c) What is the range, that is, how far does the ball move horizontally before reaching the starting height?

Soln: $R = x(t) - x_0$

$R = x_0 + v_{0x} t - x_0$ ← $t = 2t_{max}$

$R = v_{0x}(2t_{max})$

$R = v_0 \cos\theta \left(2 \frac{v_0 \sin\theta}{g}\right)$

$= \frac{v_0^2}{g}(2 \sin\theta \cos\theta)$

trig identity: $2\sin\theta\cos\theta = \sin 2\theta$

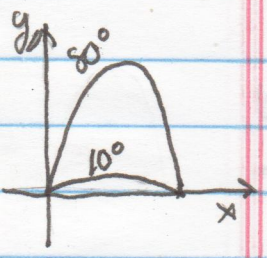
$$R = \frac{v_0^2}{g} \sin 2\theta$$

So $R = \frac{30^2}{9.8} \sin(2 \cdot 20^\circ) = \boxed{59.0\ m}$

Note:

1) Which θ gives maximum range, R?

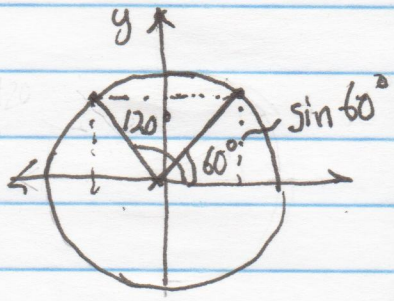
$\theta = 45^\circ$ gives $\sin(90^\circ) = 1$



2) There are two θ's for a given range!

Ex) $\theta = 30^\circ \rightarrow \sin 2\cdot 30 = \sin 60^\circ = \frac{\sqrt{3}}{2}$

if $\theta = 60^\circ \rightarrow \sin 2\cdot 60 = \sin 120^\circ = \frac{\sqrt{3}}{2}$



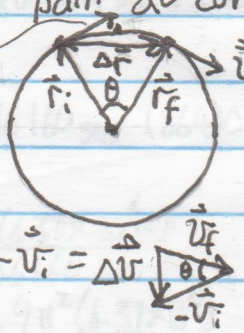
* These angles are symmetric about 45°!

Q: What θ gives same range as $\theta = 10^\circ$?

$\theta = 80^\circ$

Uniform Circular Motion = motion in a circular path at const. speed.

* Speed $v = |\vec{v}|$ ← inst. velocity
* Velocity is not constant
* $\vec{a} \neq 0$ since $\vec{v}$ is changing
* $\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$
* $\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \vec{a}_c = \frac{v^2}{r}(-\hat{r})$
* $a_c = |\vec{a}_c|$ is centripetal acceleration



See PPT#2

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \text{ so } \frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta r}{\Delta t} = \frac{v}{r} \cdot v = \frac{v^2}{r}$$

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \text{ so } \frac{\Delta \vec{v}}{\Delta t} = \frac{v \Delta \vec{r}}{r \Delta t} = \frac{v \cdot v}{r} = \frac{v^2}{r}$$