# The Classification of Stellar Spectra
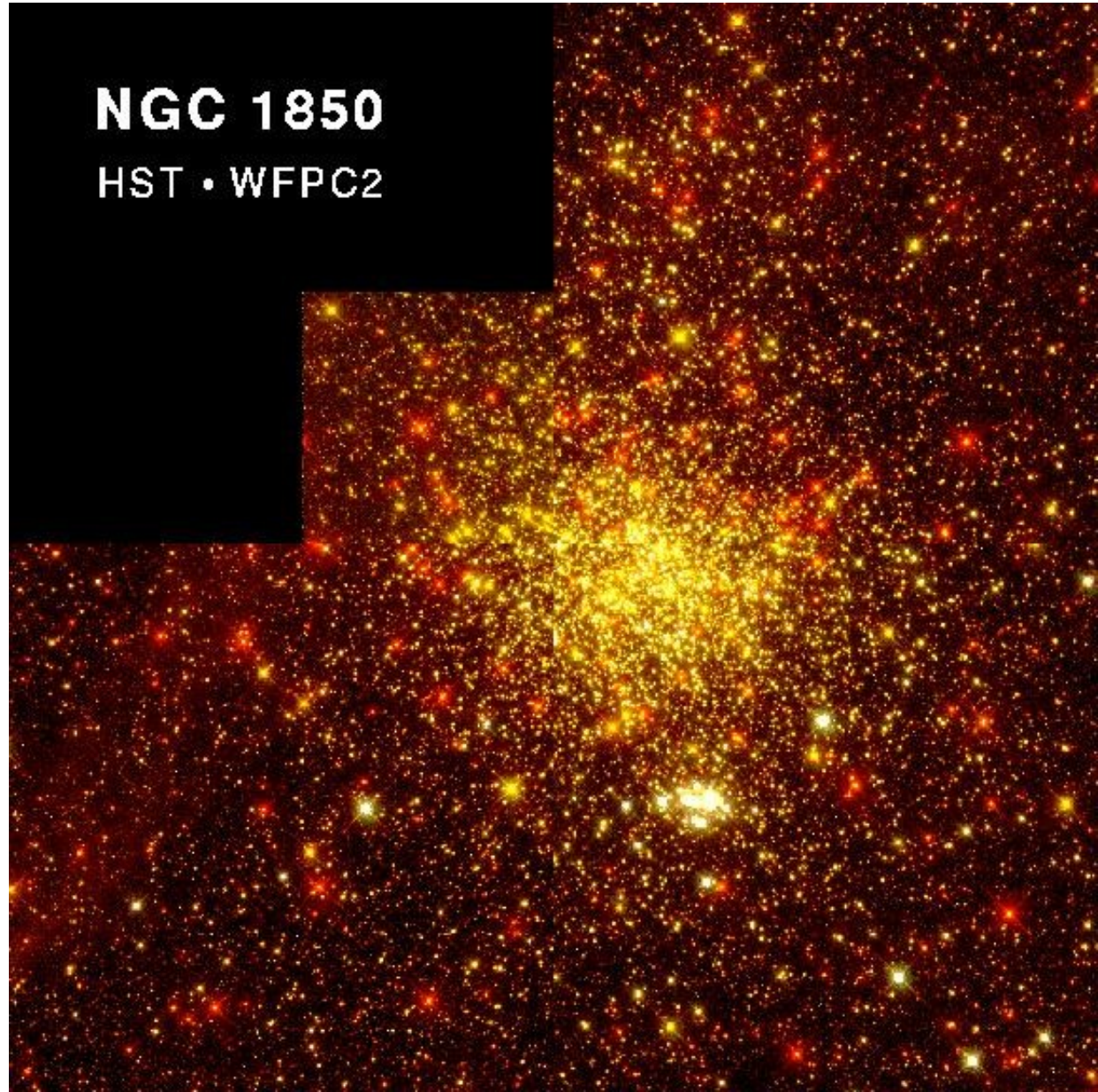
## Chapter 8

Star Clusters in the Large Magellanic Cloud



NGC 1850
HST • WFPC2
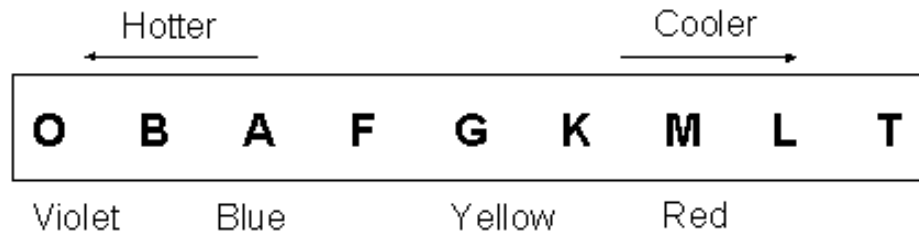
http://www.seds.org/hst/NGC1850.html

# The Classification of Stellar Spectra

- Classification scheme developed before the physics
- Parameters that could be used to classify stars
  - Apparant brightness (bad idea)
  - Luminosity (Intrinsic brightness)
  - Temperature (Color)
  - Spectra (absorption lines)
  - Mass (only for binaries)

- The Henry Draper Catalogue
  - Contained >100,000 spectral classifications from A.J. Cannon and others from Harvard
  - Used OBAFGKM

The "Computers" of the Harvard College Observatory



http://cannon.sfsu.edu/%7Egmarcy/cswa/history/pick.html

# The Classification of Stellar Spectra

- Originally organized by strength of H Balmer lines (A,B,...).
- Atomic physics allowed connection to temperature to be made.
- Spectral Types:

| Hotter | | | | | | Cooler | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| O | B | A | F | G | K | M | L | T |

Violet      Blue      Yellow      Red

Early type → late type

L and T are more modern additions – Brown Dwarfs. R N S also used after M.

- – Subdivisions in tenths: 0 → 9 (early → late, hot → cool) within a Spectral Type).  E.g., A0 is hotter than A5.
- – The Sun is a G2 – an early G-type star
  - G – yellow star (continuum peak in green/yellow)
    - – H lines weak
    - – Ca II (singly ionized) lines continue becoming stronger
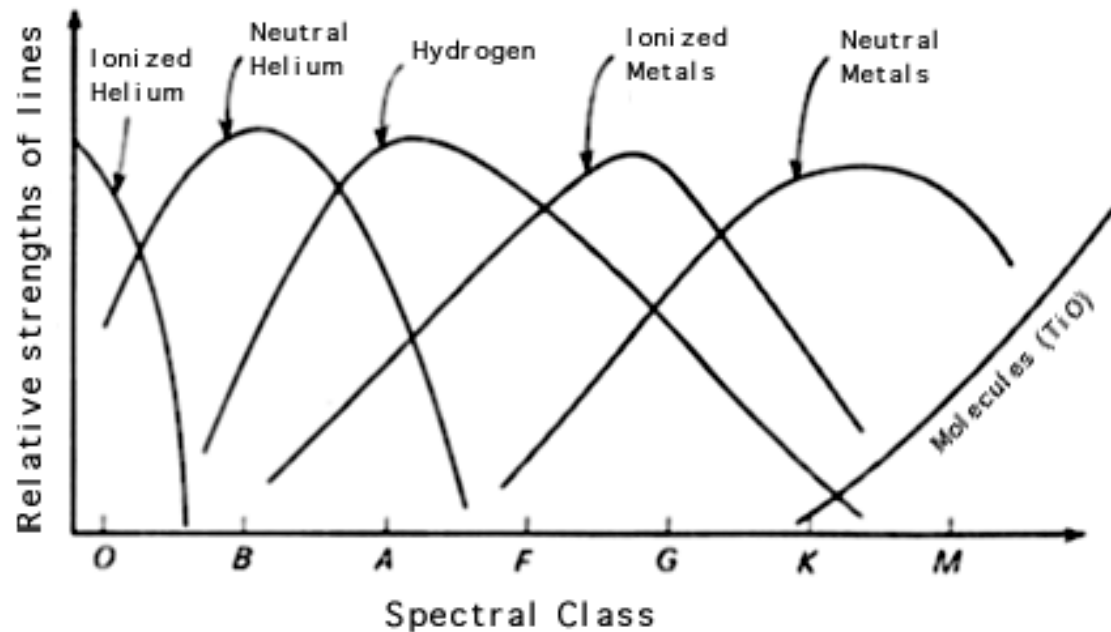    - – Fe I, other *neutrals* metal lines become stronger

# O to G example
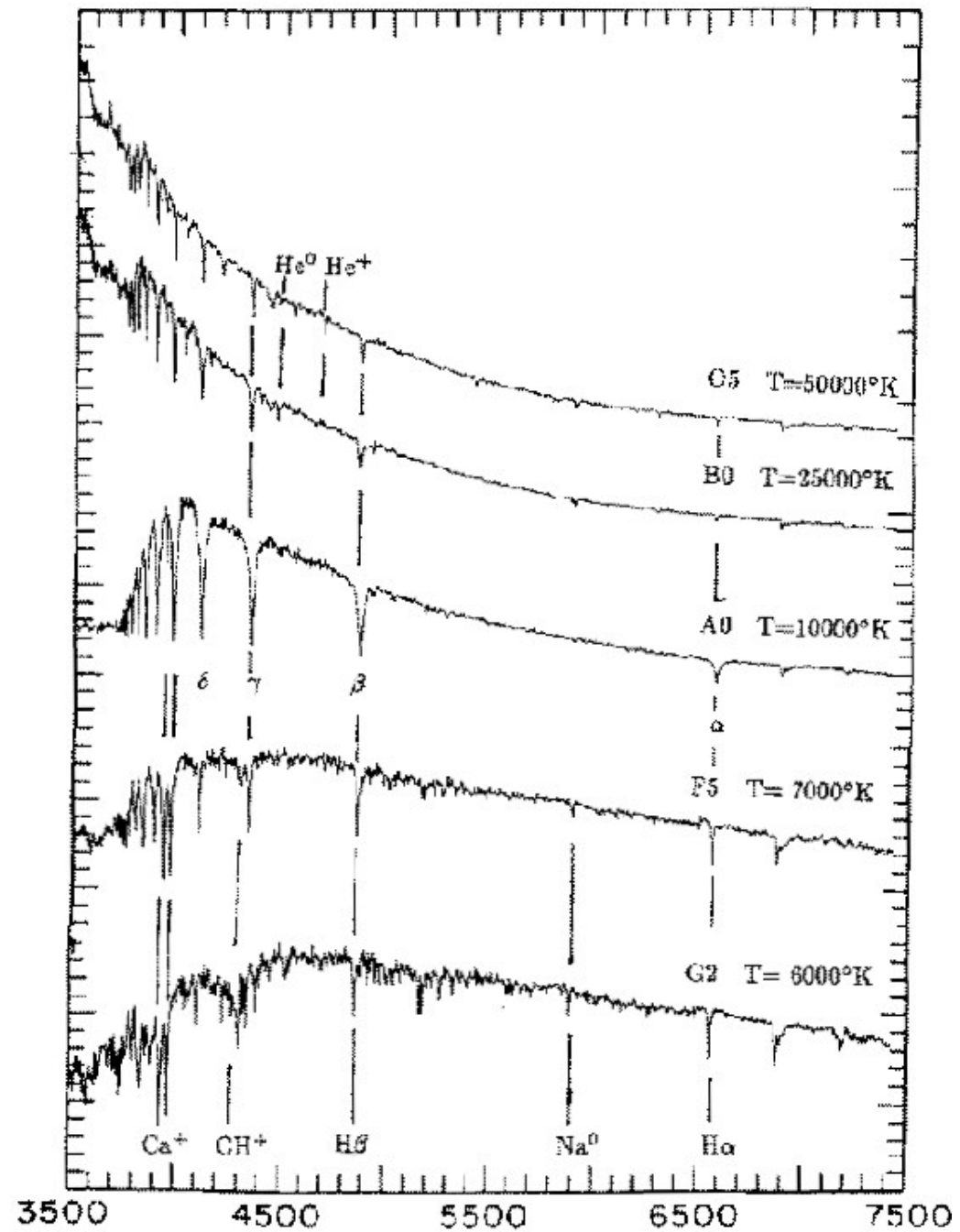
# G to M example
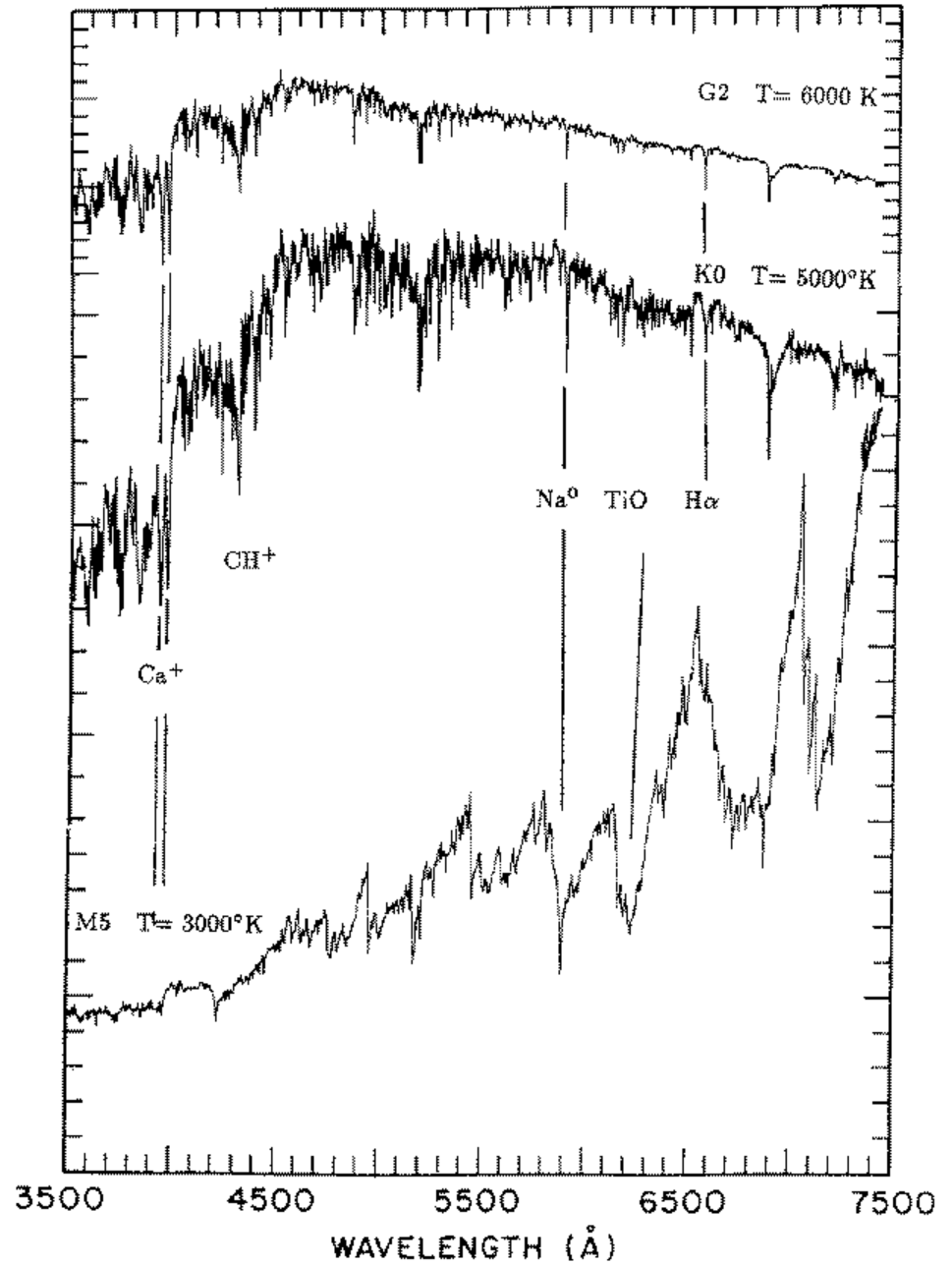
# The Formation of Spectral Lines

- Question: What causes the differences in the observed spectra??
  - [Absorption by intervening material.  Earth's atmos., ISM.]
  - Composition
  - Temperature
  - Surface gravity / pressure
- Answer:
  - Temperature is the main factor

# The Formation of Spectral Lines

- Big Question of Ch.8: Why are the H balmer lines strongest for A stars, which seem to have T_surf = 10,000K?

- To find answer:

  - Need Ch.5's info about the Bohr atom … energy levels.

  - Need Kirchoff's laws → our gas is the upper "atmosphere" of the star.

  - Need statistical mechanics – study of large numbers of particles that can occupy different states
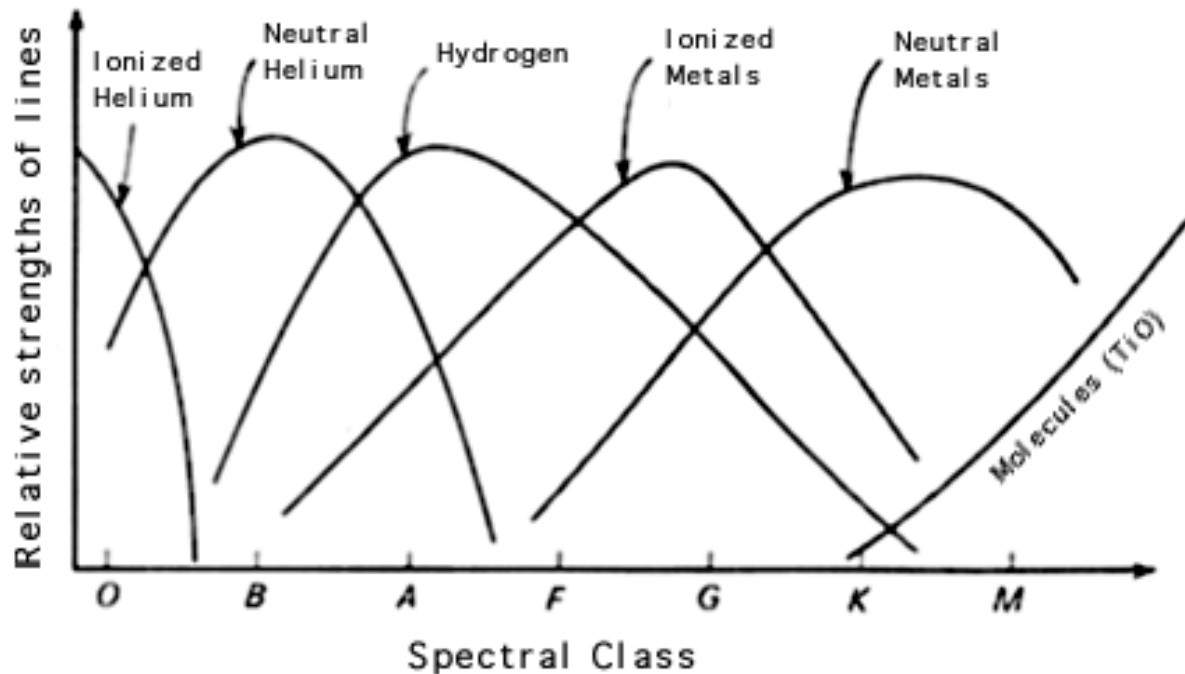
# The Formation of Spectral Lines

- Distribution of electrons in different atomic orbitals depends on temperature

- Electrons can jump up in energy by absorption of a photon OR collision with a particle!  So KE of surrounding particles important.

- What is the probability of finding an electron in a particular orbital?
  - Answer with Statistical Mechanics…
  - Maxwell-Boltzmann (velocity) Distribution
    - Assumes thermal equilibrium
    - Number of gas particles per unit volume have a speed between v and v+dv

$$n_v \, dv = n \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} e^{-\frac{1}{2} mv^2 / kT} \; 4\pi \; v^2 dv$$

# Maxwell-Boltzmann Distribution

$$n_v\,dv = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{1}{2}mv^2/kT}\;4\pi\,v^2\,dv$$

- Most probable speed
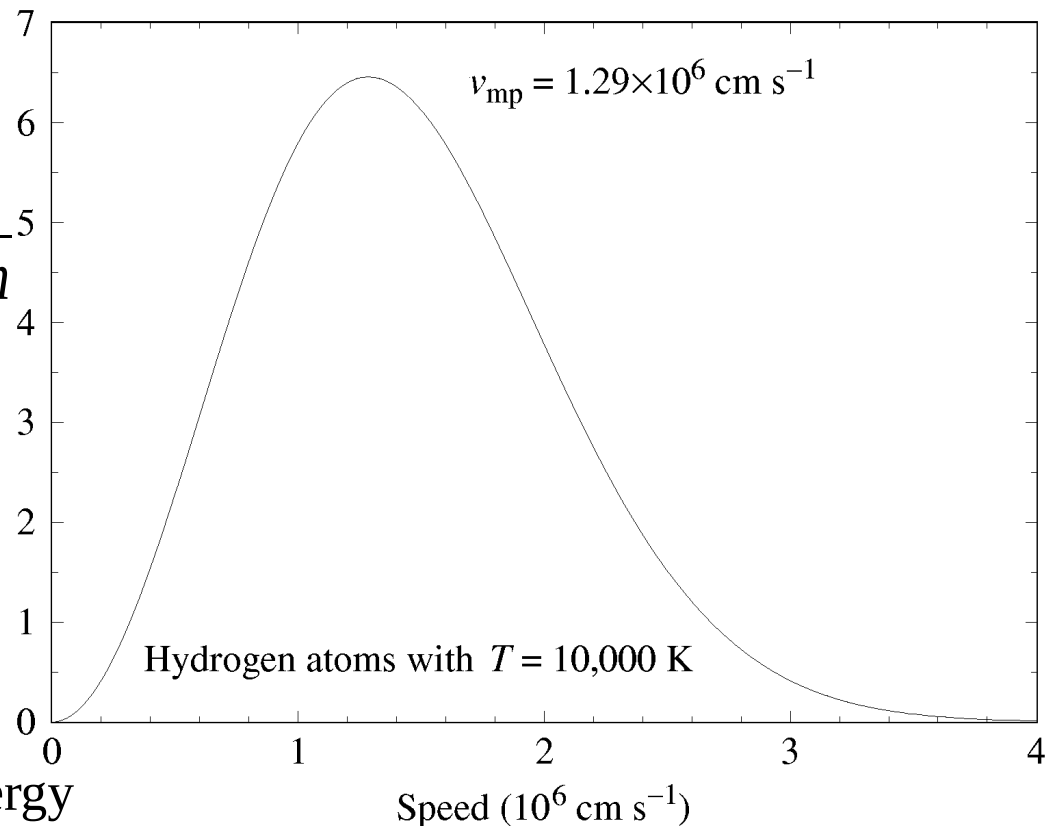
$$v_{mp} = \sqrt{\frac{2kT}{m}} = 1.4\sqrt{kT/m}$$

- Root-mean-square

$$v_{rms} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{kT/m}$$

- Average

$$v_{avg} = \sqrt{\frac{8kT}{\pi m}} = 1.6\sqrt{kT/m}$$

- Collisional energy causes a distribution of electrons among the atomic orbitals (Kinetic Energy → Potential Energy

$v_{mp} = 1.29 \times 10^6 \text{ cm s}^{-1}$

Hydrogen atoms with $T = 10{,}000$ K

Speed ($10^6$ cm s$^{-1}$)

$n_v/n$ ($10^{-7}$ s cm$^{-1}$)

# Boltzmann Factor

- The higher the energy of a state, the less likely it will be occupied

$$P_a \propto e^{\frac{-E_a}{kT}}$$

  – For the Maxwell-Boltzmann distribution, the energy is Kinetic Energy

$$P_v \propto e^{-\frac{1}{2}mv^2/kT}$$

- The "$kT$" term is associated with the thermal energy of the "gas" as a whole

- Ratio of Probabilities for two different states (and energies)

$$\frac{P_b}{P_a} = \frac{e^{\frac{-E_b}{kT}}}{e^{\frac{-E_a}{kT}}} = e^{\frac{-\left(E_b - E_a\right)}{kT}}$$

# Degeneracy Factor

- An energy (eigenvalue) is associated with each set of quantum numbers (eigenstate or eigenfunction)

- *Degenerate States* have different quantum numbers but the same energy

- Modify the Boltzmann factor $$P_a \propto g_a\, e^{\frac{-E_a}{kT}}$$

  - The probability of being in any of the $g_a$ degenerate states with energy $E_a$
    - $g_a$ is the <u>*degeneracy*</u> or <u>*statistical weight*</u> of state $a$

- Ratio of probabilities between states with two different energies
$$\frac{P_b}{P_a} = \frac{g_b}{g_a}\, e^{\frac{-\left(E_b - E_a\right)}{kT}}$$

# Degeneracy Factor

- Details of quantum mechanics determines the energies and quantum numbers…

- Visit the following site on the next page and browse…

- Quantum numbers for Hydrogen $\{n, l, m_l, m_s\}$
  - Table 8.2

| State | $n$ Principal quantum number n | $l$ Orbital quantum number | $m_l$ Magnetic quantum number | $m_s$ Spin quantum number | Maximum number of electrons | |
|---|---|---|---|---|---|---|
| 1s | 1 | 0 | 0 | $+\frac{1}{2},-\frac{1}{2}$ | 2 | |
| 2s | 2 | 0 | 0 | $+\frac{1}{2},-\frac{1}{2}$ | 2 | 8 |
| 2p | 2 | 1 | $-1,0,+1$ | $+\frac{1}{2},-\frac{1}{2}$ | 6 | |
| 3s | 3 | 0 | 0 | $+\frac{1}{2},-\frac{1}{2}$ | 2 | |
| 3p | 3 | 1 | $-1,0,+1$ | $+\frac{1}{2},-\frac{1}{2}$ | 6 | 18 |
| 3d | 3 | 2 | $-2,-1,0,1,2$ | $+\frac{1}{2},-\frac{1}{2}$ | 10 | $=2n^2$ |

# Boltzmann Equation

- Number of atoms in a particular state $a$

$$N_a = NP_a$$

$N$ = total number of atoms
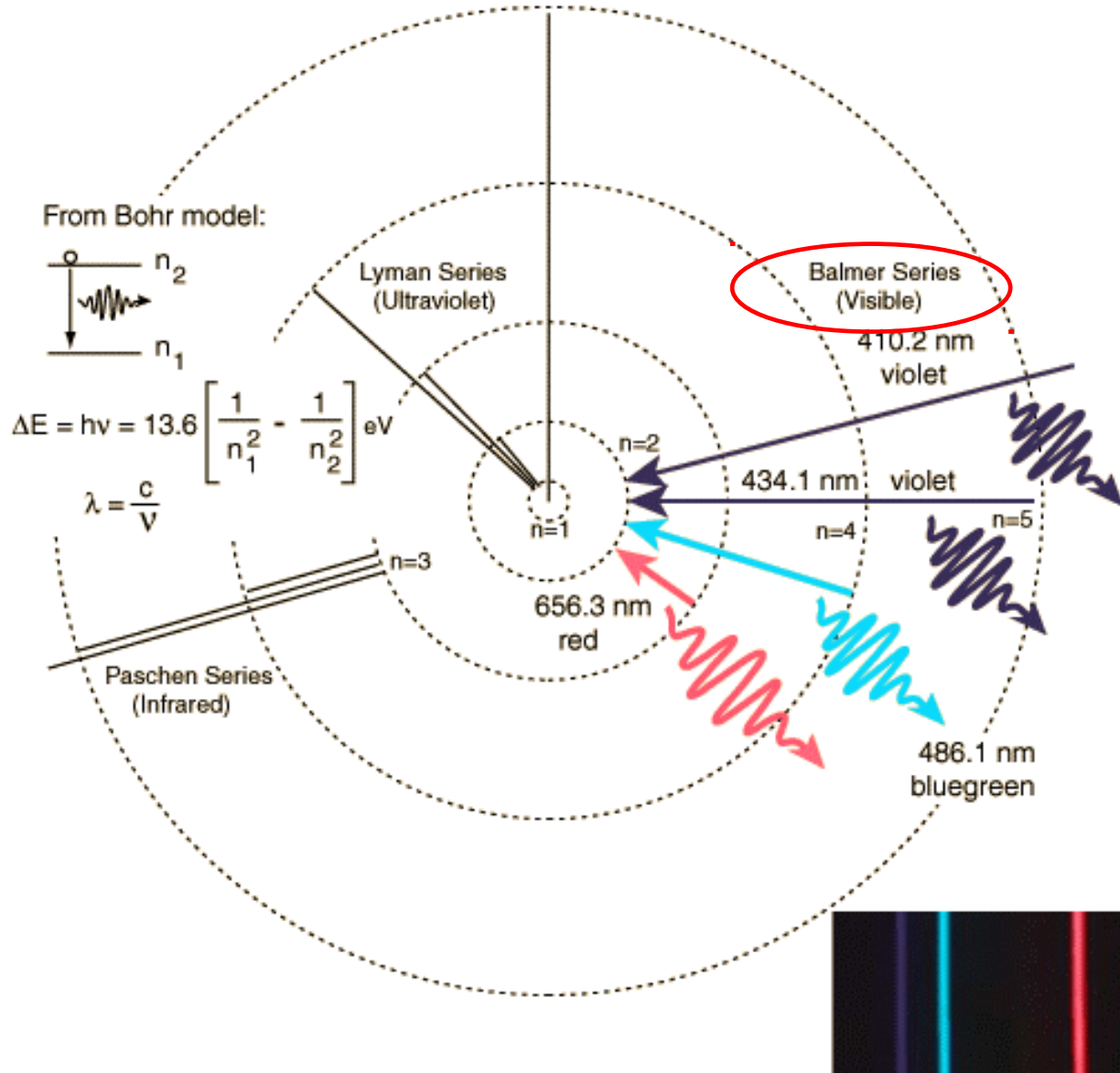
$N_a$ = number of atoms in state $a$

$P_a$ = probability of being in state $a$

$$\Rightarrow \frac{N_b}{N_a} = \frac{g_b}{g_a} e^{\frac{-\left(E_b - E_a\right)}{kT}}$$

Hydrogen Atom Examples

# Hydrogen Atom

- Balmer series absorption spectra is an upward transition from $n = 2$

- Observation: this series has a peak absorption spectrum at ~9520 K.

# Hydrogen Atom Populations

- We just saw that not many Hydrogen atoms are in the *n=1* state at 9520 K!

  – Shouldn't the line strength keep growing as the temperature increases since there is a higher probability for an H atom to be in the *n=2* state?!?!
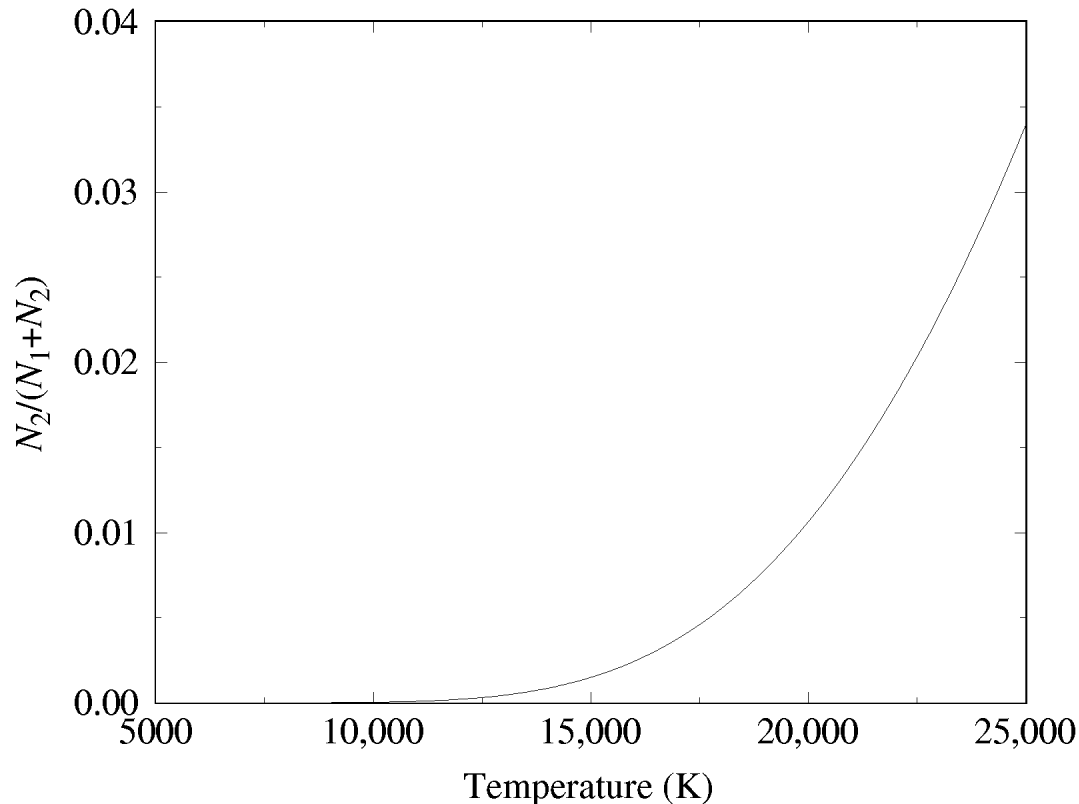
# Partition Function

- We also have to figure in all states that have a significant population

- For one state we have: $\quad P_1 \propto g_1\, e^{\frac{-E_1}{kT}}$

- Ratio between two states: $\quad \dfrac{P_2}{P_1} = \dfrac{g_2\, e^{\frac{-E_2}{kT}}}{g_1\, e^{\frac{-E_1}{kT}}} = \dfrac{g_2}{g_1}\, e^{\frac{-\left(E_2 - E_1\right)}{kT}}$

- Ratio of state *2* to *all* other states with reference to the ground state:

$$\frac{P_2}{P_{all}} = \frac{g_b\, e^{\frac{-(E_2 - E_1)}{kT}}}{g_1\, e^{\frac{-(E_1 - E_1)}{kT}} + g_2\, e^{\frac{-(E_2 - E_1)}{kT}} + g_3\, e^{\frac{-(E_3 - E_1)}{kT}} + \cdots} = \frac{g_2\, e^{\frac{-(E_2 - E_1)}{kT}}}{Z}$$

# Partition Function

- This tell us how many states are accessible or available at a given temperature (thermal energy)

$$Z = g_1 e^{\frac{-(E_1 - E_1)}{kT}} + g_2 e^{\frac{-(E_2 - E_1)}{kT}} + g_3 e^{\frac{-(E_3 - E_1)}{kT}} + \cdots$$

$$= g_1 + \sum_i g_i e^{\frac{-(E_i - E_1)}{kT}}$$

- The higher the temperature, the more states that are available

- At zero K, everything will be in the ground state
  - KT$\rightarrow$ 0 so exponent terms are e$^{-\infty}$ = 0.
  - Z diverges with n, but upper energy levels involve overlapping e- with neighboring atoms.

# Partition Function and Atoms

- We also have to handle ionization!
- Nomenclature:  H I – neutral hydrogen

    H II – singly ionized hydrogen

    He I – neutral Helium

    He II – singly ionized Helium

    He III – doubly ionized Helium

- Ionization Energy for H I to H II

$$\chi_I = 13.6\,eV$$

  - Rather than $N_2/N_1 \rightarrow \infty$, the atom will ionize before this happens

# Saha Equation

- Determines the ratio of numbers of ionized atoms
- Need distinct partition functions since energy levels of atoms are different for different ionization stages
  - $Z_i$ is the initial stage of ionization
  - $Z_{i+1}$ is the final stage of ionization
- Ratio of the number of atoms in each of these stages

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left( \frac{2\pi \, m_e \, kT}{h^2} \right)^{\frac{3}{2}} e^{-\chi_i kT}$$

  - $n_e$ is the electron density (an ideal gas of electrons)
    - Electron pressure $\quad P_e = n_e kT$

    - Electrons recombine with H II to give H I

# Ionized Hydrogen Atoms

- Fraction of hydrogen atoms that are ionized
- If we have H II, we can't have the Balmer series!
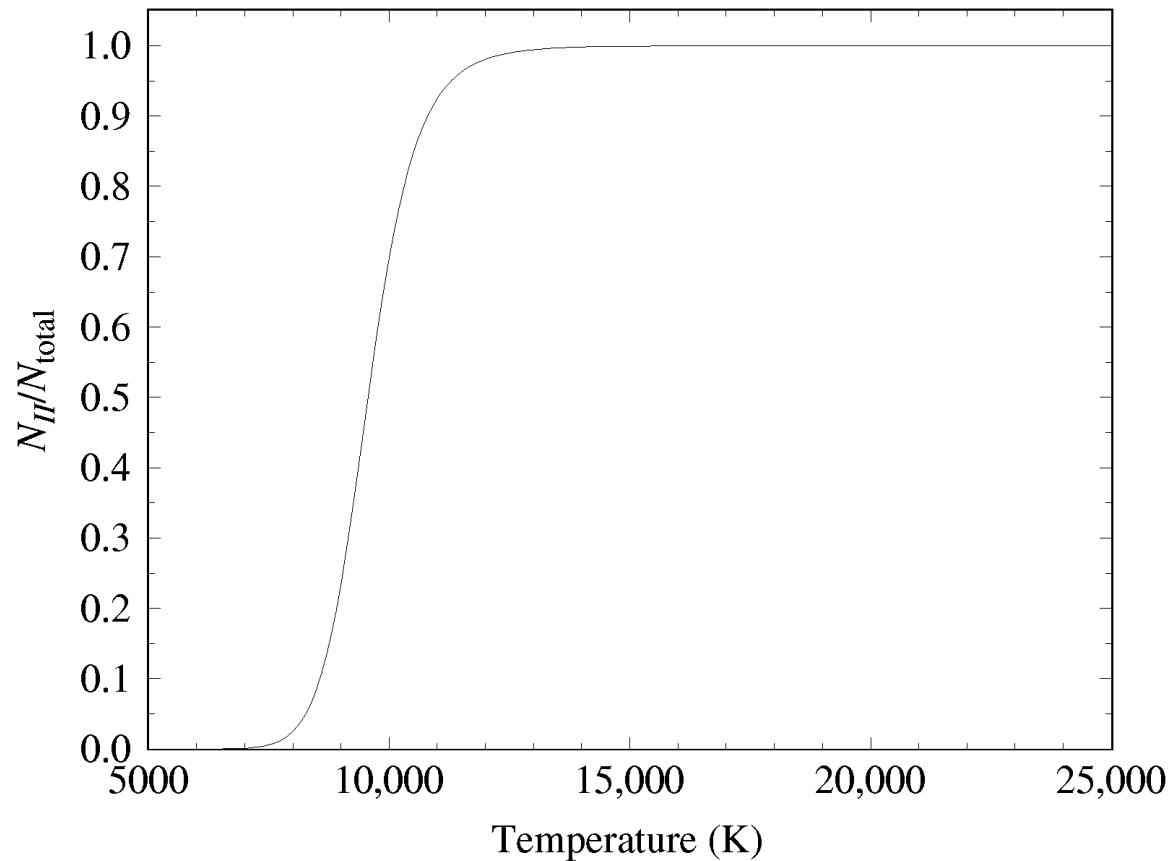
# H I $n = 2$ population

$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_I + N_2}\right)\left(\frac{N_I}{N_I + N_{II}}\right)$$

Fraction of non-ionized hydrogen Atoms in the $n = 2$ state

Fraction of non-ionized hydrogen atoms

Excitation
Boltzmann equation

Ionization
Saha equation

$e$

$n=1$
$n=2$
$n=3$
$n=4$

$H_\beta$

$e$

$H_\alpha$

$n=1$
$n=2$
$n=3$
$n=4$

$n=1$
$n=2$
$n=3$
$n=4$
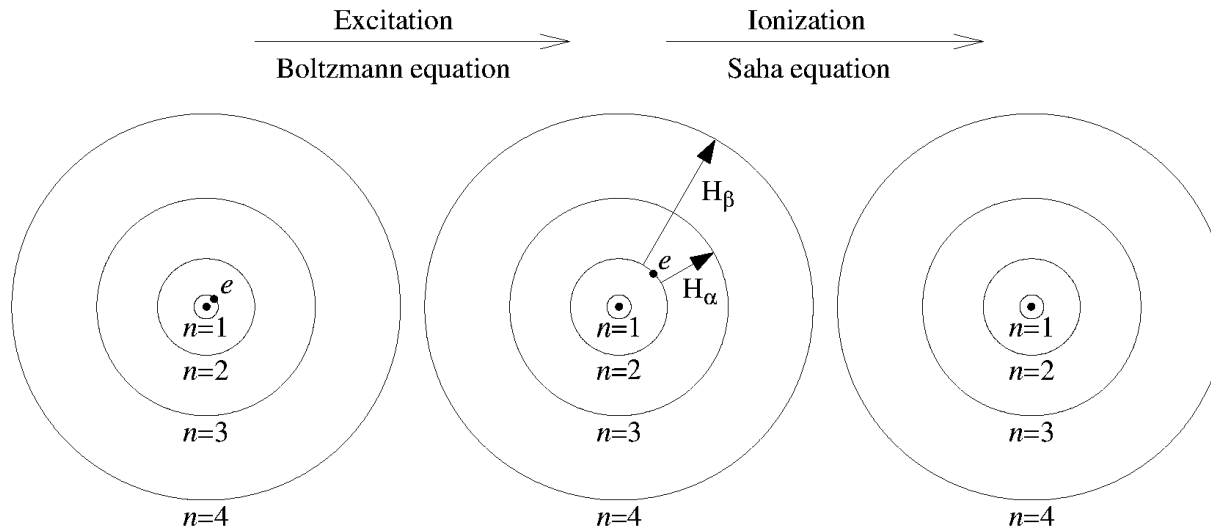
# H I $n = 2$ population

- Includes the Boltzmann factor, partition function and ionization
- Population peak at 9520 K, in agreement with observation of the Balmer series
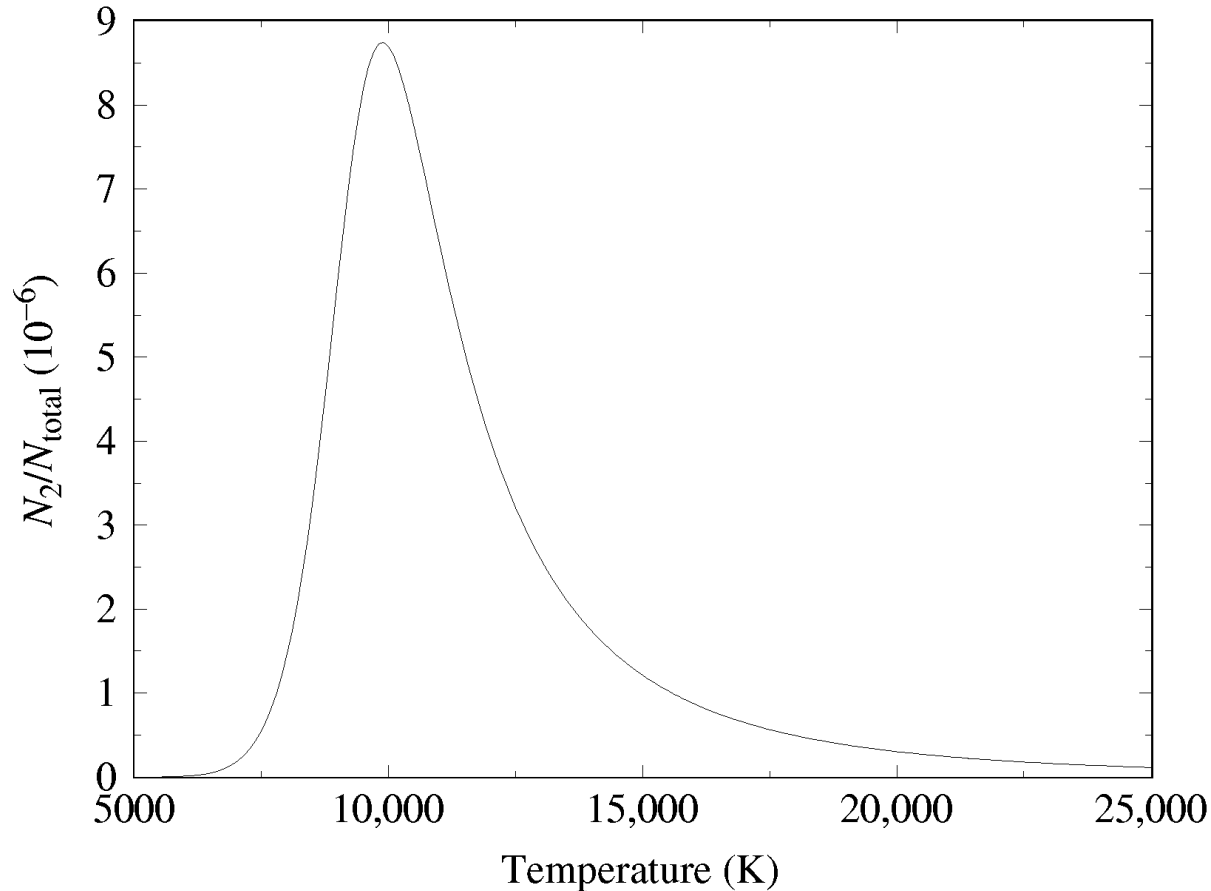
# Example 8.3

- Degree of ionization in a stellar atmosphere of pure hydrogen for the temperature range of 5000-25000 K $\dfrac{N_{II}}{N_{Total}}$

- Given electron pressure $P_e = 200 \dfrac{dyne}{cm^2} = 20 \, N/m^2$

- Saha equation $$\frac{N_{II}}{N_I} = \frac{2kTZ_{II}}{P_e Z_I} \left( \frac{2\pi \, m_e kT}{h^2} \right)^{\frac{3}{2}} e^{-\chi_i kT}$$

- Must determine the partition functions
  - Hydrogen ion is a proton, so $Z_{II} = 1$
  - Neutral hydrogen over this temp range

    $\Delta E = E_2 - E_1 = 10.2$ eV

    $\Delta E \gg kT$, then $\quad e^{-\Delta E/kT} \ll 1$

    $$\Rightarrow Z_I = g_1 + \sum_i g_i \, e^{\frac{-(E_i - E_1)}{kT}} \quad g_1 = 2$$

$T := 5000\text{K}$

$k \cdot T = 0.43 \, \text{eV}$

$T := 25000\text{K}$

$k \cdot T = 2.15 \, \text{eV}$

# Example 8.3

- Degree of Ionization

$$\frac{N_{II}}{N_I} = \frac{2\,kT\,(1)}{P_e\,(2)}\left(\frac{2\pi\,m_e\,kT}{h^2}\right)^{\frac{3}{2}}e^{-\chi_i kT}$$

$$\frac{N_{II}}{N_I + N_{II}} =$$

$$\frac{N_{II}/N_I}{1 + N_{II}/N_I}$$

Most of the ionization occurs over a 3000 K region

Partial ionization zone
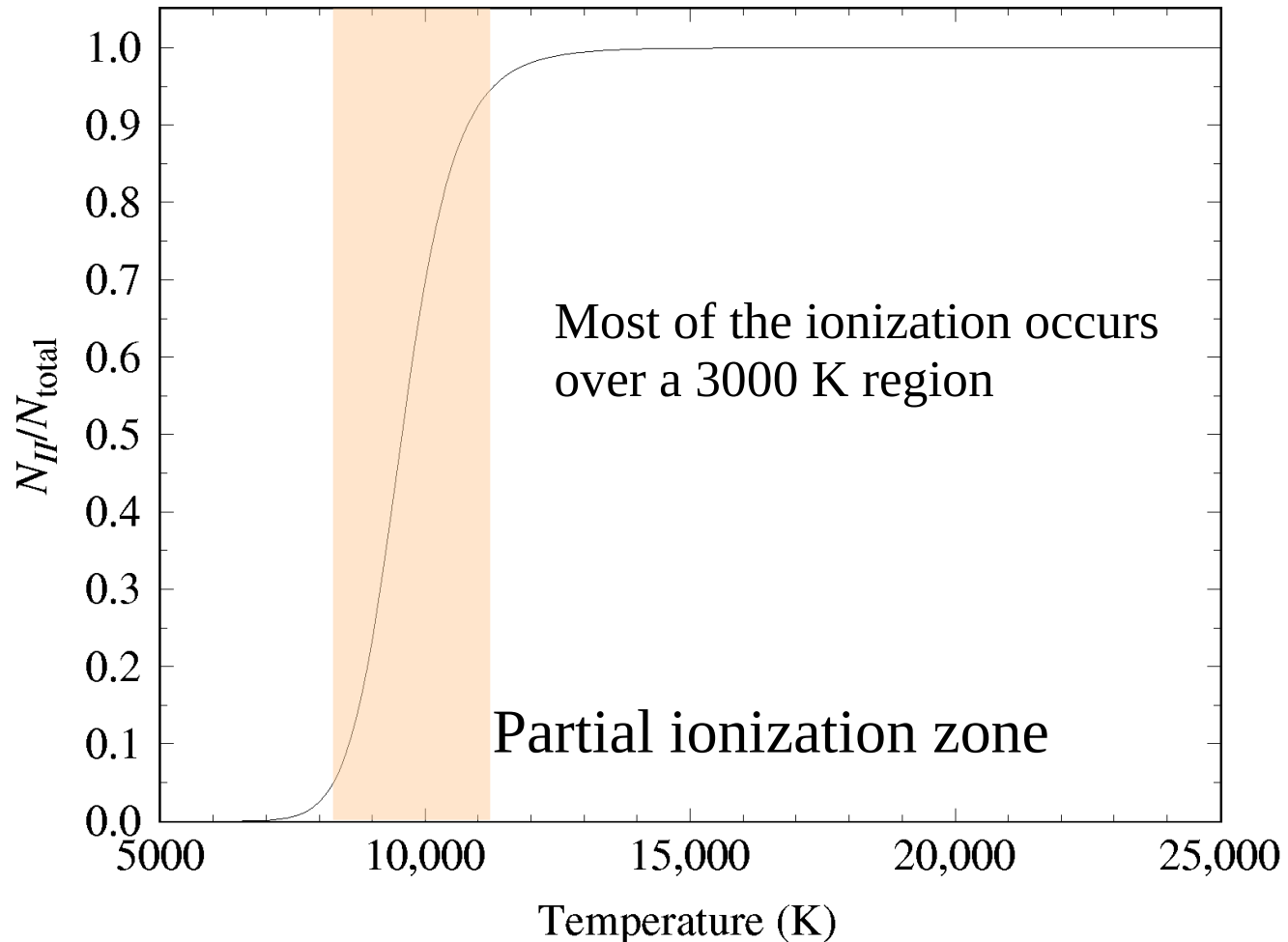
# Problem 8.7

- Evaluate the first three terms of the partition function for 10000K

Partition Function:   Counting the first ten states...   Energy:   $E(n) := \dfrac{-13.6\text{eV}}{n^2}$   Degeneracy:   $g(n) := 2 \cdot n^2$

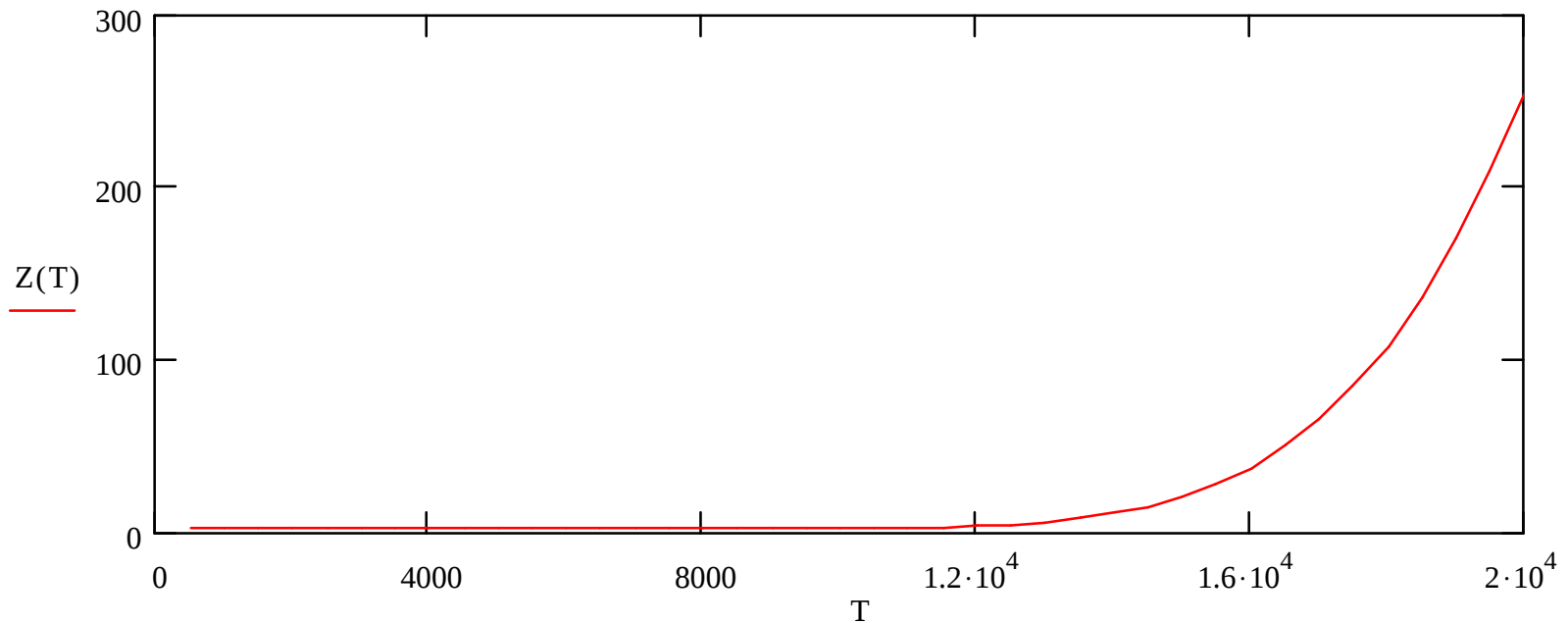$$f_B(n, T) := \exp\left[\frac{-(E(n) - E(1))}{k\,T}\right]$$

$$Z(T) := \sum_{n=1}^{10} \left(g(n) \cdot f_B(n, T)\right)$$

$T := 0, 500.. 20000$

$Z(6000\text{K}) = 2.0000$        $Z(10000\text{K}) = 2.0002$        $Z(15000\text{K}) = 2.0292$
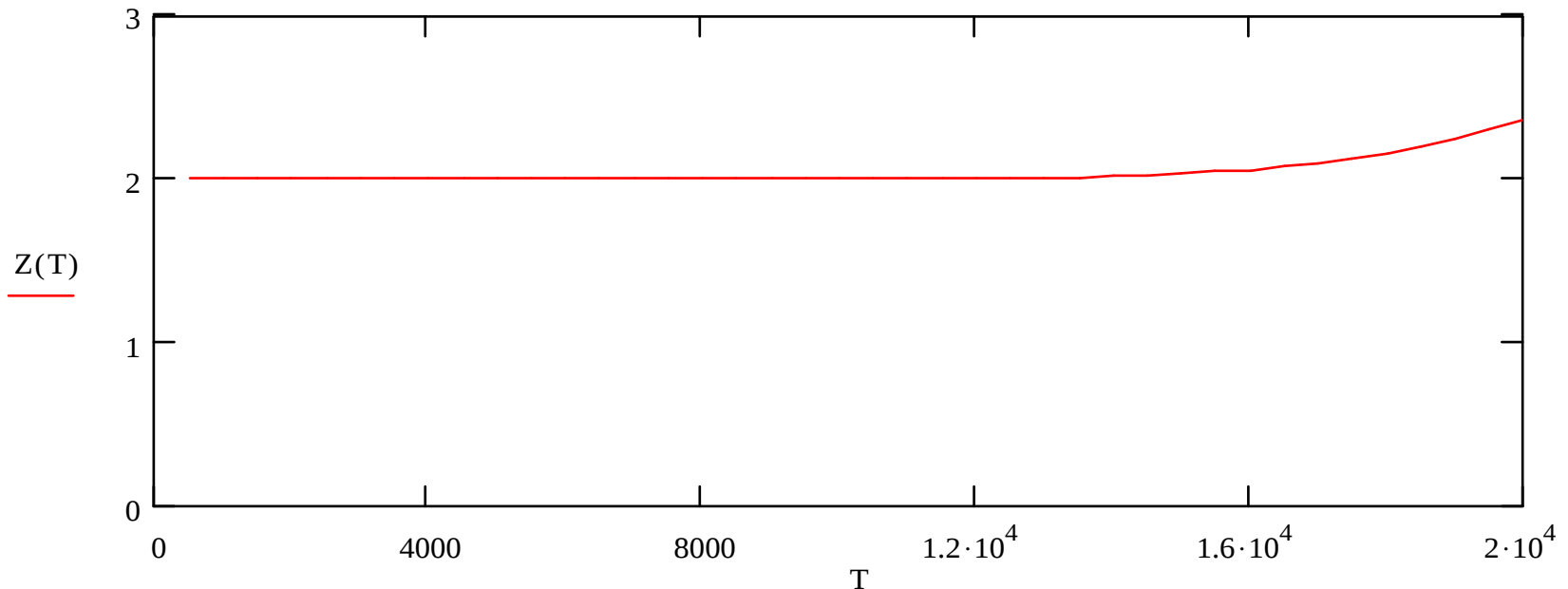
# Problem 8.8

- The partition function diverges at $n \rightarrow \infty$
  - Why do we ignore large $n$?

Partition Function:   Counting the first 100 states...   Energy: $\quad E(n) := \dfrac{-13.6\,eV}{n^2}$   Degeneracy: $\quad g(n) := 2 \cdot n^2$

$$f_B(n, T) := \exp\left[\dfrac{-(E(n) - E(1))}{k\,T}\right] \qquad\qquad Z(T) := \sum_{n=1}^{100} \left( g(n) \cdot f_B(n, T) \right) \qquad\qquad T := 0, 500 .. 20000$$

$$Z(6000K) = 2.0000 \qquad Z(10000K) = 2.0952 \qquad Z(15000K) = 20.2988$$

# Problem 8.8

Partition Function:    Counting the first 1000 states...    <u>Energy</u>:    $E(n) := \dfrac{-13.6\text{eV}}{n^2}$    <u>Degeneracy</u>:    $g(n) := 2 \cdot n^2$

$$f_B(n, T) := \exp\left[\dfrac{-(E(n) - E(1))}{k \, T}\right] \qquad Z(T) := \sum_{n=1}^{1000} \left(g(n) \cdot f_B(n, T)\right) \qquad T := 0, 500 \ldots 20000$$

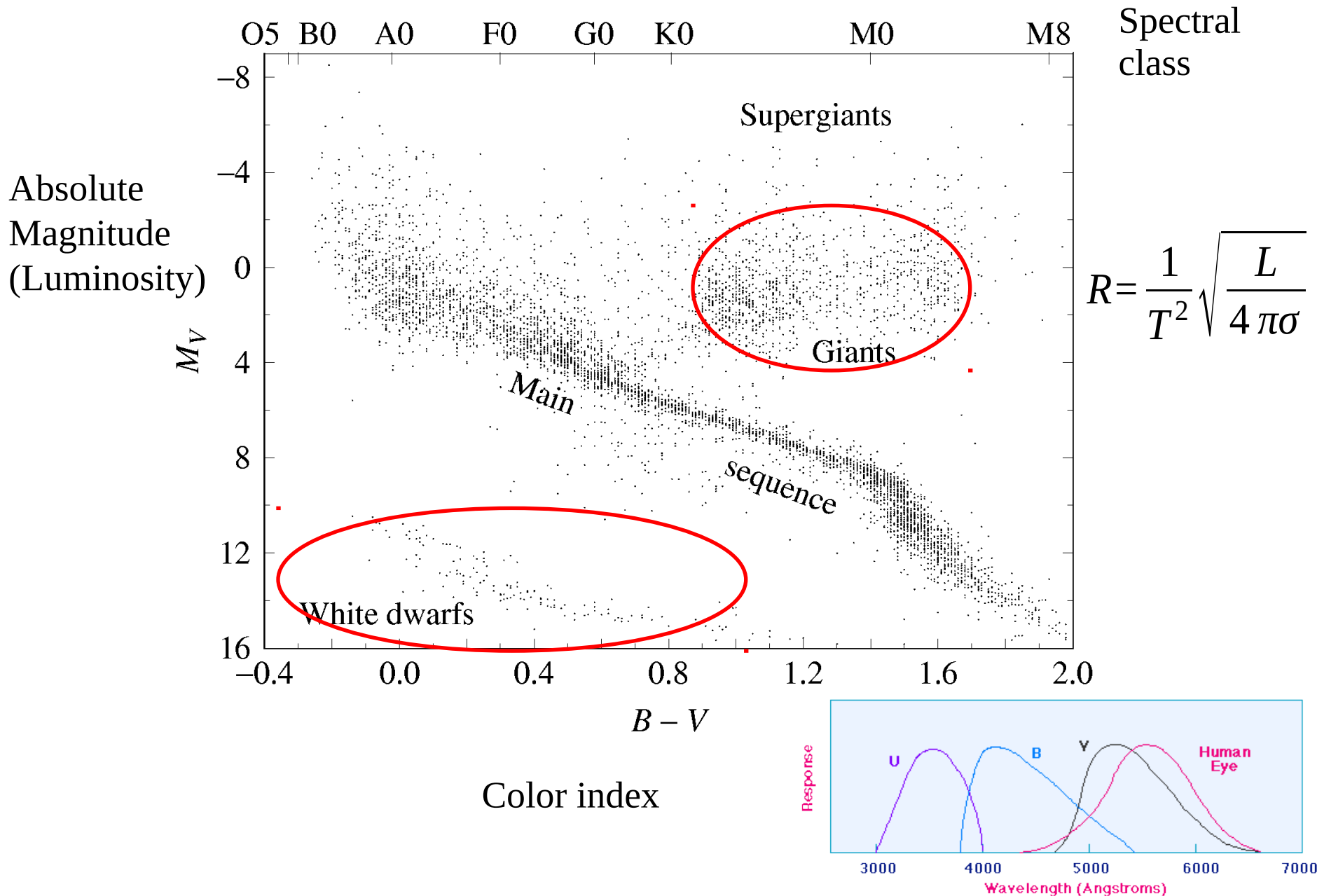$Z(6000\text{K}) = 2.0025 \qquad Z(10000\text{K}) = 95.4311 \qquad Z(15000\text{K}) = 1.7998 \times 10^4$
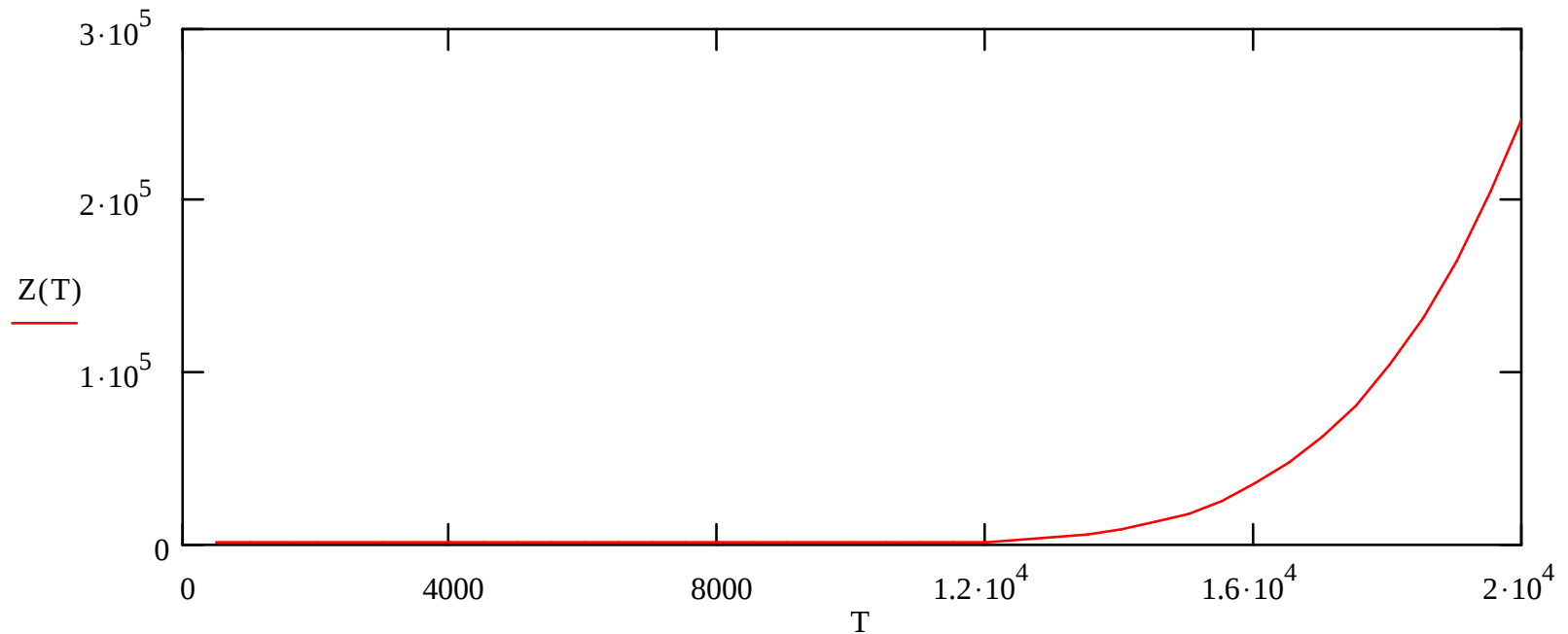


- Ionization
- Unphysical orbital size    $r_n = a_o \, n^2$

# Example 8.4

- Surface of the Sun has 500,000 hydrogen atoms per calcium atom, but calcium absorption lines are much stronger than the Balmer series lines.

- The Boltzmann and Saha equations reveal that there are 400× more Ca atoms in the ground electronic state than in the $n=2$ hydrogen state.

- Calcium is not more abundant

- Differences are due to sensitive temperature dependence

# Hertzsprung-Russell (H-R) Diagram



Spectral class

Absolute Magnitude (Luminosity)

$M_V$

O5  B0  A0  F0  G0  K0  M0  M8

−8
−4
0
4
8
12
16

−0.4  0.0  0.4  0.8  1.2  1.6  2.0

$B - V$

Supergiants

Giants

Main sequence

White dwarfs

$$R = \frac{1}{T^2} \sqrt{\frac{L}{4\pi\sigma}}$$

Color index

Response

U  B  V  Human Eye

3000  4000  5000  6000  7000

Wavelength (Angstroms)
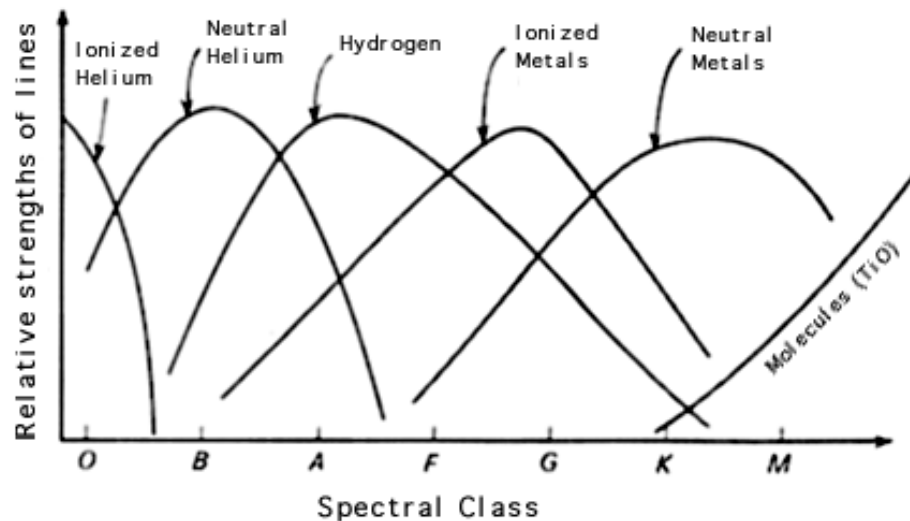
**A colorful H–R Diagram**
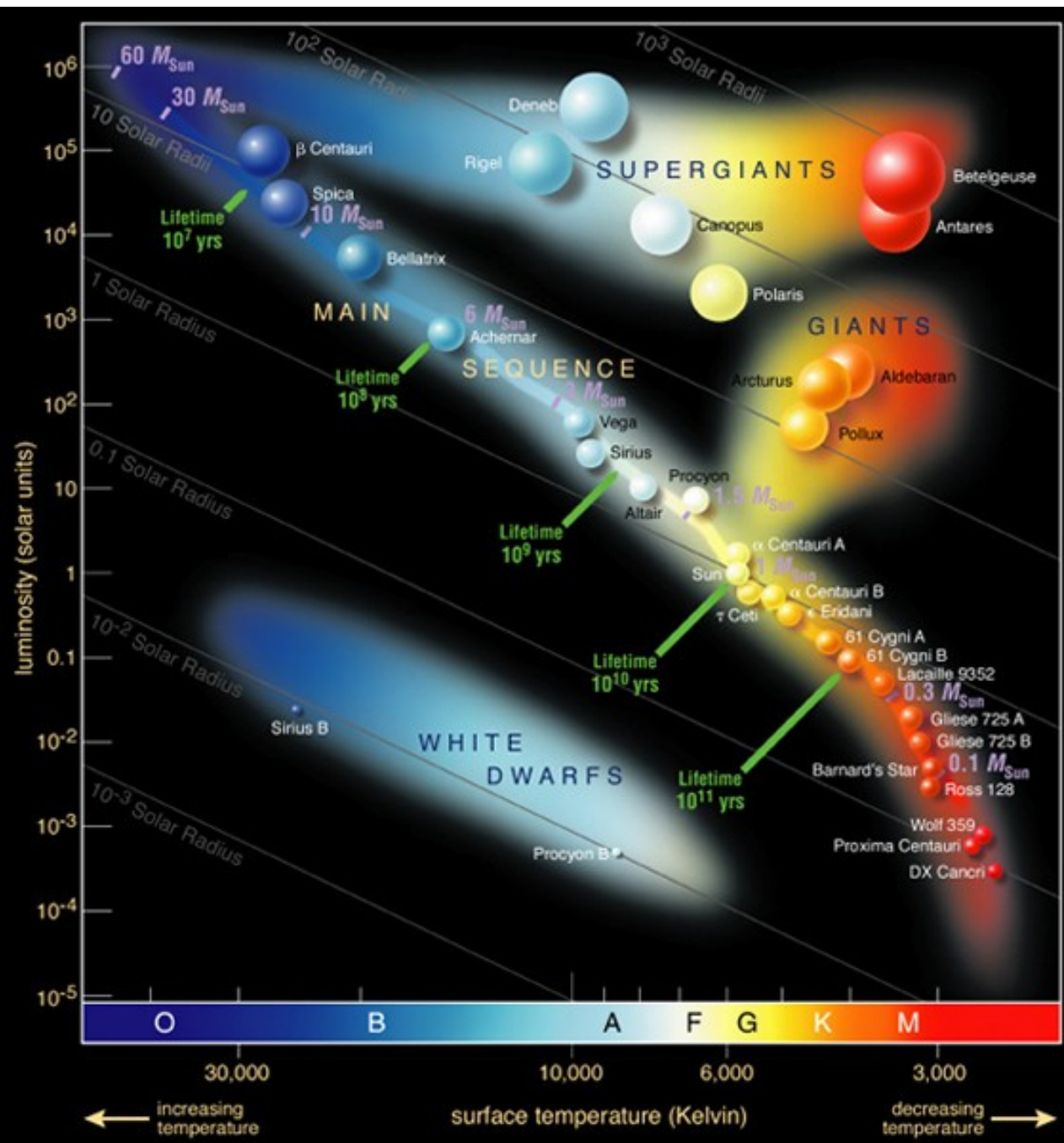
Betelgeuse

Antares

Sun (1 pixel)

Jupiter is invisible at this scale

Sirius  Pollux

Arcturus

Rigel

Aldebaran

Jupiter

Pluto
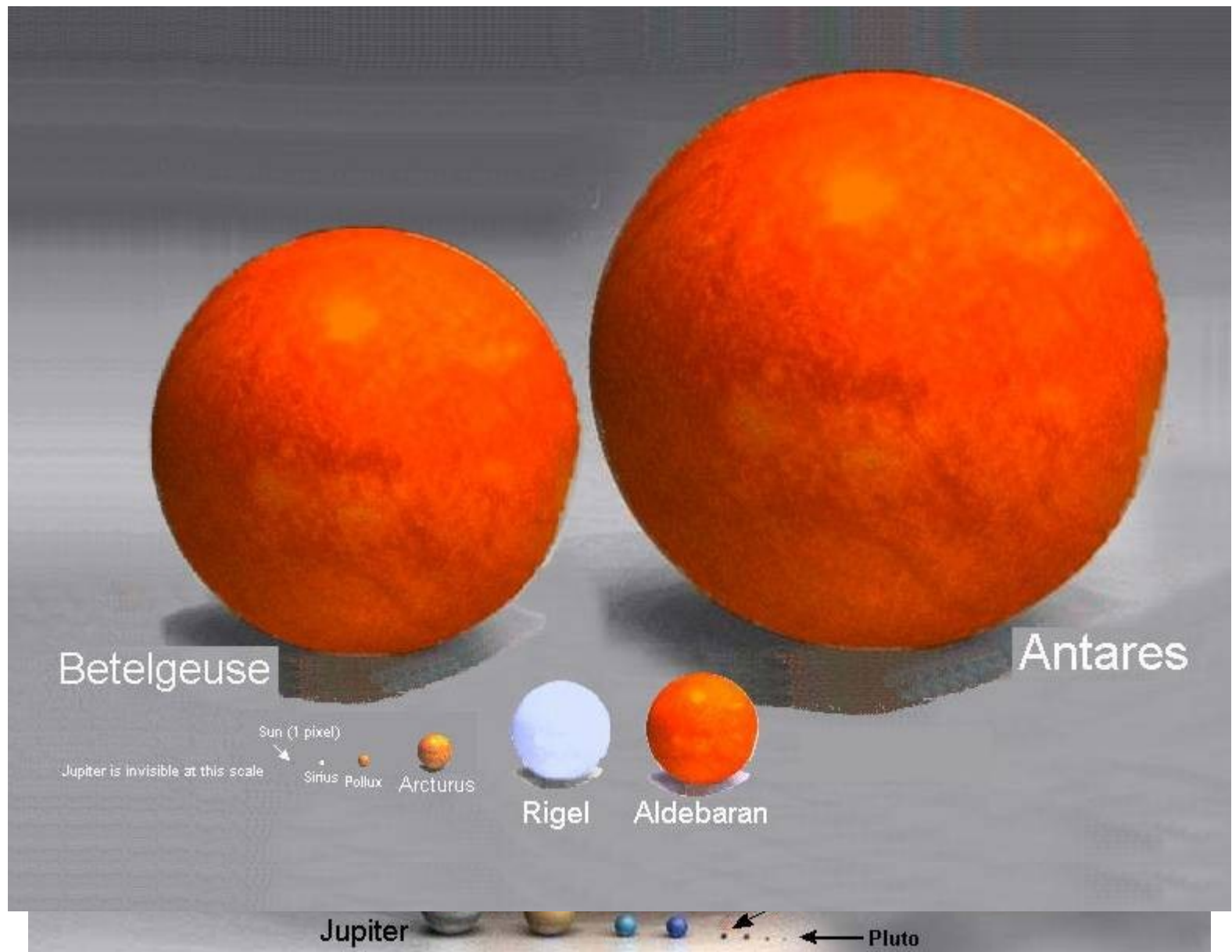
# How temperature relates to color index

……………………
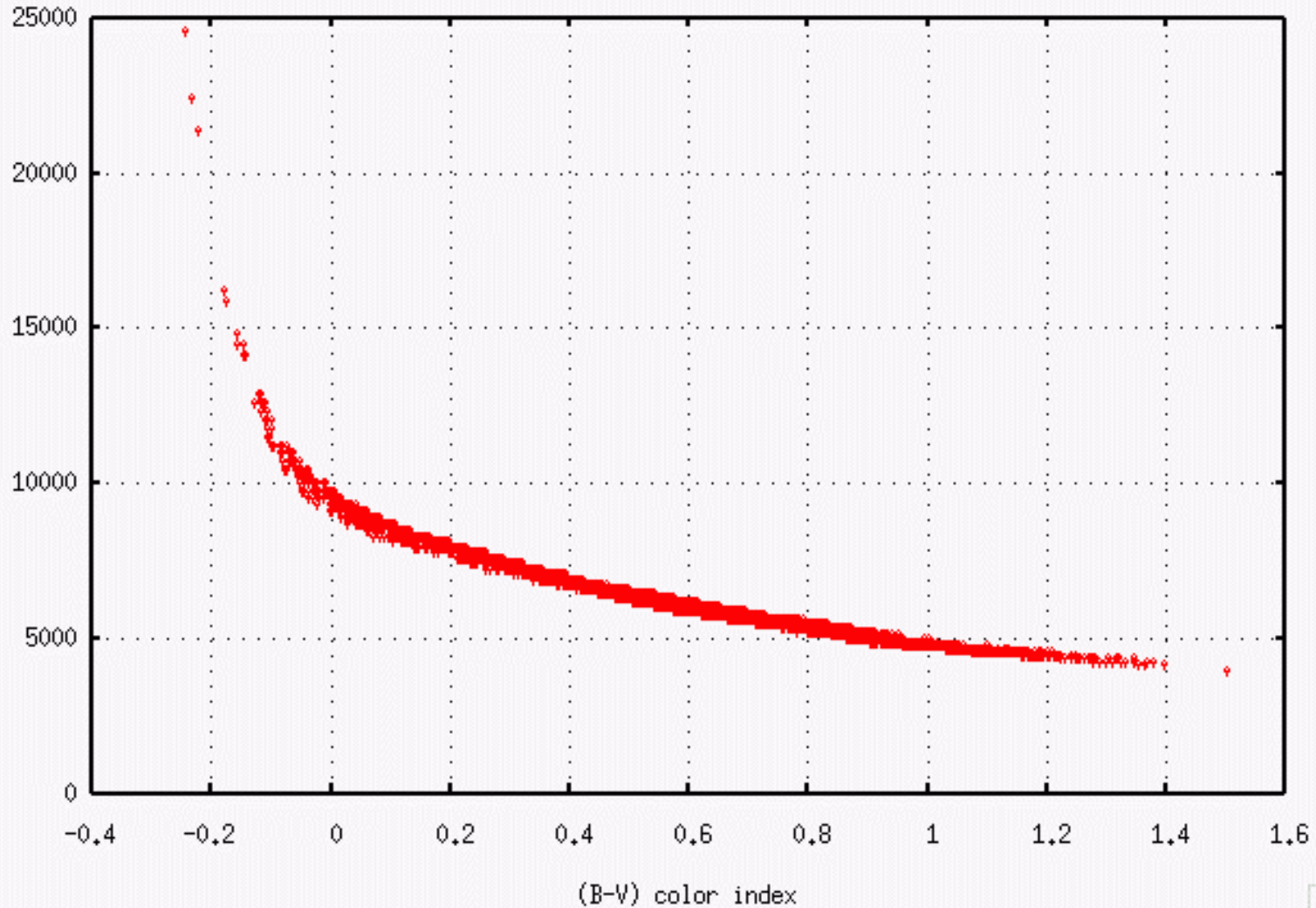
# Hertzsprung-Russell (H-R) Diagram

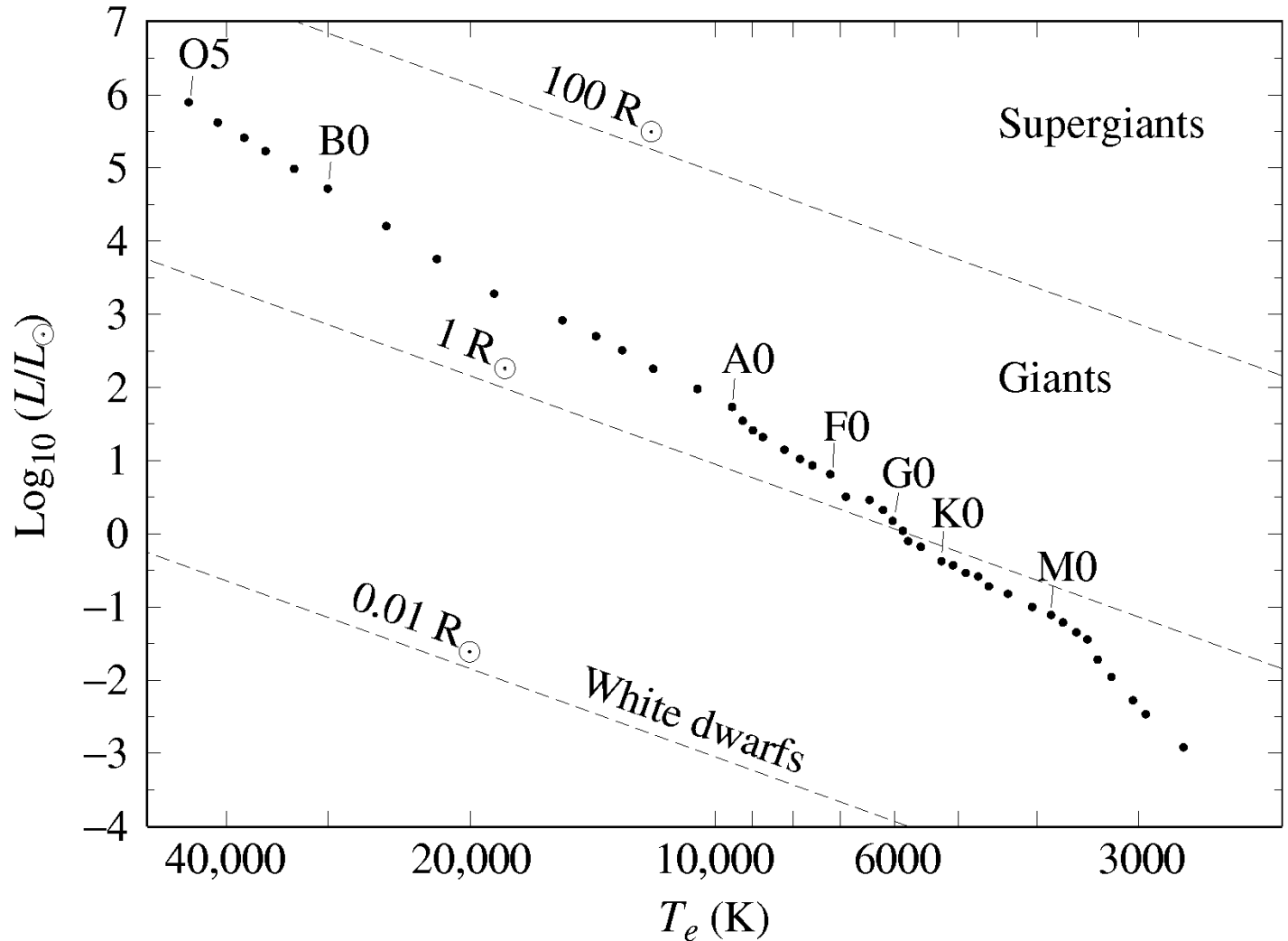- Luminosity and Temperature rather than Magnitude and Color Index
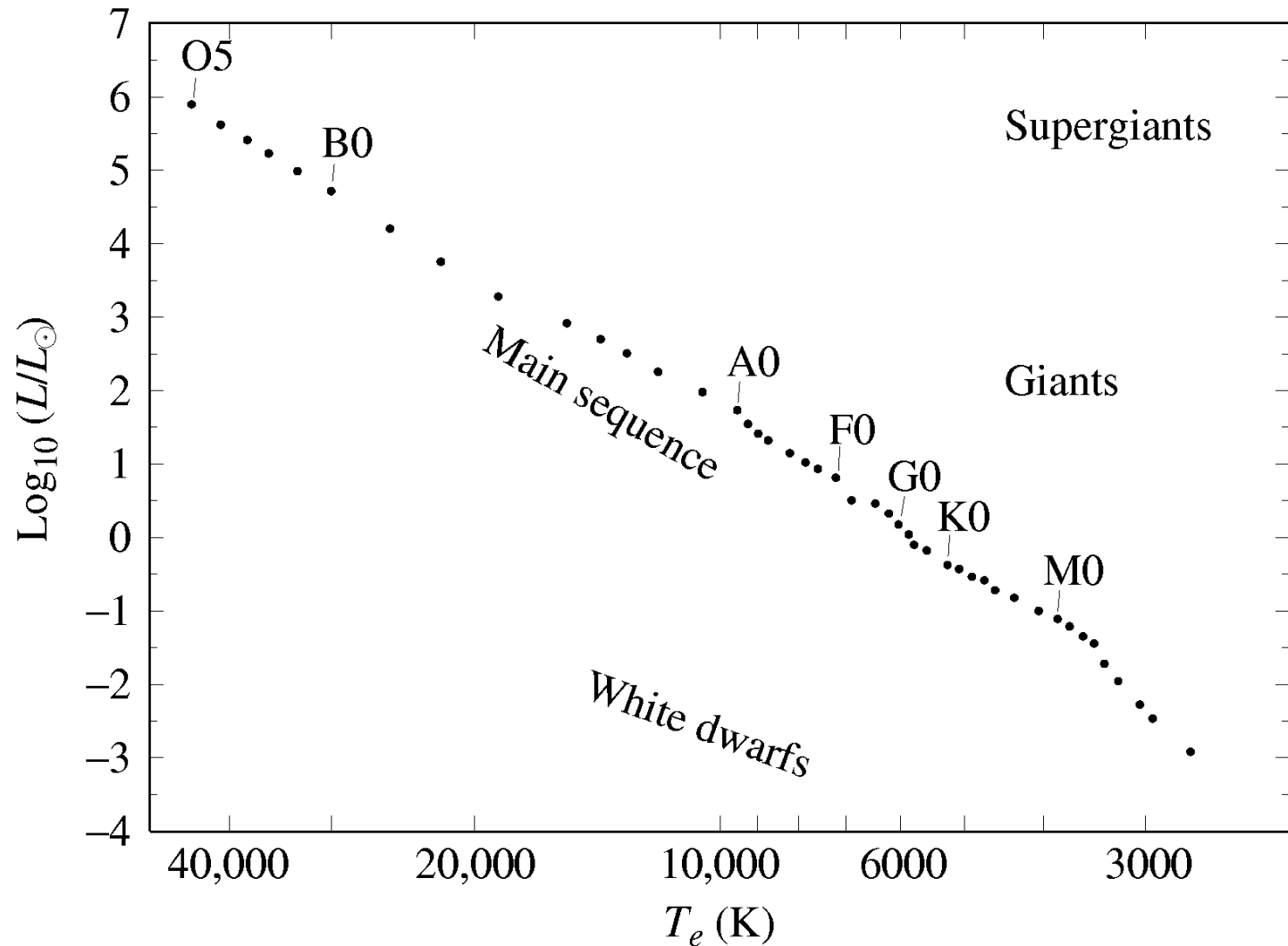


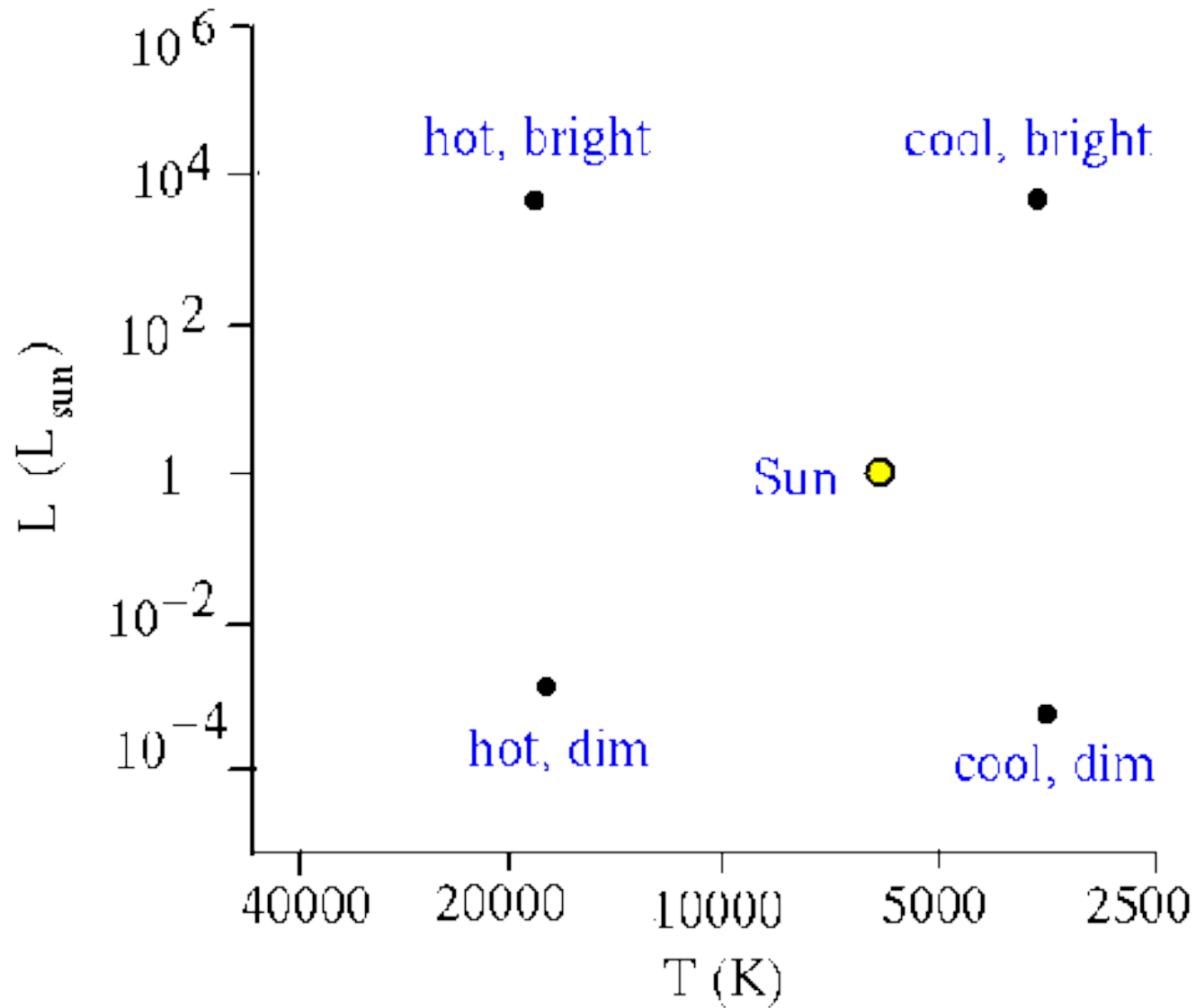The "theorists H-R Diagram". Nuclear processes vary along MS.
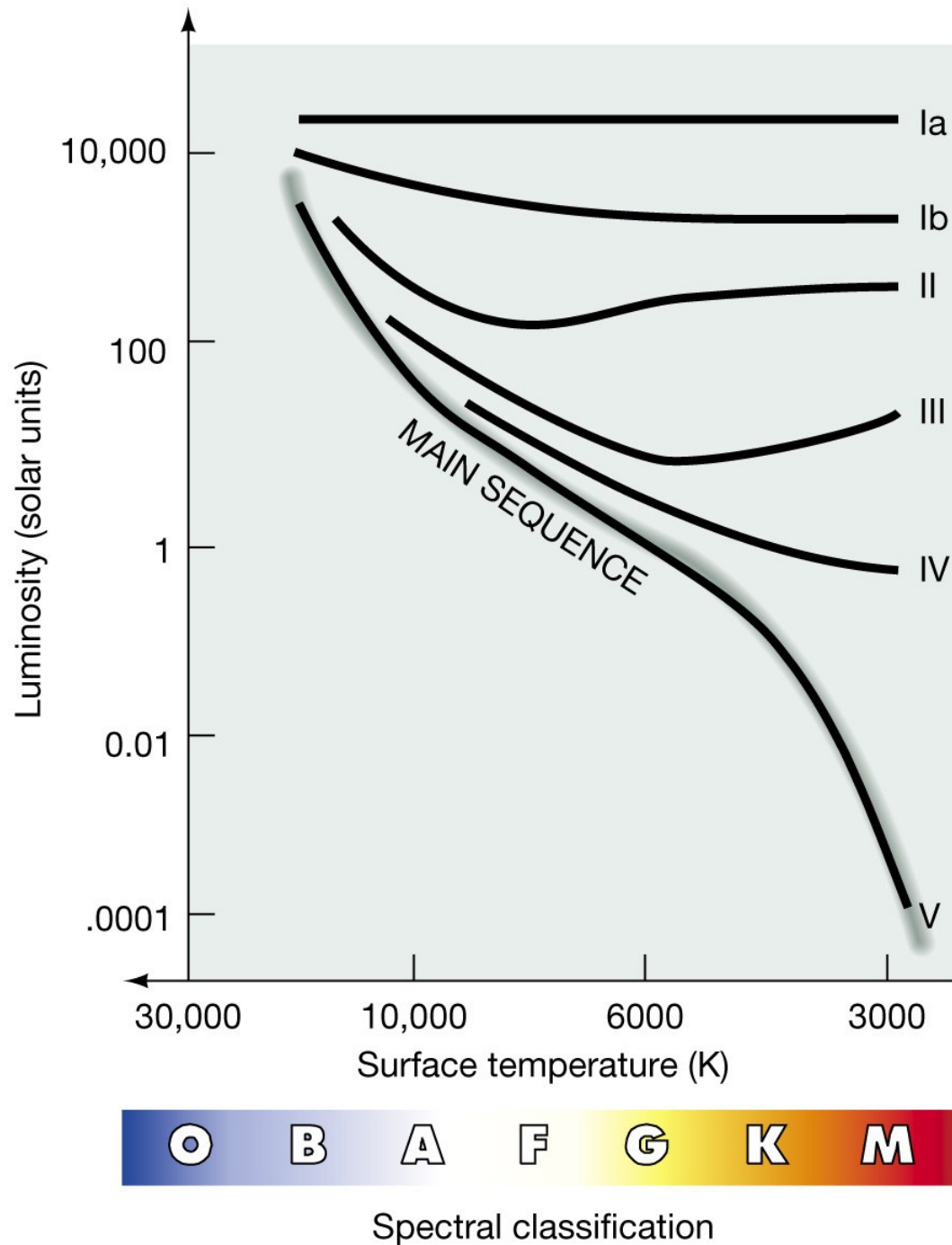
# Hertzsprung-Russell (H-R) Diagram

- Star Radius

$$R = \frac{1}{T^2}\sqrt{\frac{L}{4\pi\sigma}}$$

# Hertzsprung-Russell (H-R) Diagram

**Luminosity Classes**

Spectral classification
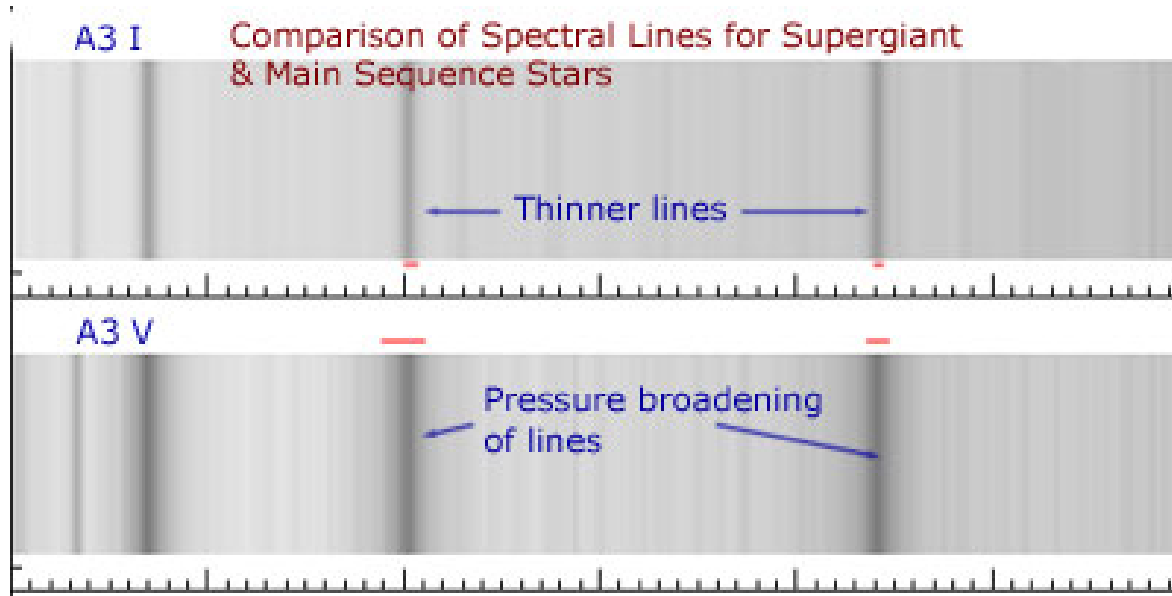
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# Stellar Luminosity Classes

| Class | Description |
| --- | --- |
| Ia | Bright supergiants |
| Ib | Supergiants |
| II | Bright giants |
| III | Giants |
| IV | Subgiants |
| V | Main-sequence stars and dwarfs |

**TABLE 17.3   Stellar Luminosity Classes**

Some define VI and wd (or D)

# Luminosity classes can be discerned by line widths.



Comparison of Spectral Lines for Supergiant & Main Sequence Stars

A3 I

Thinner lines

A3 V
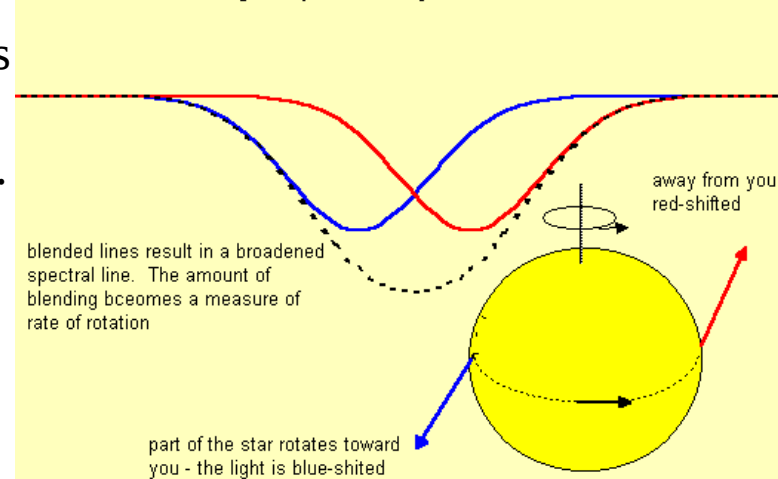
Pressure broadening of lines

Courtesy ANU.



**Pressure Broadening** is greater for the smaller, denser star. The giant star has a more tenuous atmosphere amd will produce narrower spectral lines
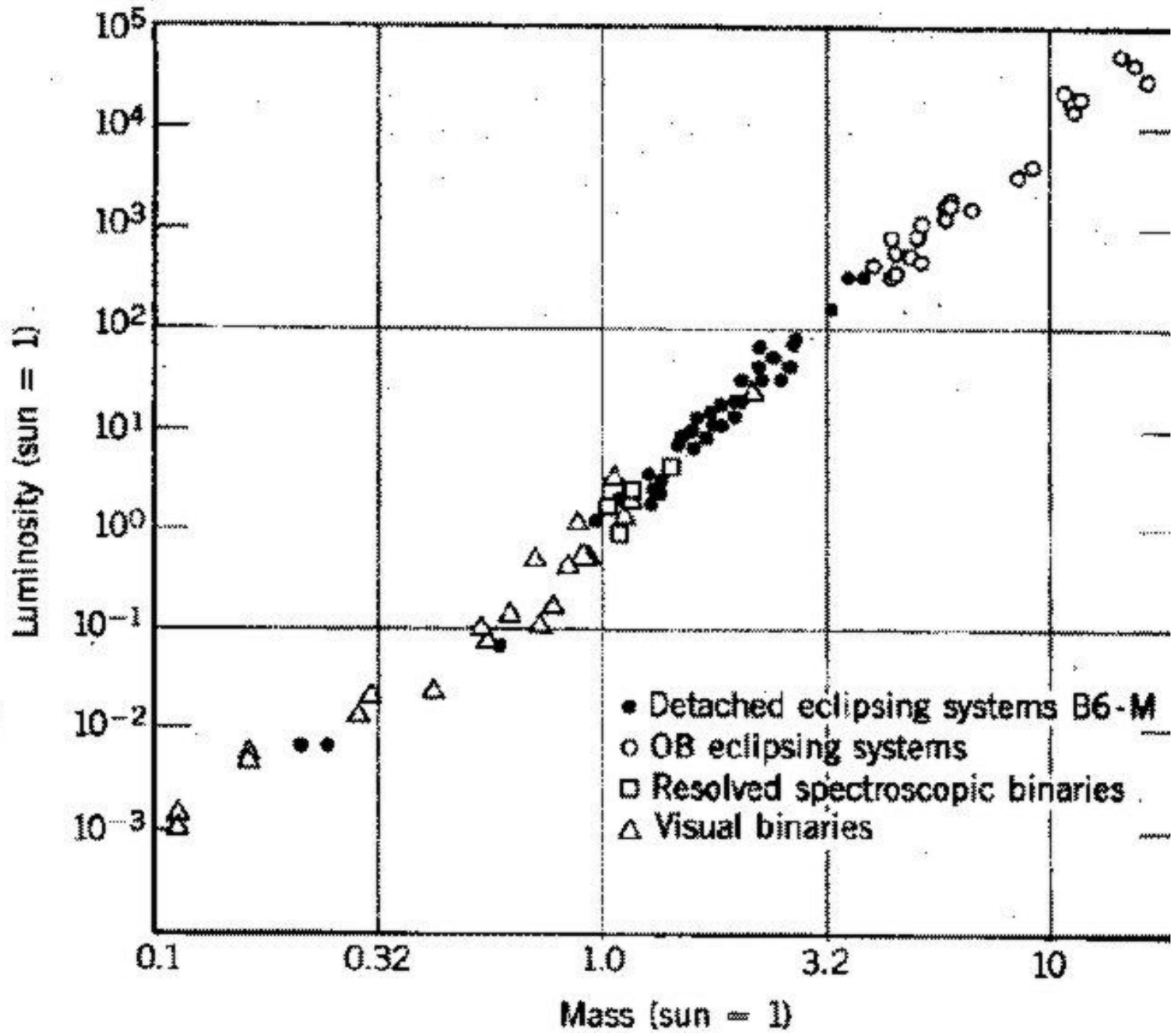
giant star - more tenuous atmosphere

dwarf star: denser atmosphere
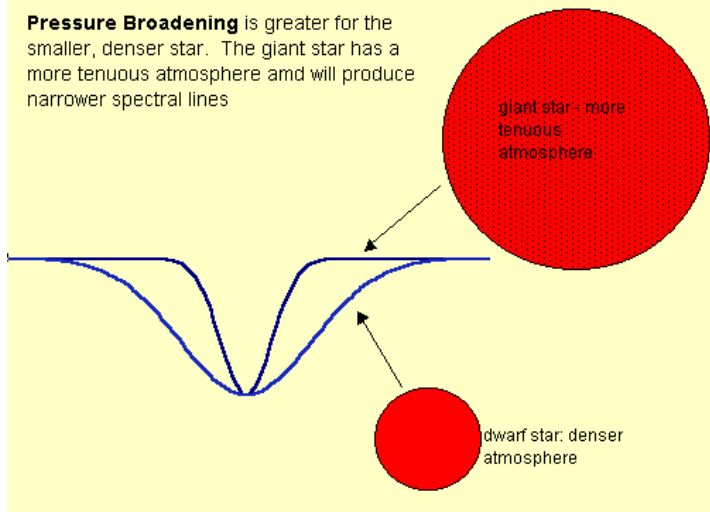
However,
Other processes
Can
Broaden lines...



**Rotational Broadening**: a way of measuring stellar rotation

away from you red-shifted

blended lines result in a broadened spectral line. The amount of blending bceomes a measure of rate of rotation

part of the star rotates toward you - the light is blue-shited
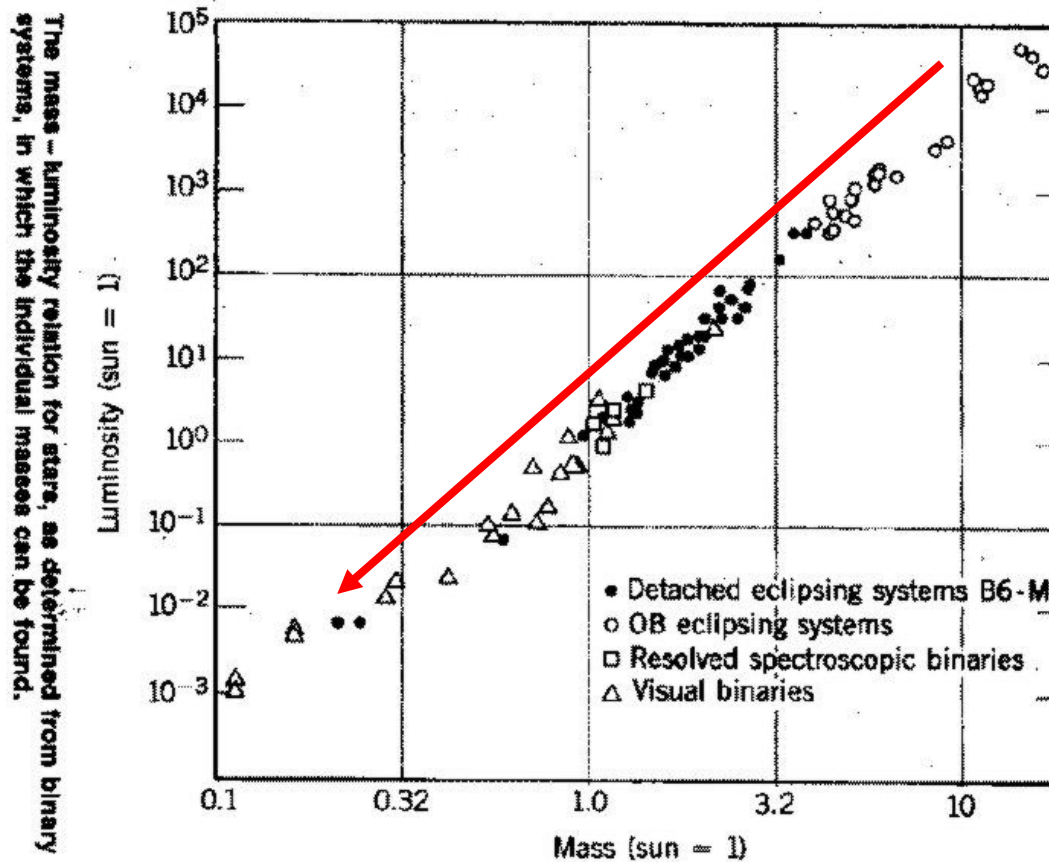
# Mass-Luminosity Relation from Binary Systems



The mass – luminosity relation for stars, as determined from binary systems, in which the individual masses can be found.

Luminosity (sun = 1)

- • Detached eclipsing systems B6-M
- ○ OB eclipsing systems
- □ Resolved spectroscopic binaries
- △ Visual binaries

Mass (sun = 1)
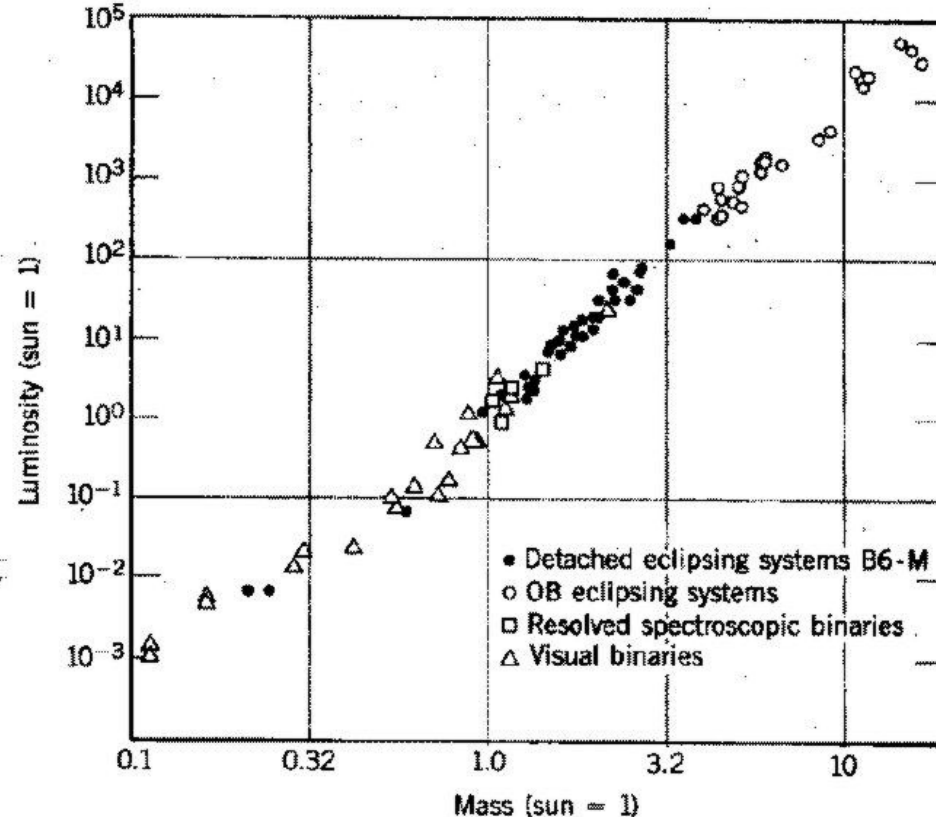
# Mass-Luminosity Relation

- Early theories had "early" O-type (bright, hot, massive) stars evolving to "old" M-type stars (dim, cool, less massive)
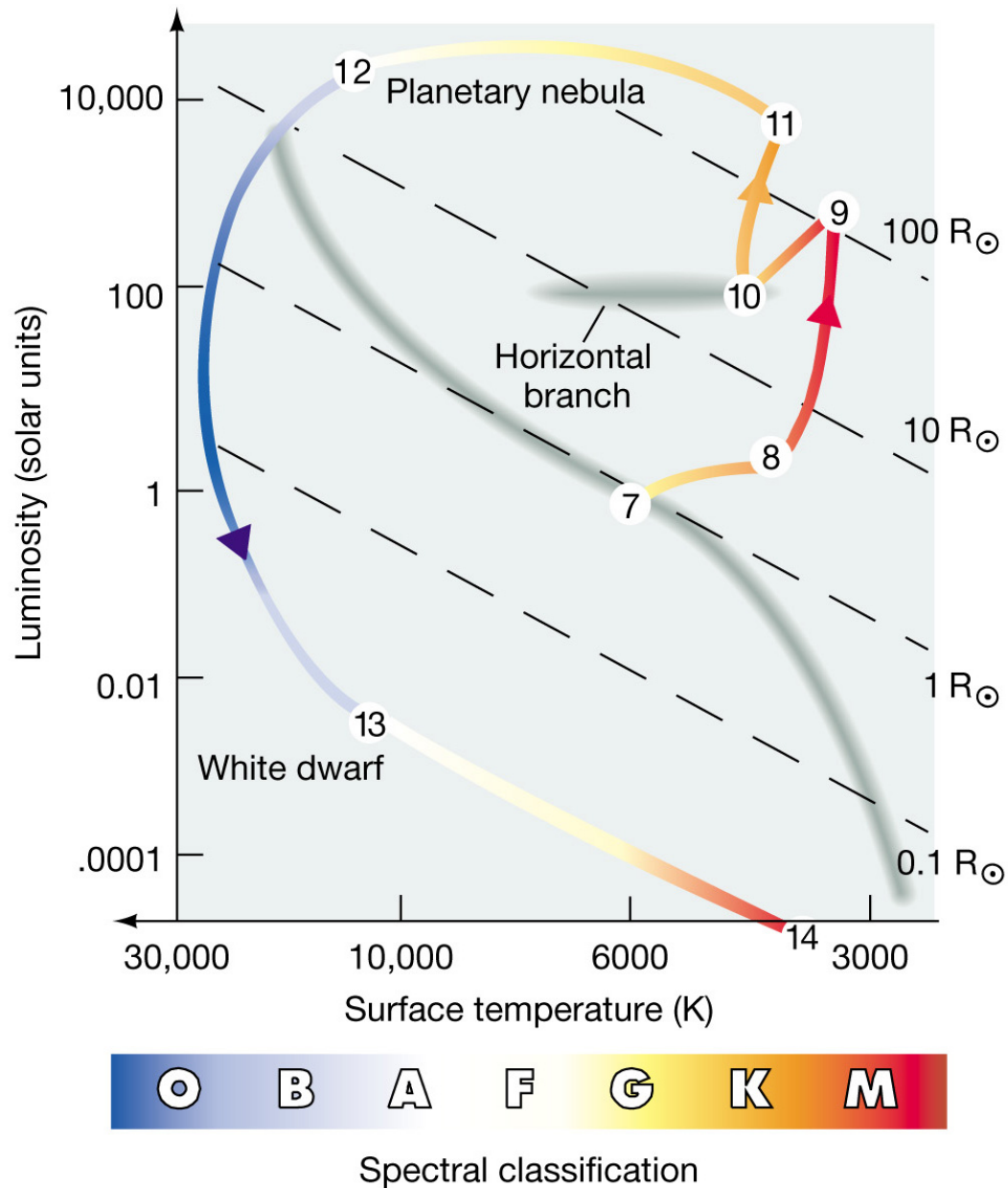
# Mass-Luminosity Relation

- Luminosity (power output) comes from nuclear fusion at the core of the stars.

- L increases dramatically with M:  $L \sim M^{3.5}$ (from M-L relation)

- From this, we can derive a lifetime for stars on the Main Seq.:
  - Lifetime = Fuel/(Rate of burning fuel)
  - Lifetime = M/L
  - Lifetime = $M/M^{3.5} = M^{-2.5}$

- Actually, $L \sim M^4$ for M>1, so

- Lifetime $\sim M^{-3}$
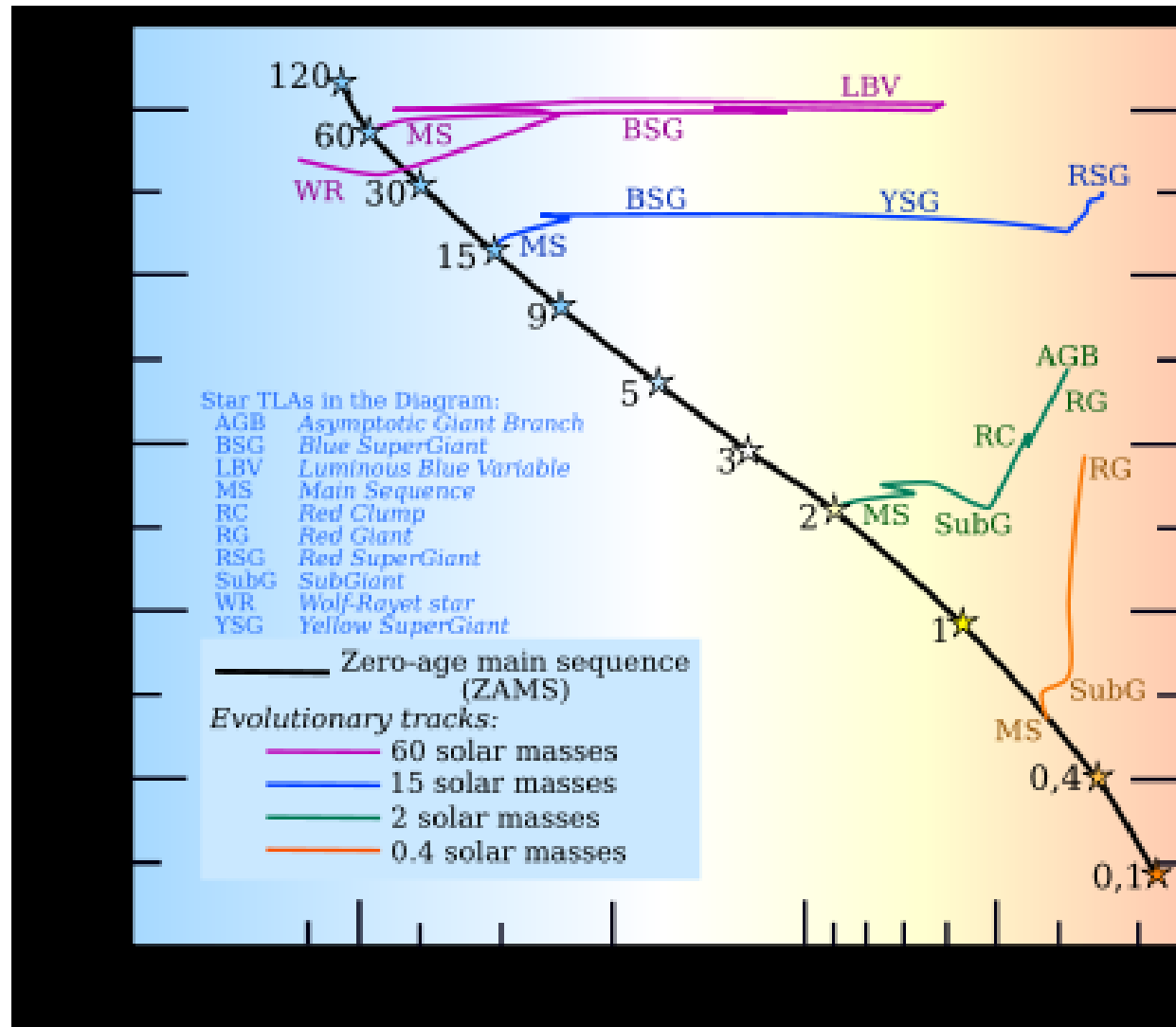


The mass – luminosity relation for stars, as determined from binary systems, in which the individual masses can be found.
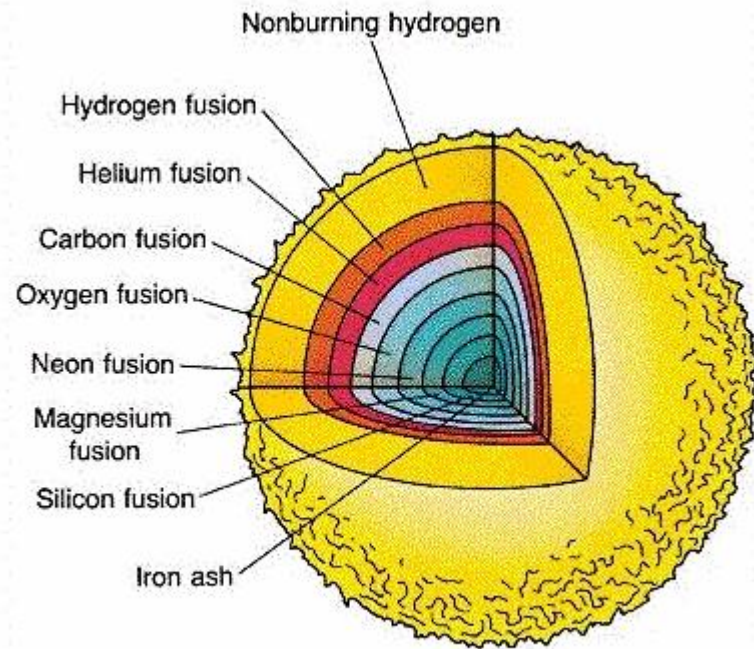
Luminosity (sun = 1)

$10^5$
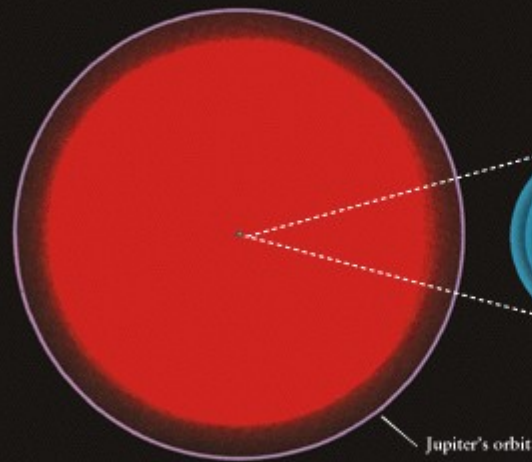$10^4$
$10^3$
$10^2$
$10^1$
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$

- Detached eclipsing systems B6-M
- OB eclipsing systems
- Resolved spectroscopic binaries
- Visual binaries

0.1    0.32    1.0    3.2    10

Mass (sun = 1)

# Evolution of 1 M$_\odot$ star.

# Evolutionary tracks.

# Massive star fusion



Nonburning hydrogen
Hydrogen fusion
Helium fusion
Carbon fusion
Oxygen fusion
Neon fusion
Magnesium fusion
Silicon fusion
Iron ash



1500 million km

Hydrogen-burning shell
Helium-burning shell
Carbon-burning shell
Neon-burning shell
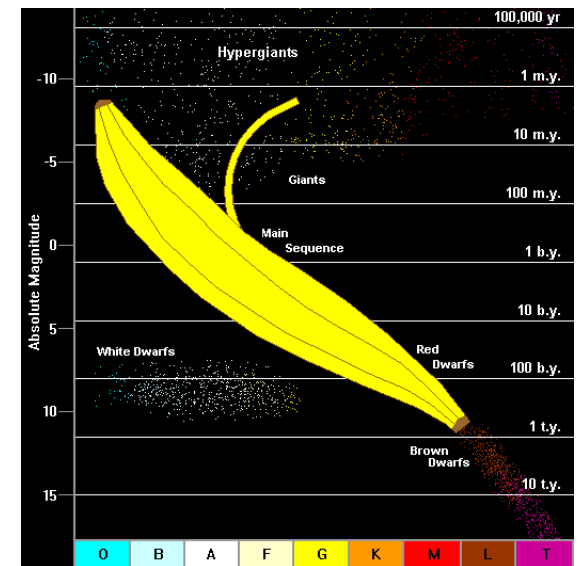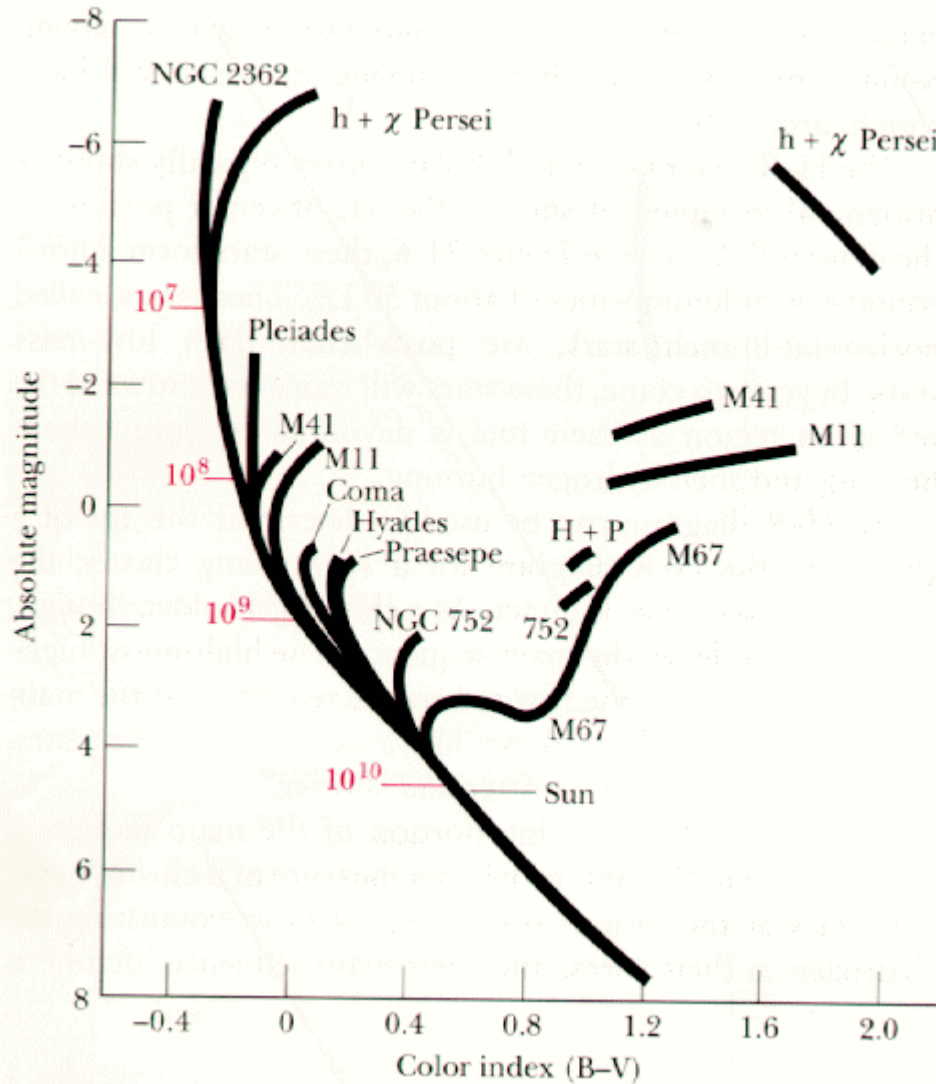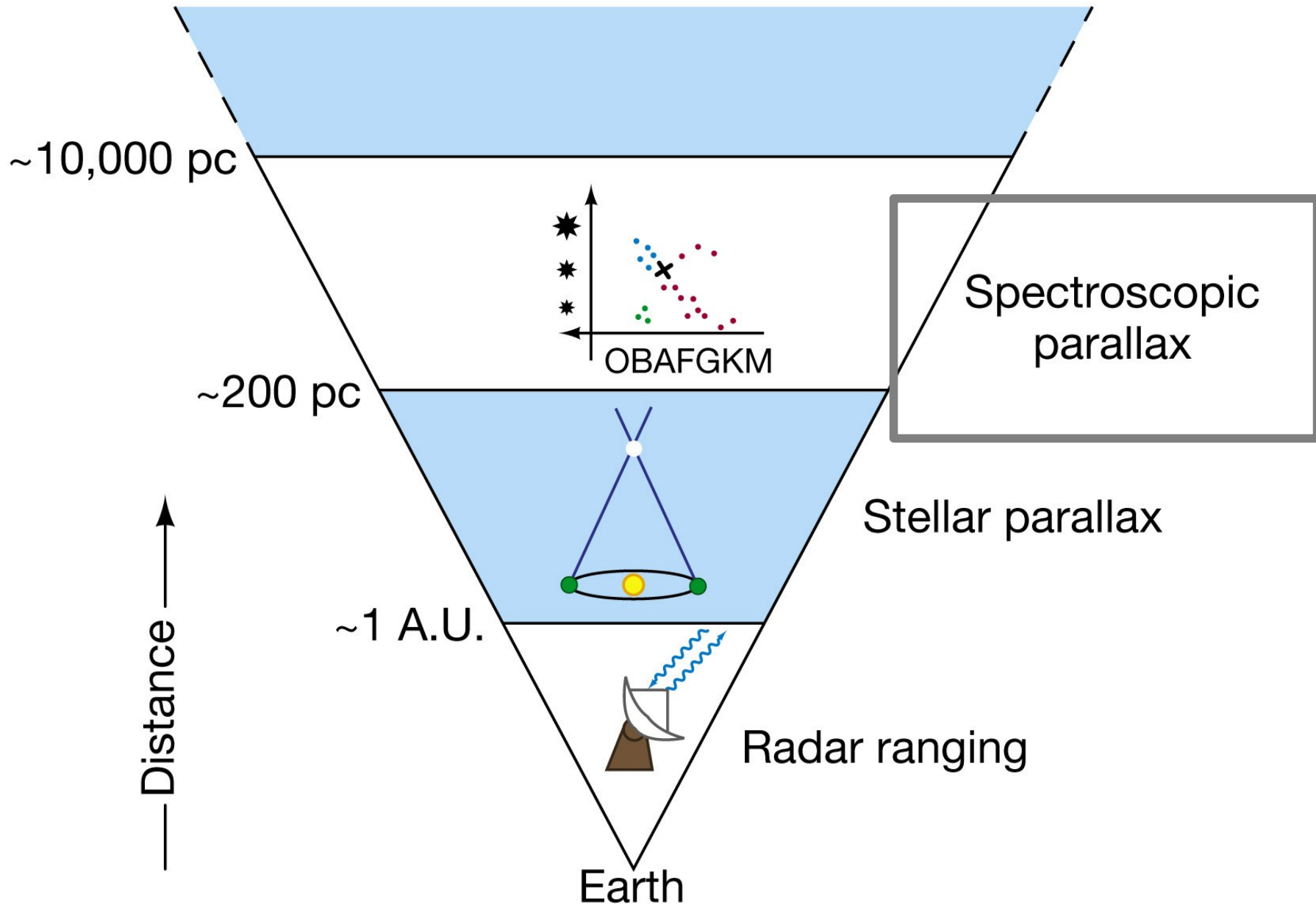Oxygen-burning shell
Silicon-burning shell
Iron core

Jupiter's orbit

# HR Diagrams of star clusters.

# Figure 17-17
## Stellar Distances



~10,000 pc

Spectroscopic parallax

OBAFGKM

~200 pc

Stellar parallax

~1 A.U.

Distance

Radar ranging

Earth

# Spectroscopic "Parallax"

- Method to determine a stars distance
  - Determine the spectral class and luminosity class.
  - Measure apparent magnitude.
    - Correct for crowding
    - Correct for extinction
  - Read the absolute magnitude from the H-R diagram
  - Compare to apparent magnitude to determine distance: $d = 10^{(m-M-A+5)/5}$

# Stellar and Spectroscopic Parallax

**Stellar Parallax** works out to 200pc (ground), 1000 pc (Hipparcos)

**Spectroscopic Parallax** works for stars for which a good spectrum can be observed (about 8 kpc), but ...

• **Not precise for individual stars, especially giants**

• **Entire clusters of stars works better! ("main-sequence fitting")**
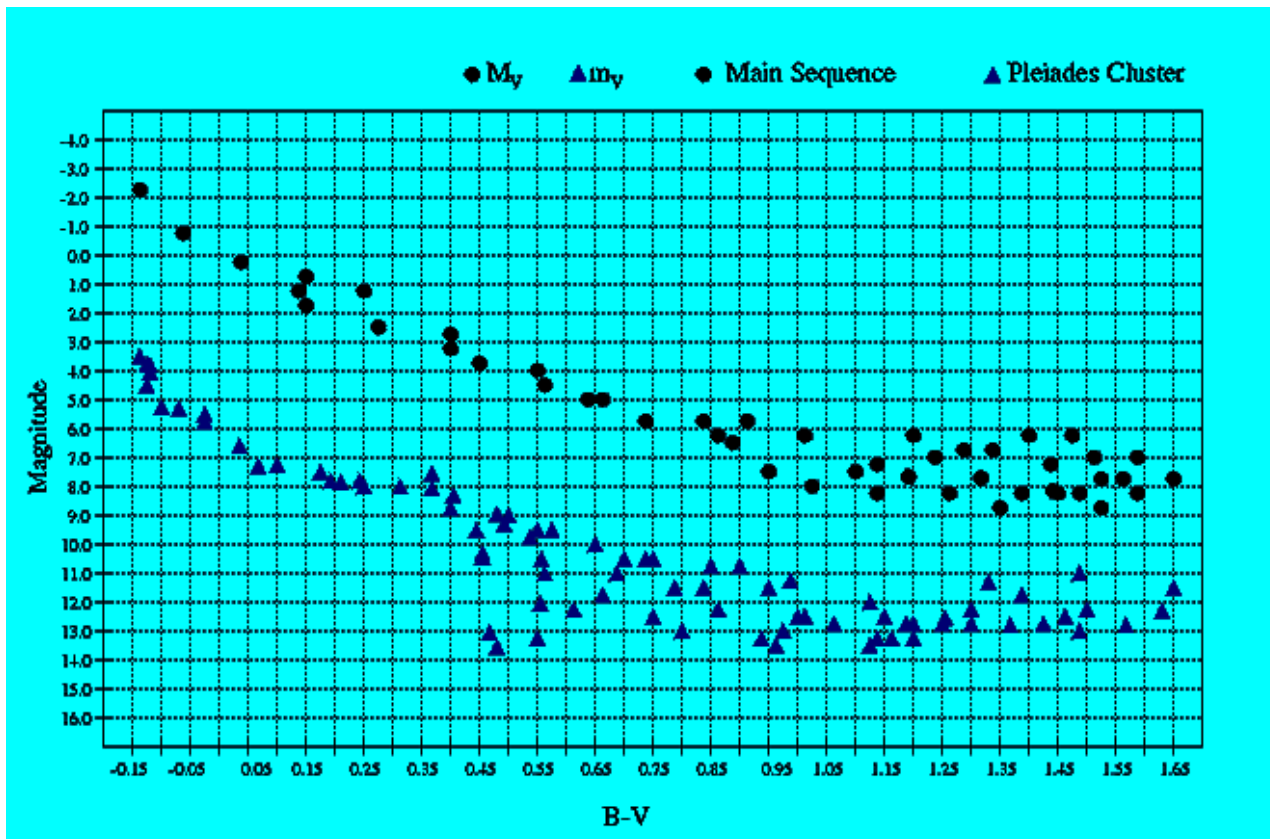


$$m-M=5\log(d/10)$$

Spec Parallax assumes, for example, that all A0V stars have the same M.
That makes A0V stars "standard candles".