PHYS 2321
Week5-6: Exam I / Capacitance

W6 Day 1 Outline

1) Hwk: Ch. 24 MiscQ. 1-13 odd   Due Friday
   Prob. 1,5,6, (more to come)
2) Review Exam I (mean = 15.9/32)
3) Capacitors and capacitance
   a. Examples of capacitors
   b. $C \equiv Q/\Delta V$ = charge per volt across capacitor
   b. $C = A\varepsilon_0/d$ for parallel plate capacitor

Notes:

PHYS 2321
Week6: Capacitance

W6 Day 2 Outline

1) Hwk: Ch. 24 MiscQ. 1-13 odd   Due Friday
   Prob. 1,5,6,7,10,11,13,16,21,22,24,38,39,40,53
2) Capacitors and capacitance
   a. Examples of capacitors
   b. $C \equiv Q/\Delta V$ = charge per volt across capacitor
   c. $C = A\varepsilon_0/d$ for parallel plate capacitor
   d. Calculating capacitance
   e. Parallel and series connections

Notes:

Try Practice Quiz for Ch. 24.

Types of capacitors

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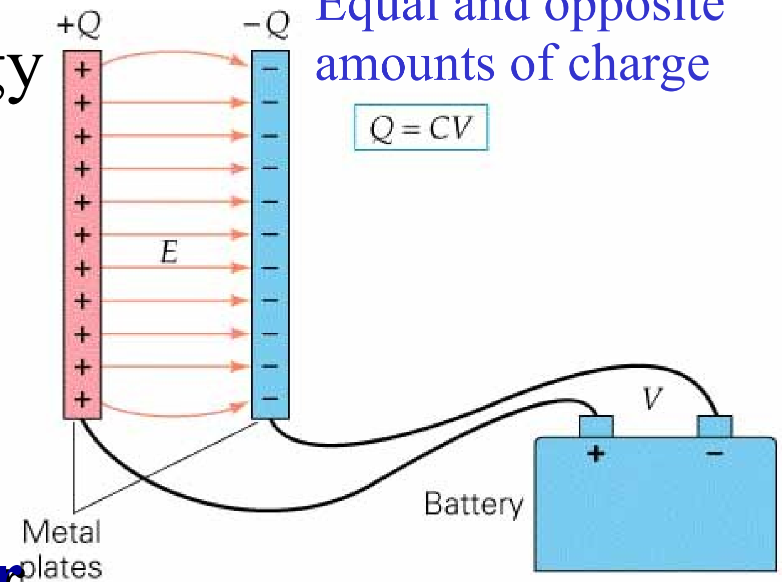
Capacitors

- Charge separation stores energy
- Parallel Plates
  - Uniform E field
  - V=ΔV between plates

Equal and opposite amounts of charge

$\boxed{Q = CV}$



(a) Parallel-plate capacitor

- **Capacitance: the "charge per volt" on a capacitor**

$$C \equiv \frac{Q}{V}$$

Capacitance governs ...
How much charge is required to produce 1 volt "on" the capacitor (Q=CV),
What the potential difference will be if +-Q of charge is on the plates. (V=Q/C)
Units: 1 Farad = 1 Coulomb/Volt

Compute Capacitance of P.Plate Capacitor

1) Determine E-field between plates.

$$E=\frac{\sigma}{2\epsilon_0}+\frac{\sigma}{2\epsilon_0}=\frac{\sigma}{\epsilon_0}$$

2) Find $|\Delta V|$.

$$\Delta V=Ed\equiv\frac{\sigma d}{\epsilon_0}=\frac{Qd}{A\epsilon_0}$$

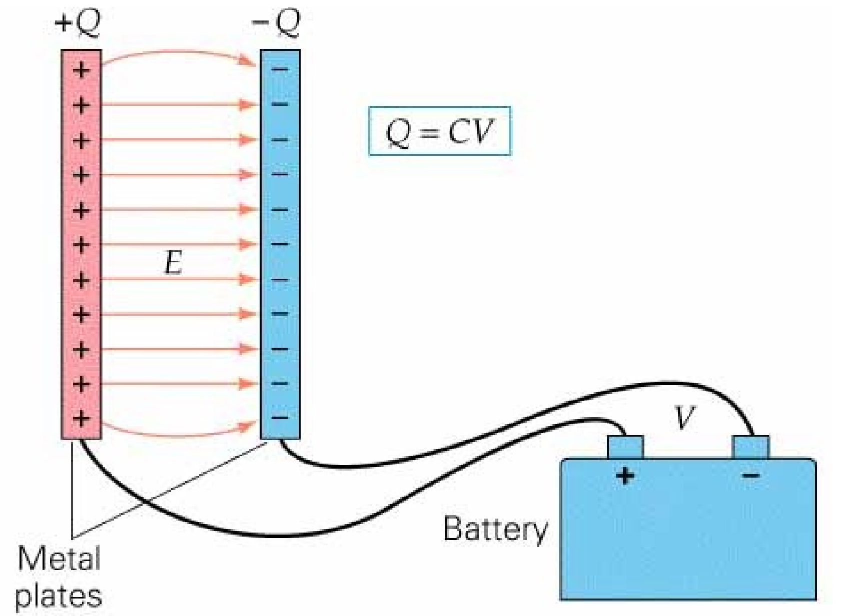
3) Insert into $C=Q/\Delta V$, and

4) Eliminate Q:

$$C=\frac{QA\epsilon_0}{Qd}=\frac{A\epsilon_0}{d}$$

5) Add k for dielectric:

$$\boxed{C=\kappa\frac{A\epsilon_0}{d}}$$



(a) Parallel-plate capacitor

For Parallel Plates only

$$\varepsilon_o=8.85\times10^{-12}\frac{C^2}{N\cdot m^2}$$

Calculating Capacitance

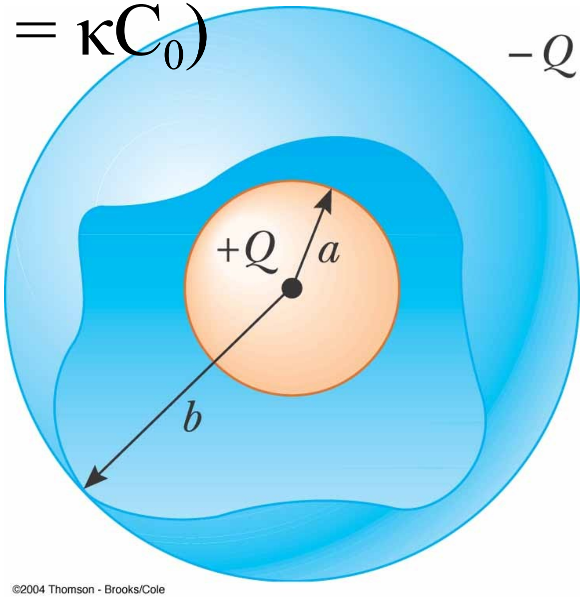
1) Determine E inside capacitor. (May need Gauss's law.)

$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{s}$$

2) Determine ΔV between the plates.
3) Insert ΔV into $C = Q/\Delta V$.
4) Cancel the Q's.
5) Consider the dielectric filler.   ($C = \kappa C_0$)

Example: concentric spheres

$$C_{spheres} = \frac{ab}{k_e(b-a)}$$



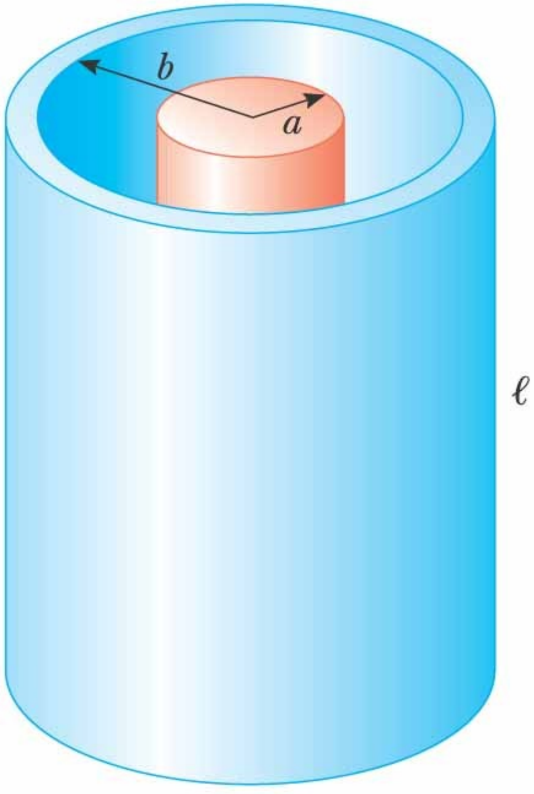
Calculating Capacitance

Other examples:

1) Parallel plates $C = \frac{A\epsilon_0}{d}$

2) Concentric cylinders $\frac{l}{2k_e \ln(b/a)} = \frac{2\pi l\epsilon_0}{\ln(b/a)}$

3) single sphere $C = 4\pi\epsilon_0 R$

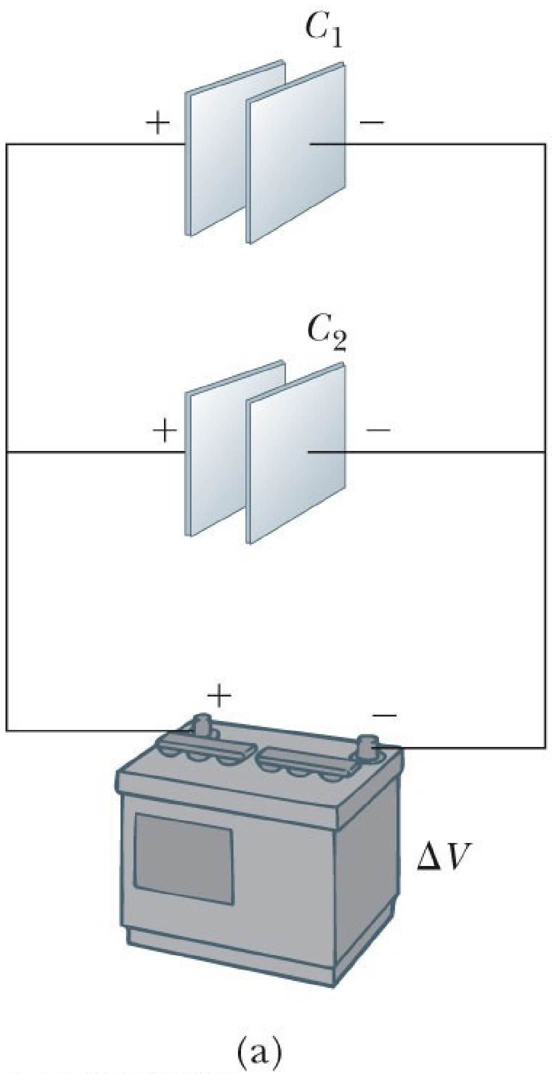


(a)

Example: concentric cylinders

Combining Capacitors

- Two fundamental arrangements: parallel and series.
- Characteristics of a parallel connection:

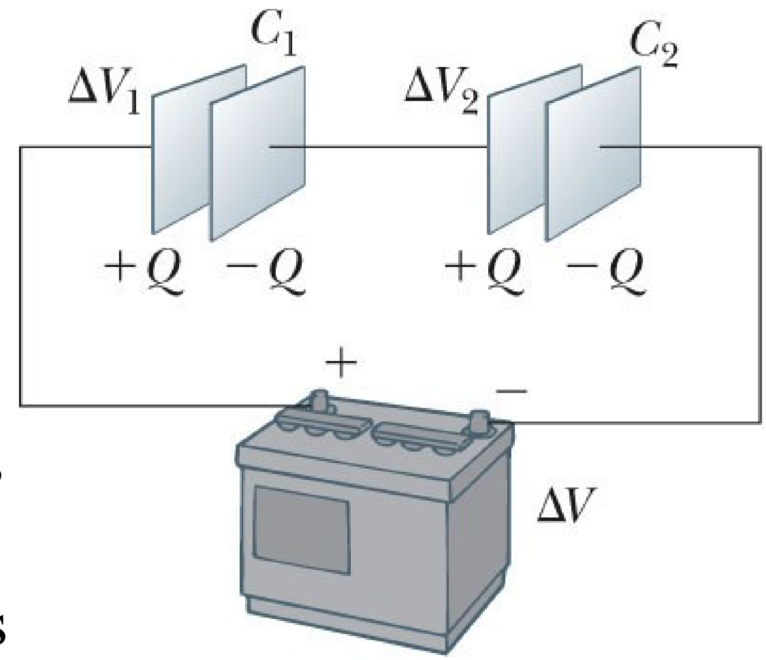


1) The same ΔV is across each capacitor.
2) If C1≠C2, than Q1≠Q2. (In general, the charges are unequal.)
3) Since the total area of the plates is greater for 2 capacitors than 1, the equivalent capacitance goes up.
4) For N capacitors in parallel:

$$Ceq = C_1+C_2+C_3+...+C_N$$

Combining Capacitors

- Characteristics of a series connection:



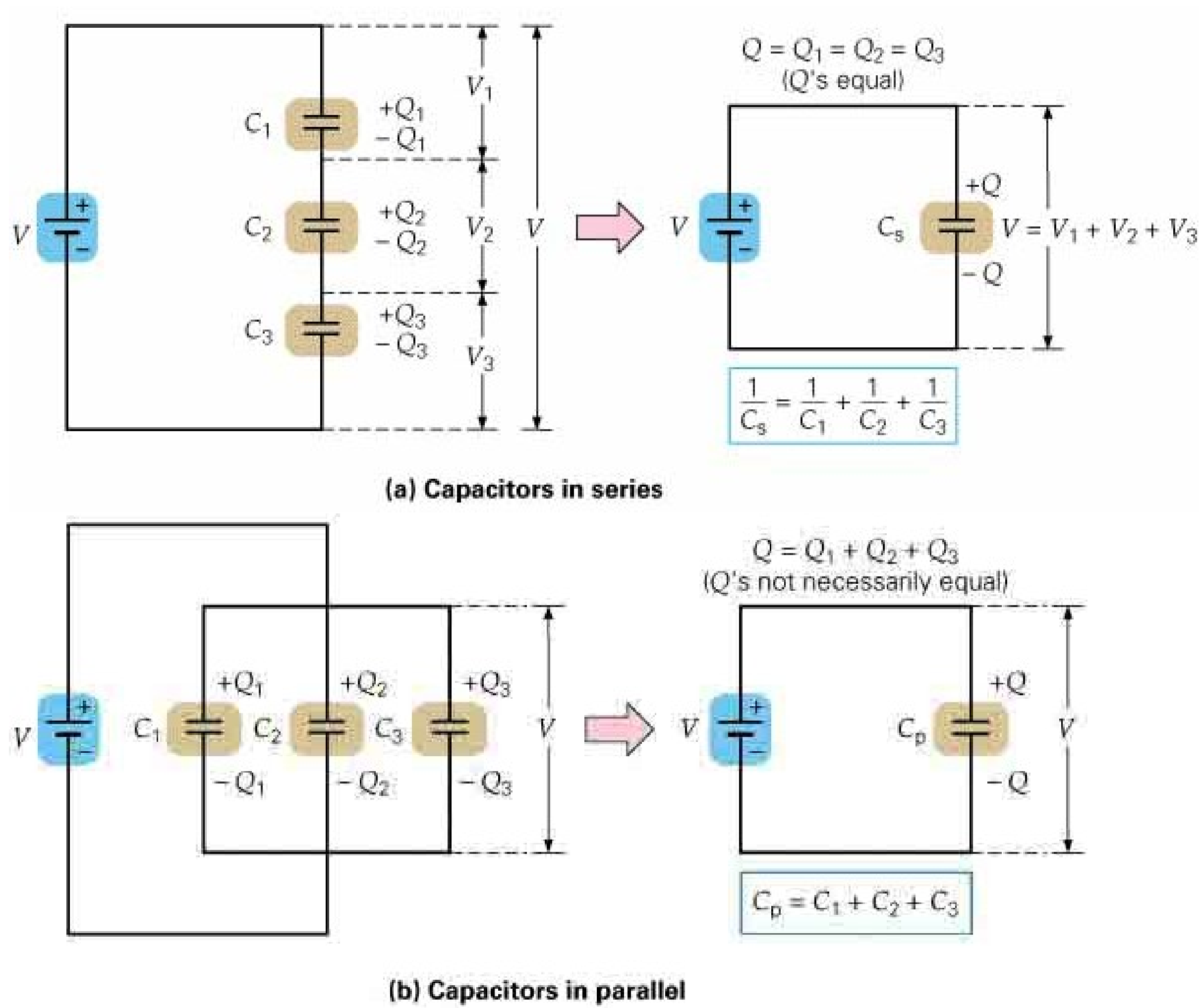
(a)

1) The same charge Q exists on each plate!
2) If $C_1 \neq C_2$, then $\Delta V_1 \neq \Delta V_2$. (In general, voltages are different on each cap..)
3) The sum of the ΔV's is the ΔV across the battery.
4) The equivalent capacitance is less than even the smallest capacitor in series.
5) For N capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

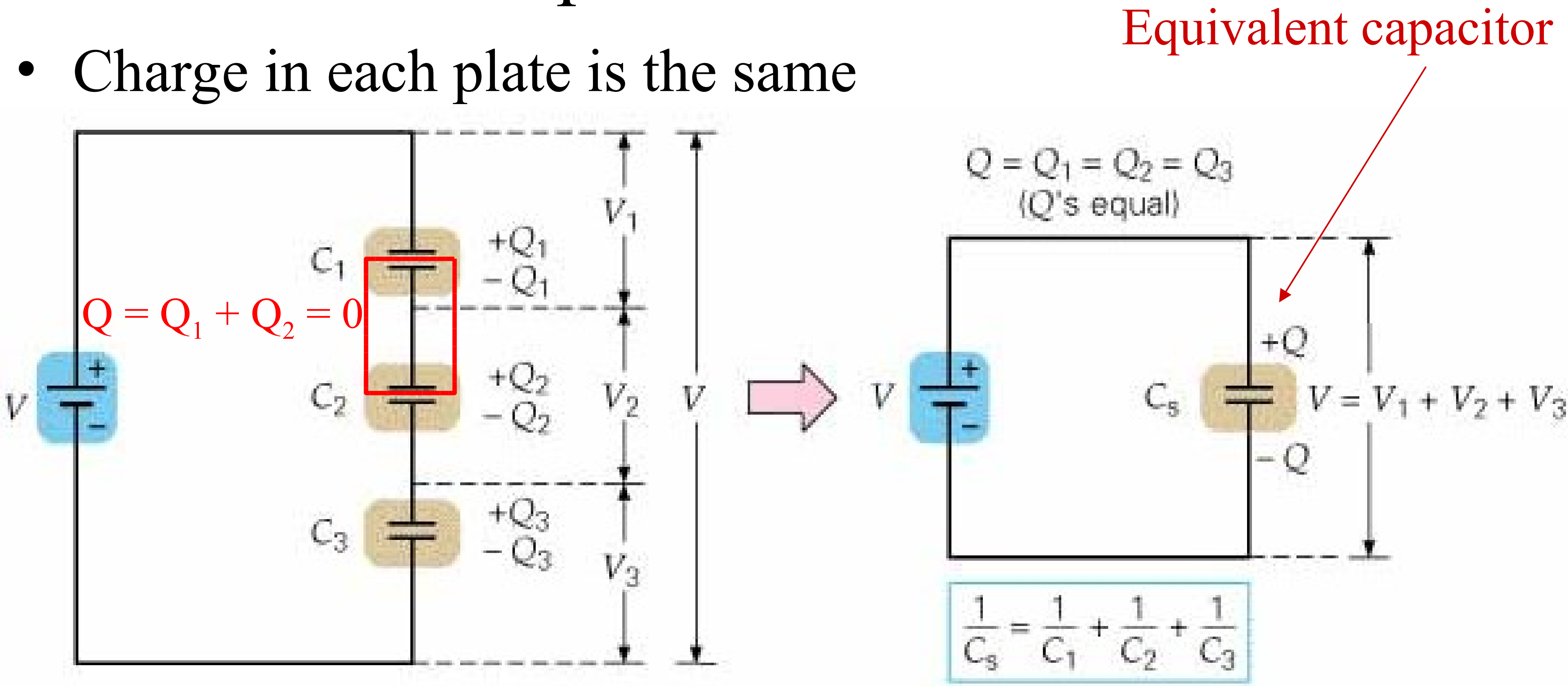
Combining Capacitors

- Goal: combine them into one *equivalent capacitance*



Capacitors in Series

- Charge in each plate is the same



(a) Capacitors in series

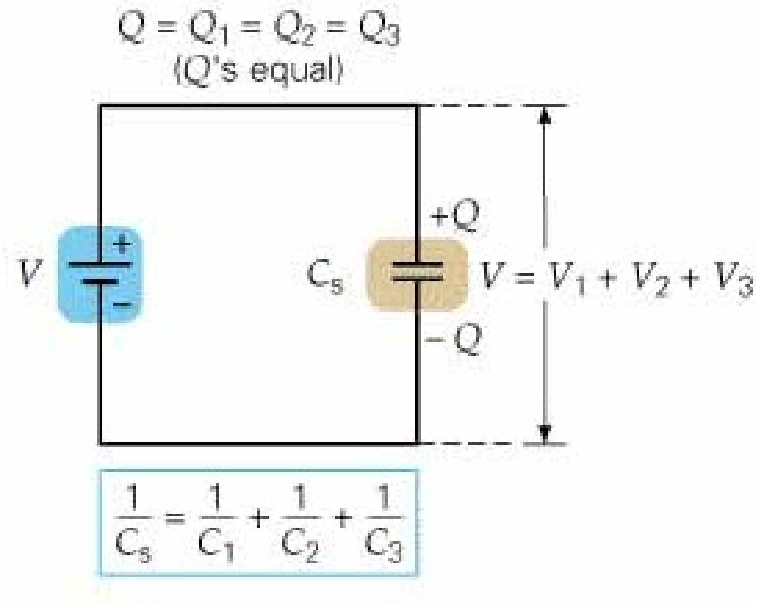
$$C_1 = \frac{Q}{V_1},\ C_2 = \frac{Q}{V_2},C_3 = \frac{Q}{V_3}$$

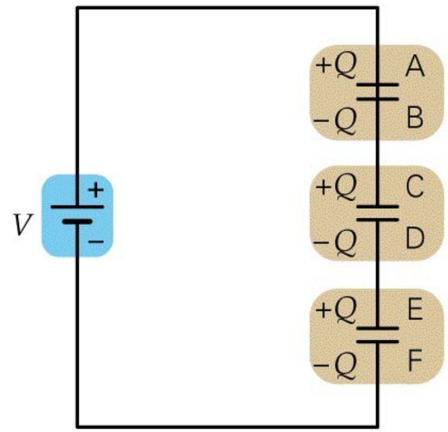
$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3}$$

$$V = V_1 + V_2 + V_3, V = \frac{Q}{C_s}$$

Capacitors in Series

- Usually get C_s to get Q, then figure out the V's





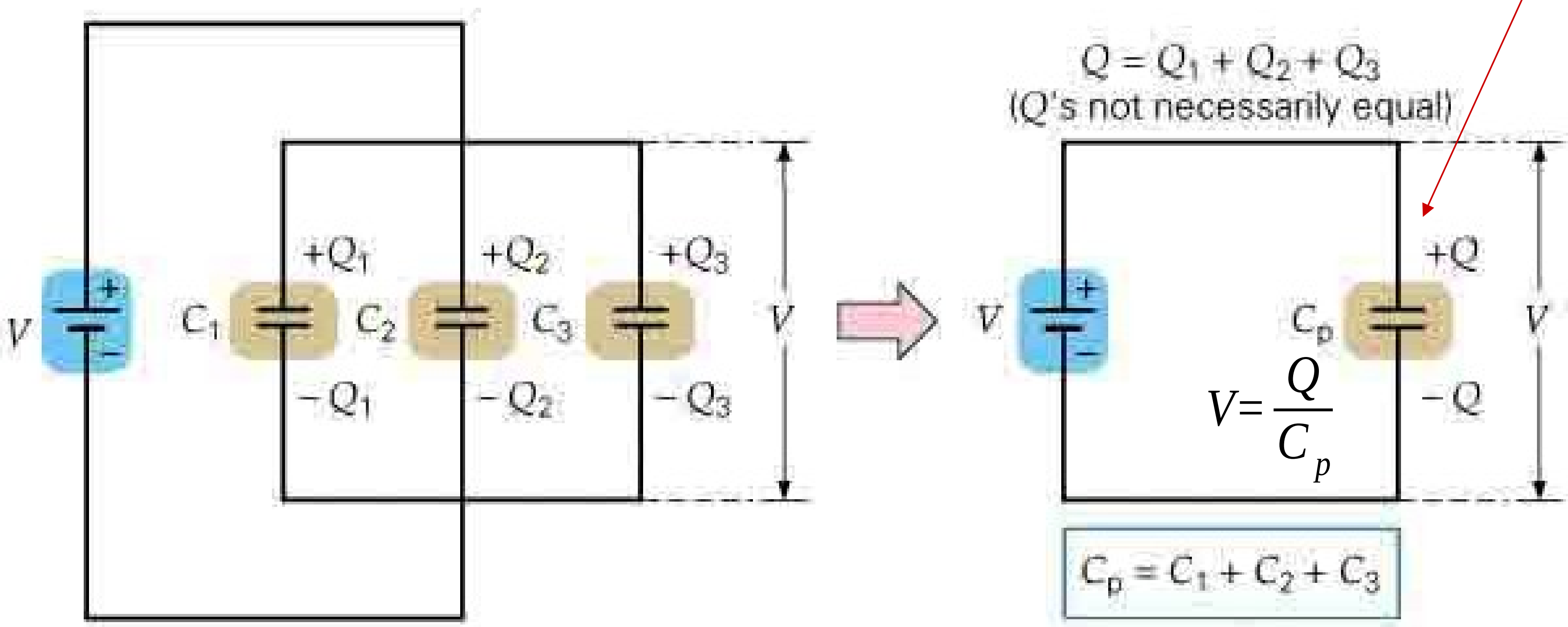
- Now expand the circuit back out

$$V_1 = \frac{Q}{C_1} \quad V_2 = \frac{Q}{C_2} \quad V_3 = \frac{Q}{C_3}$$

Capacitors in Parallel

- Voltage on each plate is the same $V = V_1 = V_2 = V_3$

Equivalent capacitor



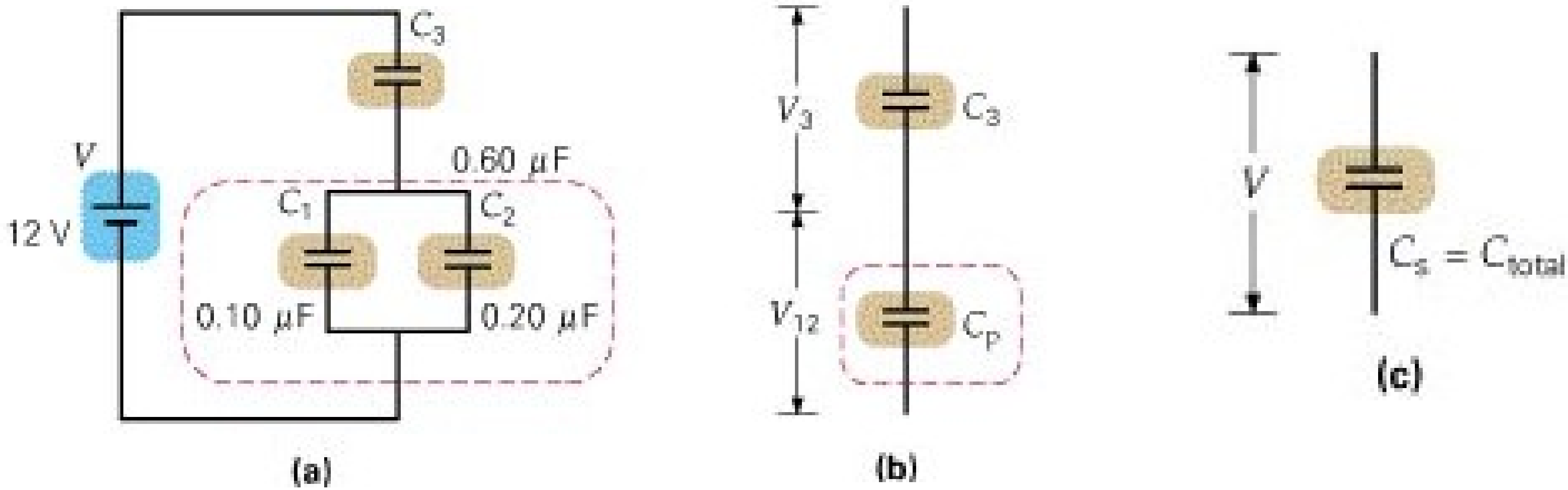
$$V = \frac{Q}{C_p}$$

(b) Capacitors in parallel

$$Q = Q_1 + Q_2 + Q_3 \qquad Q_1 = C_1 V \;\; Q_2 = C_2 V \;\; Q_3 = C_3 V$$

Combinations of Series and Parallel

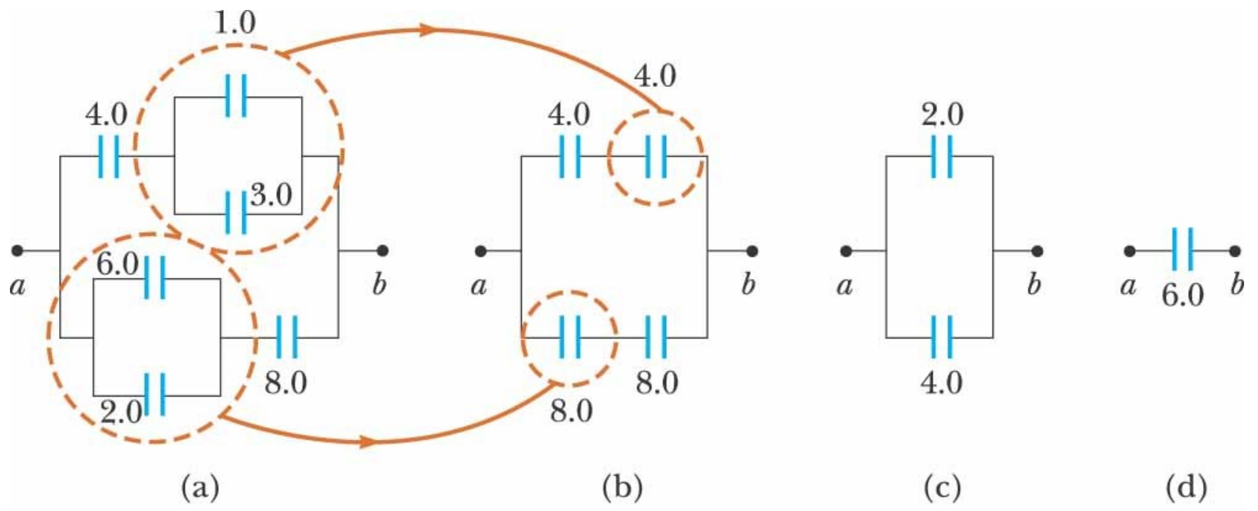
- Get to one equivalent capacitor



First combine C_1 and C_2 that are in parallel
Second combine C_{12} (C_p in diagram) and C_3 in series

CANNOT DO THE FOLLOWING:
C_3 and C_1 in series, then C_2 in parallel or series
C_3 and C_2 in series, then C_1

Equivalent Capacitance, Example



The 1.0-μF and 3.0-μF capacitors are in parallel as are the 6.0-μF and 2.0-μF capacitors

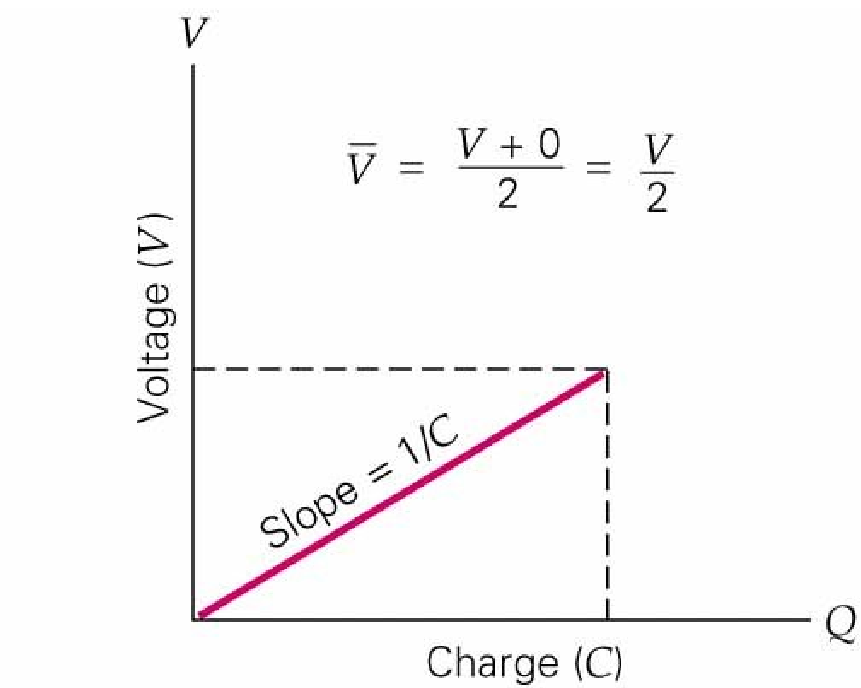
These parallel combinations are in series with the capacitors next to them

The series combinations are in parallel and the final equivalent capacitance can be found

Energy in a capacitor

- It takes energy to separate charge (& create E-field)
- The battery provides it, but that involves chemistry.
- Instead imagine charging plates "by hand"
  - Work by hand to move charge *dq*: $dW = dq\Delta V = dq\,q/C$
  - Total work to move all charge:

$$W = \int_0^Q \frac{q}{C}\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$



OR … area under V vs Q is energy!

$$\boxed{U_c = \frac{1}{2}QV = \frac{Q^2}{2C} = \frac{1}{2}CV^2}$$

Energy of a charged capacitor

Dielectrics

- Capacitor charged and **battery disconnected**
  - Q is constant - nowhere to go!

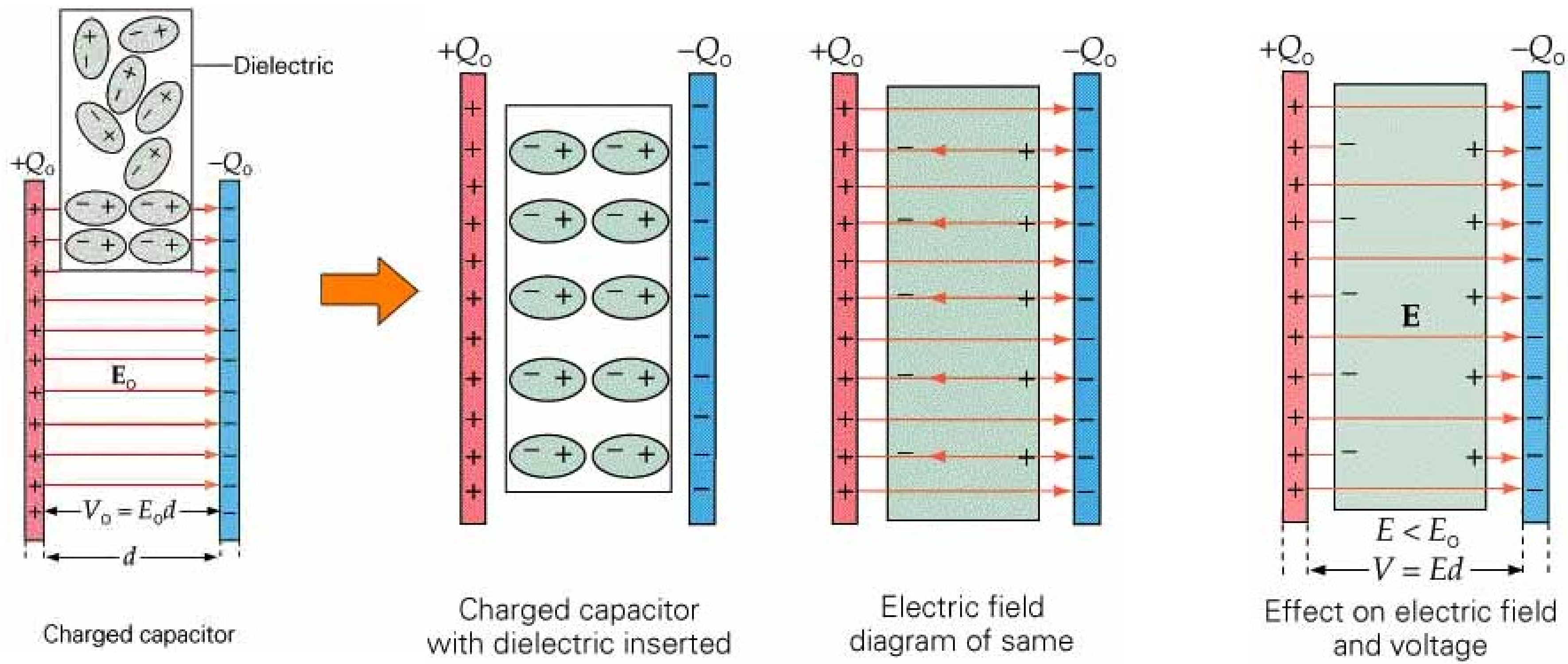


TABLE 26.1

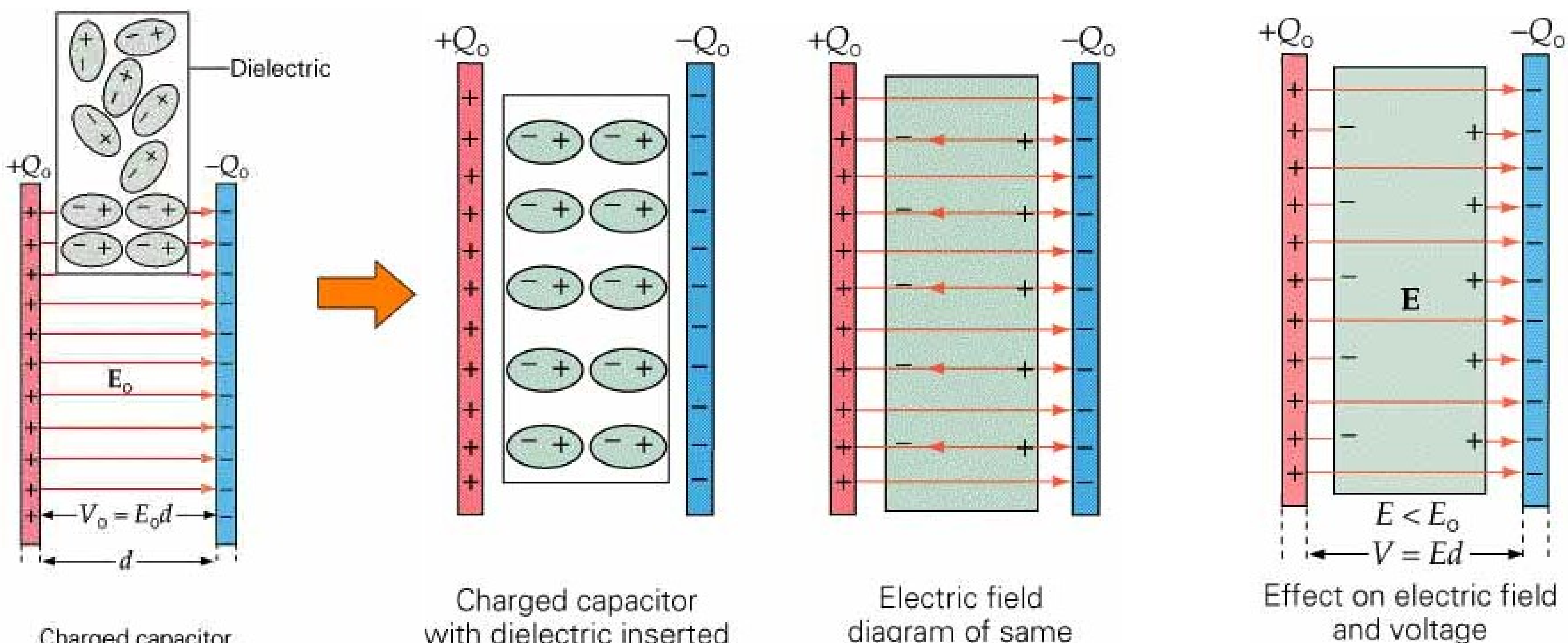
Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

Dielectrics

- Determine whether κ is greater than or less than 1:
  1) Molecules in dielectric get polarized
     2) Electric field between plates is reduced (Q is constant)
     3) ΔV across plates is reduced (remember ΔV = -Ed)
     4) U=1/2 QΔV is reduced, so must be U=1/2 Q/C
     5) Q is constant, so C must have increased.
     6) $C = \kappa C_0$ so $\kappa > 1$



Dielectrics

- Parallel Plates

$$C_o = \varepsilon_o \frac{A}{d}$$

$$C = \varepsilon \frac{A}{d} = K\varepsilon_o \frac{A}{d}$$

Dielectric **permittivity** $\varepsilon = K\varepsilon_o$