Classical Wave Description of Light

James Clerk Maxwell

- Related charges, currents, E-fields, and Mag-fields
- Unified Electricity and Magnetism into <u>four</u> equations
  - Electric Fields from charges
  - Electromagnetic Induction
    - Electric fields from changing magnetic fields
  - No magnetic monopoles (N/S pairs)
  - Electromagnet
    - Magnetic fields from moving charges (currents)

- Predicted traveling waves in Electric and Magnetic Fields (E-M waves) when <u>charges accelerate</u> or currents change
- No medium needed – happens in "empty space"
  - Ether theory (medium for E-M waves) has been disproved
    - the Greeks 5th element

Maxwell’ s Equations

- Gauss’ s Law $$\nabla \cdot \vec{E} = \frac{1}{\varepsilon_o} \rho$$
- Faraday’ s Law $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
- No magnetic monopoles $$\nabla \cdot \vec{B} = 0$$
- Ampere’ s law with Maxwell’ s correction $$\nabla \times \vec{B} = \mu_o \vec{J} + -\mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}$$

E & M Waves

- Classical wave equation

$$\nabla^2 f = \frac{1}{v^2{}_o} \frac{\partial^2 f}{\partial t^2}$$

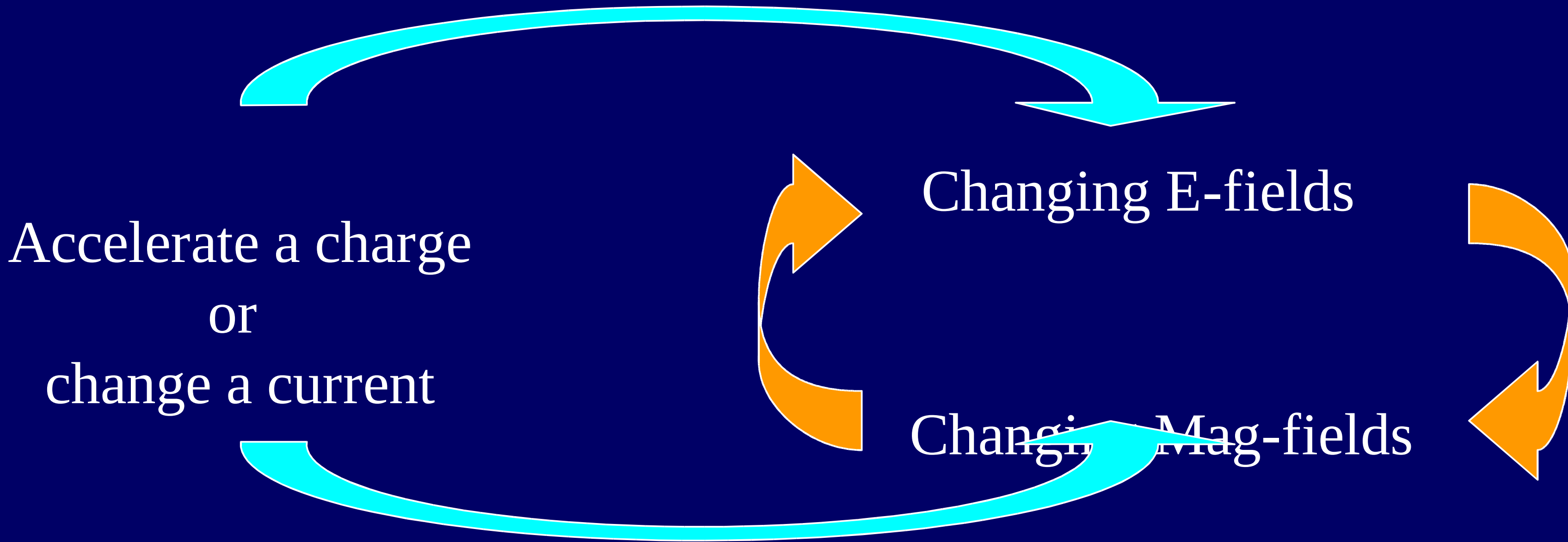
- From Maxwell' s Eqs

$$\nabla^2 \vec{E} = \mu_o \varepsilon_o \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu_o \varepsilon_o \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = 2.99792458 \times 10^8 \ {}^{m}\!/\!{}_{s}$$

E-M Waves



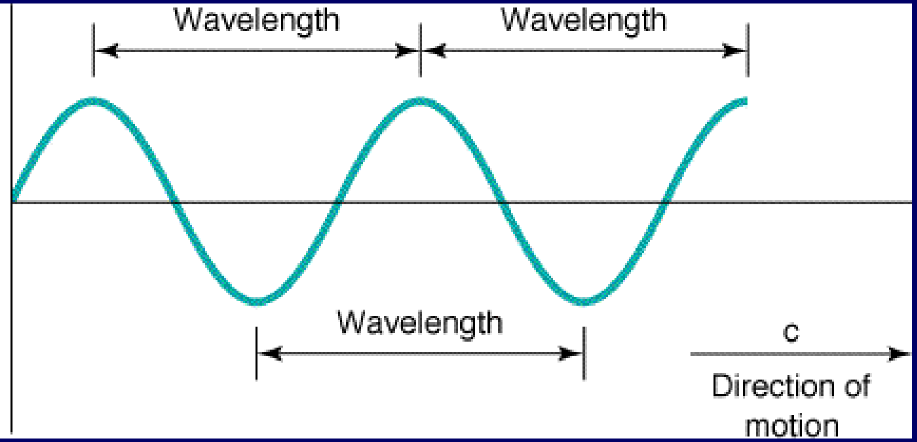
Self-propagating E-M wave

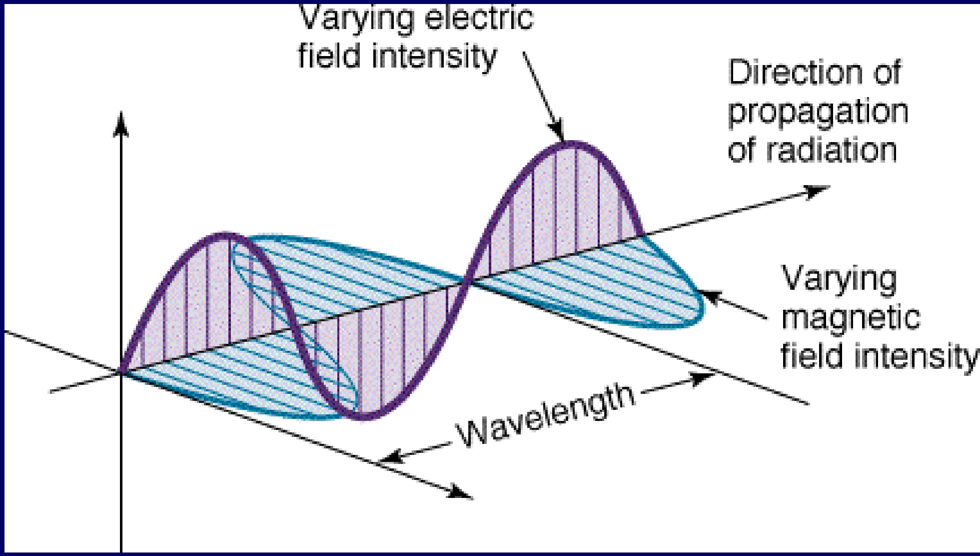
Wave Nature of Light

- Some Properties:
  - No medium needed (very strange)

    speed = c = 299792458 × 10^8 m/s
  - Transverse wave
    - Polarization
      - The E-field interacts in matter (electrons)

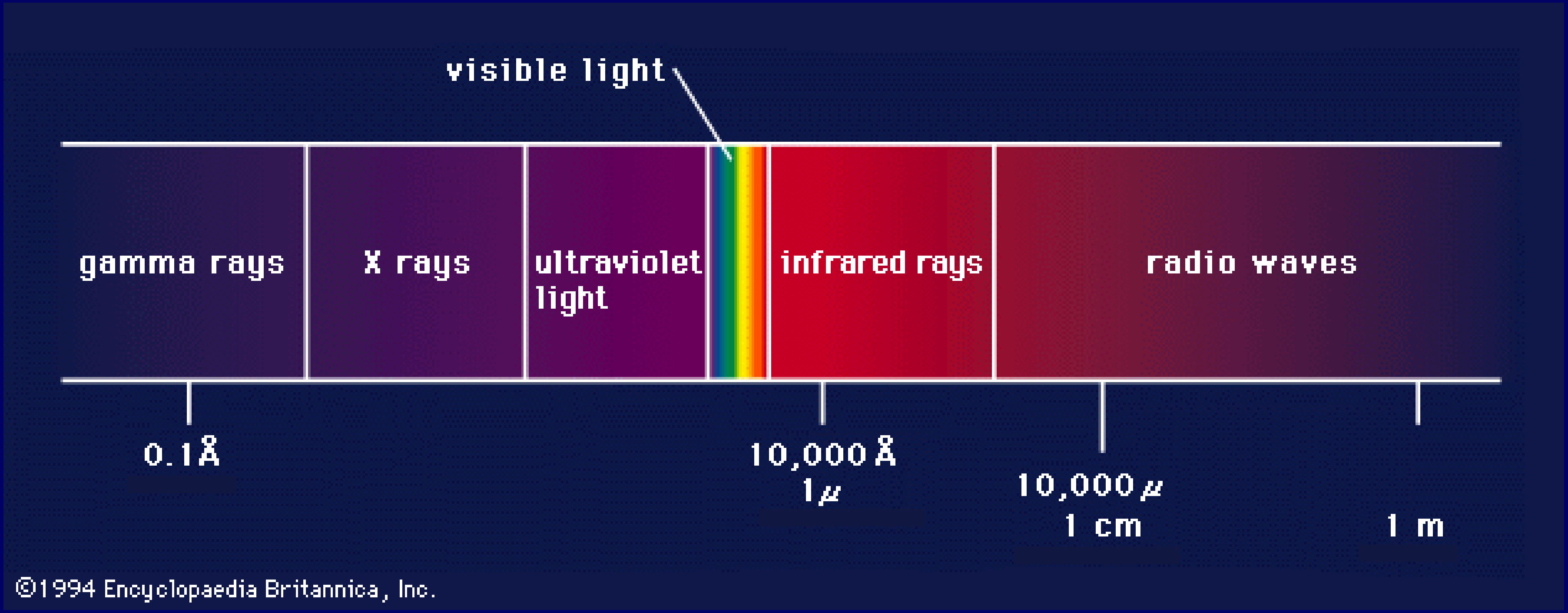
$$c = \lambda f$$



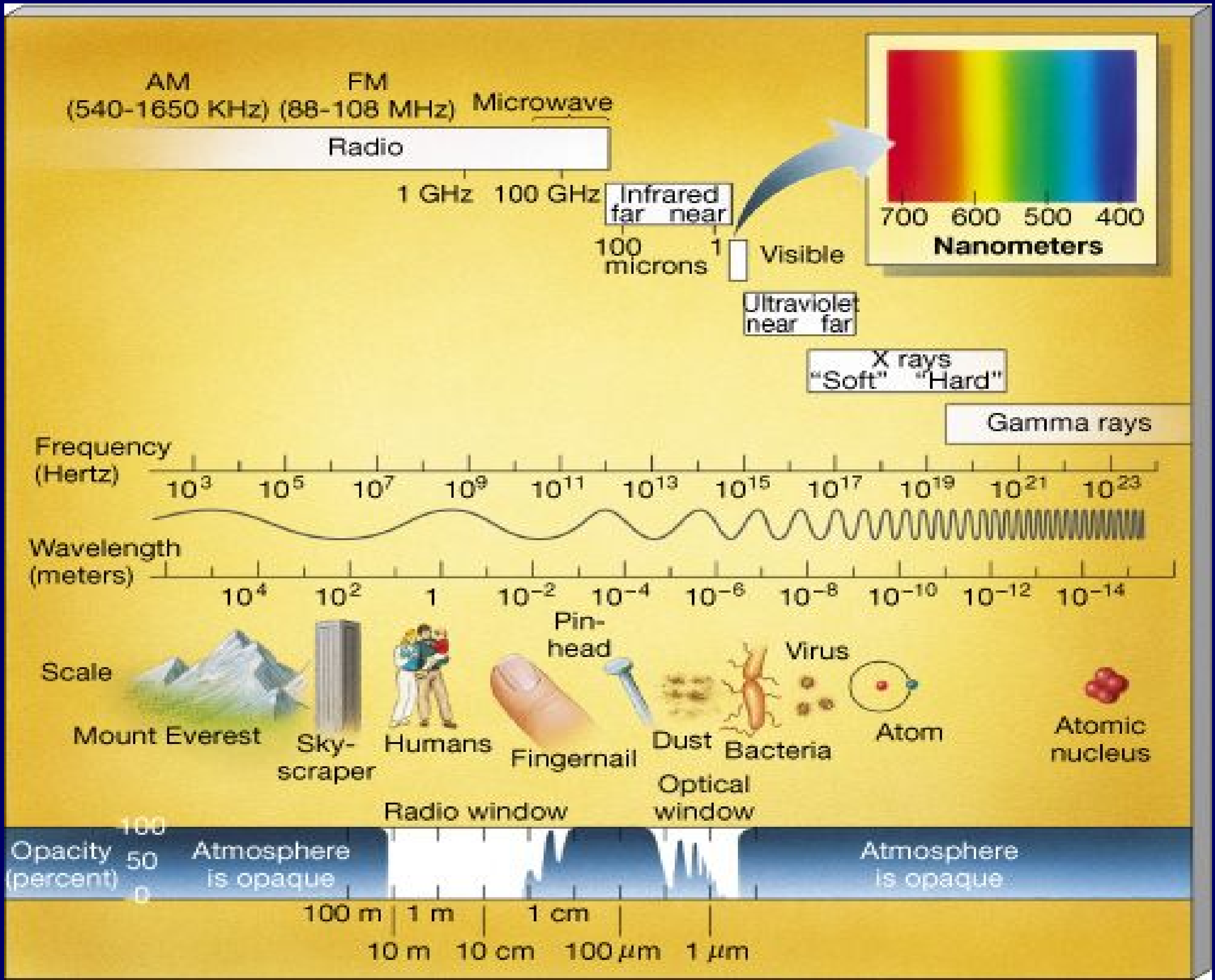


Wave Nature of Light

$$c = \lambda f$$

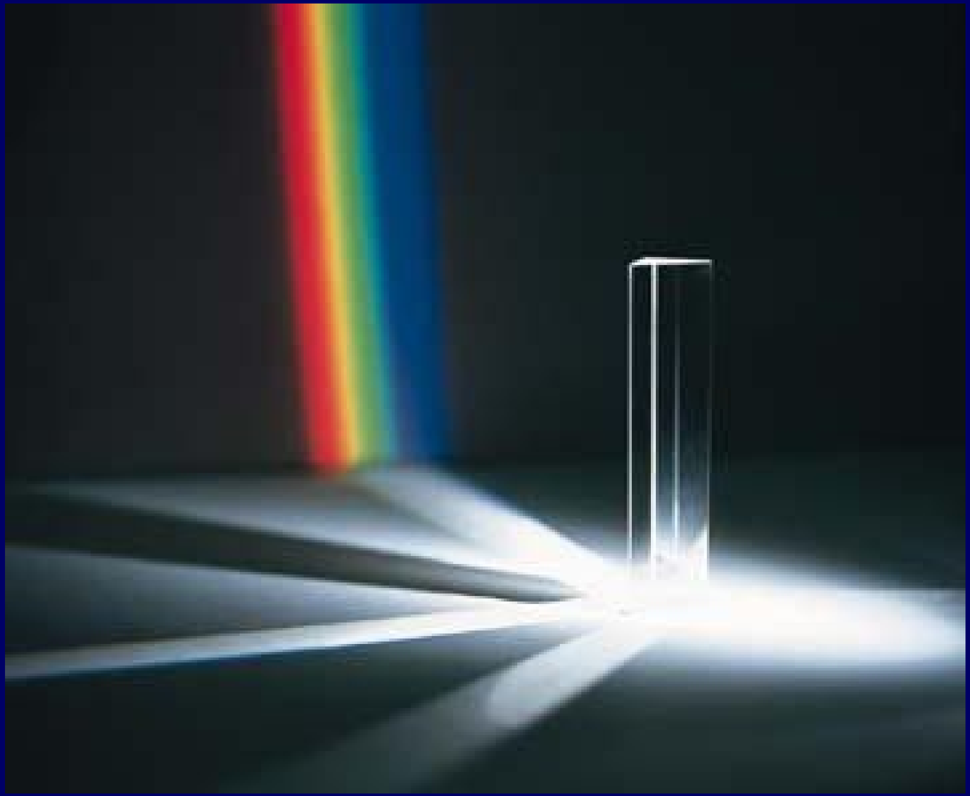


Wave Nature of Light



The Visible Spectrum

- White Light
  - mixture of all colors
  - ROY G. BIV
  - long λ to short λ
  - low *f* to high *f*
  - low Energy to high



$$c = \lambda f$$

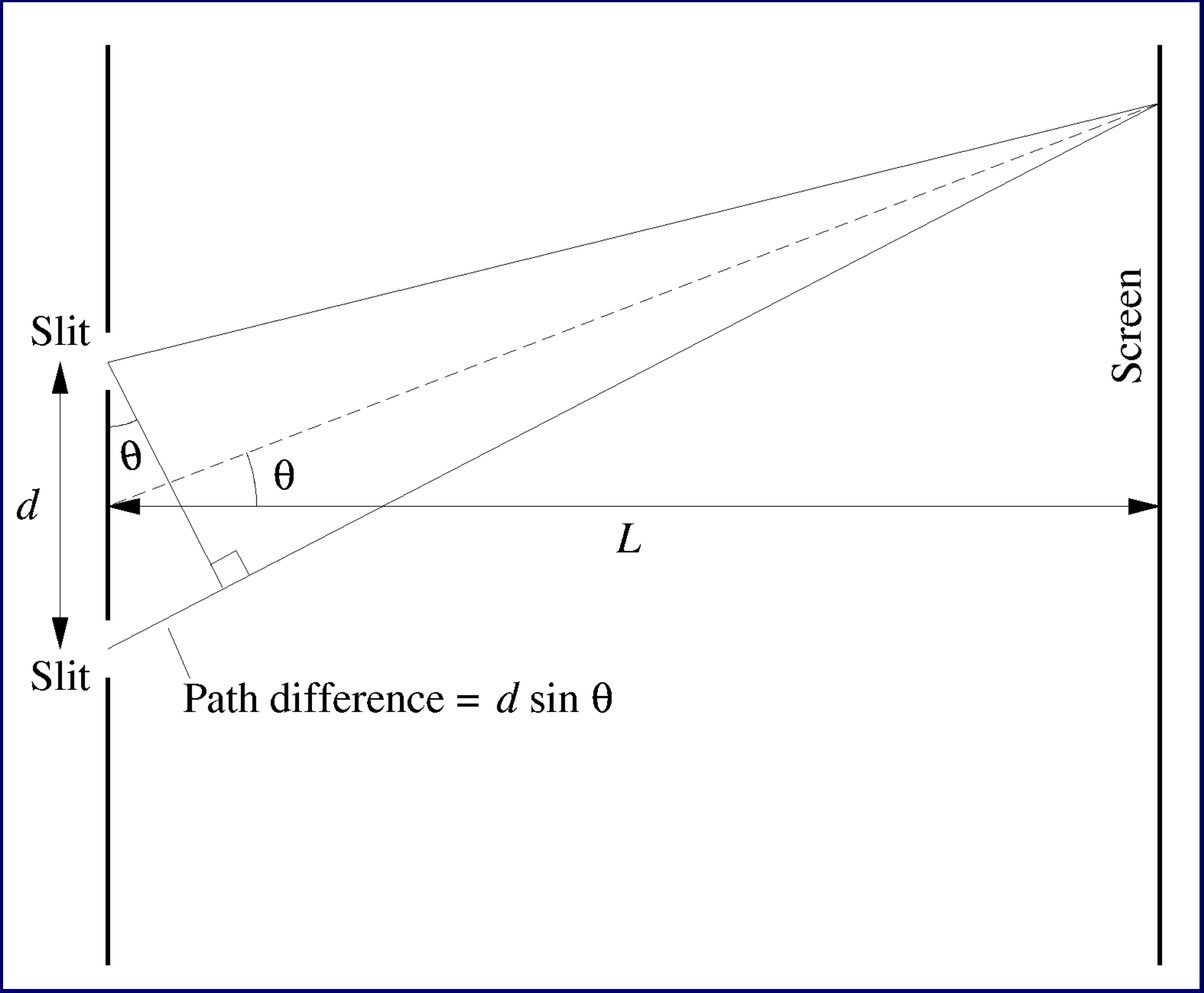
$$0.7\times10^{-6} - 0.4\times10^{-6}\,m \text{ or } 700 - 400\,nm$$

$$1nm = 1\times10^{-9}\,m$$

Evidence of Waves: Young's Double Slit Experiment

Two narrow slits –> diffraction

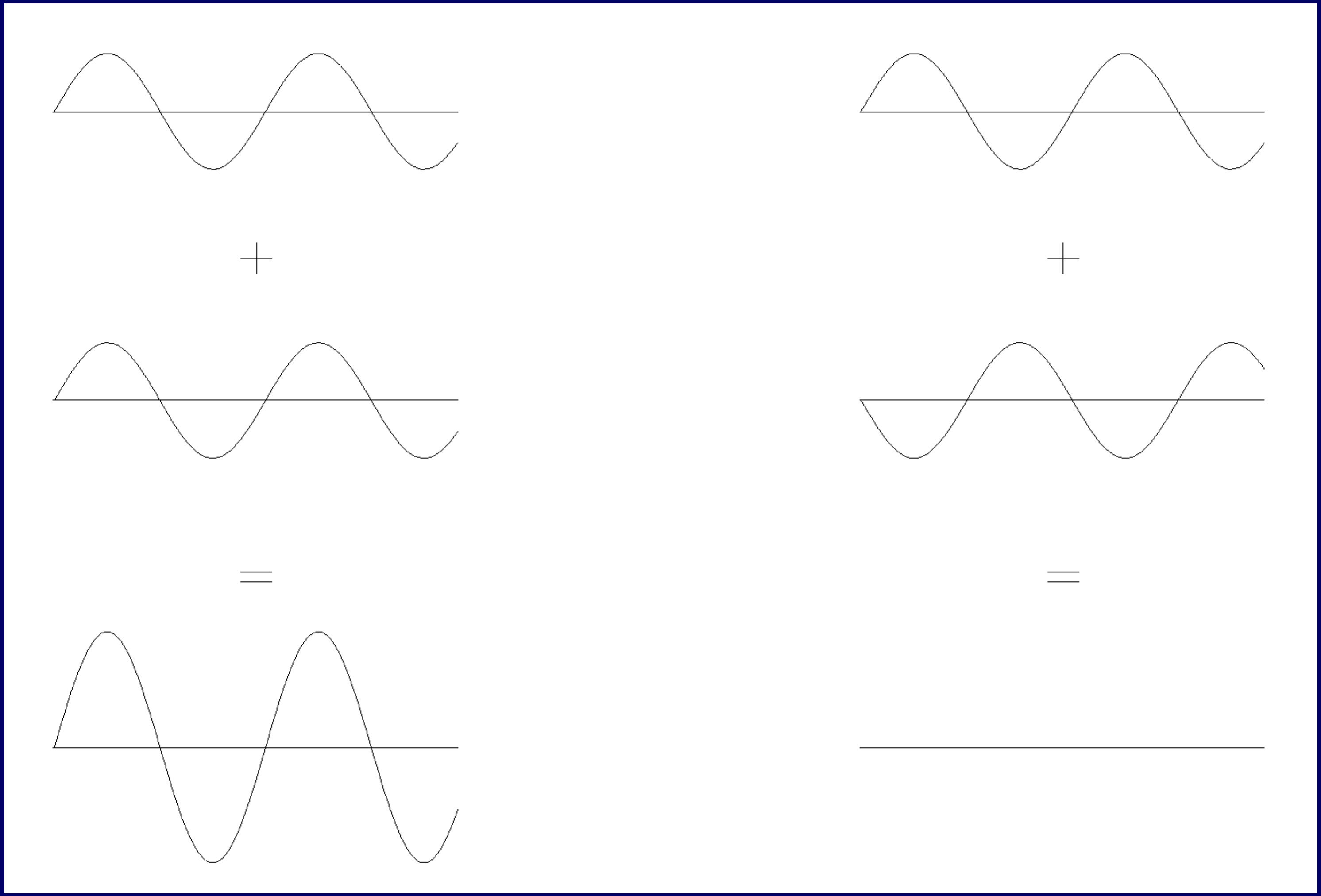
L >> d



Superposition

Constructive interference

Destructive interference



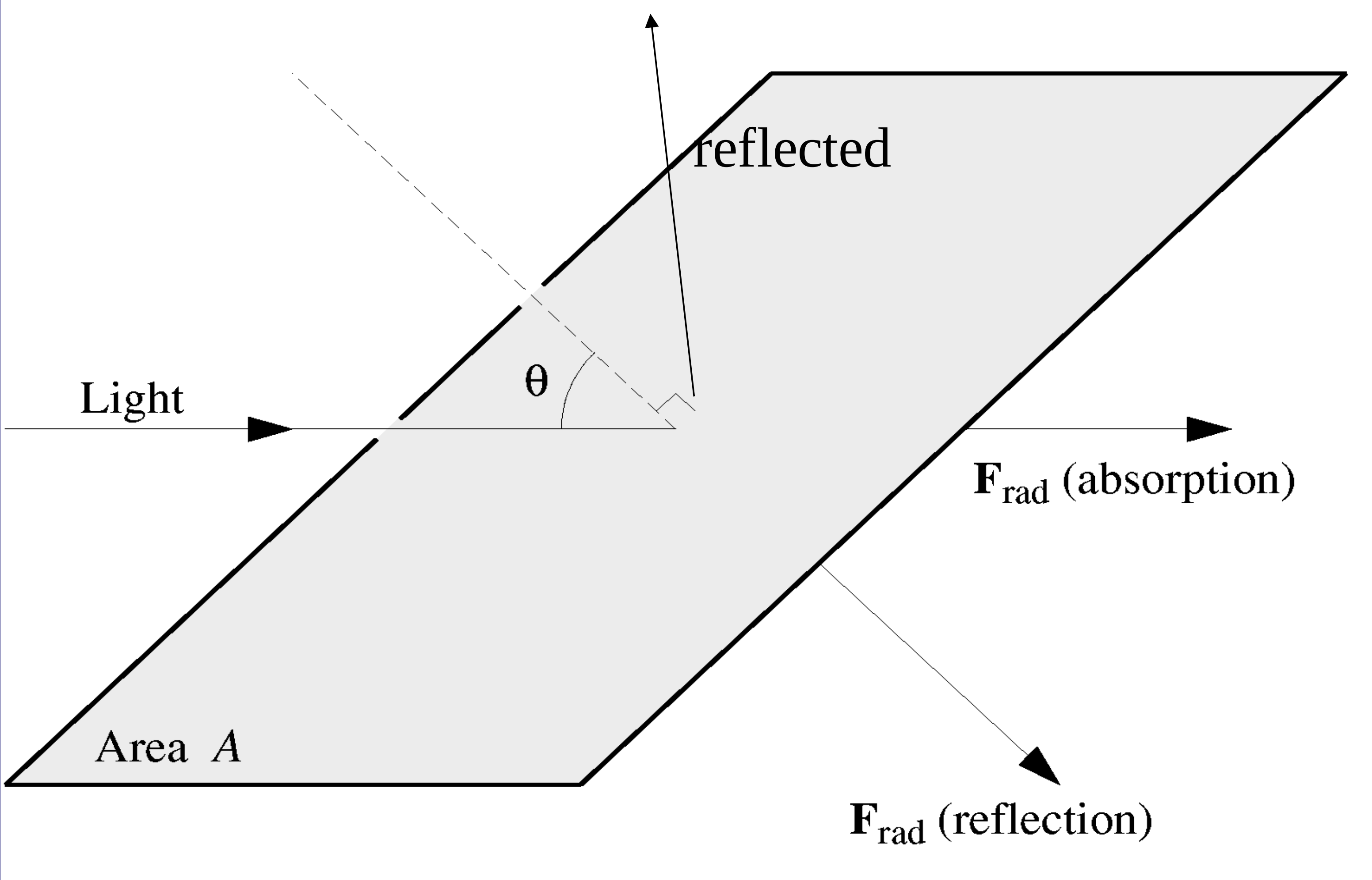
Poynting Vector, **S**

- A vector pointing in the direction of propagation
- Magnitude is equal to the amount of energy per unit time that crosses a unit area oriented perpendicular to the direction of propagation of the wave
- Need to take a time average since waves vary harmonically with time (sin and cosine functions)
  - Average over one period $\langle S \rangle = \frac{c}{(8\pi)} E_0 B_0$ Erg/s/cm^2
- This is for a particular wavelength, its a *monochromatic* flux
  - Need contributions from all wavelengths to get the radiant flux

- NOTE: Light carries both energy and momentum, but does not have a rest mass

Radiation Pressure

- Result of the momentum carried by the light.
  - Depends on reflection or absorption (both could happen)

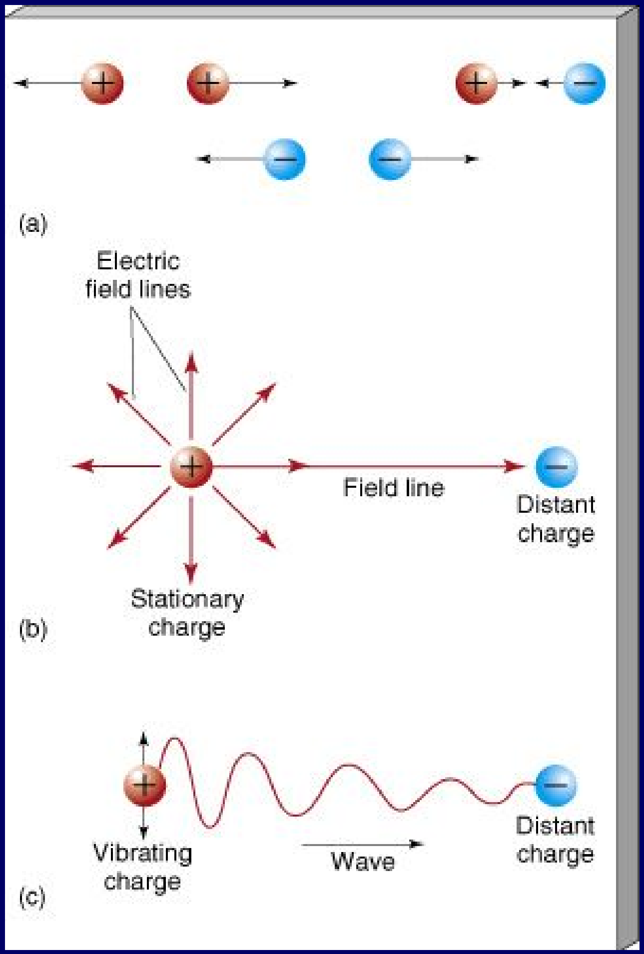


Radiation Pressure

- Important role in stability of stars – equilibrium with gravity
- May also significantly effect interstellar “dust”

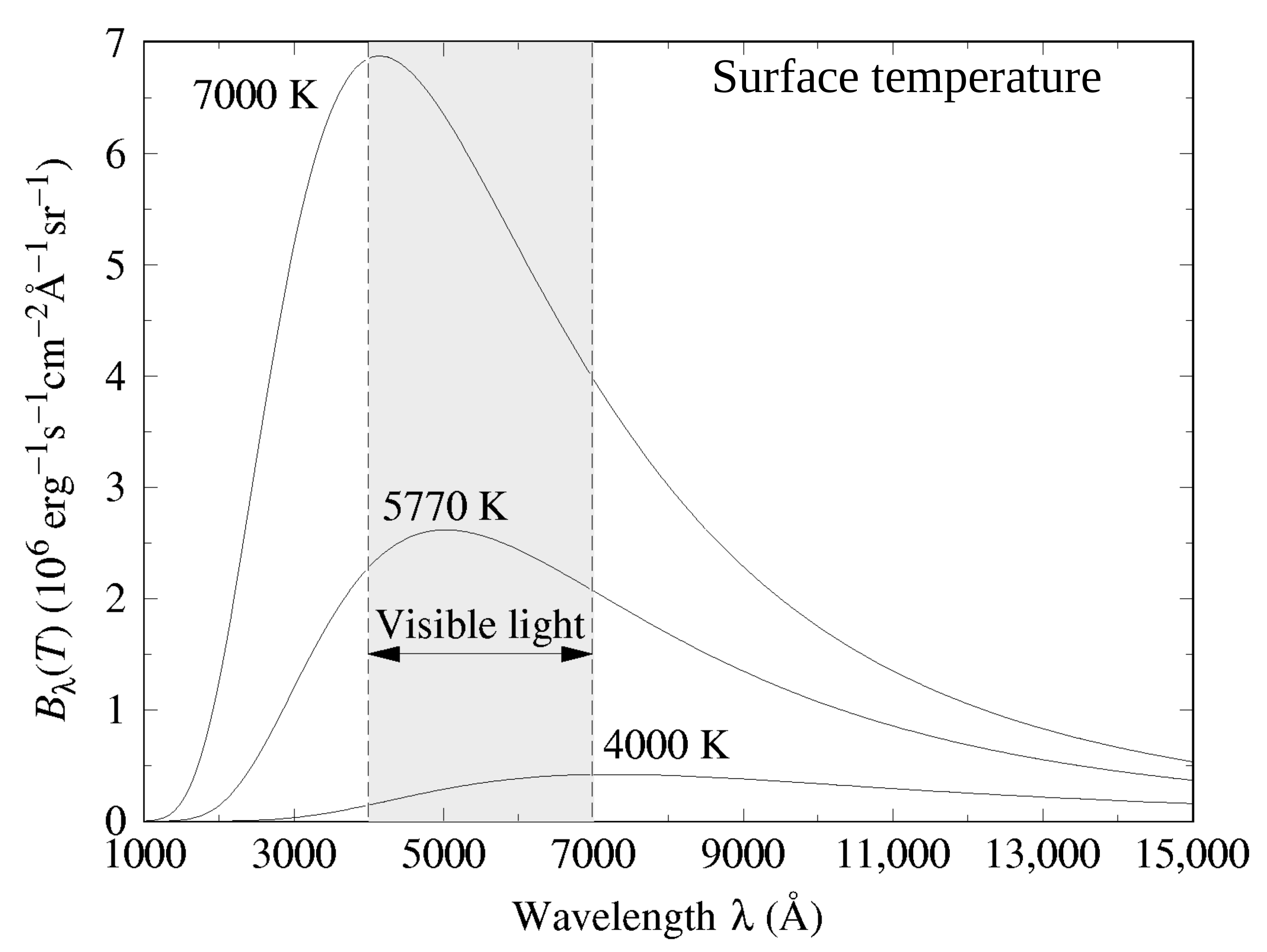
Blackbodies

- All objects above absolute zero emit radiation
  - Molecules and atoms are constantly in motion (thermal energy)
    - Accelerating charges   E & M radiation
  - Radiation (amount and type) will be temperature dependent
- Blackbody
  - Absorbs all incident radiation (no reflection)
  - Emits blackbody radiation, dependent on the objects temperature
- Stars and Planets are very close to being ideal blackbodies



$$B_\lambda(T) = \frac{(2hc^2/\lambda^5)}{(e^{(hc/\lambda kT)} - 1)}$$

Blackbody emission



Wien's Displacement Law

- Relates the surface temperature to the wavelength at the peak of the spectrum

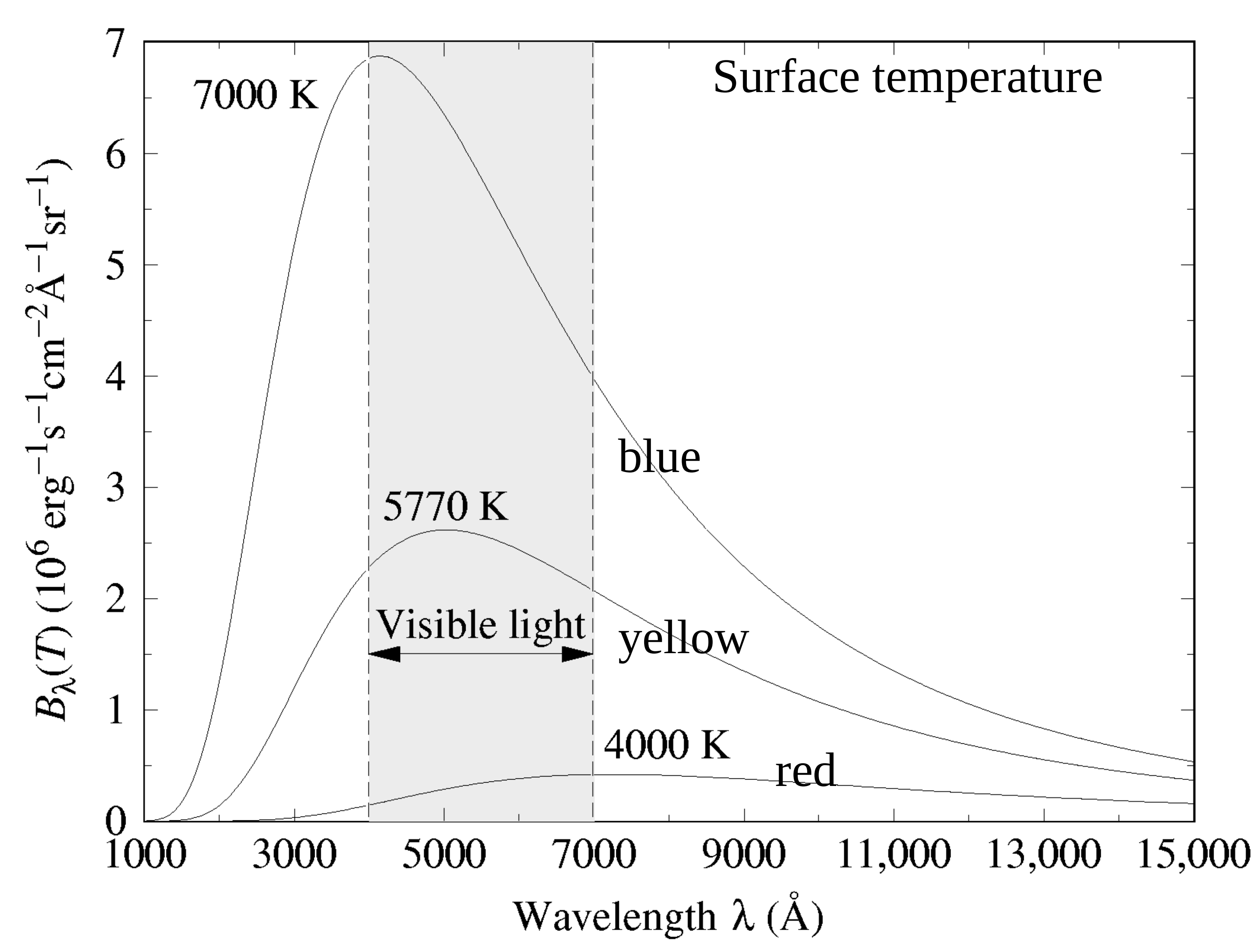
$$\lambda_{\text{max}}\, T = 0.290\ \text{cm K}$$

- Helps explain the color of stars

- Alternatively written as

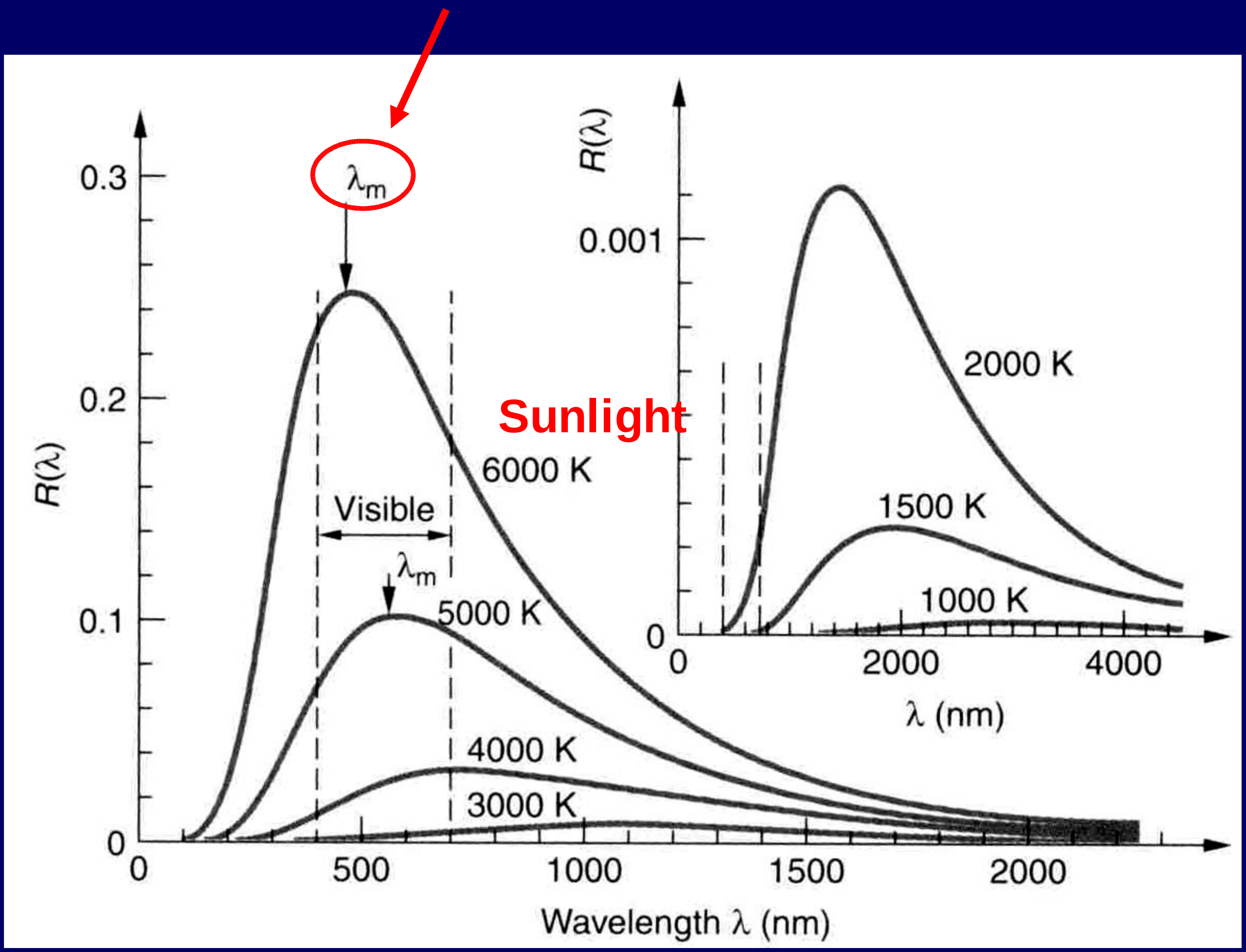
$$\lambda_{\text{max}}\, T = (5000\ \text{Å})(5800\ \text{K})$$

Blackbody Emission



Blackbody: Simple Wien's Law

$$\lambda_{max} = \frac{0.029}{T} cm \cdot K \propto \frac{1}{T}$$



Spectral Distribution depends only on Temperature

Stefan-Boltzmann Equation

- Total energy depends on temperature
- Empirically derived by Stefan
- Derived from first principles (thermo and E&M laws) by Boltzmann
- Luminosity, *L* [ergs/sec]

$$L = \varepsilon \, \sigma \, A \, T^4$$

A = area of blackbody

T = Temperature in Kelvin

σ = Stefan-Boltzmann constant

ε = emissivity, $0 \leq \varepsilon \leq 1$

ε = 1 is a prefect blackbody

we' ll assume this

Stefan-Boltzmann Equation

- For a <u>star of radius *R*</u> the surface area is $4\pi R^2$, so the Luminosity is

$$L = A\sigma T^4 = 4\pi R^2 \sigma T^4$$

- To get the <u>*radiant flux*</u> at the <u>surface</u> of the star, we divide by the area, $4\pi R^2$, so

$$F = \frac{4\pi R^2 \sigma T^4}{4\pi R^2} = \sigma T^4$$

- At a known <u>distance, *d*,</u> from the star, the radiant flux becomes

$$F = \frac{4\pi R^2 \sigma T^4}{4\pi d^2} = \sigma T^4 \left(\frac{R}{d}\right)^2$$

- Since stars are not perfect blackbodies, the temperature is often called the *effective temperature* T_e (when $\varepsilon < 1$)

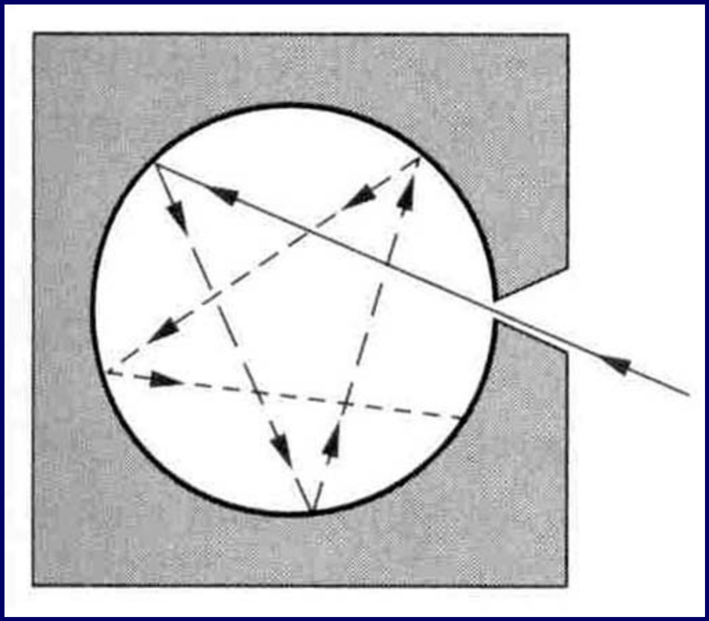
Hertz Quote

- “What is light? Since the time of Young and Fresnel we know that it is wave motion. We know the velocity of the waves, we know their lengths, and we know that they are transverse; in short, our knowledge of the geometrical conditions of the motion is complete. A doubt about these things is no longer possible; a refutation of these views is inconceivable to a physicist. The wave theory of light is, from the point of human beings, certainty.”

- The battle was mainly against Newton that believed light is made of particles (corpuscles). Hertz is saying the great Newton got one wrong…but that is not the end of the story …Newton and Hertz are both right!

Blackbody Radiation: E&M Waves from “hot” object_

- **Temperature** of a body is proportional to its average translational kinetic energy
  - **Increasing** Temp: Energy (EM waves) **absorbed** via oscillating atoms
  - **Decreasing** Temp: Energy (EM waves) **emitted** via oscillating electrons
    - Emitted Energy = Thermal Radiation (red ~ 500 ^{o}C)
  - **Constant** Temp: **Equal** rates of energy absorption and emission
- **Ideal Blackbody**: absorbs ALL incident radiation and re-emits it again



“ Ideal” Blackbody

Only absorbed and emitted radiation, no reflected radiation

Spectral Blackbody: Distribution Definitions

u = Energy Density $\quad u(\lambda) = \frac{4\pi}{c} B_\lambda(T)$ $\quad$ u is in ergs/cm^3

where B_λ = total radiation emitted per unit time per unit area in the solid angle $d\Omega$

Energy Density Distribution Function

$$u(\lambda) = E_{ave}\ n(\quad)$$

E_{ave} = average energy/mode (IMPORTANT QUANTITY)

$n(\lambda)$ = # oscillation modes = $8\pi\ \lambda^{-4}$ (independent of cavity shape)

#modes count the standing waves

boundary conditions, nodes at the walls (E field = 0)

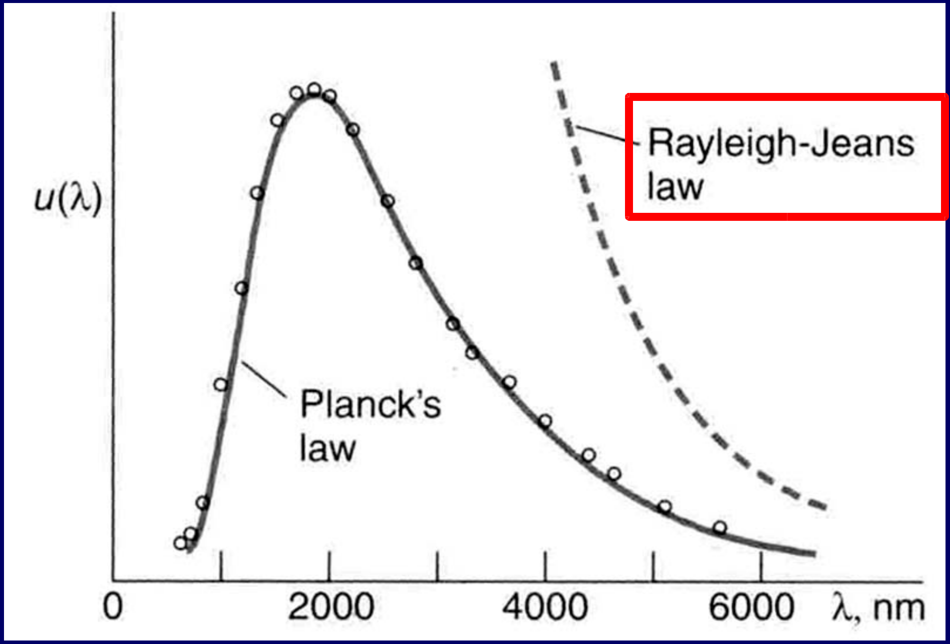
Spectral Blackbody: Rayleigh-Jeans Equation

$$u(\lambda) = kT\left(8 \quad {}^{-4}\right)$$

$E_{ave} = kT$ (Boltzmann distribution) and $n(\lambda) = 8\pi\lambda^{-4}$

$k = 1.38 \times 10^{-23}$ J/K (Boltzmann's constant)

$$\int_0^{\infty} \lambda\, u(\lambda)\, d \quad \rightarrow \infty$$



UV Catastrophe!

(explodes for small λ)

Spectral Blackbody: Planck' s Law

- Planck' s Law initially found empirically (trial and error!)
- Quantize the E&M radiation (photons)
  - Minimum energy

$$E_v = hv = {}^{hc}\!/_{\lambda}$$

where h = Planck' s Constant = 6.266 x 10^{-27} erg·s

- This is used in replacing the classical *kT* expression for the average energy in a mode

$$E_v = nhv\,, \quad n = 0, 1, 2, 3$$

  - Avoids the catastrophe – the entire hot object does not have enough energy to emit one quanta of EM waves

Spectral Blackbody: Derivation of Planck's Law

- **OLD** (Classical from Boltzmann/ Raleigh-Jeans)

$$f(E) = Ae^{\frac{-E}{kT}} \qquad E_{ave} = \int EAe^{\frac{-E}{kT}}\,dE = \boxed{kT}$$

- **NEW** (Quantum from Planck)

$$f_n(E_n) = Ae^{\frac{-En}{kT}} \qquad E_{ave} = \sum E_n Ae^{\frac{-E_n}{kT}} = \boxed{\frac{\left(hc/\lambda\right)}{e^{\frac{hc/\lambda}{kT}} - 1}}$$

where $E_n = nhv$

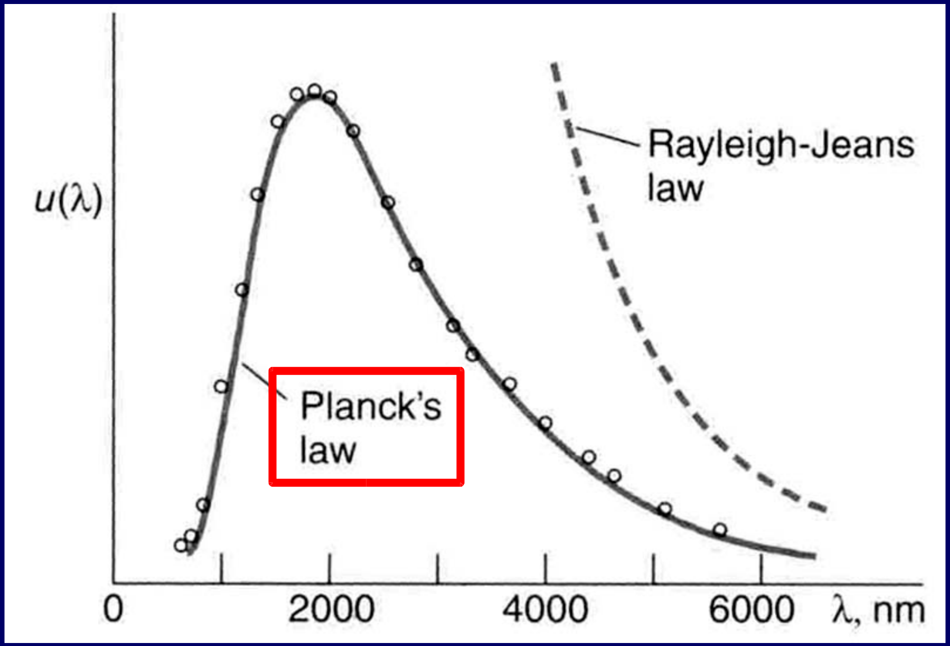
- Assumption of **<u>Quantization is CRITICAL</u>!**

Spectral Blackbody: Planck's Law

- Planck's Law initially found empirically (trial and error!)
- Quantize the E&M radiation (photons)

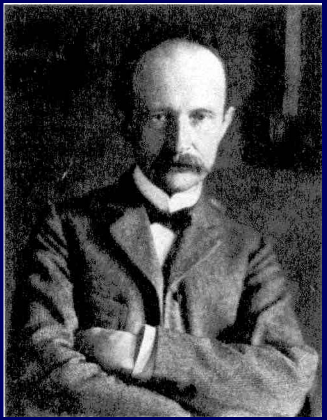
$$E_v = hv = {hc}/{\lambda}$$

$$u(\lambda) = \frac{\left(hc/\lambda \right)\left(8\pi\lambda^{-4} \right)}{e^{\frac{hc/\lambda}{kT}} - 1}$$



- $E_{ave} = (h\nu)[\exp(h\nu/kT) - 1]^{-1}$ and $n(\lambda) = 8\pi\lambda^{-4}$
- Energy of a photon: $\mathbf{E = hc/\lambda}$ and $c = \nu\lambda$

Spectral Blackbody: Limits of Planck's Law



$$u(\lambda) = \frac{\left(hc/\lambda\right)\left(8\pi\lambda^{-4}\right)}{e^{\frac{hc/\lambda}{kT}} - 1}$$

- Limit of **<u>Large λ</u>** (or small energy E)

$$e^{\frac{hc/\lambda}{kT}} \approx 1 + \frac{hc/\lambda}{kT} + \ldots$$

Taylor's Series for small exponent

$$u(\lambda \to \infty) \to \boxed{\left(8 \quad {}^{-4}\right) kT}$$

Rayleigh-Jeans Equation

- Limit of **<u>Small λ</u>** (or large energy E)

$$u(\lambda \to 0) \to \lambda^{-5} e^{\frac{-hc/\lambda}{kT}} \to \boxed{0}$$

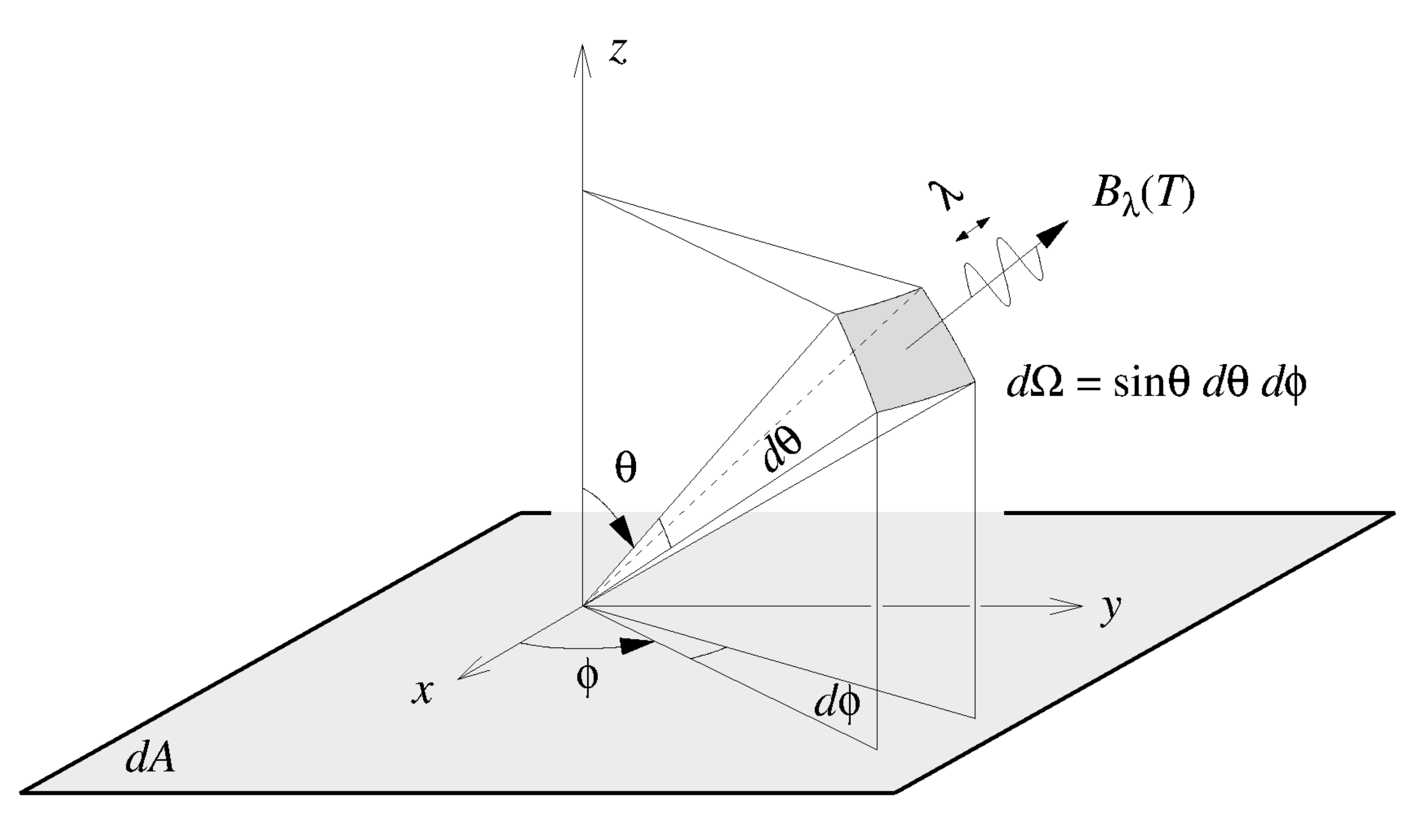
Planck's Law

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Energy from a surface element.
Units:

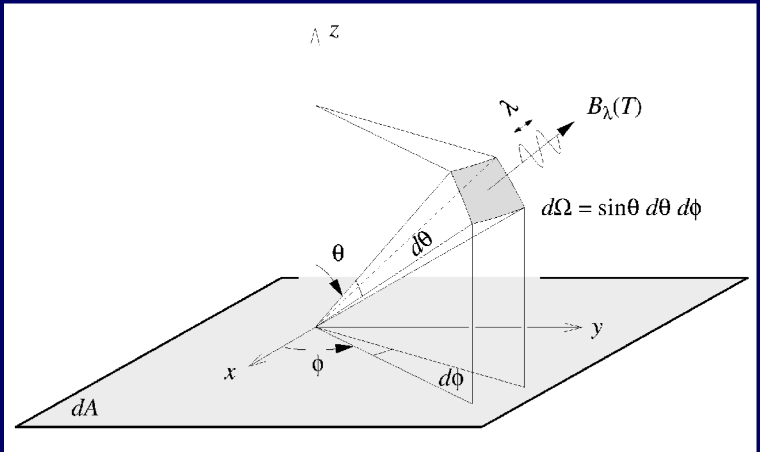
$$\frac{erg}{cm^2 \cdot s \cdot d\Omega \cdot d\lambda}$$



Planck's Law

- Observations: Radiant flux and apparent magnitude
- Star Properties: Radius, temperature
- Need to integrate over:
  - Area (sphere)
  - solid angle (from the *flat* infinitesimal surface element)
    - Isotropic – no preferred direction
- Monochromatic Luminosity

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$



$$L_\lambda d\lambda = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \left(B_\lambda \, d\lambda\right) \left(dA \, \cos\theta\right)\left(\sin\theta \, d\theta \, d\phi\right)$$

$$= 4\pi^2 R^2 B_\lambda \, d\lambda$$

From λ to $\lambda + d\lambda$

Planck' s Law

- Integrating the monochromatic luminosity over all wavelengths gives us the luminosity, which is related to the Stefan-Boltzmann equation:

$$\left.\begin{aligned} L = \int_0^\infty L_\lambda \, d\lambda = \int_0^\infty 4\pi^2 R^2 B_\lambda \, d\lambda \\ L = A\sigma T^4 \end{aligned}\right\} \Rightarrow \int_0^\infty \pi B_\lambda \, d\lambda = \sigma T^4$$

- Monochromatic Flux

$$F_\lambda d\lambda = \frac{L_\lambda}{4\pi r^2} = B_\lambda d\lambda \left(\frac{R}{r}\right)^2$$

The Color Index

- M_{bol} or m_{bol} is at all wavelengths and is called the bolometric magnitude
- Monochromatic flux integrated over a wavelength range
- Standard filters for the UBV system (there are other systems)

  - U is ultraviolet $\lambda_{center} = 3650$ Å

    $\Delta\lambda = 680$ Å

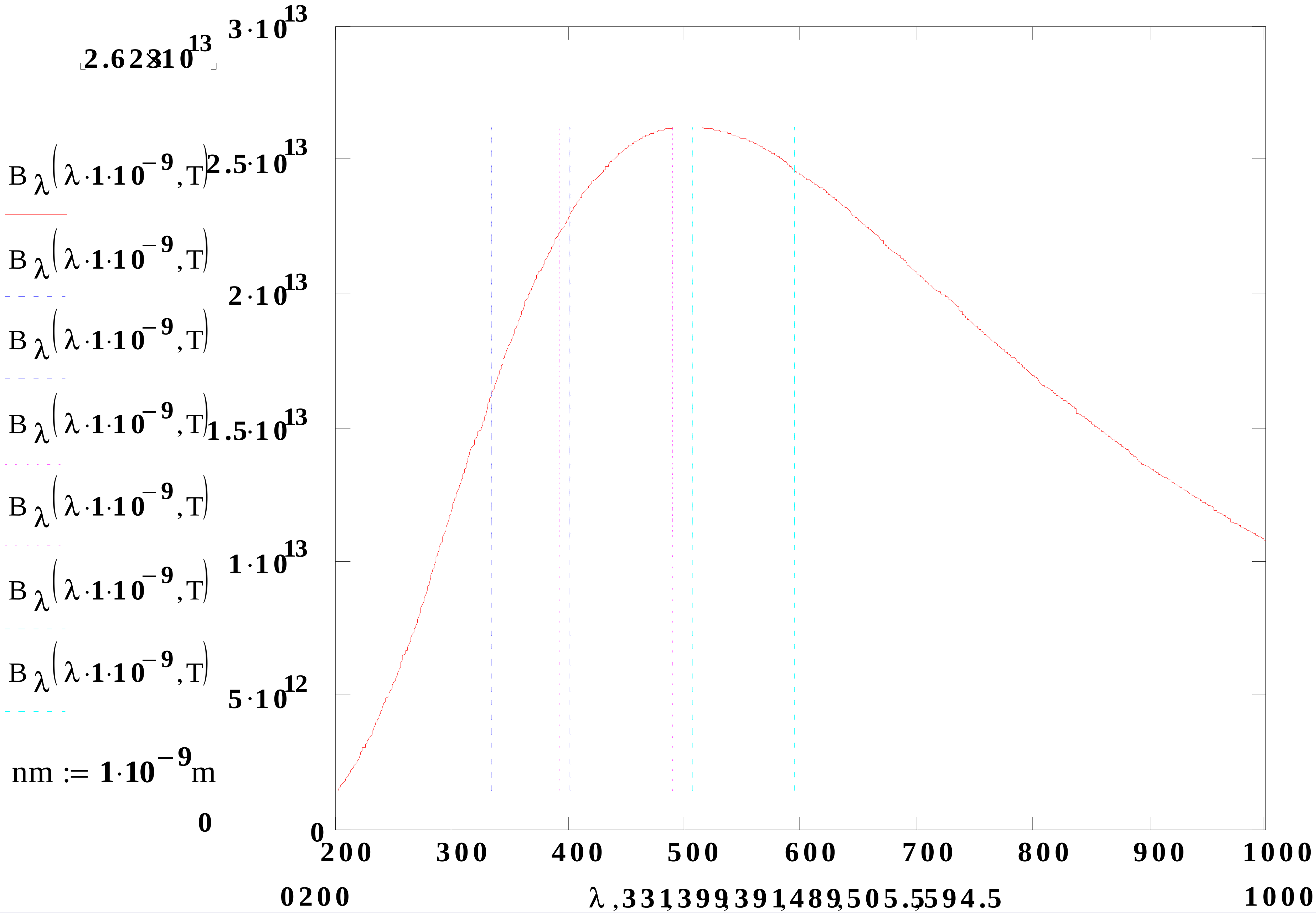
  - B is blue $\lambda_{center} = 4400$ Å

    $\Delta\lambda = 980$ Å

  - V is visible $\lambda_{center} = 5500$ Å

    $\Delta\lambda = 890$ Å

The Color Index



The Color Index

- Knowing the distance, the absolute color magnitudes can be determined, M_U, M_B, M_V.

$$m - M = 5\log_{10}\left(\frac{d}{10\ pc}\right)$$

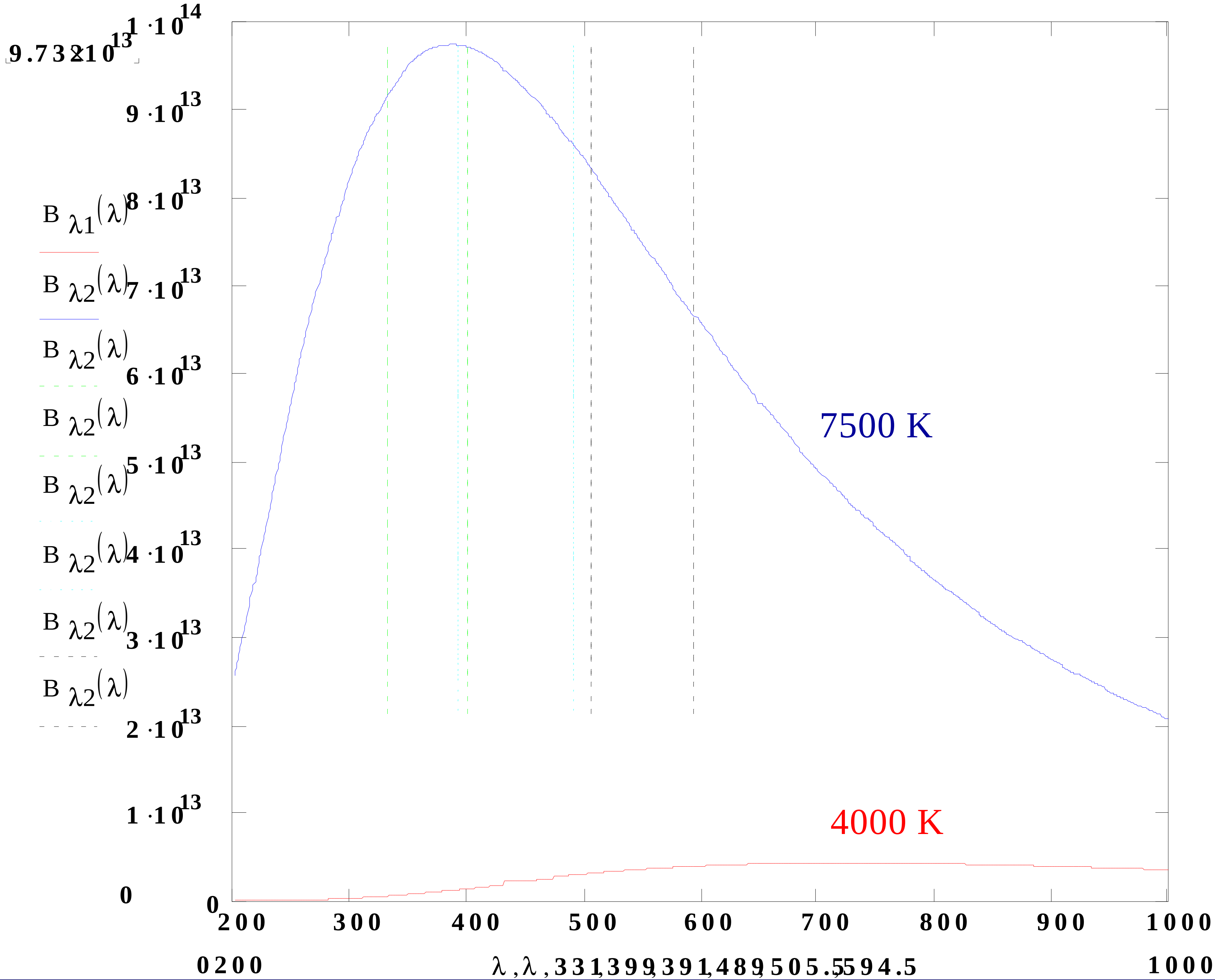
- Apparent magnitudes are: U, B, and V (instead of *m*)
- Color Indices
  - Independent of distance!

$$U - B = M_U - M_B$$

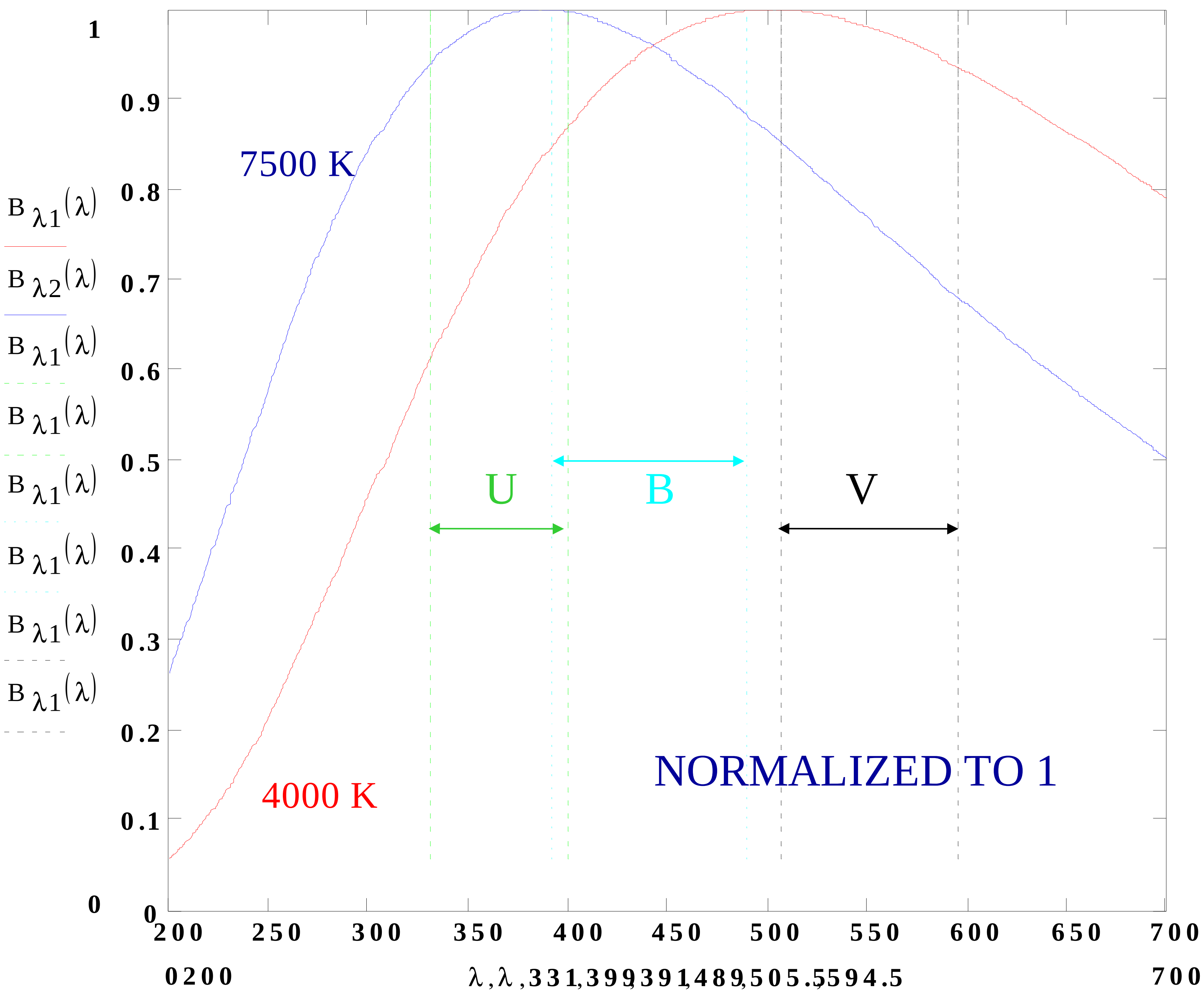
$$B - V = M_B - M_V$$

- Smaller (B-V) is bluer
  - Stellar magnitudes decrease with increasing brightness
- Bolometric correction, BC:

$$BC = m_{bol} - V = M_{bol} - M_V$$

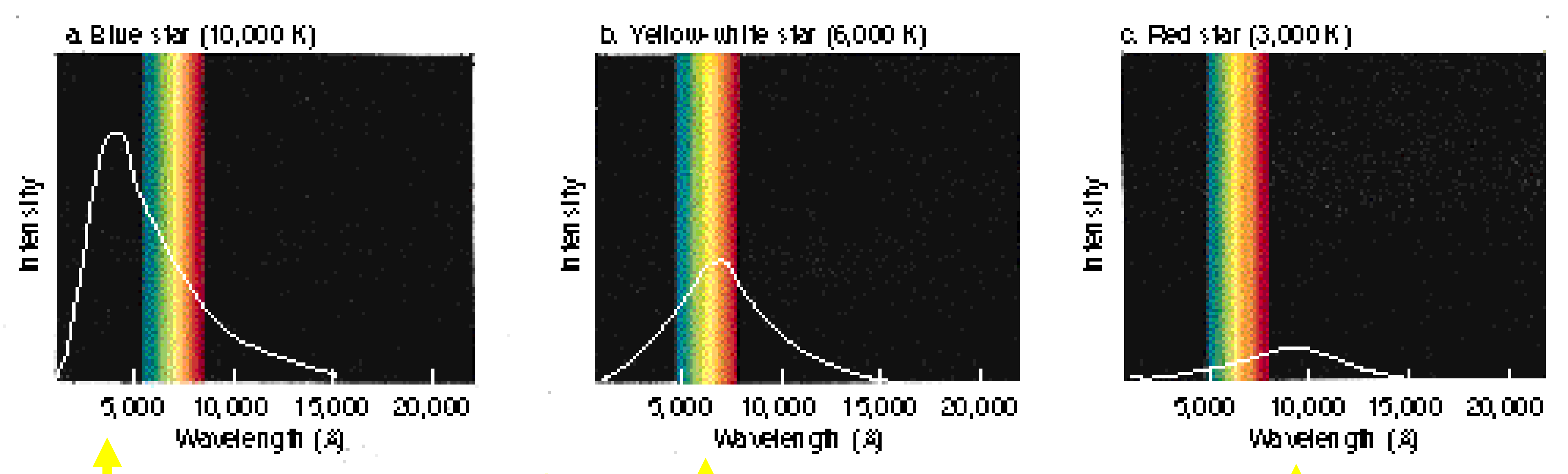


7500 K
4000 K


7500 K
4000 K
U
B
V
NORMALIZED TO 1
λ, λ, 331, 399, 391, 489, 505.5, 594.5

Temperature of Stars

- Determined by type of em radiation



Peak in UV

Peak in visible

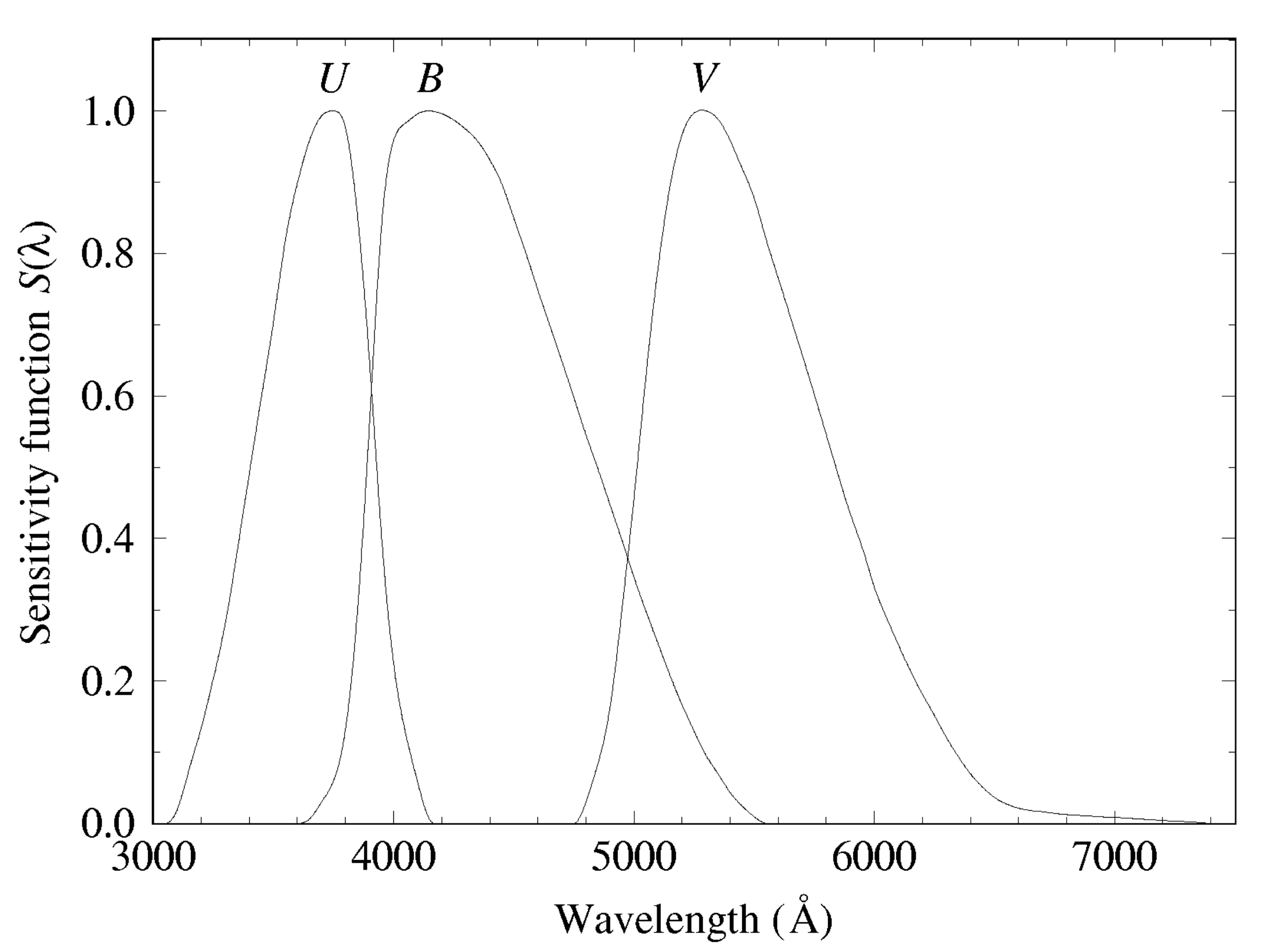
Peak in IR

The Color Index

- Sensitivity Functions, $S(\lambda)$ $\qquad m_1 - m_2 = -2.5\log_{10}\left(\frac{F_1}{F_2}\right)$

$$U = -2.5\log_{10}\left(\int_0^\infty F_\lambda S_U \, d\lambda\right) + C_U$$

$U = 0$ for Vega determines C_U



Example 3.6 – Sirius

- Brightest star in the sky

  U = -1.50, B = -1.46, V = -1.46

  $U - V$ = -0.04

  $B - V$ = 0.00

  Brightest at UV wavelengths, T_e = 9910 K:

$$\lambda_{max} = \frac{0.29\, cm \cdot K}{9910\ K} = 2.926 \times 10^{-5}\ cm = 2.926 \times 10^{-7}\ m = 2926\ \text{Å}$$

  Bolometric correction is: BC = -0.09, so its apparent bolometric magnitude is $m_{bol} = V + BC = -1.46 + (-0.09) = -1.55$

  (this is brighter than Vega)