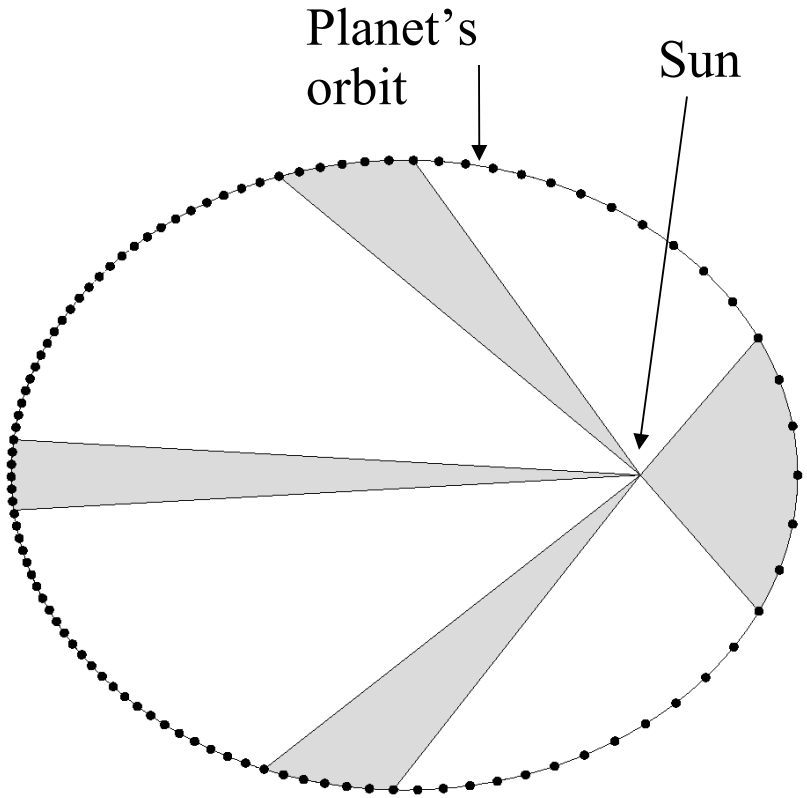
Kepler's Laws of Motion

- 1609 in Astronomica Nova (The New Astronomy)

- First Law – A planet orbits the Sun in an ellipse, with the Sun at one focus of the ellipse.
- Second Law – A line connecting a planet to the Sun sweeps out equal areas in equal time intervals
  - Several areas associated with the time interval of "six" are shown
    - They all have equal areas



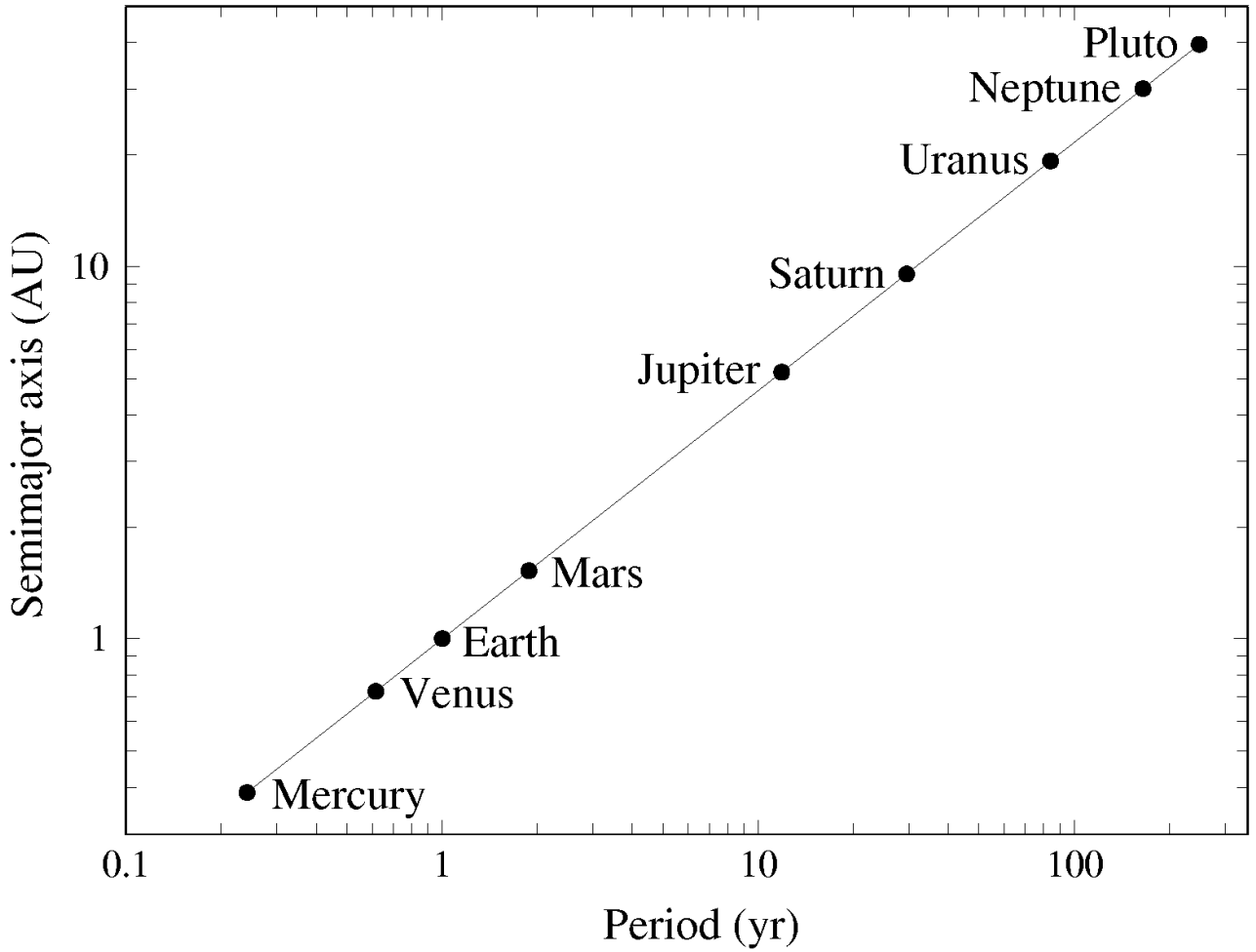
Kepler's Third Law of Motion

From *Harmonica Mundi* (1619) (Harmony of the Worlds)

$$P^2 = a^3$$

P = orbital period

a = semimajor axis



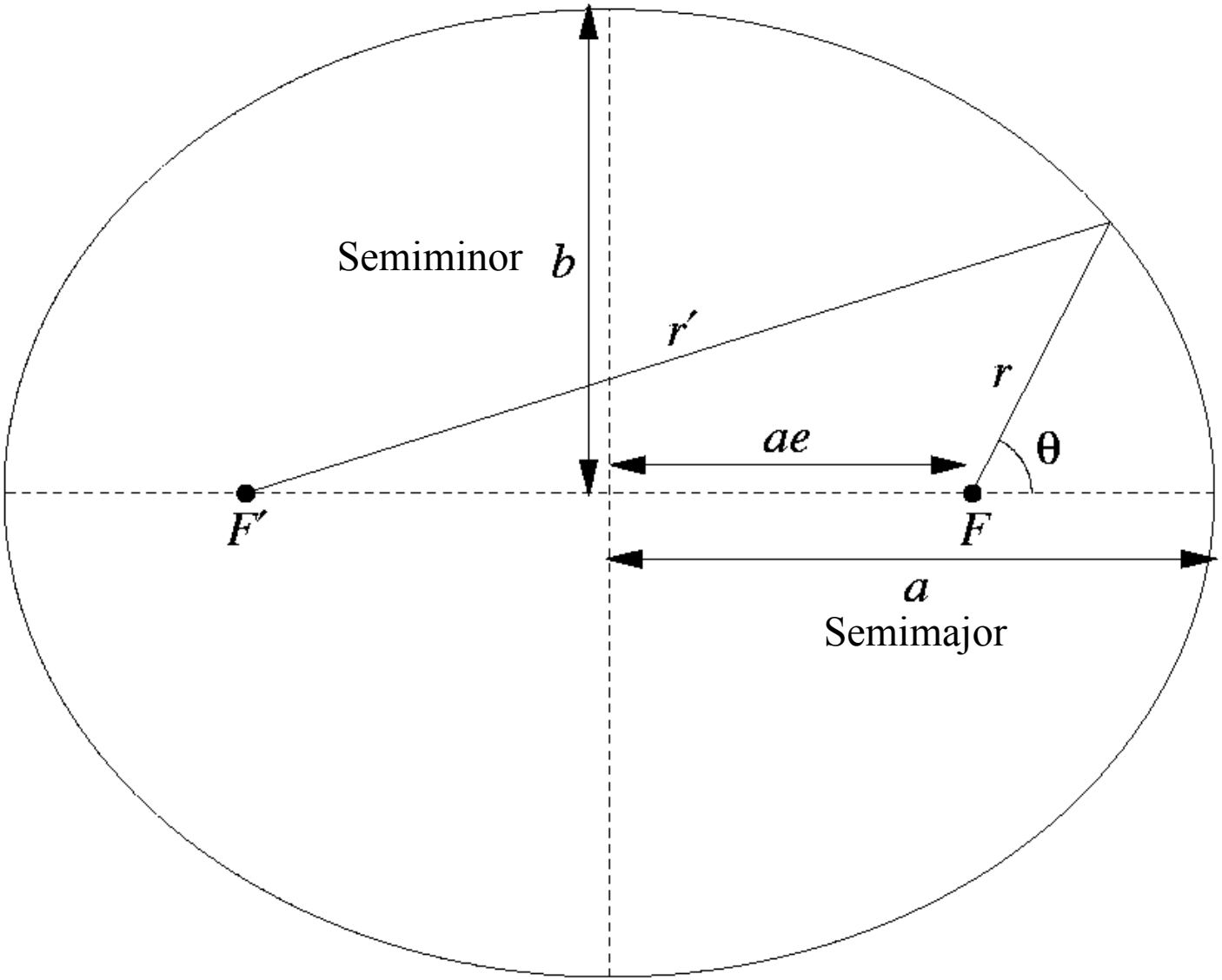
"Power law" slope is 2/3:

$$\log\left(P^2\right) = \log\left(a^3\right)$$

$$2\log\left(P\right) = 3\log\left(a\right)$$

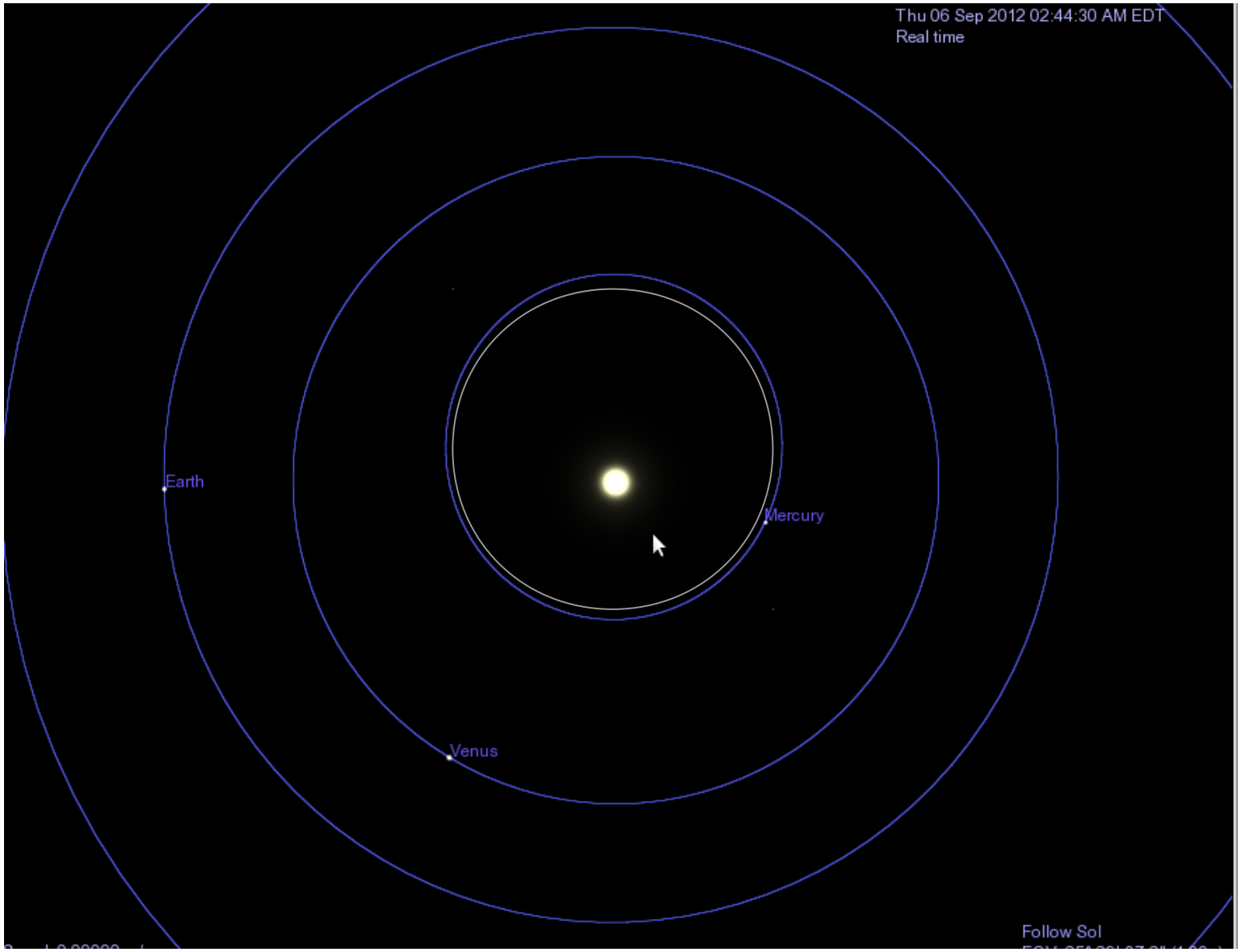
$$\log\left(a\right) = \frac{2}{3}\log\left(P\right)$$

Ellipses



Ellipses – actual orbits

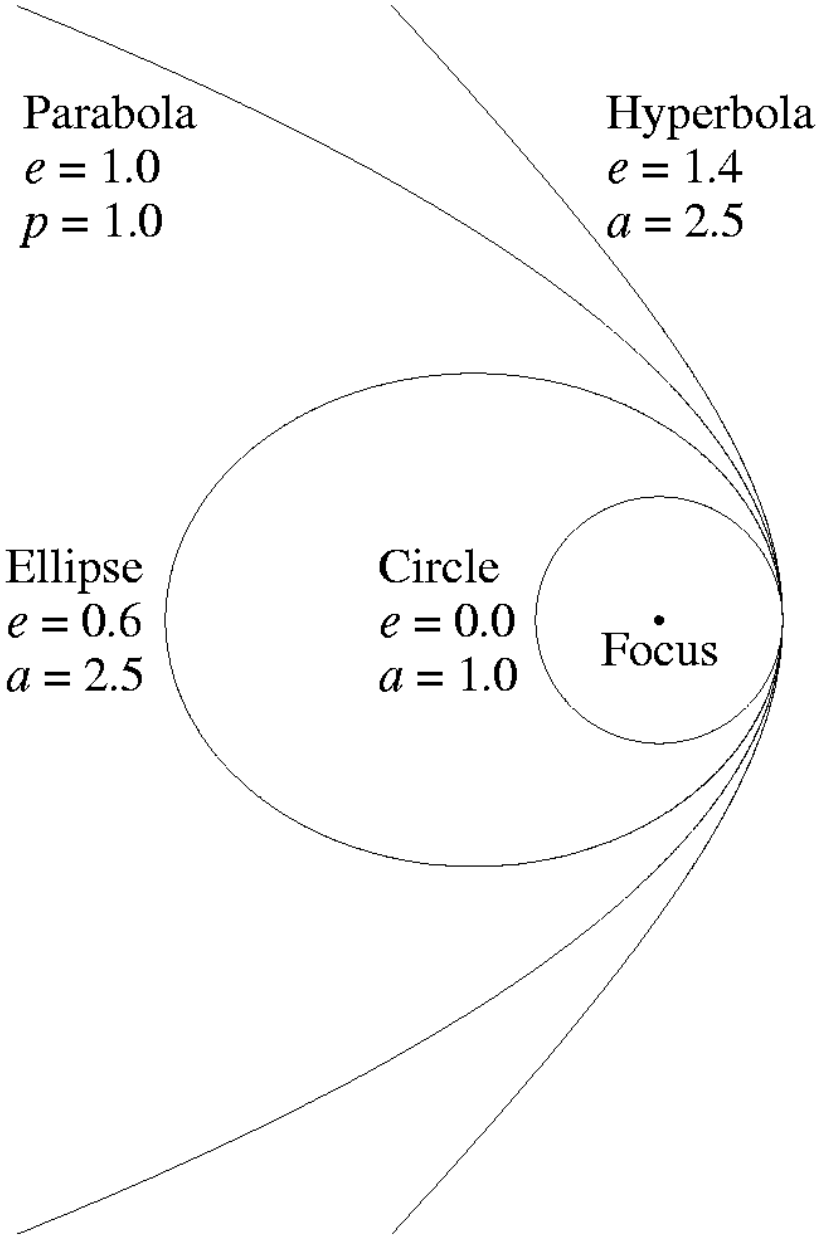
September 2012



Conic Sections

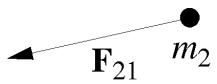
- All are possible in celestial mechanics
- Orbits are Ellipses

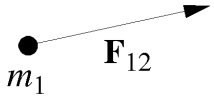
r = constant	e = 0	Circle
$r = \frac{a\left(1-e^2\right)}{1+e\cos\theta}$	0 ≤ e < 1	ellipse
$r = \frac{2p}{1+\cos\theta}$	e = 1	parabola
$r = \frac{a\left(e^2-1\right)}{1+e\cos\theta}$	e > 1	hyperbola



Newton's Laws of Motion

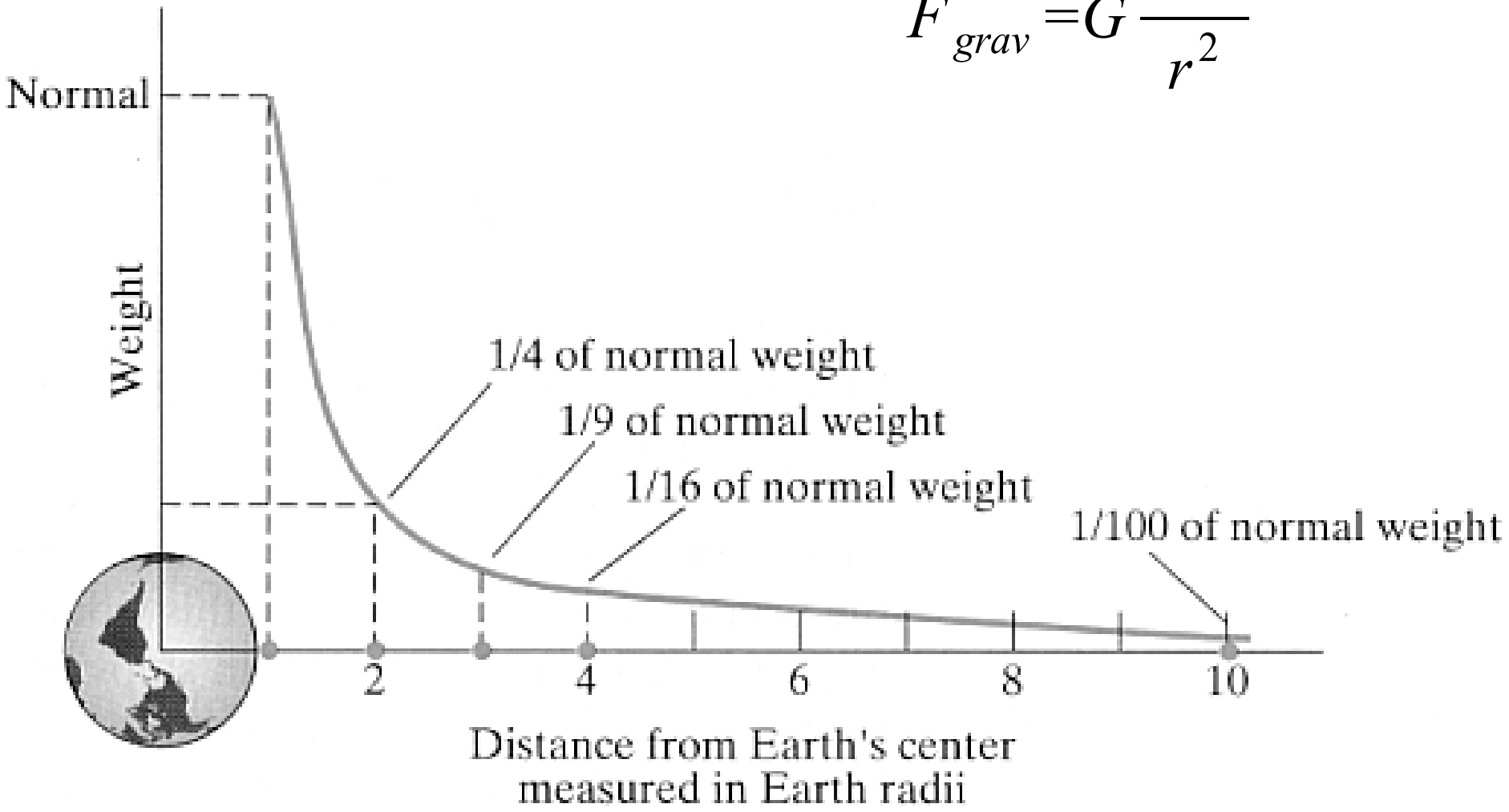
- Brachistochrone problem…
- 1st Law – Law of inertia
  - An object at rest remains at rest and an object in uniform motion remains in uniform motion unless acted upon by an unbalanced force.
  - An ***inertial reference frame*** is needed for 1st law to be valid
  - A non-inertial reference frame is being accelerated (e.g. In car going around a curve you feel a fictitious force)
- 2nd Law – $\mathbf{a} = \mathbf{F}_{net}/m$ or $\mathbf{F}_{net} = m\mathbf{a}$
  - The net force (sum of all forces) acting on an object is proportional to the object's mass and its resultant acceleration.
  - Inertial mass, m, does not appear to be different from gravitational mass
- 3rd Law
  - For every action there is an equal but opposite reaction





Universal Law of Gravitation

$$F_{grav} = G\frac{Mm}{r^2}$$

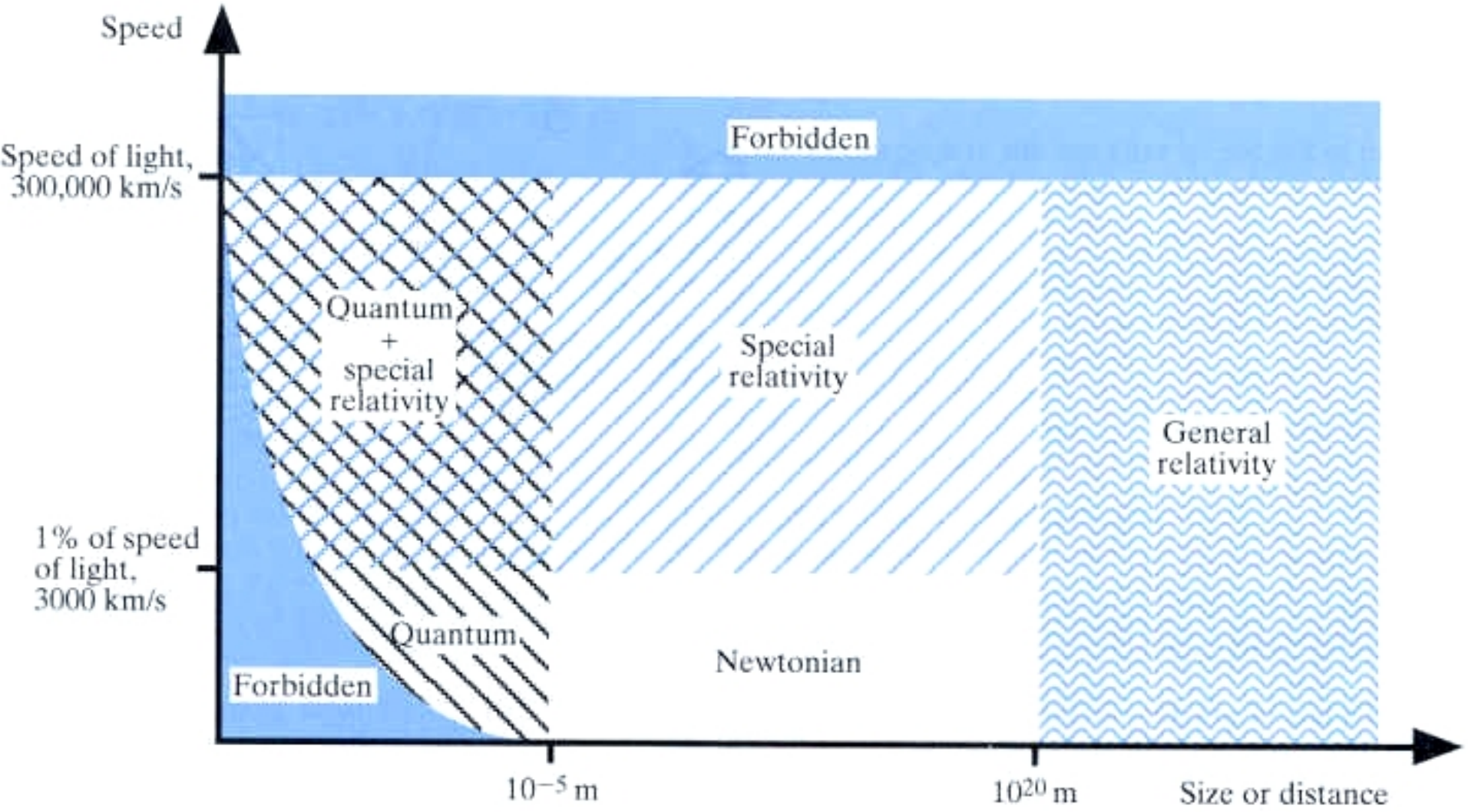


An inverse-square law.

(Light and sound intensity drop off same way.)

Beyond Newton

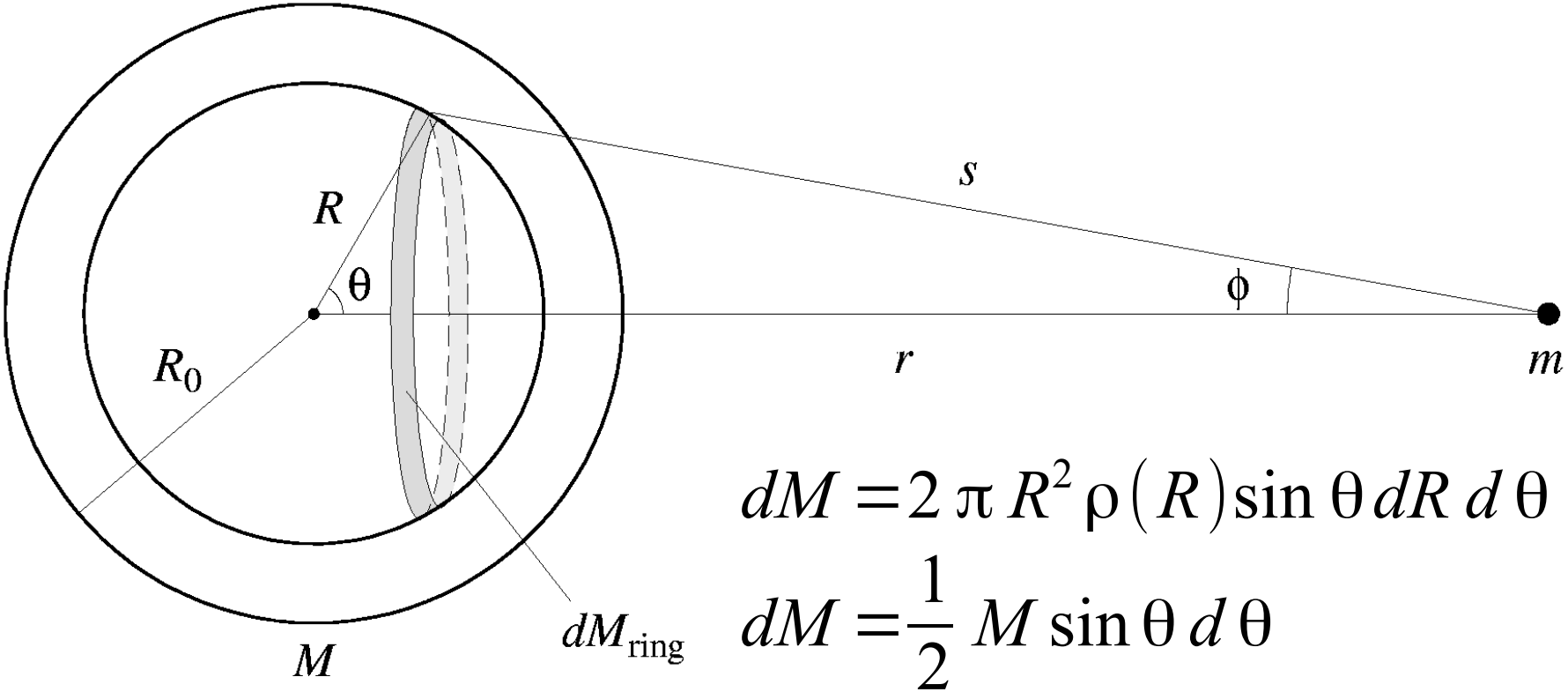
- Gravity passed every test until ~1890s
- Newton's gravity and motion is incorrect when …



Shell theorems for gravity:

) The Force on *m* due to a uniform shell of mass is the same as the force due to a point mass at the center of the shell with the same total mass as the shell.

) The force of gravity inside of a uniform shell is zero.



$$dM = 2\pi R^2 \rho(R) \sin\theta \, dR \, d\theta$$

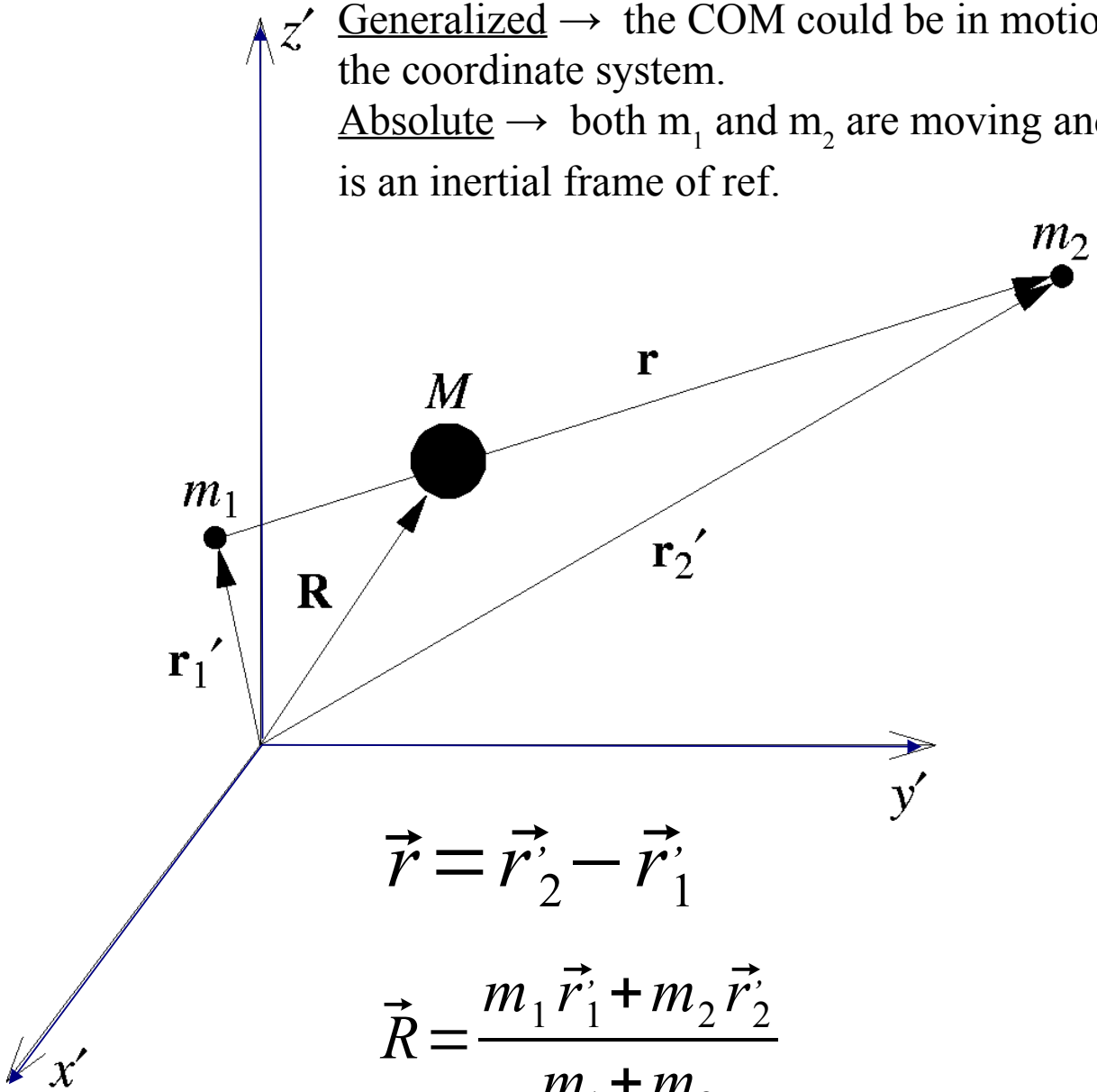
$$dM = \frac{1}{2} M \sin\theta \, d\theta$$

Binary Orbits

Generalized, absolute coordinates.

Generalized → the COM could be in motion relative to the coordinate system.

Absolute → both m_1 and m_2 are moving and the coord sys is an inertial frame of ref.



$$\vec{r} = \vec{r}_2' - \vec{r}_1'$$

$$\vec{R} = \frac{m_1 \vec{r}_1' + m_2 \vec{r}_2'}{m_1 + m_2}$$

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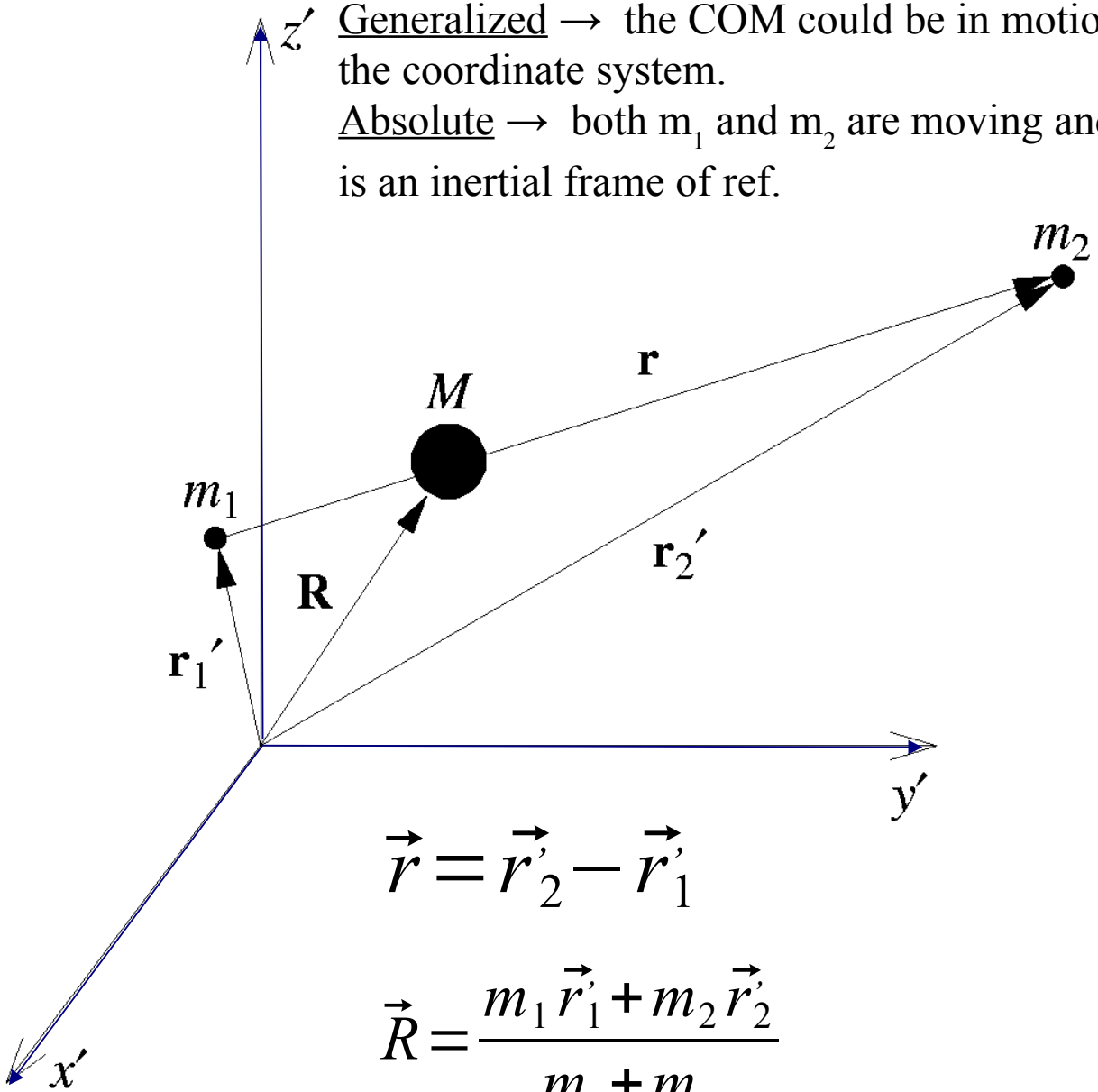

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Binary Orbits

Absolute coordinates.

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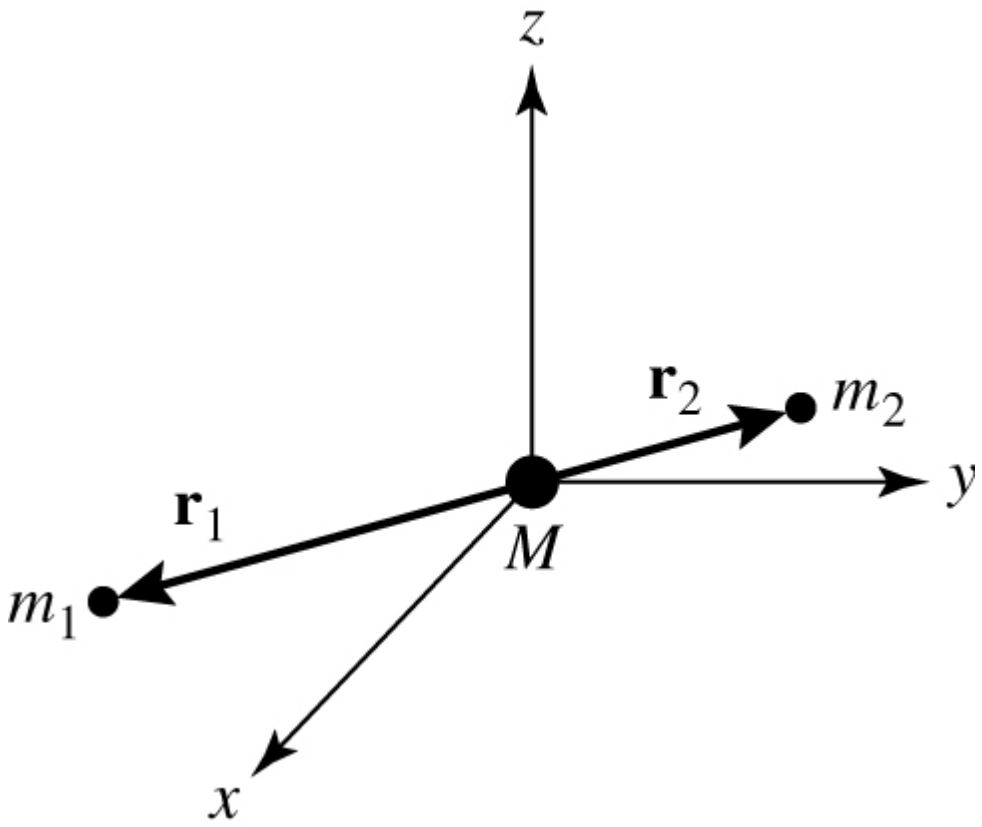
The COM is usually at the origin labeled with the total mass $M = m_1+m_2$.

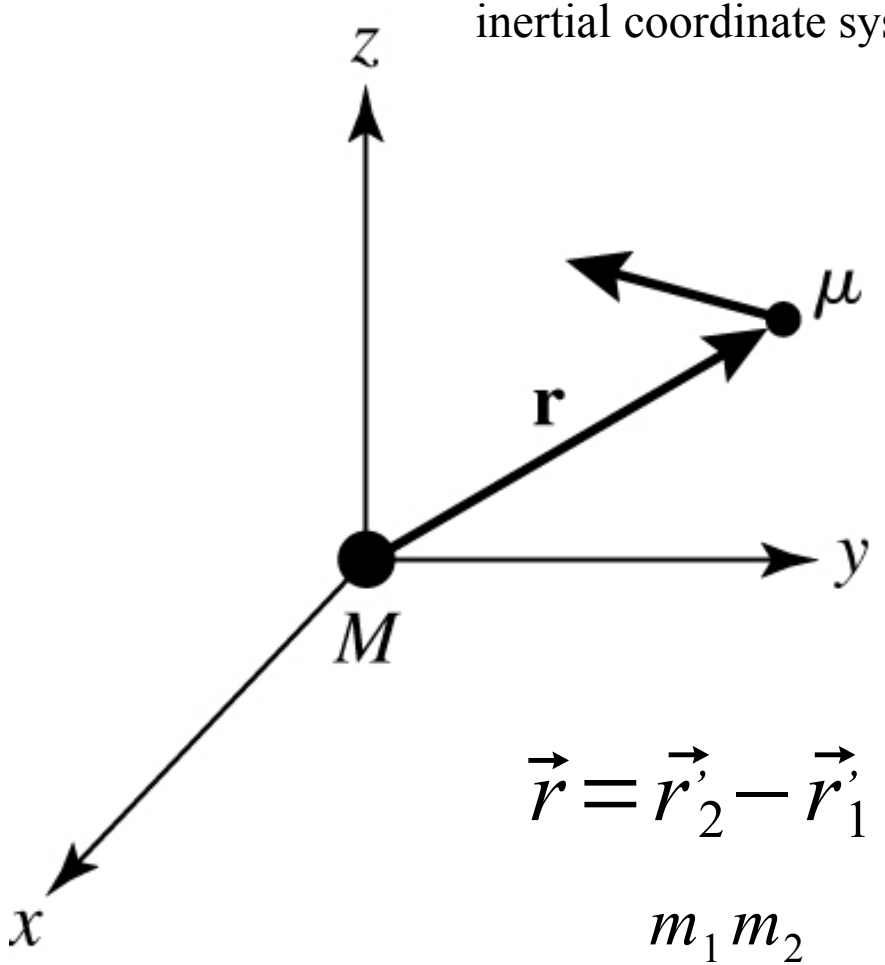


$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

Binary Orbits

Relative coordinates.

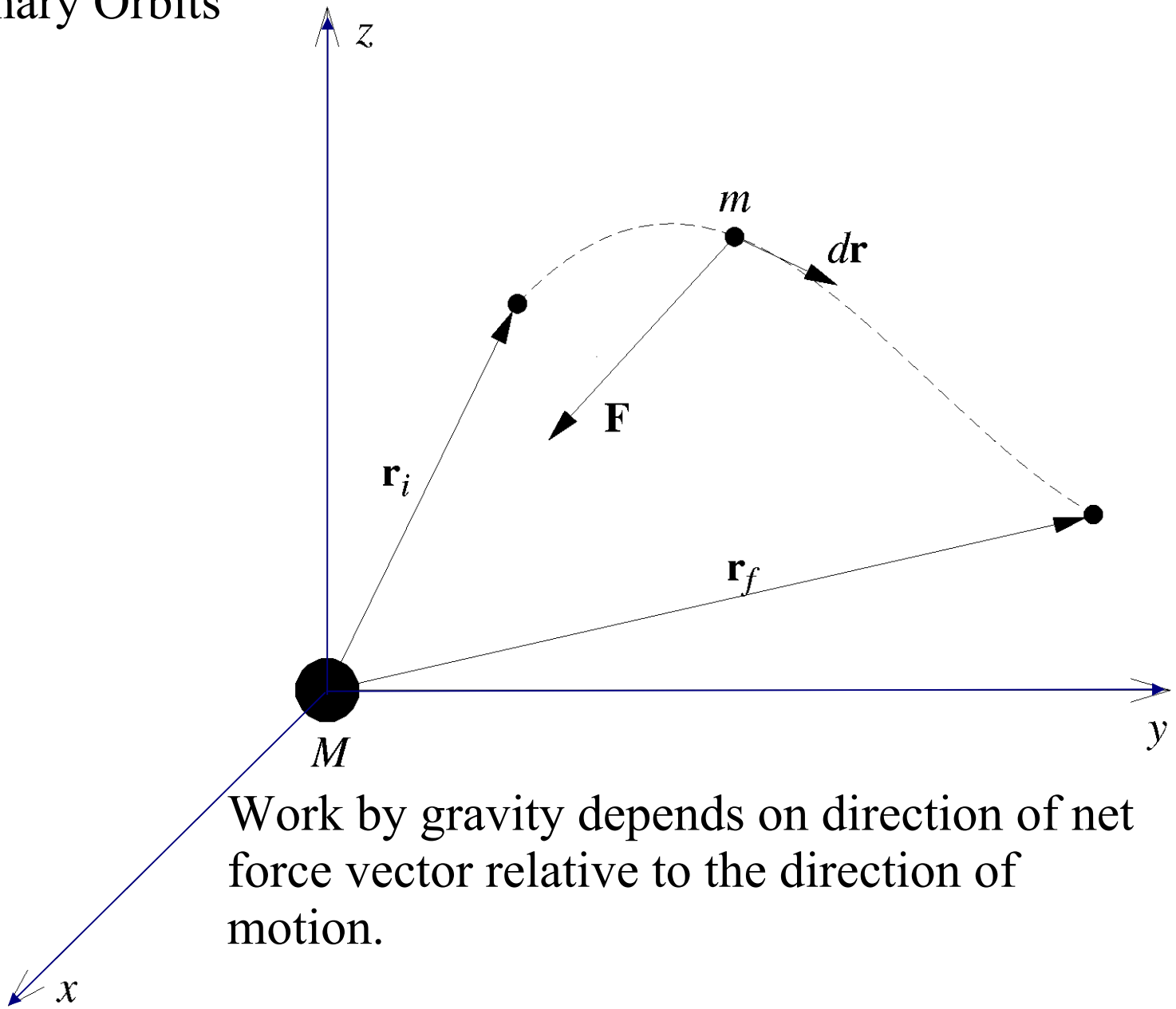
Relative → shows orbit of moving, reduced mass μ around a stationary total mass M. Since both masses move in inertial coordinate systems, this would have to be a non-inertial coordinate system.



$$\vec{r} = \vec{r}_2' - \vec{r}_1'$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Binary Orbits



Work by gravity depends on direction of net force vector relative to the direction of motion.