Chapter 16

Wave Motion

PHYS 2321
Week 12: Wave Motion

Day 3 Outline

1) Hwk: Ch. 14 Skim
   Ch. 15 P. 1,2,6,7, (+more) MiscQ 1-9 Due Wed
   Read 15.1-15.9
   Try Ch. 15 (wave motion) practice quiz
2) Review Simple Harmonic Oscillations (Ch.14)
3) Sinusoidal Wave Terms
   Demos
4) Wave functions

Notes: Exam II returned on Mon
Last day to Withdraw (see me about Exam II at 3 pm)

PHYS 2321
Week 13: Wave Motion

Day 1 Outline

1) Hwk: Ch. 14 Skim Ch. 15 Read 15.1-15.9
   Ch. 15 P. 1,2,6,7,15,16,19,23,24,25,44,45,
   MisconQ 1-9 Due Wed
   Try Ch.15 and 16 practice quizzes
2) Return Exam II
3) Wave speed
4) Wave functions
5) Energy and intensity of waves

Notes: Exam II mean = 20.5/35 (59%)
Will spend ~1 day on sound Ch.16.

Waves

Wave: a travelling disturbance or variation in a medium or field which carries energy.

Types:

Mechanical	Electromagnetic	Gravitational(!)
sound	visible light, IR	inspiralling BHs
seismic	microwaves, radio,	
water	x-rays, gamma rays	
string		

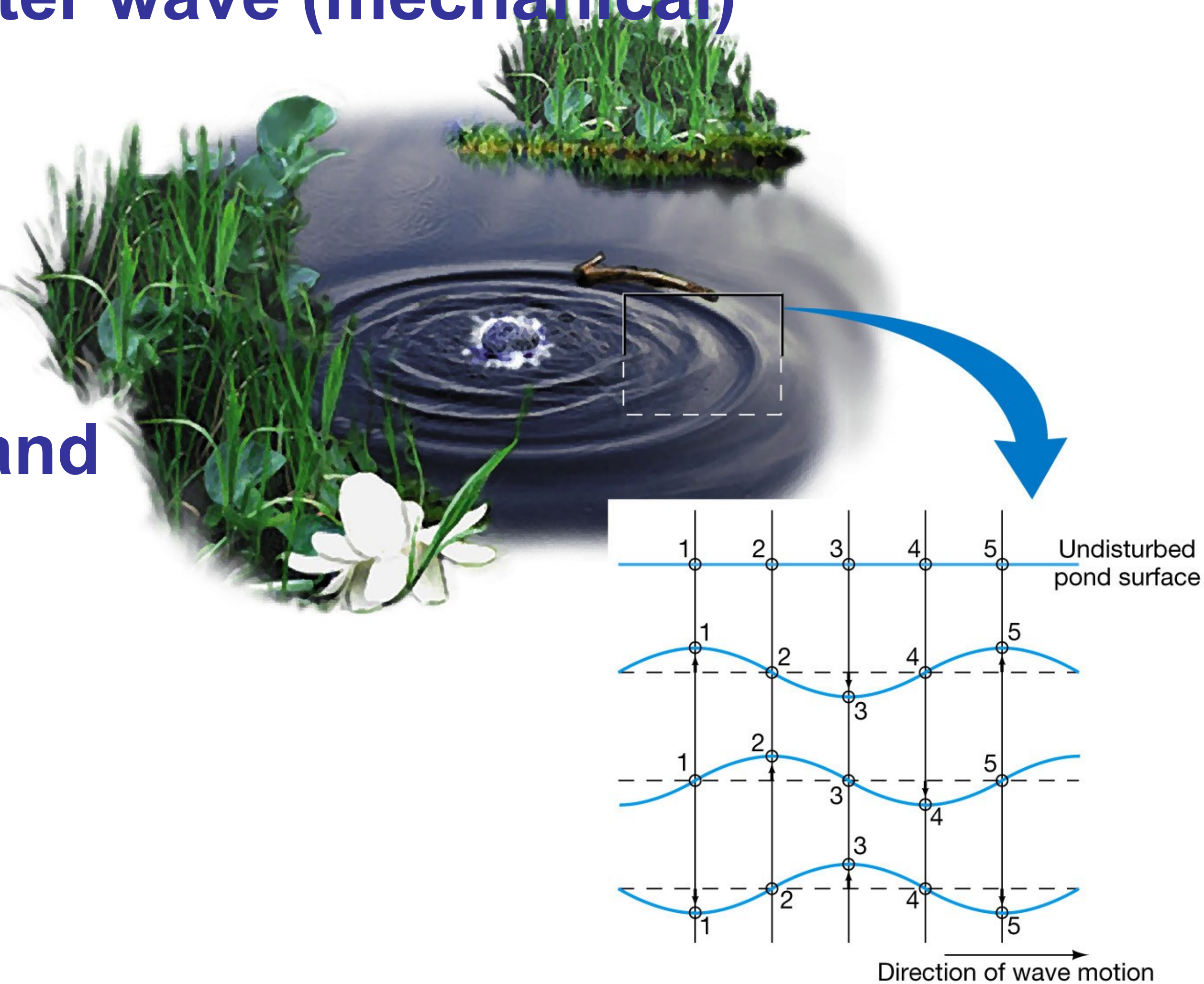
What do they have in common?

Waves - terminology

Example: water wave (mechanical)

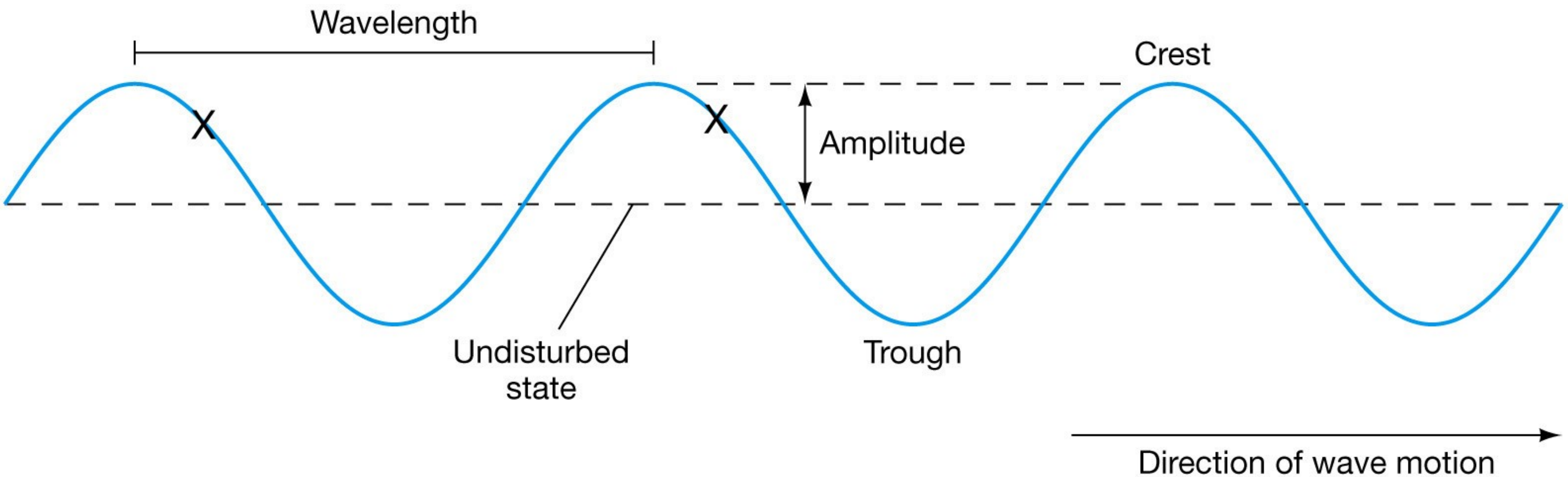
Water just moves up and down

Wave travels and can transmit energy



Waves - terminology

Sine waves: **waves described by a sine or cosine function. Also called: *"sinusoidal"***



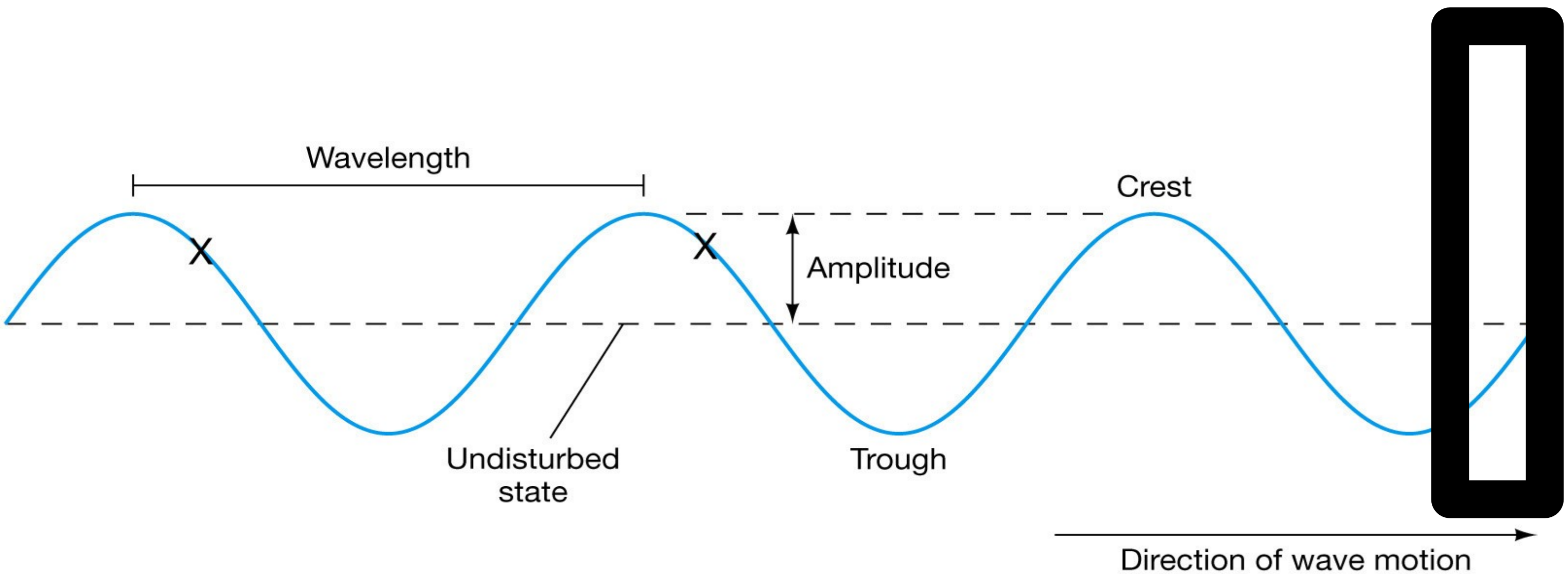
This graph shows <u>amplitude versus position</u>, but <u>amplitude versus time</u> is ALSO a sinusoidal graph!

Waves - terminology

Frequency: number of wave crests that pass a given point per second

Period: time between passage of successive crests

Relationship: Frequency = 1 / Period



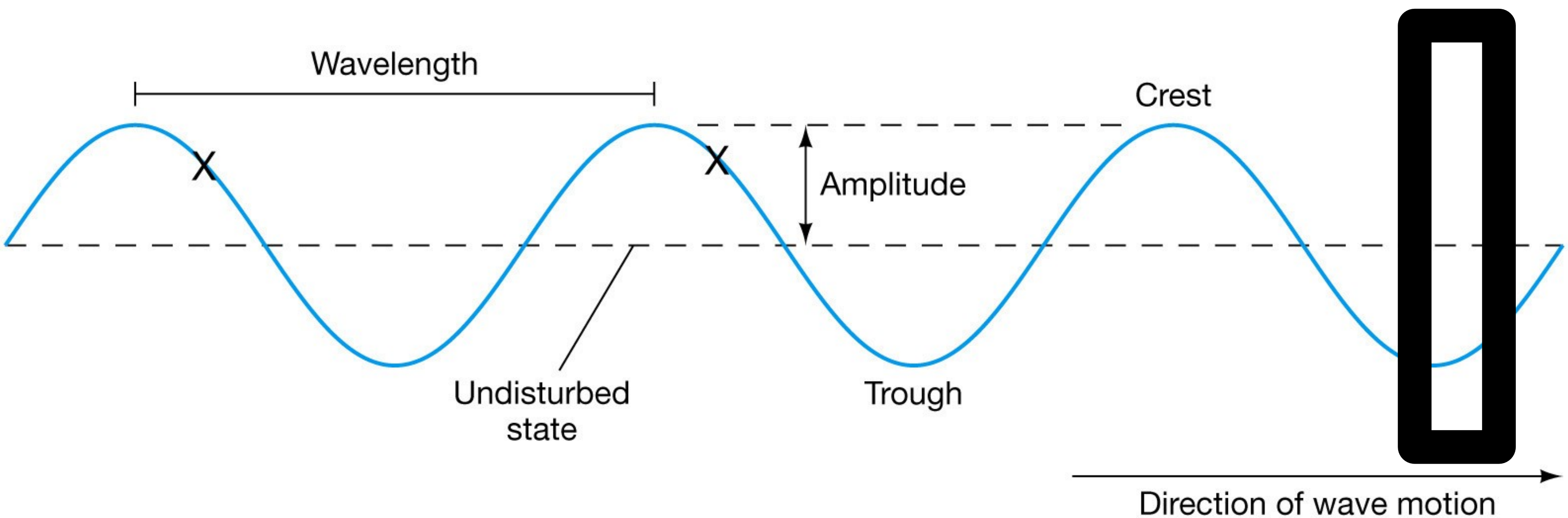
Waves - terminology

Wavelength: **distance between successive crests**

Velocity: speed at which crests move

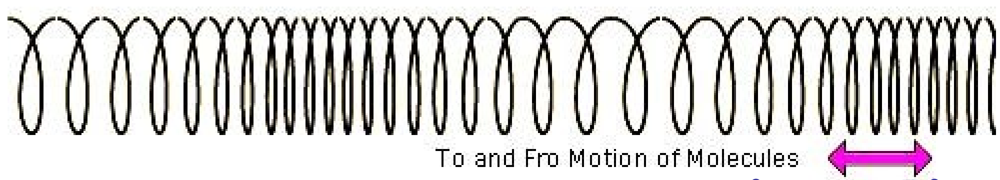
$$Velocity = Wavelength/Period$$

$$Velocity = Wavelength * frequency$$

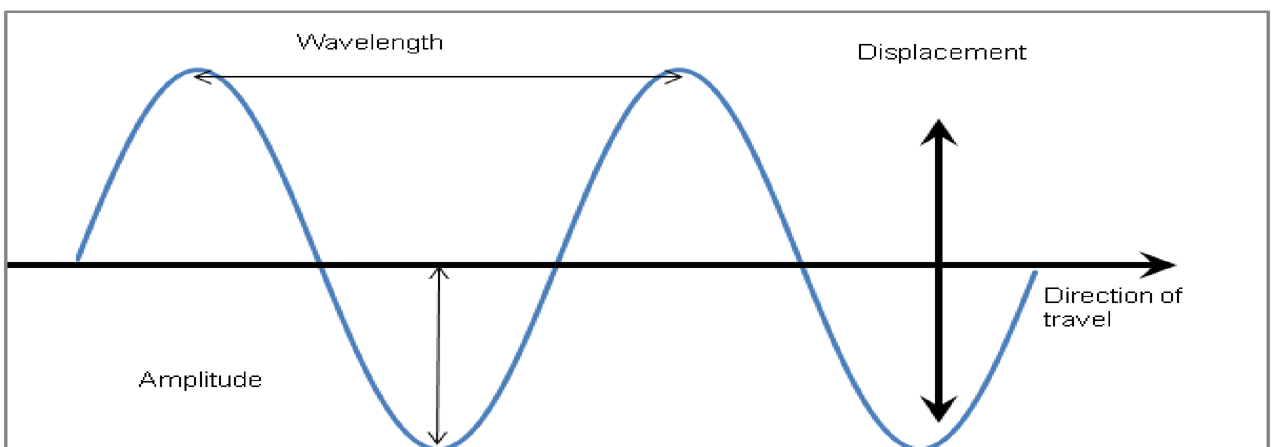


Waves - terminology

Longitudinal wave: propagates in a direction parallel to the displacement of the medium



Transverse wave: propagates in a direction perpendicular (or transverse) to the displacement of the medium



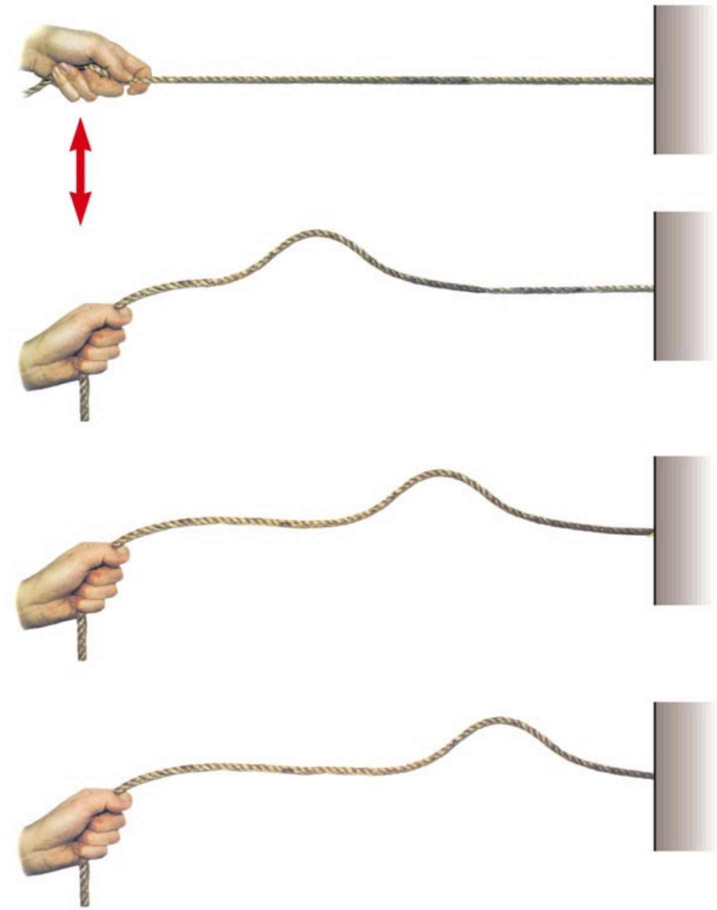
DEMO: long. and transv. waves in a SLINKY! Standing waves!

Mechanical Wave Requirements

- Some source of disturbance
- A medium that can be disturbed
- Some physical mechanism through which elements of the medium can influence each other

Pulse on a String

- The wave is generated by a flick on one end of the string
- The string is under tension
- A single bump is formed and travels along the string
  - The bump is called a **pulse**

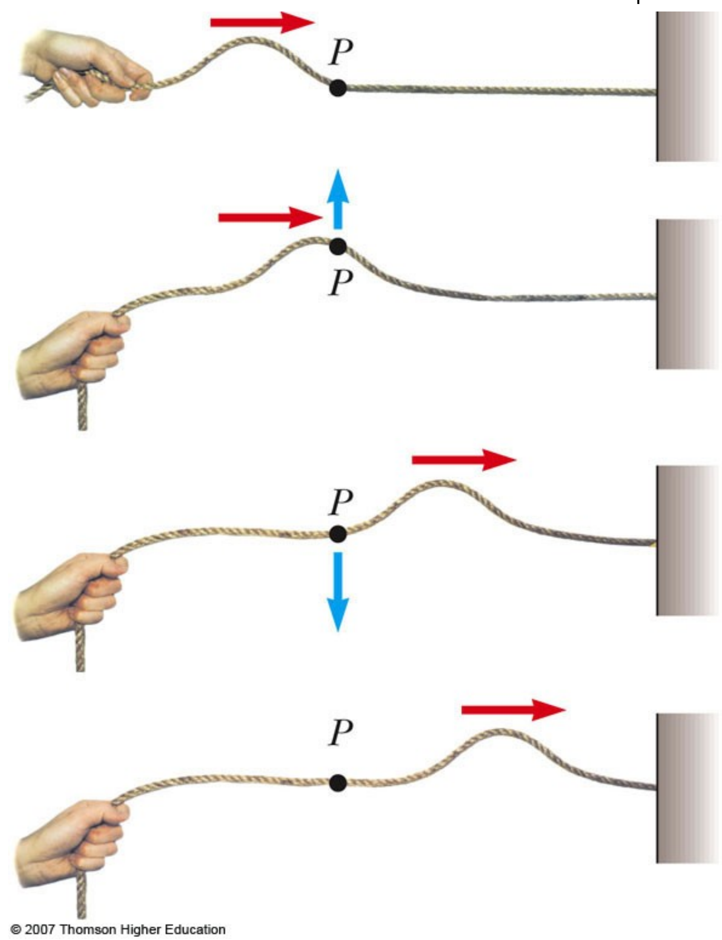


Pulse on a String

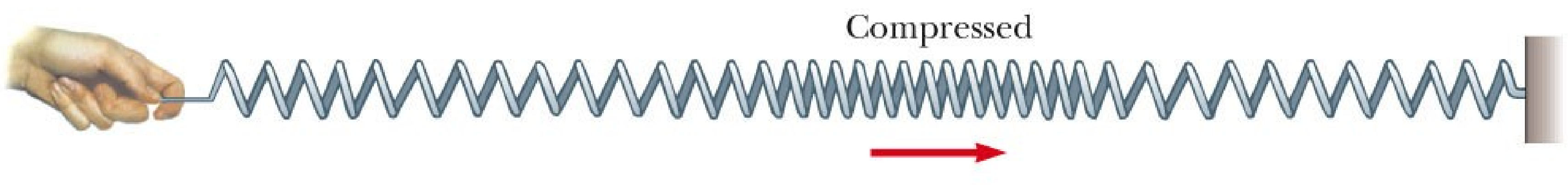
- The string is the medium through which the pulse travels
- The pulse has a definite height
- The pulse has a definite speed of propagation along the medium
- The shape of the pulse changes very little as it travels along the string
- A continuous flicking of the string would produce a periodic disturbance which would form a wave

Transverse Wave

- A traveling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation is called a **transverse wave**
- The particle motion is shown by the blue arrow
- The direction of propagation is shown by the red arrow

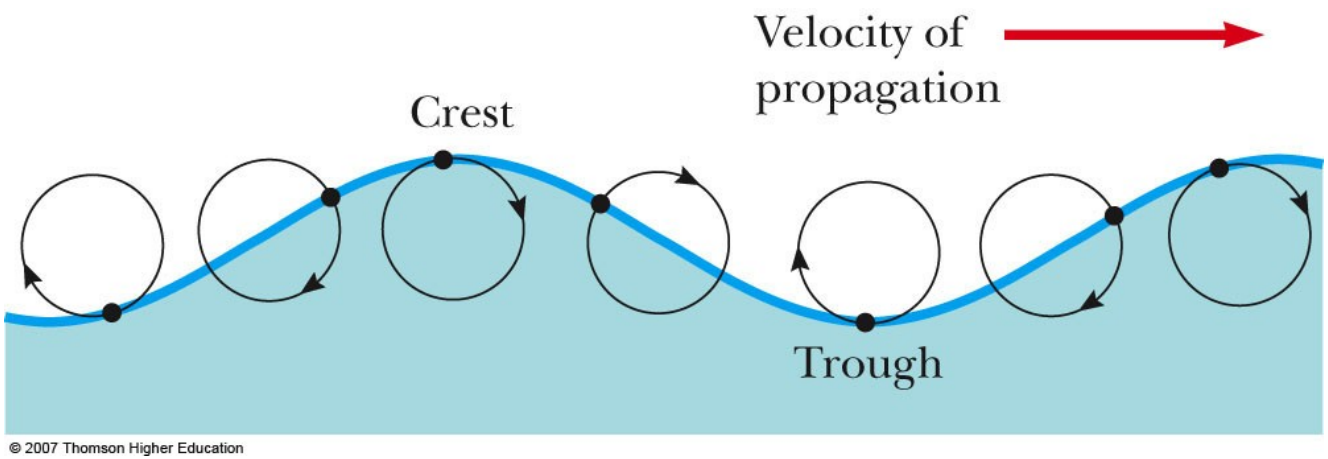


Longitudinal Wave



- A traveling wave or pulse that causes the elements of the disturbed medium to move parallel to the direction of propagation is called a **longitudinal wave**
- The displacement of the coils is parallel to the propagation

Complex Waves



- Some waves exhibit a combination of transverse and longitudinal waves
- Surface water waves are an example
- Use the active figure to observe the displacements

Example: Earthquake Waves

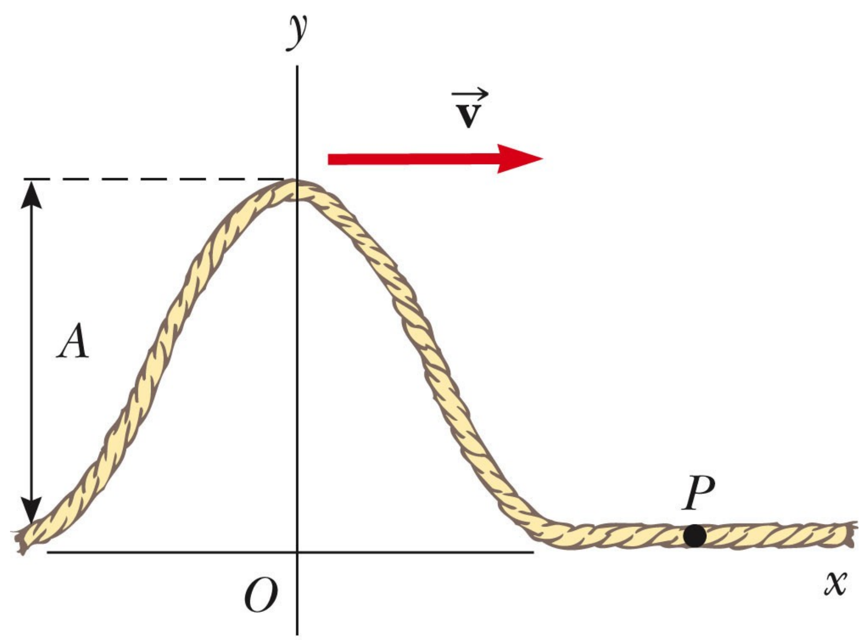
- P waves
  - “P” stands for primary
  - Fastest, at 7 – 8 km / s
  - Longitudinal
- S waves
  - “S” stands for secondary
  - Slower, at 4 – 5 km/s
  - Transverse
- A seismograph records the waves and allows determination of information about the earthquake’s place of origin

Traveling Pulse

- The shape of the pulse at $t = 0$ is shown
- The shape can be represented by

  $y\,(x,0) = f\,(x)$

  - This describes the transverse position y of the element of the string located at each value of x at $t = 0$

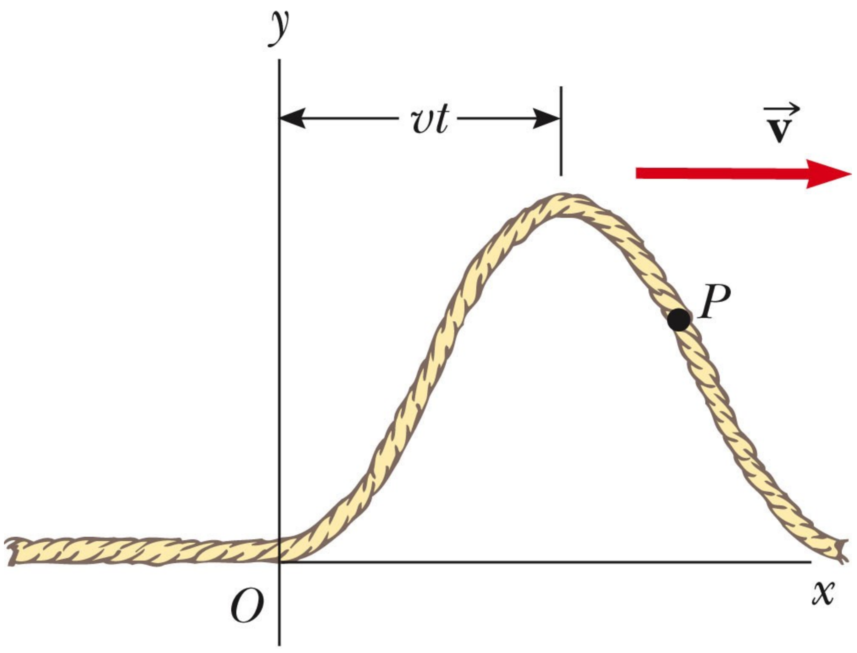


(a) Pulse at $t = 0$

Traveling Pulse, 2

- The speed of the pulse is v
- At some time, t, the pulse has traveled a distance vt
- The shape of the pulse does not change
- Its position is now

  $y = f\,(x - vt)$



(b) Pulse at time t

Traveling Pulse, 3

- For a pulse traveling to the right
  - $y(x, t) = f(x - vt)$
- For a pulse traveling to the left
  - $y(x, t) = f(x + vt)$
- The function *y* is also called the **wave function**:

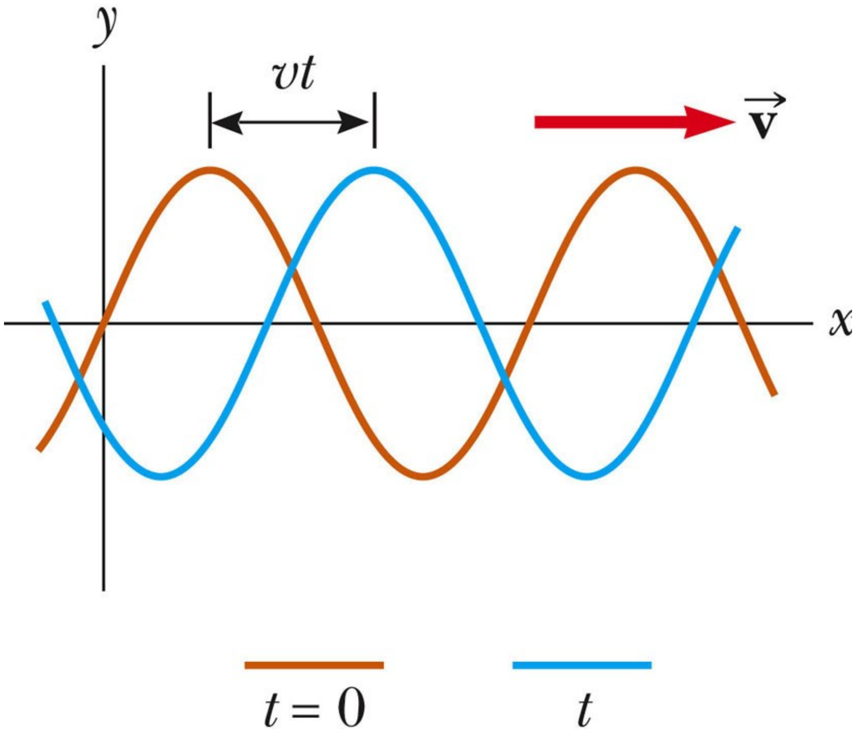
  $y(x, t)$
- The wave function represents the *y* coordinate of any element located at position *x* at any time *t*
  - The *y* coordinate is the transverse position

Traveling Pulse, final

- If *t* is fixed then the wave function is called the **waveform**
  - It defines a curve representing the actual geometric shape of the pulse at that time

Sinusoidal Waves

- The wave represented by the curve shown is a **sinusoidal wave**
- It is the same curve as sin θ plotted against θ
- This is the simplest example of a periodic continuous wave
  - It can be used to build more complex waves



Sinusoidal Waves, cont

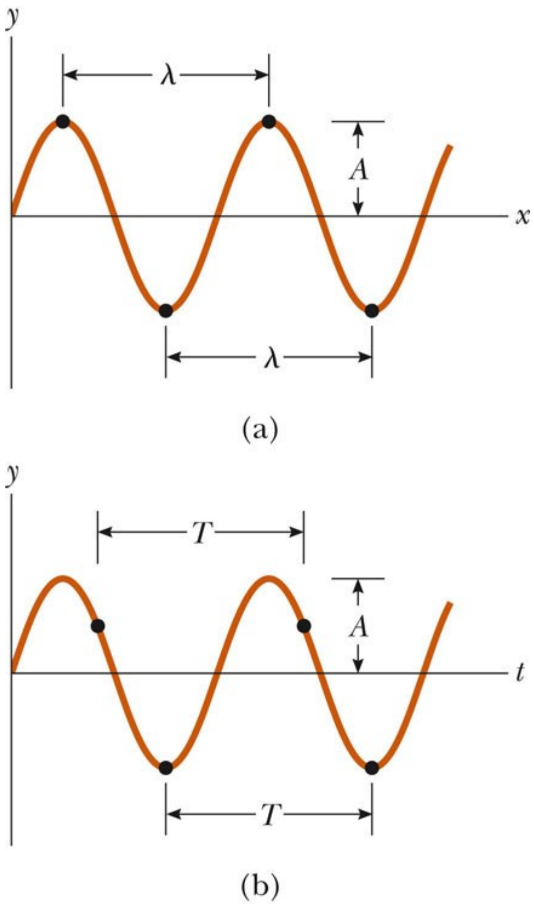
- The wave moves toward the right
  - In the previous example, the brown wave represents the initial position
  - As the wave moves toward the right, it will eventually be at the position of the blue curve
- Each element moves up and down in simple harmonic motion
- It is important to distinguish between the motion of the wave and the motion of the particles of the medium

Wave Model

- The wave model is a new simplification model
  - Allows to explore more analysis models for solving problems
  - An ideal wave has a single frequency
  - An ideal wave is infinitely long
  - Ideal waves can be combined

Terminology: Amplitude and Wavelength

- The **crest** of the wave is the location of the maximum displacement of the element from its normal position
  - This distance is called the **amplitude**, A
- The **wavelength**, λ, is the distance from one crest to the next



Terminology: Wavelength and Period

- More generally, the wavelength is the minimum distance between any two identical points on adjacent waves
- The period, T , is the time interval required for two identical points of adjacent waves to pass by a point
  - The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium

Terminology: Frequency

- The **frequency**, f, is the number of crests (or any point on the wave) that pass a given point in a unit time interval
  - The time interval is most commonly the second
  - The frequency of the wave is the same as the frequency of the simple harmonic motion of one element of the medium

Terminology: Frequency, cont

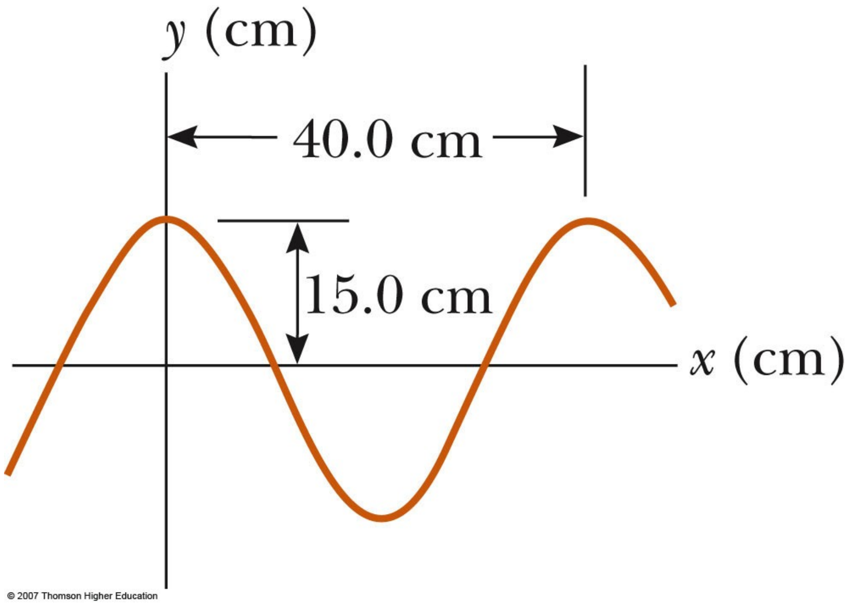
- The frequency and the period are related

$$f = \frac{1}{T}$$

- When the time interval is the second, the units of frequency are s^{-1} = Hz
  - Hz is a hertz

Terminology, Example

- The wavelength, λ, is 40.0 cm
- The amplitude, A, is 15.0 cm
- The wave function can be written in the form $y = A\cos(kx - \omega t)$



Speed of Waves

- Waves travel with a specific speed
  - The speed depends on the properties of the medium being disturbed
- The wave function is given by

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

  - This is for a wave moving to the right
  - For a wave moving to the left, replace x – vt with x + vt

Wave Function, Another Form

- Since speed is distance divided by time,

  $v = \lambda / T$
- The wave function can then be expressed as

$$y(x,t) = A \sin 2\pi \left[\frac{x}{\lambda} - \frac{t}{T} \right]$$

- This form shows the periodic nature of y
  - y can be used as shorthand notation for y(x, t)

Wave Equations

- We can also define the angular wave number (or just wave number), *k*

$$k=\frac{2\pi}{\lambda}$$

- The angular frequency can also be defined

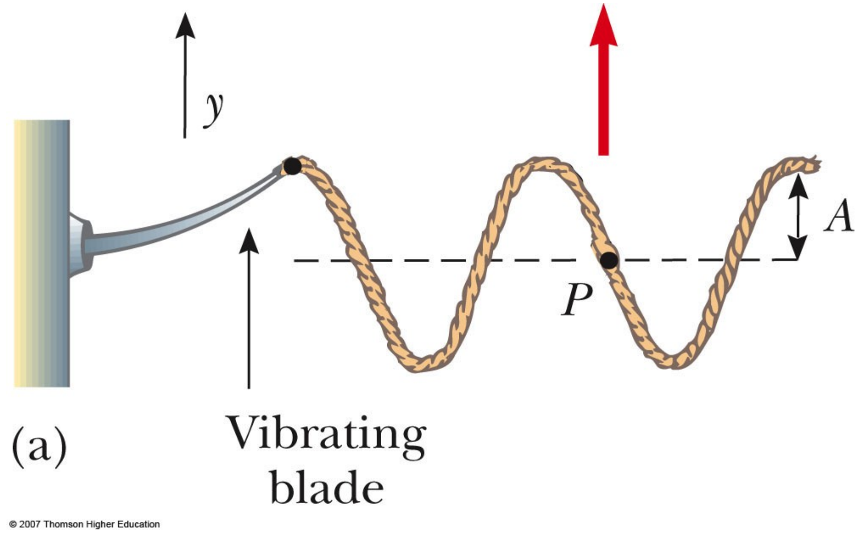
$$\omega=\frac{2\pi}{T}=2\pi f$$

Wave Equations, cont

- The wave function can be expressed as
  $y = A \sin (k x - \omega t)$
- The speed of the wave becomes $v = \lambda f$
- If $y \neq 0$ at $t = 0$ and x=0, the wave function can be generalized to
  $y = A \sin (k x - \omega t + \phi)$
  where ϕ is called the phase constant

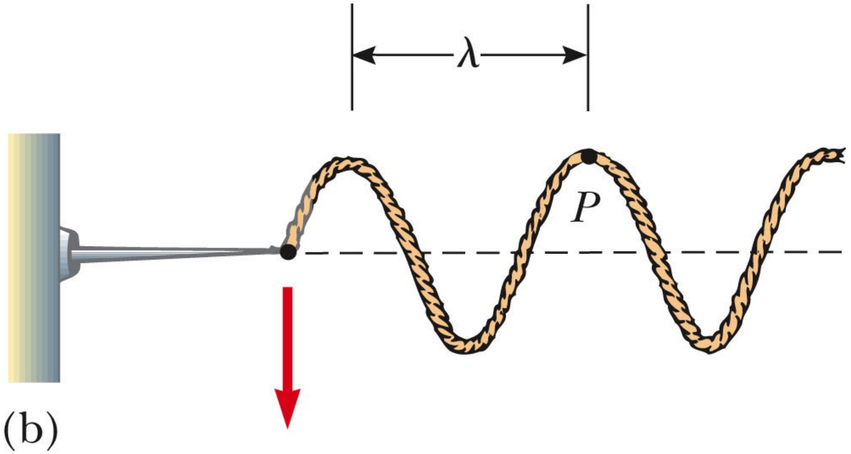
Sinusoidal Wave on a String

- To create a series of pulses, the string can be attached to an oscillating blade
- The wave consists of a series of identical waveforms
- The relationships between speed, velocity, and period hold



Sinusoidal Wave on a String, 2

- Each element of the string oscillates vertically with simple harmonic motion
  - For example, point *P*
- Every element of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of the oscillation of the blade

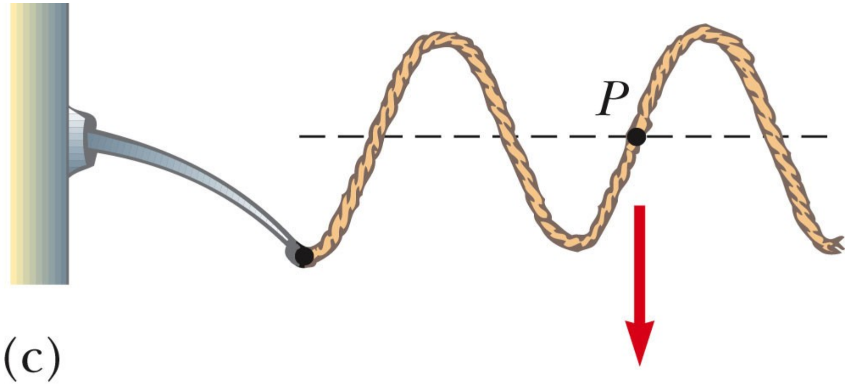


Sinusoidal Wave on a String, 3

- The transverse speed of the element is

$$v_y = \left.\frac{dy}{dt}\right]_{x=\text{constant}}$$

- or $v_y = -\omega A \cos(kx - \omega t)$
- This is different than the speed of the wave itself

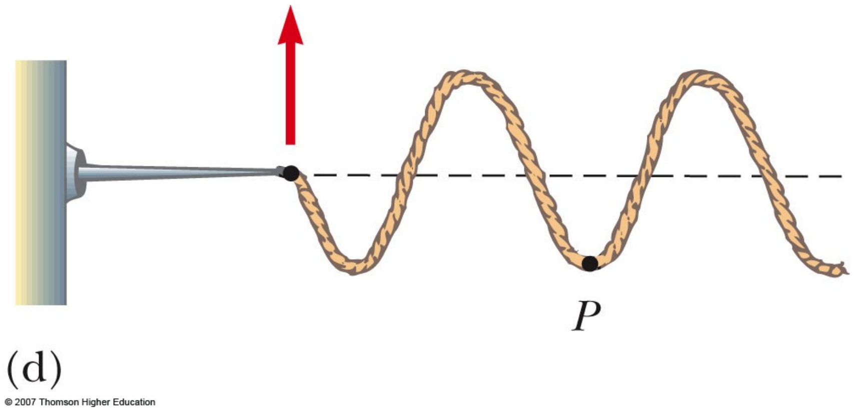


(c)

Sinusoidal Wave on a String, 4

- The transverse acceleration of the element is

$$a_y = \left.\frac{dv_y}{dt}\right]_{x=\text{constant}}$$



- or $a_y = -\omega^2 A \sin(kx - \omega t)$

Sinusoidal Wave on a String, 5

- The maximum values of the transverse speed and transverse acceleration are
  - $v_{y,\ max} = \omega A$
  - $a_{y,\ max} = \omega^2 A$
- The transverse speed and acceleration do not reach their maximum values simultaneously
  - v is a maximum at $y = 0$
  - a is a maximum at $y = \pm A$

Speed of a Wave on a String

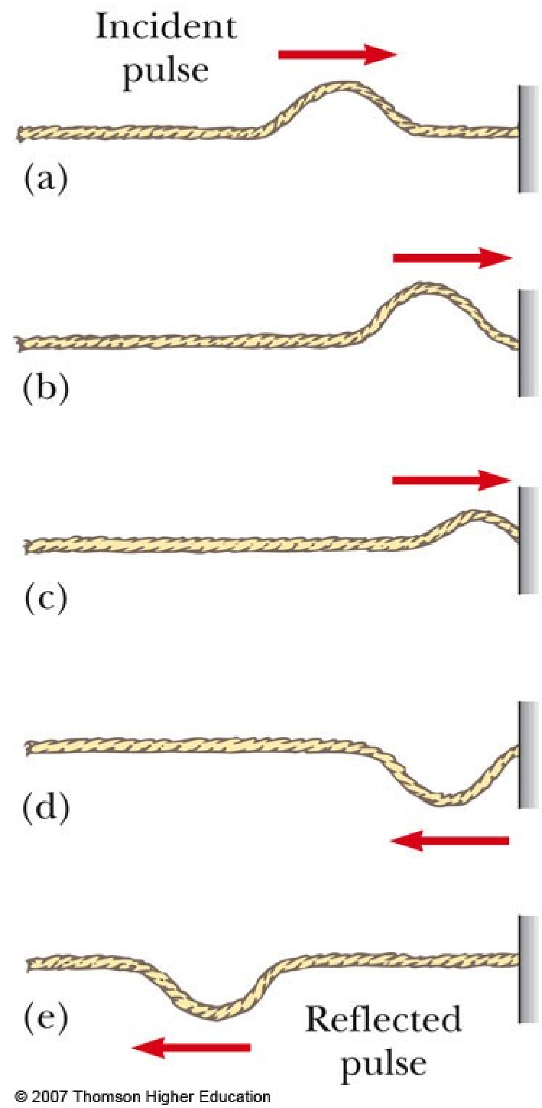
- The speed of the wave depends on the physical characteristics of the string and the tension to which the string is subjected

$$v = \sqrt{\frac{tension}{mass/length}} = \sqrt{\frac{T}{\mu}}$$

- This assumes that the tension is not affected by the pulse
- This does not assume any particular shape for the pulse

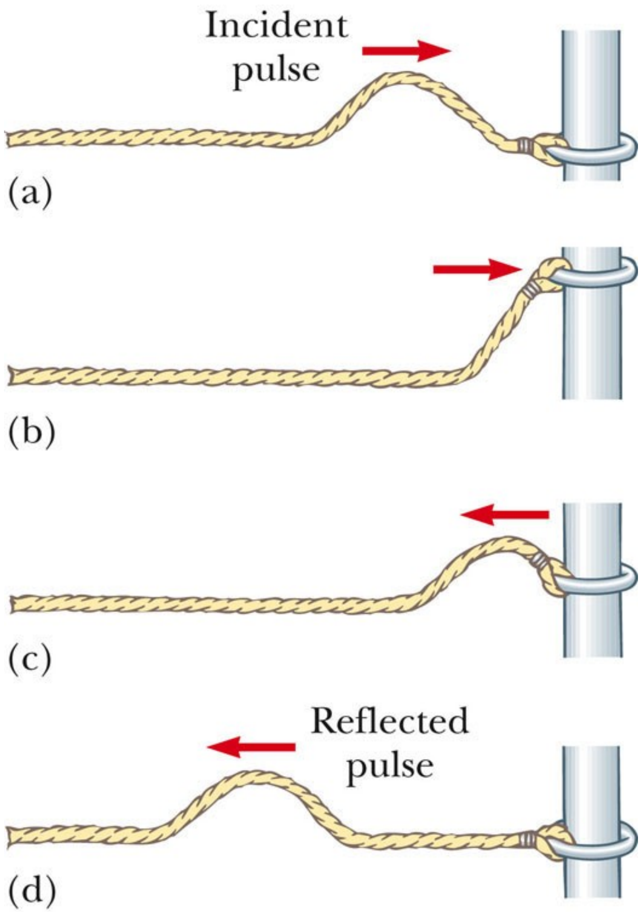
Reflection of a Wave, Fixed End

- When the pulse reaches the support, the pulse moves back along the string in the opposite direction
- This is the **reflection** of the pulse
- The pulse is inverted



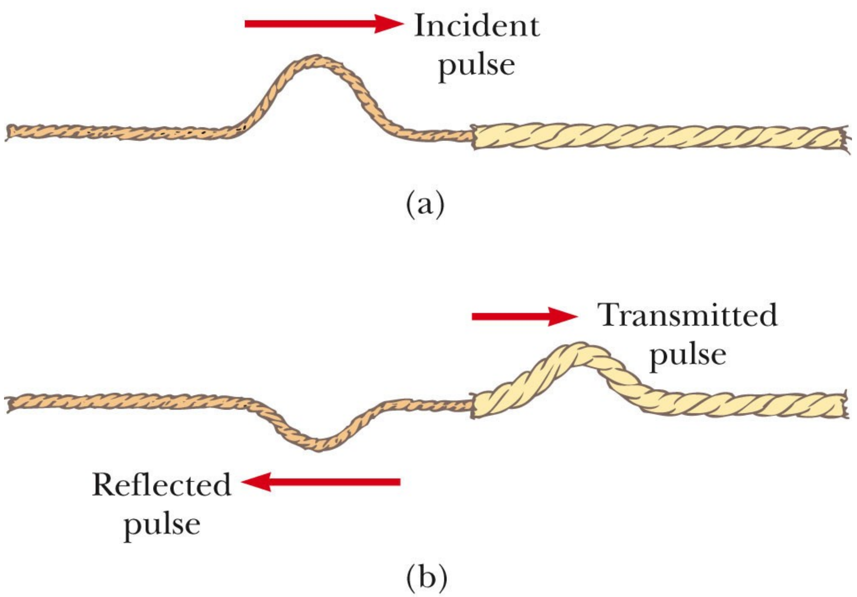
Reflection of a Wave, Free End

- With a free end, the string is free to move vertically
- The pulse is reflected
- The pulse is not inverted
- The reflected pulse has the same amplitude as the initial pulse



Transmission of a Wave

- When the boundary is intermediate between the last two extremes
  - Part of the energy in the incident pulse is reflected and part undergoes **transmission**
    - Some energy passes through the boundary

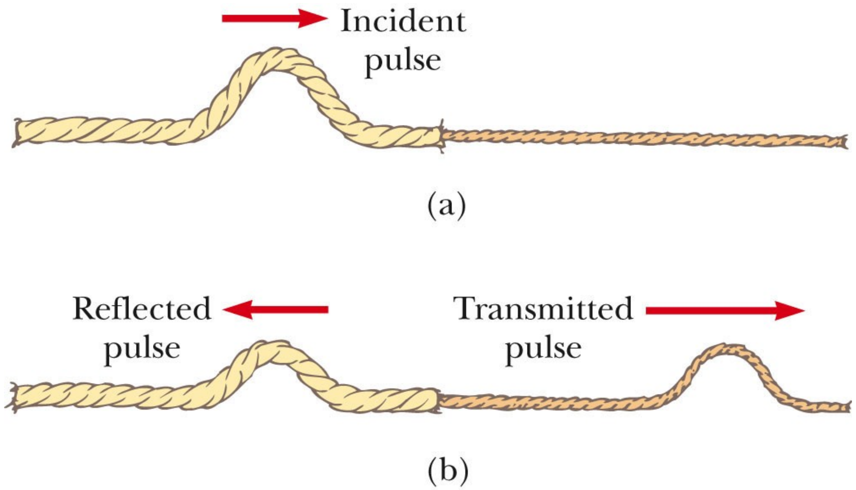


Transmission of a Wave, 2

- Assume a light string is attached to a heavier string
- The pulse travels through the light string and reaches the boundary
- The part of the pulse that is reflected is inverted
- The reflected pulse has a smaller amplitude

Transmission of a Wave, 3

- Assume a heavier string is attached to a light string
- Part of the pulse is reflected and part is transmitted
- The reflected part is not inverted



Transmission of a Wave, 4

- Conservation of energy governs the pulse
  - When a pulse is broken up into reflected and transmitted parts at a boundary, the sum of the energies of the two pulses must equal the energy of the original pulse
- When a wave or pulse travels from medium A to medium B and $v_A > v_B$, it is inverted upon reflection
  - B is denser than A
- When a wave or pulse travels from medium A to medium B and $v_A < v_B$, it is not inverted upon reflection
  - A is denser than B

Energy in Waves in a String

- Waves transport energy when they propagate through a medium
- We can model each element of a string as a simple harmonic oscillator
  - The oscillation will be in the *y*-direction
- Every element has the same total energy

Energy, cont.

- Each element can be considered to have a mass of dm
- Its kinetic energy is $dK = \frac{1}{2} (dm) v_y^2$
- The mass dm is also equal to μdx
- The kinetic energy of an element of the string is $dK = \frac{1}{2} (\mu \, dx) v_y^2$

Energy, final

- Integrating over all the elements, the total kinetic energy in one wavelength is $K_\lambda = \frac{1}{4}\mu\omega^2 A^2\lambda$
- The total potential energy in one wavelength is $U_\lambda = \frac{1}{4}\mu\omega^2 A^2\lambda$
- This gives a total energy of
  - $E_\lambda = K_\lambda + U_\lambda = \frac{1}{2}\mu\omega^2 A^2\lambda$

Power Associated with a Wave

- The power is the rate at which the energy is being transferred:

$$P = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2}\mu\omega^2 A^2 \lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 v$$

- The power transfer by a sinusoidal wave on a string is proportional to the
  - Frequency squared
  - Square of the amplitude
  - Wave speed

The Linear Wave Equation (SKIPPED)

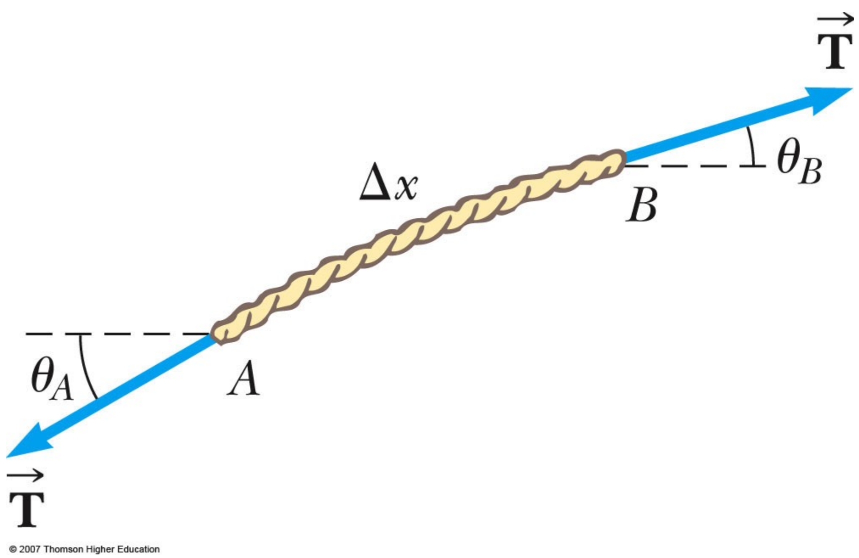
- The wave functions $y\ (x, t)$ represent solutions of an equation called the **linear wave equation**
- This equation gives a complete description of the wave motion
- From it you can determine the wave speed
- The linear wave equation is basic to many forms of wave motion

Linear Wave Equation Applied to a Wave on a String

- The string is under tension *T*
- Consider one small string element of length Δx
- The net force acting in the *y* direction is

$$\Sigma F_y \approx T(\tan\theta_B - \tan\theta_A)$$

  - This uses the small-angle approximation



Linear Wave Equation Applied to Wave on a String

- Applying Newton's Second Law gives

$$\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\left(\partial y/\partial x\right)_B - \left(\partial y/\partial x\right)_A}{\Delta x}$$

- In the limit as $\Delta x \to 0$, this becomes

$$\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$$

- This is the linear wave equation as it applies to waves on a string

Linear Wave Equation, General

- The equation can be written as

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

- This applies in general to various types of traveling waves
  - y represents various positions
    - For a string, it is the vertical displacement of the elements of the string
    - For a sound wave, it is the longitudinal position of the elements from the equilibrium position
    - For em waves, it is the electric or magnetic field components

Linear Wave Equation, General cont

- The linear wave equation is satisfied by any wave function having the form

  $y = f(x \pm vt)$
- The linear wave equation is also a direct consequence of Newton's Second Law applied to any element of a string carrying a traveling wave