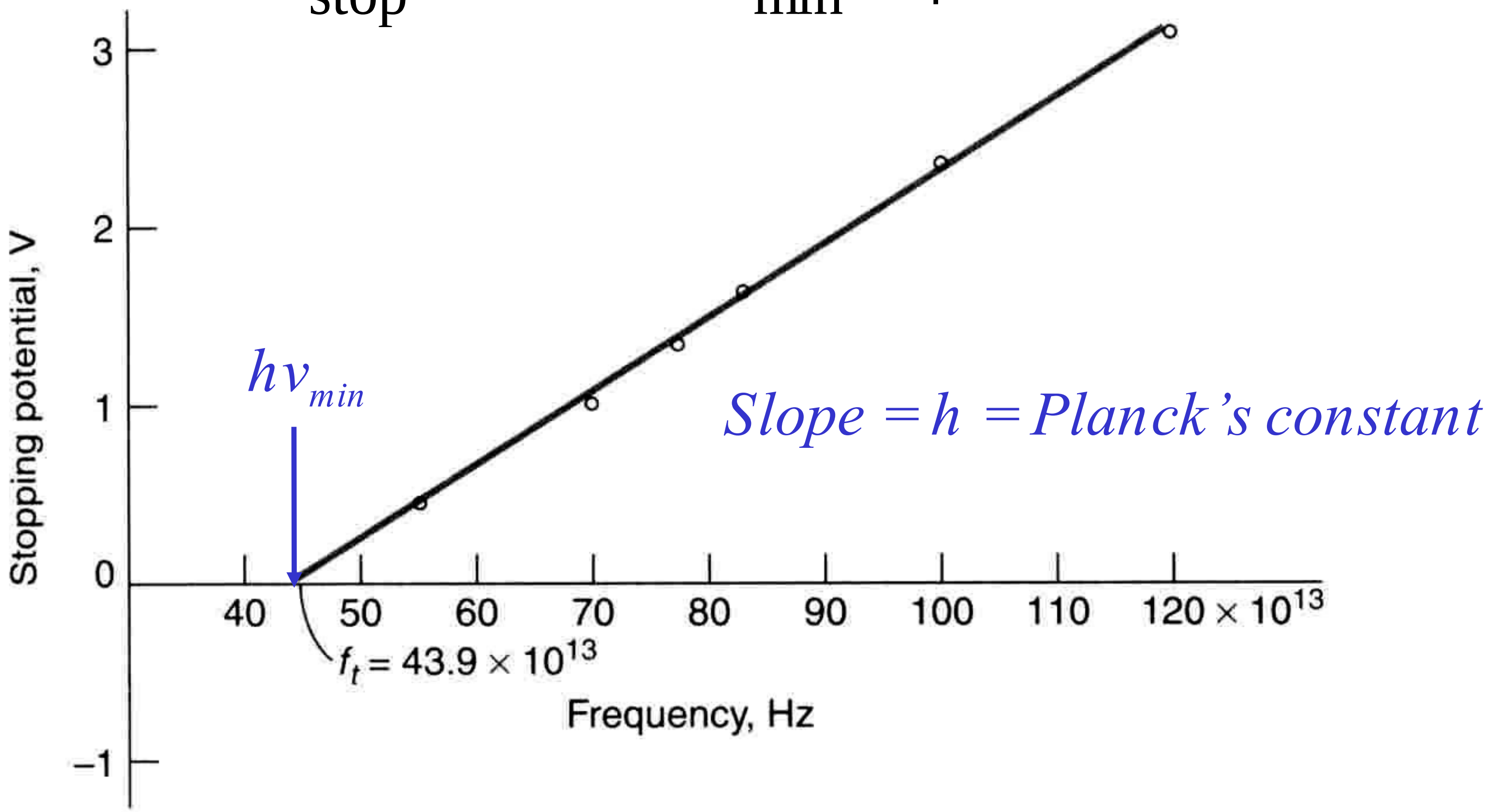




Photoelectric Effect: V_{stop} vs. Frequency

$$eV_{stop} = hv - \varphi$$

$$V_{stop} = \mathbf{0} \Rightarrow hv_{min} = \varphi$$



Photoelectric Effect Problem

If the work function of a metal is 2.0 eV,

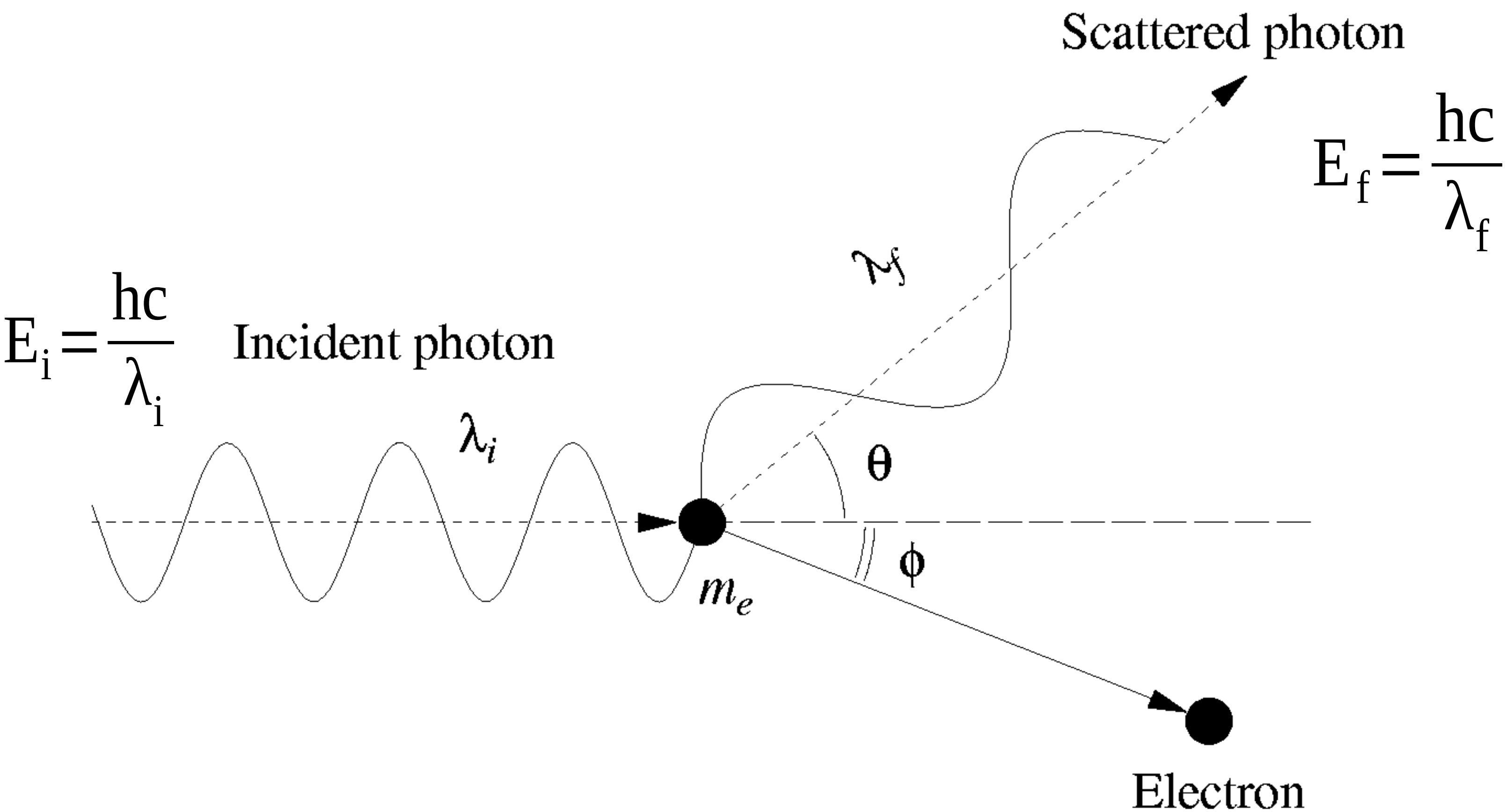
a) find the maximum wavelength λ_m capable of causing the photo-electric effect, and,

b) find the stopping potential if $\lambda = \lambda_m / 2$

Compton Scattering: “Particle-like” Behavior of Photon

Concept: Photon scatters off electron losing energy and momentum to the electron. The λ_f of scattered photon depends on θ

- **Conservation of relativistic momentum and Energy!**
- **No mass for the photon but it has momentum!!!**



Compton Scattering: Equation

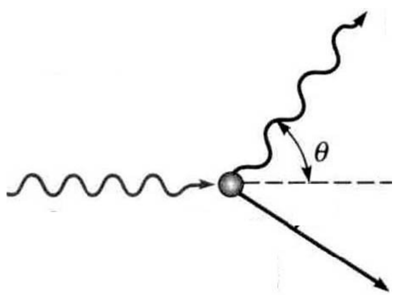
Photon OUT

Scattering Angle

$$\Delta\lambda = \lambda_f - \lambda_i = \underbrace{\frac{h}{m_e c}}\left(1 - \cos\theta\right)$$

Photon IN

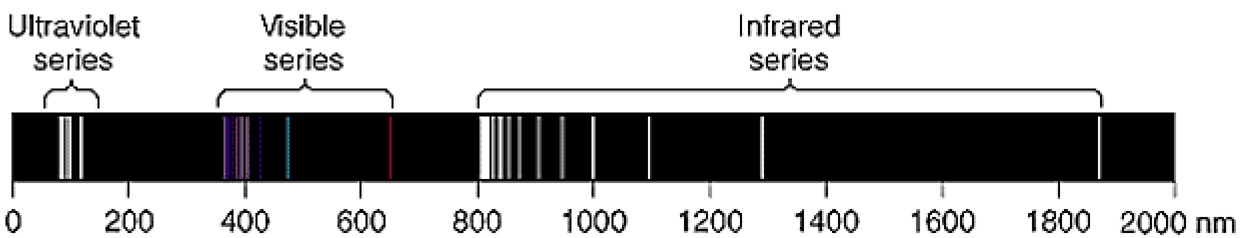
Critical λ_c = 0.0024 nm for e^-



- ***Limiting Values***
  - **No scattering:** $\theta = 0^o \rightarrow \cos 0^o = 1 \rightarrow \Delta\lambda = 0$
  - **"Bounce Back":** $\theta = 180^o \rightarrow \cos 180^o = -1 \rightarrow \Delta\lambda = 2\lambda_c$
- **Difficult to observe unless λ is small (i.e. $\Delta\lambda/\lambda > 0.01$)**

Atomic Spectra

- 1885 - Balmer observed Hydrogen Spectrum
  - **Found empirical formula for discrete wavelengths**
  - **Later generalized by Rydberg for simple ionized atoms**



$$\frac{1}{\lambda}=R_H\left(\frac{1}{2^2}-\frac{1}{n^2}\right)\ with\ 2<n$$

Atomic Spectra: Rydberg Formula

$$\frac{1}{\lambda} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \; with \; m < n$$

- Gives λ for any lower level *m* and upper level *n* of Hydrogren.
- Rydberg constant $R_H \sim 1.097 \times 10^7 \ m^{-1}$
- ***m*** = 1 (Lyman), 2 (Balmer), 3 (Paschen)

- Example for ***n = 2 to m = 1*** transition:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}\left(1.097 \times 10^7 \, \mathrm{m}^{-1}\right)$$

$$\Rightarrow \lambda = 121.6 \ \mathrm{nm \ Ultraviolet}$$

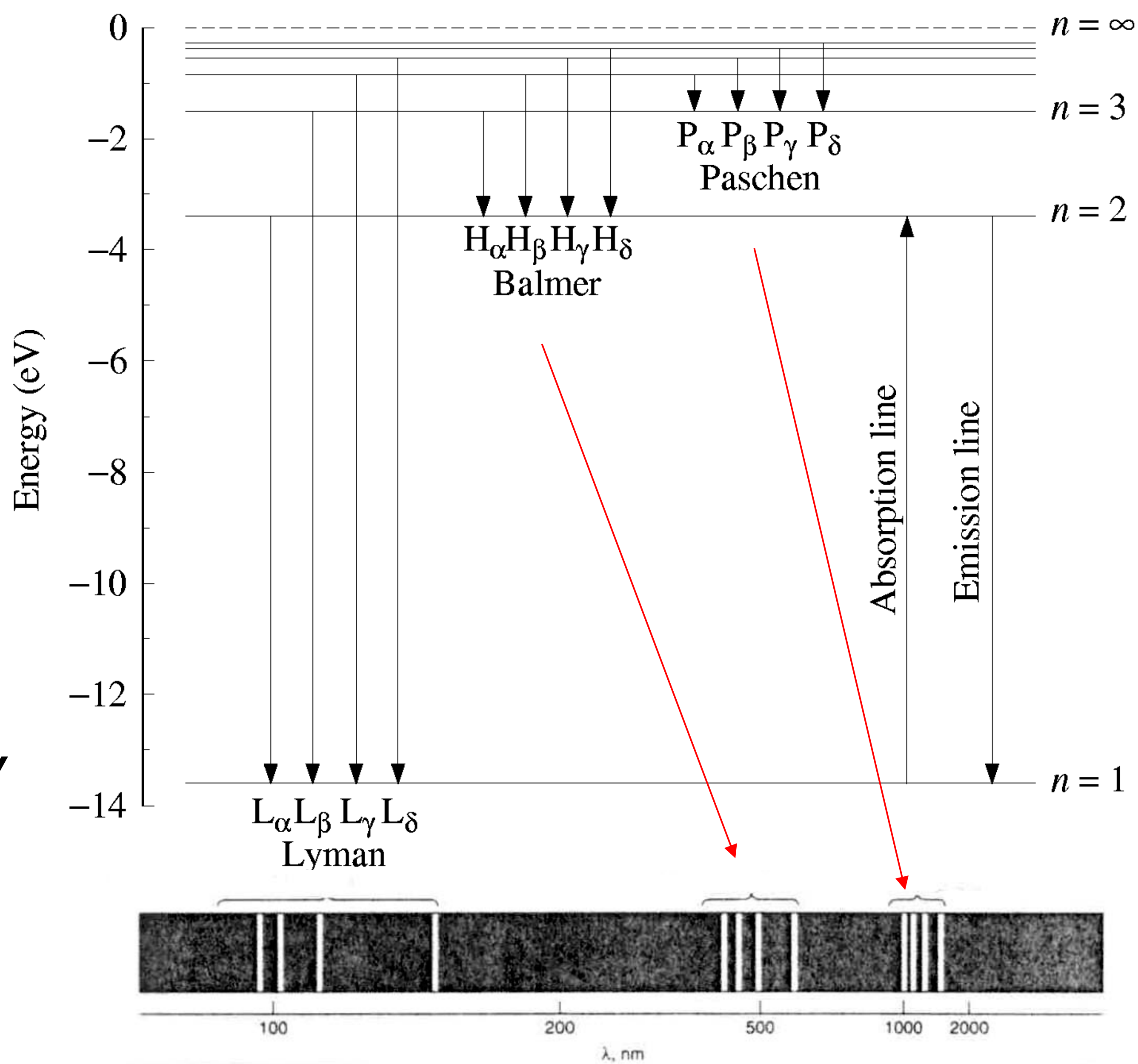
Atomic Spectra: Hydrogen Energy Levels

E_∞ = 0 eV

Energy

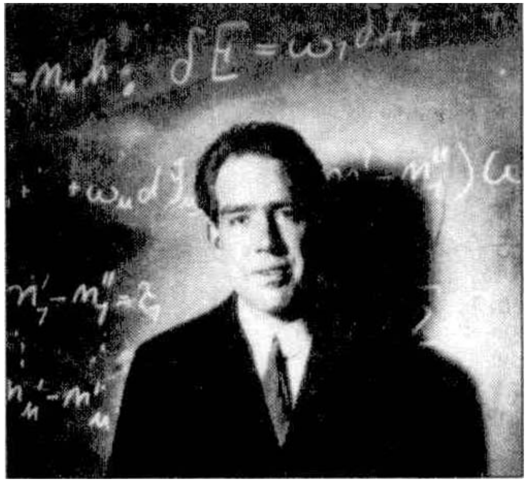
$$E_n \propto \frac{-1}{n^2}$$

E_1 = -13.6 eV



Bohr Model

- 1913 – Bohr proposed quantized model of the H atom to predict the observed spectrum.
- Problem: Classical model of the electron "orbiting" nucleus is unstable. Why unstable?
  - **Electron experiences (centripetal) acceleration.**
  - **Accelerated electron emits radiation.**
  - **Radiation leads to energy loss.**
  - **Electron quickly "crashes" into nucleus.**



Bohr Model: Quantization

- ***Solution:*** Bohr proposed two "quantum" postulates
  - **Electrons exist in stationary orbits (no radiation) with quantized angular momentum.**

$$L_n = mvr = n\hbar$$

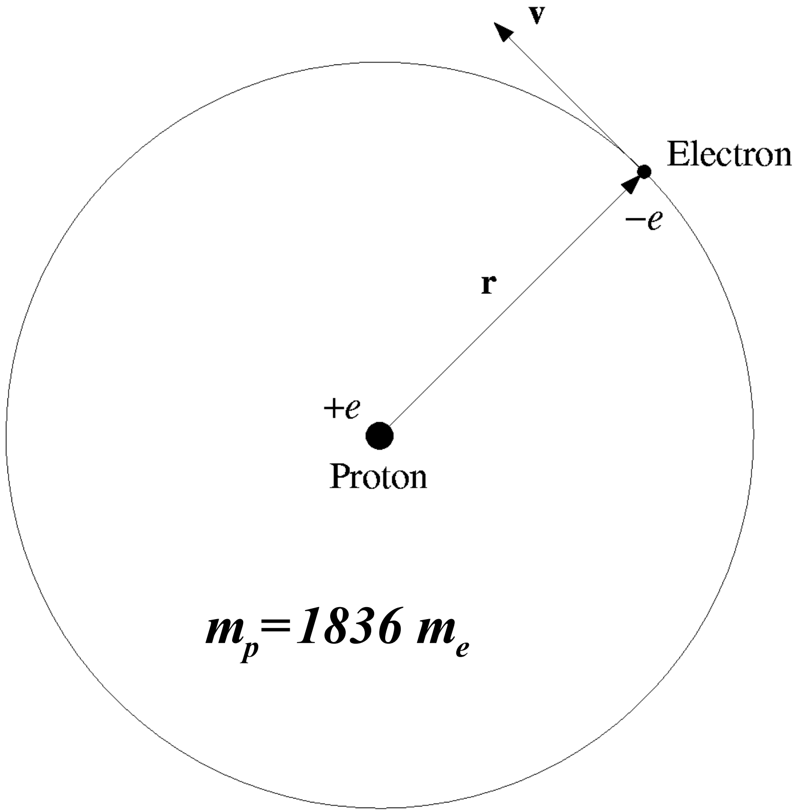
$$\left(\hbar = \frac{h}{2\pi} = 6.58\mathrm{X}10^{-16}\,\mathrm{eV\cdot s}\right)$$

  - **Atom radiates with quantized frequency *v* (or energy E) only when the electron makes a transition between two stationary states.**

$$hv = \frac{hc}{\lambda} = E_i - E_f$$

Planetary Mechanics Applied to the H Atom

- Consider the attractive electrostatic force and circular motion



$$\vec{F} = \frac{q_1 q_2}{r^2}\hat{r} = \mu\frac{v^2}{r}\hat{r}$$

Note: in cgs, *e* = *4.803x10^-10* esu

$$\frac{q_1 q_2}{r^2} = -\mu\frac{v^2}{r}$$

$$\frac{-e^2}{r^2} = -\mu\frac{v^2}{r}$$

$$\frac{1}{2}\mu v^2 = \frac{1}{2}\frac{e^2}{r} = K \qquad \textbf{Kinetic energy}$$

$$U = -2K = -\frac{e^2}{r} \qquad \textbf{Potential energy}$$

$$\mu = \frac{m_e m_p}{M} \approx m_e$$

$$M = m_e + m_p \approx m_p$$

Planetary Mechanics Applied to the H Atom

- **Introduce Bohr's quantized angular momentum** $L = \mu v r = n\hbar$ **(wrong)**

$$K = \frac{1}{2}\frac{e^2}{r} = \frac{1}{2}\mu v^2 = \frac{1}{2}\frac{(\mu v r)^2}{\mu r^2} = \frac{1}{2}\frac{(n\hbar)^2}{\mu r^2}$$

- **Solving for *r*** $$r_n = \frac{\hbar^2}{\mu e^2} n^2 = a_0 n^2$$ ***a_0* is the Bohr radius**

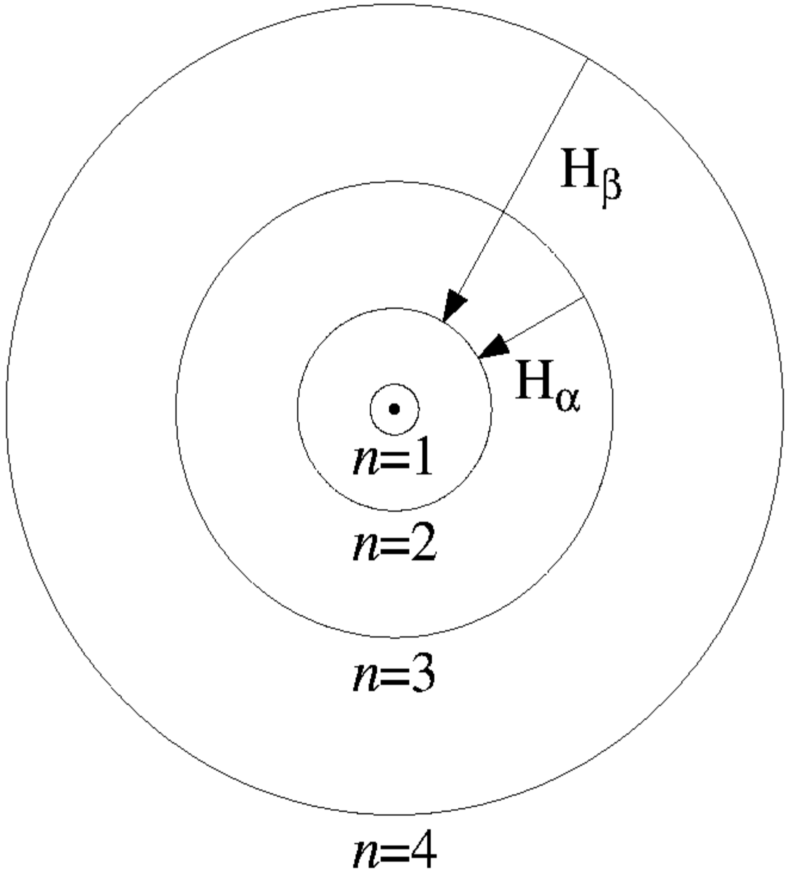
- **Get the Total Energy in terms of *n*.** **(Recall *E_{tot}=<U>/2*)**

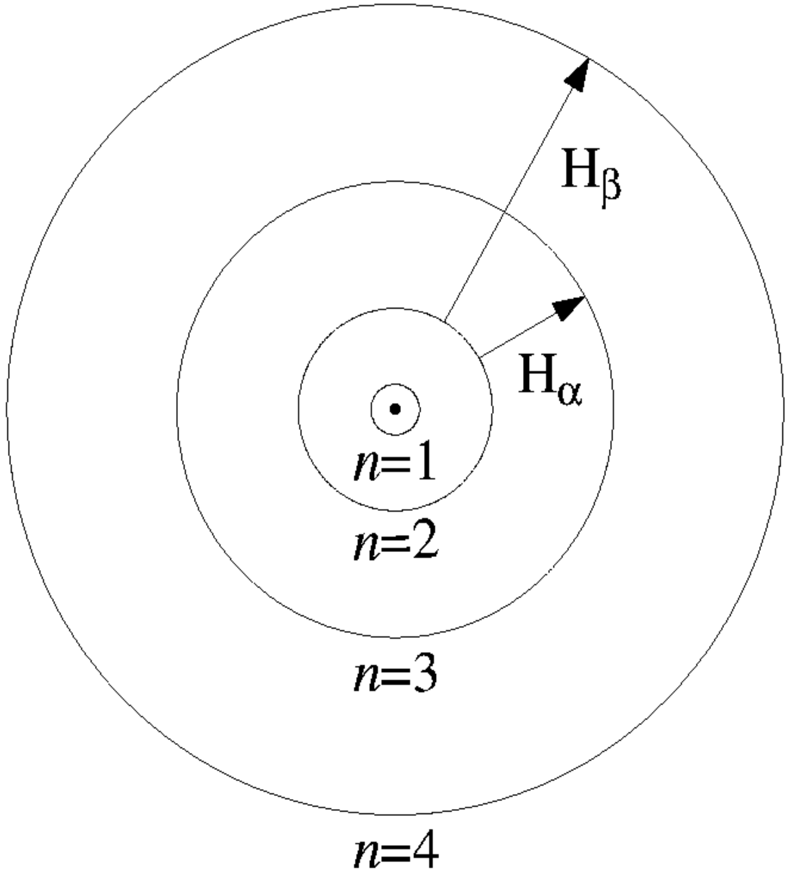
$$E_n = -\frac{1}{2}\frac{e^2}{r} = -\frac{\mu e^4}{2\hbar^2}\frac{1}{n^2} = \frac{-13.6\ \mathrm{eV}}{n^2} = \frac{-E_0}{n^2}$$

- **Principle quantum number, *n = 1, 2, 3, …***

Bohr Model: Transitions

- Transitions predicted by Bohr yield general Rydberg formula





Bohr Model Problem: Unknown Transition

If the wavelength of a transition in the Balmer series **for a** He^+ **atom is** 121 nm, **then find the corresponding transition, i.e. initial and final n values.**

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right) = R(2)^2\left(\frac{1}{(2)^2} - \frac{1}{n_i^{\,2}}\right)$$

where Z = 2 for He and n_f = 2 for Balmer

$$\frac{1}{4R\lambda} = \left(\frac{1}{4} - \frac{1}{n_i^{\,2}}\right)$$

$$n_i = \left(\frac{1}{4} - \frac{1}{4R\lambda}\right)^{-1/2} = \left(\frac{1}{4} - \frac{1}{4(1.1\times10^{7}\,m^{-1})(121\times10^{-9}\,m)}\right)^{-1/2} = \underline{4}$$

Bohr Model Problem: Ionization Energy

Suppose that a He atom (Z=2) in its ground state (n = 1) absorbs a photon whose wavelength is λ = 41.3 nm. Will the atom be ionized?

- ***Find the energy of the incoming photon and compare it to the ground state ionization energy of helium, or E_0 from n=1 to ∞.***

$$E = \frac{hc}{\lambda} = \frac{1240\ eV\ nm}{41.3\ nm} = \underline{30\ eV}$$

$$E_0(He) = Z^2 \times E_0(H) = \left(2^2\right)\left(13.6\ eV\right) = 54.4\ eV$$

- ***The photon energy (30 eV) is less than the ionization energy (54 eV), so the electron will NOT be ionized.***

Bohr Model Problem: Series Limit (book)

Find the shortest wavelength **that can be emitted by the** Li^{++} ion**.**

- ***The shortest λ (or highest energy) transition occurs for the highest initial state ($n_i = \infty$) to the lowest final state ($n_f = 1$).***

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right]$$

where Z = 3 for Li, $n_i = \infty$, and $n_f = 1$ for shortest λ

$$\frac{1}{\lambda} = (1.1\times 10^7\, m^{-1})(3)^2\left[\frac{1}{(1)^2} - \frac{1}{(\infty)^2}\right] = 10.1\ nm$$

Particle/Wave Duality - Part 2

PART 1

- ***Electrons*** as discrete ***Particles***
  - **Measurement of** e/m **(CRT) and** e **(oil-drop expt.)**
- ***Photons*** as discrete ***Particles***
  - Blackbody Radiation**: Temp. Relations & Spectral Distribution**
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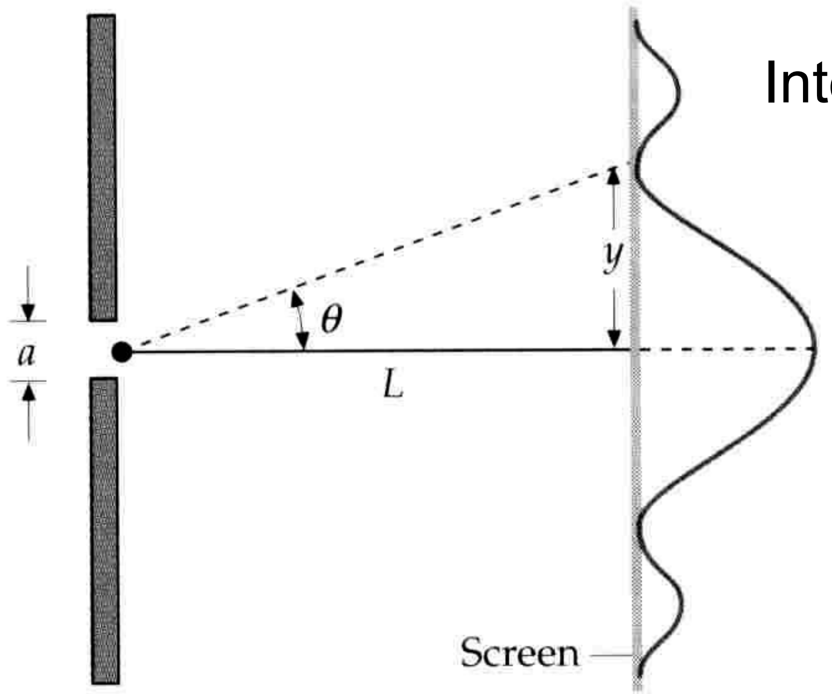
PART 2

- ***Wave Behavior:*** Diffraction and Interference
- ***Photons*** as ***Waves***: $\lambda = hc / E$
  - **X-ray Diffraction (Bragg's Law)**
- ***Electrons*** as ***Waves***: $\lambda = h / p$
  - **Low-Energy Electron Diffraction (LEED)**

Wave Property: Single-Slit Diffraction

$$\textbf{Minima:}\quad n\lambda = a\sin\theta$$

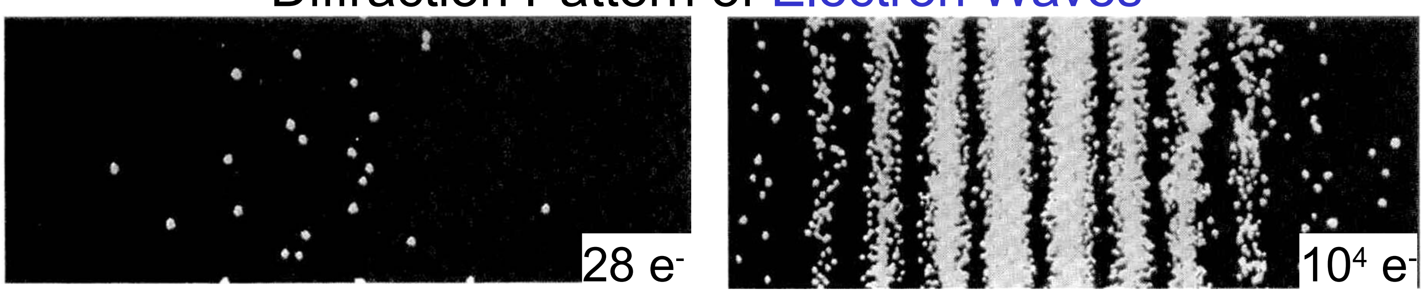
a = slit width



Intensity

$(Sin(x) / x)^2$
maxima

Diffraction Pattern of Electron Waves



Electrons: Wave-like Behavior

- Every particle has a wavelength given by: $$\lambda = \frac{h}{p}$$
- Question: **Why don't we observe effects of particle waves (i.e., diffraction and interference) in day-to-day life?**
- Answer: **Wavelengths of most macroscopic objects are too small to interact with slits, BUT atomic-sized objects DO behave like waves!**

Macroscopic – ping pong ball

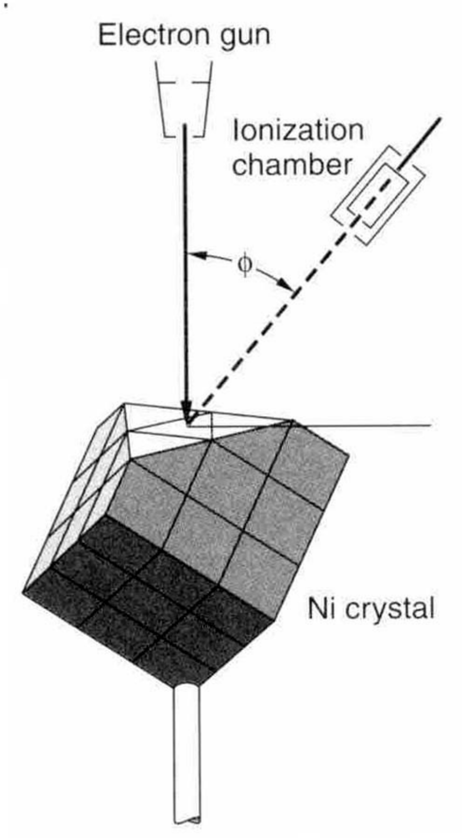
$$\lambda = \frac{h}{mv} = \frac{\mathbf{6.63\times10^{-34}}\,\mathrm{J\cdot s}}{\left(\mathbf{2\times10^{-3}}\,\mathrm{kg}\right)\left(\mathbf{5m/}\,\mathrm{s}\right)} = \mathbf{6.6}\times10^{-32}\,\mathrm{m}\ \left(\textbf{immeasurably small!}\right)$$

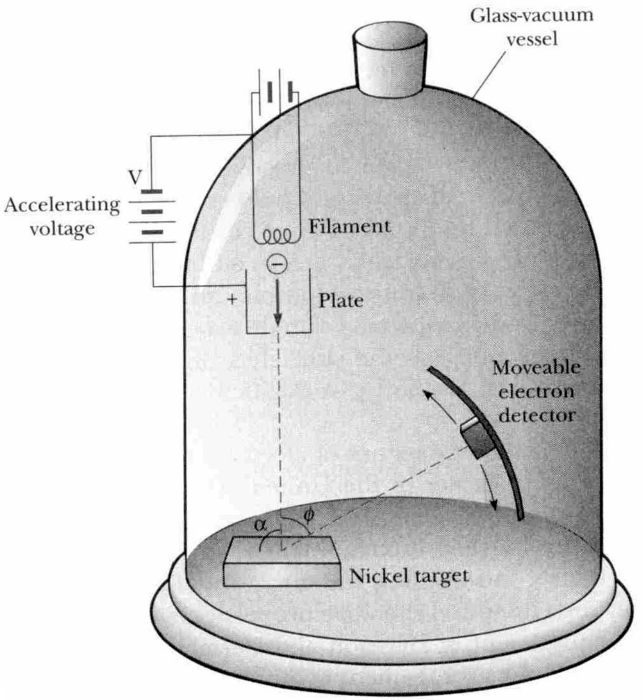
Microscopic – "slow electron" (1% speed of light)

$$\lambda = \frac{h}{mv} = \frac{\mathbf{6.63\times10^{-34}}\,\mathrm{J\cdot s}}{\left(\mathbf{9.1\times10^{-31}}\,\mathrm{kg}\right)\left(\mathbf{10^{6}}\,\mathrm{m/s}\right)} = \mathbf{7.3}\times10^{-10}\,\mathrm{m}\left(\textbf{atomic dimension}\right)$$

Electron Diffraction: Wave-like Behavior

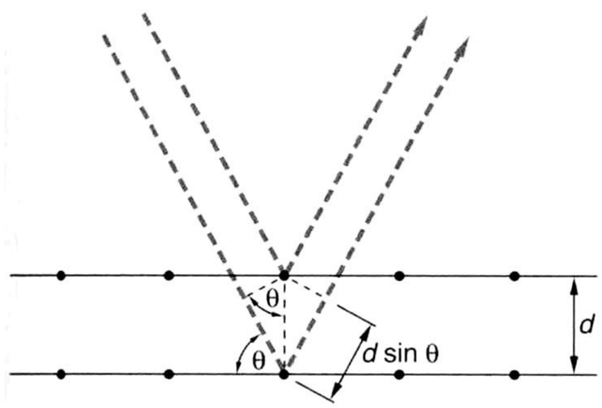
- 1927 – Davisson and Germer studied the diffraction of an electron beam from a nickel crystal surface and observed discrete spots (maxima).
- Modern day technique now: Low Energy Electron Diffraction ***(LEED).***

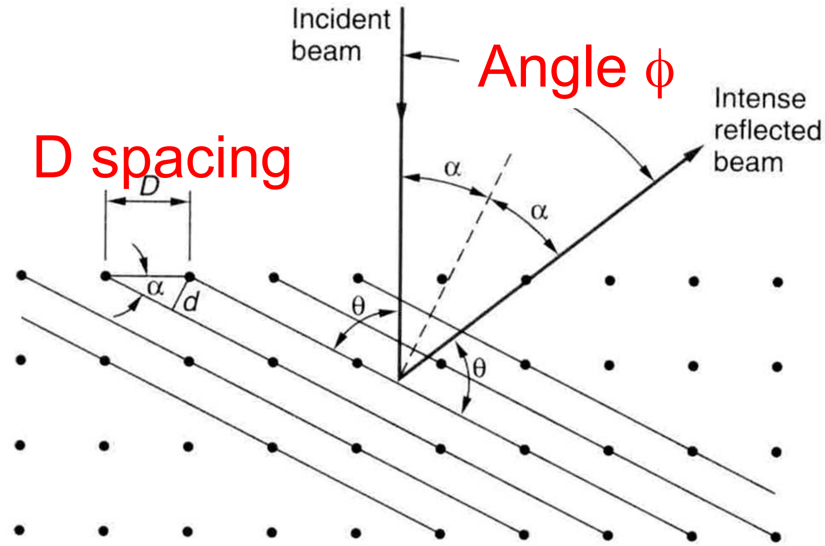




Electron Diffraction: LEED Equation

Concept: Use Bragg's Law for X-ray scattering and then substitute appropriate angles, where λ is now the electron wavelength.





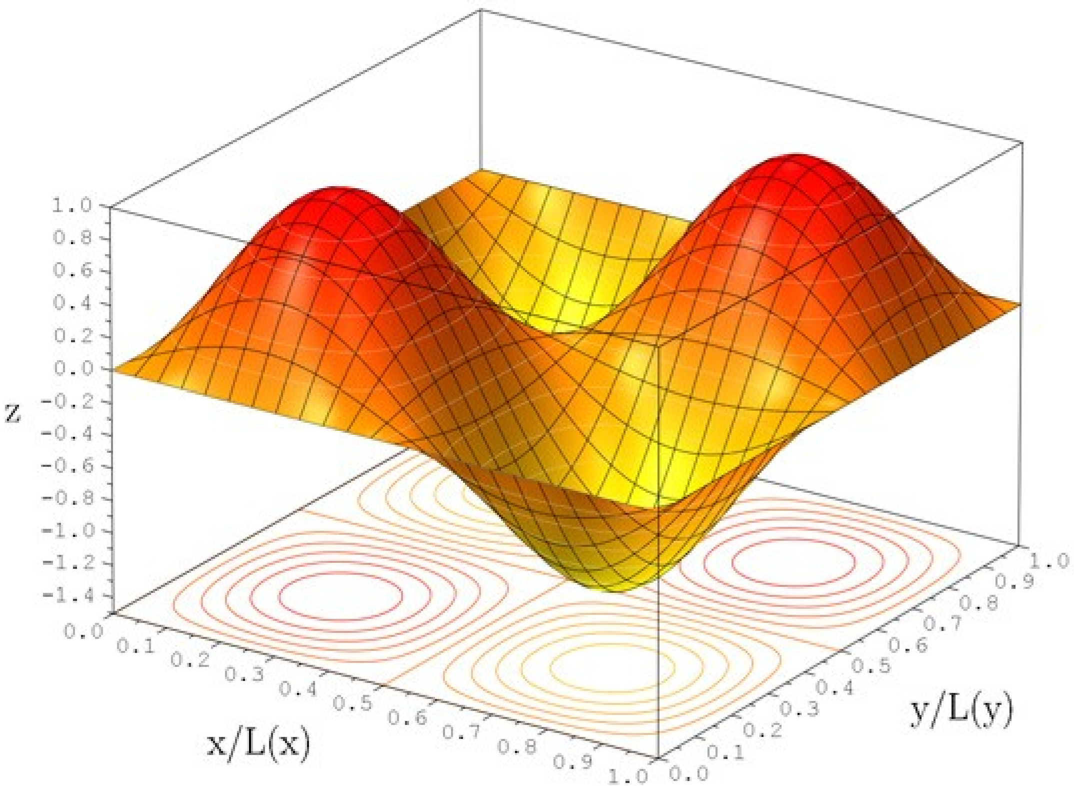
$$n\lambda = 2\underline{d}\,\underline{\sin\theta} = 2D\,\underline{\sin\alpha\cos\alpha} = D\sin 2\alpha$$

Dsinα cosα ½sin2α by trig

$$n\lambda = D\sin 2\alpha = D\sin\varphi$$

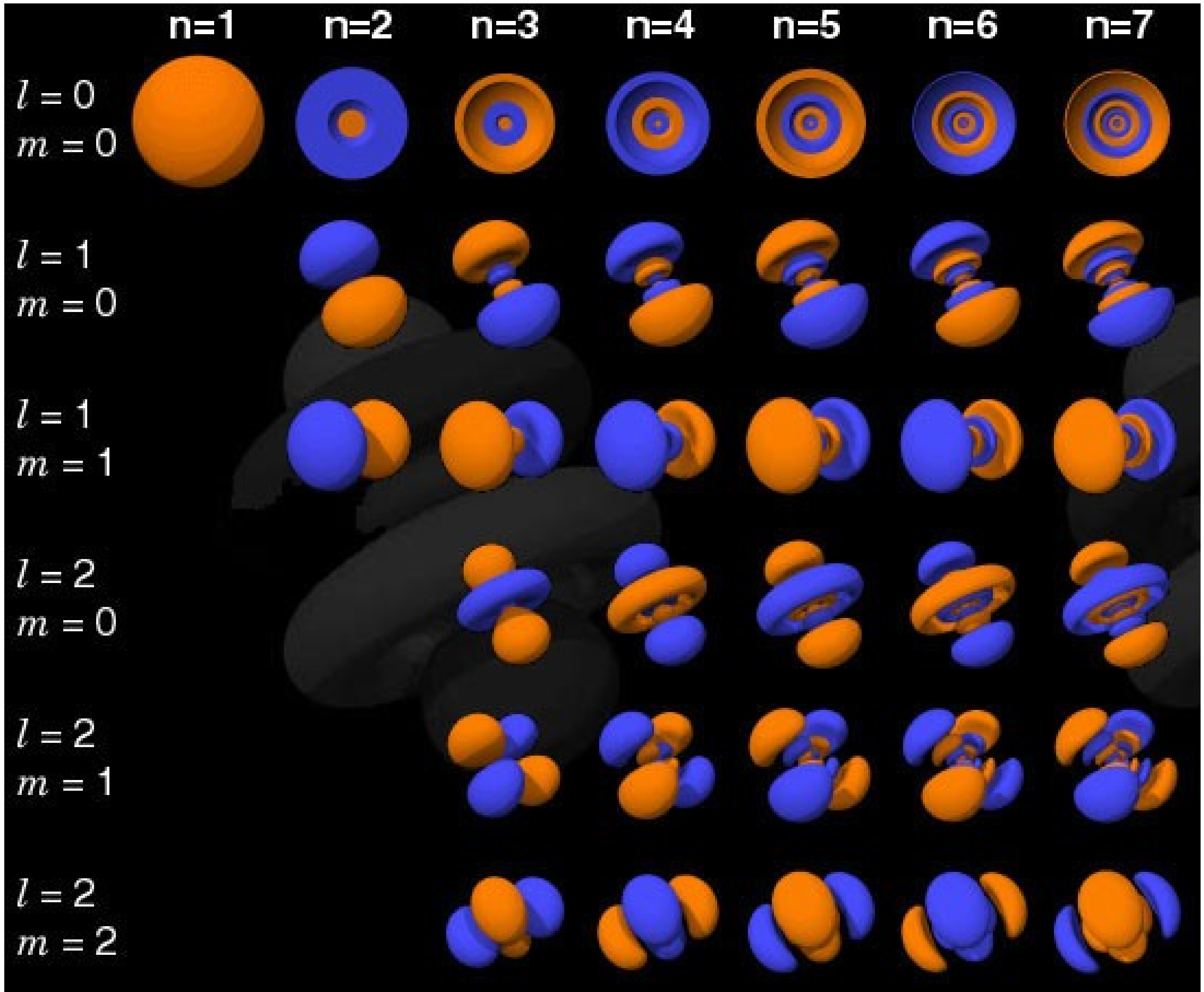
Wave/Particle Duality

- The particle wavefunction, ψ, is the "probability amplitude" (see figure "Z"), a complex number.
- Probability density = $|\Psi|^2$ gives the probability of where we might find the particle. (this must be positive)
- Can have destructive and constructive interference



Wave/Particle Duality

- This picture shows some of the possible electron probability densities for different quantum states of the H atom.
- Electron “clouds”



- Probability “clouds”
  - **kind of the opposite of the “Plum Pudding” model**