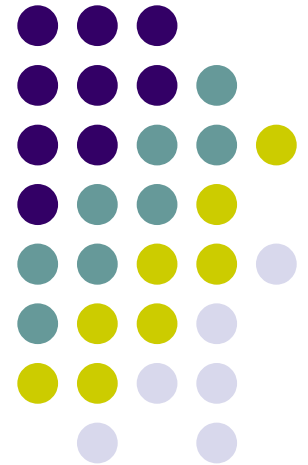


Chapter 15

Wave Motion



PHYS 2321

Week 12: Wave Motion



Day 3 Outline

- 1) Hwk: Ch. 14 Skim Ch. 15 Read 15.1-15.9
Ch. 15 P. 1,2,6,7,15,16,19,23,24,25,44,45
MiscQ 1-9 Due Wed
- 2) Review Simple Harmonic Oscillations (Ch.14)
- 3) Sinusoidal Wave Terms
Demos
- 4) Wave functions

Notes: Exam II will be returned on Mon

Last day to Withdraw

Try Ch. 15 (wave motion) practice quiz

Waves



Wave: a travelling disturbance or variation in a medium or field which carries energy.

Types:

Mechanical

Electromagnetic

Gravitational(!)

sound

visible light, IR

inspiralling BHs

seismic

microwaves, radio,

water

x-rays, gamma rays

string

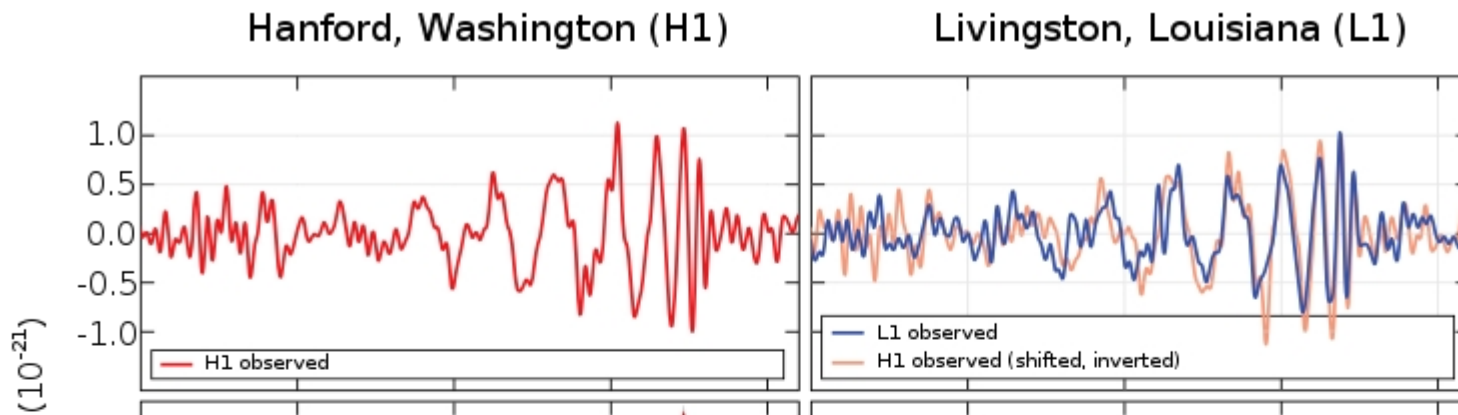
What do they have in common?



The first gravitational wave detection: by the LIGO consortium on Feb 11, 2016.

Source: inspiralling binary black holes. One $29 M_{\odot}$ and one $36 M_{\odot}$. 1.3×10^9 LY away. Produced one $62 M_{\odot}$ BH.

Power: momentarily greater than all of the stars in the observable universe. $3 M_{\odot}$ converted into gravitational wave energy in ~ 0.2 seconds.



The “chirp”

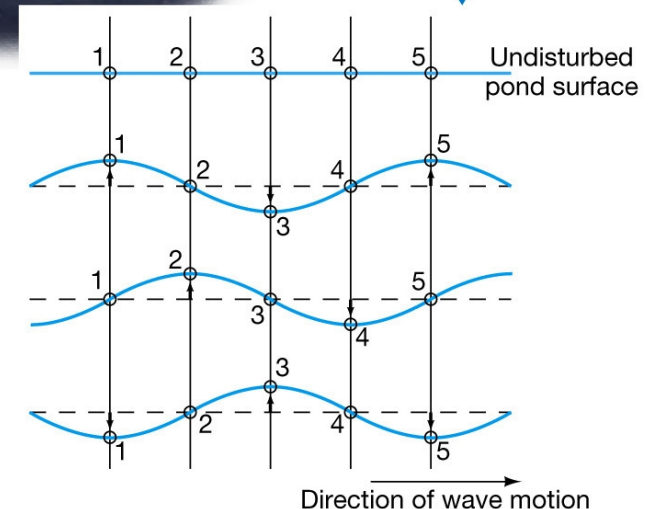
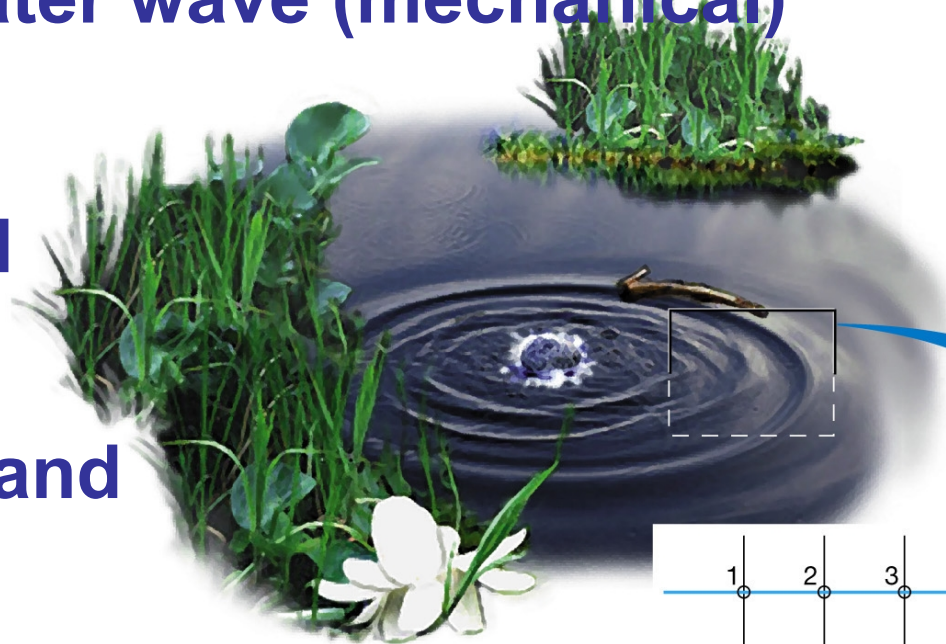
Waves - terminology



Example: water wave (mechanical)

Water just moves up and down

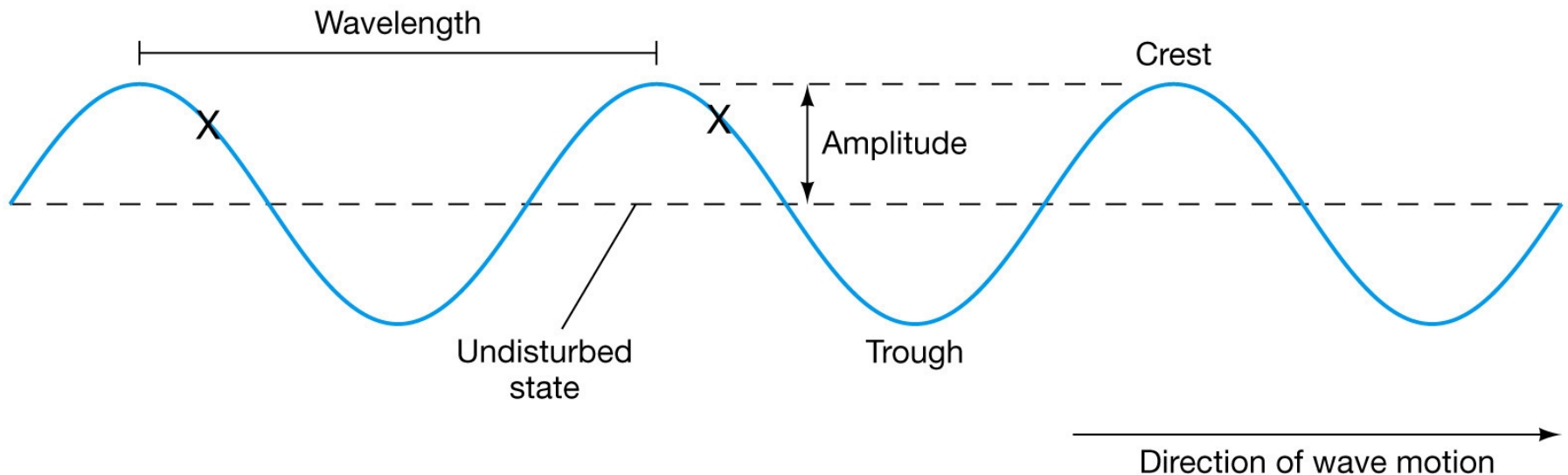
Wave travels and can transmit energy



Waves - terminology



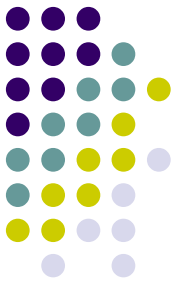
Sine waves: waves described by a sine or cosine function. Also called: “sinusoidal”



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This graph shows amplitude versus position, but amplitude versus time is ALSO a sinusoidal graph!

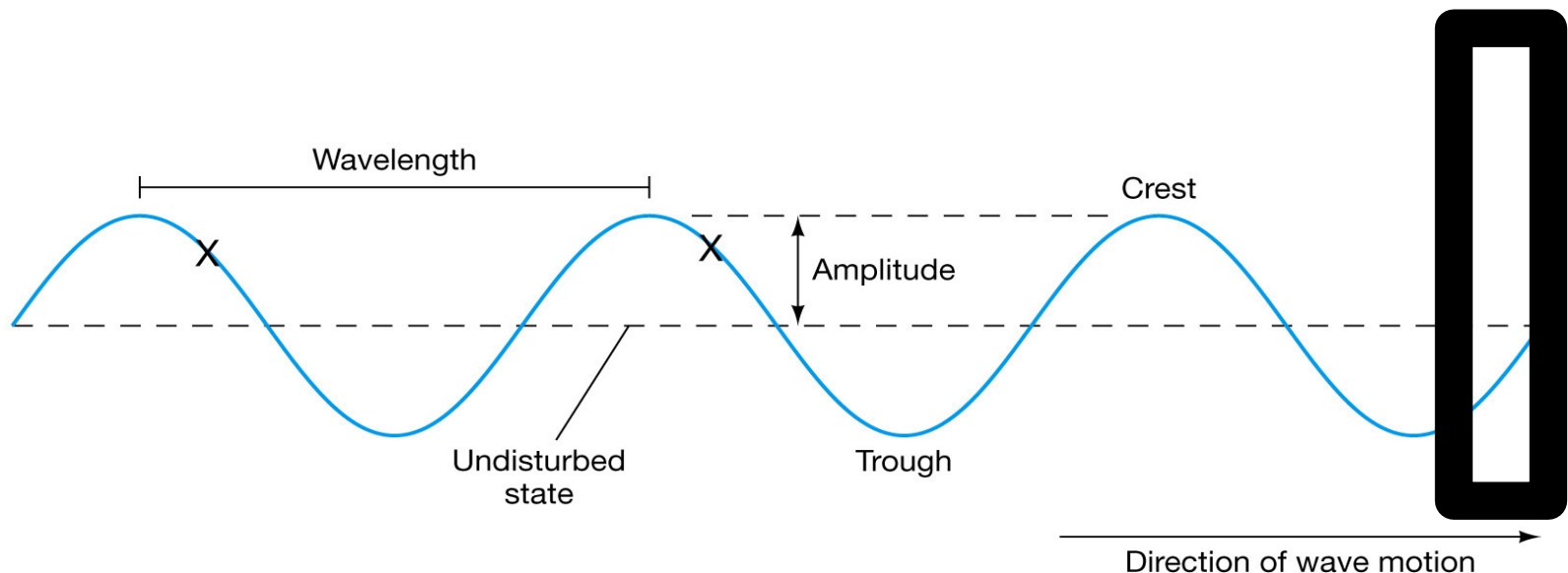
Waves - terminology



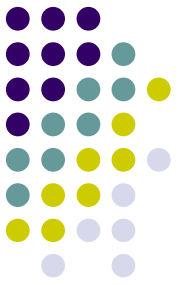
Frequency: number of wave crests that pass a given point per second

Period: time between passage of successive crests

Relationship: $\text{Frequency} = 1 / \text{Period}$



Waves - terminology

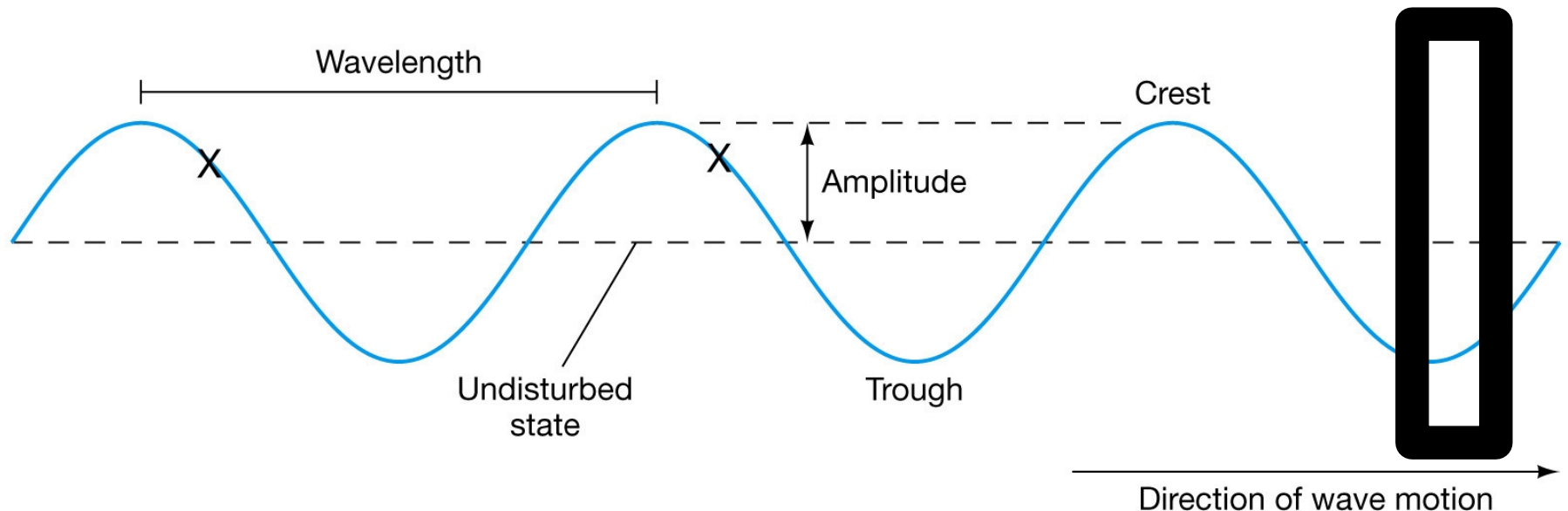


Wavelength: distance between successive crests

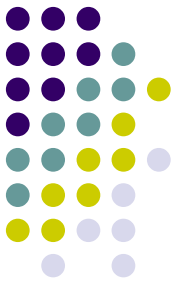
Velocity: speed at which crests move

$$\text{Velocity} = \text{Wavelength} / \text{Period}$$

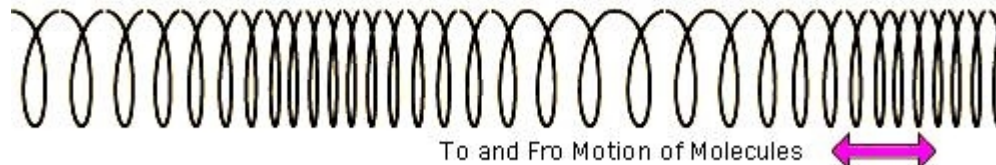
$$\text{Velocity} = \text{Wavelength} * \text{frequency}$$



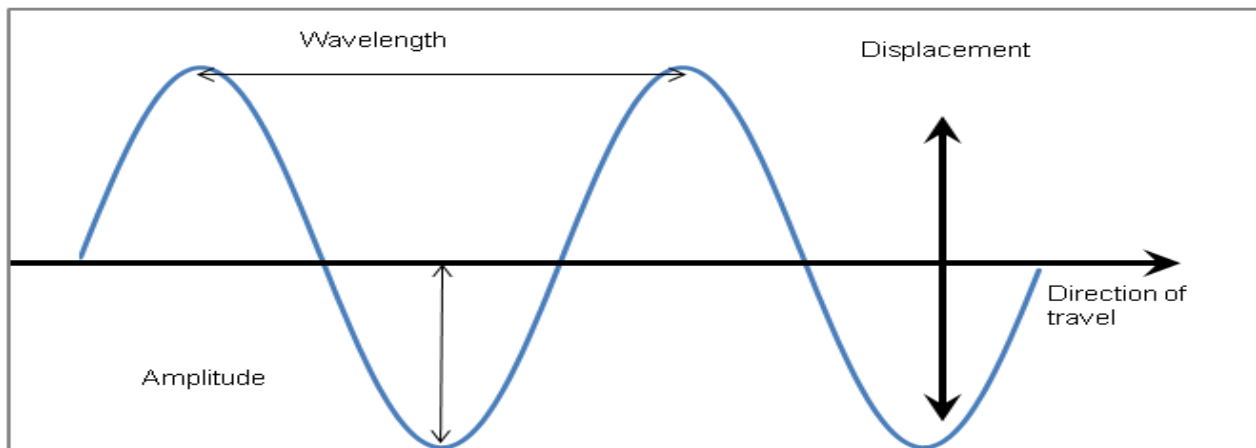
Waves - terminology



Longitudinal wave: propagates in a direction parallel to the displacement of the medium



Transverse wave: propagates in a direction perpendicular (or transverse) to the displacement of the medium



DEMO: long. and transv. waves in a SLINKY! Standing waves!

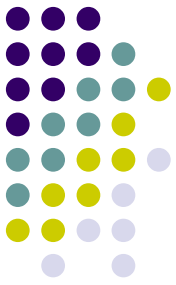
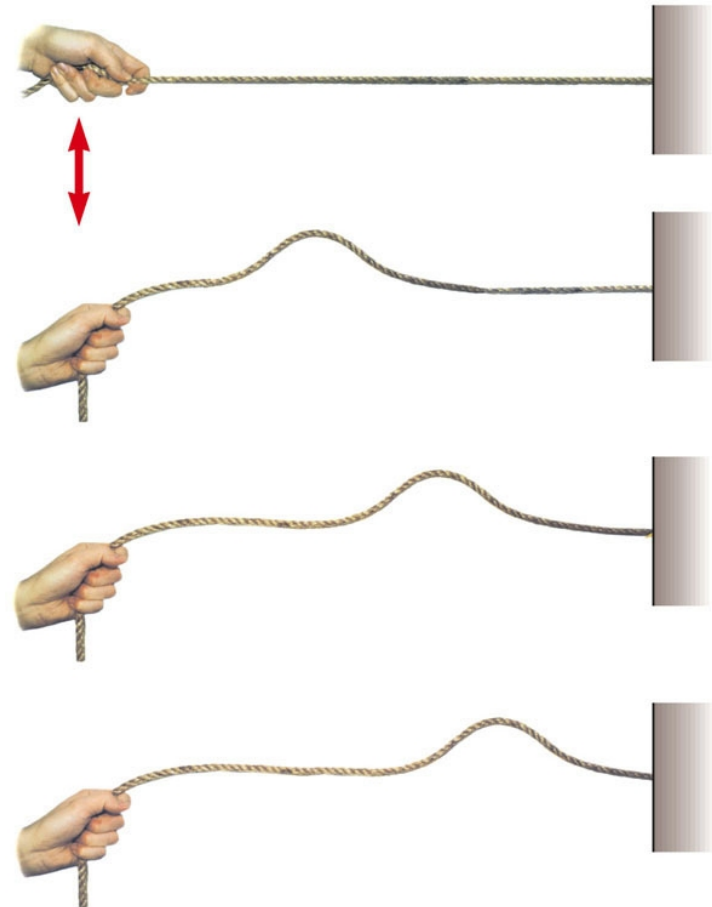
Mechanical Wave Requirements



- Some source of disturbance
- A medium that can be disturbed
- Some physical mechanism through which elements of the medium can influence each other

Pulse on a String

- The wave is generated by a flick on one end of the string
- The string is under tension
- A single bump is formed and travels along the string
 - The bump is called a **pulse**



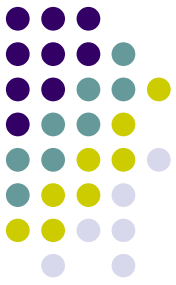
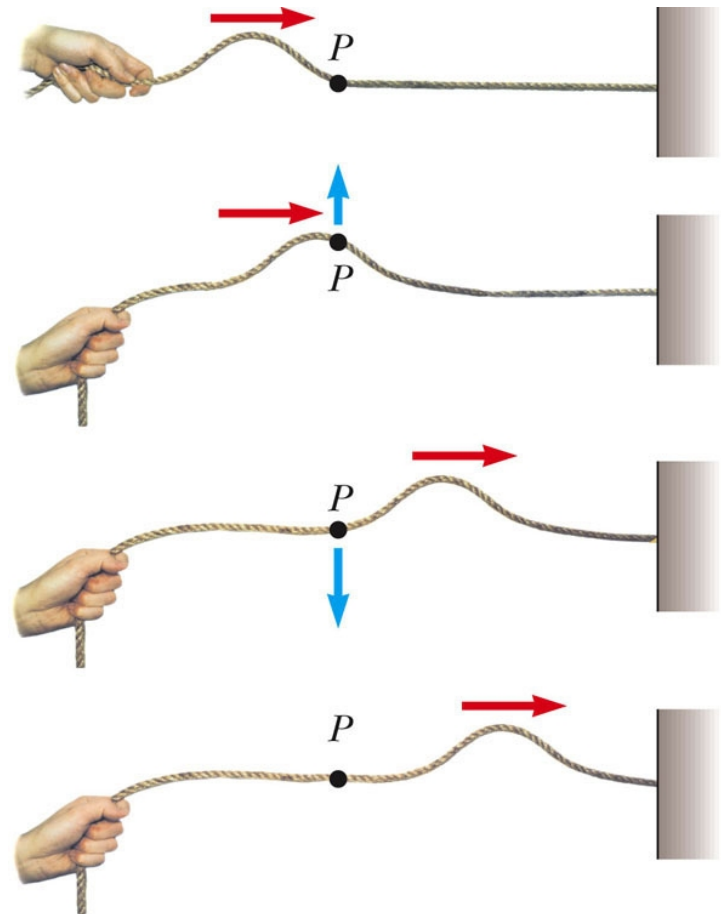


Pulse on a String

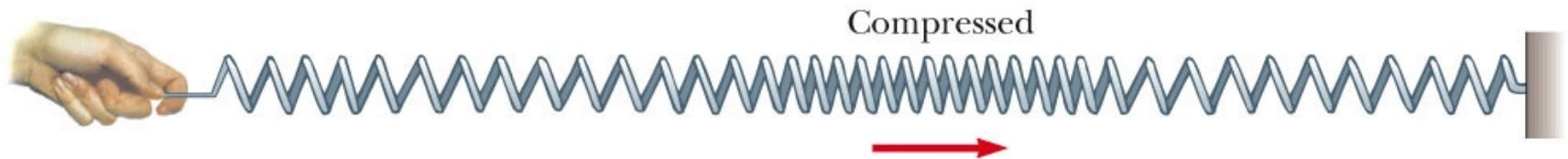
- The string is the medium through which the pulse travels
- The pulse has a definite height
- The pulse has a definite speed of propagation along the medium
- The shape of the pulse changes very little as it travels along the string
- A continuous flicking of the string would produce a periodic disturbance which would form a wave

Transverse Wave

- A traveling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation is called a **transverse wave**
- The particle motion is shown by the blue arrow
- The direction of propagation is shown by the red arrow



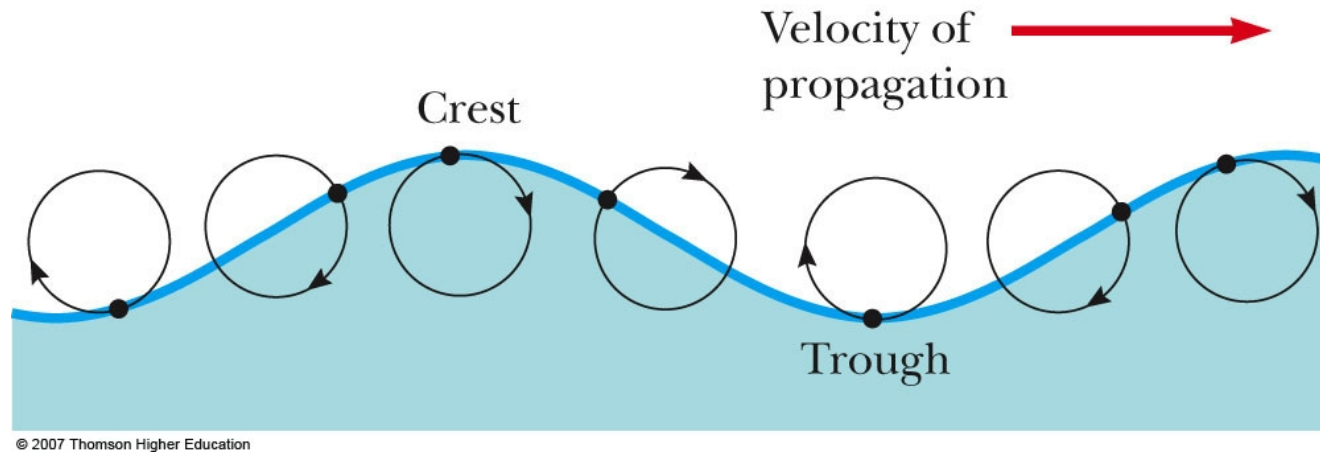
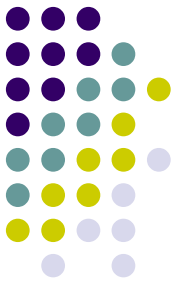
Longitudinal Wave



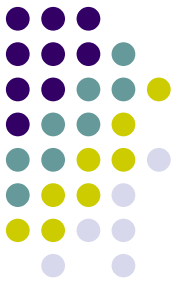
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- A traveling wave or pulse that causes the elements of the disturbed medium to move parallel to the direction of propagation is called a **longitudinal wave**
- The displacement of the coils is parallel to the propagation

Complex Waves



- Some waves exhibit a combination of transverse and longitudinal waves
- Surface water waves are an example
- Use the active figure to observe the displacements



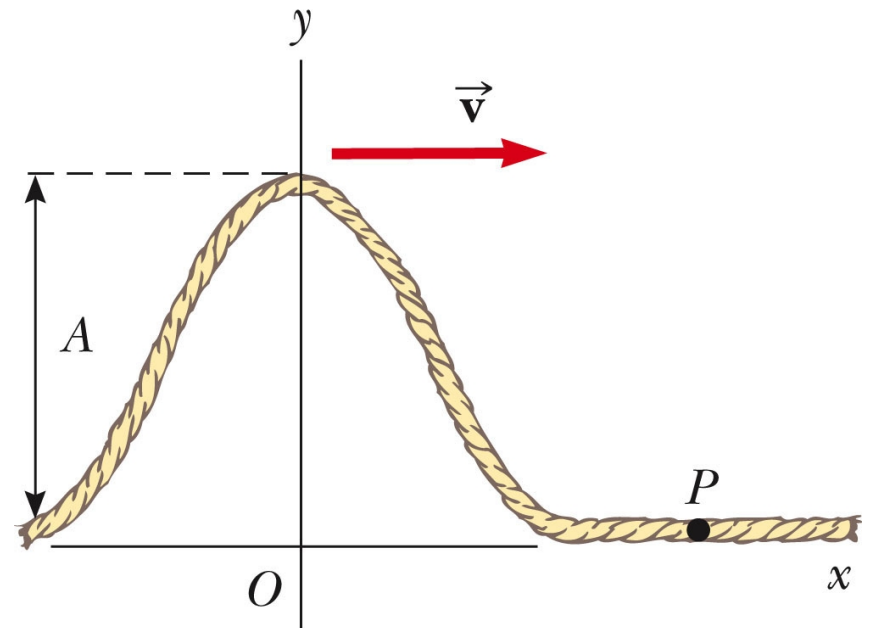
Example: Earthquake Waves

- P waves
 - “P” stands for primary
 - Fastest, at 7 – 8 km / s
 - Longitudinal
- S waves
 - “S” stands for secondary
 - Slower, at 4 – 5 km/s
 - Transverse
- A seismograph records the waves and allows determination of information about the earthquake’s place of origin



Traveling Pulse

- The shape of the pulse at $t = 0$ is shown
- The shape can be represented by $y(x, 0) = f(x)$
 - This describes the transverse position y of the element of the string located at each value of x at $t = 0$

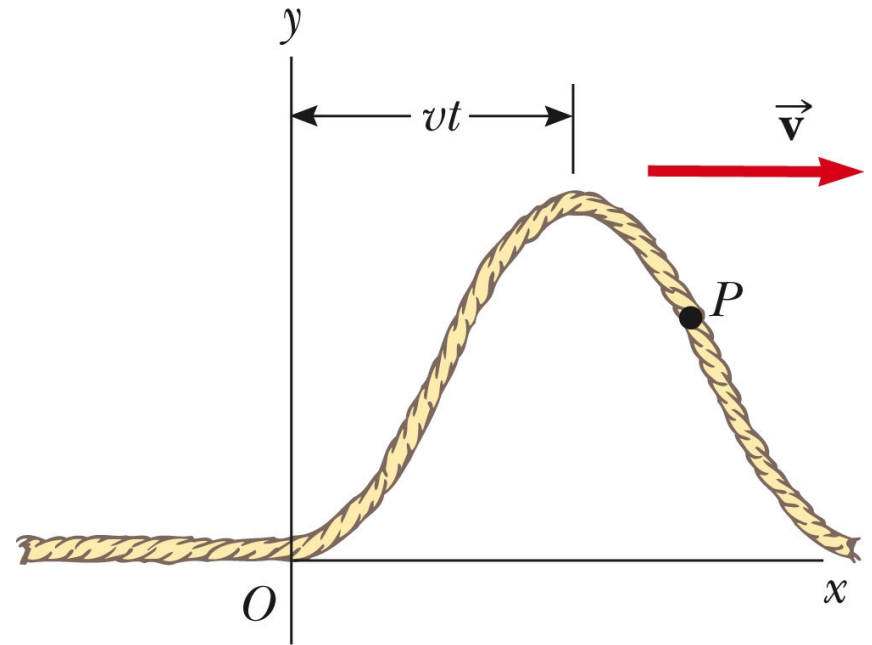


(a) Pulse at $t = 0$

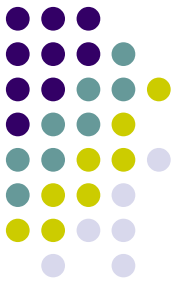
Traveling Pulse, 2



- The speed of the pulse is v
- At some time, t , the pulse has traveled a distance vt
- The shape of the pulse does not change
- Its position is now
 $y = f(x - vt)$



(b) Pulse at time t



Traveling Pulse, 3

- For a pulse traveling to the right
 - $y(x, t) = f(x - vt)$
- For a pulse traveling to the left
 - $y(x, t) = f(x + vt)$
- The function y is also called the **wave function**:
 $y(x, t)$
- The wave function represents the y coordinate of any element located at position x at any time t
 - The y coordinate is the transverse position



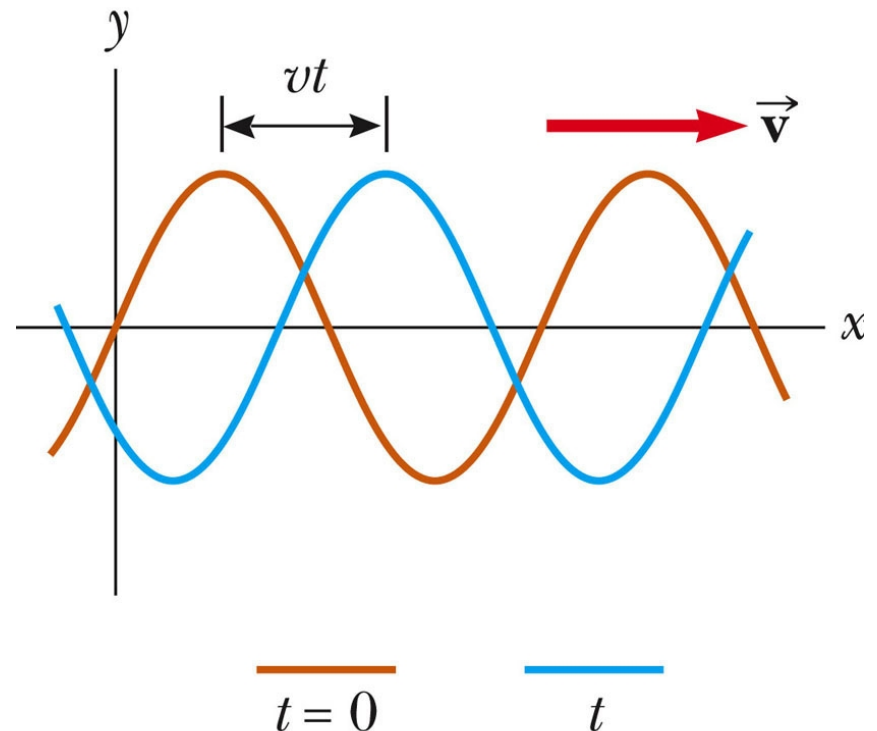
Traveling Pulse, final

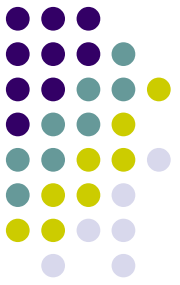
- If t is fixed then the wave function is called the **waveform**
 - It defines a curve representing the actual geometric shape of the pulse at that time

Sinusoidal Waves



- The wave represented by the curve shown is a **sinusoidal wave**
- It is the same curve as $\sin \theta$ plotted against θ
- This is the simplest example of a periodic continuous wave
 - It can be used to build more complex waves





Sinusoidal Waves, cont

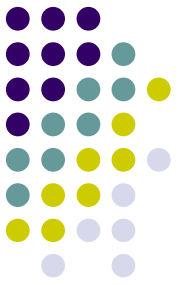
- The wave moves toward the right
 - In the previous example, the brown wave represents the initial position
 - As the wave moves toward the right, it will eventually be at the position of the blue curve
- Each element moves up and down in simple harmonic motion
- It is important to distinguish between the motion of the wave and the motion of the particles of the medium



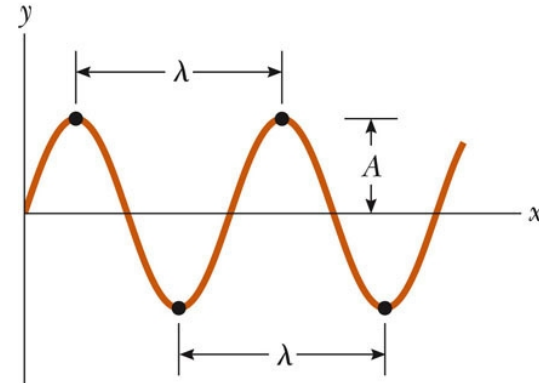
Wave Model

- The wave model is a new simplification model
 - Allows to explore more analysis models for solving problems
 - An ideal wave has a single frequency
 - An ideal wave is infinitely long
 - Ideal waves can be combined

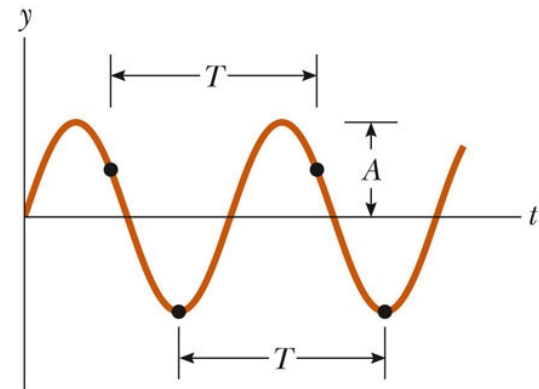
Terminology: Amplitude and Wavelength



- The **crest** of the wave is the location of the maximum displacement of the element from its normal position
 - This distance is called the **amplitude**, A
- The **wavelength**, λ , is the distance from one crest to the next

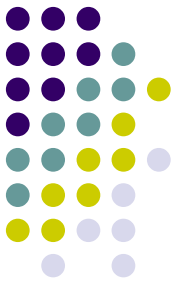


(a)

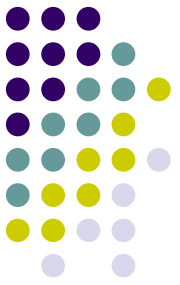


(b)

Terminology: Wavelength and Period

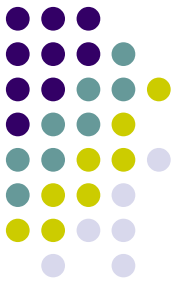


- More generally, the wavelength is the minimum distance between any two identical points on adjacent waves
- The period, T , is the time interval required for two identical points of adjacent waves to pass by a point
 - The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium



Terminology: Frequency

- The **frequency**, f , is the number of crests (or any point on the wave) that pass a given point in a unit time interval
 - The time interval is most commonly the second
 - The frequency of the wave is the same as the frequency of the simple harmonic motion of one element of the medium



Terminology: Frequency, cont

- The frequency and the period are related

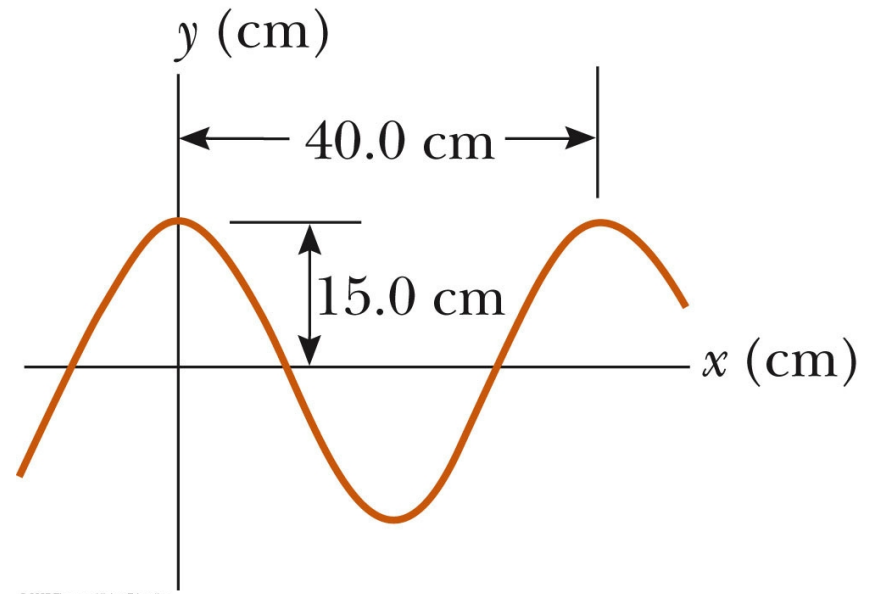
$$f = \frac{1}{T}$$

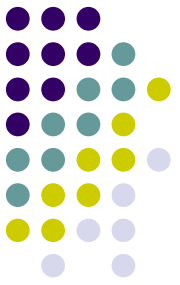
- When the time interval is the second, the units of frequency are $\text{s}^{-1} = \text{Hz}$
 - Hz is a hertz



Terminology, Example

- The wavelength, λ , is 40.0 cm
- The amplitude, A , is 15.0 cm
- The wave function can be written in the form $y = A \cos(kx - \omega t)$





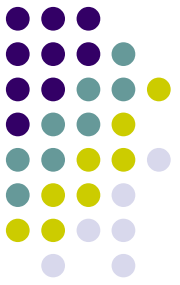
Speed of Waves

- Waves travel with a specific speed
 - The speed depends on the properties of the medium being disturbed

- The wave function is given by

$$y(x,t) = A \sin \left[\frac{2\pi}{\lambda} (x - vt) \right]$$

- This is for a wave moving to the right
- For a wave moving to the left, replace $x - vt$ with $x + vt$



Wave Function, Another Form

- Since speed is distance divided by time,
 $v = \lambda / T$
- The wave function can then be expressed as

$$y(x, t) = A \sin 2\pi \left[\frac{x}{\lambda} - \frac{t}{T} \right]$$

- This form shows the periodic nature of y
 - y can be used as shorthand notation for $y(x, t)$



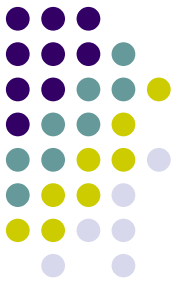
Wave Equations

- We can also define the angular wave number (or just wave number), k

$$k = \frac{2\pi}{\lambda}$$

- The angular frequency can also be defined

$$\omega = \frac{2\pi}{T} = 2\pi f$$



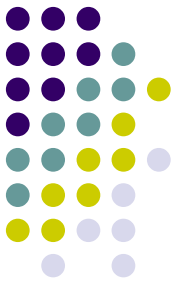
Wave Equations, cont

- The wave function can be expressed as
$$y = A \sin (k x - \omega t)$$
- The speed of the wave becomes $v = \lambda f$
- If $y \neq 0$ at $t = 0$ and $x=0$, the wave function can be generalized to

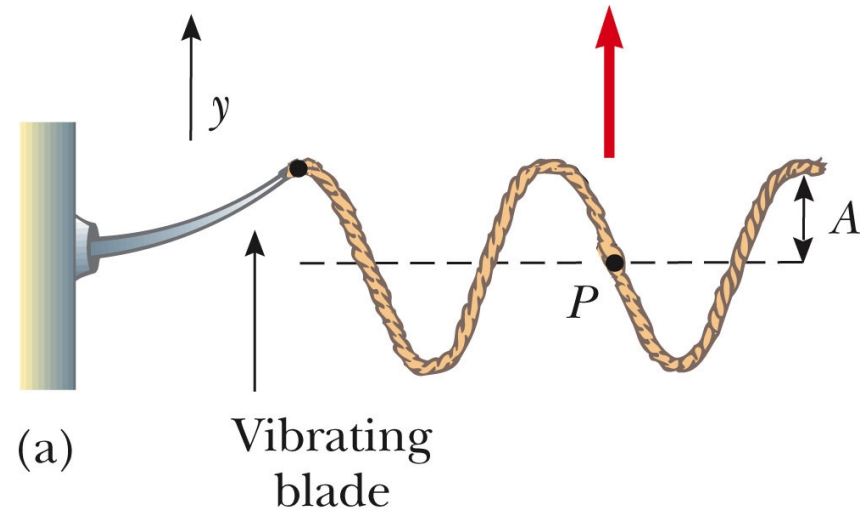
$$y = A \sin (k x - \omega t + \phi)$$

where ϕ is called the phase constant

Sinusoidal Wave on a String



- To create a series of pulses, the string can be attached to an oscillating blade
- The wave consists of a series of identical waveforms
- The relationships between speed, velocity, and period hold

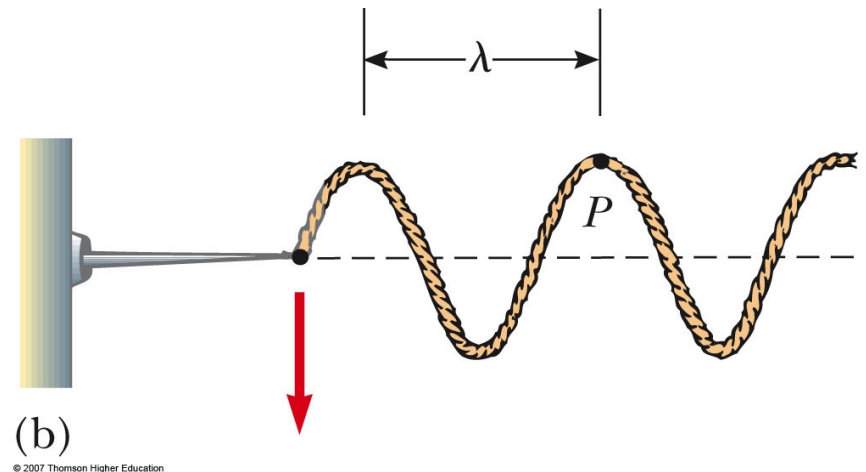


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Sinusoidal Wave on a String, 2



- Each element of the string oscillates vertically with simple harmonic motion
 - For example, point P
- Every element of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of the oscillation of the blade



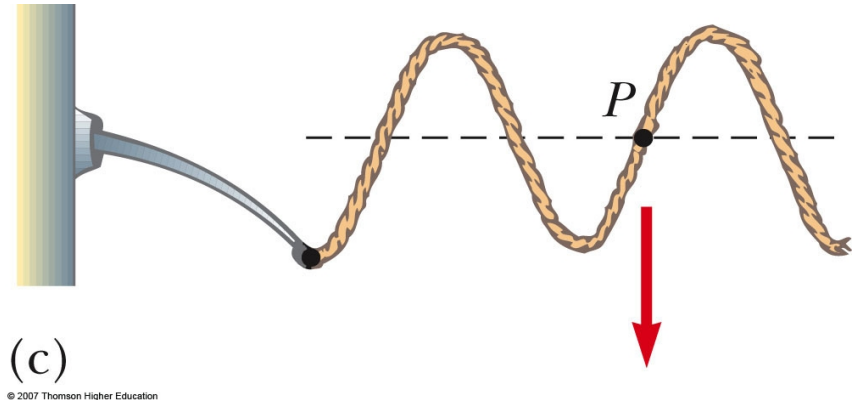
Sinusoidal Wave on a String, 3



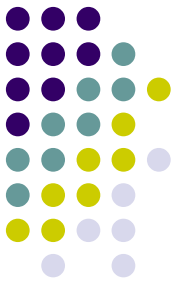
- The transverse speed of the element is

$$v_y = \left. \frac{dy}{dt} \right]_{x=\text{constant}}$$

- or $v_y = -\omega A \cos(kx - \omega t)$
- This is different than the speed of the wave itself

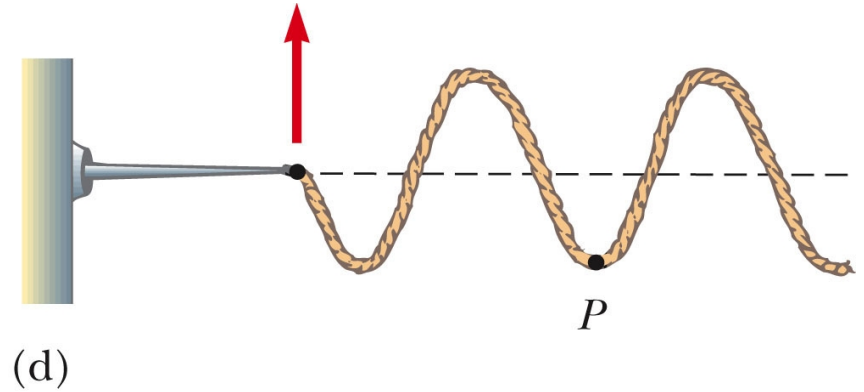


Sinusoidal Wave on a String, 4



- The transverse acceleration of the element is

$$a_y = \left. \frac{dv_y}{dt} \right]_{x=\text{constant}}$$



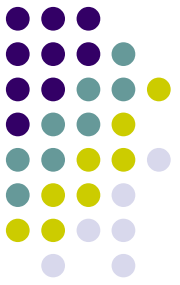
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- or $a_y = -\omega^2 A \sin(kx - \omega t)$

Sinusoidal Wave on a String, 5



- The maximum values of the transverse speed and transverse acceleration are
 - $v_{y, \max} = \omega A$
 - $a_{y, \max} = \omega^2 A$
- The transverse speed and acceleration do not reach their maximum values simultaneously
 - v is a maximum at $y = 0$
 - a is a maximum at $y = \pm A$



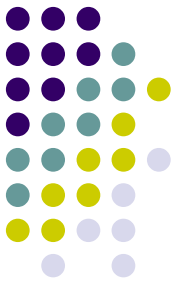
Speed of a Wave on a String

- The speed of the wave depends on the physical characteristics of the string and the tension to which the string is subjected

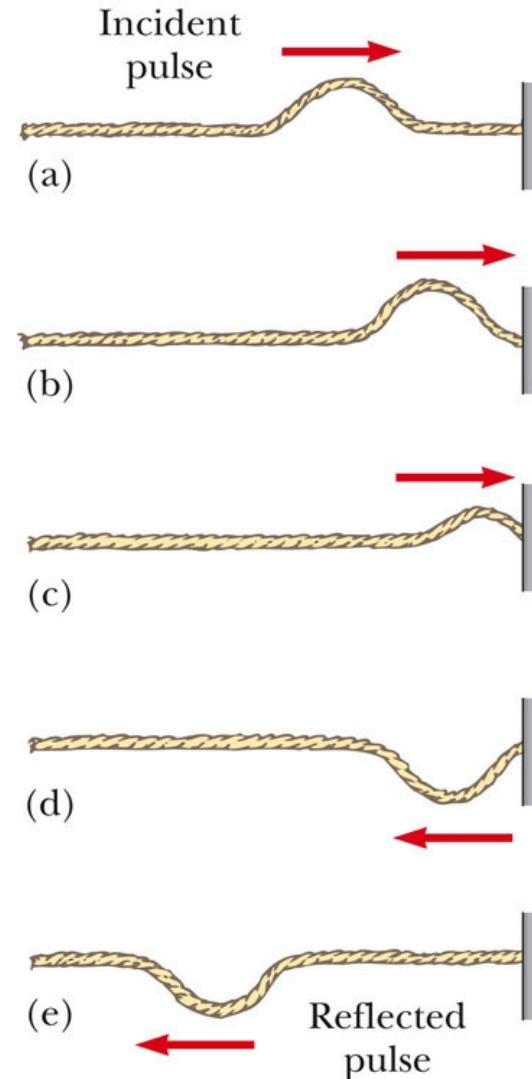
$$v = \sqrt{\frac{\text{tension}}{\text{mass/length}}} = \sqrt{\frac{T}{\mu}}$$

- This assumes that the tension is not affected by the pulse
- This does not assume any particular shape for the pulse

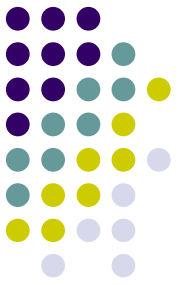
Reflection of a Wave, Fixed End



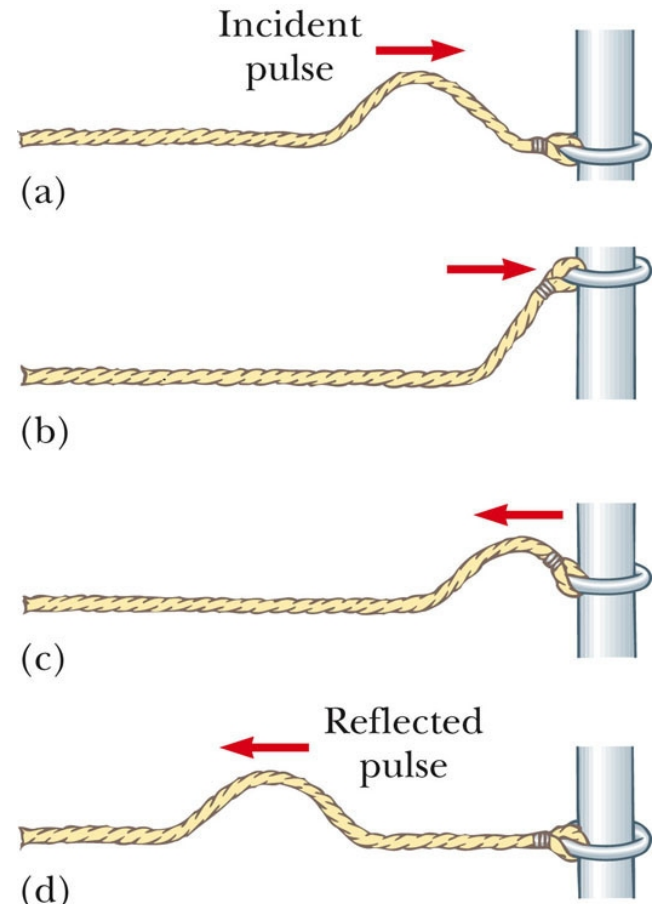
- When the pulse reaches the support, the pulse moves back along the string in the opposite direction
- This is the **reflection** of the pulse
- The pulse is inverted



Reflection of a Wave, Free End



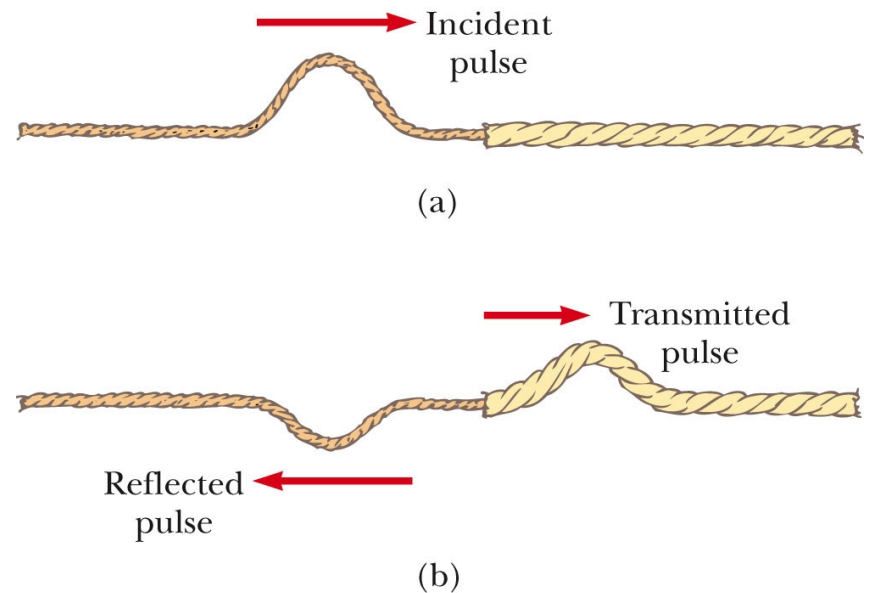
- With a free end, the string is free to move vertically
- The pulse is reflected
- The pulse is not inverted
- The reflected pulse has the same amplitude as the initial pulse

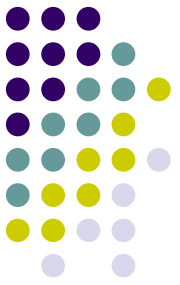




Transmission of a Wave

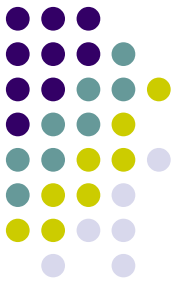
- When the boundary is intermediate between the last two extremes
 - Part of the energy in the incident pulse is reflected and part undergoes **transmission**
 - Some energy passes through the boundary





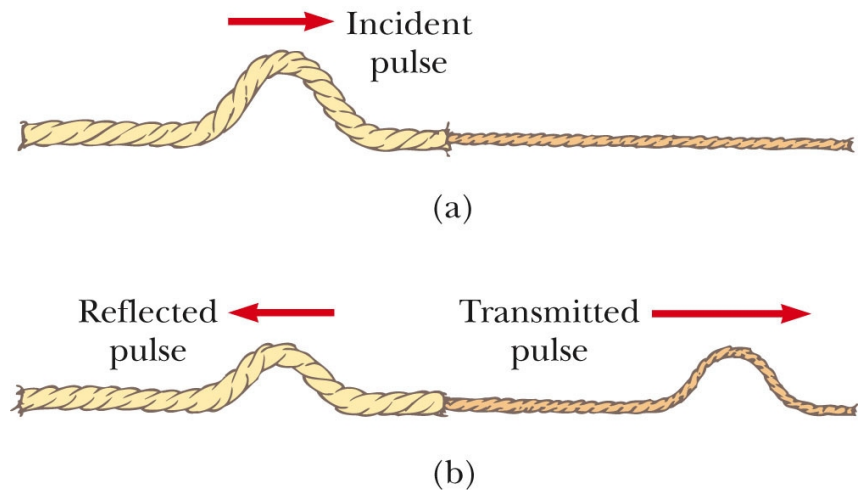
Transmission of a Wave, 2

- Assume a light string is attached to a heavier string
- The pulse travels through the light string and reaches the boundary
- The part of the pulse that is reflected is inverted
- The reflected pulse has a smaller amplitude



Transmission of a Wave, 3

- Assume a heavier string is attached to a light string
- Part of the pulse is reflected and part is transmitted
- The reflected part is not inverted



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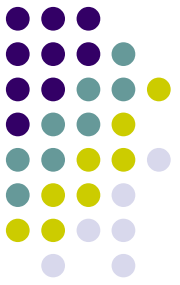
Transmission of a Wave, 4

- Conservation of energy governs the pulse
 - When a pulse is broken up into reflected and transmitted parts at a boundary, the sum of the energies of the two pulses must equal the energy of the original pulse
- When a wave or pulse travels from medium A to medium B and $v_A > v_B$, it is inverted upon reflection
 - B is denser than A
- When a wave or pulse travels from medium A to medium B and $v_A < v_B$, it is not inverted upon reflection
 - A is denser than B

Energy in Waves in a String



- Waves transport energy when they propagate through a medium
- We can model each element of a string as a simple harmonic oscillator
 - The oscillation will be in the y -direction
- Every element has the same total energy



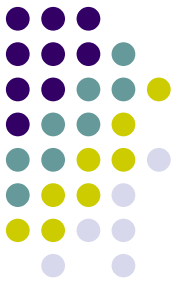
Energy, cont.

- Each element can be considered to have a mass of dm
- Its kinetic energy is $dK = \frac{1}{2} (dm) v_y^2$
- The mass dm is also equal to μdx
- The kinetic energy of an element of the string is $dK = \frac{1}{2} (\mu dx) v_y^2$



Energy, final

- Integrating over all the elements, the total kinetic energy in one wavelength is $K_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$
- The total potential energy in one wavelength is $U_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$
- This gives a total energy of
 - $E_\lambda = K_\lambda + U_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$



Power Associated with a Wave

- The power is the rate at which the energy is being transferred:

$$P = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2} \mu \omega^2 A^2 \lambda}{T} = \frac{1}{2} \mu \omega^2 A^2 v$$

- The power transfer by a sinusoidal wave on a string is proportional to the
 - Frequency squared
 - Square of the amplitude
 - Wave speed