

Physics 2311 – Physics I, Week 4

Dr. J. Pinkney

Outline for Day W3,D1

1D kinematics

Equations of uniform acceleration, including free-fall

Examples

2D kinematics - Vectors

Homework

Ch. 2 Prob. 2,3,5-7,14,23-27,35-38,53-56 for 2-day

Ch. 3 P. 1,3,6,7,10,11,19,20,23,24,

32,33,37,38,39 Do by next Mon.

Notes: Lab this week: “Graphs and Tracks”

Under NEW STUFF: “Exam like problems” for Ch. 1
And “Week 3-5” practice quiz.

Motion in 1D.

Equations for Uniform Acceleration

A) [Text: 2-12a] $\vec{v}_f = \vec{v}_i + \vec{a}t$

B) [Text: 2-12d] $\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$

C) [Text: 2-8] $\vec{x}_f = \vec{x}_i + \frac{\vec{v}_i + \vec{v}_f}{2}t$

D) [Text: 2-12b] $\vec{x}_f = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$

E) [Text: 2-12c] $v_f^2 - v_i^2 = 2 a (x_f - x_i)$

Motion in 1D.

Examples using Equations for Uniform Acceleration

1) A car passes $x=10\text{m}$ at $t=0$ going 10 m/s with a constant accelerating of 4 m/s^2 . Where will the car be in 5 seconds?

$$\vec{x}_f = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

2) A car accelerates uniformly, starting at $v_i=5\text{ m/s}$ at $x_i=20$, and reaching $x_f=100$ only 5 seconds later. How fast did it cross the $x_f=100\text{ m}$ mark?

$$\vec{x}_f = \vec{x}_i + \frac{\vec{v}_i + \vec{v}_f}{2} t$$

Motion in 1D.

Examples using Equations for Uniform Acceleration

3) A rock thrown down a well at 10 m/s reaches the bottom at 40 m/s. What was the average velocity? (Uniform acceleration of 9.8 m/s² downward.)

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

4) Find the depth of the well in the previous problem, assuming the rock was thrown straight down.

$$v_f^2 - v_i^2 = 2 a (x_f - x_i)$$

Motion in 1D.

Examples using Equations for Uniform Acceleration

5) A car on ice is sliding backwards to the left at 5 m/s while accelerating uniformly to the right at 3 m/s². What is its velocity after 7 seconds?

$$\vec{v}_f = \vec{v}_i + \vec{a} t$$

Physics 2311 – Physics I, Week 4

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Outline for Day W3,D2

1D kinematics: free-fall example

2D kinematics – Vectors

Definitions and Eqns of uniform acceleration

Radial and tangential acceleration in “curvy” motion

Projectile motion

Homework

Ch. 3 P. 1,3,6,7,10,11,19,20,23,24,
32,33,37,38,39 Do by Mon.

Notes: Lab this week: “Graphs and Tracks”

Under NEW STUFF: “Exam like problems” for Ch. 1
And “Week 3-5” practice quiz.

Motion in 1D.

Free Fall problems

Assumes downward acceleration, g , near the surface of a planet (usually Earth!)

The Equations for Uniform Acceleration apply!

Assumes no air resistance or other forces on the object.

Object can be moving downwards OR upwards during free fall!

Motion in 1D.

Free Fall problems

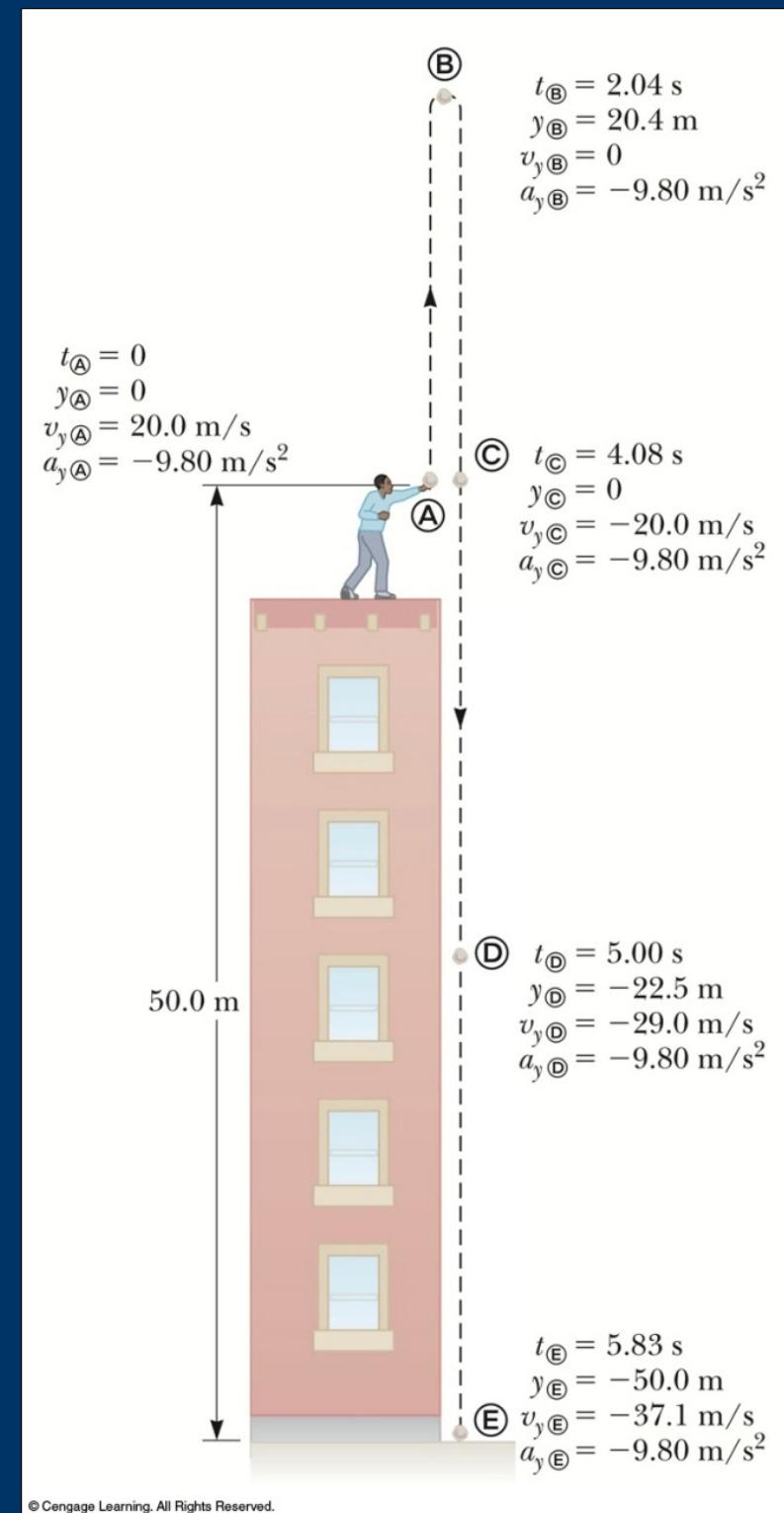
Initial velocity at A is upward (+) and acceleration is $-g$ (-9.8 m/s^2).

At B, the velocity is 0 and the acceleration is $-g$ (-9.8 m/s^2).

At C, the velocity has the same magnitude as at A, but is in the opposite direction.

The displacement is -50.0 m (it ends up 50.0 m below its starting point).

$$\vec{y}_f = \vec{y}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$



Motion in 1D.

Free Fall problems

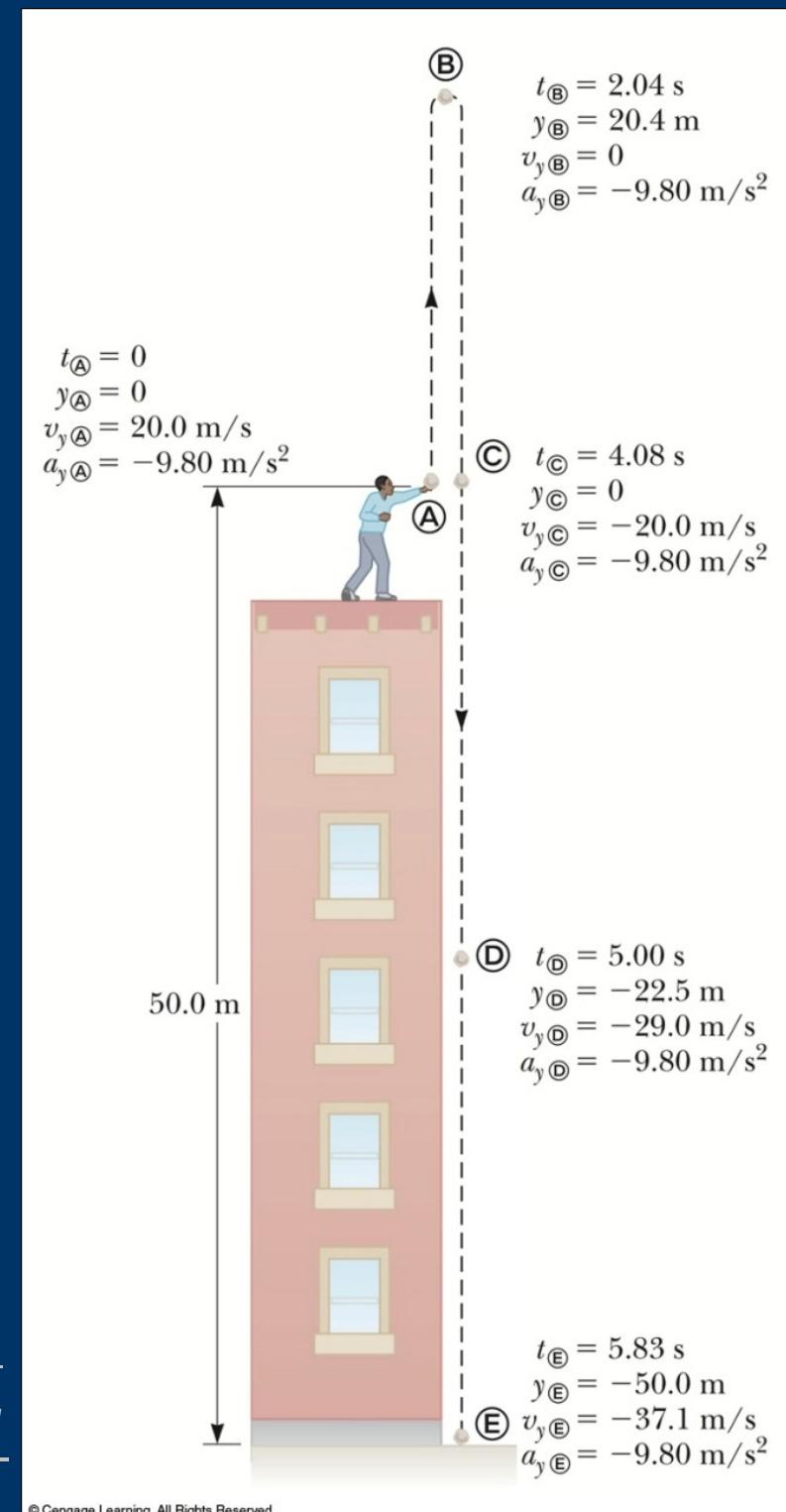
Example) Verify that the ball hits the ground at $t_f=5.83$ seconds if it is thrown from an initial height of $y_i=0$ upwards at $v_i=20$ m/s.

Also given: $y_f=-50$ m, $a=-9.8$ m/s²

Use: $\vec{y}_f = \vec{y}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$
 $-50 = 0 + 20t - 4.9t^2$

Quadratic eqn. $0=-4.9t^2 + 20t + 50$

$$0 = at^2 + bt + c$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Motion in 2 Dimensions

Goals of Week 3:

- Learn how to work with vectors
- Rewrite 1D equations for 2D (and 3D) cases
- Acceleration in curvy motion – tangential and centripetal[†]
- Apply 2D equations to projectile motion.
- Calculate range, maximum height & t_{\max} for trajectories
- Relative velocity

[†] or “radial”

Motion in 2D. Vectors

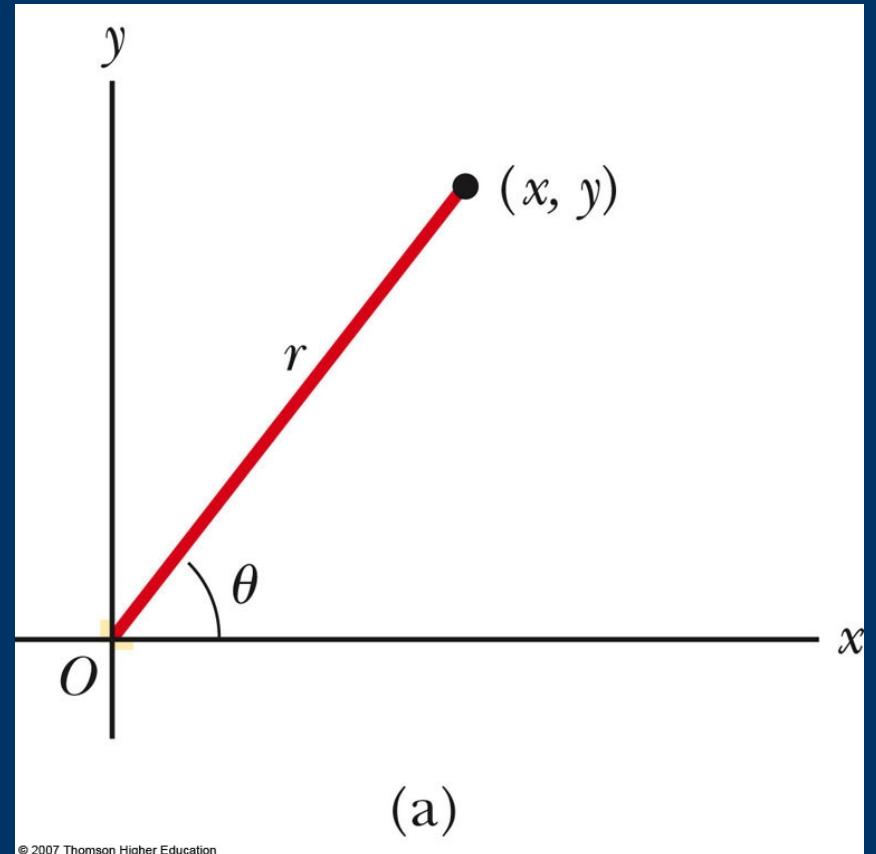
2D “Cartesian” coordinate systems provide 2 ways to specify the position of a point:

1) Polar coordinates

Points are labeled (r, θ)

2) Cartesian coordinates

Points are labeled (x, y)



Conversions

Polar to cartesian:

$$x = r \cos \theta, \quad y = r \sin \theta$$

Cartesian to polar:

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1} \frac{y}{x}$$

Motion in 2D. Vectors

Similarly, vectors can be expressed in two main ways: *polar* and *vector component form*.

Example) Vector \mathbf{r} is a *position vector* because it has its tail at the origin.

Vector \mathbf{A} is not a position vector, but it's a vector.

However, these vectors are *equal* because they have the same length and direction. Thus,

$$\mathbf{A} = \mathbf{r} = 5\hat{i} + 3\hat{j}$$

and this is the *vector component form*.

Convert \mathbf{A} and \mathbf{r} into *polar vectors*...

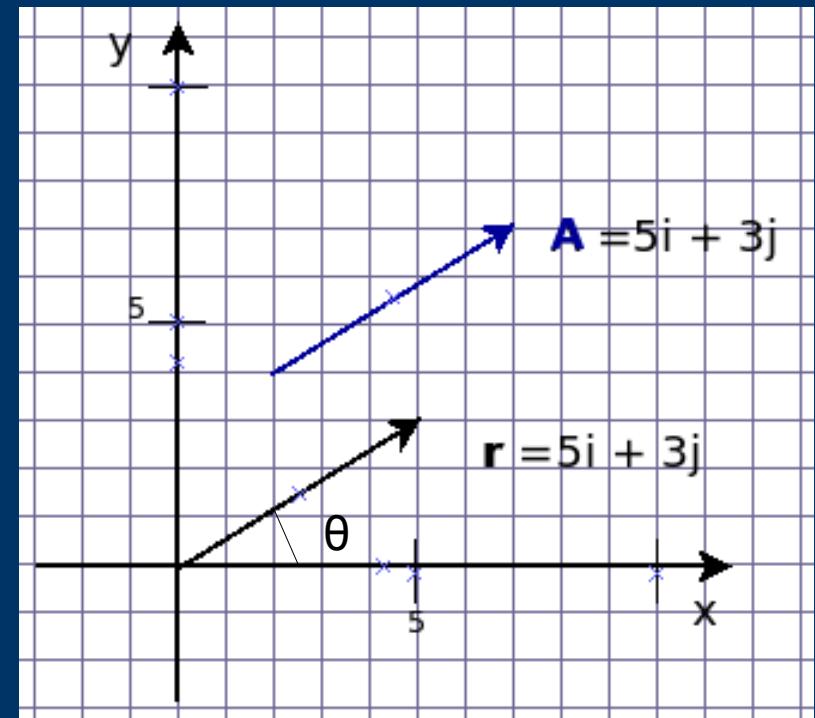
The length of both is

$$|\mathbf{A}|=|\mathbf{r}| = (5^2+3^2)^{1/2} = 5.83$$

The angle between the vector and the $+x$ -axis is

$$\theta = \tan^{-1}(3/5) = 30.96^\circ$$

Therefore, \mathbf{A} is $(r,\theta) = (5.83, 30.96^\circ)$



Motion in 2D. Vectors

Careful with finding the angle using the inverse tangent (\tan^{-1}) function on a calculator!

First, make sure calculator is set on degrees instead of radians.

Second, if the vector points towards quadrants 2 or 3, you must add 180° (or subtract 180° to keep the angle less than 360°) to the calculator's answer:

$$\theta = \tan^{-1} (y/x) + 180^\circ$$

If the vector points towards quadrants 1 or 2,
It is ok to use:

$$\theta = \tan^{-1} (y/x)$$

ALWAYS define θ to be measured CCW from the $+x$ axis to the position vector!

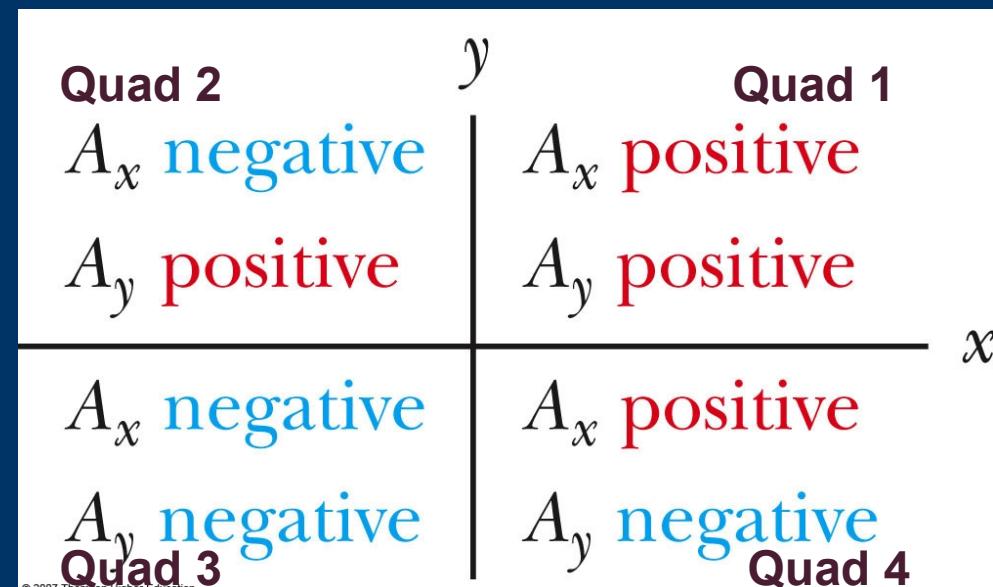
Then expect the position vector to have:

$$\text{Quad 1 } 0 < \theta < 90^\circ$$

$$\text{Quad 2 } 90 < \theta < 180^\circ$$

$$\text{Quad 3 } 180 < \theta < 270^\circ$$

$$\text{Quad 4 } 270 < \theta < 360^\circ$$



Motion in 2D. Vectors

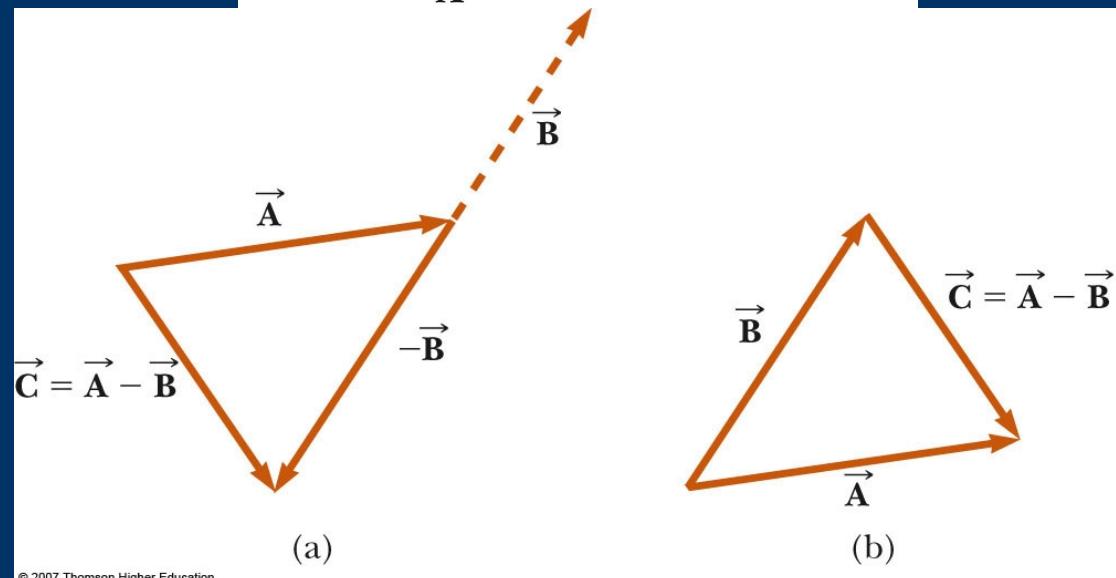
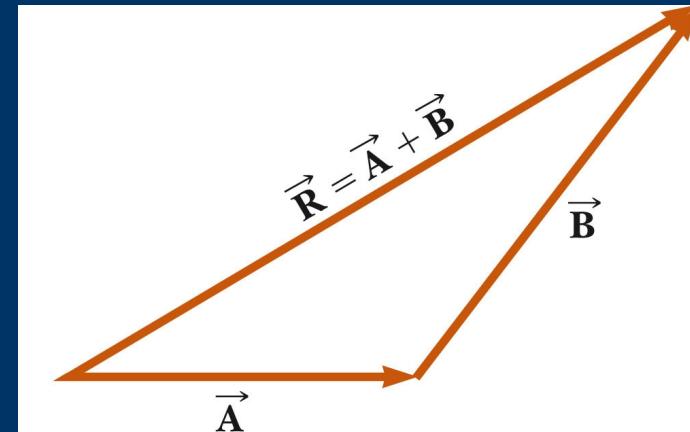
Vector addition and subtraction

Graphical approach

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

$$\mathbf{C} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

Tip-to-tail method



Algebraic approach

$$\mathbf{A} = A_x \hat{i} + A_y \hat{j}$$

$$\mathbf{B} = B_x \hat{i} + B_y \hat{j}$$

$$\mathbf{A} + \mathbf{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

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Motion in 2D.

Using vectors for position.

Top: position vector \mathbf{A} in 2-D.
Vector components are:

$$A_x = |\mathbf{A}| \cos \theta$$

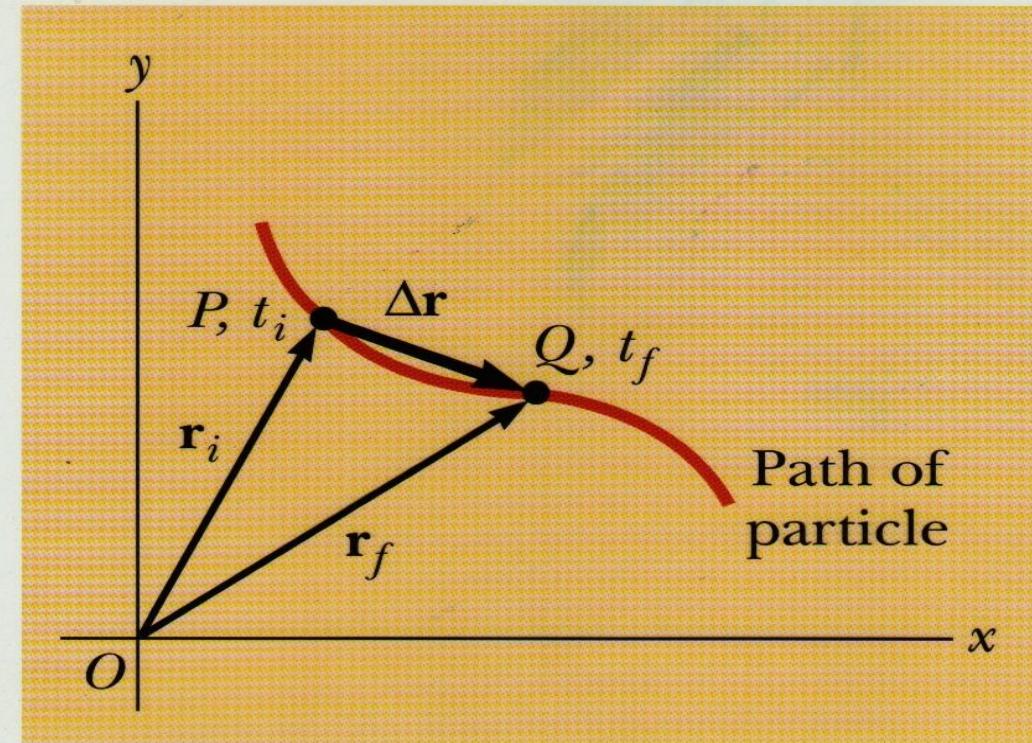
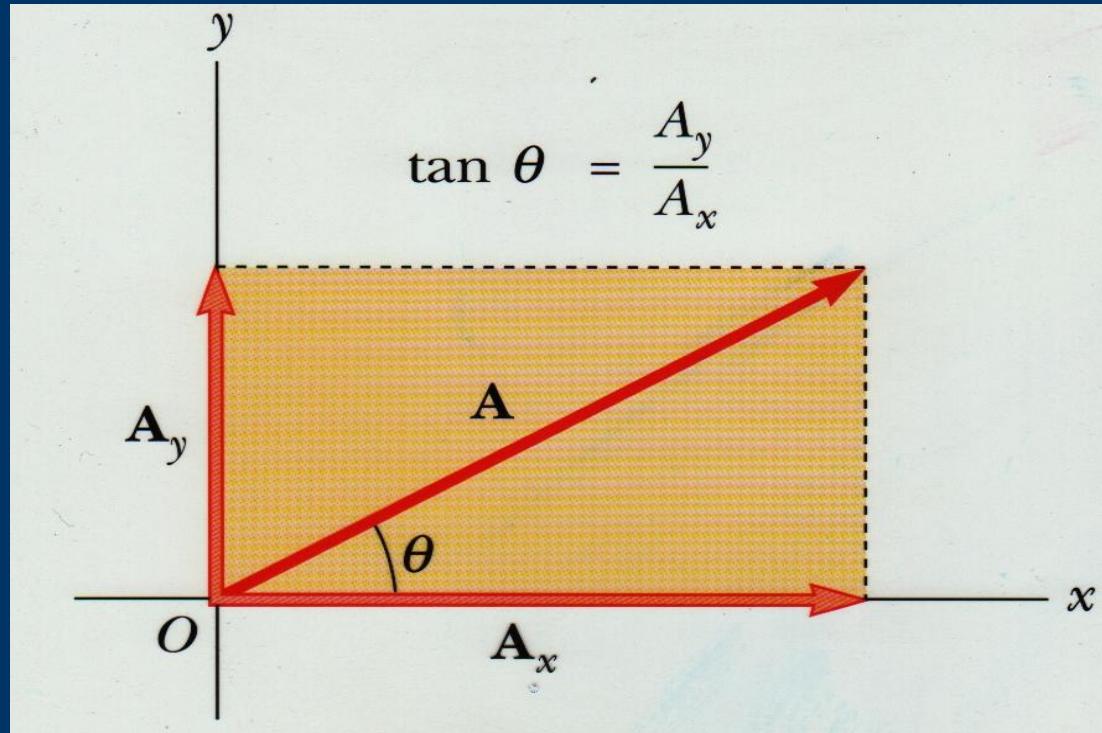
$$A_y = |\mathbf{A}| \sin \theta$$

$$\text{so } \mathbf{A} = A_x \hat{i} + A_y \hat{j}$$

$$|\mathbf{A}| = (A_x^2 + A_y^2)^{1/2}$$

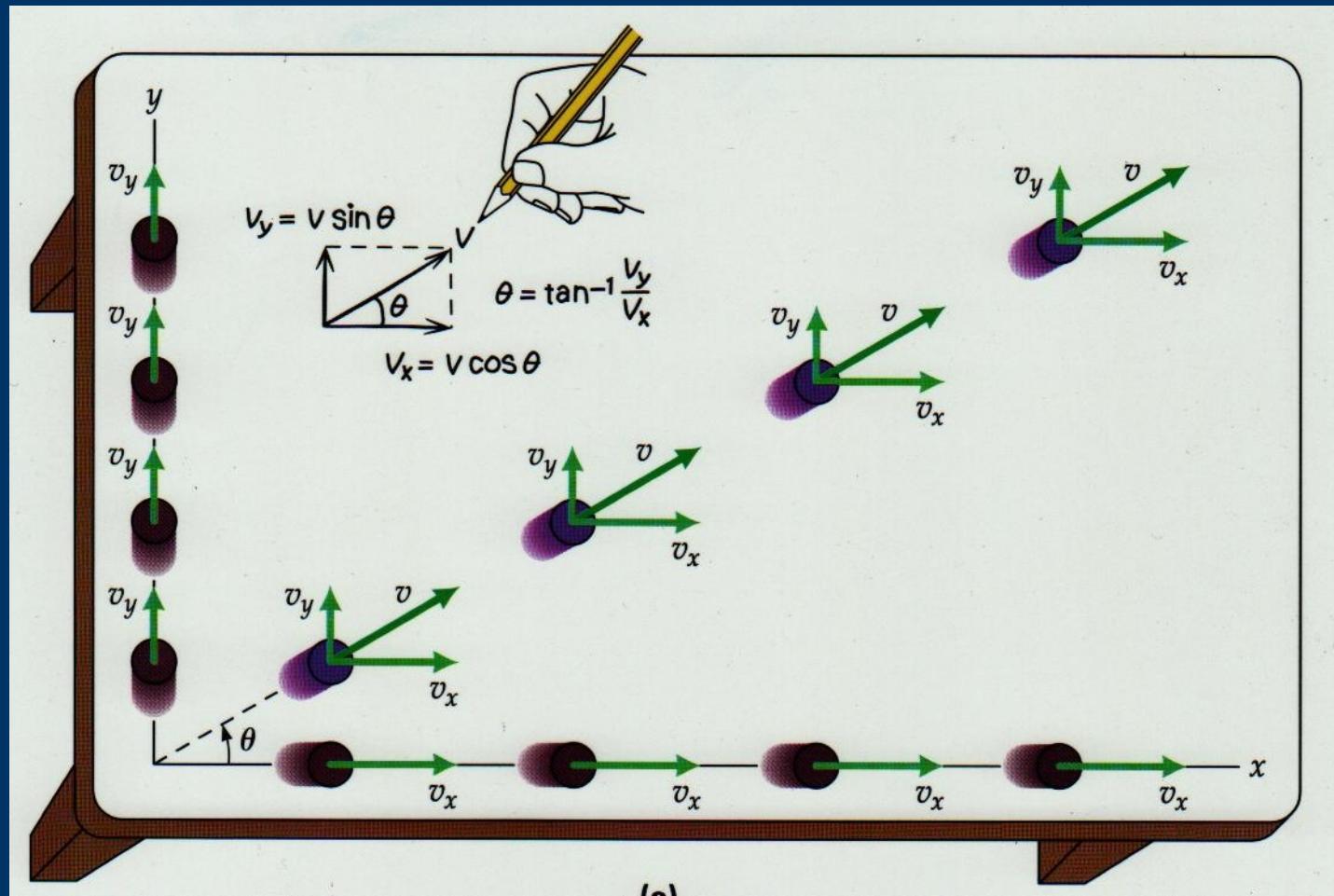
Bottom: change of the position vector \mathbf{r} gives a displacement $\Delta \mathbf{r}$.

Remember, $\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$



Motion in 2D.

Using vectors for velocity in 2-D.



Notice that this motion is all in a straight line and so could be expressed with 1 dimension (using a rotated axis).

Motion in 2-D (and beyond)

Definitions

Definitions ...

(Most of these are very similar to the Ch. 2 equations)

Position vector:

$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$$

Displacement:

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

Average velocity:

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

. Instantaneous velocity:

$$\vec{v}_{inst} = \frac{d\vec{r}}{dt}$$

Average acceleration:

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

. Instantaneous acceleration:

$$\vec{a}_{inst} = \frac{d\vec{v}}{dt}$$

Equations of Uniform acceleration

Final velocity

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

Average Velocity

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

Position as function of time:

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}t$$

Position as function of time:

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

Velocity change related to position change:

$$\vec{v}_f \cdot \vec{v}_f - \vec{v}_i \cdot \vec{v}_i = 2 \vec{a} \cdot (\vec{r}_f - \vec{r}_i)$$

Physics 2311 – Physics I, Week 4

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Outline for Day W3,D3

1D kinematics: free-fall example

2D kinematics:

Radial and tangential acceleration in “curvy” motion

Projectile motion

Homework

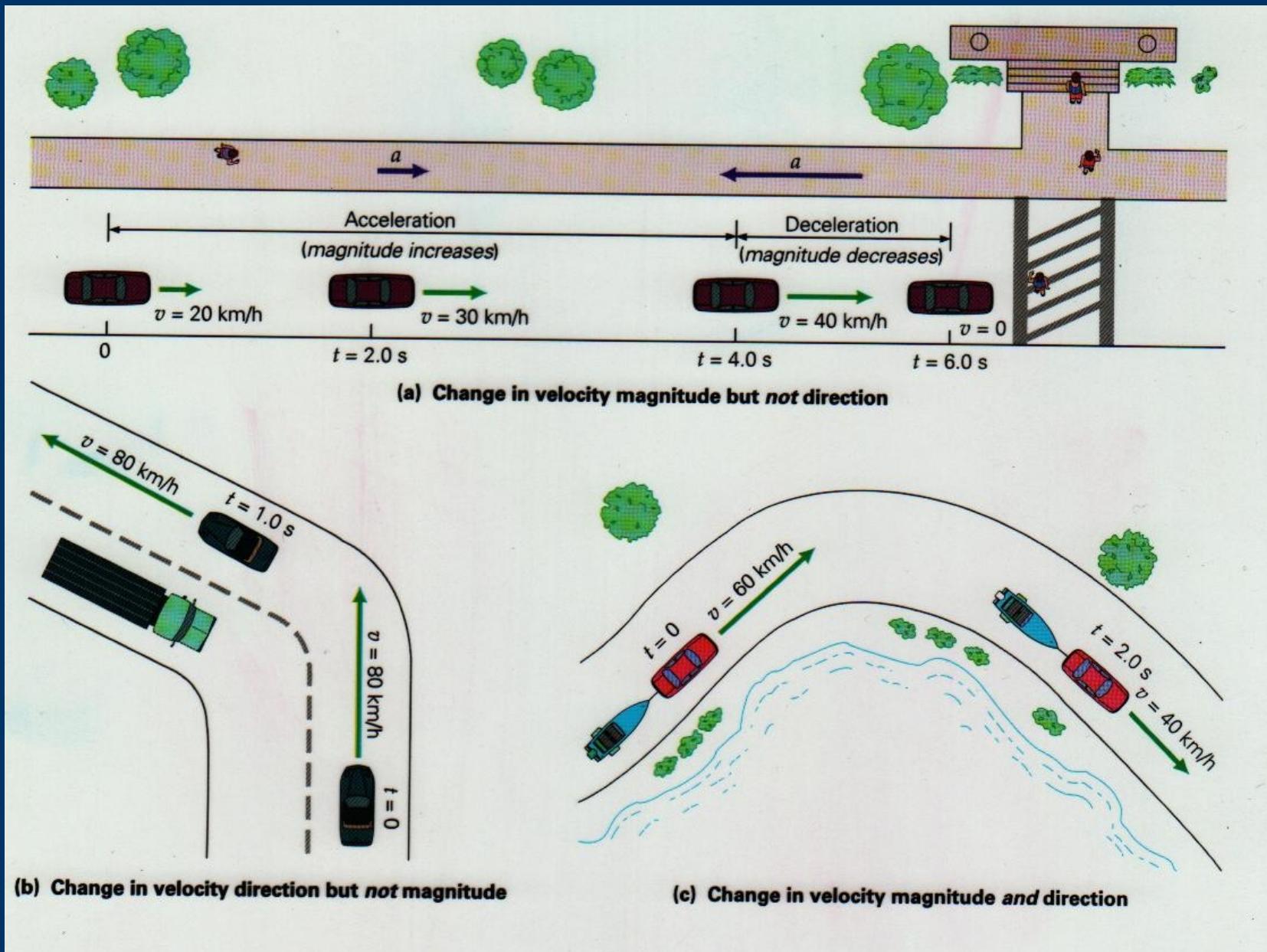
Ch. 3 P. 1,3,6,7,10,11,19,20,23,24,
32,33,37,38,39 Do by Mon.

Notes: Lab this week: “Graphs and Tracks”

Under NEW STUFF: “Exam like problems” for Ch. 2
and “Key to exam-like problems for Ch 1&2”

Top: Motion in 1D

Bottom: Motion in 2D.



Show 4.16.swf - acceleration has a radial and tangential component.

Motion in 2 dimensions. General motion.

In the most general case, there could be acceleration in both the x and y directions:

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

And so the 2D position is:

$$\mathbf{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$

and the 2D velocity is:

$$\mathbf{v}(t) = dx/dt\hat{i} + dy/dt\hat{j} = (v_{0x} + a_x t)\hat{i} + (v_{0y} + a_y t)\hat{j}$$

and the 2D acceleration is:

$$\mathbf{a}(t) = dv/dt = a_x\hat{i} + a_y\hat{j}$$

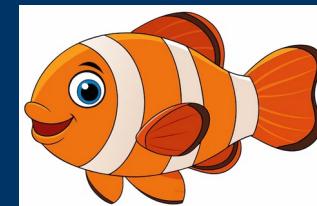
This is equivalent to:

$$\vec{r}_f = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a} t^2$$

with:

$$\mathbf{r}_0 = x_0\hat{i} + y_0\hat{j}$$

Example: swimming fish problem...



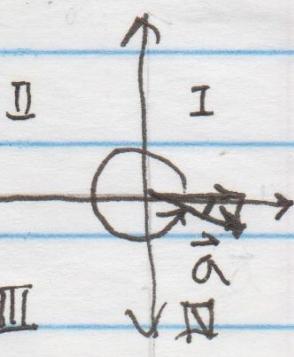
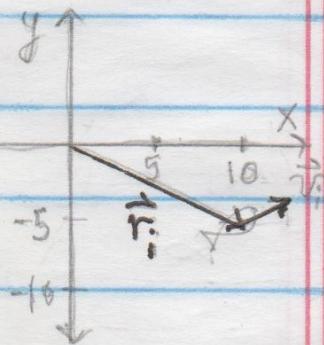
Ex) A fish swims in a horizontal plane with $\vec{v}_i = 4\hat{i} + \hat{j}$ m/s at a position vector of $\vec{r}_i = 10\hat{i} - 4\hat{j}$ m. The fish swims with uniform acceleration $\vec{a} = \underline{\quad}$ for $t = 20$ seconds until $\vec{v}_f = 20\hat{i} - 5\hat{j}$ m/s.

a) What is \vec{a} ?

$$\text{Sol'n: since } \vec{a} \text{ is uniform, } \vec{a} = \vec{a}_{\text{avg}} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{(20\hat{i} - 5\hat{j}) - (4\hat{i} + \hat{j})}{20} \\ = \frac{(16\hat{i} - 6\hat{j})}{20} \\ = 0.8\hat{i} - 0.3\hat{j} \text{ m/s}^2$$

b) Find direction of \vec{a} .

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-0.3}{0.8}\right) = [-20.6^\circ] \\ (\text{or } \theta = 339.4^\circ)$$



Fish problem (cont.)

c) If \vec{a} is maintained, where is fish at $t=25s$, and in what direction is \vec{v} ?

$$\begin{aligned}\vec{r}(t=25) &= \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2 && (\text{new } 2.10) \\ &= (10\hat{i} - 4\hat{j}) + (4\hat{i} + 1\hat{j})t + \frac{1}{2}(0.8\hat{i} - 0.3\hat{j})t^2 \\ &= [10 + 4(25) + 0.4(25)^2]\hat{i} + [-4 + 1(25) - \frac{0.3}{2}(25)^2]\hat{j}\end{aligned}$$

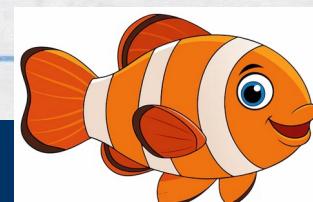
$$\boxed{\vec{r}(t=25) = 360\hat{i} - 72.75\hat{j} \text{ m}}$$

And $\vec{v}(t=25) = \vec{v}_i + \vec{a}t$ (new 2.12)

$$= (4\hat{i} + \hat{j}) + (0.8\hat{i} - 0.3\hat{j})(25)$$

$$= [4 + 0.8(25)]\hat{i} + [1 - 0.3(25)]\hat{j}$$

$$\vec{v}_f = 24\hat{i} - 6.5\hat{j} \quad \text{so } \theta_v = \tan^{-1}\left(\frac{-6.5}{24}\right) = \boxed{-15.15^\circ}$$

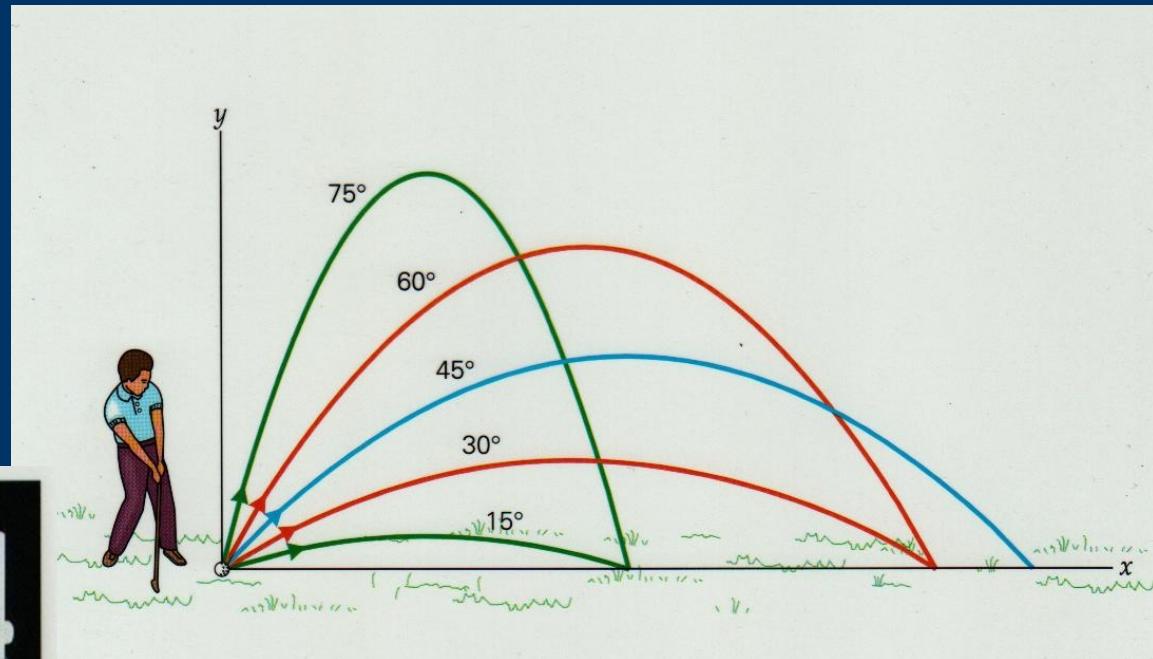
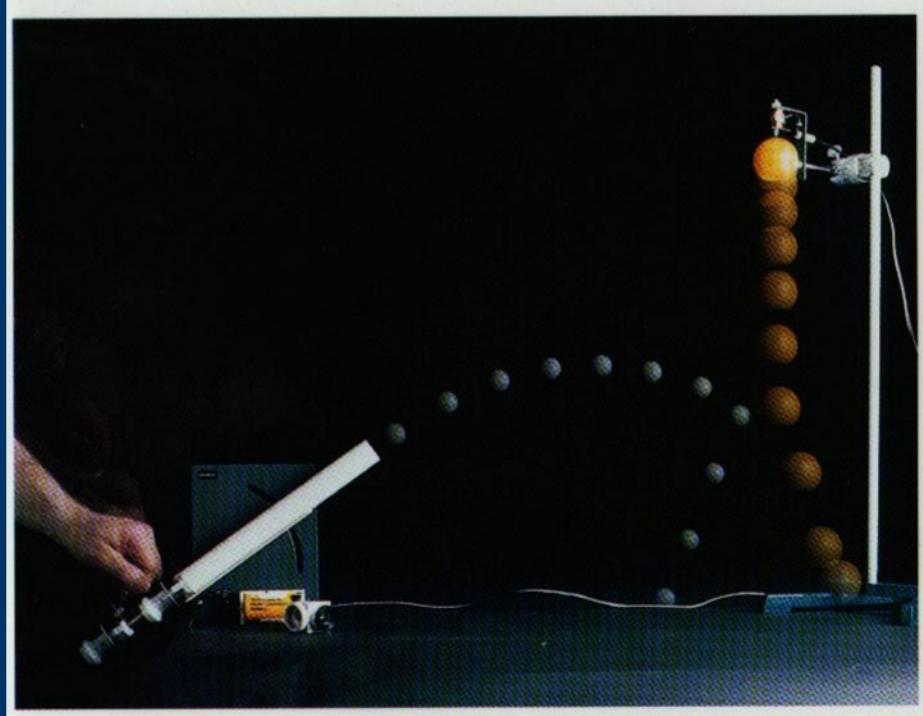


Motion in 2 dimensions. Projectile Motion.

P.M. is 2-D motion when the only acceleration is due to gravity.
That is:

$$\mathbf{a} = 0\hat{i} - g\hat{j}$$

This leads to parabolic trajectories.

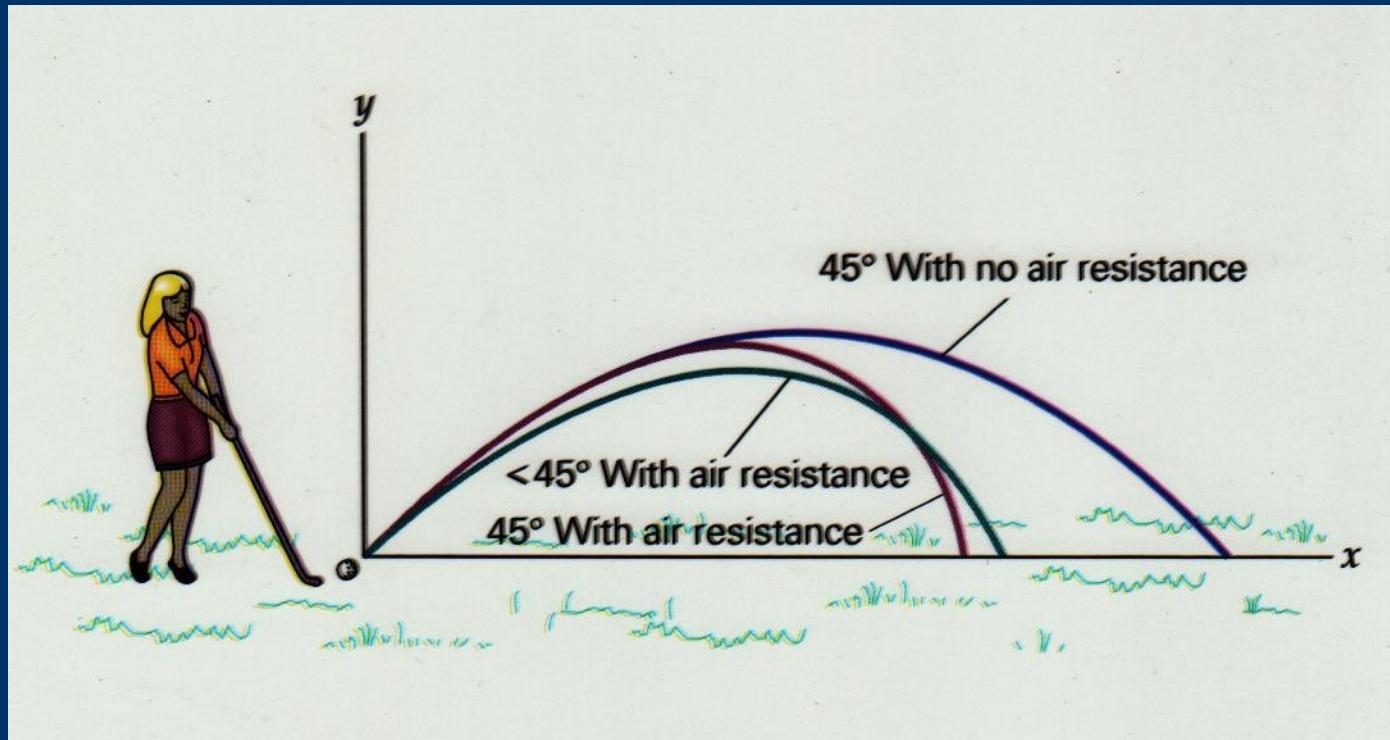


Notice that 2 initial angles lead to the same final *range*, except 45 degrees.

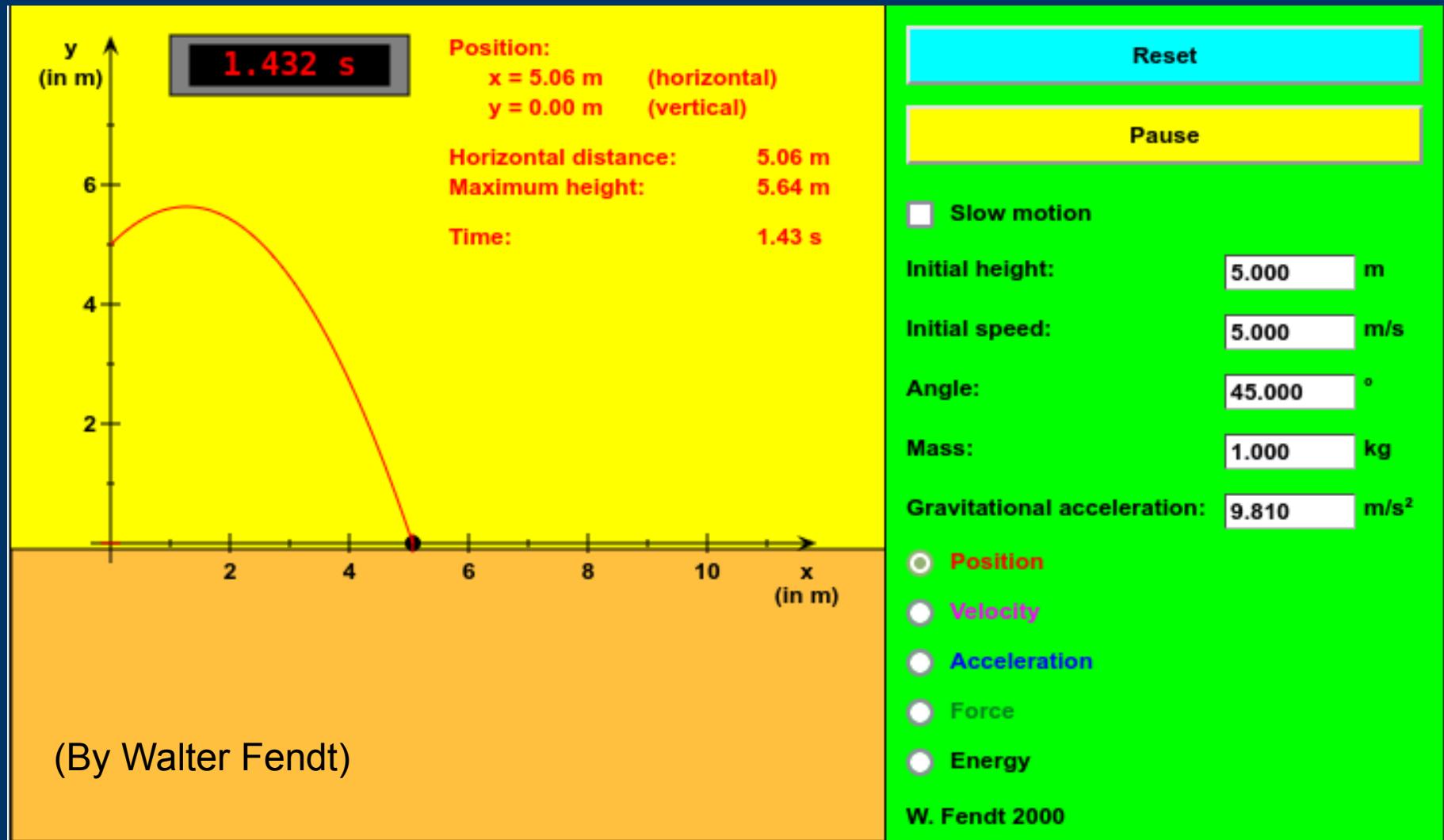
$$R = \frac{v_0^2 \sin 2\theta}{g}$$

PHYS 2311 Motion in 2 dimensions. Projectile Motion.

Actual trajectories: parabolas distorted by air resistance (drag).



Motion in 2 dimensions. Projectile Motion



Trajectories are specified with an initial position, velocity (or speed & inclination angle), and acceleration.

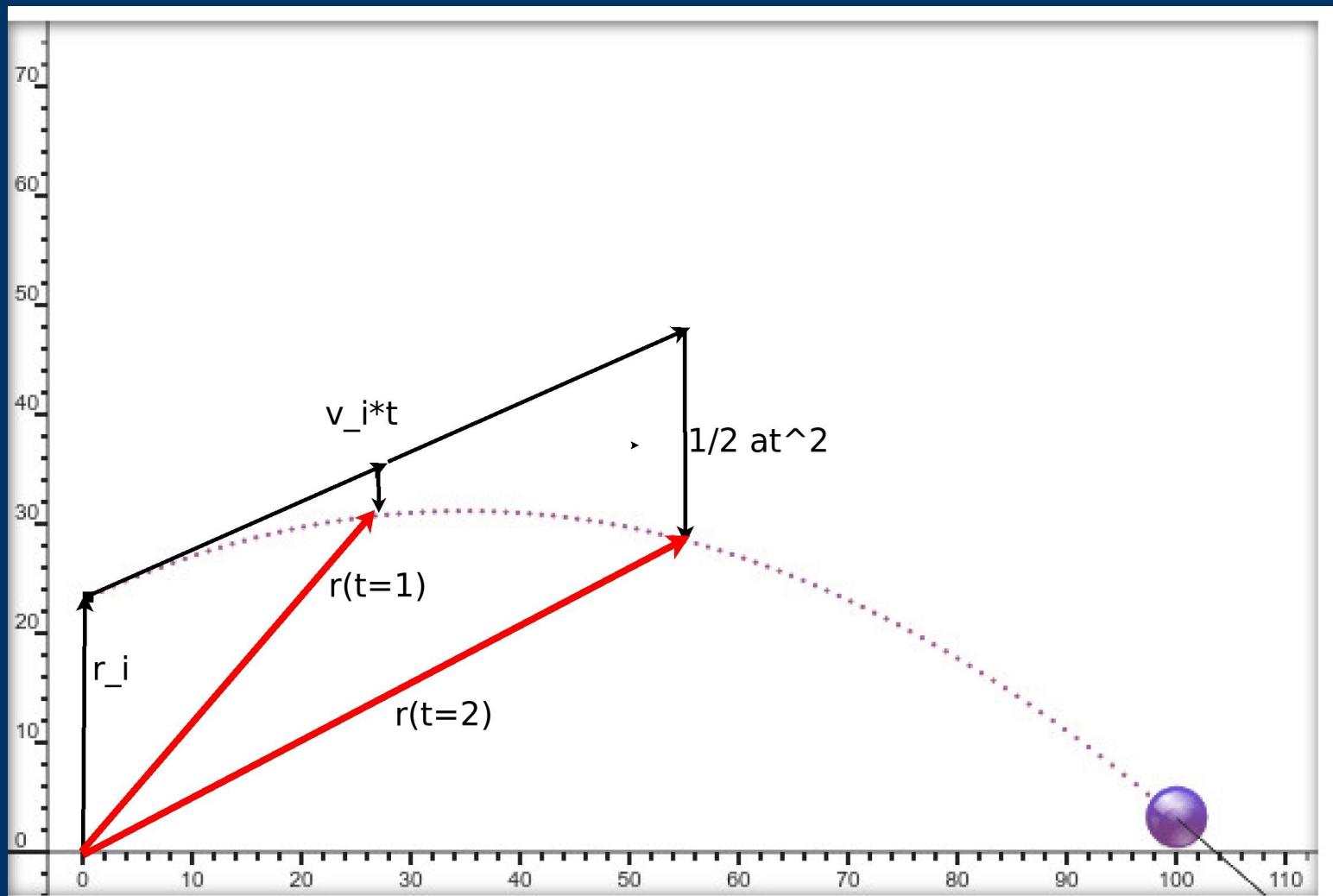
Motion in 2 dimensions. Projectile Motion

Trajectories: the position vector (red) is a sum of 3 vectors!

$$\vec{r}(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

... or 2 vectors:

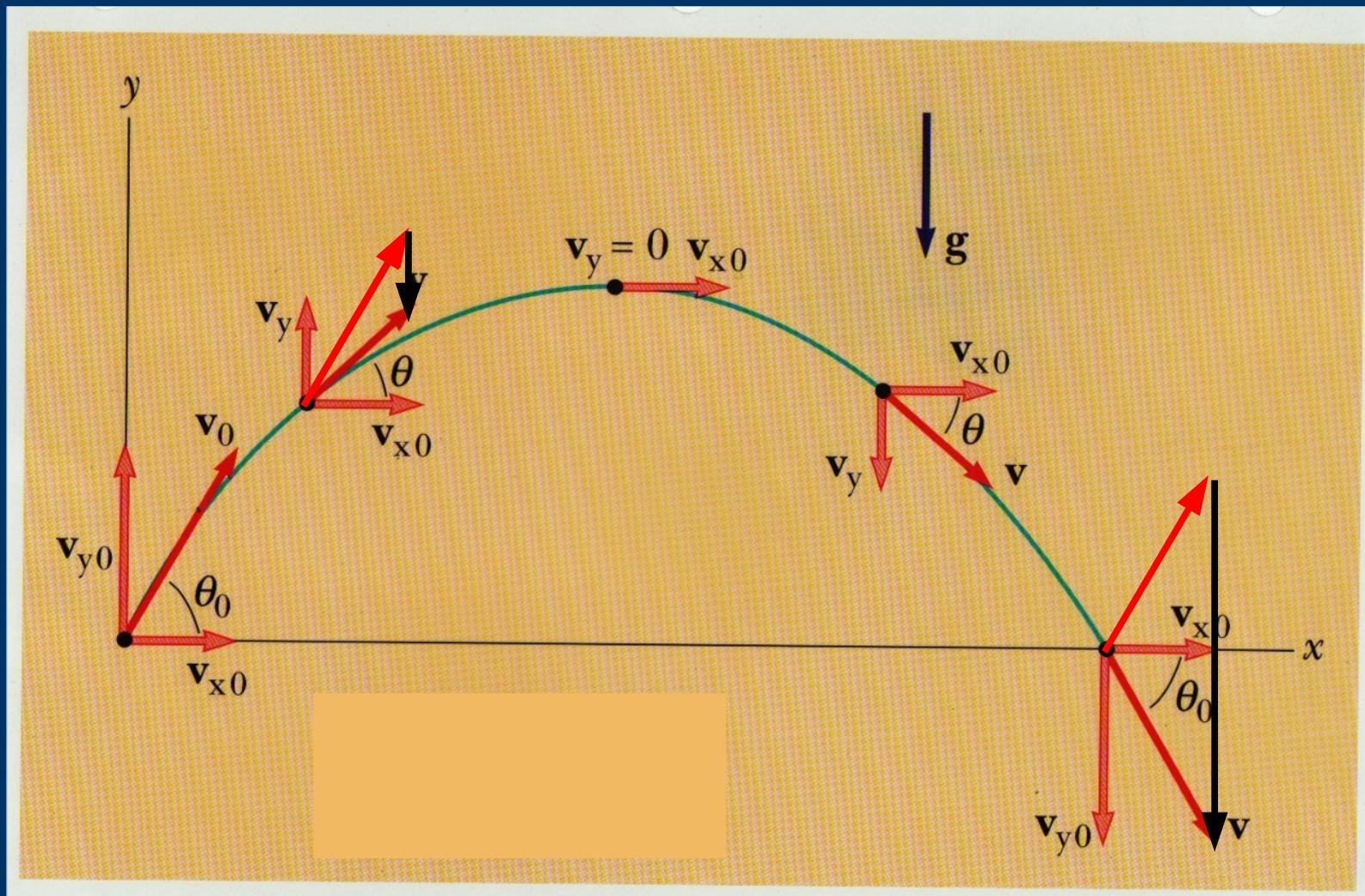
$$\vec{r}(t) = x(t) \hat{i} + y(t) \hat{j}$$



Motion in 2 dimensions. Projectile Motion

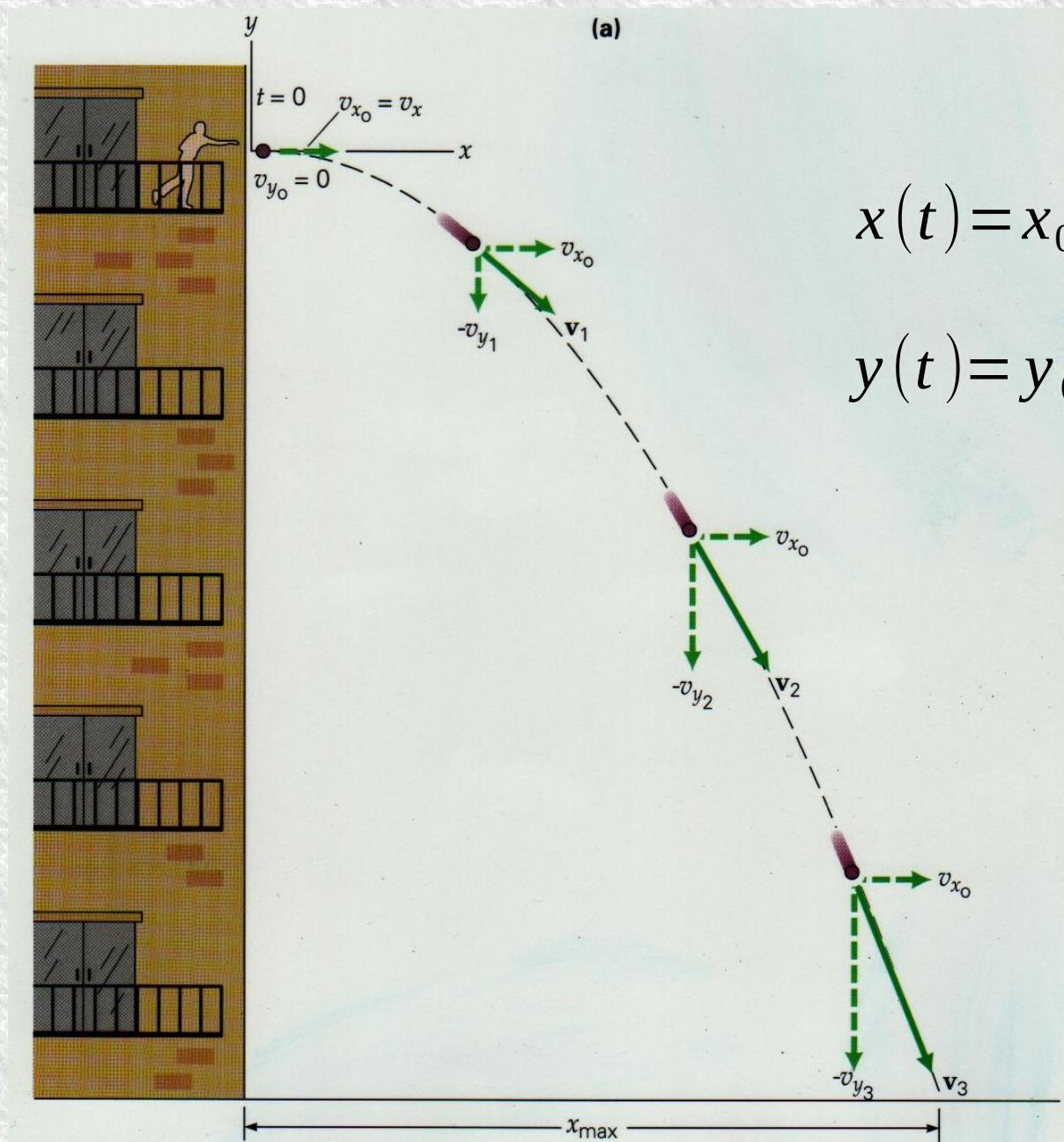
Trajectories: the velocity vector is a sum of 2 vectors.

$$\vec{v}(t) = \vec{v}_0 + \vec{a} t \quad \text{or} \quad \vec{v}(t) = v_{x,0} \hat{i} + v_y \hat{j}$$



P231 Week 3

Independence of x and y components

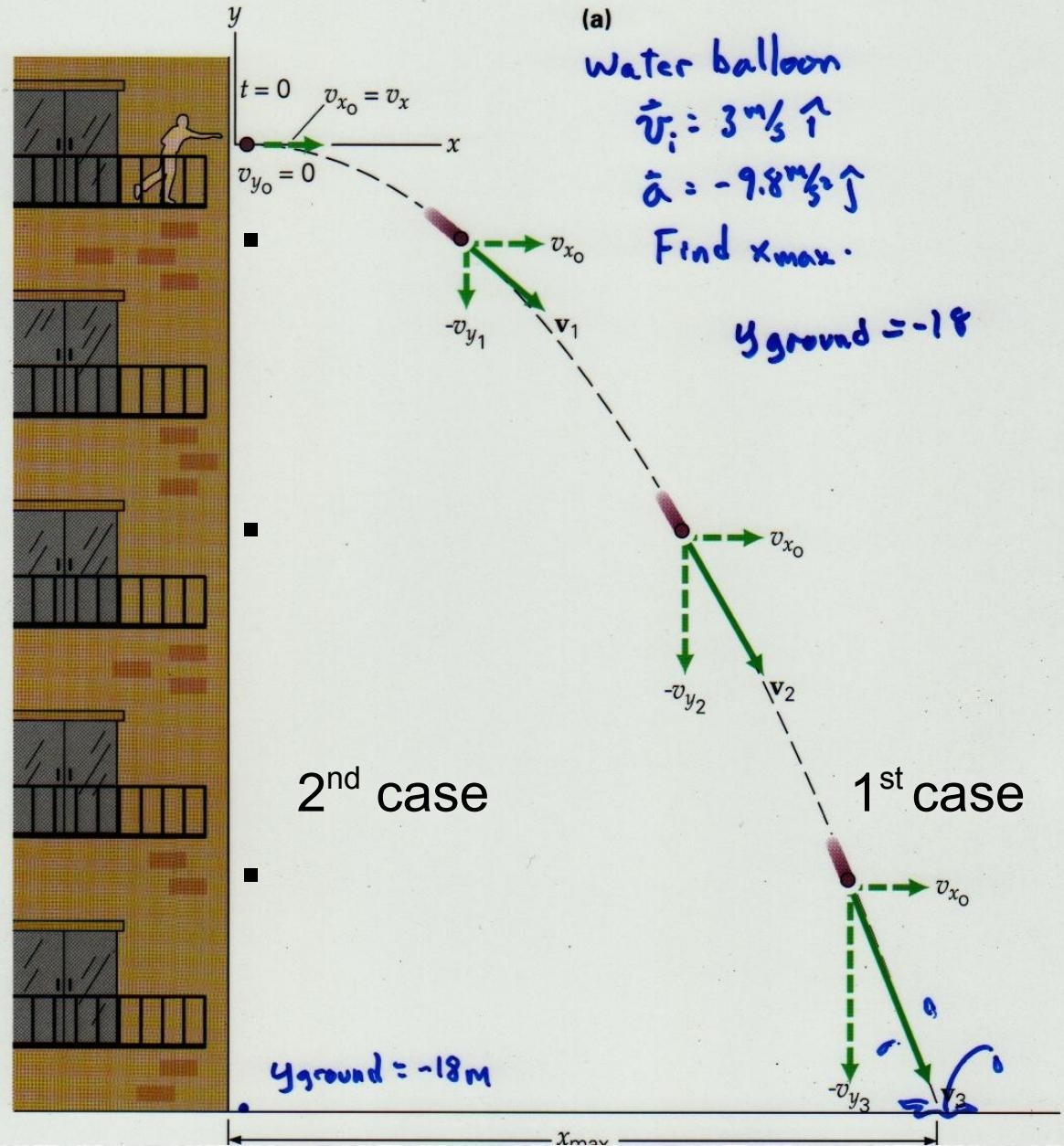


P231 Week 3

Independence of x and y components

If dropped from rest, the vertical progress is identical!

1st case: $v_{x_0} = v_x$
 2nd case: $v_{x_0} = 0$



Motion in 2-D

Projectile Motion formulas

Time to reach max height:

$$t_{max} = \frac{v_i \sin \theta_i}{g}$$

(v_i is the magnitude of the initial velocity)

Maximum height:

$$h_{max} = \frac{v_i^2 \sin^2 \theta}{2g}$$

Range:

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

To derive t_{max} , ask yourself “at what time does the vertical speed reach 0?”

$$v_y = 0 = v_{0y} + a_y t_{max} \quad \text{with } a_y = -g \text{ and } v_{0y} = v_i \sin \theta$$

To derive h_{max} , use $y_{max} = y(t_{max}) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$

To derive R, use $R = x(t=2t_{max}) = x_0 + v_{0x} t$