W8, D1

Hwk Ch.7 P. 1, 3, 5, 8, 9, 11, 16, 21, 29, 31, 35 } Due today, 11:59

Ch. 8 P. 1, 2, 3, 7, 8, 10, 11, 12, 16, 18, 21 } Due 3/15/21 (next Mon.)

Read 8.1 - 8.5

Notes: Midterms: 50% Exam I, 25% Hwk, 25% Quiz

Last homework (Ch.6) Graded (G-Drive) $\mu = 7.4/10$ (incl. zeros)

Checked #10 and 31

Hwks 3-5 imported to Moodle Gradebook

No Quiz This week

TODAY: Grav. Potential Energy

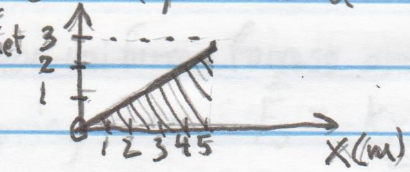
Conservative Forces

Determining $F_x$, $F_y$ & $F_z$ from $U(x,y,z)$

Stable & Unstable equilibrium.

□ Grav. Potential Energy [Back 2 pages]

Ex) The net force on a particle varies with position:




The particle starts with $v_i = 0$ at $x = 0$.

If its mass = 4kg, what is its speed at x = 5m?

$W_{ext} = \Delta K$

Area under curve $= \frac{1}{2} m (v_f^2 - v_i^2)$

$\frac{1}{2}(3N)(5m) = \frac{1}{2}(4kg)(v_f^2 - 0)$

$\frac{15\ Nm}{4\ kg} = v_f^2 \qquad v_f = \sqrt{3.75} = \boxed{1.94\ m/s}$

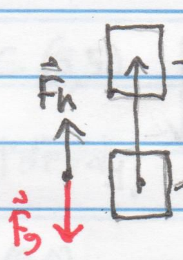
Gravitational Potential Energy $\boxed{U_g = mgh}$

* $h = 0$ at some arbitrary height, e.g. a table top.
* $U_g$ is really the energy of the mass + Earth system.

[Transp. T-51 "Reference point..."]

3/5/21 STOPPED HERE → [Transp. T-49]

Ex) What is $W_{hand}$ when it lifts a 0.2 Kg can by $\Delta h = 0.6$ m at constant speed?




$W_{hand} = F_h \Delta h$

$W_{hand} = \Delta U_g$

$= mg\Delta h = (0.2)(9.8)(0.6m) = \boxed{1.176J}$

Also $W_{grav} = -W_{hand} = -\Delta U_g$

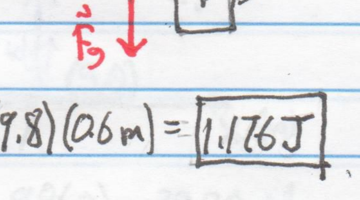
$= \boxed{-1.176J}$

*Gravity is an "internal force" so $W_{int} = -\Delta U$

Conservative Forces

obey: 1) Work done is independent of path

2) Work = 0 over a closed path (loop)




Ex) Gravity is a conservative force

* $\vec{F}_g = mg(-\hat{j})$ is constant with position

[DEMO: 7.15.swf]

Conservative Forces (cont.)

* Some non-uniform forces are conservative

Ex) Spring force $\vec{F}_s = -kx\hat{i}$

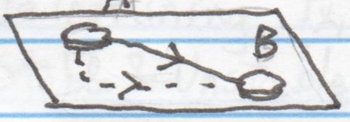
Ex) ?? force $\vec{F} = 2x^2\hat{i} + 3y^2\hat{j}$ N

* Some non-uniform forces are non-conservative

Ex) $\vec{F} = 3y^2\hat{i} + 2x^2\hat{j}$

Ex) $\vec{F} = 2y\hat{i} + x^2\hat{j}$ (Prob. 33)

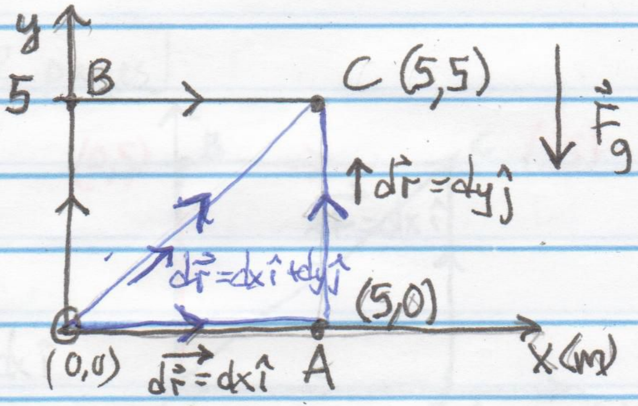
* Friction and air resistance are non-conservative




More work for curved path!

Ex) Prob. 31

A 4 kg particle moves from origin to Ⓒ (5.0, 5.0) along 3 different paths. $F_g$ acts in $-y$ direction. Find $W_{grav}$...




a) for path OAC

$$W_g = \int_0^A \vec{F}_g \cdot d\vec{r} + \int_A^C \vec{F}_g \cdot d\vec{r} \quad \text{with } \vec{F}_g = 4 \cdot 9.8(-\hat{j}) = -39.2\hat{j}\ \text{N}$$

$$= \int_{(0,0)}^{(5,0)} (-39.2)\hat{j} \cdot dx\hat{i} \quad \int_{(5,0)}^{(5,5)} (-39.2)\hat{j} \cdot dy\,\hat{j}$$

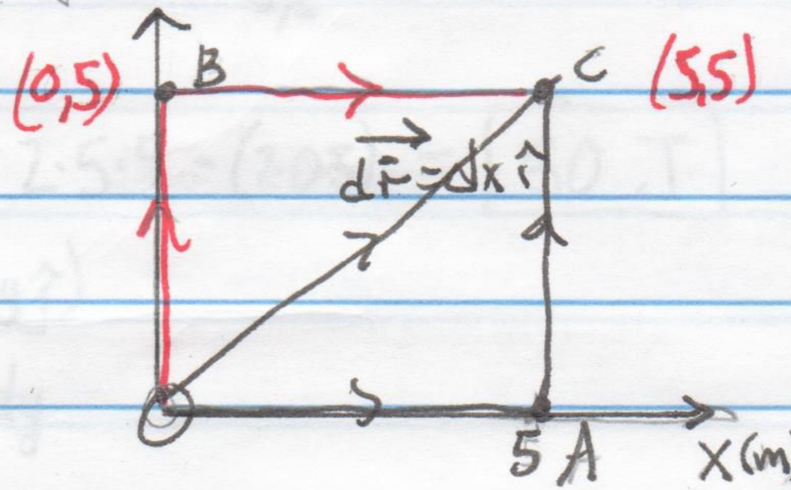
$$W_g = \int 0 + \int_{(5,0)}^{(5,5)} -39.2\,dy = -39.2\,y\Big|_0^{y=5} = -39.2(5) = \boxed{-196\ \text{J}}$$

b) for path O-C: $W_g = \int_{(0,0)}^{(5,5)} (-39.2\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$

$$= \int_{0,0}^{5,5} 0 - 39.2\,dy = \int_0^5 -39.2\,dy = -39.2(5) = \boxed{-196\ \text{J}}$$

- Grav. Potential Energy [Back 2 pages]
- Conservative Forces (cont.)

Example (cont.)




c) Path OBC

$$W_g = \int_{(0,0)}^{(0,5)} -392\hat{j}\cdot dy\,\hat{j} + \int_{(0,5)}^{(5,5)} -392\hat{j}\cdot dx\,\hat{i}$$

$$= \int_0^5 -39.2\,dy + \int_{(0,5)}^{(5,5)} 0 = -39.2y\Big|_0^5 = \boxed{-196\text{ J}}$$

* All paths have identical $W_{grav}$ because $F_g$ is conservative

Q1) Suppose $f_k$ is also acting to resist motion along paths OAC, OBC, & OC.

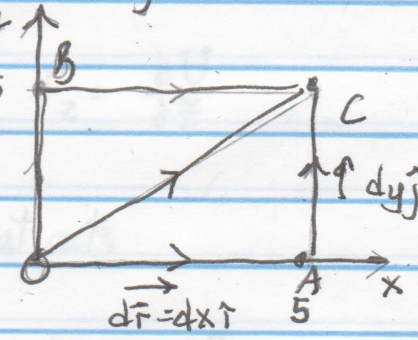
a) Does $W_{grav}$ change? Ans: No

b) Does $W_{hand}$ change? Ans: Yes!

c) Which path(s) require most $W_{hand}$?

Ans: $W_{OAC} = W_{OBC} > W_{OC}$

Ex) Suppose the force $\vec{F} = 2y\,\hat{i} + x^2\hat{j}$ is acting instead of $\vec{F}_g$.




a) Find $W_{OAC}$

$$W_{OAC} = \int_{0,0}^{5,0} (2y\hat{i} + x^2\hat{j})\cdot dx\,\hat{i} + \int_{5,0}^{5,5} (2y\hat{i} + x^2\hat{j})\cdot dy\,\hat{j}$$

$$= \int_{0,0}^{5,0} 2y\,dx + \int_{5,0}^{5,5} x^2\,dy$$

$$= 2yx\Big|_{0,0}^{5,0} + x^2y\Big|_{0,0}^{5,5}$$

$$= \underline{2\cdot 0\cdot 5 - 0} + 5^2\cdot 5 - 0 = 0 + (125 - 0) = \boxed{125\text{ J}}$$

b) Find $W_{OBC} = \int_{0,0}^{0,5} (2y\hat{i} + x^2\hat{j})\cdot dy\,\hat{j} + \int_{0,5}^{5,5} (2y\hat{i} + x^2\hat{j})\cdot dx\,\hat{i}$

$$= \int_{0,0}^{0,5} x^2\,dy + \int_{0,5}^{5,5} 2y\,dx$$

$$= x^2y\Big|_{0,0}^{0,5} + 2yx\Big|_{0,5}^{5,5}$$

$$= 0 + 2\cdot 5\cdot 5 - (2\cdot 0\cdot 5) = \boxed{50\text{ J}}$$

c) Find $W_{OC} = \int_{0,0}^{5,5} (2y\hat{i} + x^2\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$

$$= \int_{0,0}^{5,5} 2y\,dx + \int_{0,0}^{5,5} x^2\,dy$$

STOPPED ⟶

$$= 2yx\Big|_{0,0}^{5,5} + x^2y\Big|_{0,0}^{5,5} = 2\cdot 5\cdot 5 + 5^2\cdot 5$$

$$= 50 + 125 = \boxed{175\text{ J}}$$

Summary of Conservative forces

* If $\vec{F}(x,y,z) = f(x)\hat{i} + f(y)\hat{j} + f(z)\hat{k}$ it is conservative
* If $\vec{F}(x,y,z) = f(y,z)\hat{i} + f(x,z)\hat{j} + f(x,y)\hat{k}$ it is non-conservative

Determine $F_x$, $F_y$ & $F_z$ from $U(x,y,z)$

$$* \quad F_x = -\frac{\partial U}{\partial x}, \quad F_y = -\frac{\partial U}{\partial y}, \quad F_z = -\frac{\partial U}{\partial z}$$

(P. 36) Ex) $U = 3x^3y - 7x$ Find the force that acts at the point $(x,y)$.

Soln: $F_x = \frac{-\partial U}{\partial x} = -\frac{\partial}{\partial x}[3x^3y - 7x] = -[9x^2y - 7]$

$= -9x^2y + 7$

$F_y = -\frac{\partial U}{\partial y} = \frac{-\partial}{\partial y}[3x^3y - 7x] = -[3x^3 + 0]$

$= -3x^3$

So $\boxed{\vec{F}(x,y) = (-9x^2y + 7)\hat{\imath} - 3x^3\hat{\jmath}}$

Ex) A conserv. force acts on a 5kg particle according to $F_x = 2x + 4$. What is the corresponding potential energy function?

(and $F_y = 0$)

Soln: $F_x = -\frac{\partial U}{\partial x}$

$2x + 4 = -\frac{\partial U}{\partial x}$

$-2x - 4 = \frac{\partial U}{\partial x} = \frac{dU}{dx}$

$\int (-2x-4)\,dx = \int dU$

$-x^2 - 4x + C = U_x$

unknown constant of integration

Also, $U_y = C$ so that $-\frac{\partial U_y}{\partial y} = 0$

Energy Diagrams and Equilibrium of a System