Chapter 15

Wave Motion

PHYS 2321
Week 13: Wave Motion / Sound

Day 1 Outline

1) Hwk: Ch. 15 Read 15.1-15.9 (less on 15.5), 16.1-16.3
   Ch. 15 P. 1,2,6,7,15,16,19,23,24,25,44,45
   MiscQ 1-9 Do by Mon aft. brk
   Ch. 16 P. 2,3,5,7,11,17,20 Due Wed aft. brk

2) Exam II on Ch. 24-29
   * Spread out, caps off
   * Eqns on last page. Scratch paper available.
   * Pencils and calculators available
   * Finish by 12:52
   * Come forward for questions
   * Show work – especially formulas

Notes:

Try Ch. 15 (wave motion) practice quiz

PHYS 2321
Week 13: Wave Motion / Sound

Day 2 Outline

1) Hwk: Ch. 15 Read 15.1-15.9 (less on 15.5), 16.1-16.3
   Ch. 15 P. 1,2,6,7,15,16,19,23,24,25,44,45
   MiscQ 1-9 Do by Mon aft. brk
   Ch. 16 P. 2,3,5,7,11,17,20 Due Wed aft. brk
2) Exam II return. Mean = 23.2/35
3) Waves

Notes:

Try Ch. 15 (wave motion) practice quiz

Waves

Wave: **a travelling disturbance or variation in a medium or field which carries energy.**

What do all types of waves have in common?

They transmit energy.

They can carry information.

They all have amplitudes, speeds, wavelengths and frequencies.

Types of Waves

Mechanical waves

Some physical medium is being disturbed

A restorative force acts between neighboring elements of the medium

Examples: in air (sound), on strings, in Earth (seismic)

Electromagnetic waves

No medium required!

Accelerating charge creates changing E and B fields

Examples: light, radio waves, x-rays, IR, gamma rays

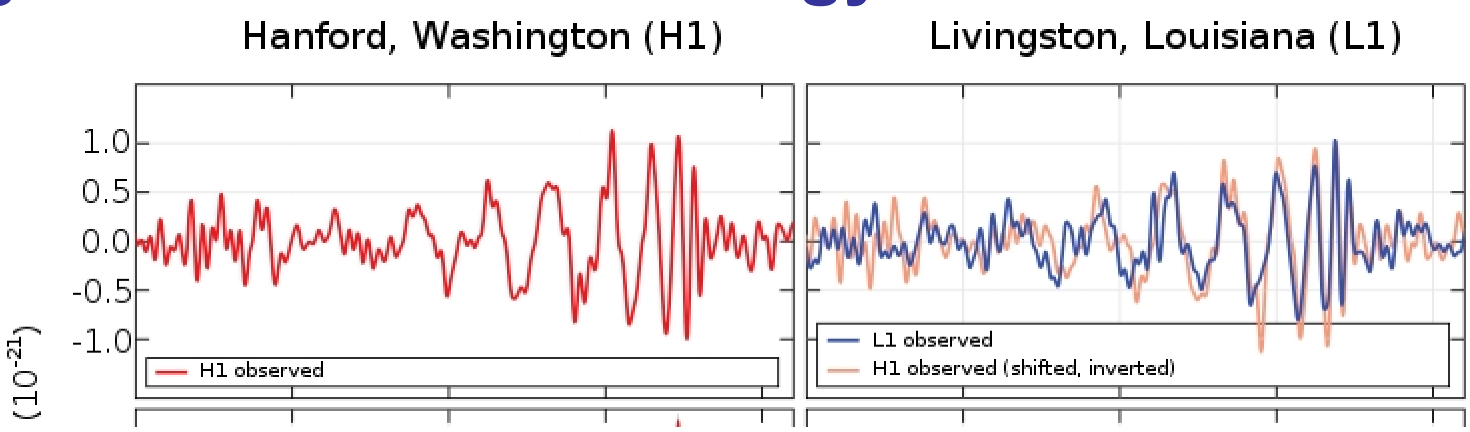
Gravitational waves (since 2016)

No medium required. Ex: inspiraling black holes!

The first gravitational wave detection: by the LIGO consortium on Feb 11, 2016.

Source: inspiralling binary black holes. One 29 $M_\odot$ and one 36 $M_\odot$. 1.3×10^9 LY away. Produced one 62 $M_\odot$ BH.

Power: momentarily greater than all of the stars in the observable universe. 3 $M_\odot$ converted into gravitational wave energy in ~0.2 seconds.



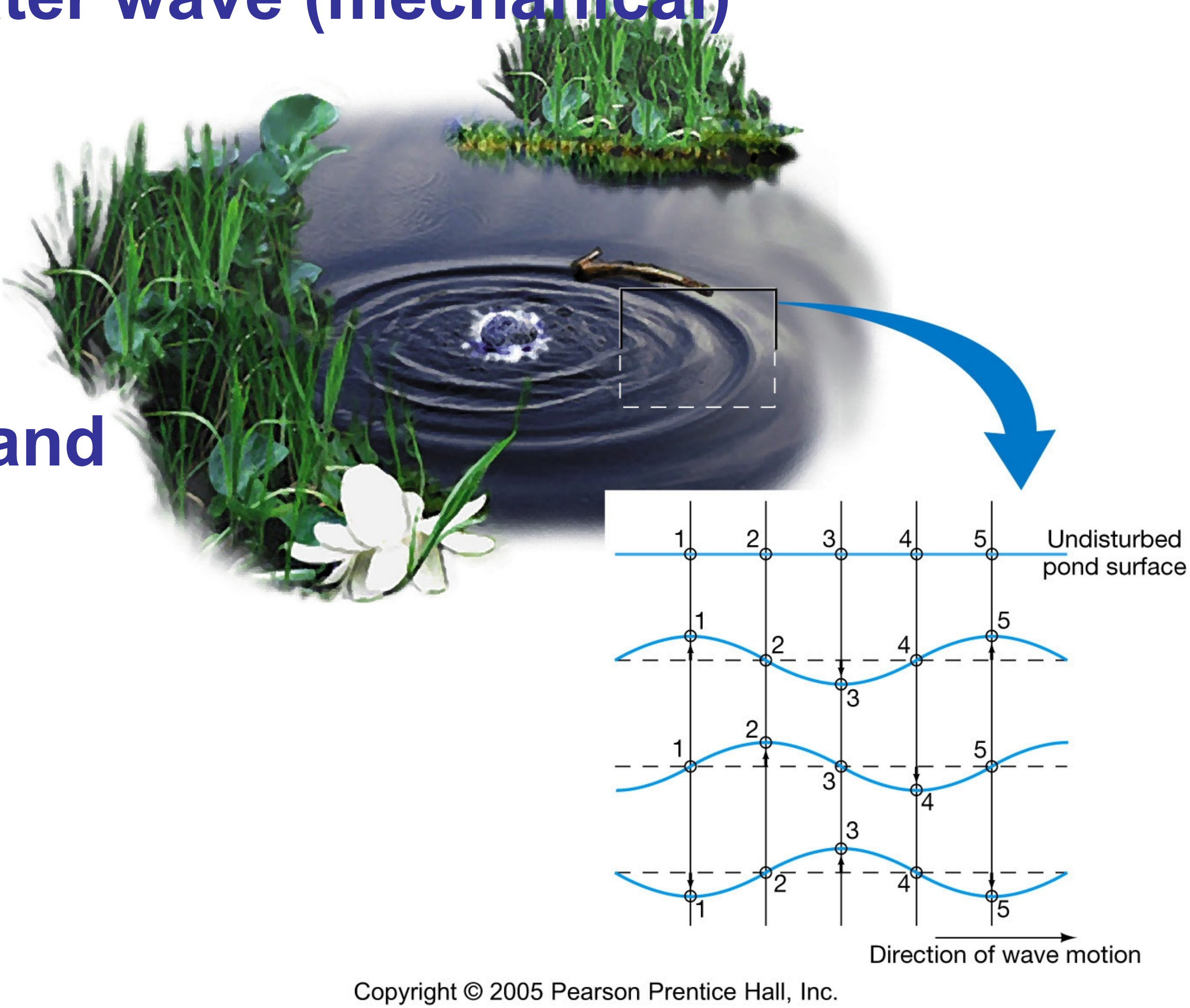
The "chirp"

Waves - terminology

Example: water wave (mechanical)

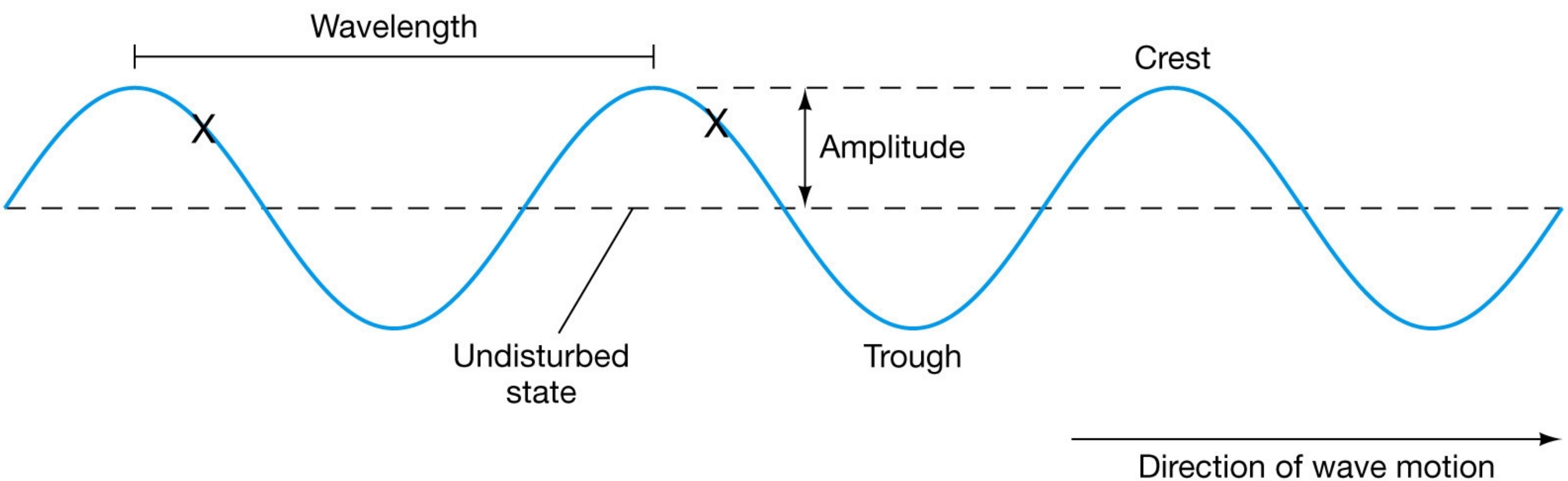
Water just moves up and down

Wave travels and can transmit energy



Waves - terminology

Sine waves: **waves described by a sine or cosine function. Also called: *"sinusoidal"***



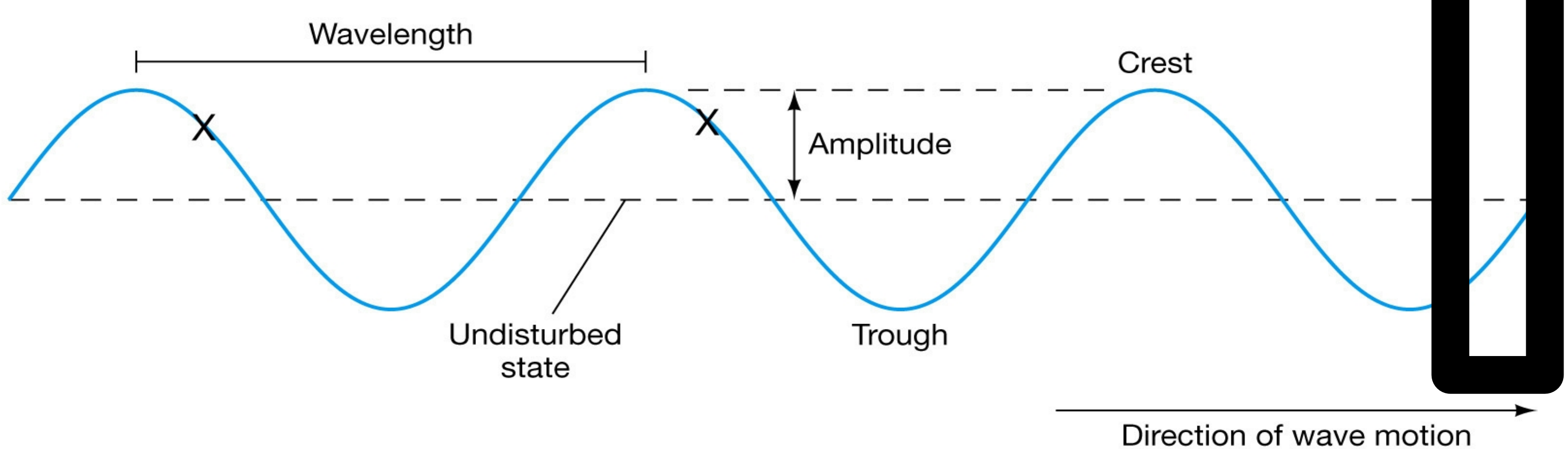
This graph shows <u>amplitude versus position</u>, but <u>amplitude versus time</u> is ALSO a sinusoidal graph!

Waves - terminology

Frequency: number of wave crests that pass a given point per second

Period: time between passage of successive crests

Relationship: **Frequency = 1 / Period**



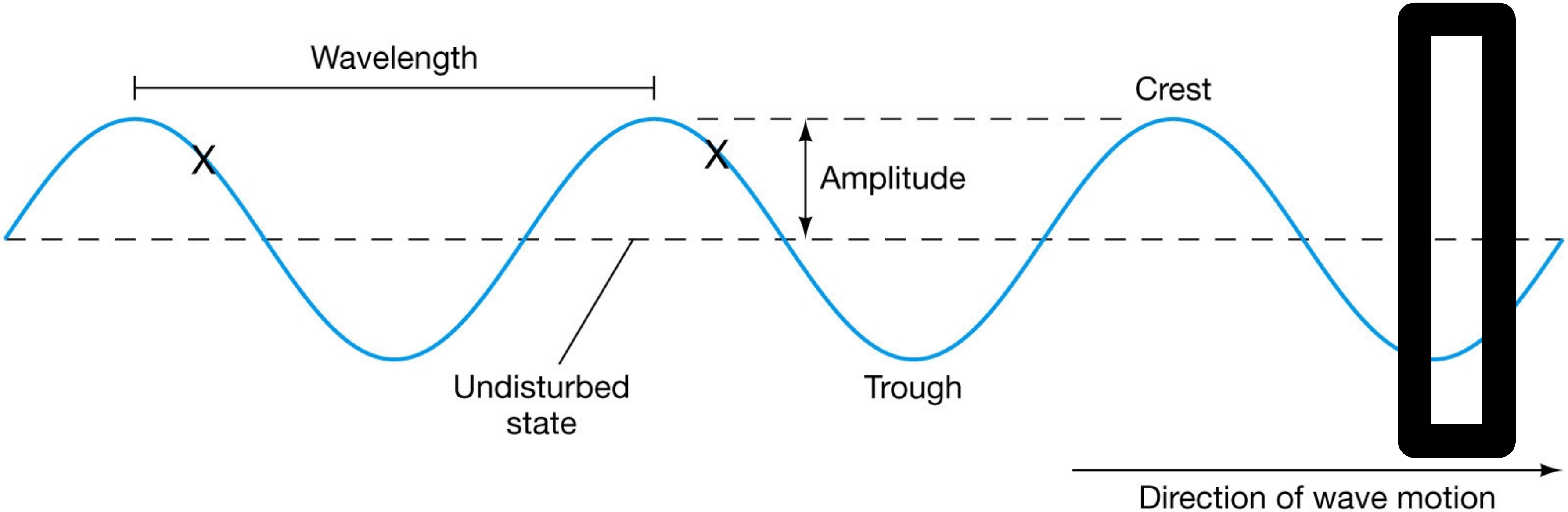
Waves - terminology

Wavelength: distance between successive crests

Velocity: speed at which crests move

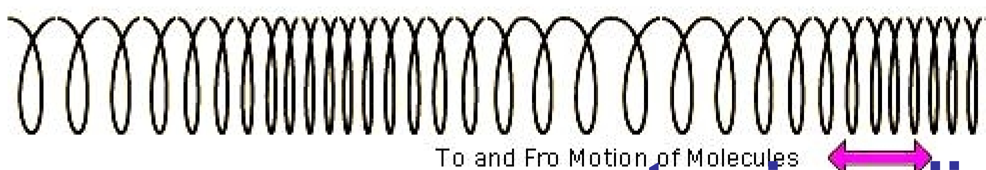
Velocity = Wavelength/Period

Velocity = Wavelength * frequency

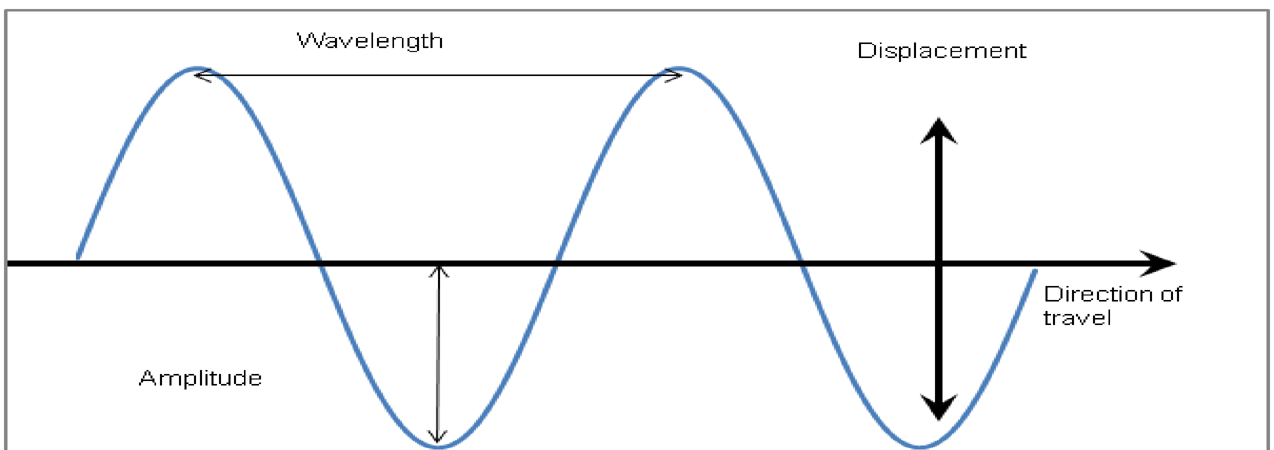


Waves - terminology

Longitudinal wave: propagates in a direction parallel to the displacement of the medium



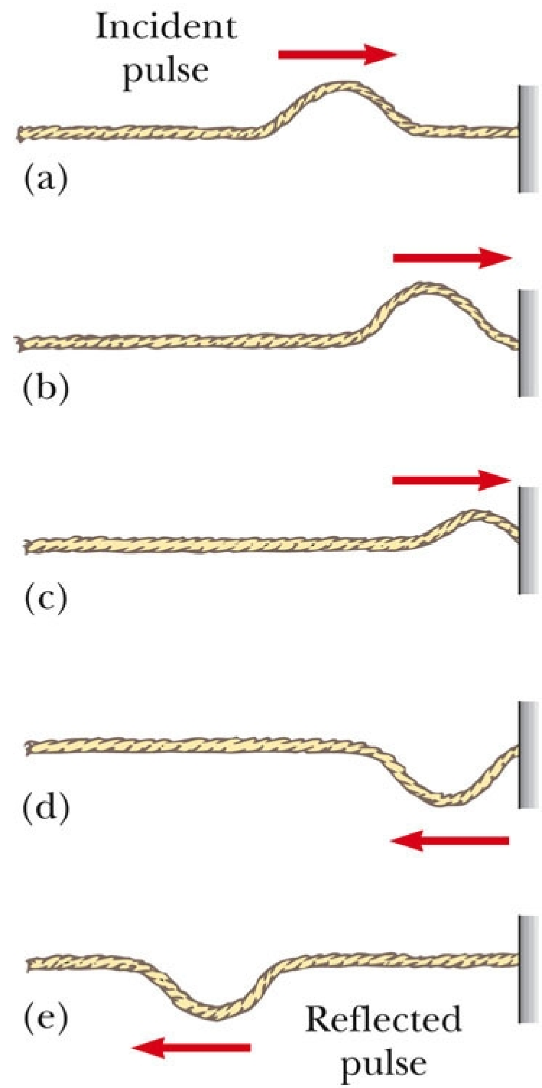
Transverse wave: propagates in a direction perpendicular (or transverse) to the displacement of the medium



DEMO: long. and transv. waves in a SLINKY! Standing waves!

Reflection of a Wave, Fixed End

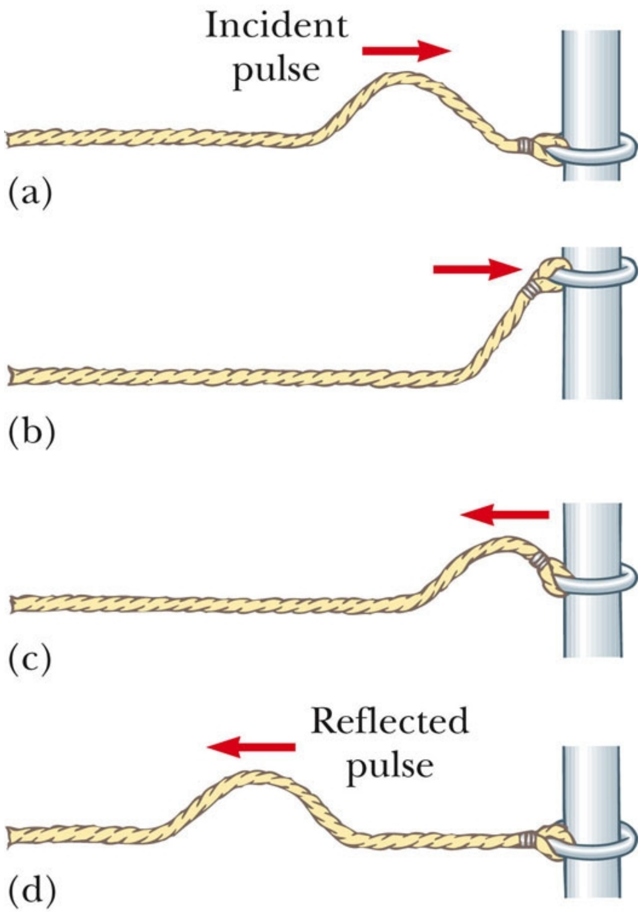
- When the pulse reaches the support, the pulse moves back along the string in the opposite direction
- This is the **reflection** of the pulse
- The pulse is inverted



DEMO: Pulses on white elastic cord.
Speed $v \propto \sqrt{T/\mu}$

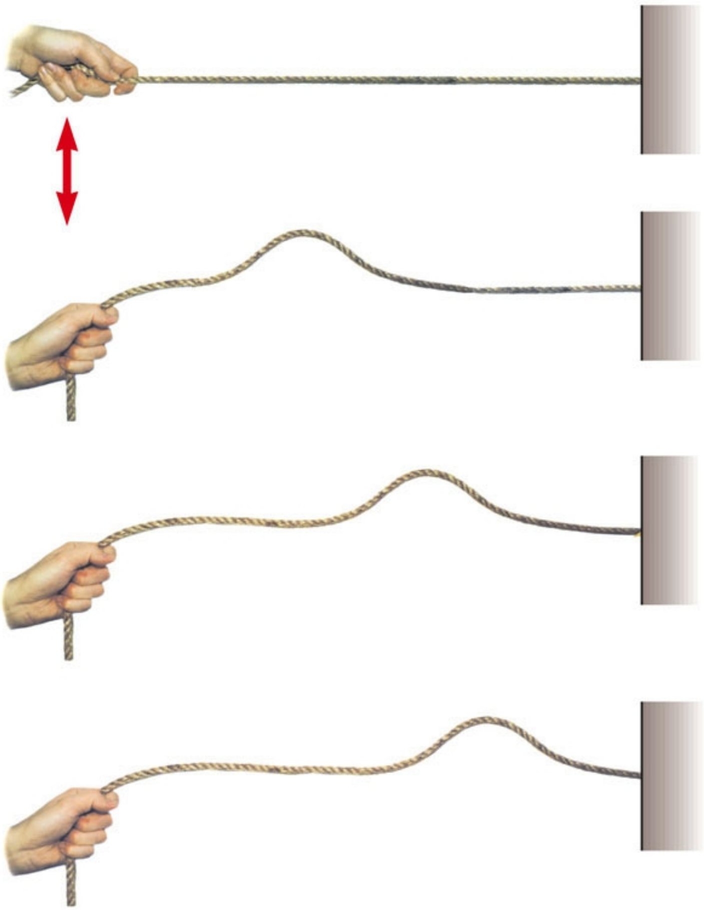
Reflection of a Wave, Free End

- With a free end, the string is free to move vertically
- The pulse is reflected
- The pulse is not inverted
- The reflected pulse has the same amplitude as the initial pulse

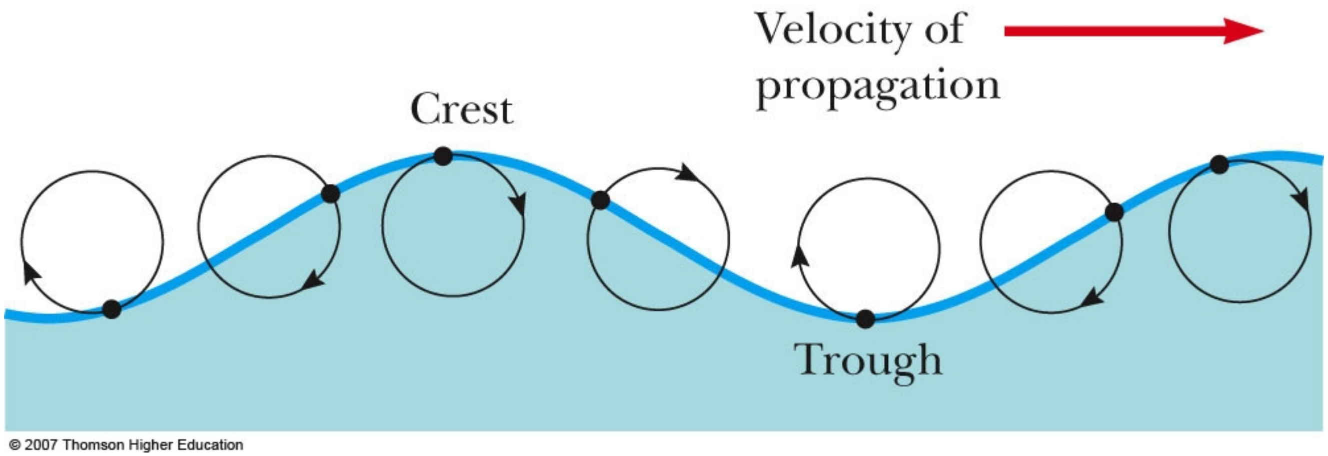


Pulse on a String

- The wave is generated by a flick on one end of the string
- The string is under tension
- A single bump is formed and travels along the string
  - The bump is called a **pulse**



Complex Waves



- Some waves exhibit a combination of transverse and longitudinal waves
- Surface water waves are an example
- Use the active figure to observe the displacements

Example: Earthquake Waves

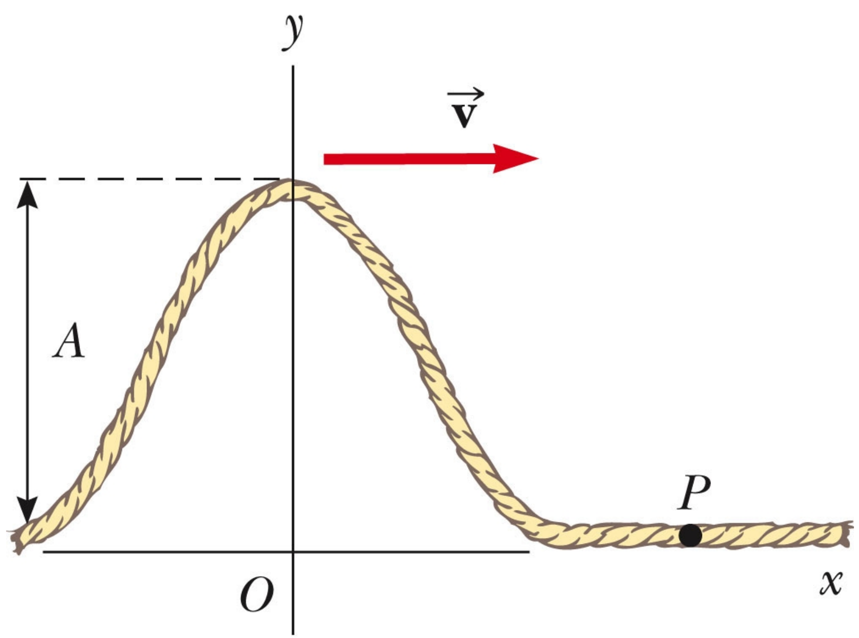
- P waves
  - "P" stands for primary
  - Fastest, at 7 – 8 km / s
  - Longitudinal
- S waves
  - "S" stands for secondary
  - Slower, at 4 – 5 km/s
  - Transverse
- A seismograph records the waves and allows determination of information about the earthquake's place of origin

Traveling Pulse

- The shape of the pulse at $t = 0$ is shown
- The shape can be represented by

  $y\ (x,0) = f\ (x)$

  - This describes the transverse position y of the element of the string located at each value of x at $t = 0$

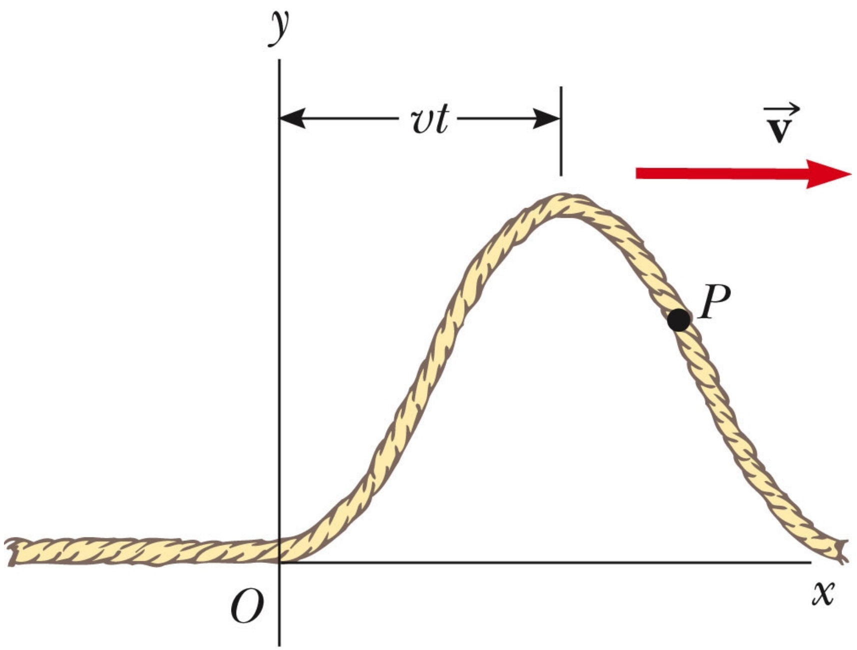


(a) Pulse at $t = 0$

Traveling Pulse, 2

- The speed of the pulse is *v*
- At some time, *t*, the pulse has traveled a distance *vt*
- The shape of the pulse does not change
- Its position is now

  $y = f(x - vt)$



(b) Pulse at time t

Traveling Pulse, 3

- For a pulse traveling to the right
  - $y(x, t) = f(x - vt)$
- For a pulse traveling to the left
  - $y(x, t) = f(x + vt)$
- The function *y* is also called the **wave function**:

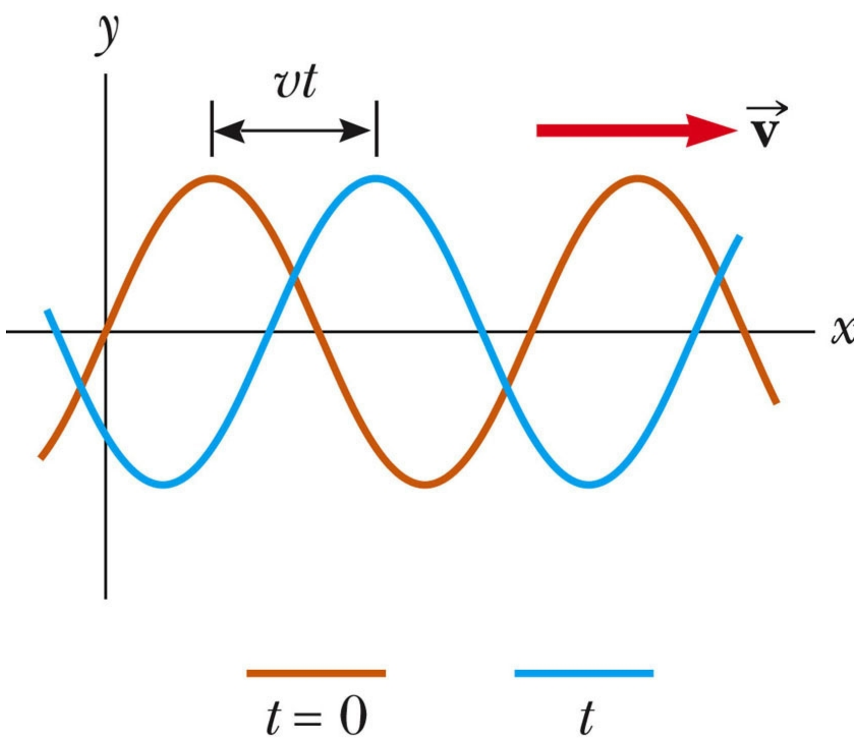
  $y(x, t)$
- The wave function represents the *y* coordinate of any element located at position *x* at any time *t*
  - The *y* coordinate is the transverse position

Traveling Pulse, final

- If *t* is fixed then the wave function is called the **waveform**
  - It defines a curve representing the actual geometric shape of the pulse at that time

Sinusoidal Waves

- The wave represented by the curve shown is a **sinusoidal wave**
- It is the same curve as sin θ plotted against θ
- This is the simplest example of a periodic continuous wave
  - It can be used to build more complex waves

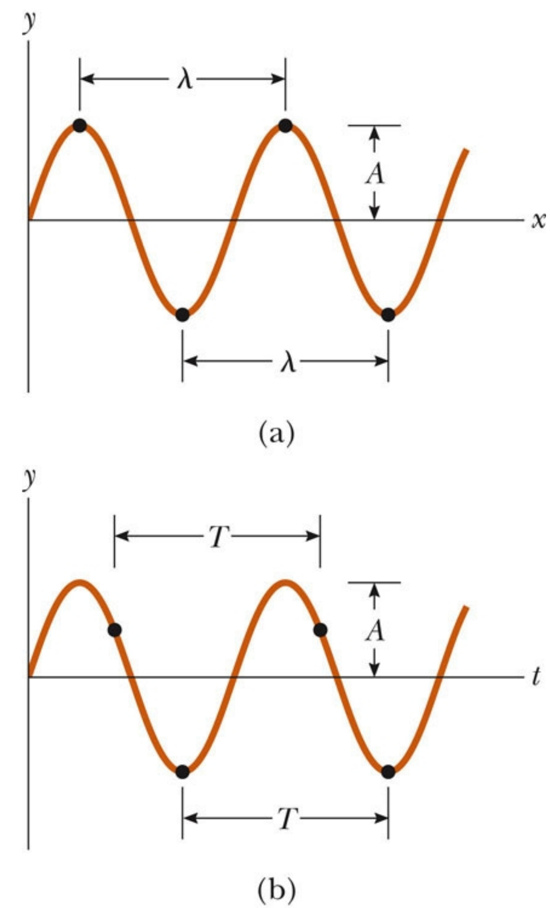


Sinusoidal Waves, cont

- The wave moves toward the right
  - In the previous example, the brown wave represents the initial position
  - As the wave moves toward the right, it will eventually be at the position of the blue curve
- Each element moves up and down in simple harmonic motion
- It is important to distinguish between the motion of the wave and the motion of the particles of the medium

Terminology: Amplitude and Wavelength

- The **crest** of the wave is the location of the maximum displacement of the element from its normal position
  - This distance is called the **amplitude**, *A*
- The **wavelength**, λ, is the distance from one crest to the next



Terminology: Wavelength and Period

- More generally, the wavelength is the minimum distance between any two identical points on adjacent waves
- The period, T , is the time interval required for two identical points of adjacent waves to pass by a point
  - The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium

Terminology: Frequency

- The **frequency**, f, is the number of crests (or any point on the wave) that pass a given point in a unit time interval
  - The time interval is most commonly the second
  - The frequency of the wave is the same as the frequency of the simple harmonic motion of one element of the medium

Terminology: Frequency, cont

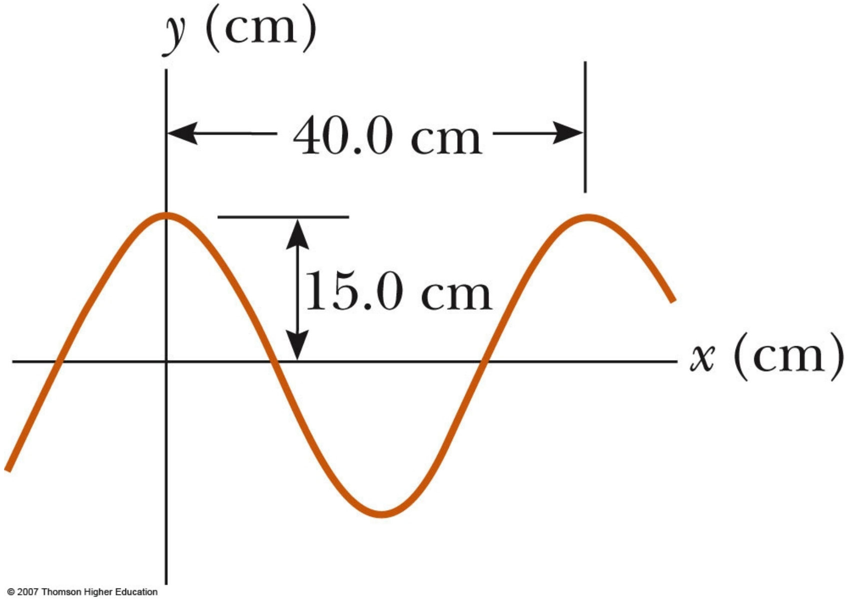
- The frequency and the period are related

$$f = \frac{1}{T}$$

- When the time interval is the second, the units of frequency are s^{-1} = Hz
    - Hz is a hertz

Terminology, Example

- The wavelength, λ, is 40.0 cm
- The amplitude, *A*, is 15.0 cm
- The wave function can be written in the form $y = A \cos(kx - \omega t)$



Speed of Waves

- Waves travel with a specific speed
  - The speed depends on the properties of the medium being disturbed
- The wave function is given by

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

  - This is for a wave moving to the right
  - For a wave moving to the left, replace x – vt with x + vt

Wave Function, Another Form

- Since speed is distance divided by time,

  $v = \lambda / T$
- The wave function can then be expressed as

$$y(x,t) = A\sin 2\pi\left[\frac{x}{\lambda} - \frac{t}{T}\right]$$

- This form shows the periodic nature of y
  - y can be used as shorthand notation for y(x, t)

Wave Equations

- We can also define the angular wave number (or just wave number), *k*

$$k = \frac{2\pi}{\lambda}$$

- The angular frequency can also be defined

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Wave Equations, cont

- The wave function can be expressed as

  $y = A \sin (k x - \omega t)$

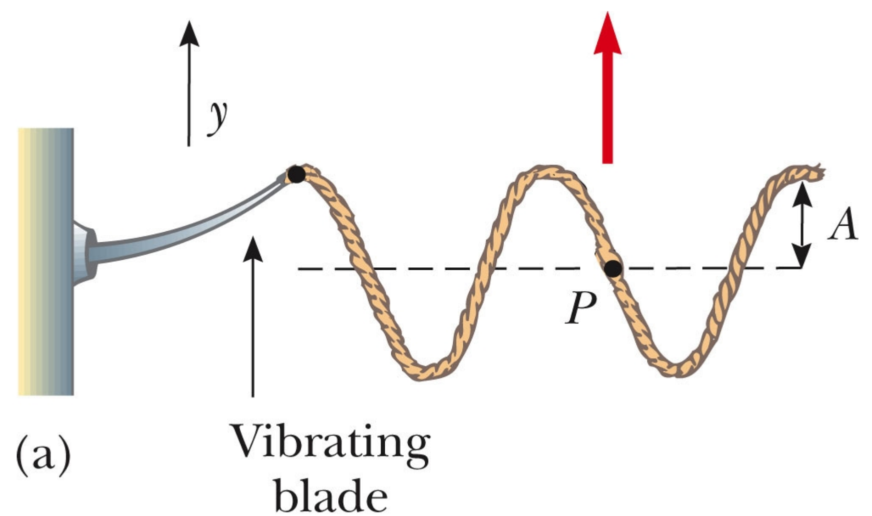
- The speed of the wave becomes $v = \lambda f$
- If $y \neq 0$ at $t = 0$ and x=0, the wave function can be generalized to

  $y = A \sin (k x - \omega t + \phi)$

  where ϕ is called the phase constant

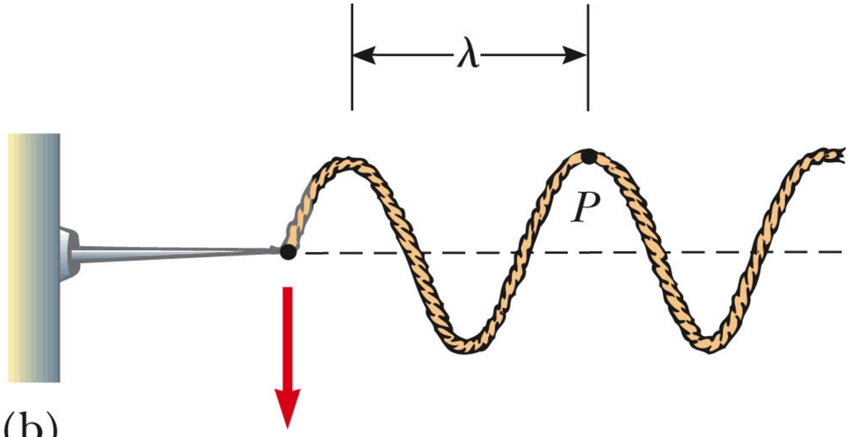
Sinusoidal Wave on a String

- To create a series of pulses, the string can be attached to an oscillating blade
- The wave consists of a series of identical waveforms
- The relationships between speed, velocity, and period hold



Sinusoidal Wave on a String, 2

- Each element of the string oscillates vertically with simple harmonic motion
  - For example, point *P*
- Every element of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of the oscillation of the blade

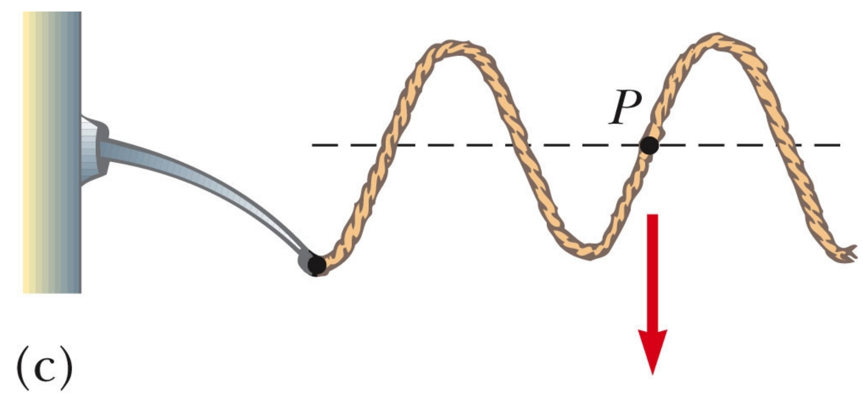


Sinusoidal Wave on a String, 3

- The transverse speed of the element is

$$v_y = \left.\frac{dy}{dt}\right]_{x = \text{constant}}$$

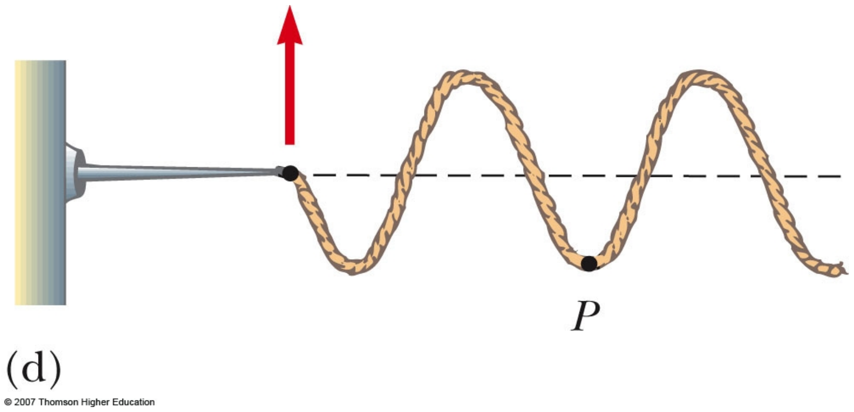
- or $v_y = -\omega A \cos(kx - \omega t)$
- This is different than the speed of the wave itself



Sinusoidal Wave on a String, 4

- The transverse acceleration of the element is

$$a_y = \left.\frac{dv_y}{dt}\right]_{x=\text{constant}}$$



- or $a_y = -\omega^2 A \sin(kx - \omega t)$

Sinusoidal Wave on a String, 5

- The maximum values of the transverse speed and transverse acceleration are
  - $v_{y,\ max} = \omega A$
  - $a_{y,\ max} = \omega^2 A$
- The transverse speed and acceleration do not reach their maximum values simultaneously
  - v is a maximum at y = 0
  - a is a maximum at y = ±A

Speed of a Wave on a String

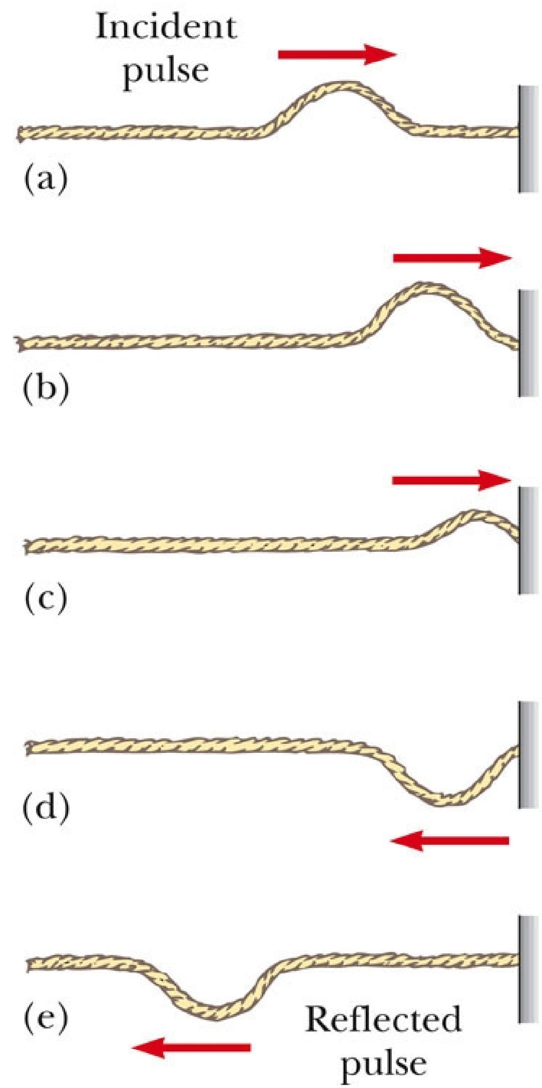
- The speed of the wave depends on the physical characteristics of the string and the tension to which the string is subjected

$$v = \sqrt{\frac{tension}{mass/length}} = \sqrt{\frac{T}{\mu}}$$

- This assumes that the tension is not affected by the pulse
- This does not assume any particular shape for the pulse

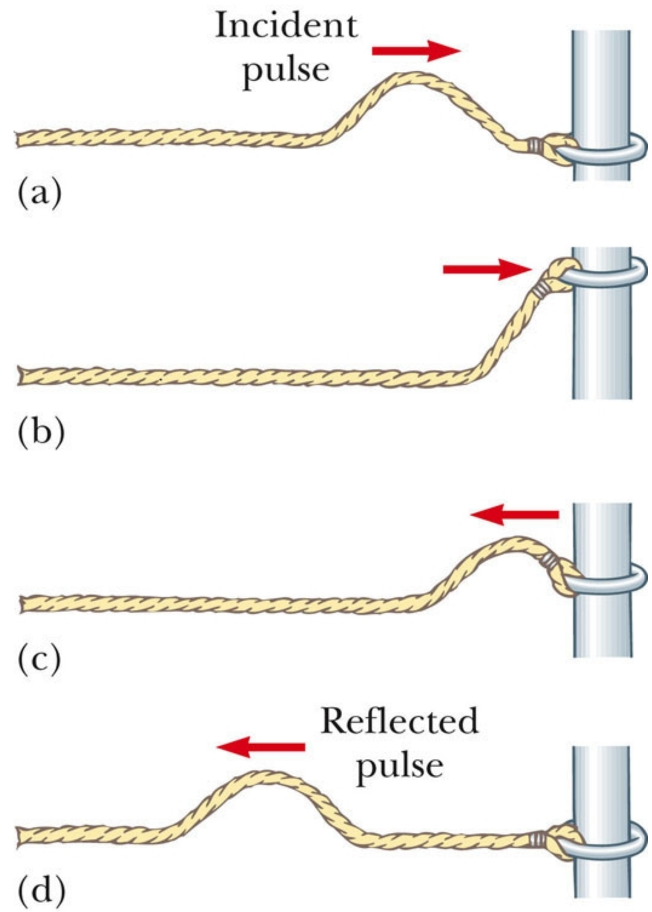
Reflection of a Wave, Fixed End

- When the pulse reaches the support, the pulse moves back along the string in the opposite direction
- This is the **reflection** of the pulse
- The pulse is inverted



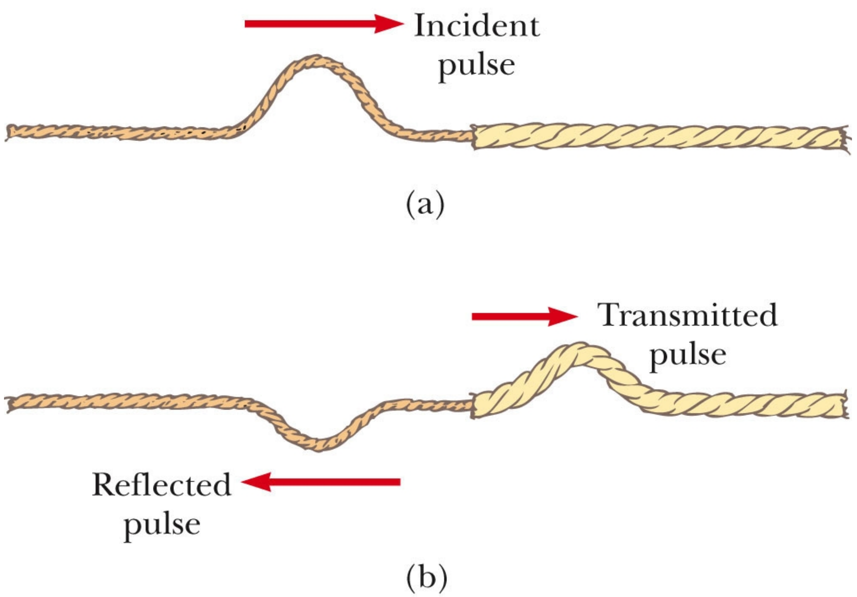
Reflection of a Wave, Free End

- With a free end, the string is free to move vertically
- The pulse is reflected
- The pulse is not inverted
- The reflected pulse has the same amplitude as the initial pulse



Transmission of a Wave

- When the boundary is intermediate between the last two extremes
  - Part of the energy in the incident pulse is reflected and part undergoes **transmission**
    - Some energy passes through the boundary

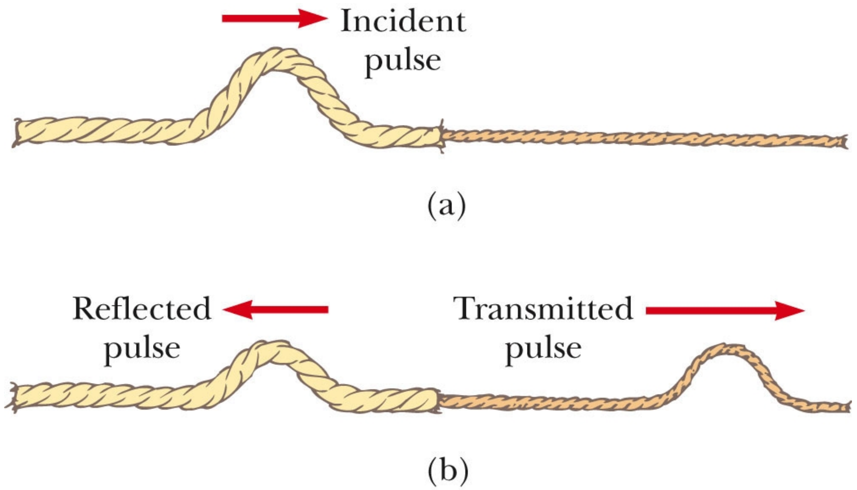


Transmission of a Wave, 2

- Assume a light string is attached to a heavier string
- The pulse travels through the light string and reaches the boundary
- The part of the pulse that is reflected is inverted
- The reflected pulse has a smaller amplitude

Transmission of a Wave, 3

- Assume a heavier string is attached to a light string
- Part of the pulse is reflected and part is transmitted
- The reflected part is not inverted



Transmission of a Wave, 4

- Conservation of energy governs the pulse
  - When a pulse is broken up into reflected and transmitted parts at a boundary, the sum of the energies of the two pulses must equal the energy of the original pulse
- When a wave or pulse travels from medium A to medium B and $v_A > v_B$, it is inverted upon reflection
  - B is denser than A
- When a wave or pulse travels from medium A to medium B and $v_A < v_B$, it is not inverted upon reflection
  - A is denser than B

Energy in Waves in a String

- Waves transport energy when they propagate through a medium
- We can model each element of a string as a simple harmonic oscillator
  - The oscillation will be in the *y*-direction
- Every element has the same total energy

Energy, cont.

- Each element can be considered to have a mass of *dm*
- Its kinetic energy is $dK = ½ (dm)\, v_y^2$
- The mass *dm* is also equal to $\mu\, dx$
- The kinetic energy of an element of the string is $dK = ½ (\mu\ dx)\, v_y^2$

Energy, final

- Integrating over all the elements, the total kinetic energy in one wavelength is $K_\lambda = \frac{1}{4}\mu\omega^2 A^2\lambda$
- The total potential energy in one wavelength is $U_\lambda = \frac{1}{4}\mu\omega^2 A^2\lambda$
- This gives a total energy of
  - $E_\lambda = K_\lambda + U_\lambda = \frac{1}{2}\mu\omega^2 A^2\lambda$

Power Associated with a Wave

- The power is the rate at which the energy is being transferred:

$$P = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2}\mu\omega^2 A^2 \lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 v$$

- The power transfer by a sinusoidal wave on a string is proportional to the
  - Frequency squared
  - Square of the amplitude
  - Wave speed