Physics 2311 – Physics I, Week 4
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Outline for Day W3,D3

1D kinematics
  Equations of uniform acceleration, including free-fall
  Examples
2D kinematics - Vectors

Homework
Ch. 2 Prob. 2,3,5-7,14,23-27,35-38,53-56 for 2-day
Ch. 3 P. 1,3,6,7,10,11,19,20,23,24,
 32,33,37,38,39 Do by next Mon.

Notes: Lab this week: Exp. 2 “Graphs and Tracks”
  Under NEW STUFF:
   “Week 3-5” practice quiz.

Motion in 1D.

Equations for Uniform Acceleration

A) [Text: 2-12a] $\vec{v}_f = \vec{v}_i + \vec{a}\,t$

B) [Text: 2-12d] $\vec{v}_{avg} = \dfrac{\vec{v}_i + \vec{v}_f}{2}$

C) [Text: 2-8] $\vec{x}_f = \vec{x}_i + \dfrac{\vec{v}_i + \vec{v}_f}{2}\,t$

D) [Text: 2-12b] $\vec{x}_f = \vec{x}_0 + \vec{v}_0\,t + \dfrac{1}{2}\vec{a}\,t^2$

E) [Text: 2-12c] $v_f^2 - v_i^2 = 2a\left(x_f - x_i\right)$

Motion in 1D.

Examples using Equations for Uniform Acceleration

1) A car passes x=10m at t=0 going 10 m/s with a constant accelerating of 4 m/s^2 . Where will the car be in 5 seconds?

$$\vec{x}_f = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

2) A car accelerates uniformly, starting at v_i=5 m/s at x_i=20, and reaching x_f=100 only 5 seconds later. How fast did it cross the x_f=100 m mark?

$$\vec{x}_f = \vec{x}_i + \frac{\vec{v}_i + \vec{v}_f}{2} t$$

Motion in 1D.

Examples using Equations for Uniform Acceleration

3) A rock thrown down a well at 10 m/s reaches the bottom at 40 m/s. What was the average velocity? (Uniform acceleration of 9.8 m/s^2 downward.)

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

4) Find the depth of the well in the previous problem, assuming the rock was thrown straight down.

$$v_f^2 - v_i^2 = 2a(x_f - x_i)$$

Motion in 1D.

Examples using Equations for Uniform Acceleration

5) A car on ice is sliding backwards to the left at 5 m/s while accelerating uniformly to the right at 3 m/s^2. What is its velocity after 7 seconds?

$$\vec{v}_f = \vec{v}_i + \vec{a}\,t$$

Motion in 1D.

Free Fall problems

Assumes downward acceleration, g, near the surface of a planet (usually Earth!)

The Equations for Uniform Acceleration apply!

Assumes no air resistance or other forces on the object.

Object can be moving downwards OR upwards during free fall!

Motion in 1D.

Free Fall problems

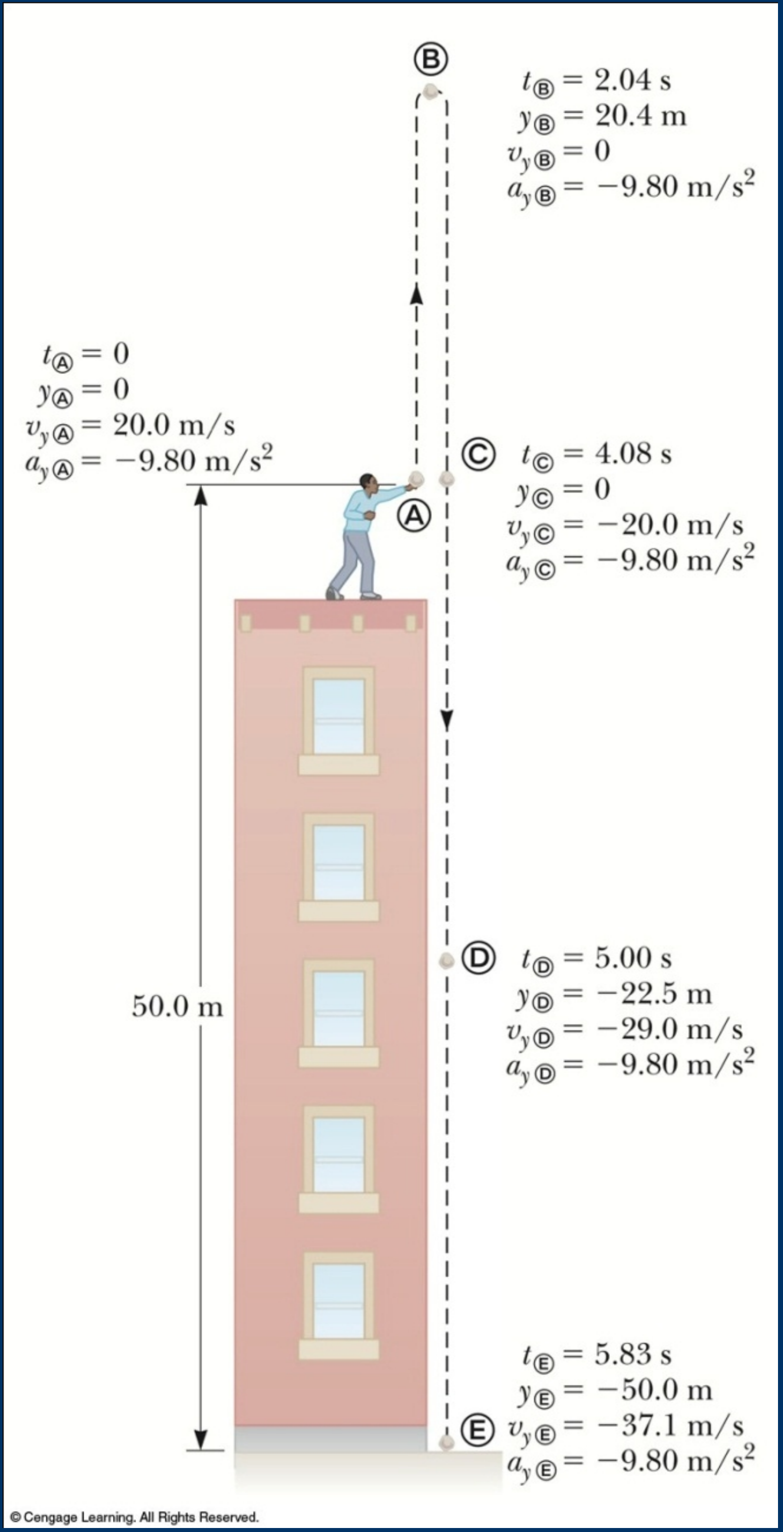
Initial velocity at A is upward (+) and acceleration is -*g* (-9.8 m/s²).

At B, the velocity is 0 and the acceleration is -*g* (-9.8 m/s²).

At C, the velocity has the same magnitude as at A, but is in the opposite direction.

The displacement is –50.0 m (it ends up 50.0 m below its starting point).

$$\vec{y}_f = \vec{y}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}\, t^2$$



Motion in 1D.

Free Fall problems

Example) Verify that the ball hits the ground at t_f=5.83 seconds if it is thrown from an initial height of y_i=0 upwards at v_i=20 m/s.

Also given: y_f=-50 m, a=-9.8 m/s^2

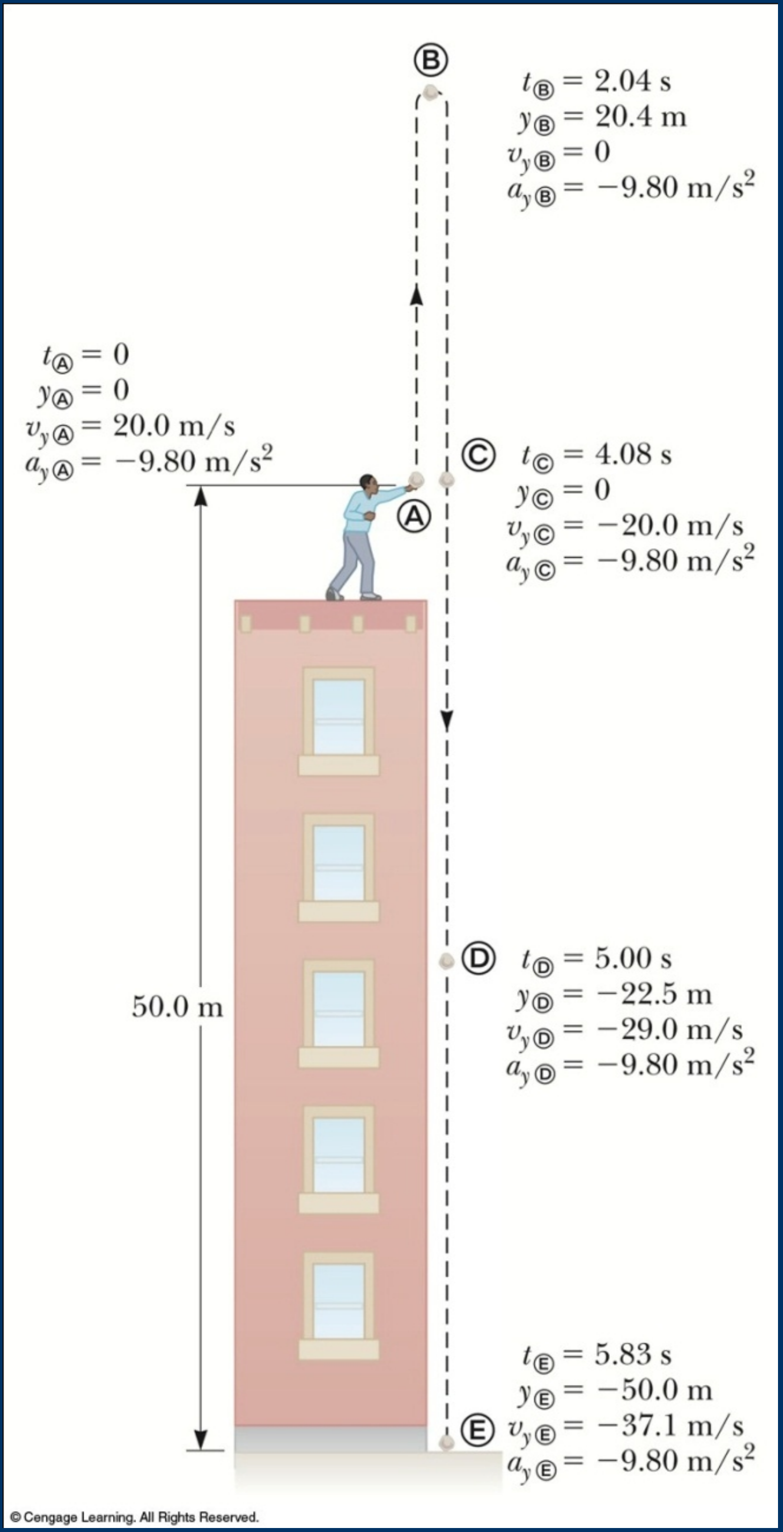
Use: $\vec{y}_f = \vec{y}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}\,t^2$

$-50 = 0 + 20t - 4.9t^2$

Quadratic eqn. $0 = -4.9t^2 + 20t + 50$

$0 = at^2 + bt + c$

Quadratic formula: $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



Motion in 2 Dimensions

Goals of Week 4:

- Learn how to work with vectors
- Rewrite 1D equations for 2D (and 3D) cases
- Acceleration in curvy motion – tangential and centripetal†
- Apply 2D equations to projectile motion.
- Calculate range, maximum height & t_{max} for trajectories
- Relative velocity

† or "radial"

Motion in 2D. Vectors

2D "Cartesian" coordinate systems provide 2 ways to specify the position of a point:

1) **Polar coordinates**
Points are labeled (r,θ)

2) **Cartesian coordinates**
Points are labeled (x,y)

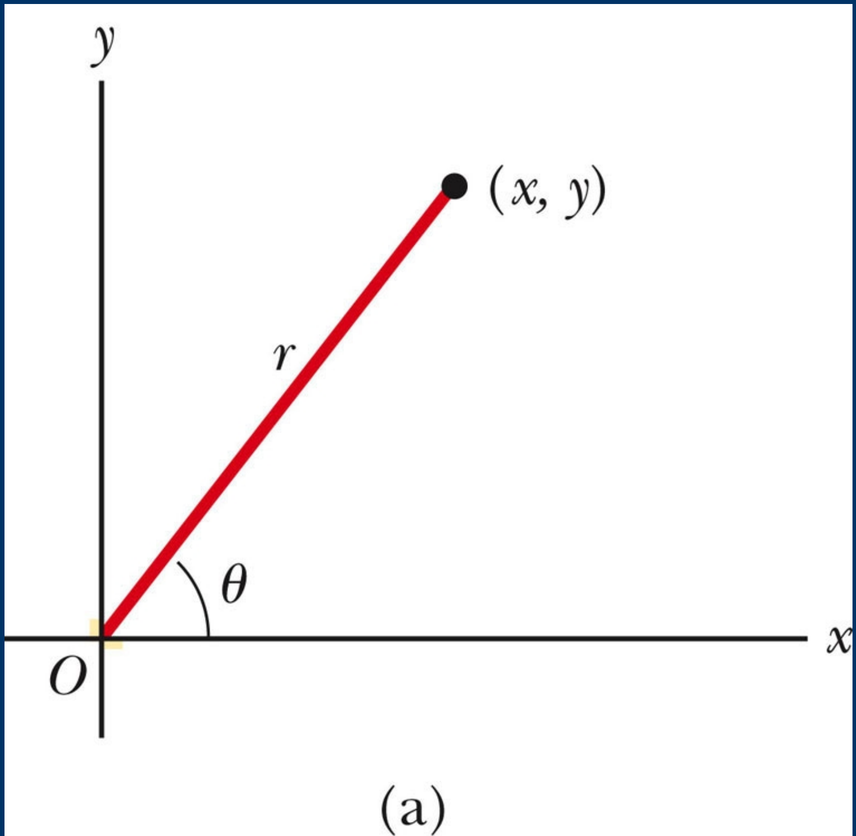
Conversions

Polar to cartesian:

$$x = r\cos\theta \,,\quad y = r\sin\theta$$

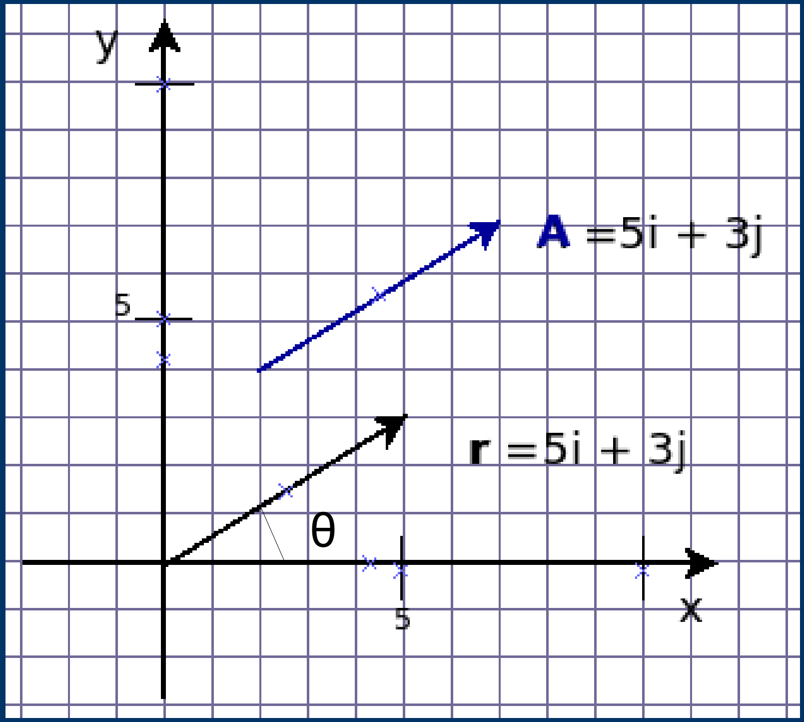
Cartesian to polar:

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\frac{y}{x}$$



Motion in 2D. Vectors

Similarly, vectors can be expressed in two main ways: *polar* and *vector component* form.



Example) Vector **r** is a *position vector* because it has its tail at the origin.

Vector **A** is not a position vector, but it's a vector.

However, these vectors are *equal* because they have the same length and direction. Thus,

$$\mathbf{A} = \mathbf{r} = 5\hat{\imath} + 3\hat{\jmath}$$

and this is the *vector component* form.

Convert **A** and **r** into *polar vectors*...

The length of both is

$$|\mathbf{A}|=|\mathbf{r}| = (5^2+3^2)^{1/2} = 5.83$$

The angle between the vector and the +x-axis is

$$\theta = \tan^{-1}(3/5) = 30.96°$$

Therefore, **A** is $(r,\theta) = (5.83, 30.96°)$

Motion in 2D. Vectors

Careful with finding the angle using the inverse tangent ($\tan^{-1}$) function on a calculator!

First, make sure calculator is set on degrees instead of radians.

Second, if the vector points towards quadrants 2 or 3, you must add 180° (or subtract 180° to keep the angle less than 360°) to the calculator's answer:

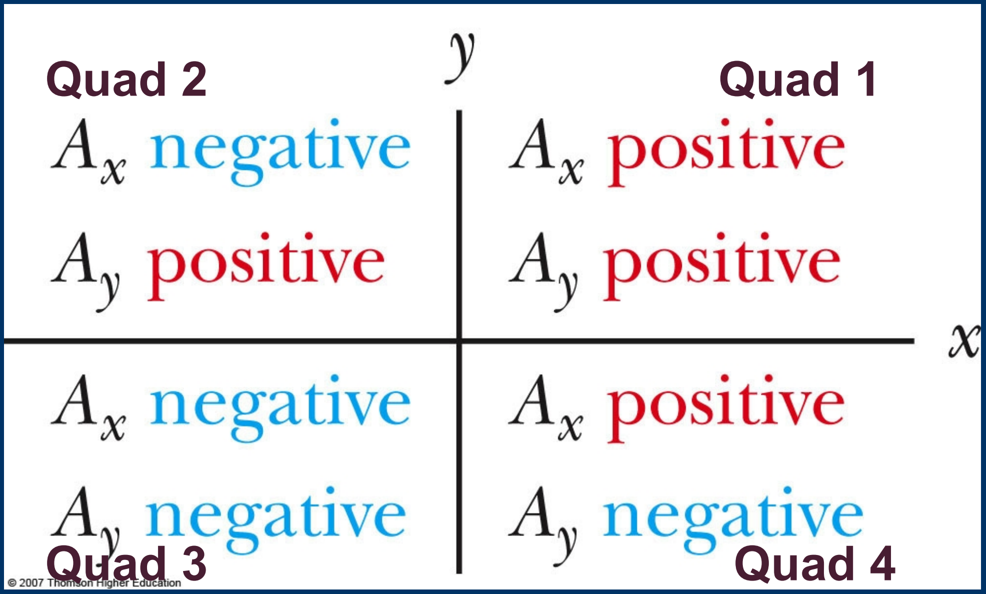
$$\theta = \tan^{-1}(y/x) + 180°$$

If the vector points towards quadrants 1 or 2,
It is ok to use:

$$\theta = \tan^{-1}(y/x)$$

ALWAYS define θ to be measured CCW from the +x axis to the position vector!
Then expect the position vector to have:

Quad 1 $0 < \theta < 90°$
Quad 2 $90 < \theta < 180°$
Quad 3 $180 < \theta < 270°$
Quad 4 $270 < \theta < 360°$



Motion in 2D. Vectors

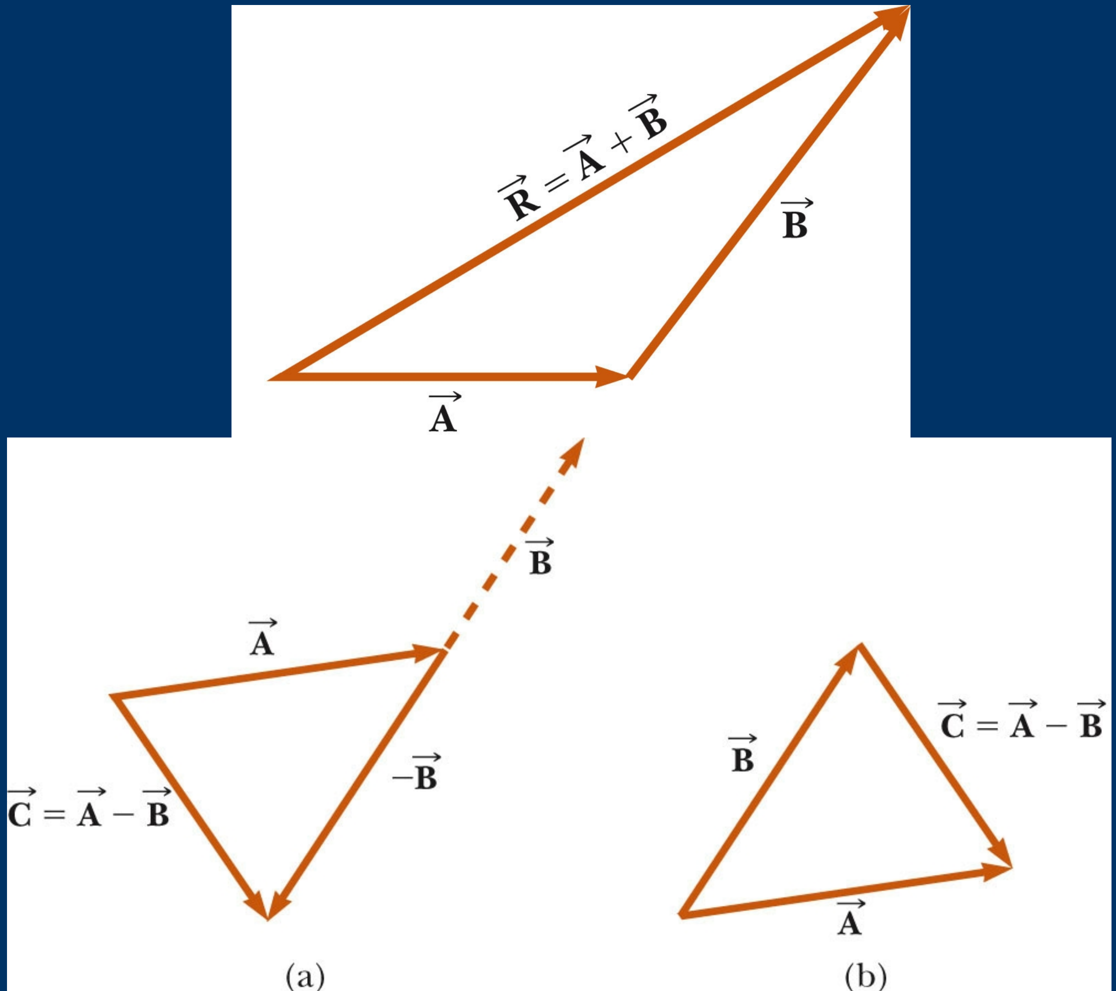
Vector addition and subtraction

Graphical approach

R = **A** + **B**

C = **A** – **B** = **A** + (-**B**)

Tip-to-tail method



Algebraic approach

A = A_x î + A_y ĵ

B = B_x î + B_y ĵ

A+B = (Ax+Bx)î + (Ay+By)ĵ

Motion in 2D.

Using vectors for <u>position</u>.

Top: position vector **A** in 2-D.
Vector components are:

$A_x = |\mathbf{A}|\cos\theta$

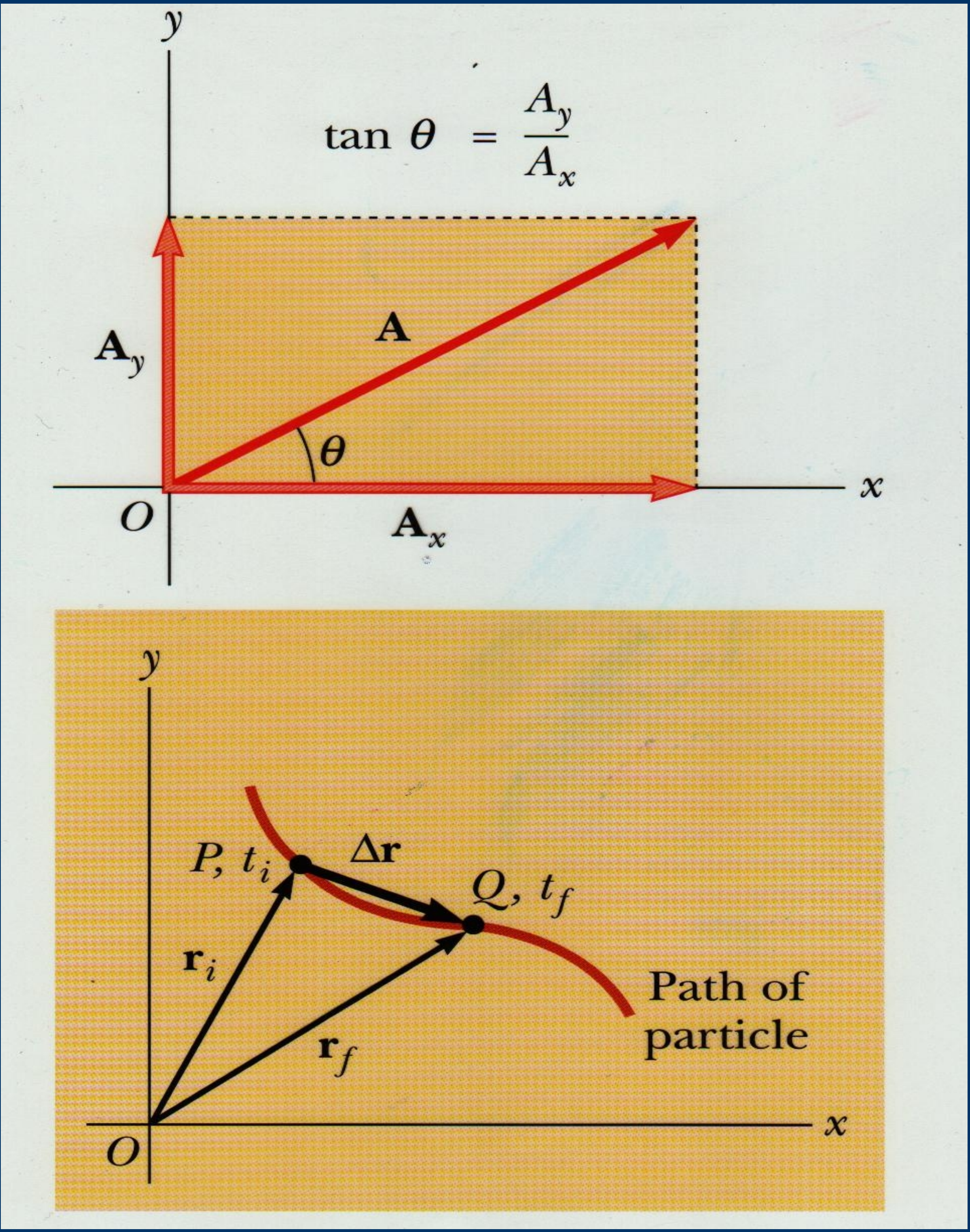
$A_y = |\mathbf{A}|\sin\theta$

so $\mathbf{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$

$|\mathbf{A}| = (A_x^{\,2} + A_y^{\,2})^{1/2}$

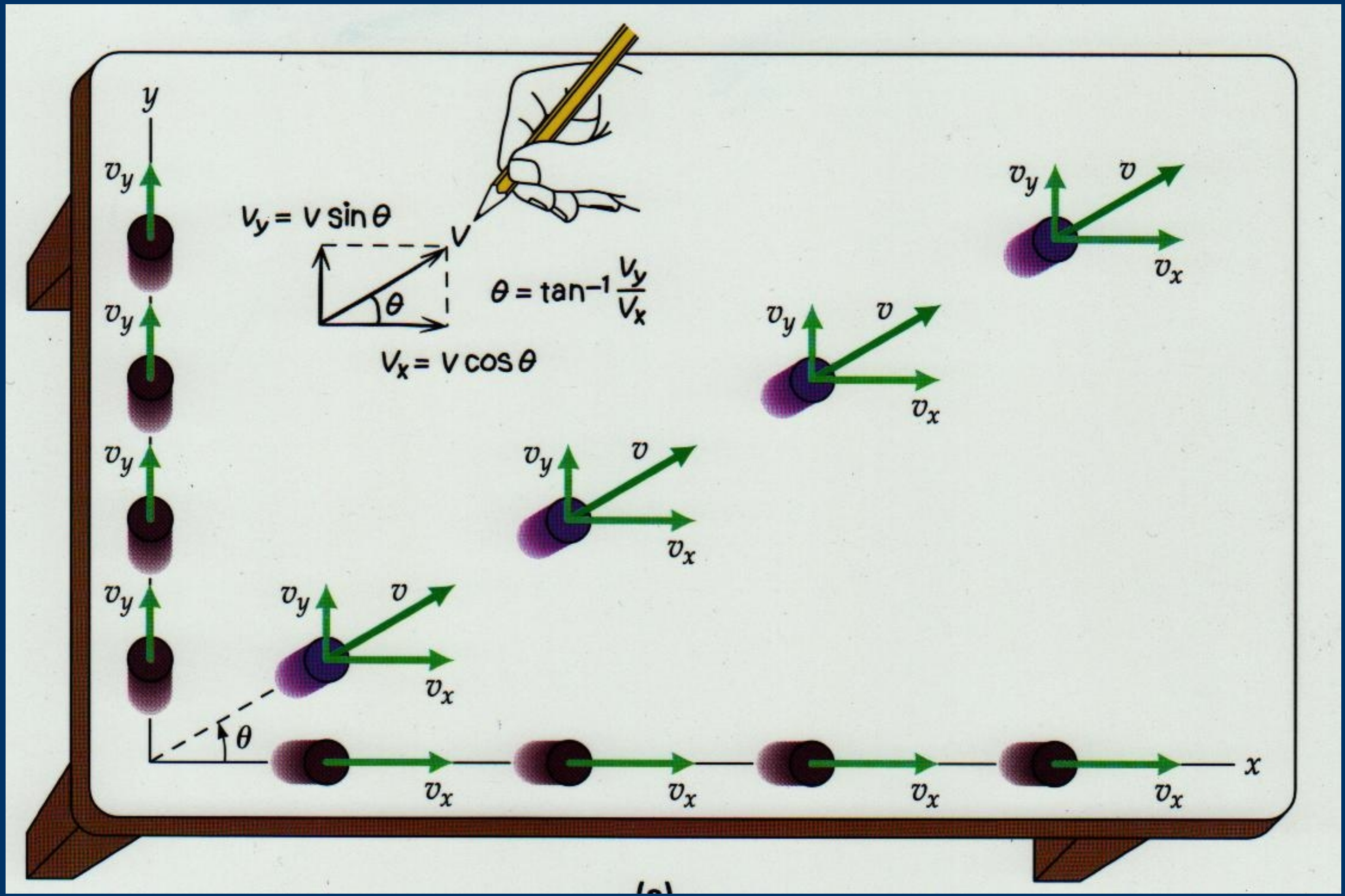
Bottom: change of the position vector **r** gives a displacement Δ**r**.

Remember, $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$



Motion in 2D.

Using vectors for velocity in 2-D.



Notice that this motion is all in a straight line and so could be expressed with 1 dimension (using a rotated axis).

Motion in 2-D (and beyond)
Definitions

Definitions ... (Most of these are very similar to the Ch. 2 equations)

Position vector: $\boxed{\vec{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}}$

Displacement: $\boxed{\Delta\vec{r}=\vec{r}_f-\vec{r}_i}$

Average velocity: $\boxed{\vec{v}_{avg}=\dfrac{\Delta\vec{r}}{\Delta t}}$. Instantaneous velocity: $\boxed{\vec{v}_{inst}=\dfrac{d\vec{r}}{dt}}$

Average acceleration: $\boxed{\vec{a}_{avg}=\dfrac{\Delta\vec{v}}{\Delta t}}$. Instantaneous acceleration: $\boxed{\vec{a}_{inst}=\dfrac{d\vec{v}}{dt}}$

Equations of Uniform acceleration

Final velocity $\boxed{\vec{v}_f=\vec{v}_i+\vec{a}\,t}$

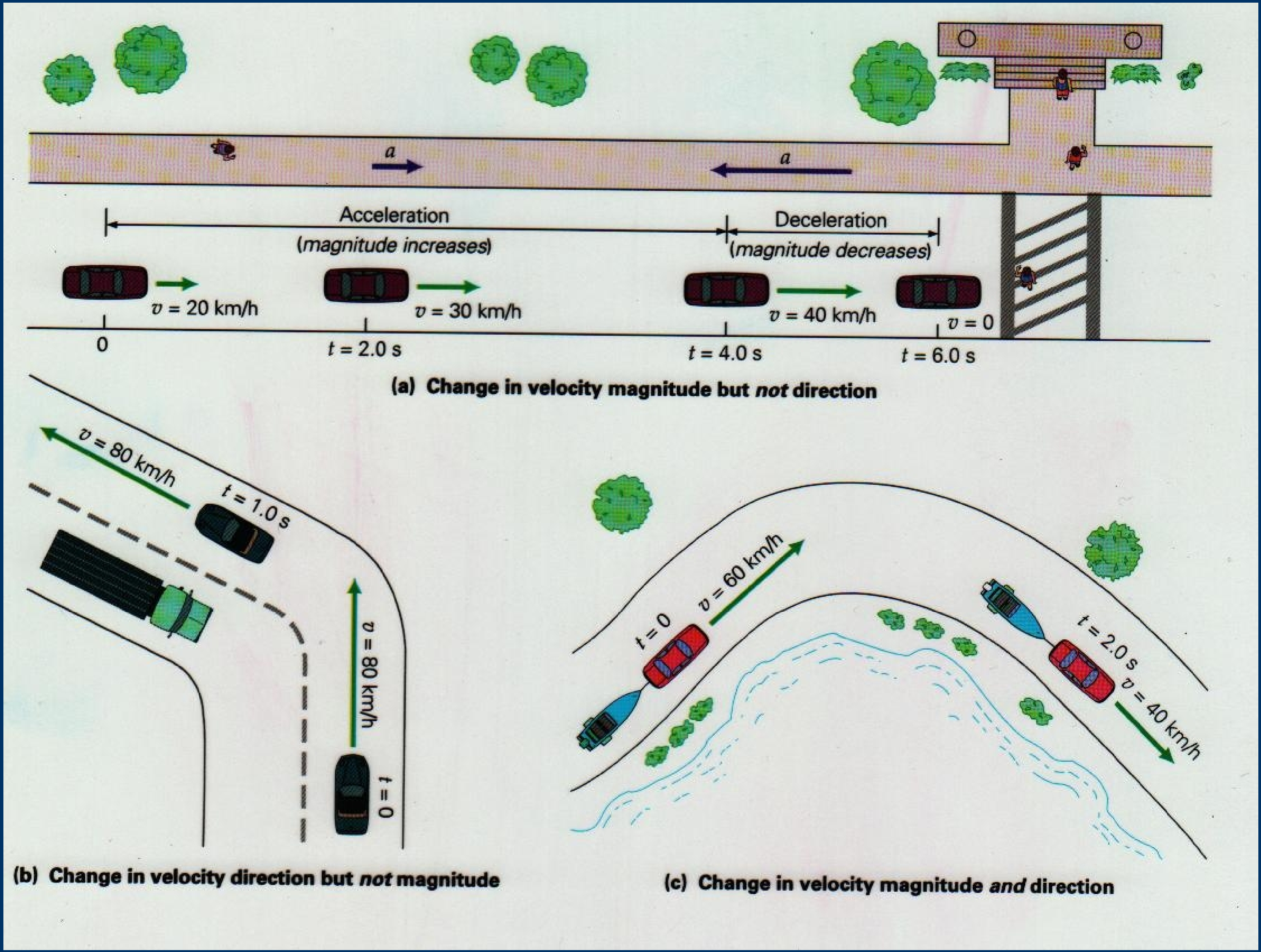
Average Velocity $\boxed{\vec{v}_{avg}=\dfrac{\vec{v}_i+\vec{v}_f}{2}}$

Position as function of time: $\boxed{\vec{r}_f=\vec{r}_i+\vec{v}_{avg}\,t}$

Position as function of time: $\boxed{\vec{r}_f=\vec{r}_i+\vec{v}_i t+\dfrac{1}{2}\vec{a}\,t^2}$

Velocity change related to position change: $\boxed{\vec{v}_f\cdot\vec{v}_f-\vec{v}_i\cdot\vec{v}_i=2\,\vec{a}\cdot(\vec{r}_f-\vec{r}_i)}$

Top: Motion in 1D
Bottom: Motion in 2D.



(a) Change in velocity magnitude but *not* direction

(b) Change in velocity direction but *not* magnitude

(c) Change in velocity magnitude *and* direction

Show 4.16.swf - acceleration has a radial and tangential component.

Motion in 2 dimensions. General motion.

In the most general case, there could be acceleration in both the x and y directions:

$$x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$
$$y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

And so the 2D position is: This is equivalent to: with:

$\mathbf{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$ $\quad \vec{r}_f = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a}\, t^2$ $\quad \mathbf{r}_0 = x_0\hat{\imath} + y_0\hat{\jmath}$

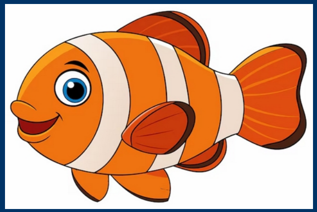
and the 2D velocity is:

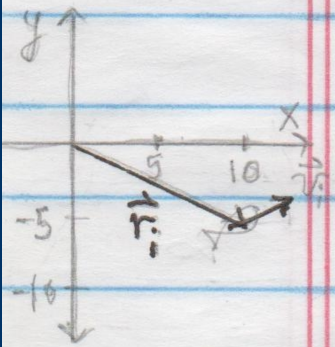
$v(t) = dx/dt\ \hat{\imath} + dy/dt\ \hat{\jmath} = (v_{0x} + a_x t)\,\hat{\imath} + (v_{0y} + a_y t)\,\hat{\jmath}$

and the 2D acceleration is:

$a(t) = dv/dt = a_x\,\hat{\imath} + a_y\,\hat{\jmath}$

Example: swimming fish problem...





Ex) A fish swims in a horizontal plane with $\vec{v}_i = 4\hat{i} + 1\hat{j}$ m/s at a position vector of $\vec{r}_i = 10\hat{i} - 4\hat{j}$ m. The fish swims with uniform acceleration $\vec{a} =$ ____ for $t = 20$ seconds until $\vec{v}_f = 20\hat{i} - 5\hat{j}$ m/s.

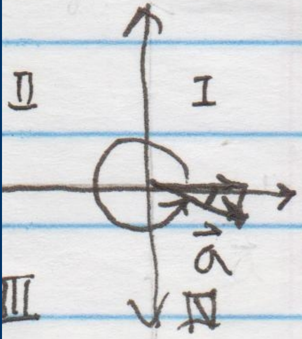
a) What is $\vec{a}$?

Sol'n: since $\vec{a}$ is uniform, $\vec{a} = \vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{20}$

$$= \frac{(20\hat{i} - 5\hat{j}) - (4\hat{i} + \hat{j})}{20}$$

$$= (16\hat{i} - 6\hat{j})/20$$

$$= 0.8\hat{i} - 0.3\hat{j} \text{ m/s}^2$$



b) Find direction of $\vec{a}$.

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-0.3}{0.8}\right) = \boxed{-20.6°}$$

(or $\theta = 339.4°$)

Fish problem (cont.)

d) If $\vec{a}$ is maintained, where is fish at $t = 25s$, and in what direction is $\vec{v}$?

$$\vec{r}(t{=}25) = \vec{r}_i + \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2 \qquad (\text{new } \cancel{2.10}\ 3.13)$$

$$= (10\hat{i} - 4\hat{j}) + (4\hat{i} + 1\hat{j})t + \tfrac{1}{2}(0.8\hat{i} - 0.3\hat{j})t^2$$

$$= [10 + 4(25) + 0.4(25)^2]\hat{i} + [-4 + 1(25s) - \tfrac{0.3}{2}(25)^2]\hat{j}$$

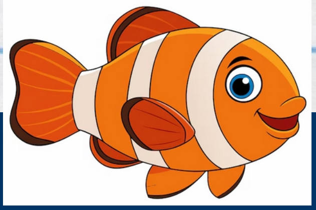
$$\vec{r}(t{=}25) = 360\hat{i} \quad - 72.75\hat{j} \ \text{m}$$

And $\vec{v}(t{=}25) = \vec{v}_i + \vec{a}t$ $\qquad (\text{new } \cancel{2.13}\ 3.12)$

$$= (4\hat{i} + \hat{j}) + (0.8\hat{i} - 0.3\hat{j})(25)$$

$$= [4 + 0.8(25)]\hat{i} + [1 - 0.3(25)]\hat{j}$$

$$\vec{v}_f = 24\hat{i} \quad - 6.5\hat{j} \qquad \text{so } \theta_v = \tan^{-1}\left(\frac{-6.5}{24}\right) = \boxed{-15.15^\circ}$$

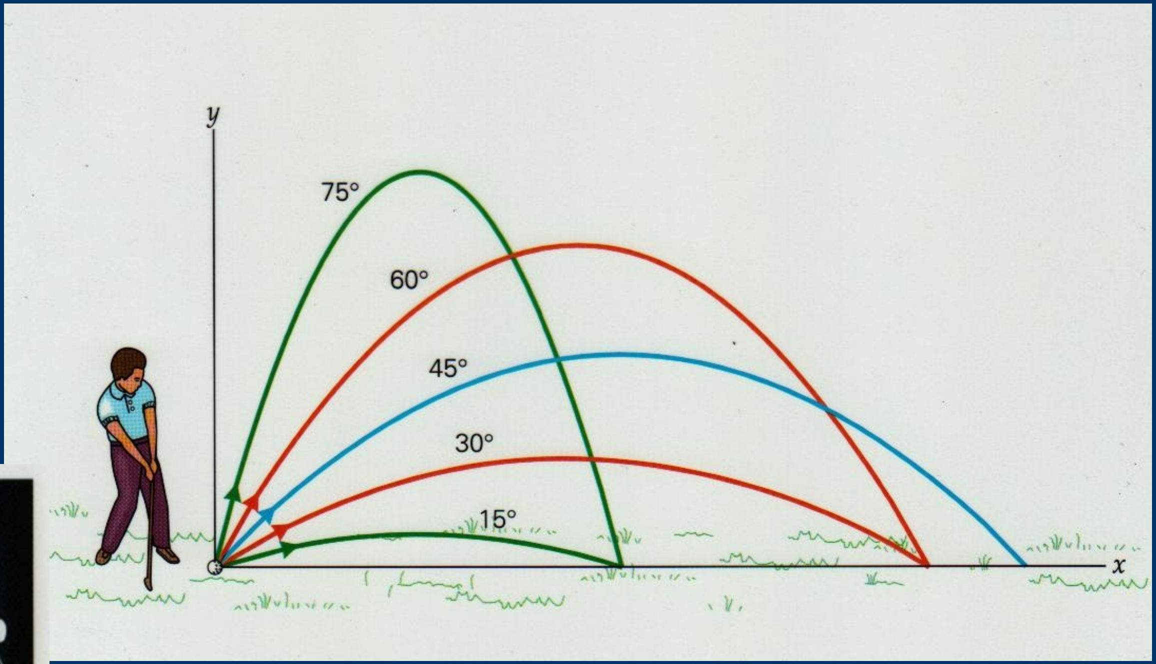


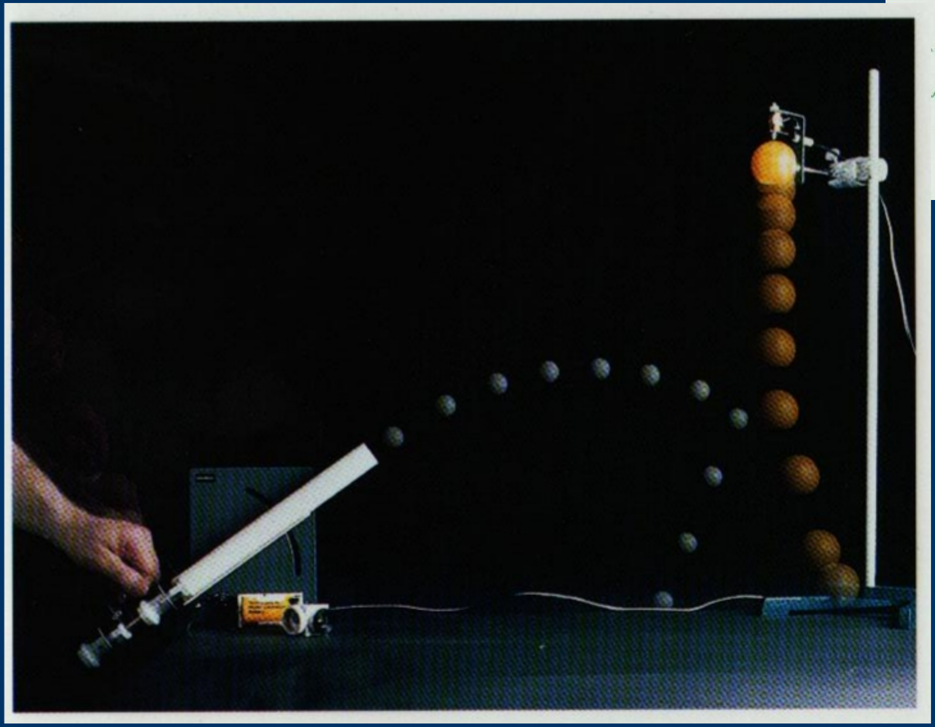
Motion in 2 dimensions. **Projectile Motion.**

<u>P.M.</u> is 2-D motion when the only acceleration is due to gravity.
That is:

$$\mathbf{a} = 0\hat{\imath} - g\,\hat{\jmath}$$

This leads to parabolic trajectories.



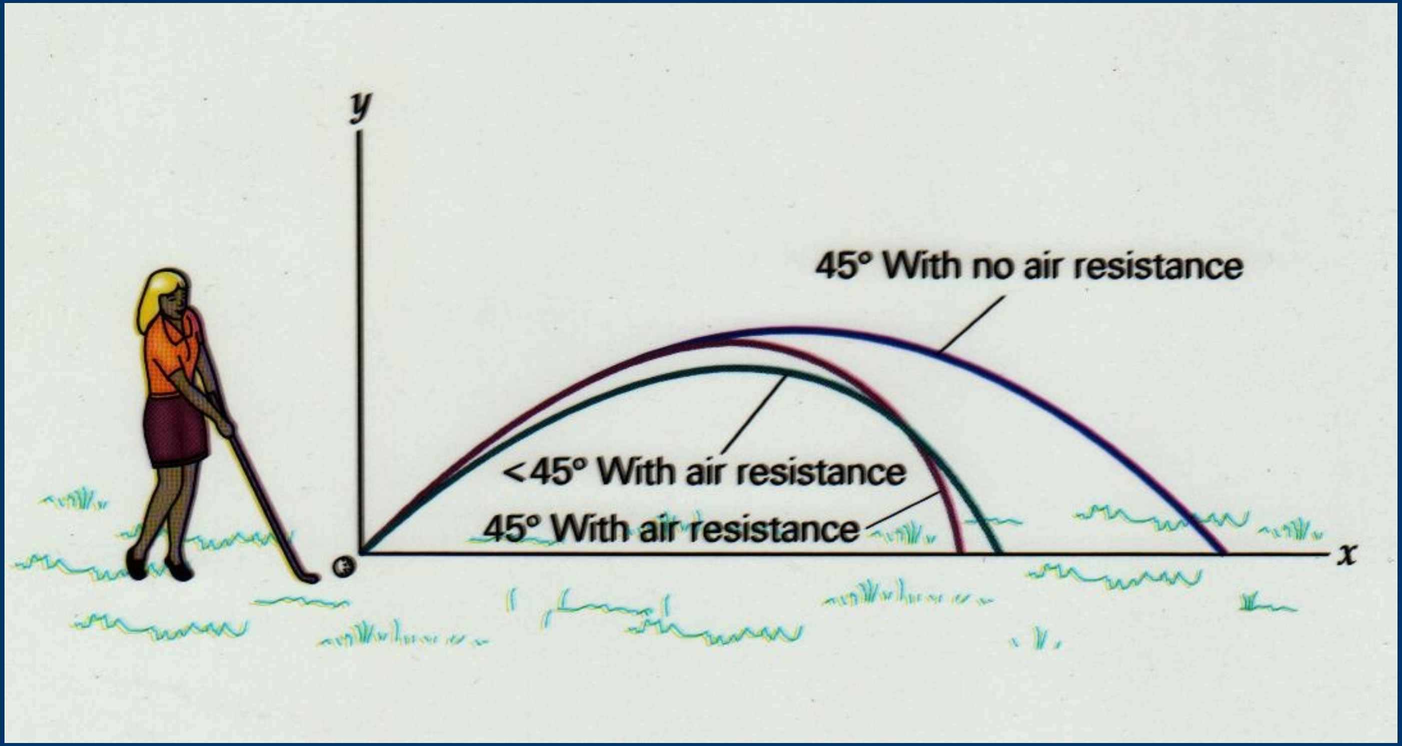


Notice that 2 initial angles lead to the same final *range*, except 45 degrees.

$$R = \frac{v_0^{\,2} \sin 2\theta}{g}$$

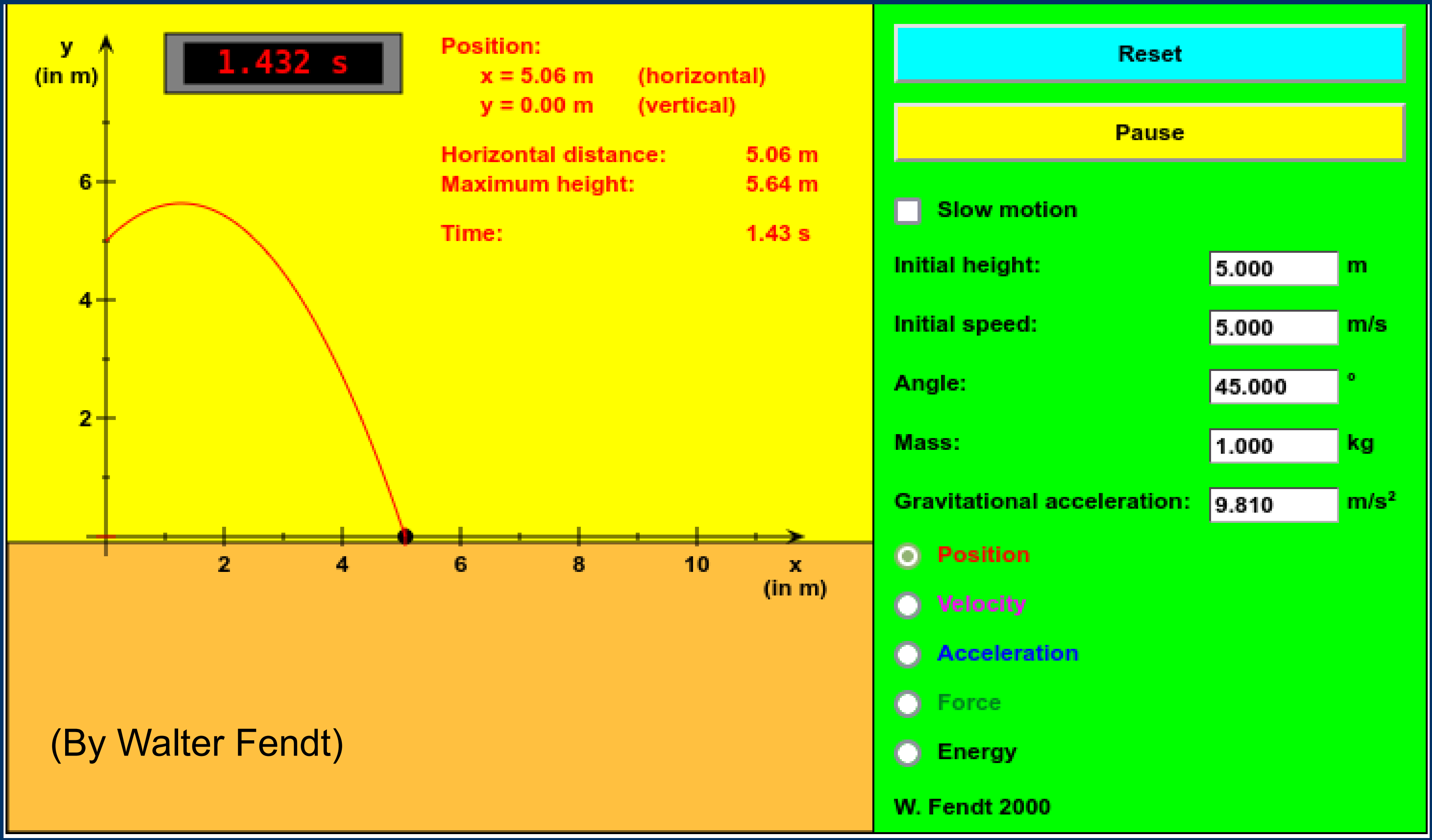
PHYS 2311 Motion in 2 dimensions. **Projectile Motion.**

Actual trajectories: parabolas distorted by air resistance (drag).



Motion in 2 dimensions. Projectile Motion

Interactive simulations (see also ophysics.com/k8.html)

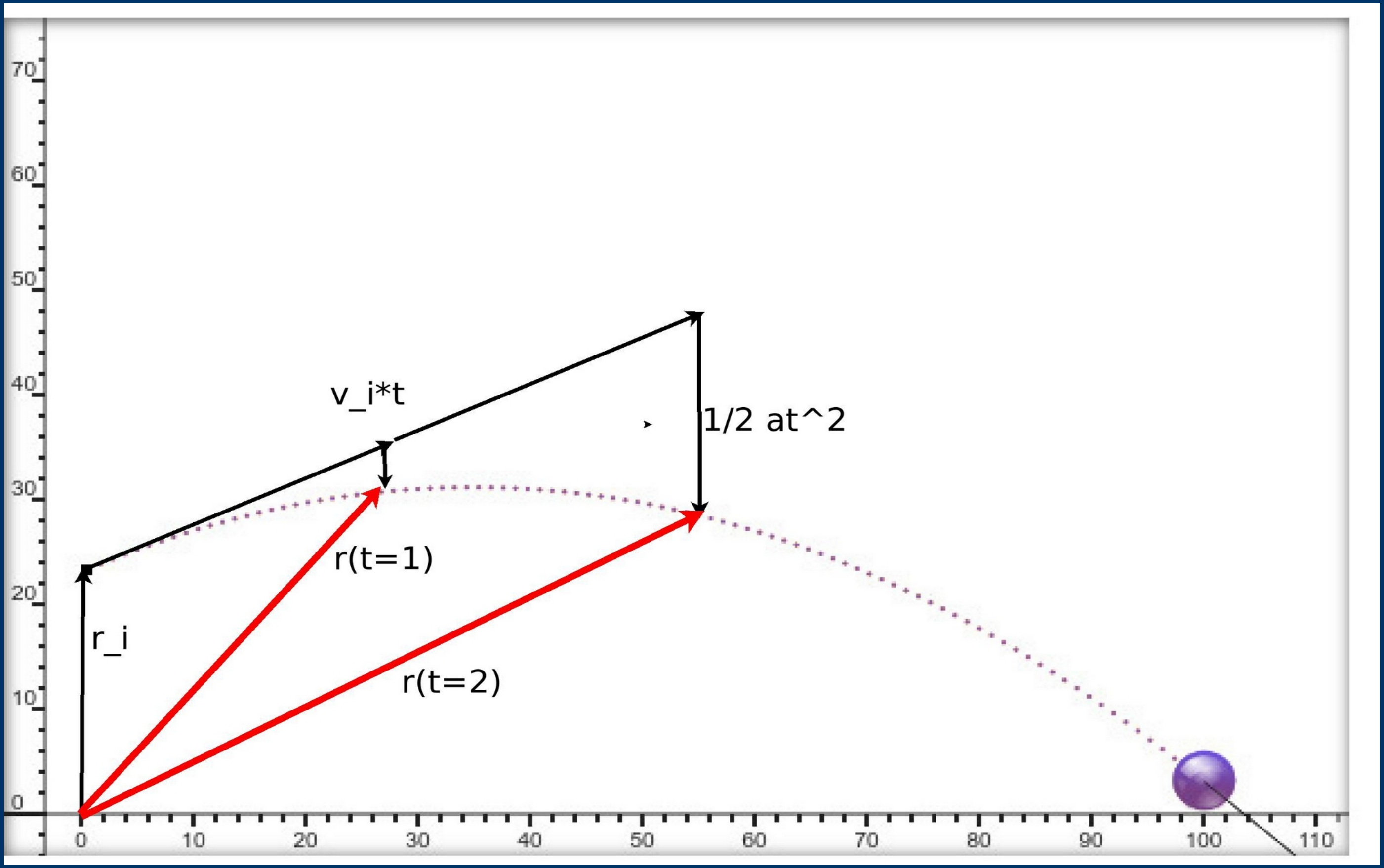


Trajectories are specified with an initial position, velocity (or speed & inclination angle), and acceleration.

Motion in 2 dimensions. Projectile Motion

Trajectories: the position vector (red) is a sum of 3 vectors.

$$\vec{r}(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$



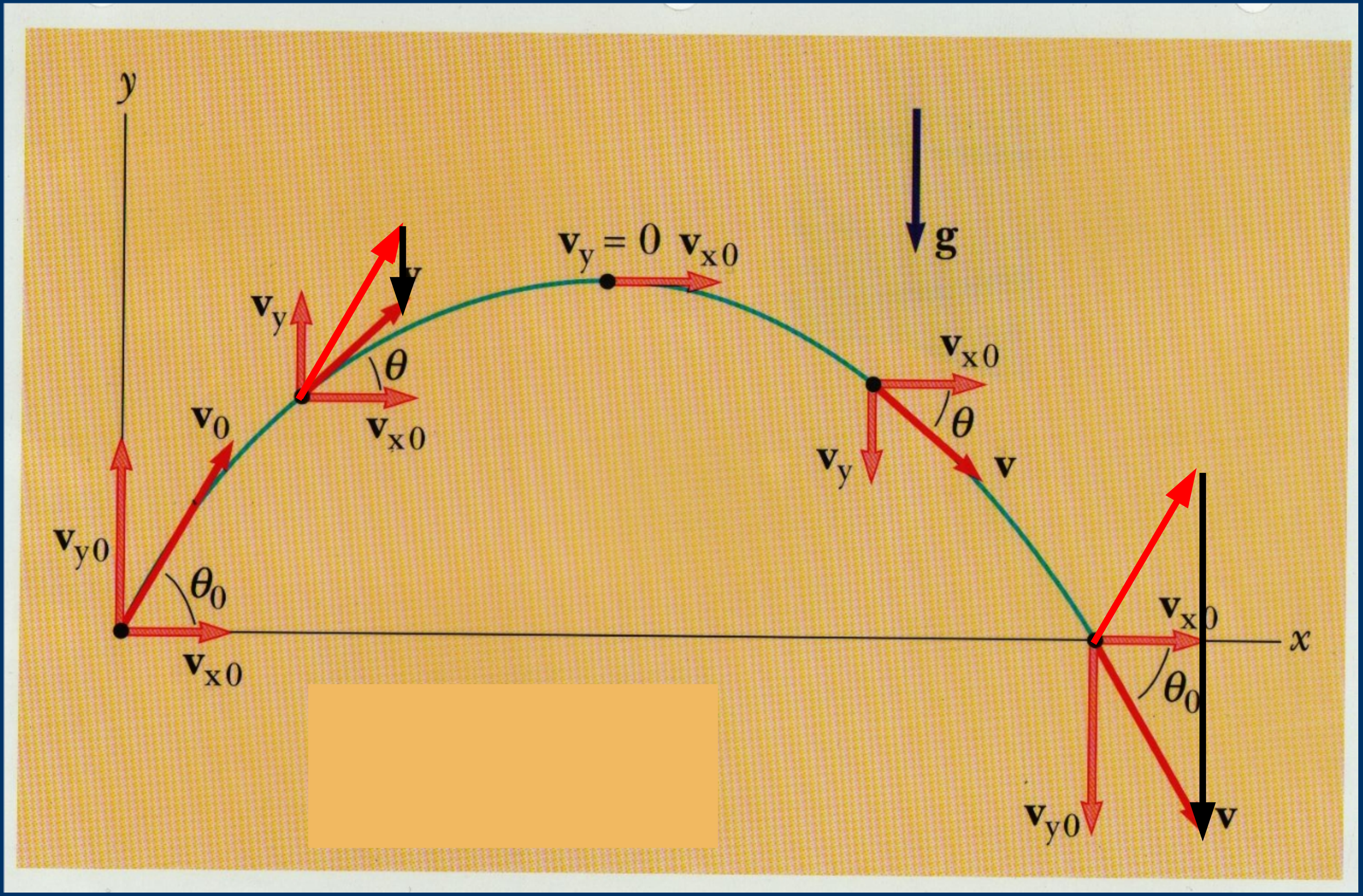
… or a sum of 2 vector components: $\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$

Demo: ophysics.com/k8.html

Motion in 2 dimensions. **Projectile Motion**

Trajectories: the velocity vector is a sum of 2 vectors.

$$\vec{v}(t)=\vec{v}_0+\vec{a}\,t \qquad \text{or} \qquad \vec{v}(t)=v_{x,0}\,\hat{i}+v_y\,\hat{j}$$

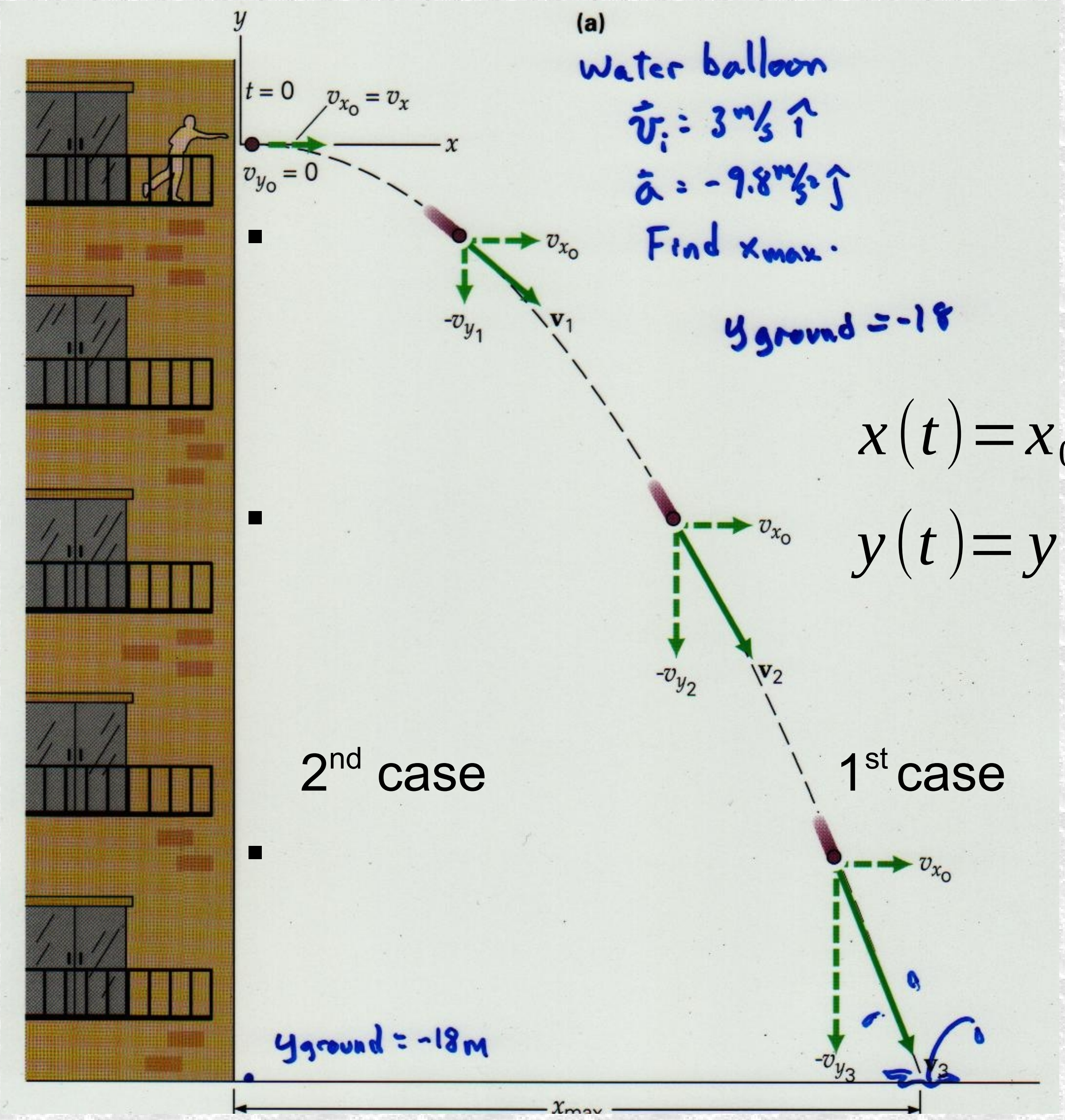


P231 Week 3
The independence of x and y components

If dropped from rest, the vertical progress is identical!

1st case: $v_{x0}=v_x$

2nd case: $v_{x0}=0$



$$x(t) = x_0 + v_{x,0}\,t$$

$$y(t) = y_0 + v_{y,0}\,t - \frac{g}{2}t^2$$

Motion in 2-D

Projectile Motion formulas

Time to reach max height: $\boxed{t_{max}=\frac{v_i \sin\theta_i}{g}}$ (v_i is the magnitude of the initial velocity)

Maximum height: $\boxed{h_{max}=\frac{v_i^2 \sin^2\theta}{2g}}$

Range: $\boxed{R=\frac{v_i^2 \sin 2\theta}{g}}$

To derive t_{max}, ask yourself "at what time does the vertical speed reach 0?"

$v_y=0=v_{0y}+a_y t_{max}$ with $a_y = -g$ and $v_{0y} = v_i \sin\theta$

To derive h_{max}, use $y_{max} = y(t_{max})=y_0+v_{oy}t+1/2\, a_y t^2$ (assumes $y_0=0$)

To derive Range, use $R = x(t=2t_{max}) = x_0+v_{ox}t$. (Need $2\sin\theta\cos\theta=\sin2\theta$)

PHYS 2311 Motion in 2 dimensions. Projectile Motion.

P 3.32) A tiger leaps horizontally from a 7.5-m high rock with a speed of 3.0 m/s. How far from the base of the rock will she land?

Find x_{land} Givens: $a=-9.8\,\hat{j}$ m/s^2 $y_0=7.5$ m, $v_i = 3.0\,\hat{i} + 0\,\hat{j}$ m/s

Set up: find t_{land} with $y_f = y_i + v_{oy}t - 4.9t^2$

Then find x_{land} with $x_{land} = x_0 + v_{ox}t$

P. 3.37) A firehose held near the ground shoots water at a speed of 6.5 m/s. At what angles should the nozzle point in order that the water land 2.5 m away? Why are there two different angles? Sketch the 2 trajectories.

Find θ Givens: $|v_i| = 6.5$ m/s, $a=-9.8\,\hat{j}$ m/s^2 $x_0=y_0=y_f=0$ m, $x_f=2.5$ m

Set up: use Range formula: $R=(v_i^2\sin 2\theta)/g$ with $R=x_f=2.5$ m

$\sin^{-1}(0.58)=2\theta=35.4° \rightarrow \theta=17.7°$. But $\sin^{-1}(0.58)$ also $=180-35.4° \rightarrow \theta=72.3°$