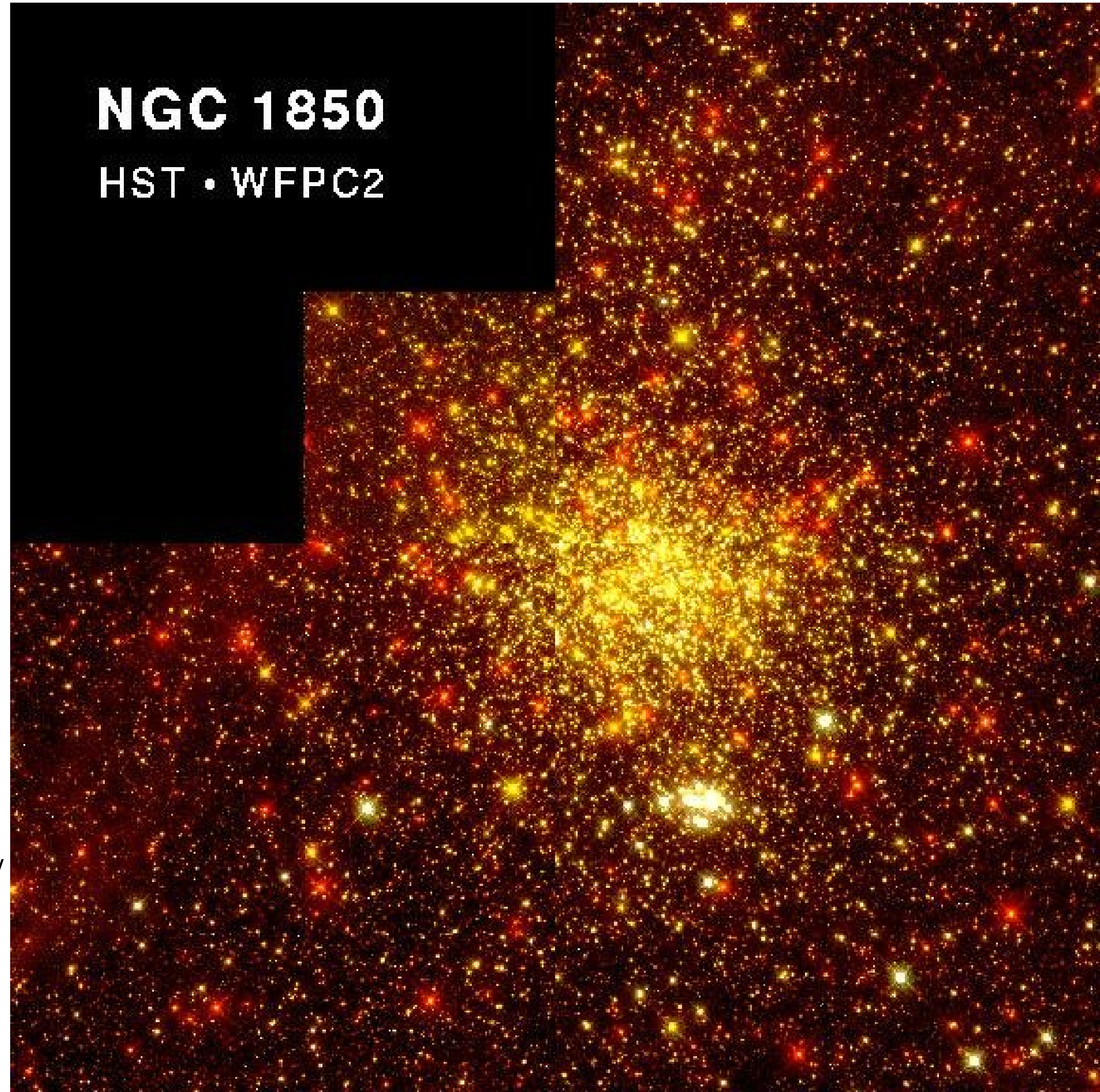
# The Classification of Stellar Spectra

Chapter 8

Star Clusters in the Large Magellanic Cloud




http://www.seds.org/hst/NGC1850.html

# The Classification of Stellar Spectra

- Classification scheme developed before the physics
- Parameters that could be used to classify stars
  - –Apparant brightness (bad idea)
  - –Luminosity (Intrinsic brightness)
  - –Temperature (Color)
  - –Spectra (absorption lines)
  - –Mass (only for binaries)

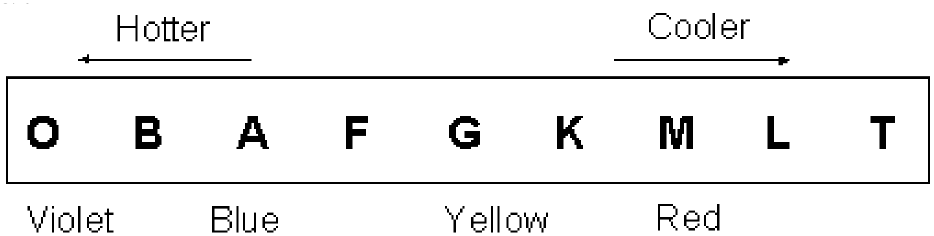
- The Henry Draper Catalogue
  - –Contained >100,000 spectral classifications from A.J. Cannon and others from Harvard
  - –Used OBAFGKM

The Harvard "Computers" of the Harvard College Observatory

http://cannon.sfsu.edu/%7Egmarcy/cswa/history/pick.html

# The Classification of Stellar Spectra

- Originally organized by strength of H Balmer lines (A,B,...).
- Atomic physics allowed connection to temperature to be made.
- Spectral Type

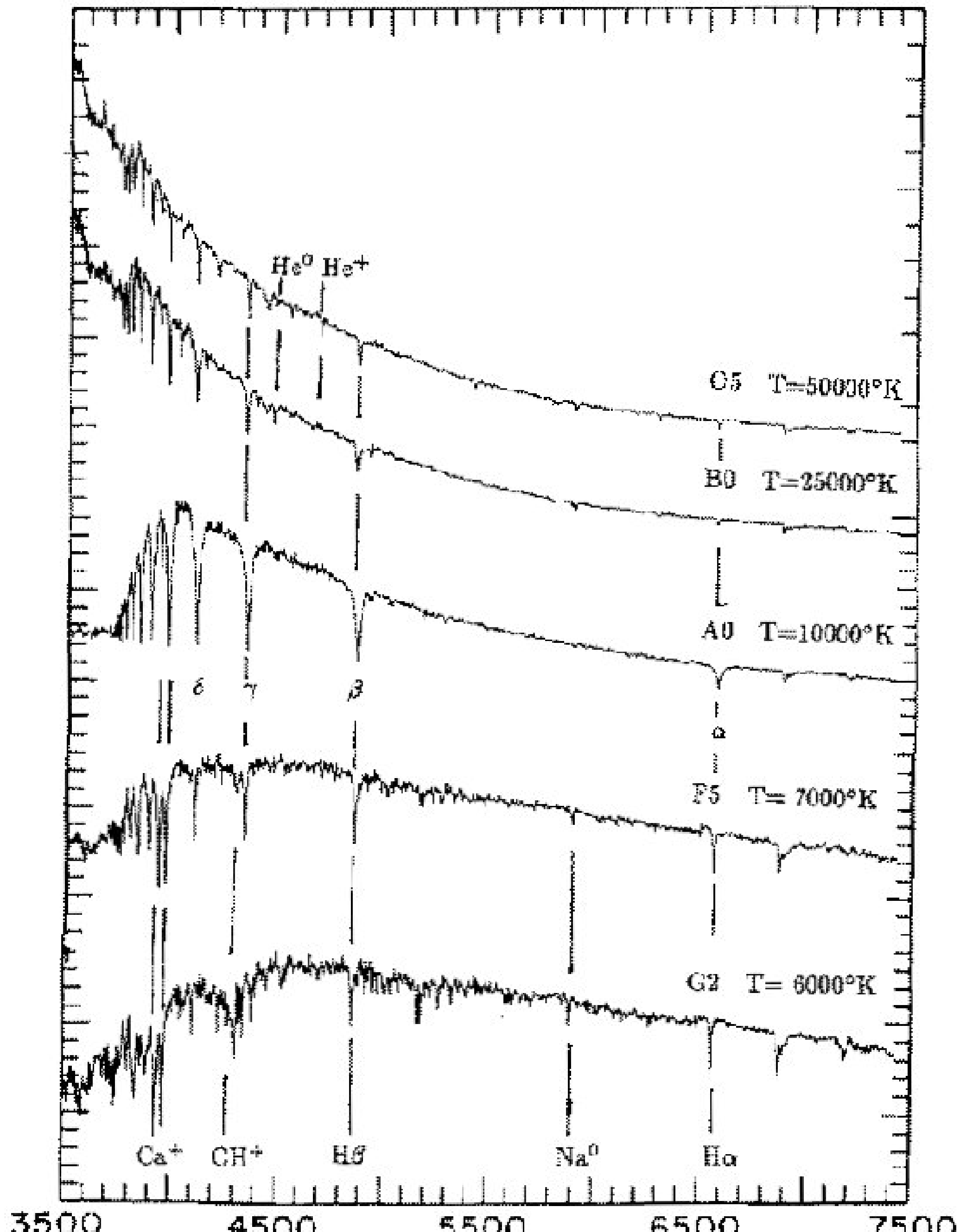



Early type → late type

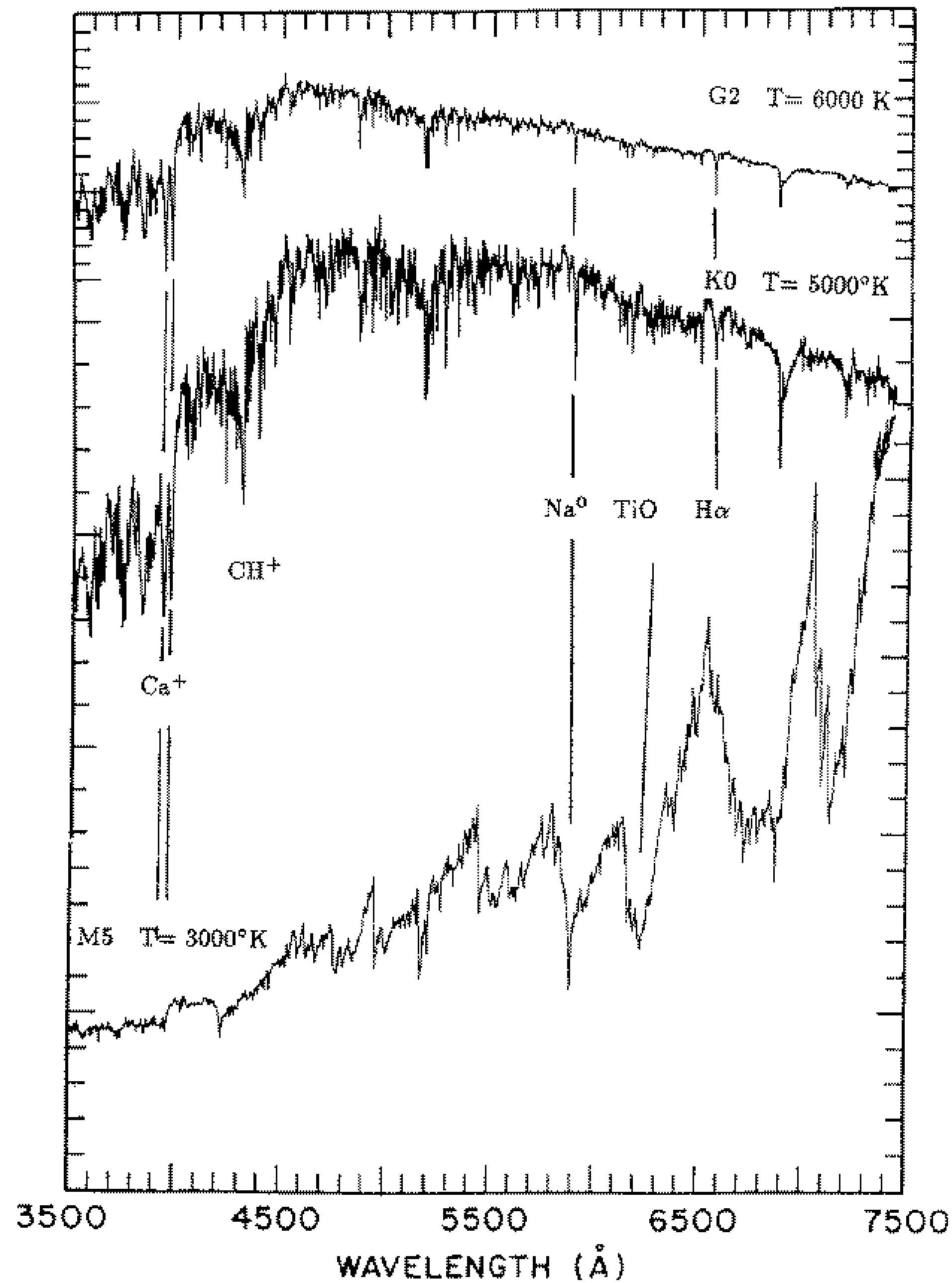
L and T are more modern additions – Brown Dwarfs. R N S also used after M.

  - Subdivisions in tenths: 0 → 9 (early → late, hot → cool) within a Spectral Type). E.g., A0 is hotter than A5.
  - The Sun is a G2 – an early G-type star
    - G – yellow star (continuum peak in green/yellow)
      - H lines weak
      - Ca II (singly ionized) lines continue becoming stronger
      - Fe I, other *neutrals* metal lines become stronger

# O to G example

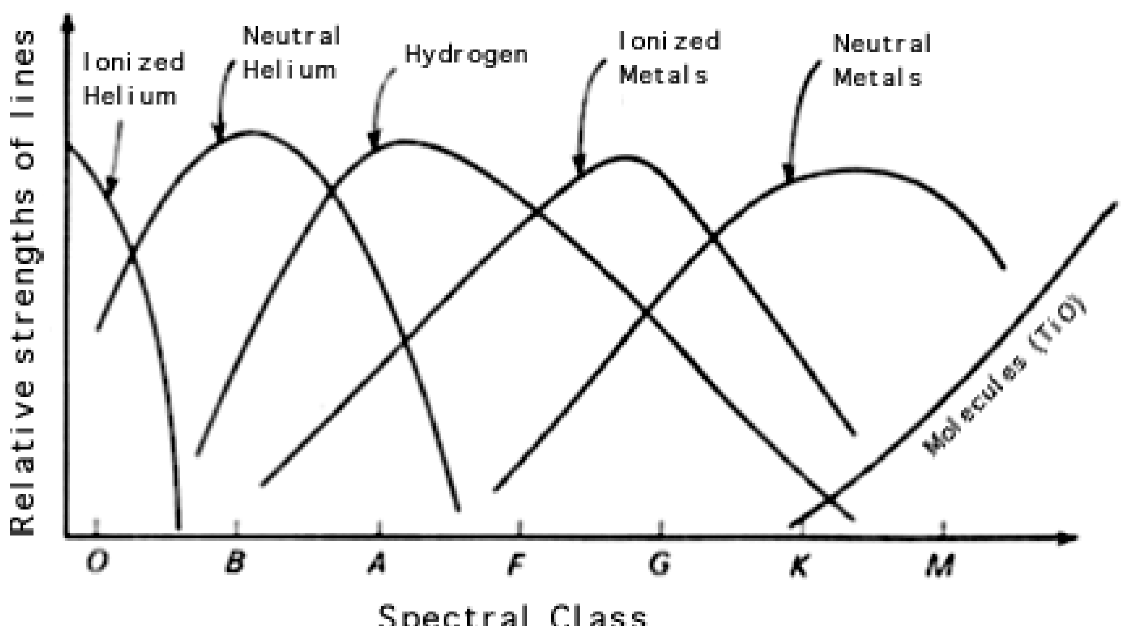


# G to M example



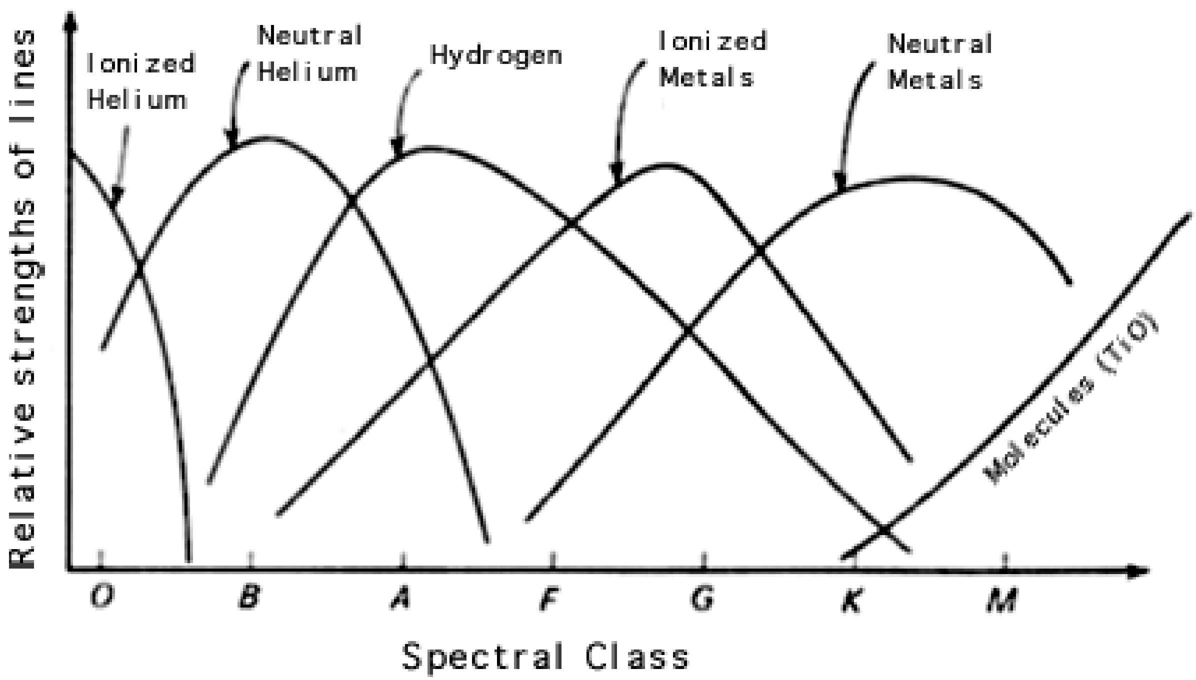
# The Formation of Spectral Lines

- Question: What causes the differences in the observed spectra??
  - [Absorption by intervening material.  Earth's atmos., ISM.]
  - Composition
  - Temperature
  - Surface gravity / pressure
- Answer:
  - Temperature is the main factor



# The Formation of Spectral Lines

- Big Question of Ch.8: Why are the H balmer lines strongest for A stars, which seem to have T_surf = 10,000K?
- To find answer:
  - Need Ch.5's info about the Bohr atom … energy levels.
  - Need Kirchoff's laws → our gas is the upper “atmosphere” of the star.
  - Need statistical mechanics – study of large numbers of particles that can occupy different states



# The Formation of Spectral Lines

- Distribution of electrons in different atomic orbitals depends on temperature
- Electrons can jump up in energy by absorption of a photon OR collision with a particle! So KE of surrounding particles important.

- What is the probability of finding an electron in a particular orbital?
  - Answer with Statistical Mechanics…
  - Maxwell-Boltzmann (velocity) Distribution
    - Assumes thermal equilibrium
    - Number of gas particles per unit volume have a speed between v and v+dv

$$n_v\,dv = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{1}{2}mv^2/kT}\,4\pi\,v^2\,dv$$

# Maxwell-Boltzmann Distribution

$$n_v\, dv = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{1}{2}mv^2/kT}\, 4\pi\, v^2\, dv$$

- Most probable speed

$$v_{mp} = \sqrt{\frac{2kT}{m}} = 1.4\sqrt{kT/m}$$

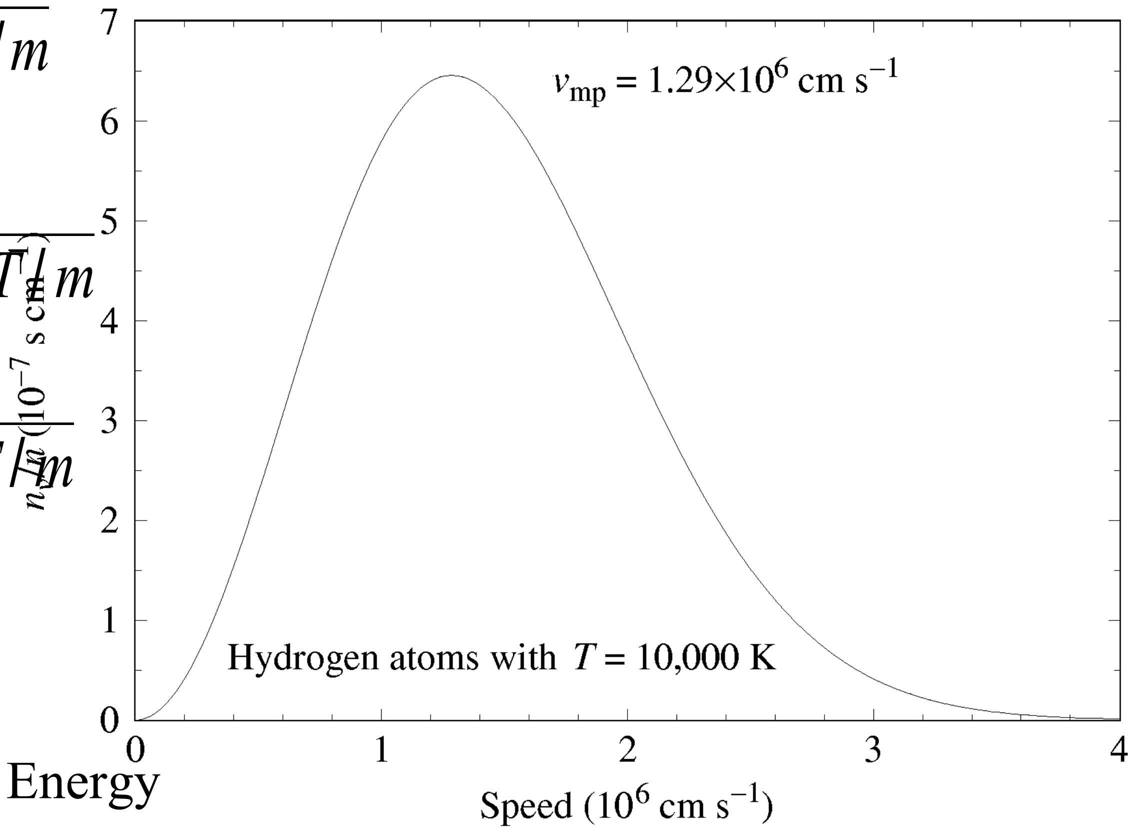
- Root-mean-square

$$v_{rms} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{kT/m}$$

- Average

$$v_{avg} = \sqrt{\frac{8kT}{\pi m}} = 1.6\sqrt{kT/m}$$

- Collisional energy causes a distribution of electrons among the atomic orbitals (Kinetic Energy → Potential Energy)



# Boltzmann Factor

- The higher the energy of a state, the less likely it will be occupied

$$P_a \propto e^{\frac{-E_a}{kT}}$$

  – For the Maxwell-Boltzmann distribution, the energy is Kinetic Energy

$$P_v \propto e^{-\frac{1}{2}mv^2/kT}$$

- The "*kT*" term is associated with the thermal energy of the "gas" as a whole
- Ratio of Probabilities for two different states (and energies)

$$\frac{P_b}{P_a} = \frac{e^{\frac{-E_b}{kT}}}{e^{\frac{-E_a}{kT}}} = e^{\frac{-\left(E_b - E_a\right)}{kT}}$$

# Degeneracy Factor

- An energy (eigenvalue) is associated with each set of quantum numbers (eigenstate or eigenfunction)
- *Degenerate States* have different quantum numbers but the same energy

- Modify the Boltzmann factor

$$P_a \propto g_a e^{\frac{-E_a}{kT}}$$

  - The probability of being in any of the $g_a$ degenerate states with energy $E_a$
    - $g_a$ is the *degeneracy* or *statistical weight* of state $a$

- Ratio of probabilities between states with two different energies

$$\frac{P_b}{P_a} = \frac{g_b}{g_a} e^{\frac{-\left(E_b - E_a\right)}{kT}}$$

# Degeneracy Factor

- Details of quantum mechanics determines the energies and quantum numbers…
- Visit the following site on the next page and browse…
- Quantum numbers for Hydrogen $\{n, l, m_l, m_s\}$
  - Table 8.2

| State | $n$ Principal quantum number n | $l$ Orbital quantum number | $m_l$ Magnetic quantum number | $m_s$ Spin quantum number | Maximum number of electrons | |
|---|---|---|---|---|---|---|
| 1s | 1 | 0 | 0 | $+\frac{1}{2}, -\frac{1}{2}$ | 2 | |
| 2s | 2 | 0 | 0 | $+\frac{1}{2}, -\frac{1}{2}$ | 2 | 8 |
| 2p | 2 | 1 | -1,0,+1 | $+\frac{1}{2}, -\frac{1}{2}$ | 6 | |
| 3s | 3 | 0 | 0 | $+\frac{1}{2}, -\frac{1}{2}$ | 2 | |
| 3p | 3 | 1 | -1,0,+1 | $+\frac{1}{2}, -\frac{1}{2}$ | 6 | 18 |
| 3d | 3 | 2 | -2,-1,0,1,2 | $+\frac{1}{2}, -\frac{1}{2}$ | 10 | $=2n^2$ |

# Boltzmann Equation

- Number of atoms in a particular state $a$

$$N_a = NP_a$$

$N$ = total number of atoms

$N_a$ = number of atoms in state $a$

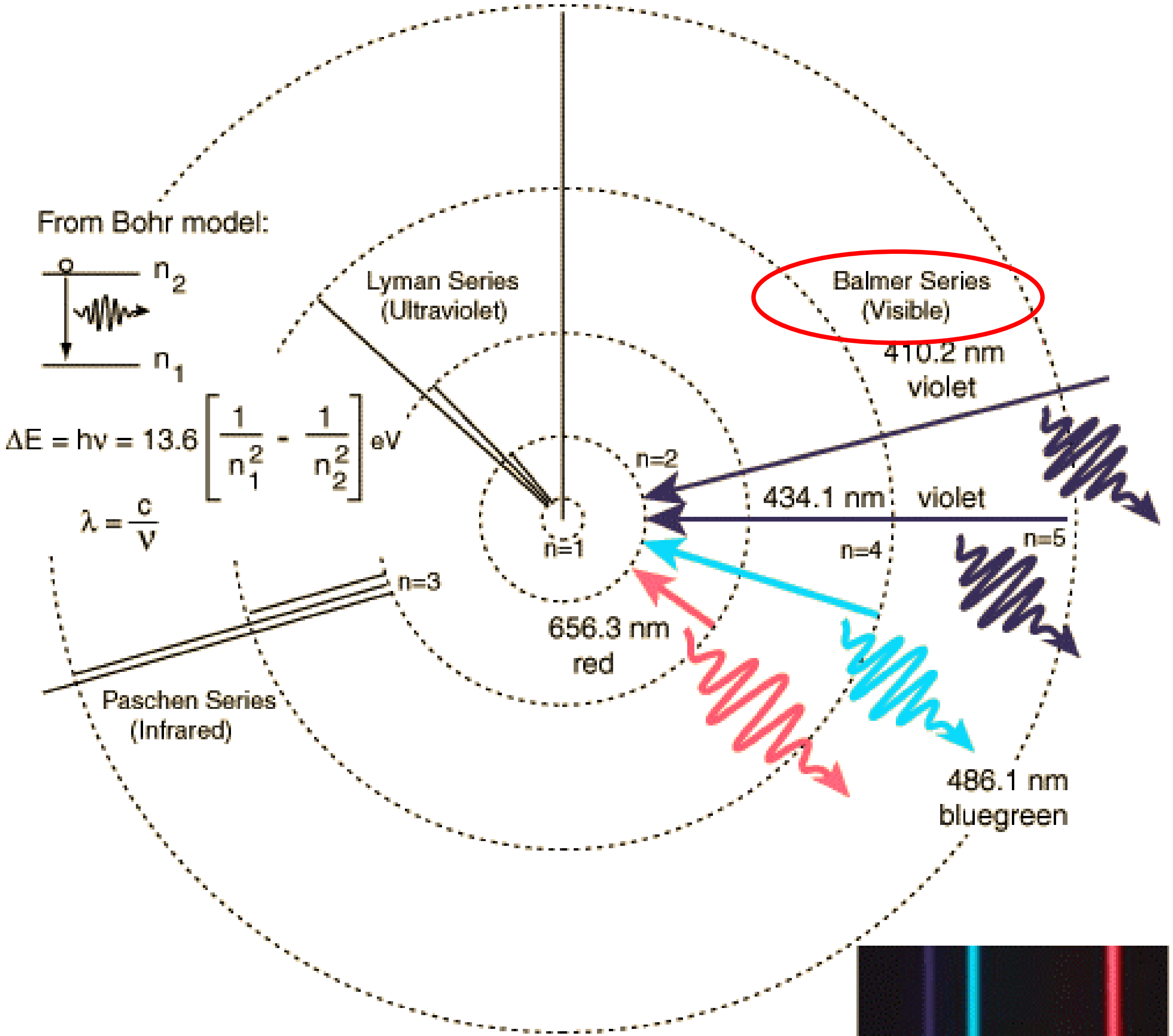
$P_a$ = probability of being in state $a$

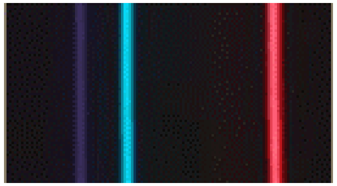
$$\Rightarrow \frac{N_b}{N_a} = \frac{g_b}{g_a} e^{\frac{-(E_b - E_a)}{kT}}$$

Hydrogen Atom Examples

# Hydrogen Atom

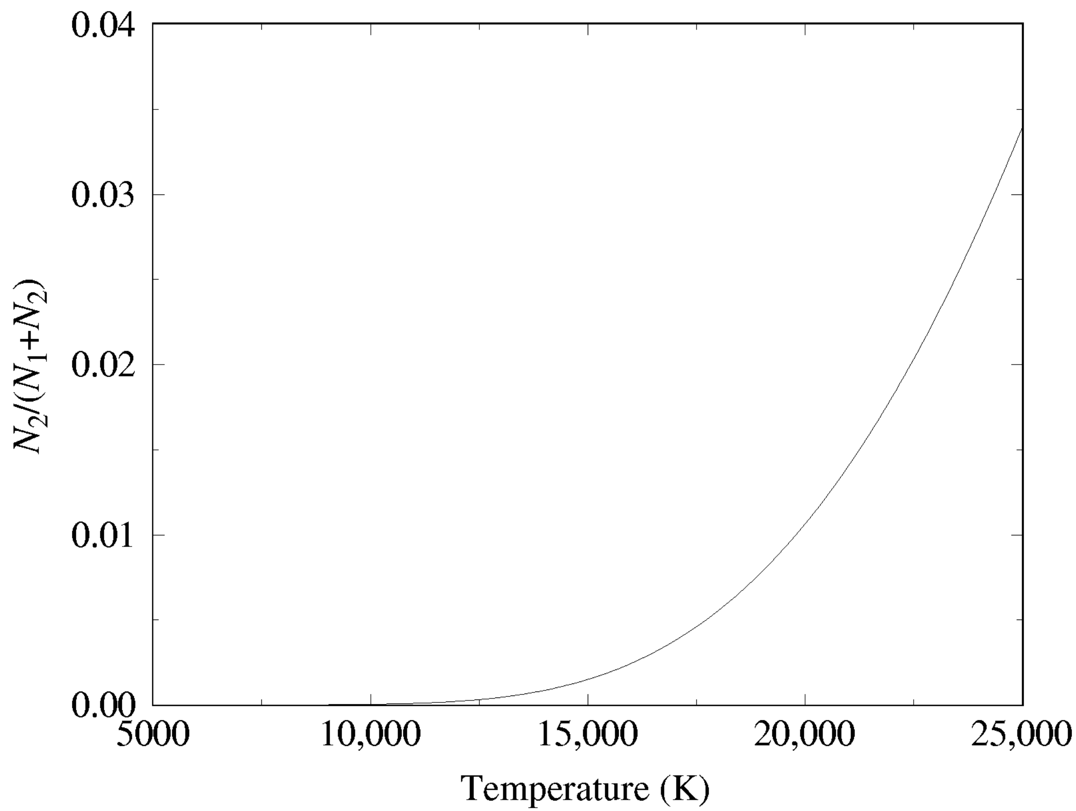
- Balmer series absorption spectra is an upward transition from $n = 2$
- Observation: this series has a peak absorption spectrum at ~9520 K.



# Hydrogen Atom Populations

- We just saw that not many Hydrogen atoms are in the *n=1* state at 9520 K!
  - Shouldn't the intensity keep growing as the temperature increases since there is a higher probability for an H atom to be in the *n=2* state?!?!



# Partition Function

- We also have to figure in all states that have a significant population
- For one state we have: $$P_1 \propto g_1 e^{\frac{-E_1}{kT}}$$
- Ratio between two states: $$\frac{P_2}{P_1} = \frac{g_2 e^{\frac{-E_2}{kT}}}{g_1 e^{\frac{-E_1}{kT}}} = \frac{g_2}{g_1} e^{\frac{-\left(E_2 - E_1\right)}{kT}}$$
- Ratio of state *2* to *all* other states with reference to the ground state:

$$\frac{P_2}{P_{all}} = \frac{g_b e^{\frac{-(E_2 - E_1)}{kT}}}{g_1 e^{\frac{-(E_1 - E_1)}{kT}} + g_2 e^{\frac{-(E_2 - E_1)}{kT}} + g_3 e^{\frac{-(E_3 - E_1)}{kT}} + \cdots} = \frac{g_2 e^{\frac{-(E_2 - E_1)}{kT}}}{Z}$$

# Partition Function

- This tell us how many states are accessible or available at a given temperature (thermal energy)

$$Z = g_1 e^{\frac{-(E_1 - E_1)}{kT}} + g_2 e^{\frac{-(E_2 - E_1)}{kT}} + g_3 e^{\frac{-(E_3 - E_1)}{kT}} + \cdots$$

$$= g_1 + \sum_i g_i e^{\frac{-(E_i - E_1)}{kT}}$$

- The higher the temperature, the more states that are available
- At zero K, everything will be in the ground state
  - Bose-Einstein Condensates

# Partition Function and Atoms

- We also have to handle ionization!
- Nomenclature: H I – neutral hydrogen

  H II – singly ionized hydrogen

  He I – neutral Helium

  He II – singly ionized Helium

  He III – doubly ionized Helium

- Ionization Energy for H I to H II

$$\chi_I = 13.6\, eV$$

  – Rather than $n \rightarrow \infty$, the atom will ionize before this happens

# Saha Equation

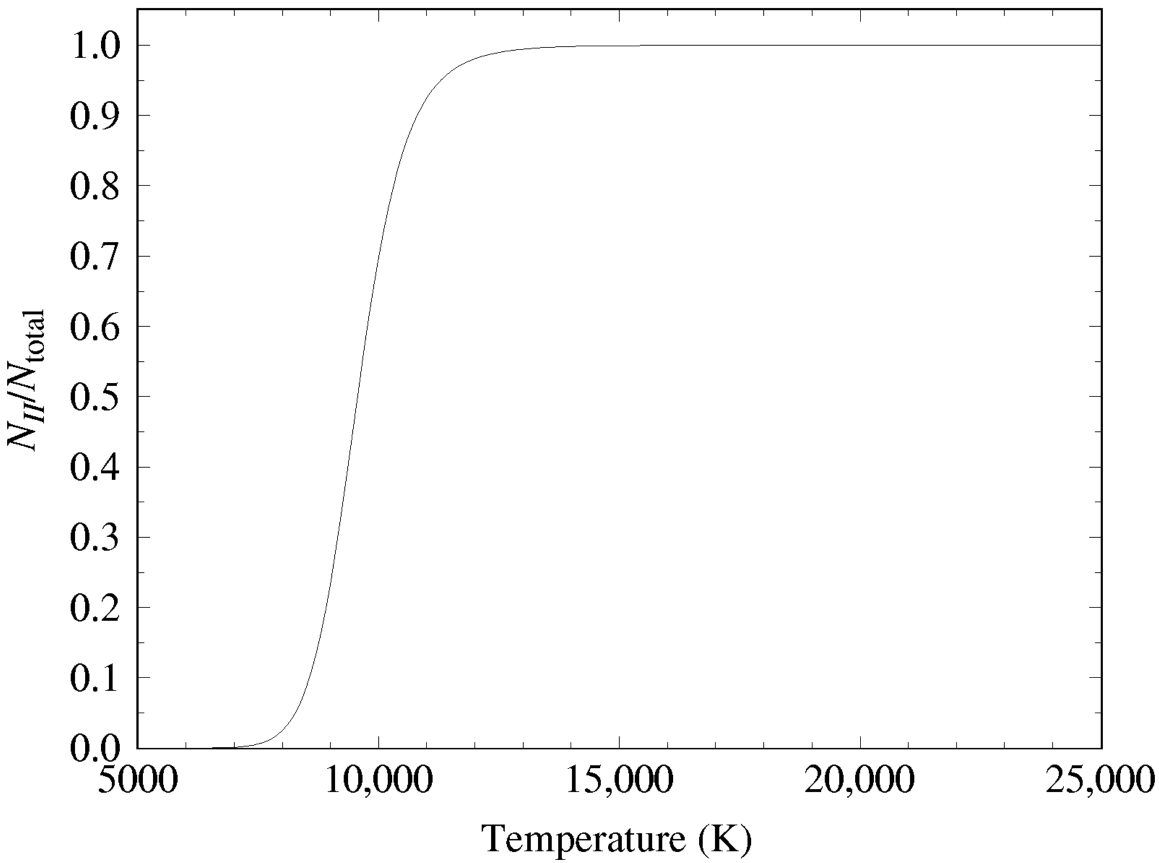
- Determines the ratio of ionized atoms
- Need partition functions since all the atoms are not in the same state
  - $Z_i$ is the initial stage of ionization
  - $Z_{i+1}$ is the final stage of ionization
- Ratio of the number of atoms in each of these stages

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi\ m_e kT}{h^2}\right)^{\frac{3}{2}} e^{-\chi_i kT}$$

  - $n_e$ is the electron density (an ideal gas of electrons)
    - Electron pressure $P_e = n_e kT$
    - Electrons recombine with H II to give H I

# Ionized Hydrogen Atoms

- Fraction of hydrogen atoms that are ionized
- If we have H II, we can't have the Balmer series!



# H I $n = 2$ population

$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_I}\right)\left(\frac{N_I}{N_I + N_{II}}\right)$$

Fraction of non-ionized hydrogen
Atoms in the $n = 2$ state

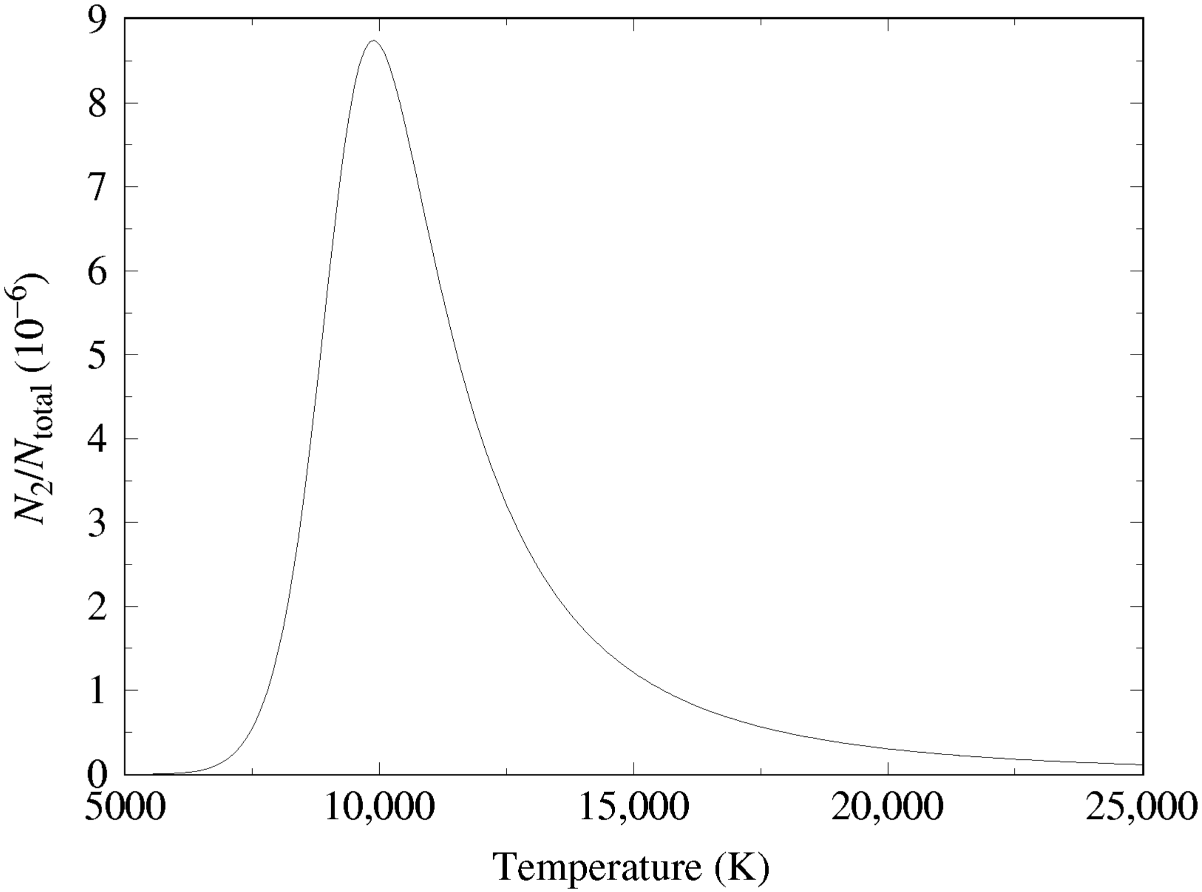
Fraction of non-ionized
hydrogen atoms

$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_I}\right)\left(\frac{1}{1 + N_{II}/N_I}\right) \xrightarrow{\quad\quad} _{N_I \approx N_1 + N_2} \left(\frac{N_2}{N_1 + N_2}\right)\left(\frac{1}{1 + N_{II}/N_I}\right)$$

$$\frac{N_2}{N_{total}} = \left(\frac{N_2/N_1}{1 + N_2/N_1}\right)\left(\frac{1}{1 + N_{II}/N_I}\right)$$

# H I $n = 2$ population

- Includes the Boltzmann factor, partition function and ionization
- Population peak at 9520 K, in agreement with observation of the Balmer series

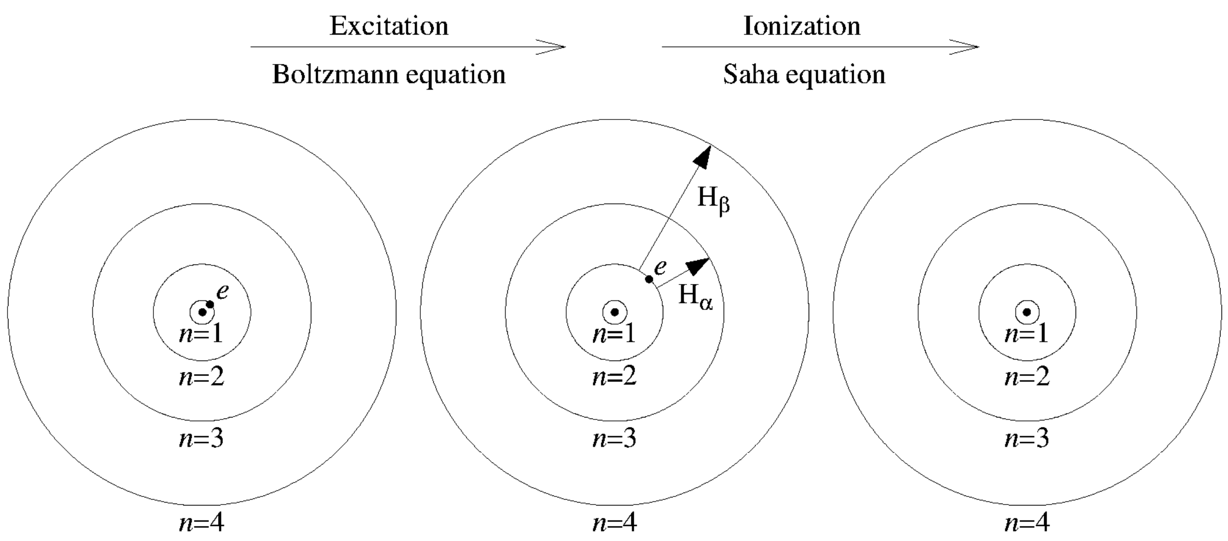


# H I $n = 2$ population

$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_I}\right)\left(\frac{N_I}{N_I + N_{II}}\right)$$

Fraction of non-ionized hydrogen Atoms in the $n = 2$ state

Fraction of non-ionized hydrogen atoms



# Example 8.3

- Degree of ionization in a stellar atmosphere of pure hydrogen for the temperature range of 5000-25000 K $\dfrac{N_{II}}{N_{Total}}$
- Given electron pressure $P_e = 200 \dfrac{dyne}{cm^2}$
- Saha equation $$\frac{N_{II}}{N_I} = \frac{2kTZ_{II}}{P_e Z_I}\left(\frac{2\pi\ m_e kT}{h^2}\right)^{\frac{3}{2}} e^{-\chi_i kT}$$
- Must determine the partition functions
  - Hydrogen ion is a proton, so $Z_{II} = 1$
  - Neutral hydrogen over this temp range

$$\Delta E = E_2 - E_1 = 10.2 \text{ eV}$$

$\Delta E >> kT$, then $e^{-\Delta E / kT} << 1$

$$\Rightarrow Z_I = g_1 + \sum_i g_i e^{\frac{-(E_i - E_1)}{kT}} \quad g_1 = 2$$

$T := 5000\text{K}$

$k \cdot T = 0.43 \text{ eV}$

$T := 25000\text{K}$

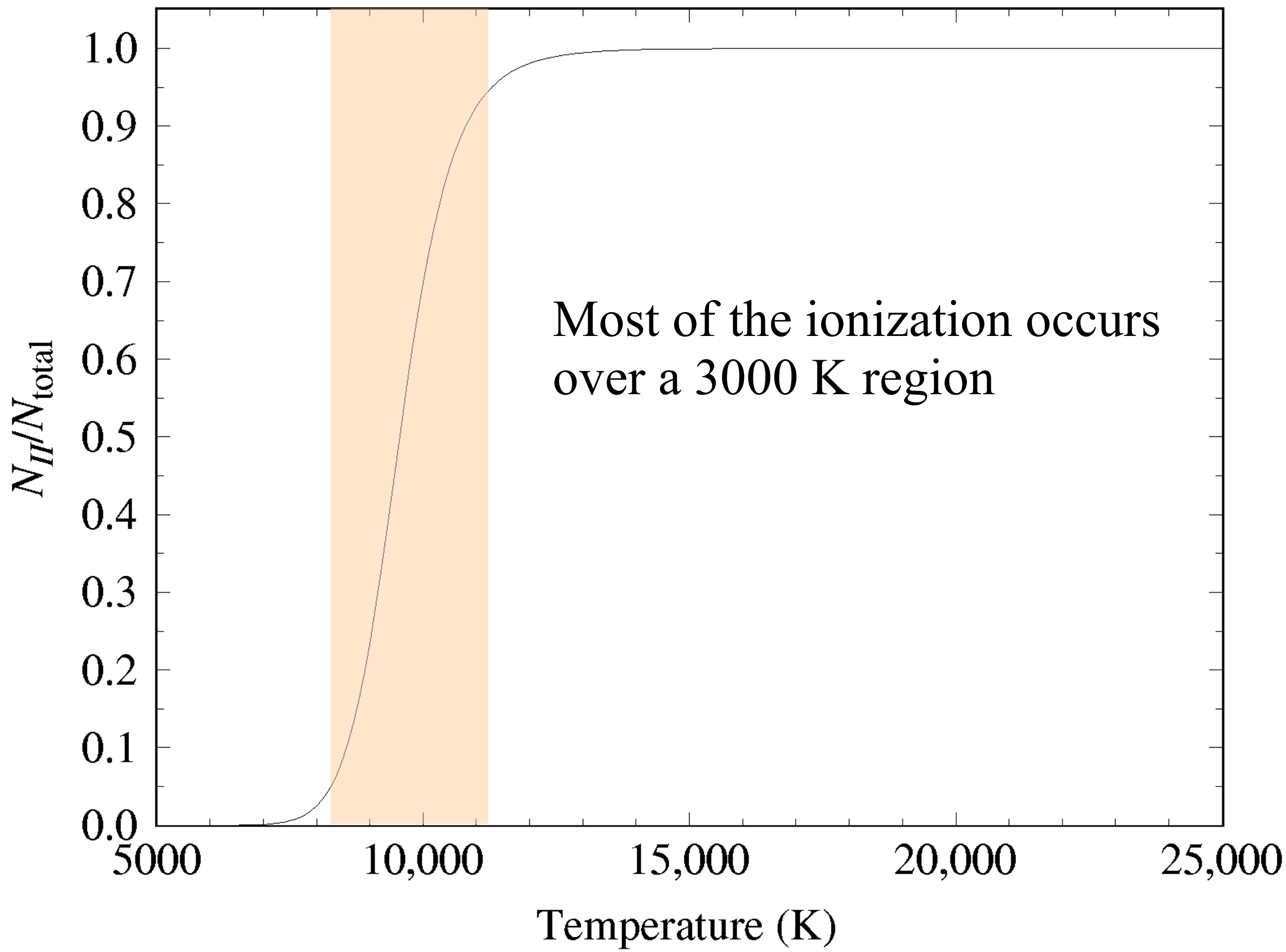
$k \cdot T = 2.15 \text{ eV}$

# Example 8.3

- Degree of Ionization

$$\frac{N_{II}}{N_I} = \frac{2kT(1)}{P_e(2)}\left(\frac{2\pi\, m_e kT}{h^2}\right)^{\frac{3}{2}} e^{-\chi_i kT}$$

$$\frac{N_{II}}{N_I + N_{II}} = \frac{N_{II}/N_I}{1 + N_{II}/N_I}$$



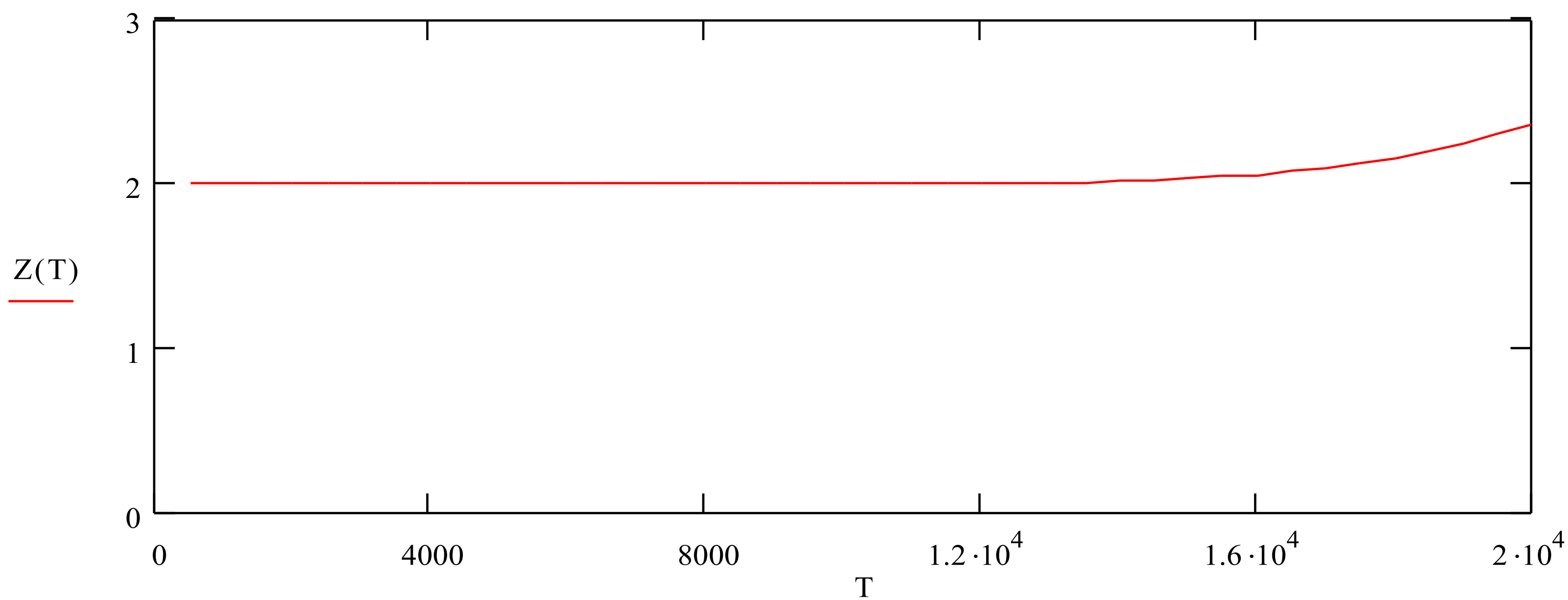
# Problem 8.7

- Evaluate the first three terms of the partition function for 10000K

Partition Function: Counting the first ten states... Energy: $E(n) := \frac{-13.6eV}{n^2}$ Degeneracy: $g(n) := 2 \cdot n^2$

$$f_B(n, T) := \exp\left[\frac{-(E(n) - E(1))}{k \cdot T}\right] \qquad Z(T) := \sum_{n=1}^{10} \left(g(n) \cdot f_B(n, T)\right) \qquad T := 0, 500 .. 20000$$

$Z(6000K) = 2.0000$ $Z(10000K) = 2.0002$ $Z(15000K) = 2.0292$



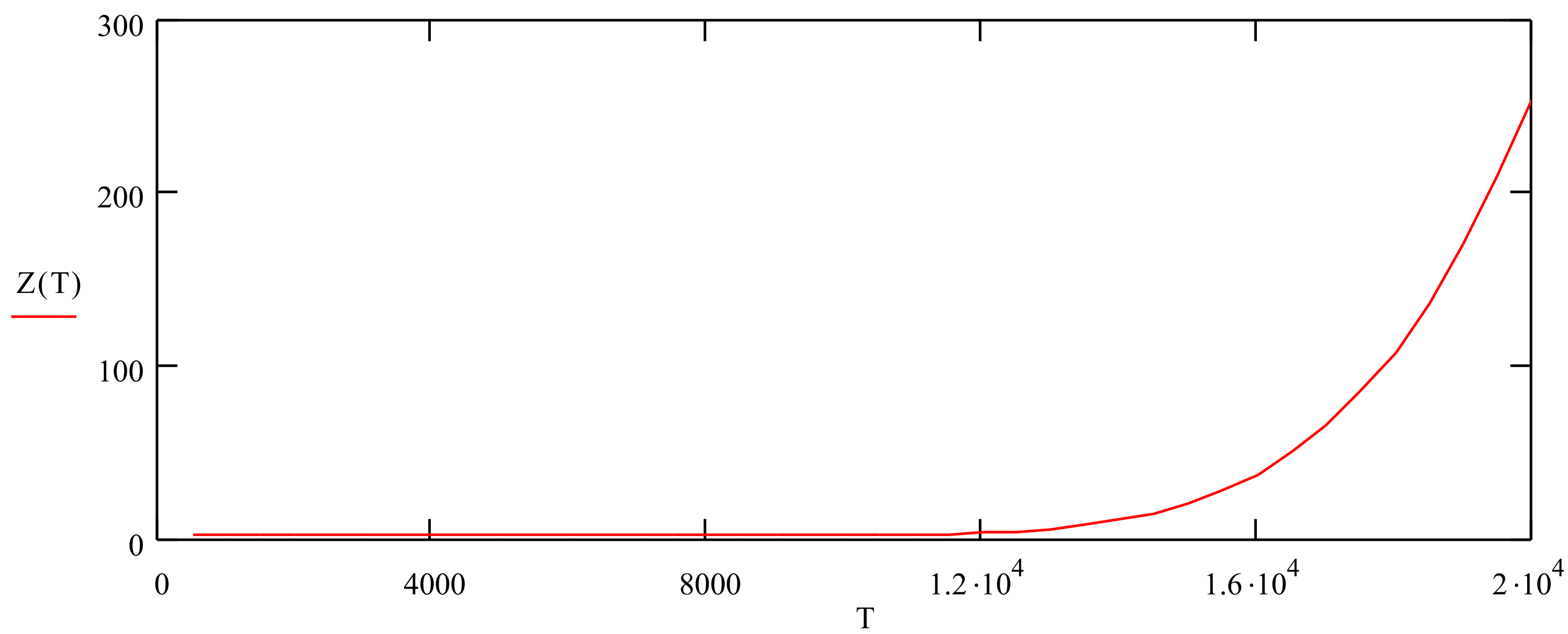
# Problem 8.8

- The partition function diverges at $n \to \infty$
  - Why do we ignore large $n$?

Partition Function: Counting the first 100 states... Energy: $E(n) := \frac{-13.6\text{eV}}{n^2}$ Degeneracy: $g(n) := 2 \cdot n^2$

$$f_B(n,T) := \exp\left[\frac{-(E(n) - E(1))}{k \cdot T}\right] \qquad Z(T) := \sum_{n=1}^{100} \left(g(n) \cdot f_B(n,T)\right) \qquad T := 0, 500..20000$$

$Z(6000\text{K}) = 2.0000$ $Z(10000\text{K}) = 2.0952$ $Z(15000\text{K}) = 20.2988$

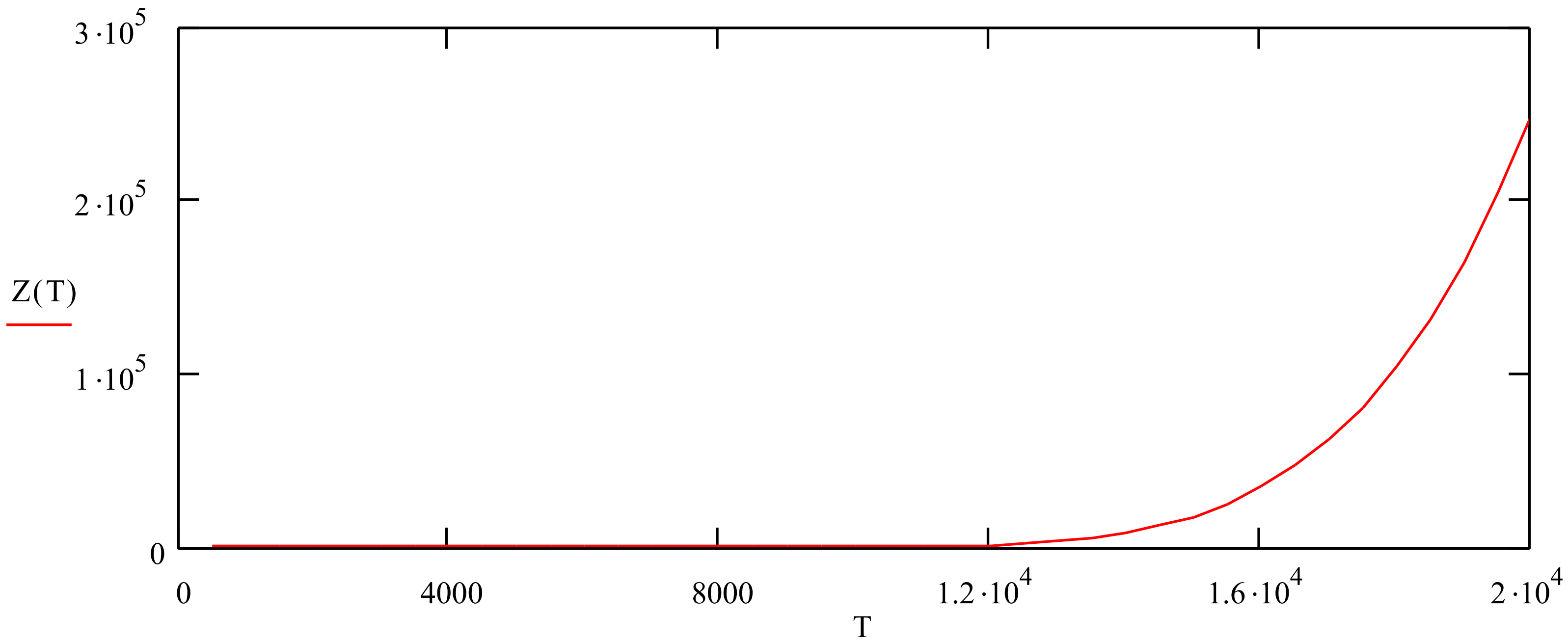


# Problem 8.8

Partition Function: Counting the first 1000 states... Energy: $E(n) := \frac{-13.6\text{eV}}{n^2}$ Degeneracy: $g(n) := 2 \cdot n^2$

$$f_B(n,T) := \exp\left[\frac{-(E(n) - E(1))}{k \cdot T}\right] \qquad Z(T) := \sum_{n=1}^{1000} \left(g(n) \cdot f_B(n,T)\right) \qquad T := 0, 500 .. 20000$$

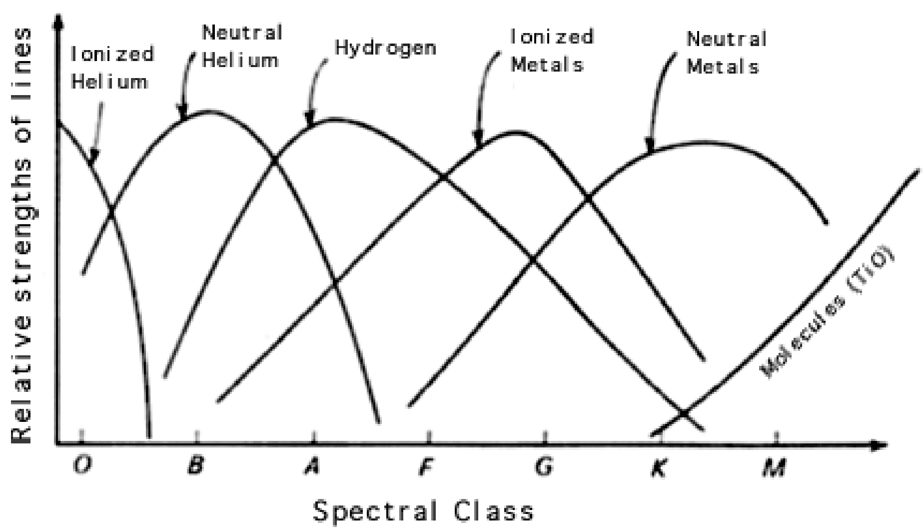
$Z(6000K) = 2.0025$ $\quad Z(10000K) = 95.4311$ $\quad Z(15000K) = 1.7998 \times 10^4$




- Ionization
- Unphysical orbital size $\quad r_n = a_o n^2$

# Example 8.4

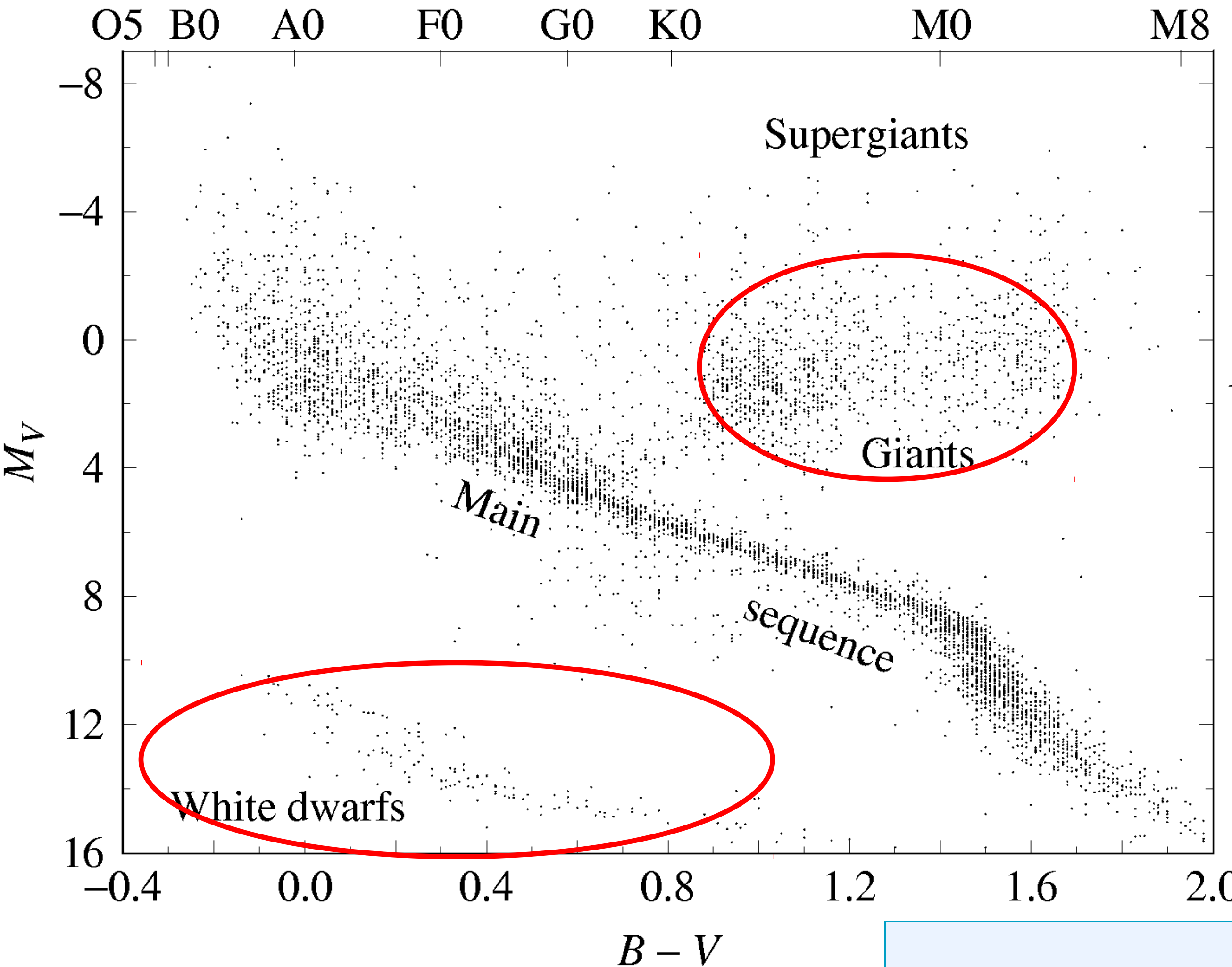
- Surface of the Sun has 500,000 hydrogen atoms per calcium atom, but calcium absorption lines are much stronger than the Balmer series lines.
- The Boltzmann and Saha equations reveal that there are 400× more Ca atoms in the ground electronic state than in the *n=2* hydrogen state.
- Calcium is not more abundant
- Differences are due to sensitive temperature dependence



# Hertzsprung-Russell (H-R) Diagram

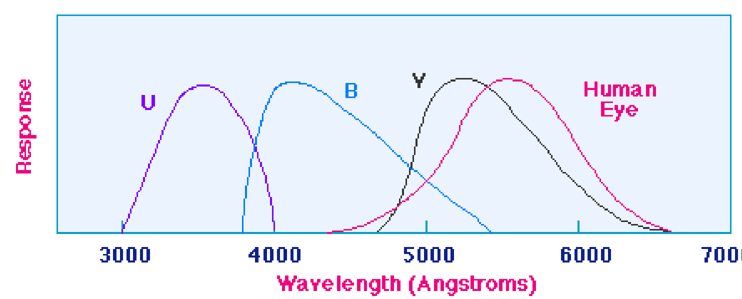
Spectral class

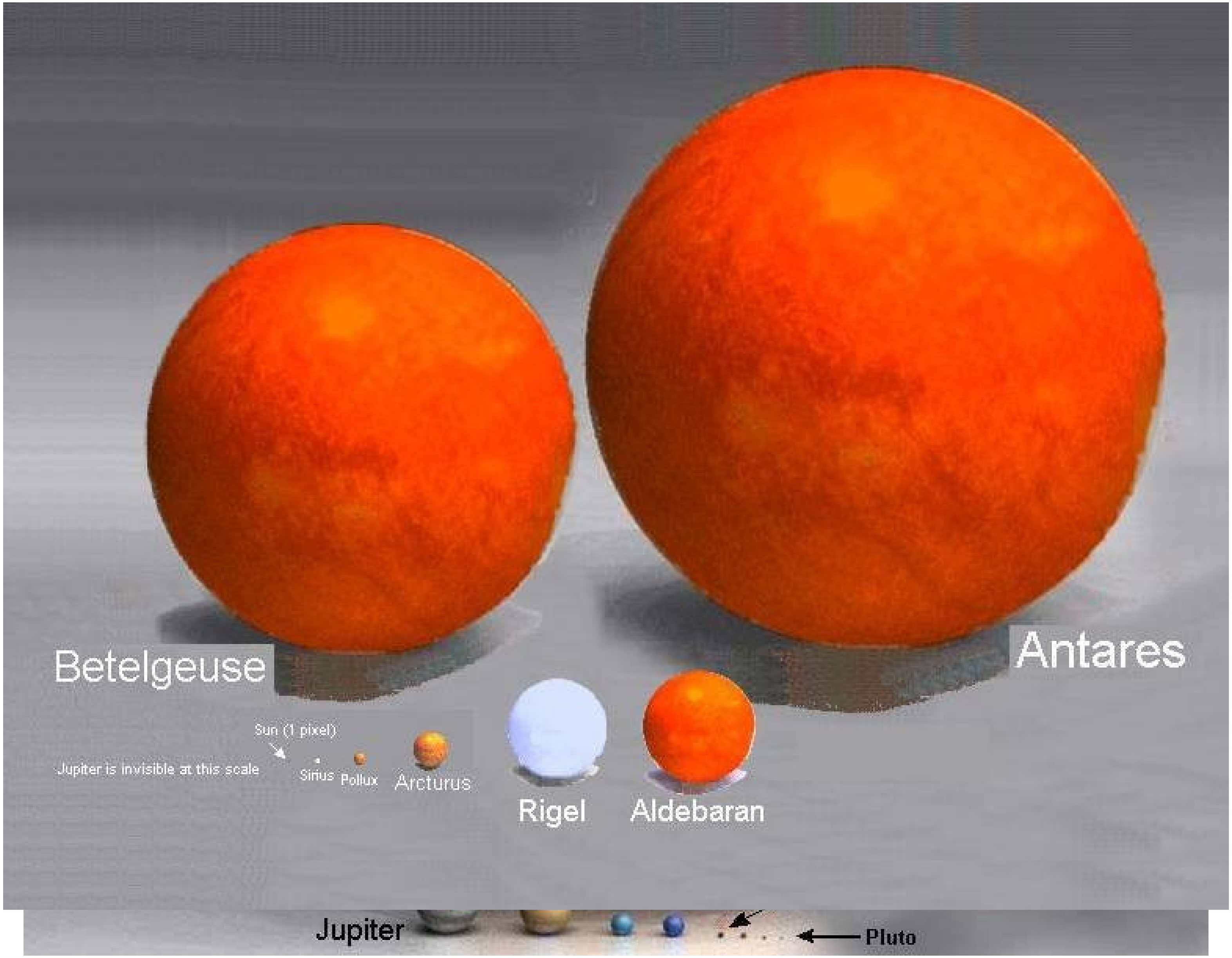
Luminosity (Absolute Magnitude)




Color index

$$R = \frac{1}{T^2}\sqrt{\frac{L}{4\pi\sigma}}$$



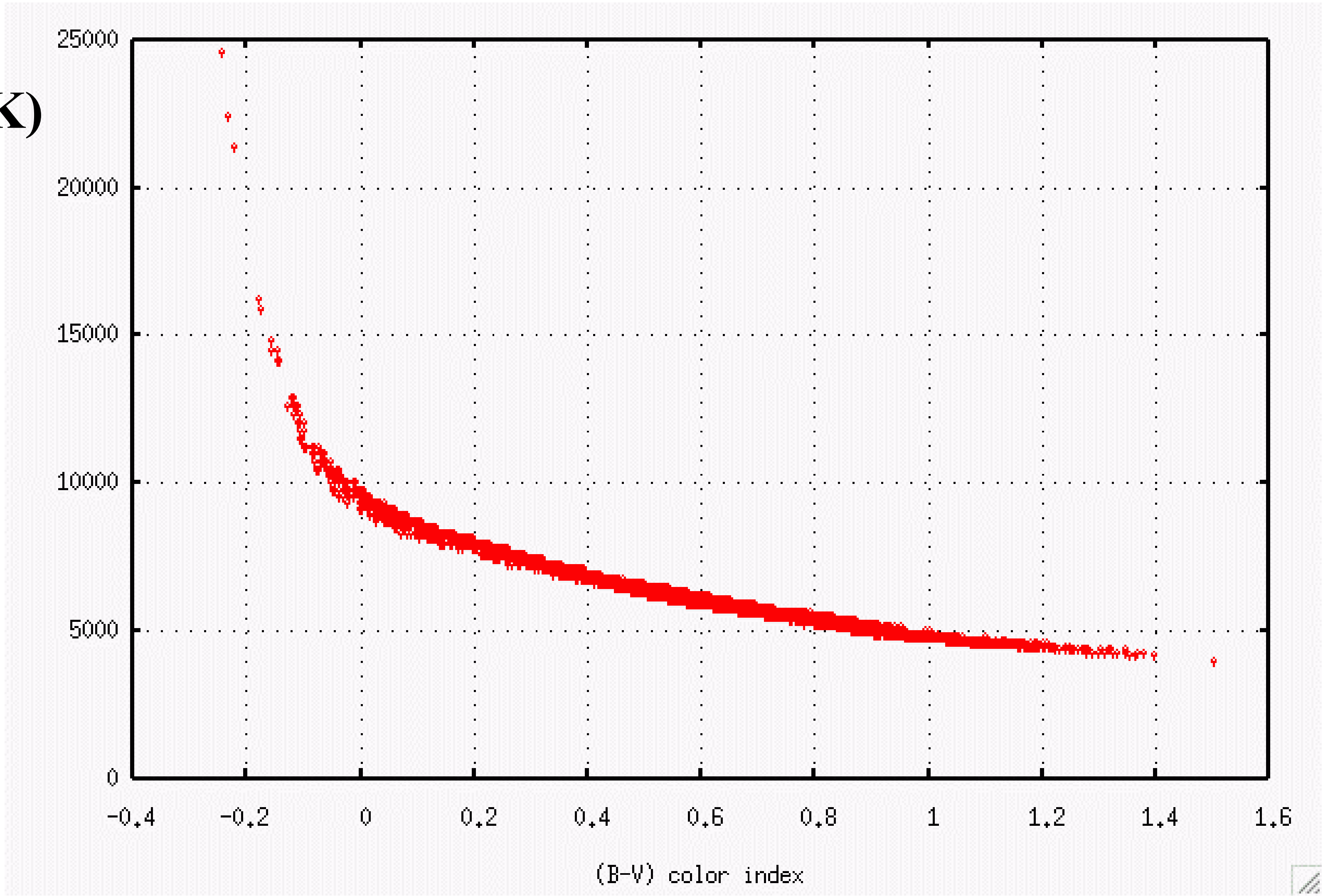


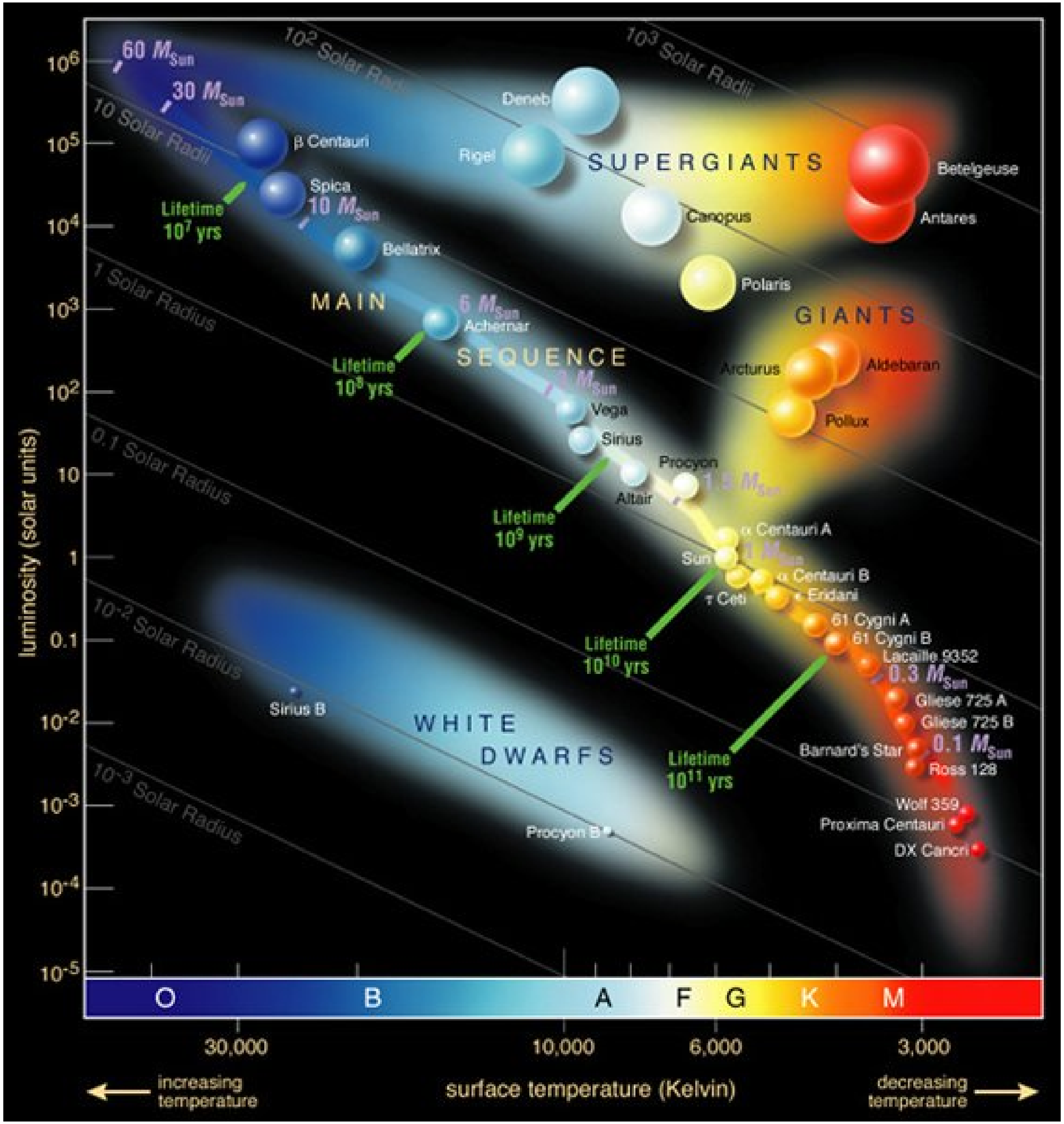
Betelgeuse
Antares
Sun (1 pixel)
Jupiter is invisible at this scale
Sirius
Pollux
Arcturus
Rigel
Aldebaran
Jupiter
Pluto

# How temperature relates to color index

……………………….

T (K)

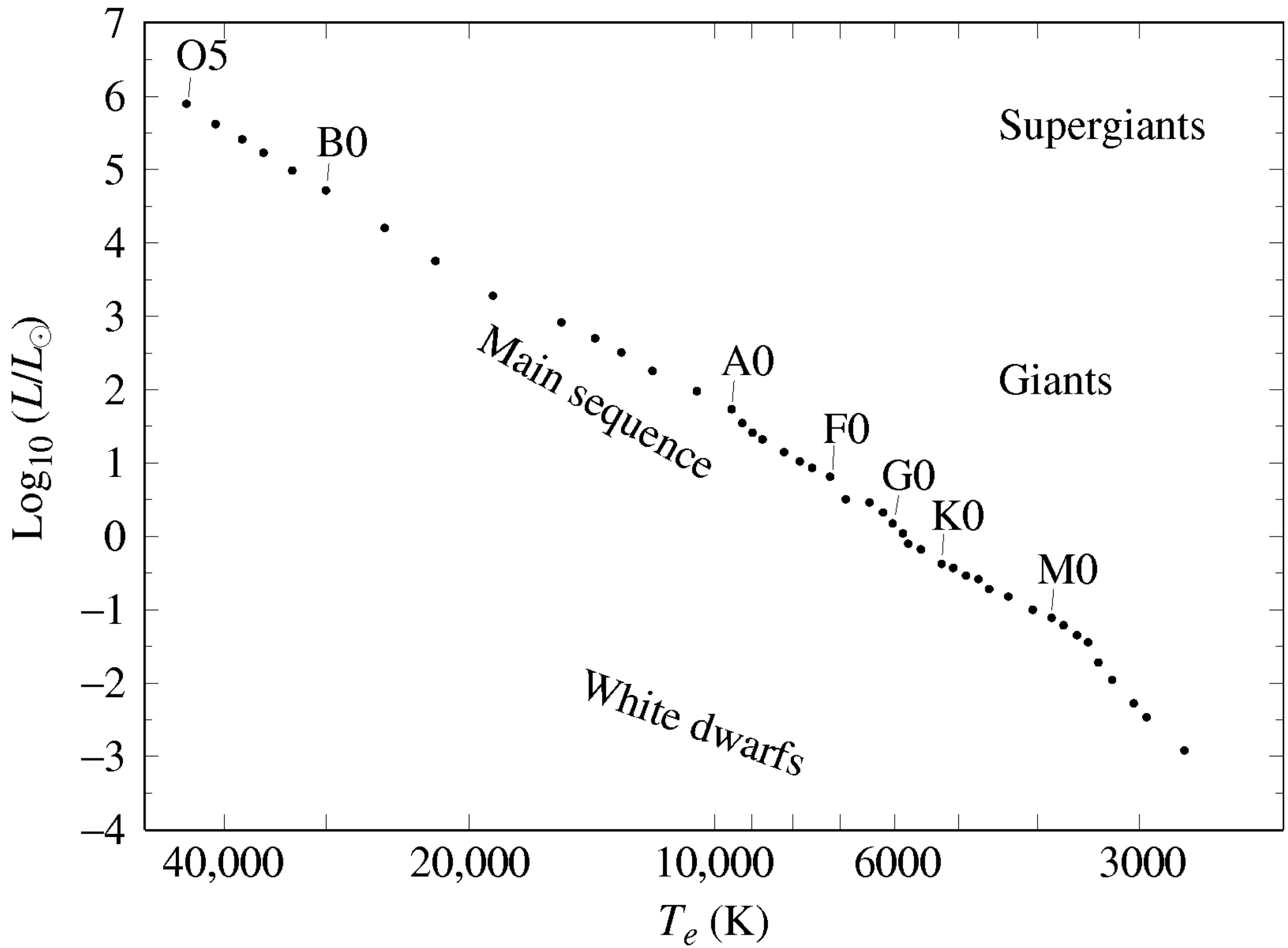





**A colorful H–R Diagram**

# Hertzsprung-Russell (H-R) Diagram

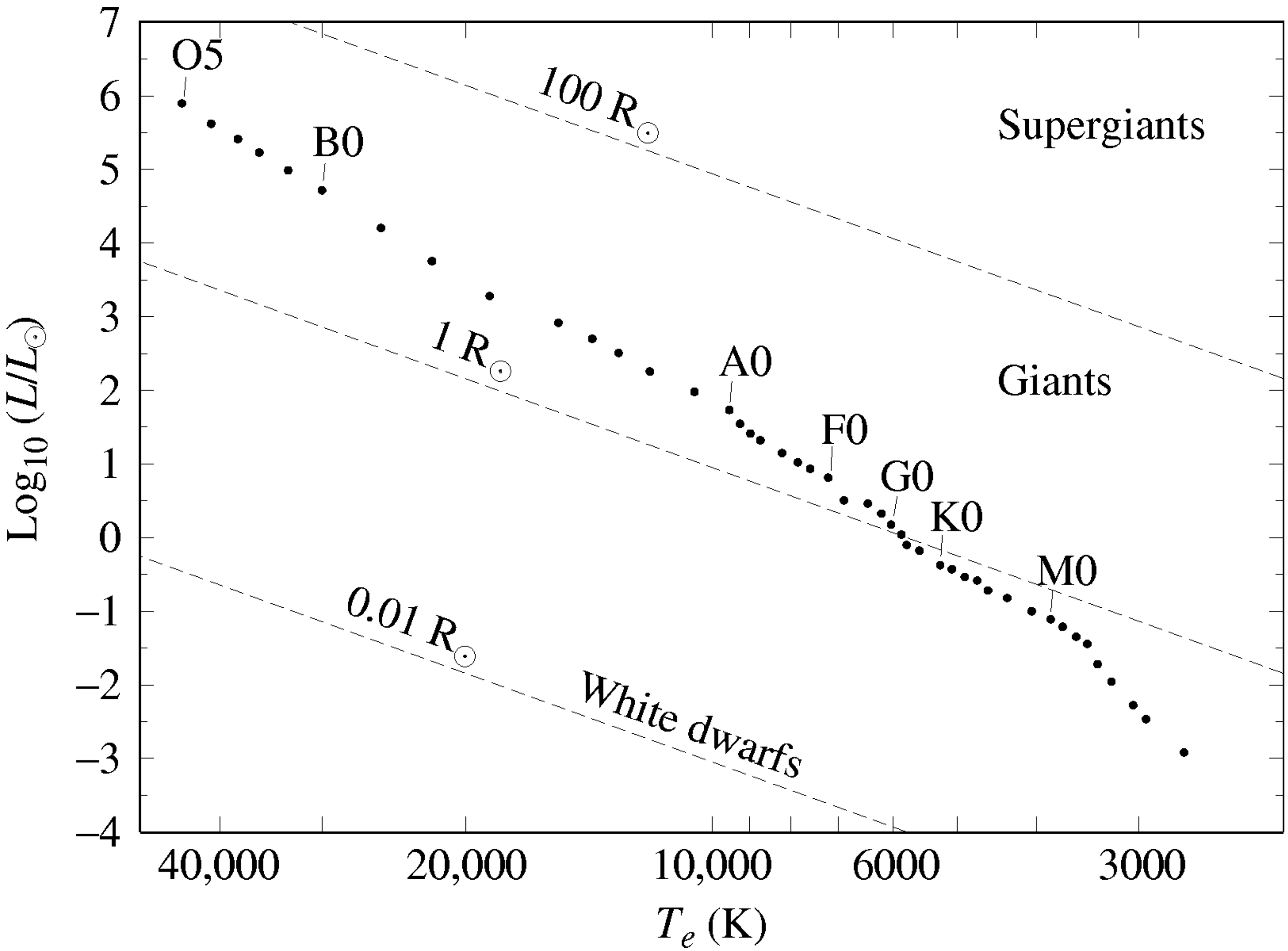
- Luminosity and Temperature rather than Magnitude and Color Index



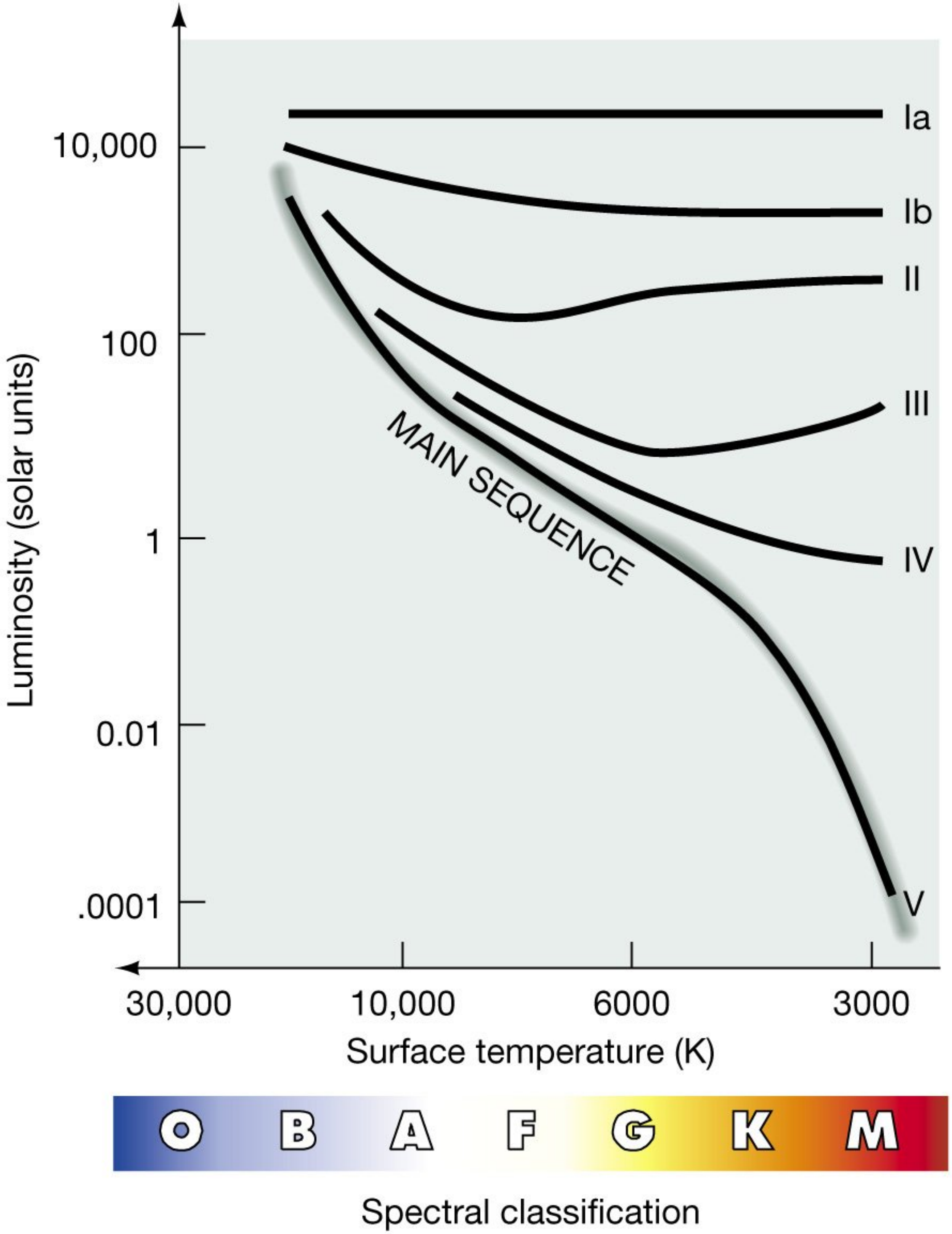
# Hertzsprung-Russell (H-R) Diagram

- Star Radius

$$R = \frac{1}{T^2}\sqrt{\frac{L}{4\pi\sigma}}$$



# Luminosity Classes



# Stellar Luminosity Classes

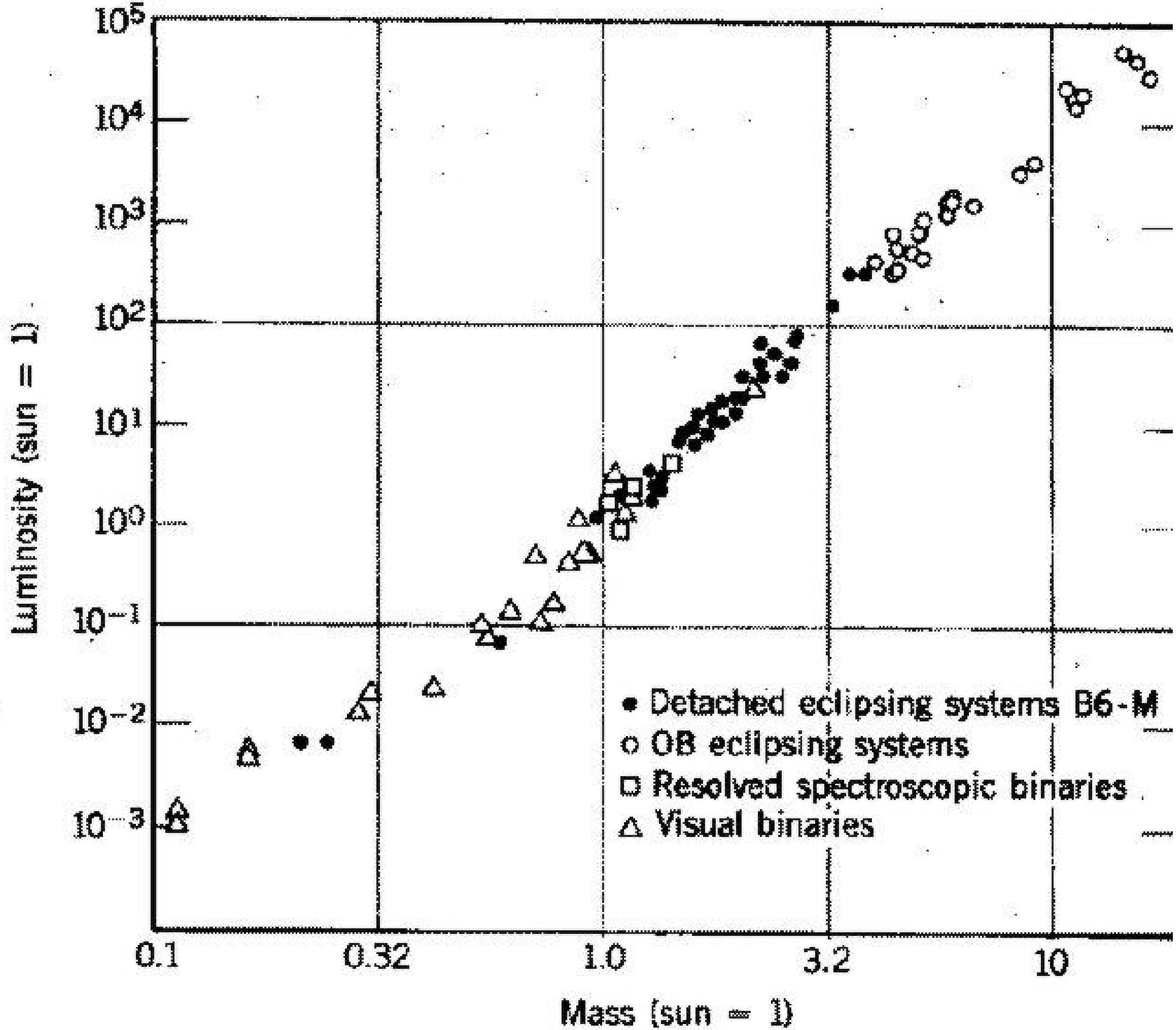
**TABLE 17.3 Stellar Luminosity Classes**

| Class | Description |
|---|---|
| Ia | Bright supergiants |
| Ib | Supergiants |
| II | Bright giants |
| III | Giants |
| IV | Subgiants |
| V | Main-sequence stars and dwarfs |



Some define VI and wd (or D)

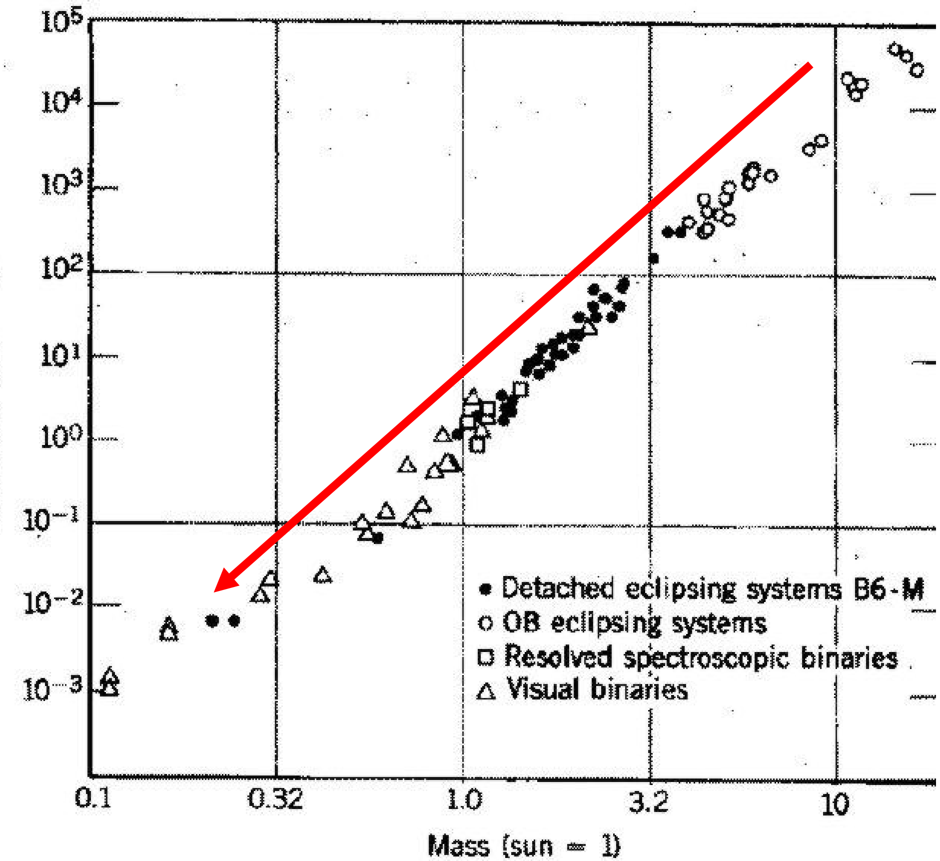
# Mass-Luminosity Relation from Binary Systems




The mass – luminosity relation for stars, as determined from binary systems, in which the individual masses can be found.

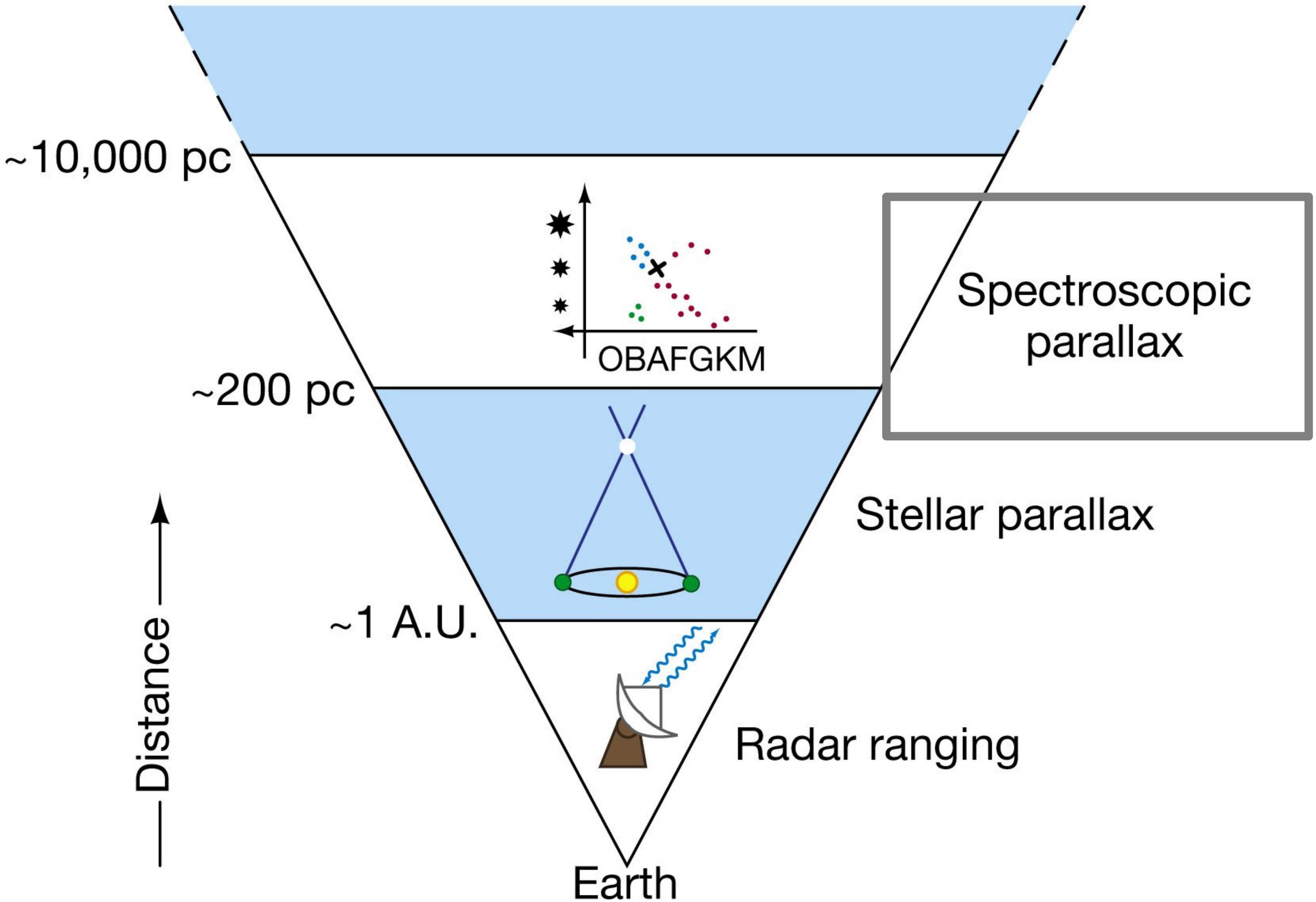
# Mass-Luminosity Relation

- Early theories had “early” O-type (bright, hot, massive) stars evolving to “old” M-type stars  (dim, cool, less massive)




The mass – luminosity relation for stars, as determined from binary systems, in which the individual masses can be found.

# Figure 17-17
# Stellar Distances



# Spectroscopic "Parallax"

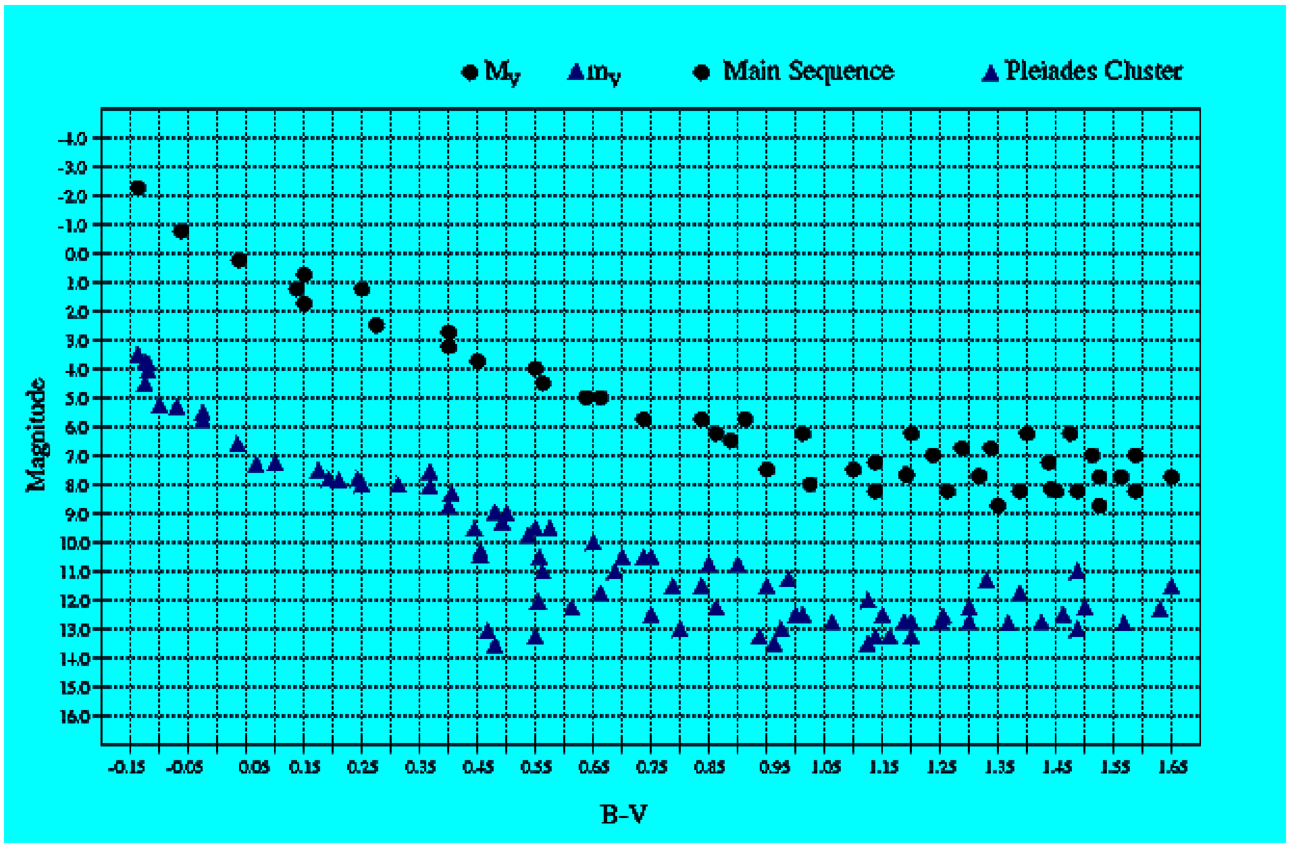
- Method to determine a stars distance
  - Determine the star's spectral class
  - Read the absolute magnitude from the H-R diagram
  - Compare to apparent magnitude to determine distance
  - Accurate to a factor of ± 1 magnitude
    - $10^{1/5} = 1.6$

# Stellar and Spectroscopic Parallax

**Stellar Parallax** works out to 200pc (ground), 1000 pc (Hipparcos)

**Spectroscopic Parallax** works for stars for which a good spectrum can be observed (about 8 kpc), but ...

- **Not precise for individual stars, especially giants**
- **Entire clusters of stars works better! ("main-sequence fitting")**




$$m - M = 5\log(d/10)$$

Spec Parallax assumes, for example, that all A0V stars have the same M.
That makes A0V stars "standard candles".