

# Physics 2311 – Physics I, Week 4

## Dr. J. Pinkney

### Outline for Day W3,D1

1D kinematics

Equations of uniform acceleration, including free-fall

Examples

2D kinematics - Vectors

### Homework

Ch. 2 Prob. 2,3,5-7,14,23-27,35-38,53-56 for 2-day

Ch. 3 P. 1,3,6,7,10,11,19,20,23,24,

32,33,37,38,39 Do by next Mon.

Notes: Lab this week: “Graphs and Tracks”

Under NEW STUFF: “Exam like problems” for Ch. 1  
And “Week 3-5” practice quiz.

## Motion in 1D.

### Equations for Uniform Acceleration

A) [Text: 2-12a]  $\vec{v}_f = \vec{v}_i + \vec{a}t$

B) [Text: 2-12d]  $\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$

C) [Text: 2-8]  $\vec{x}_f = \vec{x}_i + \frac{\vec{v}_i + \vec{v}_f}{2}t$

D) [Text: 2-12b]  $\vec{x}_f = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$

E) [Text: 2-12c]  $v_f^2 - v_i^2 = 2 a (x_f - x_i)$

## Motion in 1D.

### Examples using Equations for Uniform Acceleration

1) A car passes  $x=10\text{m}$  at  $t=0$  going  $10\text{ m/s}$  with a constant accelerating of  $4\text{ m/s}^2$ . Where will the car be in 5 seconds?

$$\vec{x}_f = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

2) A car accelerates uniformly, starting at  $v_i=5\text{ m/s}$  at  $x_i=20$ , and reaching  $x_f=100$  only 5 seconds later. How fast did it cross the  $x_f=100\text{ m}$  mark?

$$\vec{x}_f = \vec{x}_i + \frac{\vec{v}_i + \vec{v}_f}{2} t$$

## Motion in 1D.

### Examples using Equations for Uniform Acceleration

3) A rock thrown down a well at 10 m/s reaches the bottom at 40 m/s. What was the average velocity? (Uniform acceleration of 9.8 m/s<sup>2</sup> downward.)

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

4) Find the depth of the well in the previous problem, assuming the rock was thrown straight down.

$$v_f^2 - v_i^2 = 2 a (x_f - x_i)$$

## Motion in 1D.

### Examples using Equations for Uniform Acceleration

5) A car on ice is sliding backwards to the left at 5 m/s while accelerating uniformly to the right at 3 m/s<sup>2</sup>. What is its velocity after 7 seconds?

$$\vec{v}_f = \vec{v}_i + \vec{a} t$$

# Physics 2311 – Physics I, Week 4

## Dr. J. Pinkney

### Outline for Day W3,D2

1D kinematics: free-fall example

2D kinematics – Vectors

Definitions and Eqns of uniform acceleration

Radial and tangential acceleration in “curvy” motion

Projectile motion

### Homework

Ch. 3 P. 1,3,6,7,10,11,19,20,23,24,  
32,33,37,38,39 Do by Mon.

Notes: Lab this week: “Graphs and Tracks”

Under NEW STUFF: “Exam like problems” for Ch. 1  
And “Week 3-5” practice quiz.

## Motion in 1D.

### Free Fall problems

Assumes downward acceleration,  $g$ , near the surface of a planet (usually Earth!)

The Equations for Uniform Acceleration apply!

Assumes no air resistance or other forces on the object.

Object can be moving downwards OR upwards during free fall!

# Motion in 1D.

## Free Fall problems

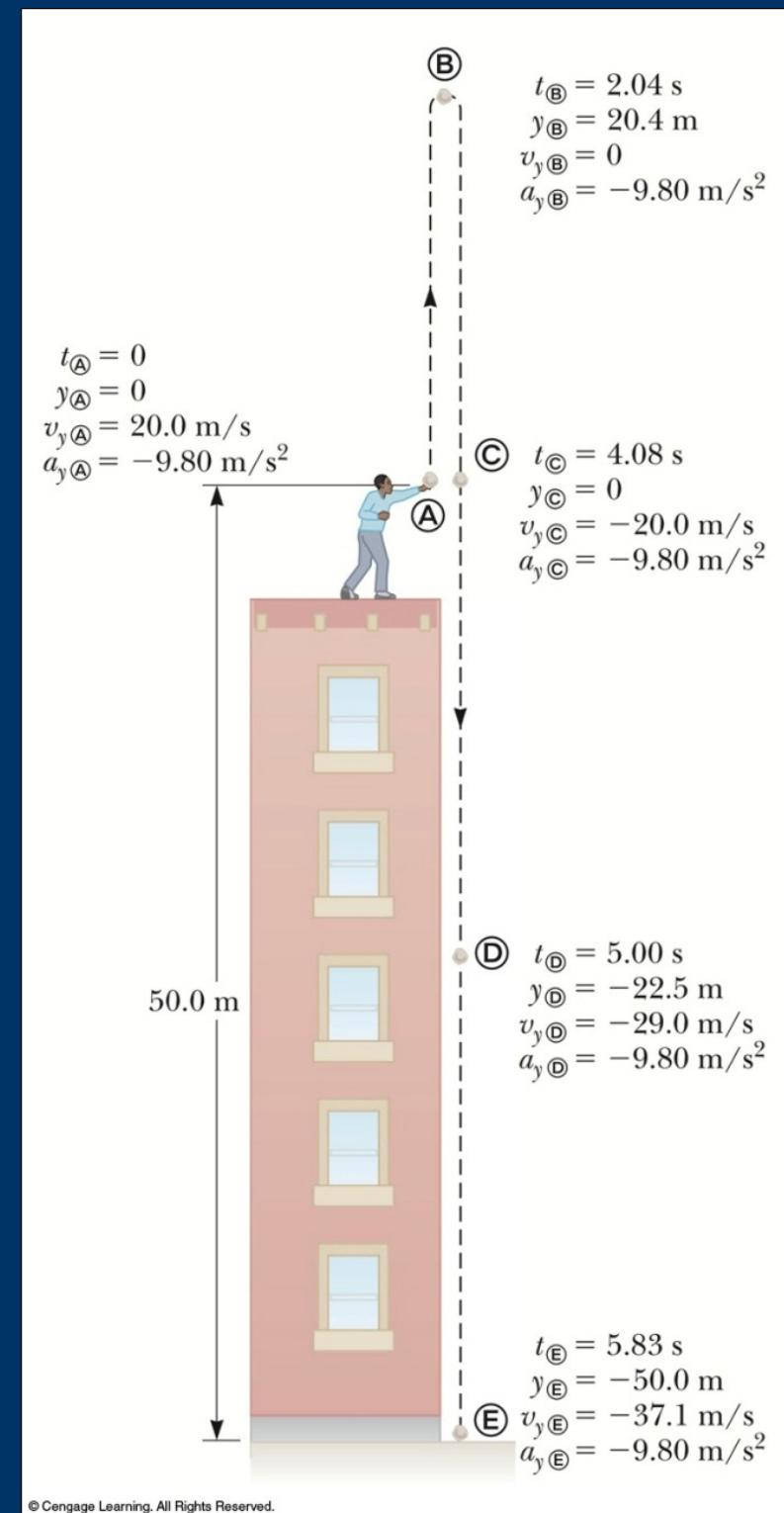
Initial velocity at A is upward (+) and acceleration is  $-g$  ( $-9.8 \text{ m/s}^2$ ).

At B, the velocity is 0 and the acceleration is  $-g$  ( $-9.8 \text{ m/s}^2$ ).

At C, the velocity has the same magnitude as at A, but is in the opposite direction.

The displacement is  $-50.0 \text{ m}$  (it ends up  $50.0 \text{ m}$  below its starting point).

$$\vec{y}_f = \vec{y}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$



# Motion in 1D.

## Free Fall problems

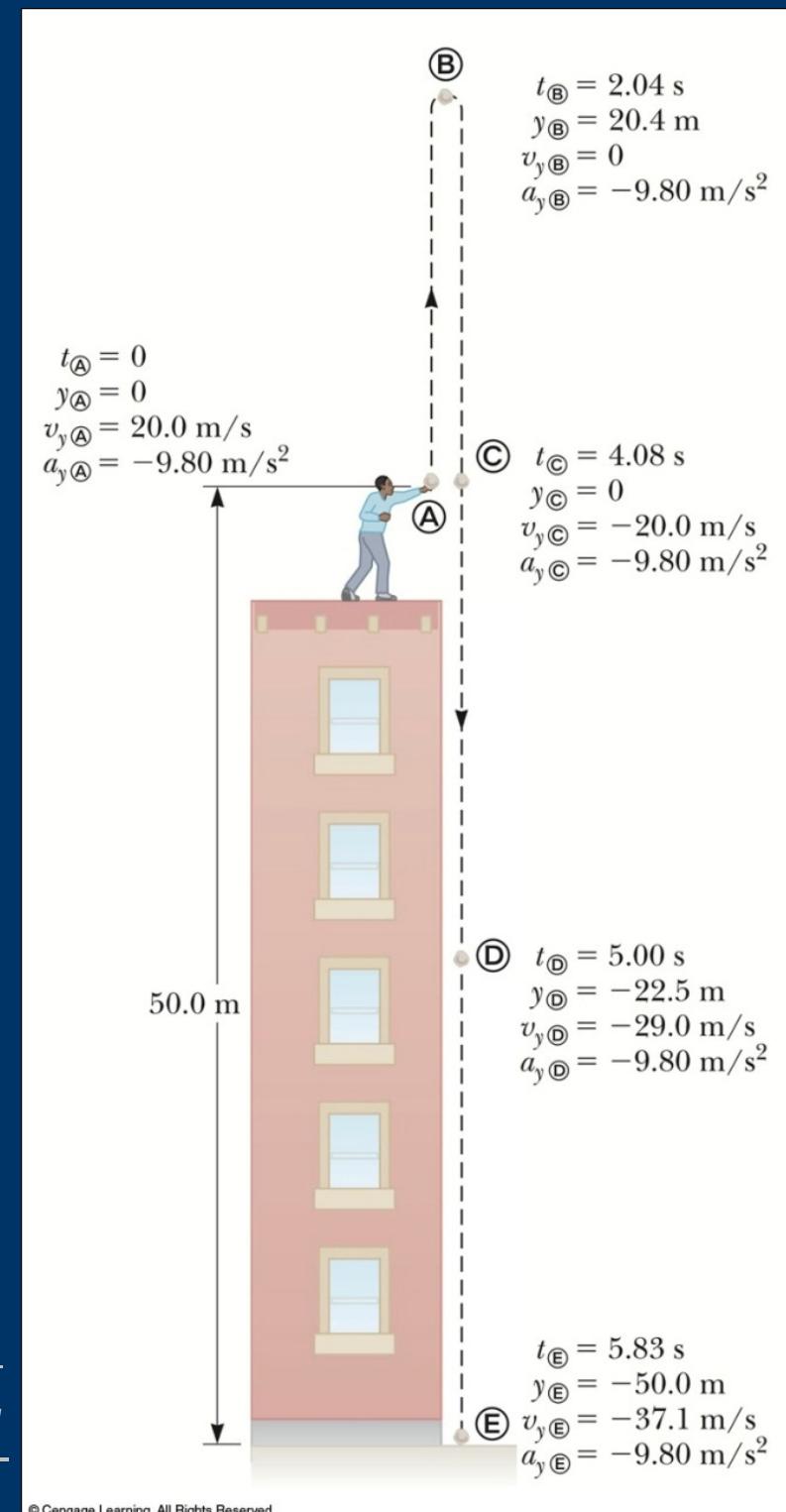
Example) Verify that the ball hits the ground at  $t_f=5.83$  seconds if it is thrown from an initial height of  $y_i=0$  upwards at  $v_i=20$  m/s.

Also given:  $y_f=-50$  m,  $a=-9.8$  m/s<sup>2</sup>

Use:  $\vec{y}_f = \vec{y}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$   
 $-50 = 0 + 20t - 4.9t^2$

Quadratic eqn.  $0=-4.9t^2 + 20t + 50$

$$0 = at^2 + bt + c$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



# Motion in 2 Dimensions

## Goals of Week 3:

- Learn how to work with vectors
- Rewrite 1D equations for 2D (and 3D) cases
- Acceleration in curvy motion – tangential and centripetal<sup>†</sup>
- Apply 2D equations to projectile motion.
- Calculate range, maximum height &  $t_{\max}$  for trajectories
- Relative velocity

<sup>†</sup> or “radial”

## Motion in 2D. Vectors

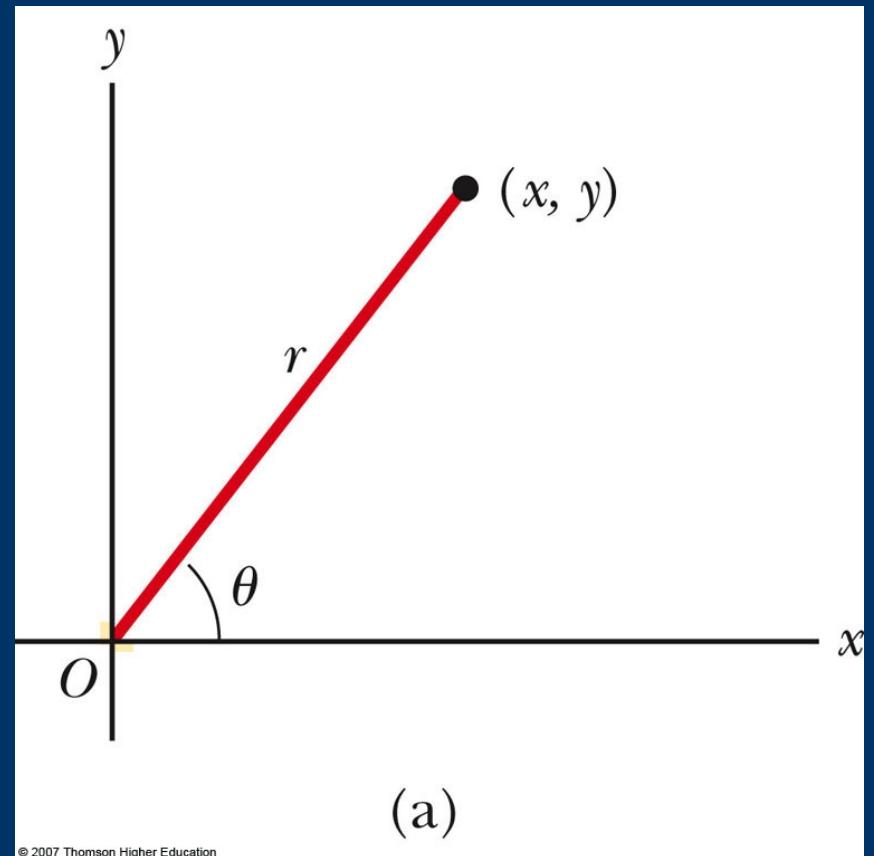
2D “Cartesian” coordinate systems provide 2 ways to specify the position of a point:

### 1) Polar coordinates

Points are labeled  $(r, \theta)$

### 2) Cartesian coordinates

Points are labeled  $(x, y)$



## Conversions

Polar to cartesian:

$$x = r \cos \theta, \quad y = r \sin \theta$$

Cartesian to polar:

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1} \frac{y}{x}$$

## Motion in 2D. Vectors

Similarly, vectors can be expressed in two main ways: *polar* and *vector component form*.

Example) Vector  $\mathbf{r}$  is a *position vector* because it has its tail at the origin.

Vector  $\mathbf{A}$  is not a position vector, but it's a vector.

However, these vectors are *equal* because they have the same length and direction. Thus,

$$\mathbf{A} = \mathbf{r} = 5\hat{i} + 3\hat{j}$$

and this is the *vector component form*.

Convert  $\mathbf{A}$  and  $\mathbf{r}$  into *polar vectors*...

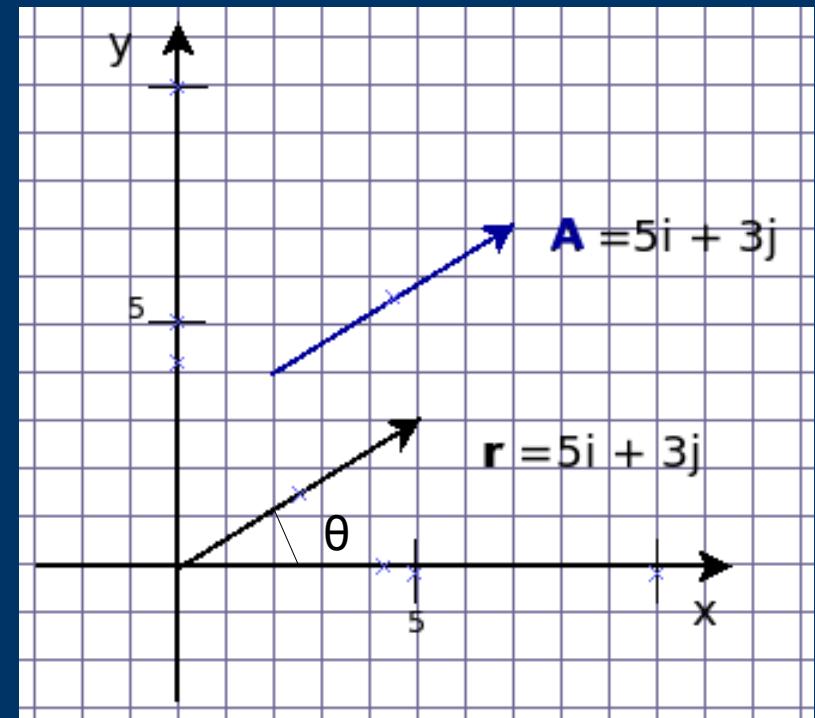
The length of both is

$$|\mathbf{A}|=|\mathbf{r}| = (5^2+3^2)^{1/2} = 5.83$$

The angle between the vector and the  $+x$ -axis is

$$\theta = \tan^{-1}(3/5) = 30.96^\circ$$

Therefore,  $\mathbf{A}$  is  $(r,\theta) = (5.83, 30.96^\circ)$



# Motion in 2D. Vectors

Careful with finding the angle using the inverse tangent ( $\tan^{-1}$ ) function on a calculator!

First, make sure calculator is set on degrees instead of radians.

Second, if the vector points towards quadrants 2 or 3, you must add  $180^\circ$  (or subtract  $180^\circ$  to keep the angle less than  $360^\circ$ ) to the calculator's answer:

$$\theta = \tan^{-1} (y/x) + 180^\circ$$

If the vector points towards quadrants 1 or 2,  
It is ok to use:

$$\theta = \tan^{-1} (y/x)$$

ALWAYS define  $\theta$  to be measured CCW from the  $+x$  axis to the position vector!

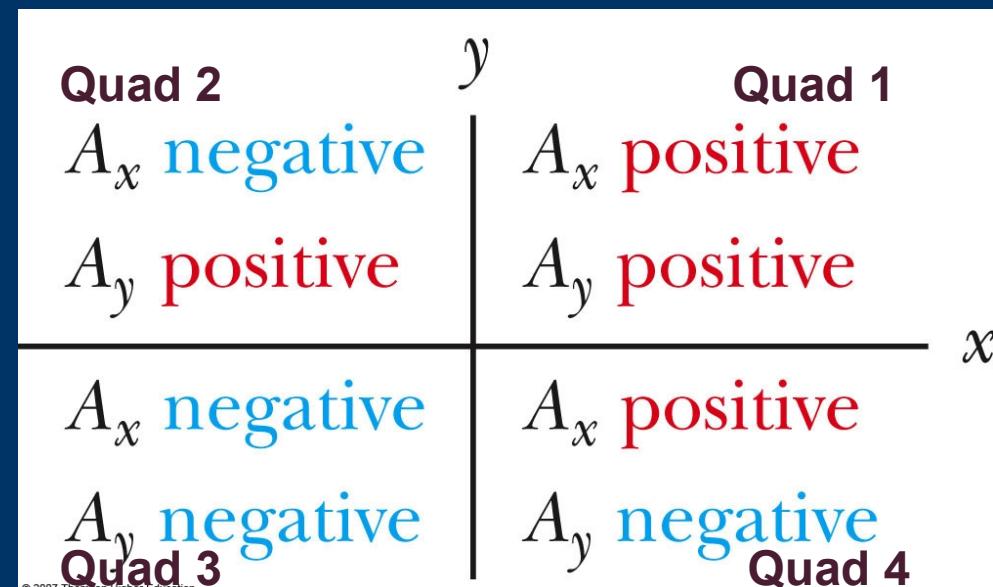
Then expect the position vector to have:

$$\text{Quad 1 } 0 < \theta < 90^\circ$$

$$\text{Quad 2 } 90 < \theta < 180^\circ$$

$$\text{Quad 3 } 180 < \theta < 270^\circ$$

$$\text{Quad 4 } 270 < \theta < 360^\circ$$



# Motion in 2D. Vectors

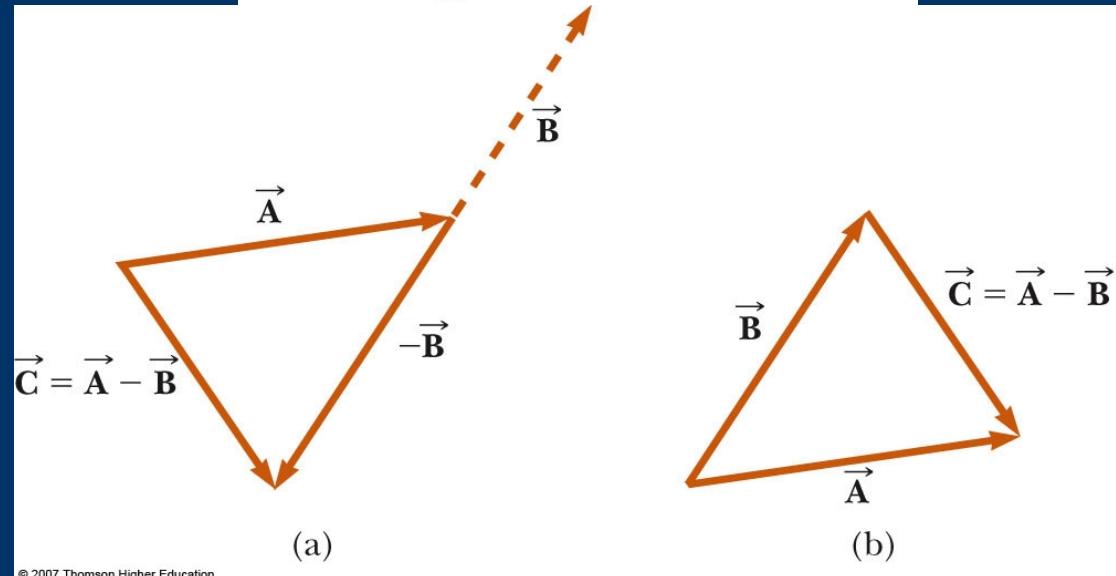
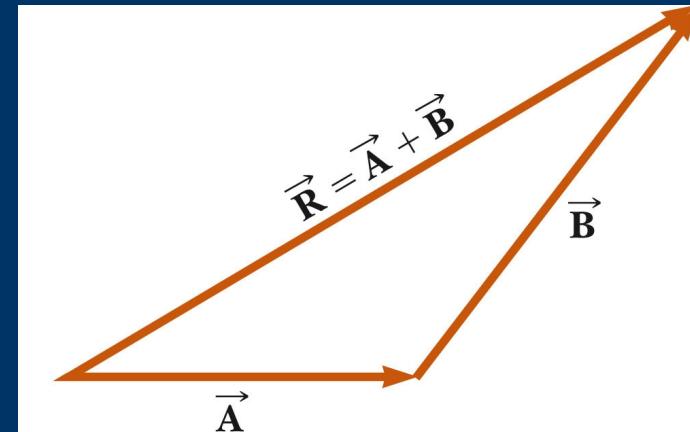
## Vector addition and subtraction

### Graphical approach

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

$$\mathbf{C} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

Tip-to-tail method



### Algebraic approach

$$\mathbf{A} = A_x \hat{i} + A_y \hat{j}$$

$$\mathbf{B} = B_x \hat{i} + B_y \hat{j}$$

$$\mathbf{A} + \mathbf{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

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## Motion in 2D.

Using vectors for position.

Top: position vector  $\mathbf{A}$  in 2-D.  
Vector components are:

$$A_x = |\mathbf{A}| \cos \theta$$

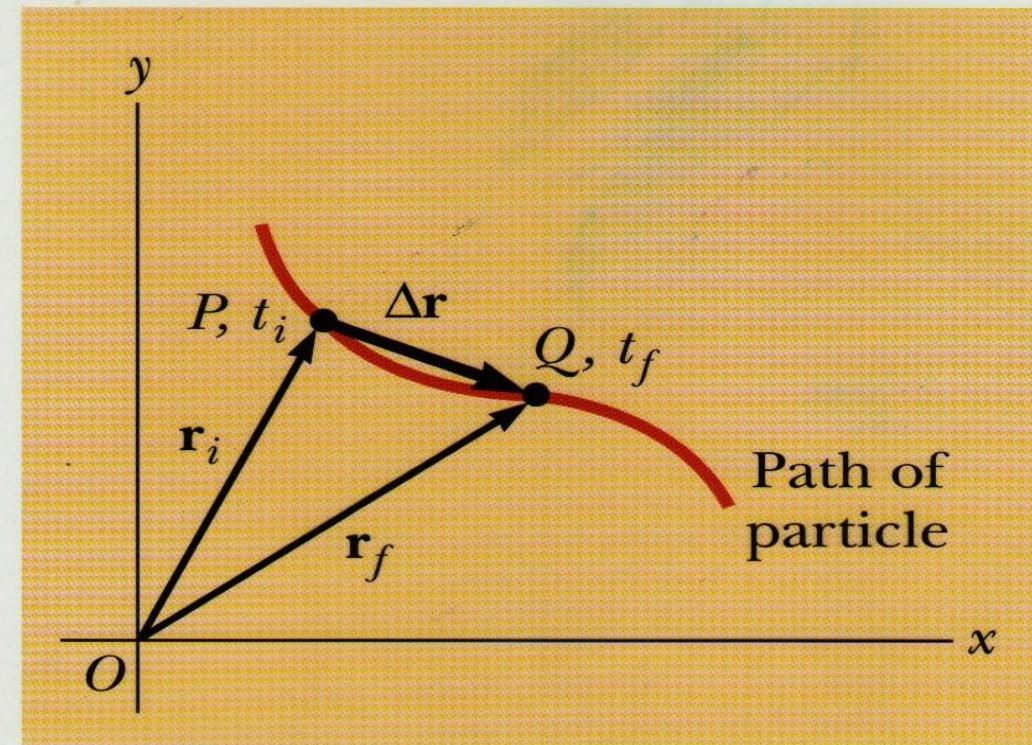
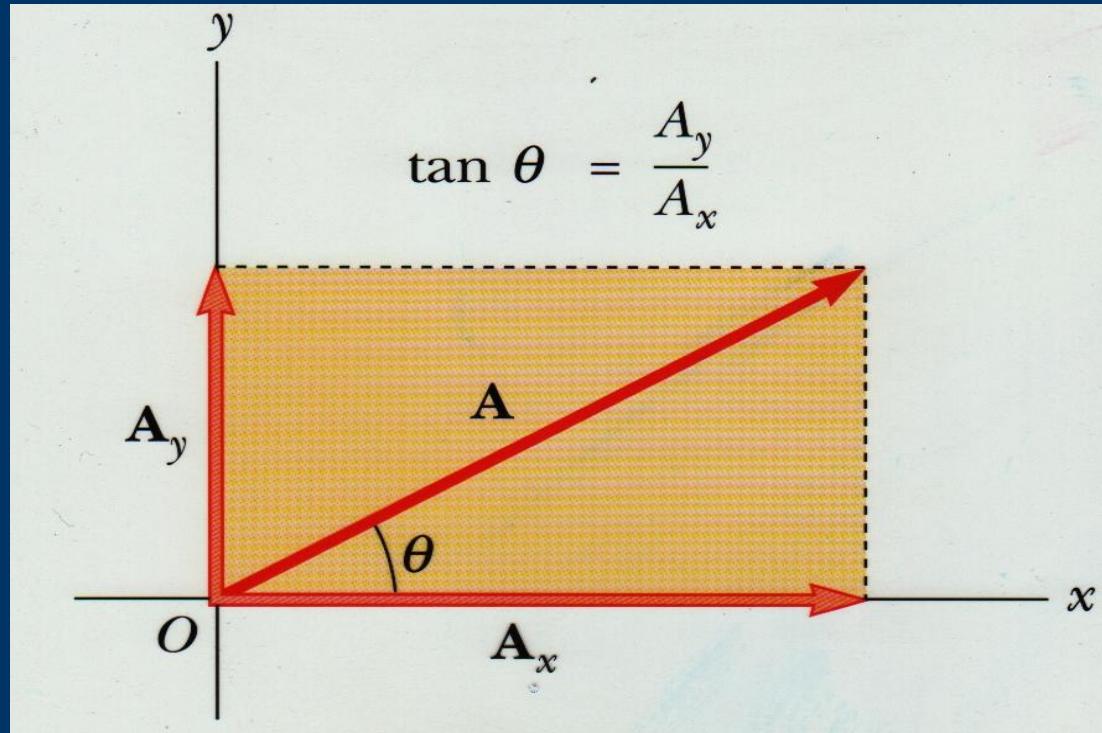
$$A_y = |\mathbf{A}| \sin \theta$$

$$\text{so } \mathbf{A} = A_x \hat{i} + A_y \hat{j}$$

$$|\mathbf{A}| = (A_x^2 + A_y^2)^{1/2}$$

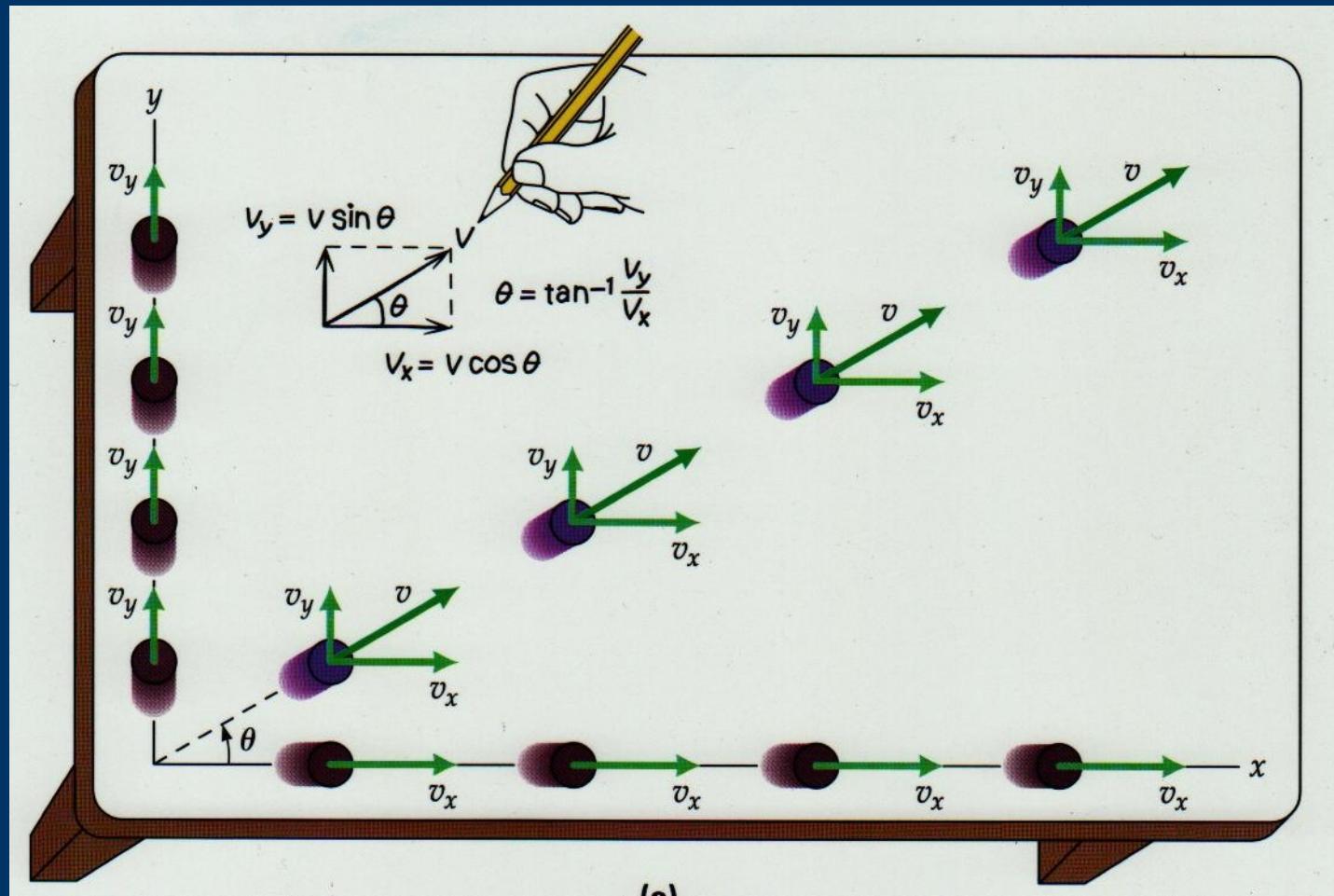
Bottom: change of the position vector  $\mathbf{r}$  gives a displacement  $\Delta \mathbf{r}$ .

Remember,  $\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$



# Motion in 2D.

Using vectors for velocity in 2-D.



**Notice that this motion is all in a straight line and so could be expressed with 1 dimension (using a rotated axis).**

# Motion in 2-D (and beyond)

## Definitions

Definitions ...

(Most of these are very similar to the Ch. 2 equations)

Position vector:

$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$$

Displacement:

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

Average velocity:

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

. Instantaneous velocity:

$$\vec{v}_{inst} = \frac{d\vec{r}}{dt}$$

Average acceleration:

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

. Instantaneous acceleration:

$$\vec{a}_{inst} = \frac{d\vec{v}}{dt}$$

## Equations of Uniform acceleration

Final velocity

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

Average Velocity

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

Position as function of time:

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} t$$

Position as function of time:

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

Velocity change related to position change:

$$\vec{v}_f \cdot \vec{v}_f - \vec{v}_i \cdot \vec{v}_i = 2 \vec{a} \cdot (\vec{r}_f - \vec{r}_i)$$

# Physics 2311 – Physics I, Week 4

Dr. J. Pinkney

## Outline for Day W3,D3

1D kinematics: free-fall example

2D kinematics:

Radial and tangential acceleration in “curvy” motion

Projectile motion

## Homework

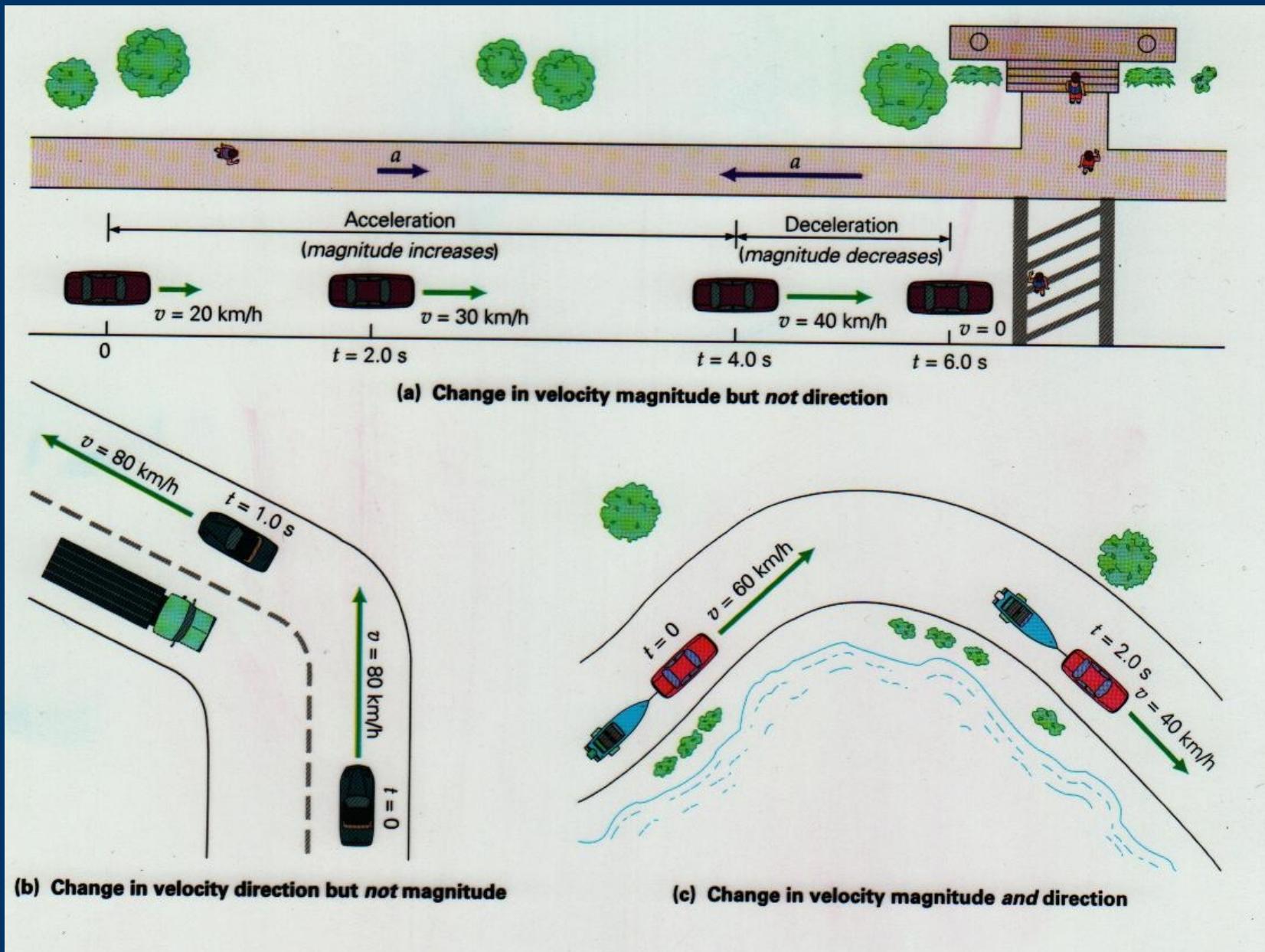
Ch. 3 P. 1,3,6,7,10,11,19,20,23,24,  
32,33,37,38,39 Do by Mon.

Notes: Lab this week: “Graphs and Tracks”

Under NEW STUFF: “Exam like problems” for Ch. 2  
and “Key to exam-like problems for Ch 1&2”

# Top: Motion in 1D

# Bottom: Motion in 2D.



Show 4.16.swf - acceleration has a radial and tangential component.

# Motion in 2 dimensions. General motion.

In the most general case, there could be acceleration in both the x and y directions:

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

And so the 2D position is:

$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

and the 2D velocity is:

$$\mathbf{v}(t) = dx/dt\hat{i} + dy/dt\hat{j} = (v_{0x} + a_x t)\hat{i} + (v_{0y} + a_y t)\hat{j}$$

and the 2D acceleration is:

$$\mathbf{a}(t) = dv/dt = a_x\hat{i} + a_y\hat{j}$$

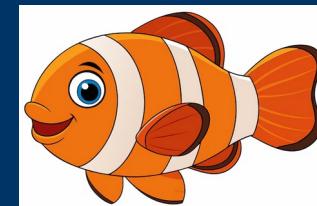
This is equivalent to:

$$\vec{r}_f = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a} t^2$$

with:

$$\mathbf{r}_0 = x_0\hat{i} + y_0\hat{j}$$

Example: swimming fish problem...



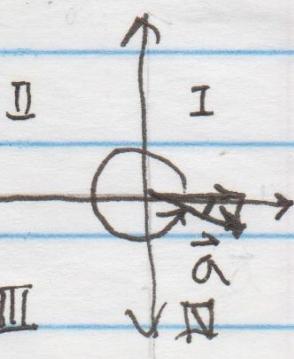
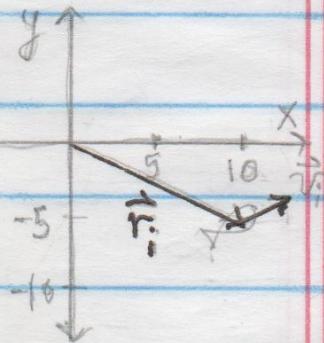
Ex) A fish swims in a horizontal plane with  $\vec{v}_i = 4\hat{i} + \hat{j}$  m/s at a position vector of  $\vec{r}_i = 10\hat{i} - 4\hat{j}$  m. The fish swims with uniform acceleration  $\vec{a} = \underline{\quad}$  for  $t = 20$  seconds until  $\vec{v}_f = 20\hat{i} - 5\hat{j}$  m/s.

a) What is  $\vec{a}$ ?

$$\text{Sol'n: since } \vec{a} \text{ is uniform, } \vec{a} = \vec{a}_{\text{avg}} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{(20\hat{i} - 5\hat{j}) - (4\hat{i} + \hat{j})}{20} \\ = \frac{(16\hat{i} - 6\hat{j})}{20} \\ = 0.8\hat{i} - 0.3\hat{j} \text{ m/s}^2$$

b) Find direction of  $\vec{a}$ .

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-0.3}{0.8}\right) = [-20.6^\circ] \\ (\text{or } \theta = 339.4^\circ)$$



## Fish problem (cont.)

c) If  $\vec{a}$  is maintained, where is fish at  $t=25s$ , and in what direction is  $\vec{v}$ ?

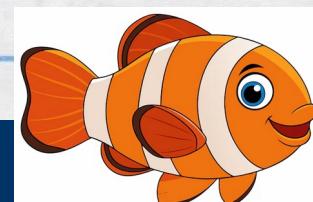
$$\begin{aligned}\vec{r}(t=25) &= \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2 && (\text{new } 2.10) \\ &= (10\hat{i} - 4\hat{j}) + (4\hat{i} + 1\hat{j})t + \frac{1}{2}(0.8\hat{i} - 0.3\hat{j})t^2 \\ &= [10 + 4(25) + 0.4(25)^2]\hat{i} + [-4 + 1(25) - \frac{0.3}{2}(25)^2]\hat{j}\end{aligned}$$

$$\boxed{\vec{r}(t=25) = 360\hat{i} - 72.75\hat{j} \text{ m}}$$

And  $\vec{v}(t=25) = \vec{v}_i + \vec{a}t$  (new 2.12)

$$\begin{aligned}&= (4\hat{i} + \hat{j}) + (0.8\hat{i} - 0.3\hat{j})(25) \\ &= [4 + 0.8(25)]\hat{i} + [1 - 0.3(25)]\hat{j}\end{aligned}$$

$$\vec{v}_f = 24\hat{i} - 6.5\hat{j} \quad \text{so } \theta_v = \tan^{-1}\left(\frac{-6.5}{24}\right) = \boxed{-15.15^\circ}$$

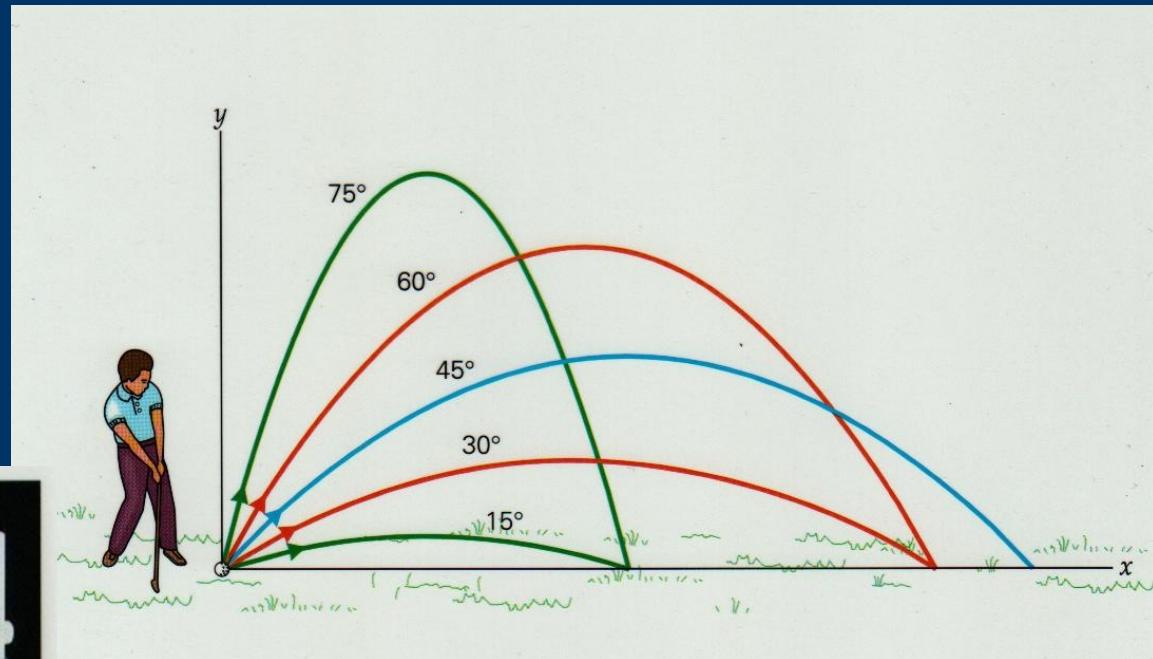
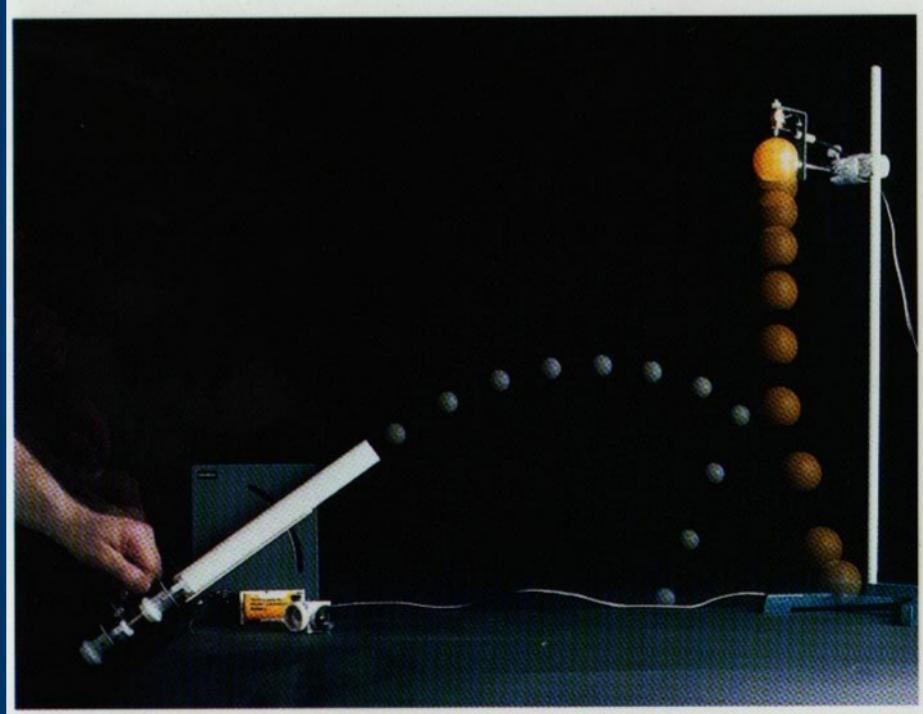


# Motion in 2 dimensions. Projectile Motion.

P.M. is 2-D motion when the only acceleration is due to gravity.  
That is:

$$\mathbf{a} = 0\hat{i} - g\hat{j}$$

This leads to parabolic trajectories.

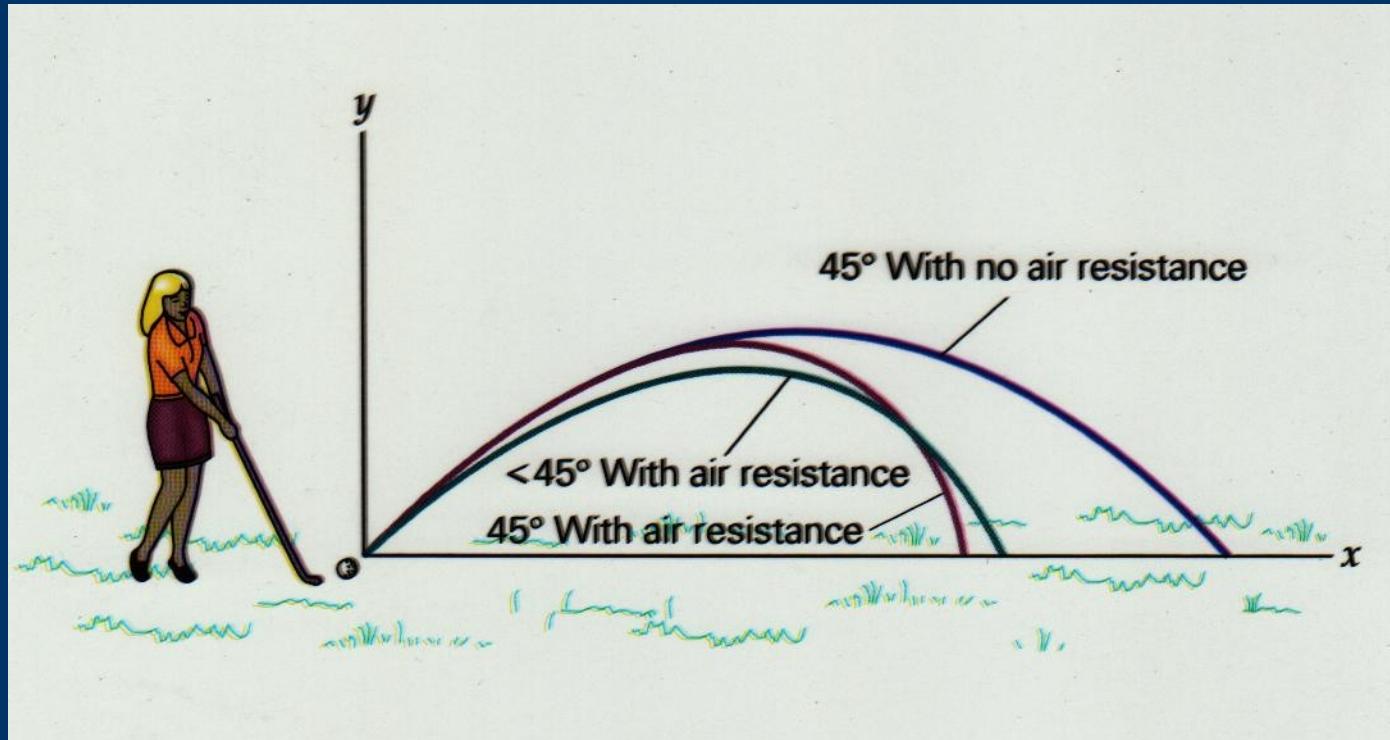


Notice that 2 initial angles lead to the same final *range*, except 45 degrees.

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

# PHYS 2311 Motion in 2 dimensions. Projectile Motion.

Actual trajectories: parabolas distorted by air resistance (drag).



# Physics 2311 – Physics I, Week 4

## Dr. J. Pinkney

### Outline for Day W4,D1

2D kinematics:

Projectile motion - simulators

Derive Range,  $h_{\max}$ ,  $t_{\max}$

Example problems.

### Homework

Ch. 3 P. 1,3,6,7,10,11,19,20,23,24,  
32,33,37,38,39 Do by today.

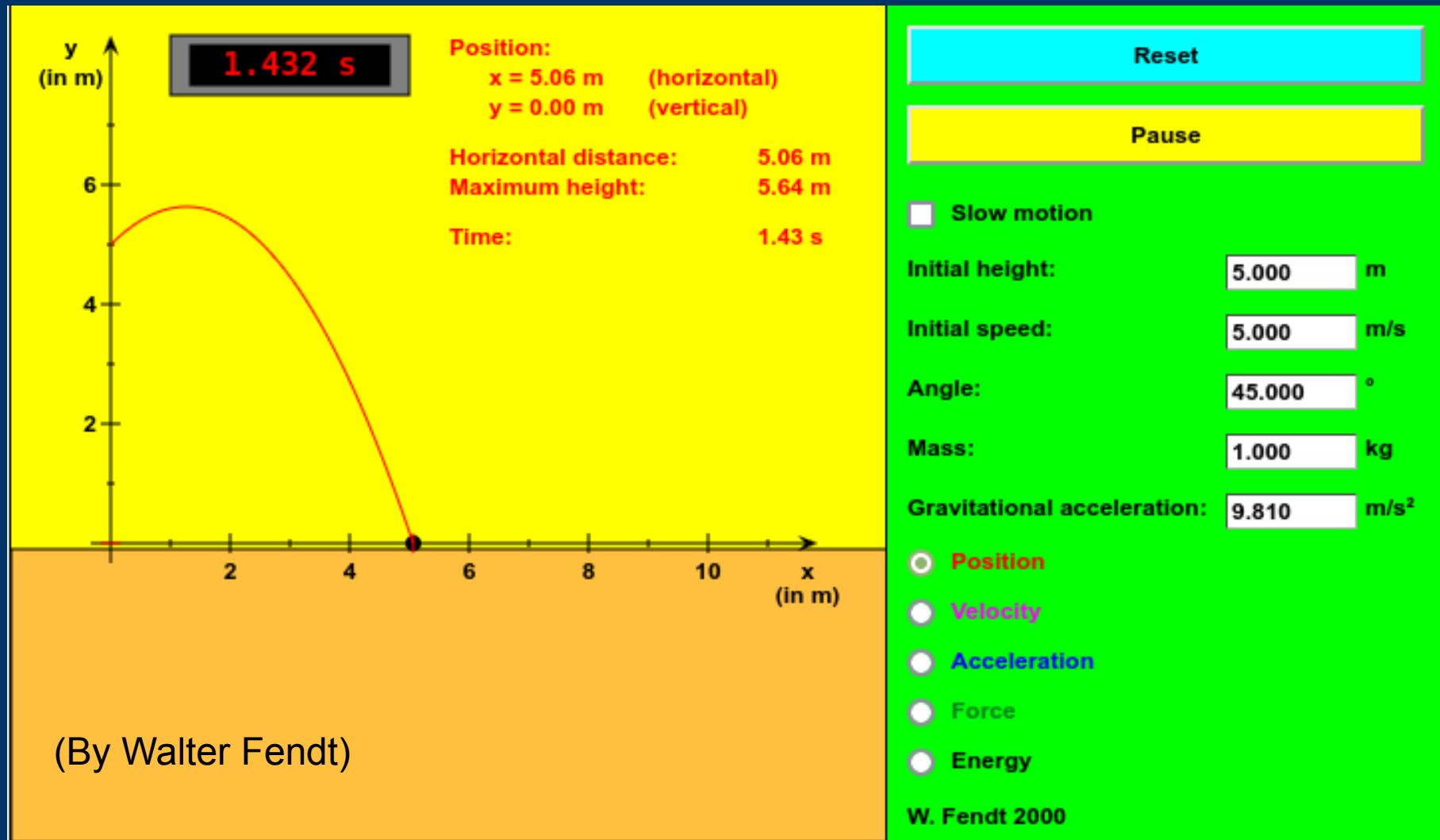
Ch. 4 P. 1-5,7,12-14,28,33,42,45,47,48  
MisQ 1-11 (odd), Read 4.1-4.8,3.9 (rel velocity)

Notes: Lab this week: “Acceleration of Gravity”  
Coming soon: “Exam like problems for Ch.3”

# Motion in 2 dimensions. Projectile Motion

Interactive simulations

(see also [ophysics.com/k8.html](http://ophysics.com/k8.html))

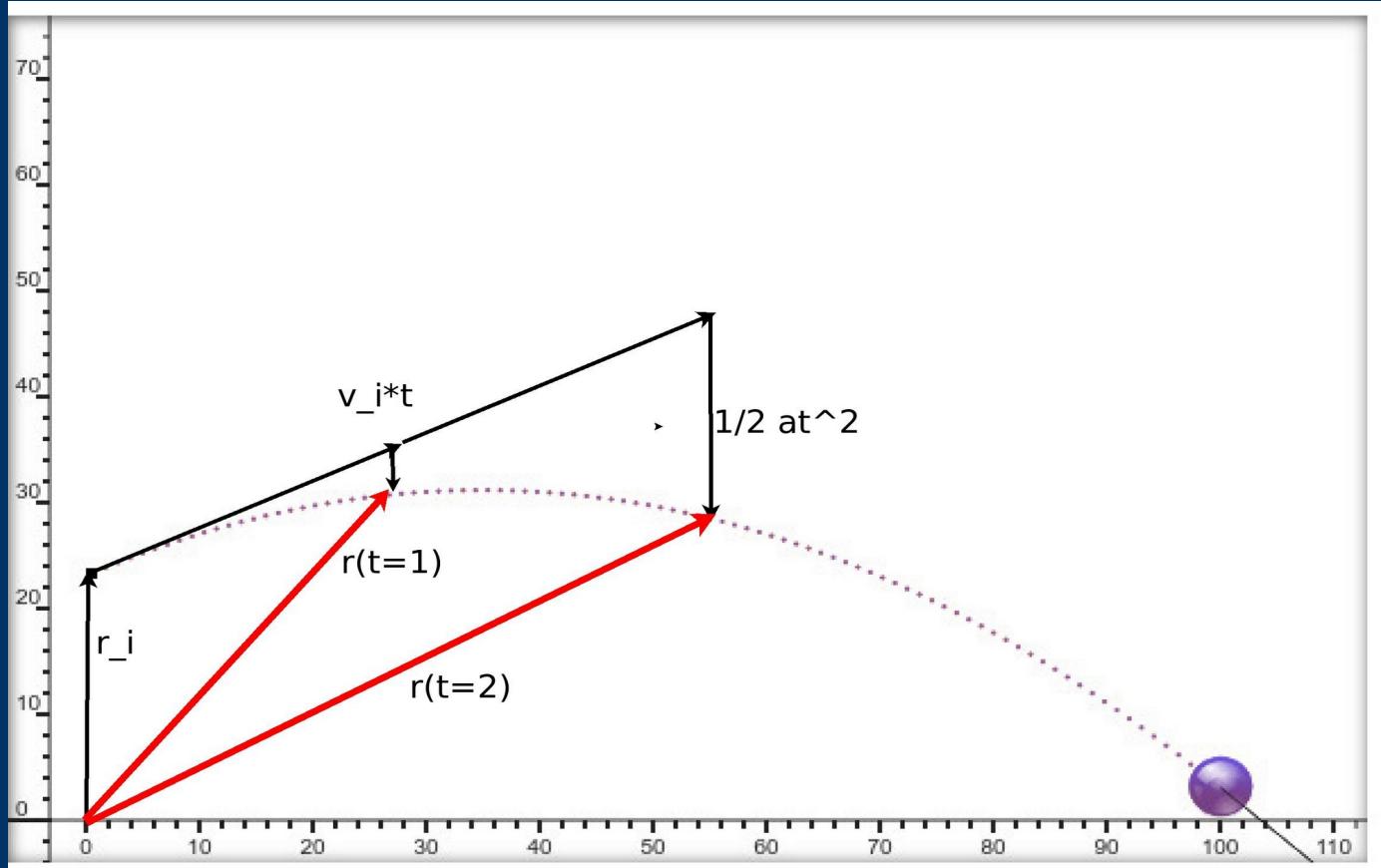


Trajectories are specified with an initial position, velocity (or speed & inclination angle), and acceleration.

# Motion in 2 dimensions. Projectile Motion

Trajectories: the position vector (red) is a sum of 3 vectors.

$$\vec{r}(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

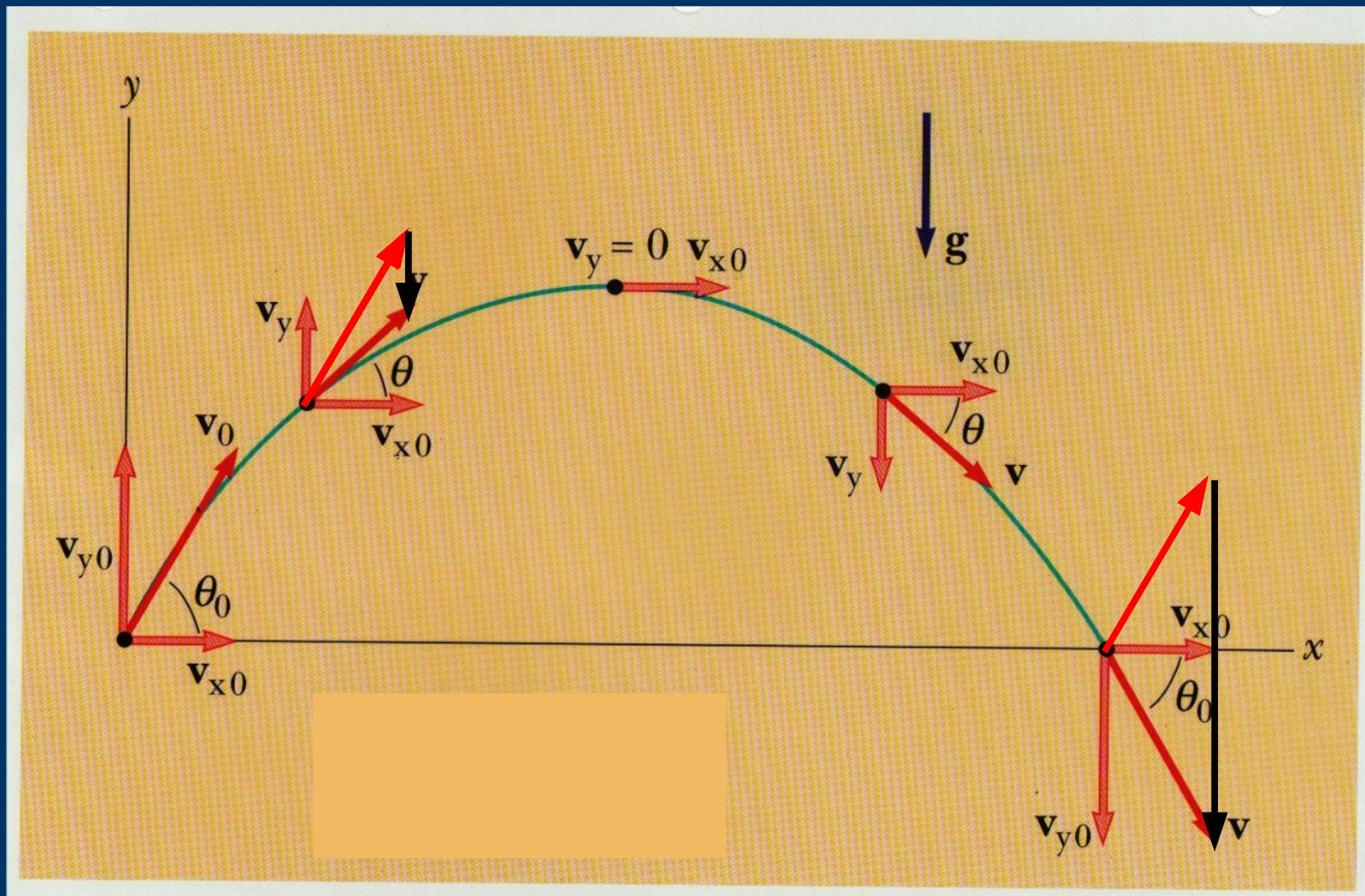


... or a sum of 2 vector components:  $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$   
Demo: [ophysics.com/k8.html](http://ophysics.com/k8.html)

# Motion in 2 dimensions. Projectile Motion

Trajectories: the velocity vector is a sum of 2 vectors.

$$\vec{v}(t) = \vec{v}_0 + \vec{a} t \quad \text{or} \quad \vec{v}(t) = v_{x,0} \hat{i} + v_y \hat{j}$$



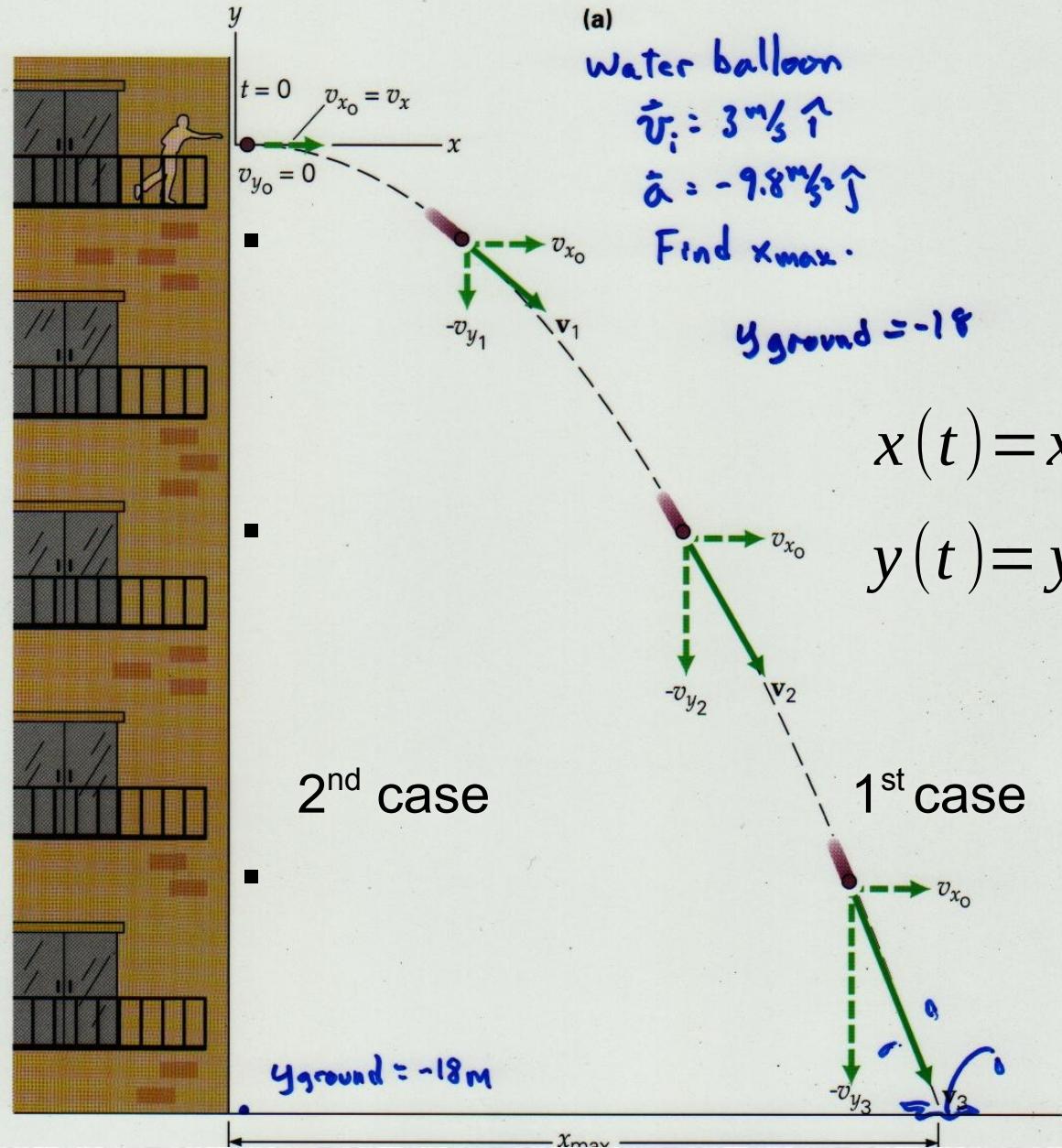
# P231 Week 3

## The independence of x and y components

If dropped from rest, the vertical progress is identical!

1<sup>st</sup> case:  $v_{x_0} = v_x$

2<sup>nd</sup> case:  $v_{x_0} = 0$



# **Motion in 2-D**

## **Projectile Motion formulas**

Time to reach max height:

$$t_{max} = \frac{v_i \sin \theta_i}{g}$$

( $v_i$  is the magnitude of the initial velocity)

Maximum height:

$$h_{max} = \frac{v_i^2 \sin^2 \theta}{2g}$$

Range:

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

To derive  $t_{max}$ , ask yourself “at what time does the vertical speed reach 0?”

$$v_y = 0 = v_{0y} + a_y t_{max} \quad \text{with } a_y = -g \text{ and } v_{0y} = v_i \sin \theta$$

To derive  $h_{max}$ , use  $y_{max} = y(t_{max}) = y_0 + v_{0y} t + 1/2 a_y t^2$  (assumes  $y_0 = 0$ )

To derive Range, use  $R = x(t=2t_{max}) = x_0 + v_{ox} t$ . (Need  $2\sin\theta\cos\theta = \sin 2\theta$ )

# PHYS 2311 Motion in 2 dimensions. Projectile Motion.



P 3.32) A tiger leaps horizontally from a 7.5-m high rock with a speed of 3.0 m/s. How far from the base of the rock will she land?

Find  $x_{\text{land}}$     Givens:  $a = -9.8 \hat{j} \text{ m/s}^2$      $y_0 = 7.5 \text{ m}$ ,  $v_i = 3.0 \hat{i} + 0 \hat{j} \text{ m/s}$

Set up: find  $t_{\text{land}}$  with  $y_f = y_i + v_{oy}t - 4.9t^2$

Then find  $x_{\text{land}}$  with  $x_{\text{land}} = x_0 + v_{ox}t$

P. 3.37) A firehose held near the ground shoots water at a speed of 6.5 m/s. At what angles should the nozzle point in order that the water land 2.5 m away? Why are there two different angles? Sketch the 2 trajectories.