The Interaction of Light and Matter

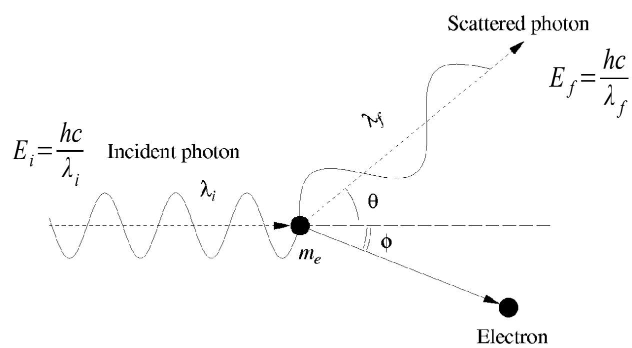
- http://webphysics.ph.msstate.edu/javamirror/ipmj/java/atomphoton/index.html
- http://webphysics.ph.msstate.edu/javamirror/ipmj/java/slitdiffr/index.html
- http://www.colorado.edu/physics/2000/quantumzone/index.html
- http://micro.magnet.fsu.edu/primer/java/doubleslit/index.html
- http://members.tripod.com/~vsg/interfer.htm
- http://micro.magnet.fsu.edu/primer/java/exciteemit/index.html
- http://www.colorado.edu/physics/2000/applets/a2.html
- http://home.a-city.de/walter.fendt/physengl/photoeffect.htm
- http://lectureonline.cl.msu.edu/~mmp/kap28/PhotoEffect/photo.htm

Outline

- (0) Compton scattering – light acts like a particle
- (1) Motivation: Why spectral lines?
  - the Birth of Spectroscopy
  - Kirchoff's Laws
- (2) The Bohr Model of the Atom
  - Need for a theory to describe spectral lines
- (3) Photons – the particle nature of light
- (4) Quantum Mechanics and the Wave-Particle Duality

Compton Scattering: "Particle-like" Behavior of Photon

Concept: Photon scatters off electron and loses energy, where resulting λ of scattered photon **depends on θ.**
Conservation of relativistic momentum and Energy!
No mass for the photon but it has momentum!!!

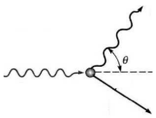


Compton Scattering: Equation

Photon OUT — Scattering Angle

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c}\left(1-\cos\theta\right)$$

Photon IN

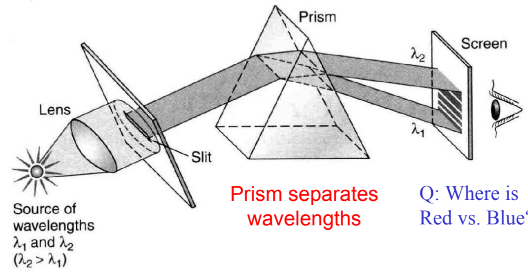


Critical λ_c= 0.0024 nm for e^-

- **Limiting Values**
  - No scattering: $\theta = 0^o \rightarrow \cos 0^o = 1 \rightarrow$ $\mathbf{\Delta\lambda = 0}$
  - "Bounce Back": $\theta = 180^o \rightarrow \cos 180^o = -1 \rightarrow$ $\mathbf{\Delta\lambda = 2\lambda_c}$
- Difficult to observe unless λ is small (i.e. $\Delta\lambda/\lambda > 0.01$)

Spectroscopy - history

- Trogg (50 million BC) – rainbow
- Newton (1642-1727) – decomposes light into spectrum and back again
- W. Herschel (1800) – discovers infrared
- J. W. Ritter (1801) – discovers ultraviolet
- W. Wollaston (1802) – discovers absorption lines in solar spectrum



Prism separates wavelengths

Q: Where is Red vs. Blue?

Spectroscopy - history

- J. Herschel, Wheatstone, Alter, Talbot and Angstrom studied spectra of terrestrial things (flames, arcs and sparks) ~1810
- Joseph Fraunhofer
  - Cataloged ~475 dark lines of the solar spectrum by 1814
  - Identifies sodium in the Sun from flame spectra in the lab!
  - Looks at other stars (connects telescope to spectroscope)
- Foucault (1848) – sees absorption lines in sodium flame with bright arc behind it.

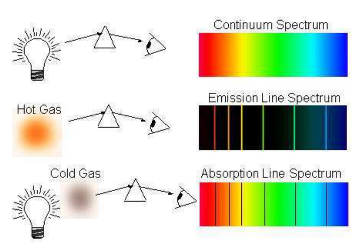
There is the need for a *new physics!*

Kirchhoff's laws (1859):

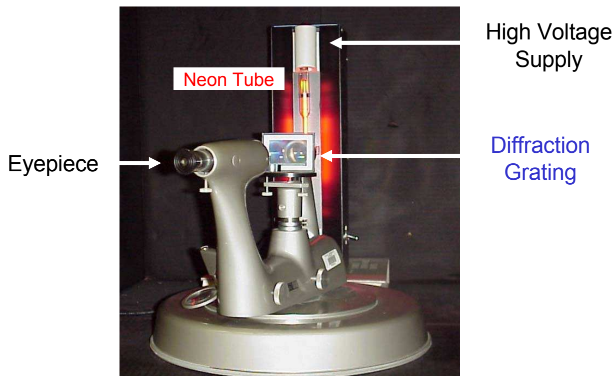
Worked with Bunsen on flame spectra

Developed a prism spectroscope

- Hot solid or dense gas, → blackbody radiation
- Hot diffuse gas → emission lines
- Cool diffuse gas in front of a blackbody → absorption lines



Atomic Spectra Lab



$$d \sin\theta = n\lambda$$

Doppler shift

- Spectral lines allow for the measurement of radial velocities
- At low velocities, $v_r << c$
  - Classical Doppler effect
    - *Radial velocity*, v_r
    - *Heliocentric correction* for Earth's motion, up to 29.8 km/s, depending on direction.
- Example: H_α is 6562.80 Å
  - Vega is measured to be 6562.50 Å
  - Coupled with the *proper motion*
    - Can determine total velocity

$$\frac{\lambda_{obs}-\lambda_{rest}}{\lambda_{rst}}=\frac{\Delta\lambda}{\lambda_{rest}}=\frac{v_r}{c}$$

$$\Delta\lambda=\frac{v_r}{c}\lambda_{rest}$$

$$v_r=c\frac{\Delta\lambda}{\lambda_{rest}}=-14\frac{km}{sec}$$

$$v_\theta=r\mu=13\frac{km}{s}$$

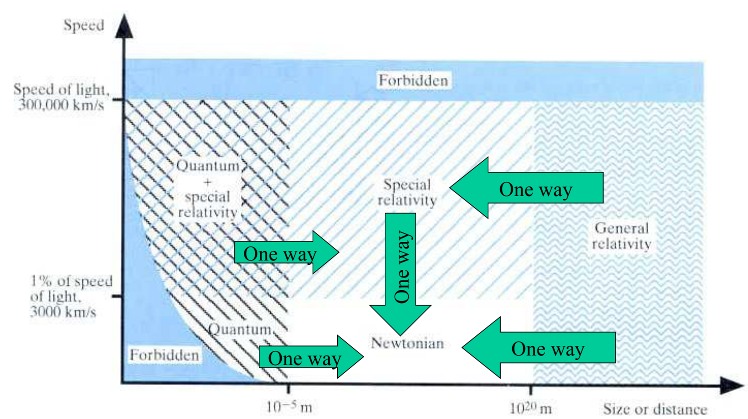
$$v=\sqrt{v_r^2+v_\vartheta^2}=19\frac{km}{s}$$

What is Modern Physics?

- Modern physics only came of age in the **1900's.**
  - Physicists discovered that Newtonian mechanics does not apply when objects move <u>very fast</u> or are <u>very small</u>!
- If things move **<u>very fast</u>** (close to the speed of light), then **<u>RELATIVISTIC</u>** mechanics is necessary.
  - Cosmic particles, atomic clocks (GPS), synchrotrons.
- If things are confined to **<u>very small</u>** dimensions (nanometer-scale), then **<u>QUANTUM</u>** mechanics is necessary.
  - Atomic orbitals, quantum heterostructures.
- **Need** a lot of these ideas to describe the universe

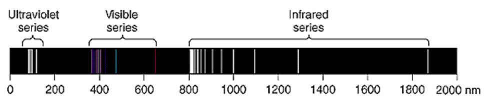
Beyond Newton

- Gravity passed every test until ~1890s
- Newton's gravity and motion is incorrect when …



Atomic Spectra

- 1885 - Balmer observed Hydrogen Spectrum
  - Found empirical formula for discrete wavelengths
  - Later generalized by Rydberg for simple ionized atoms



$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \textbf{ with } 2<n$$

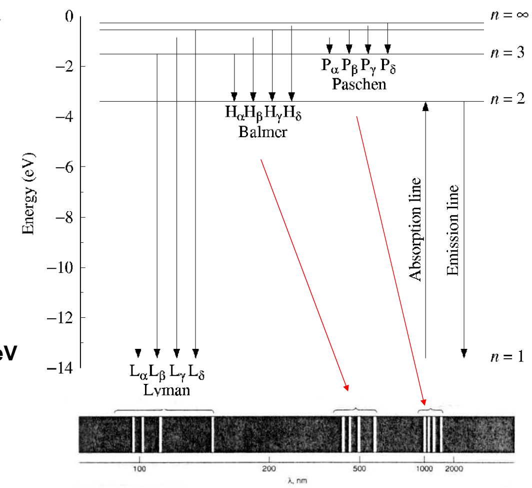
Atomic Spectra: Hydrogen Energy Levels

E_∞ = 0 eV

Energy

$$E_n \propto \frac{-1}{n^2}$$

E_1 = -13.6 eV



Atomic Spectra: Rydberg Formula

$$\frac{1}{\lambda} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \textbf{ with } m<n$$

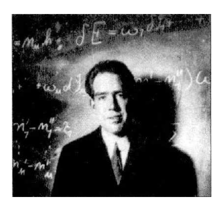
- Rydberg constant $R_H \sim 1.097 \times 10^5\ cm^{-1}$
- m = 1 (Lyman), 2 (Balmer), 3 (Paschen)
- Example for **n = 2 to m = 1** transition:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}\left(1.097\times 10^{-5}\ cm^{-1}\right)$$

$$\Rightarrow \lambda = 121.6\ nm\ \textit{Ultraviolet}$$

Bohr Model

- 1913 – Bohr proposed quantized model of the H atom to predict the observed spectrum.
- Problem: Classical model of the electron "orbiting" nucleus is unstable. Why unstable?
  - Electron experiences centripetal acceleration.
  - Accelerated electron emits radiation.
  - Radiation leads to energy loss.
  - Electron eventually "crashes" into nucleus.



Bohr Model: Quantization

- **Solution:** Bohr proposed two "quantum" postulates
  - Electrons exist in stationary orbits (no radiation) with quantized angular momentum

$$L_n = mvr = n\hbar \qquad \text{where } \hbar = \frac{h}{2\pi} = 6.58\times10^{-16}\, ev\cdot s$$

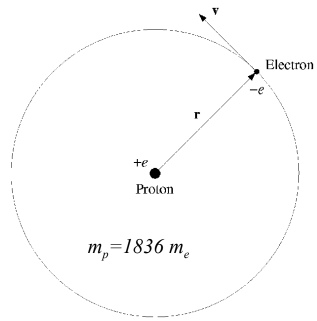
h = Planck's Constant

  - Atom radiates with quantized frequency ν (or energy E) only when the electron makes a transition between two stationary states.

$$h\nu = \frac{hc}{\lambda} = E_i - E_f$$

Planetary Mechanics Applied to the Atom

- Consider the attractive electrostatic force and circular motion



$$\vec{F} = \frac{q_1 q_2}{r^2}\hat{r} = \mu\frac{v^2}{r}\hat{r}$$

Note: in cgs, $e = 4.803\mathrm{x}10^{-10}$ esu

$$\frac{q_1 q_2}{r^2} = -\mu\frac{v^2}{r}$$

$$\frac{-e^2}{r^2} = -\mu\frac{v^2}{r}$$

$$\frac{1}{2}\mu v^2 = \frac{1}{2}\frac{e^2}{r} = K \qquad \text{Kinetic energy}$$

$$U = -2K = -\frac{e^2}{r} \qquad \text{Potential energy}$$

$$\mu = \frac{m_e m_p}{M} \approx m_e$$

$$M = m_e + m_p \approx m_p$$

Planetary Mechanics Applied to the Atom

- Introduce Bohr's quantized angular momentum $L = \mu v r = n\hbar$ (wrong)

$$K = \frac{1}{2}\frac{e^2}{r} = \frac{1}{2}\mu v^2 = \frac{1}{2}\frac{(\mu v r)^2}{\mu v r^2} = \frac{1}{2}\frac{(n\hbar)^2}{\mu v r^2}$$

- Solving for r

$$r_n = \frac{\hbar^2}{\mu e^2}n^2 = a_0 n^2$$

a_0 is the Bohr radius

- Get the Total Energy in terms of n

$$E_n = -\frac{1}{2}\frac{e^2}{r} = -\frac{\mu e^4}{2\hbar^2}\frac{1}{n^2} = \frac{-13.6\ eV}{n^2} = \frac{-E_0}{n^2}$$

- Principle quantum number, $n = 1, 2, 3, \ldots$

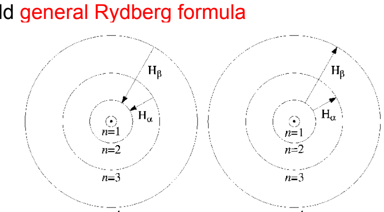
Bohr Model: Transitions

- Transitions predicted by Bohr yield general Rydberg formula

$$\delta E = -E_0\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \; -\delta E = \frac{hc}{\lambda}$$

$$\frac{1}{\lambda} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \quad \textbf{with } m<n$$

$$R_H = \frac{E_0}{hc} = \frac{\mu e^4}{4\pi\hbar^3 c}$$



  - Applies to ionized atoms with only one electron and of nuclear charge Z.

$$\frac{1}{\lambda} = Z^2\left(\frac{E_o}{hc}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{hc}{\lambda} = E_f - E_i = -Z^2 E_o\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Bohr Model Problem: Unknown Transition

If the wavelength of a transition in the **Balmer series** for a **He^+** atom is **121 nm**, then find the corresponding transition, i.e. initial and final n values.

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right) = R(2)^2\left(\frac{1}{(2)^2} - \frac{1}{n_i^{\,2}}\right)$$

where Z = 2 for He and n_f = 2 for Balmer

$$\frac{1}{4R\lambda} = \left(\frac{1}{4} - \frac{1}{n_i^{\,2}}\right)$$

$$n_i = \left(\frac{1}{4} - \frac{1}{4R\lambda}\right)^{-1/2} = \left(\frac{1}{4} - \frac{1}{4(1.1\times10^{7}\,m^{-1})(121\times10^{-9}\,m)}\right)^{-1/2} = \underline{\boldsymbol{4}}$$

Bohr Model Problem: Ionization Energy

Suppose that a He atom (Z=2) in its ground state (n = 1) absorbs a photon whose wavelength is **λ = 41.3 nm**. Will the atom be **ionized**?

➢ *Find the energy of the incoming photon and compare it to the ground state ionization energy of helium, or E_0 from n=1 to ∞.*

$$E = \frac{hc}{\lambda} = \frac{1240\ eV\ nm}{41.3\ nm} = \underline{\boldsymbol{30\ eV}}$$

$$E_0(He) = Z^2 \times E_0(H) = (2^2)(13.6\ eV) = 54.4\ eV$$

➢ *The photon energy (30 eV) is less than the ionization energy (54 eV), so the electron will NOT be ionized.*

Compton Scattering Problem

If a 0.511-MeV photon from a positron-electron annihilation scatters at [redacted] from a free electron, then find the wavelength and energy of the Compton scattered photon.

notaiton $\lambda_1 = \lambda_i$ *and* $\lambda_2 = \lambda_f$

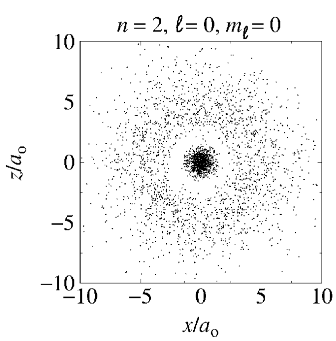
$$\lambda_2 - \lambda_1 = \lambda_C(1 - \cos\theta) = (0.00243\ \ nm)(1 - \cos 180^o) = 4.86\times10^{-3}\,nm$$

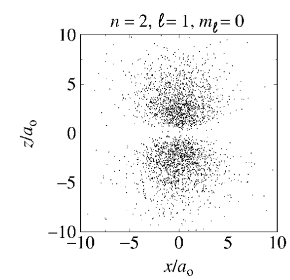
$$\lambda_1 = \frac{hc}{E_1} = \frac{1240\ \ eV\cdot nm}{0.511\times10^{6}\,eV} = \underline{2.43\times10^{-3}\,nm}$$

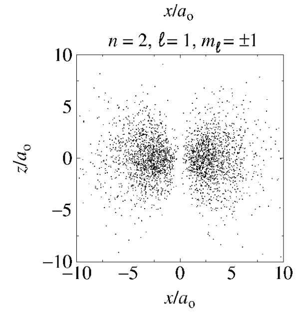
$$\lambda_2 = \lambda_1 + \Delta\lambda = 2.43\times10^{-3}\,nm + 4.86\times10^{-3}\,nm = \underline{7.29\times10^{-3}\,nm}$$

$$E_2 = \frac{hc}{\lambda_2} = \frac{1240\ \ eV\cdot nm}{7.29\times10^{-3}\,nm} = 1.70\times10^{5}\,eV\ \ (or\ \ \underline{0.17\ \ MeV})$$

- Probability "clouds"
  - kind of the opposite to the "Plum Pudding" model







Zeeman Effect

- Measure magnetic field strengths
- Solving the Schrodinger equation yields two more quantum numbers

$$L = \hbar \sqrt{l(l+1)}$$
$$L_z = m_l \hbar$$

Spectral line becomes split