(a) For an electron,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{(9.11\times10^{-31}\ \text{kg})(9.80\ \text{m/s}^2)}{-1.60\times10^{-19}\ \text{C}}\hat{\mathbf{j}}$$

$$= \boxed{-(5.58\times10^{-11}\ \text{N/C})\hat{\mathbf{j}}}$$

(b) For a proton, which is 1 836 times more massive than an electron,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{(1.67\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)}{-1.60\times10^{-19}\ \text{C}}\hat{\mathbf{j}}$$

$$= \boxed{(1.02\times10^{-7}\ \text{N/C})\hat{\mathbf{j}}}$$

P23.24 In order for the object to "float" in the electric field, the electric force exerted on the object by the field must be directed upward and have a magnitude equal to the weight of the object. Thus, $F_e = qE = mg$, and the magnitude of the electric field must be

$$E = \frac{mg}{|q|} = \frac{(3.80\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{18.0\times10^{-6}\ \text{C}} = \boxed{2.07\times10^{3}\ \text{N/C}}$$

The electric force on a negatively charged object is in the direction opposite to that of the electric field. Since the electric force must be directed upward, the electric field must be directed $\boxed{\text{downward}}$.

P23.25 We sum the electric fields from each of the other charges using Equation 23.7 for the definition of the electric field.

The field at charge q is given by

$$\vec{\mathbf{E}} = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3$$

(a) Substituting for each of the charges gives

$$\vec{\mathbf{E}} = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}\left(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°\right) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$$

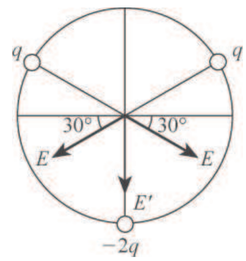
$$= \frac{k_e q}{a^2}\left[\left(2 + \frac{3}{2}\cos 45.0°\right)\hat{\mathbf{i}} + \left(\frac{3}{2}\sin 45.0° + 4\right)\hat{\mathbf{j}}\right]$$

$$= \boxed{\frac{k_e q}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

(b) The electric force on charge q is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \boxed{\frac{k_e q^2}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

P23.26 Call the fields $E = \frac{k_e q}{r^2}$ and $E' = \frac{k_e(2q)}{r^2} = 2E$ (see ANS. FIG. P23.26).



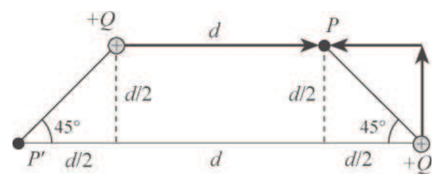
ANS. FIG. P23.26

The total field at the center of the circle has components

$$\begin{aligned}\vec{\mathbf{E}} &= (E\cos 30.0° - E\cos 30.0°)\hat{\mathbf{i}} - (E' + 2E\sin 30.0°)\hat{\mathbf{j}} \\ &= -(E' + 2E\sin 30.0°)\hat{\mathbf{j}} = -(2E + 2E\sin 30.0°)\hat{\mathbf{j}} \\ &= -2E(1+\sin 30.0°)\hat{\mathbf{j}} \\ &= -2\frac{k_e q}{r^2}(1+\sin 30.0°)\hat{\mathbf{j}} = -2\frac{k_e q}{r^2}(1.50)\hat{\mathbf{j}} = \boxed{-k_e\frac{3q}{r^2}\hat{\mathbf{j}}}\end{aligned}$$

P23.27 (a) See ANS. FIG. P23.27(a). The distance from the $+Q$ charge on the upper left is d, and the distance from the $+Q$ charge on the lower right to point P is

$$\sqrt{(d/2)^2 + (d/2)^2}$$

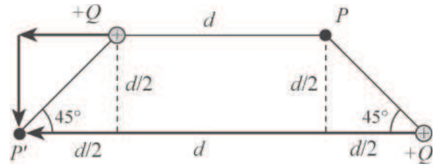


ANS. FIG. P23.27(a)

The total electric field at point P is then

$$\begin{aligned}\vec{\mathbf{E}}_P &= k_e\frac{Q}{d^2}\hat{\mathbf{i}} + k_e\frac{Q}{\left[(d/2)^2 + (d/2)^2\right]}\left(\frac{-\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}\right) \\ &= k_e\left[\frac{Q}{d^2}\hat{\mathbf{i}} + \frac{Q}{d^2/2}\left(\frac{-\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}\right)\right] \\ &= \boxed{k_e\frac{Q}{d^2}\left[\left(1-\sqrt{2}\right)\hat{\mathbf{i}} + \sqrt{2}\hat{\mathbf{j}}\right]}\end{aligned}$$

(b) See ANS. FIG. P23.27(b). The distance from the +Q charge on the lower right to point P′ is $2d$, and the distance from the $+Q$ charge on the upper right to point P' is



ANS. FIG. P23.27(b)

$$\sqrt{(d/2)^2 + (d/2)^2}$$

The total electric field at point P' is then

$$\vec{\mathbf{E}}_{P'} = k_e \frac{Q}{\left[(d/2)^2 + (d/2)^2\right]}\left(\frac{-\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{2}}\right) + k_e \frac{Q}{(2d)^2}\left(-\hat{\mathbf{i}}\right)$$

$$\vec{\mathbf{E}}_{P'} = -k_e \left[\frac{Q}{d^2/2}\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right) + \frac{Q}{4d^2}\left(-\hat{\mathbf{i}}\right)\right]$$

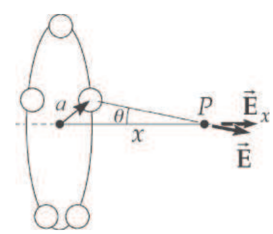
$$= -k_e \frac{Q}{4d^2}\left[\frac{8}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right) + \left(\hat{\mathbf{i}}\right)\right]$$

$$\vec{\mathbf{E}}_{P'} = \boxed{-k_e \frac{Q}{4d^2}\left[\left(1 + 4\sqrt{2}\right)\hat{\mathbf{i}} + 4\sqrt{2}\hat{\mathbf{j}}\right]}$$

P23.28 (a) One of the charges creates at P a field

$$\vec{\mathbf{E}} = E_x \hat{\mathbf{i}} = \frac{\left(k_e Q/n\right)}{a^2 + x^2}\hat{\mathbf{i}}$$

at an angle θ to the x axis as shown in ANS. FIG. P23.28. When all the charges produce the field, for $n > 1$, by symmetry the components perpendicular to the x axis add to zero.



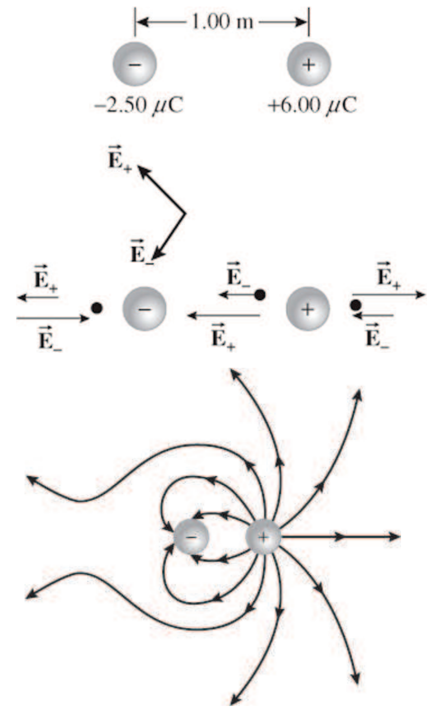
ANS. FIG. P23.28

The total field is then

$$\vec{\mathbf{E}} = nE_x\hat{\mathbf{i}} = n\left(\frac{k_e(Q/n)\hat{\mathbf{i}}}{a^2 + x^2}\cos\theta\right) = \boxed{\frac{k_e Q x \hat{\mathbf{i}}}{\left(a^2 + x^2\right)^{3/2}}}$$

(b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Because of the symmetrical arrangement of the charges, smearing the charge around the circle does not change its amount or its distance from the field point, so it does not change the field.

P23.29 The field of the positively-charged object is everywhere pointing radially away from its location. The object with negative charge creates everywhere a field pointing toward its different location. These two fields are directed along different lines at any point in the plane except for points along the extended line joining the particles; so the two fields cannot be oppositely-directed to add to zero except at some location along this line, which we take as the x axis. Observing the middle panel of ANS. FIG. P23.29, we see that at points to the left of the negatively-charged object, this particle creates field pointing to the right and the positive object creates field to the left. At some point along this segment the fields will add to zero. At locations in between the objects, both create fields pointing toward the left, so the total field is not zero. At points to the right of the positive 6-μC object, its field is directed to the right and is stronger than the leftward field of the –2.5-μC object, so the two fields cannot be equal in magnitude to add to zero. We have argued that only at a certain point straight to the left of both charges can the fields they separately produce be opposite in direction and equal in strength to add to zero.



ANS. FIG. P23.29

Let x represent the distance from the negatively-charged particle (charge q_-) to the zero-field point to its left. Then 1.00 m + x is the distance from the positive particle (of charge q_+) to this point. Each field is separately described by

$$\vec{\mathbf{E}} = k_e q \hat{\mathbf{r}}/x^2$$

so the equality in magnitude required for the two oppositely-directed vector fields to add to zero is described by

$$\frac{k_e|q_-|}{x^2} = \frac{k_e|q_+|}{(1\text{ m}+x)^2}$$

It is convenient to solve by taking the square root of both sides and cross-multiplying to clear of fractions:

$$|q_-|^{1/2}(1\text{ m}+x) = q_+^{1/2}x$$

$$1\text{ m}+x=\left(\frac{6.00}{2.50}\right)^{1/2}x=1.55\,x$$

$$1\text{ m}=0.549x$$

and $x=\boxed{1.82\text{ m}}$ to the left of the negatively-charged object.

P23.30 (a) Let s = 0.500 m be length of a side of the triangle. Call q_1 = 7.00 μC and $q_2=|-4.00\ \mu\text{C}|=4.00\ \mu\text{C}$. The electric field at the position of the 2.00-μC charge is the sum of the fields from the other two charges:

$$\vec{\mathbf{E}}=\vec{\mathbf{E}}_1+\vec{\mathbf{E}}_2=k_e\frac{q_1}{r_1^2}\hat{\mathbf{r}}_1+k_e\frac{q_2}{r_2^2}\hat{\mathbf{r}}_2$$

substituting,

$$\vec{\mathbf{E}}=k_e\frac{q_1}{s^2}\left(-\cos 60.0^\circ\,\hat{\mathbf{i}}-\sin 60.0^\circ\,\hat{\mathbf{j}}\right)+k_e\frac{q_2}{s^2}\hat{\mathbf{i}}$$

$$=\frac{k_e}{s^2}\left[\left(q_2-q_1\cos 60.0^\circ\right)\hat{\mathbf{i}}-q_1\sin 60.0^\circ\,\hat{\mathbf{j}}\right]$$

substituting numerical values,

$$\vec{\mathbf{E}}=\left[\frac{8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{(0.500\text{ m})^2}\right]$$

$$\times\left[4.00\times 10^{-6}\ \text{C}-\left(7.00\times 10^{-6}\ \text{C}\right)\cos 60.0^\circ\right]\hat{\mathbf{i}}$$

$$-\left(7.00\times 10^{-6}\ \text{C}\right)\sin 60.0^\circ\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}=\left(1.80\times 10^4\ \text{N/C}\right)\hat{\mathbf{i}}-\left(2.18\times 10^5\ \text{N/C}\right)\hat{\mathbf{j}}$$

$$=\boxed{\left(18.0\hat{\mathbf{i}}-218\hat{\mathbf{j}}\right)\ \text{kN/C}}$$

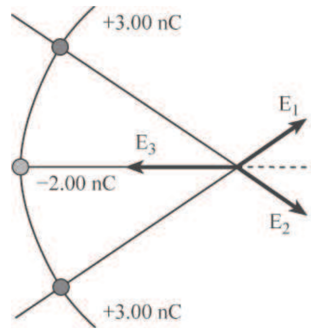
(b) The force on this charge is given by

$$\vec{\mathbf{F}}=q\vec{\mathbf{E}}=\left(2.00\times 10^{-6}\ \text{C}\right)\left(18.0\hat{\mathbf{i}}-218\hat{\mathbf{j}}\right)\ \text{kN/C}$$

$$=\boxed{\left(0.0360\hat{\mathbf{i}}-0.436\hat{\mathbf{j}}\right)\ \text{N}}$$

P23.31 Call $Q = 3.00$ nC and $q = |-2.00 \text{ nC}| = 2.00$ nC, and $r = 4.00$ cm $= 0.040\ 0$ m. Then,

$$E_1 = E_2 = \frac{k_e Q}{r^2} \quad \text{and} \quad E_3 = \frac{k_e q}{r^2}$$



ANS. FIG. P23.31

Then,

$$E_y = 0$$

$$E_x = E_{total} = 2\frac{k_e Q}{r^2}\cos 30.0° - \frac{k_e q}{r^2}$$

$$E_x = \frac{k_e}{r^2}\left(2Q\cos 30.0° - q\right)$$

$$E_x = \left[\frac{8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}{(0.040\ 0 \text{ m})^2}\right] \times\left[2\left(3.00\times 10^{-9} \text{ C}\right)\cos 30.0° - 2.00\times 10^{-9} \text{ C}\right]$$

$$= 1.80\times 10^4 \text{ N/C}$$

(a) $\boxed{1.80\times 10^4 \text{ N/C to the right}}$

(b) The electric force on a point charge placed at point P is

$$F = qE = \left(-5.00\times 10^{-9} \text{ C}\right)E = \boxed{-8.98\times 10^{-5} \text{ N (to the left)}}$$

P23.32 The first charge creates at the origin a field $\dfrac{k_e Q}{a^2}$ to the right. Both charges are on the x axis, so the total field cannot have a vertical component, but it can be either to the right or to the left. If the total field at the origin is to the right, then q must be negative:

ANS. FIG. P23.32

$$\frac{k_e Q}{a^2}\hat{\mathbf{i}} + \frac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_e Q}{a^2}\hat{\mathbf{i}} \quad \rightarrow \quad q = -9Q$$

In the alternative, if the total field at the origin is to the left,

$$\frac{k_e Q}{a^2}\hat{\mathbf{i}} + \frac{k_e q}{9a^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_e Q}{a^2}\left(-\hat{\mathbf{i}}\right) \quad \rightarrow \quad q = +27Q$$

$\boxed{\text{The field at the origin can be to the right, if the unknown charge is } -9Q, \text{ or the field can be to the left, if and only if the unknown charge is } +27Q.}$

***23.33** From the free-body diagram shown in ANS. FIG. P23.33,

$$\sum F_y = 0: \qquad T\cos 15.0° = 1.96\times 10^{-2}\ \text{N}$$

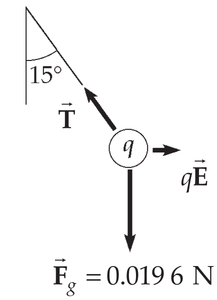
So $\quad T = 2.03\times 10^{-2}\ \text{N}$.

From $\sum F_x = 0$, we have $qE = T\sin 15.0°$,

or

$$q = \frac{T\sin 15.0°}{E} = \frac{(2.03\times 10^{-2}\ \text{N})\sin 15.0°}{1.00\times 10^{3}\ \text{N/C}}$$

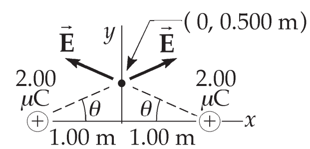
$$= 5.25\times 10^{-6}\ \text{C} = \boxed{5.25\ \mu\text{C}}$$



ANS. FIG. P23.33

***P23.34** (a) The distance from each charge to the point at $y = 0.500$ m is

$$d = \sqrt{(1.00\ \text{m})^2 + (0.500\ \text{m})^2} = 1.12\ \text{m}$$



ANS. FIG. P23.34

the magnitude of the electric field from each charge at that point is then given by

$$E = \frac{k_e q}{r^2} = \frac{(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times 10^{-6}\ \text{C})}{(1.12\ \text{m})^2} = 14\ 400\ \text{N/C}$$

The x components of the two fields cancel and the y components add, giving

$$E_x = 0 \text{ and } \qquad E_y = 2(14\ 400\ \text{N/C})\sin 26.6° = 1.29\times 10^{4}\ \text{N/C}$$

so $\boxed{\vec{\mathbf{E}} = 1.29\times 10^{4}\,\hat{\mathbf{j}}\ \text{N/C}}$.

(b) The electric force at this point is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = (-3.00\times 10^{-6}\ \text{C})(1.29\times 10^{4}\ \text{N/C}\,\hat{\mathbf{j}})$$

$$= \boxed{-3.86\times 10^{-2}\,\hat{\mathbf{j}}\ \text{N}}$$

***P23.35** (a) The electric field at the origin due to each of the charges is given by

$$\vec{\mathbf{E}}_1 = \frac{k_e|q_1|}{r_1^2}(-\hat{\mathbf{j}})$$

$$= \frac{(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times 10^{-9}\ \text{C})}{(0.100\ \text{m})^2}(-\hat{\mathbf{j}})$$

$$= -(2.70\times 10^{3}\ \text{N/C})\hat{\mathbf{j}}$$

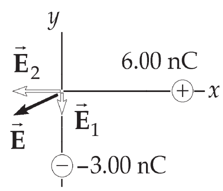
$$\vec{\mathbf{E}}_2 = \frac{k_e|q_2|}{r_2^2}(-\hat{\mathbf{i}})$$

$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.00\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2}(-\hat{\mathbf{i}})$$

$$= -(5.99\times10^2\ \text{N/C})\hat{\mathbf{i}}$$

and their sum is

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_1 = \boxed{-(5.99\times10^2\ \text{N/C})\hat{\mathbf{i}} - (2.70\times10^3\ \text{N/C})\hat{\mathbf{j}}}$$



ANS. FIG. P23.35

(b) The vector electric force is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = (5.00\times10^{-9}\ \text{C})(-599\hat{\mathbf{i}} - 2\,700\hat{\mathbf{j}})\ \text{N/C}$$

$$\vec{\mathbf{F}} = (-3.00\times10^{-6}\hat{\mathbf{i}} - 13.5\times10^{-6}\hat{\mathbf{j}})\ \text{N} = \boxed{(-3.00\hat{\mathbf{i}} - 13.5\hat{\mathbf{j}})\ \square\text{N}}$$

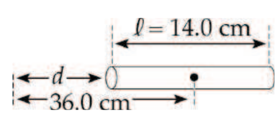
***P23.36** The electric field at any point x is

$$E = \frac{k_e q}{(x-a)^2} - \frac{k_e q}{[x-(-a)]^2} = \frac{k_e q(4ax)}{(x^2-a^2)^2}$$

When x is much, much greater than a, we find $E \approx \boxed{\dfrac{4a(k_e q)}{x^3}}$.

Section 23.5 Electric Field of a Continuous Charge Distribution

P23.37 (a) From Example 23.7, the magnitude of the electric field produced by the rod is



ANS. FIG. P23.37

$$|E| = \frac{k_e\lambda\ell}{d(\ell+d)} = \frac{k_e(Q/\ell)\ell}{d(\ell+d)} = \frac{k_e Q}{d(\ell+d)}$$

$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(22.0\times10^{-6}\ \text{C})}{(0.290\ \text{m})(0.140\ \text{m} + 0.290\ \text{m})}$$

$$E = \boxed{1.59\times10^6\ \text{N/C}}$$

(b) The charge is negative, so the electric field is directed towards its source, $\boxed{\text{to the right}}$.

P23.38 The electric field for the disk is given by

$$E = 2\pi k_e \sigma \left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

in the positive x direction (away from the disk). Substituting,

$$E = 2\pi\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(7.90\times10^{-3}\ \mathrm{C/m^2}\right) \times\left(1 - \frac{x}{\sqrt{x^2 + (0.350)^2}}\right)$$

$$= \left(4.46\times10^8\ \mathrm{N/C}\right)\left(1 - \frac{x}{\sqrt{x^2 + 0.123}}\right)$$

(a) At $x = 0.050\,0$ m,

$$E = 3.83\times10^8\ \mathrm{N/C} = \boxed{383\ \mathrm{MN/C}}$$

(b) At $x = 0.100$ m,

$$E = 3.24\times10^8\ \mathrm{N/C} = \boxed{324\ \mathrm{MN/C}}$$

(c) At $x = 0.500$ m,

$$E = 8.07\times10^7\ \mathrm{N/C} = \boxed{80.7\ \mathrm{MN/C}}$$

(d) At $x = 2.000$ m,

$$E = 6.68\times10^8\ \mathrm{N/C} = \boxed{6.68\ \mathrm{MN/C}}$$

P23.39 We may particularize the result of Example 23.8 to

$$|E| = \frac{k_e x Q}{\left(x^2 + a^2\right)^{3/2}} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(75.0\times10^{-6}\ \mathrm{C/m^2}\right)x}{\left(x^2 + 0.100^2\right)^{3/2}}$$

$$= \frac{6.74\times10^5\,x}{\left(x^2 + 0.010\,0\right)^{3/2}}$$

where we choose the x axis along the axis of the ring. The field is parallel to the axis, directed away from the center of the ring above and below it.

(a) At $x = 0.010\,0$ m, $\vec{\mathbf{E}} = 6.64\times10^6\,\hat{\mathbf{i}}\ \mathrm{N/C} = \boxed{6.64\hat{\mathbf{i}}\ \mathrm{MN/C}}$

(b) At $x = 0.050\,0$ m, $\vec{\mathbf{E}} = 2.41 \times 10^7 \hat{\mathbf{i}}$ N/C $= \boxed{24.1\hat{\mathbf{i}}\ \text{MN/C}}$

(c) At $x = 0.300$ m, $\vec{\mathbf{E}} = 6.40 \times 10^6 \hat{\mathbf{i}}$ N/C $= \boxed{6.40\hat{\mathbf{i}}\ \text{MN/C}}$

(d) At $x = 1.00$ m, $\vec{\mathbf{E}} = 6.64 \times 10^5 \hat{\mathbf{i}}$ N/C $= \boxed{0.664\hat{\mathbf{i}}\ \text{MN/C}}$

P23.40 The electric field at a distance x is
$$E_x = 2\pi k_e \sigma \left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

This is equivalent to
$$E_x = 2\pi k_e \sigma \left[1 - \frac{1}{\sqrt{1 + R^2/x^2}}\right]$$

For large x, $\dfrac{R^2}{x^2} << 1$ and
$$\sqrt{1 + \frac{R^2}{x^2}} \approx 1 + \frac{R^2}{2x^2}$$

so
$$E_x = 2\pi k_e \sigma \left(1 - \frac{1}{\left[1 + R^2/\left(2x^2\right)\right]}\right) = 2\pi k_e \sigma \frac{\left(1 + R^2/\left(2x^2\right) - 1\right)}{\left[1 + R^2/\left(2x^2\right)\right]}$$

Substitute $\sigma = \dfrac{Q}{\pi R^2}$,
$$E_x = \frac{k_e Q\left(1/x^2\right)}{\left[1 + R^2/\left(2x^2\right)\right]} = \frac{k_e Q}{x^2 + R^2/2}$$

But for $x >> R$, $\dfrac{1}{x^2 + R^2/2} \approx \dfrac{1}{x^2}$, so
$$\boxed{E_x \approx \frac{k_e Q}{x^2} \text{ for a disk at large distances}}$$

P23.41 (a) From Example 23.9,
$$E = 2\pi k_e \sigma \left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

here,
$$\sigma = \frac{Q}{\pi R^2} = \frac{5.20 \times 10^{-6}}{\pi (0.0300)^2} = 1.84 \times 10^{-3}\ \text{C/m}^2$$

the electric field is then

$$E = 2\pi k_e \sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

$$E = 2\pi\left(8.99\times 10^9\right)\left(1.84\times 10^{-3}\right)$$

$$\times\left(1 - \frac{0.00300}{\sqrt{(0.00300)^2 + (0.0300)^2}}\right)$$

$$E = \left(1.04\times 10^8\ \text{N/C}\right)\left(1 - \frac{0.00300}{\sqrt{(0.00300)^2 + (0.0300)^2}}\right)$$

$$= \boxed{9.36\times 10^7\ \text{N/C}}$$

(b) The near-field approximation gives:

$$E = 2\pi k_e \sigma = \boxed{1.04\times 10^8\ \text{N/C (about 11\% high)}}$$

(c) The electric field at this point is

$$E = \left(1.04\times 10^8\ \text{N/C}\right)\left(1 - \frac{0.300}{\sqrt{(0.300)^2 + (0.0300)^2}}\right)$$

$$= \boxed{5.16\times 10^5\ \text{N/C}}$$

(d) With this approximation, suppressing units,

$$E = k_e\frac{Q}{r^2} = \left(8.99\times 10^9\right)\left[\frac{5.20\times 10^{-6}}{(0.30)^2}\right]$$

$$= \boxed{5.19\times 10^5\ \text{N/C (about 0.6\% high)}}$$

P23.42 (a) The electric field at point P due to each element of length dx is $dE = \dfrac{k_e dq}{x^2 + d^2}$ and is directed along the line joining the element to point P. The charge element $dq = Q dx/\text{L}$. The x and y components are

ANS. FIG. P23.42

$$E_x = \int dE_x = \int dE\sin\theta$$

where $\sin\theta = \dfrac{x}{\sqrt{d^2 + x^2}}$

and

$$E_y = \int dE_y = \int dE\cos\theta \quad \text{where} \quad \cos\theta = \frac{d}{\sqrt{d^2 + x^2}}$$

Therefore,

$$E_x = -k_e \frac{Q}{L}\int_0^L \frac{x\,dx}{\left(d^2 + x^2\right)^{3/2}} = -k_e \frac{Q}{L}\left[\frac{-1}{\left(d^2 + x^2\right)^{1/2}}\right]_0^L$$

$$E_x = -k_e \frac{Q}{L}\left[\frac{-1}{\left(d^2 + L^2\right)^{1/2}} - \frac{-1}{\left(d^2 + 0\right)^{1/2}}\right]$$

$$E_x = \boxed{-k_e \frac{Q}{L}\left[\frac{1}{d} - \frac{1}{\left(d^2 + L^2\right)^{1/2}}\right]}$$

and

$$E_y = k_e \frac{Qd}{L}\int_0^L \frac{dx}{\left(d^2 + x^2\right)^{3/2}} = k_e \frac{Qd}{L}\left[\frac{x}{d^2\left(d^2 + x^2\right)^{1/2}}\right]_0^L$$

$$E_y = k_e \frac{Q}{\cancel{L}d}\left[\frac{\cancel{L}}{\left(d^2 + L^2\right)^{1/2}} - 0\right] \quad \to \quad E_y = \boxed{k_e \frac{Q}{d}\frac{1}{\left(d^2 + L^2\right)^{1/2}}}$$

(b) When $d >> L$,

$$E_x = -k_e \frac{Q}{L}\left[\frac{1}{d} - \frac{1}{\left(d^2 + L^2\right)^{1/2}}\right] \to -k_e \frac{Q}{L}\left[\frac{1}{d} - \frac{1}{\left(d^2\right)^{1/2}}\right] \to \boxed{E_x \approx 0}$$

and

$$E_y = k_e \frac{Q}{d}\frac{1}{\left(d^2 + L^2\right)^{1/2}} \to k_e \frac{Q}{d}\frac{1}{\left(d^2\right)^{1/2}} \to \boxed{E_y \approx k_e \frac{Q}{d^2}}$$

which is the field of a point charge Q at a distance d along the y axis above the charge.

P23.43 (a) Magnitude $|E| = \int \frac{k_e dq}{x^2}$, where $dq = \lambda_0 dx$

$$E = k_e\lambda_0 \int_{x_0}^{\infty} \frac{dx}{x^2} = k_e\lambda_0\left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty} = \boxed{\frac{k_e\lambda_0}{x_0}}$$

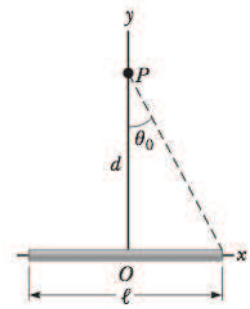
(b) The charge is positive, so the electric field points away from its source, $\boxed{\text{to the left}}$.

P23.44 (a) The electric field at point P, due to each element of length dx, is $dE = \dfrac{k_e dq}{x^2 + d^2}$ and is directed along the line joining the element to point P. By symmetry,

$$E_x = \int dE_x = 0$$

and since $dq = \lambda dx$,

$$E = E_y = \int dE_y = \int dE \cos\theta$$



ANS. FIG. P23.44

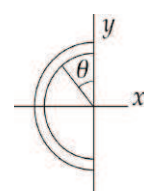
where $\cos\theta = \dfrac{y}{\sqrt{x^2 + d^2}}$.

Therefore, $E = 2k_e\lambda d \displaystyle\int_0^{\ell/2} \frac{dx}{\left(x^2 + d^2\right)^{3/2}} = \boxed{\dfrac{2k_e\lambda \sin\theta_0}{d}}$

with $\sin\theta_0 = \dfrac{\ell/2}{\sqrt{(\ell/2)^2 + d^2}}$.

(b) For a bar of infinite length, $\theta_0 = 90°$ and $E_y = \boxed{\dfrac{2k_e\lambda}{d}}$.

P23.45 Due to symmetry, $E_y = \int dE_y = 0$, and $E_x = -\int dE \sin\theta = -k_e \int \dfrac{dq \sin\theta}{r^2}$ where $dq = \lambda ds = \lambda r d\theta$; the component E_x is negative because charge $q = -7.50\ \mu\text{C}$, causing the net electric field to be directed to the left.



ANS. FIG. P23.45

$$E_x = -\frac{k_e\lambda}{r}\int_0^{\pi} \sin\theta d\theta = -\frac{k_e\lambda}{r}(-\cos\theta)\Big|_0^{\pi} = -\frac{2k_e\lambda}{r}$$

where $\lambda = \dfrac{|q|}{L}$ and $r = \dfrac{L}{\pi}$. Thus,

$$E_x = -\frac{2k_e|q|\pi}{L^2} = -\frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.50\times10^{-6}\ \text{C}\right)\pi}{(0.140\ \text{m})^2}$$

$$E_x = -2.16\times10^7\ \text{N/C}$$

(a) magnitude $E = \boxed{2.16\times10^7\ \text{N/C}}$

(b) $\boxed{\text{to the left}}$

P23.46 (a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness dx, has charge $\frac{Qdx}{h}$ and produces, at the chosen point, a field

$$d\vec{\mathbf{E}} = \frac{k_e x}{\left(x^2 + R^2\right)^{3/2}} \frac{Qdx}{h} \hat{\mathbf{i}}$$

The total field is

$$\vec{\mathbf{E}} = \int_{\text{all charge}} d\mathbf{E} = \int_{d}^{d+h} \frac{k_e Q x dx}{h\left(x^2 + R^2\right)^{3/2}} \hat{\mathbf{i}}$$

$$= \frac{k_e Q \hat{\mathbf{i}}}{2h} \int_{x=d}^{d+h} \left(x^2 + R^2\right)^{-3/2} 2x\,dx$$

integrating,

$$\vec{\mathbf{E}} = \frac{k_e Q \hat{\mathbf{i}}}{2h} \frac{\left(x^2 + R^2\right)^{-1/2}}{(-1/2)} \Bigg|_{x=d}^{d+h}$$

$$= \boxed{\frac{k_e Q \hat{\mathbf{i}}}{h} \left[\frac{1}{\left(d^2 + R^2\right)^{1/2}} - \frac{1}{\left((d+h)^2 + R^2\right)^{1/2}} \right]}$$

(b) Think of the cylinder as a stack of disks, each with thickness dx, charge $\frac{Qdx}{h}$, and charge-per-area $\sigma = \frac{Qdx}{\pi R^2 h}$. One disk produces a field

$$d\vec{\mathbf{E}} = \frac{2\pi k_e Q dx}{\pi R^2 h} \left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}} \right) \hat{\mathbf{i}}$$

So, $$\vec{\mathbf{E}} = \int_{\text{all charge}} d\vec{\mathbf{E}} = \int_{x=d}^{d+h} \frac{2k_e Q dx}{R^2 h} \left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}} \right) \hat{\mathbf{i}}$$

$$\vec{\mathbf{E}} = \frac{2k_e Q \hat{\mathbf{i}}}{R^2 h} \left[\int_{d}^{d+h} dx - \frac{1}{2} \int_{x=d}^{d+h} \left(x^2 + R^2\right)^{-1/2} 2x\,dx \right]$$

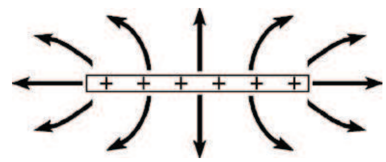
$$= \frac{2k_e Q \hat{\mathbf{i}}}{R^2 h} \left[x\Big|_{d}^{d+h} - \frac{1}{2} \frac{\left(x^2 + R^2\right)^{1/2}}{1/2} \Bigg|_{d}^{d+h} \right]$$

$$\vec{\mathbf{E}} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[d + h - d - \left((d+h)^2 + R^2\right)^{1/2} + \left(d^2 + R^2\right)^{1/2}\right]$$

$$\vec{\mathbf{E}} = \boxed{\frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[h + \left(d^2 + R^2\right)^{1/2} - \left((d+h)^2 + R^2\right)^{1/2}\right]}$$

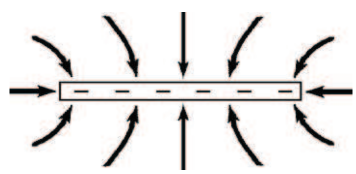
Section 23.6 Electric Field Lines

P23.47 The field lines are shown in ANS. FIG. P23.47, where we've followed the rules for drawing field lines where field lines point toward negative charge, meeting the rod perpendicularly and ending there.



ANS. FIG. P23.47

P23.48 For the positively charged disk, a side view of the field lines, pointing into the disk, is shown in ANS. FIG. P23.48.



ANS. FIG. P23.48

P23.49 Field lines emerge from positive charge and enter negative charge.

(a) The number of field lines emerging from positive q_2 and entering negative charge q_1 is proportional to their charges:

$$\frac{q_1}{q_2} = \frac{-6}{18} = \boxed{-\frac{1}{3}}$$

(b) From above, $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$.

P23.50 (a) The electric field has the general appearance shown in ANS. FIG. P23.50 below.

(b) It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second panel of ANS. FIG. P23.50 indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.

(c) You may need to review vector addition in Chapter 1. The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$.

The electric field from a point charge is

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}.$$

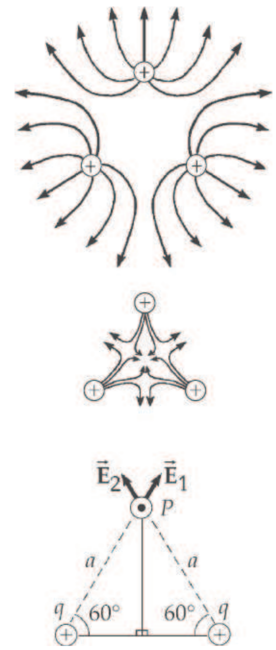
As shown in the bottom panel of ANS. FIG. P23.50,

$$\vec{\mathbf{E}}_1 = k_e \frac{q}{a^2}$$

to the right and upward at 60°, and

$$\vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}$$

to the left and upward at 60°. So,



ANS. FIG. P23.50

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right) + \left(-\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right)\right]$$

$$= k_e \frac{q}{a^2}\left[2\left(\sin 60°\hat{\mathbf{j}}\right)\right] = \boxed{1.73 k_e \frac{q}{a^2}\hat{\mathbf{j}}}$$

Section 23.7 Motion of a Charged Particle in a Uniform Electric Field

P23.51 (a) We obtain the acceleration of the proton from the particle under a net force model, with $F = qE$ representing the electric force:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{(1.602 \times 10^{-19}\ \text{C})(640\ \text{N/C})}{1.67 \times 10^{-27}\ \text{kg}} = \boxed{6.14 \times 10^{10}\ \text{m/s}^2}$$

(b) The particle under constant acceleration model gives us $v_f = v_i + at$, from which we obtain

$$t = \frac{v_f - 0}{a} = \frac{1.20 \times 10^{6}\ \text{m/s}}{6.14 \times 10^{10}\ \text{m/s}^2} = \boxed{19.5\ \mu\text{s}}$$

(c) Again, from the particle under constant acceleration model,

$$\Delta x = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(6.14 \times 10^{10}\ \text{m/s}^2)(19.5 \times 10^{-6}\ \text{s})^2$$

$$= \boxed{11.7\ \text{m}}$$

(d) The final kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\ \text{kg})(1.20\times10^{6}\ \text{m/s})^2 = \boxed{1.20\times10^{-15}\ \text{J}}$$

P23.52 (a) $|a| = \dfrac{qE}{m} = \dfrac{(1.602\times10^{-19}\ \text{C})(6.00\times10^{5}\ \text{N/C})}{1.67\times10^{-27}\ \text{kg}} = 5.76\times10^{13}\ \text{m/s},$

so $\vec{\mathbf{a}} = \boxed{-5.76\times10^{13}\,\hat{\mathbf{i}}\ \text{m/s}^2}$.

(b) $v_f^2 = v_i^2 + 2a(x_f - x_i)$

$0 = v_i^2 + 2(-5.76\times10^{13}\ \text{m/s}^2)(0.070\,0\ \text{m})$

$\vec{\mathbf{v}}_i = \boxed{2.84\times10^{6}\,\hat{\mathbf{i}}\ \text{m/s}}$

(c) $v_f = v_i + at$

$0 = 2.84\times10^{6}\ \text{m/s} + (-5.76\times10^{13}\ \text{m/s}^2)t \quad\rightarrow\quad t = \boxed{4.93\times10^{-8}\ \text{s}}$

P23.53 We use $v_f = v_i + at$, where $v_i = 0$, $t = 48.0 \times 10^{-9}$ s, and $a = F/m = eE/m$.

For the electron, $m = m_e = 9.11\times10^{-31}$ kg

and for the proton, $m = m_p = 1.67\times10^{-27}$ kg

The electric force on both particles is given by

$$F = eE = (1.60\times10^{-19}\ \text{C})(5.20\times10^{2}\ \text{N/C}) = 8.32\times10^{-17}\ \text{N}$$

Then, for the electron,

$$v_{fe} = v_{ie} + at = 0 + \left(\frac{eE}{m_e}\right)t = \left(\frac{8.32\times10^{-17}\ \text{N}}{9.11\times10^{-31}\ \text{kg}}\right)(48.0\times10^{-9}\ \text{s})$$
$$= \boxed{4.38\times10^{6}\ \text{m/s}}$$

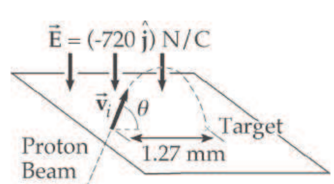
and for the proton,

$$v_{fp} = v_{ip} + at = 0 + \left(\frac{eE}{m_p}\right)t = \left(\frac{8.32\times10^{-17}\ \text{N}}{1.67\times10^{-27}\ \text{kg}}\right)(48.0\times10^{-9}\ \text{s})$$
$$= \boxed{2.39\times10^{3}\ \text{m/s}}$$

P23.54 (a) $\boxed{\text{Particle under constant velocity}}$

(b) $\boxed{\text{Particle under constant acceleration}}$

(c) The vertical acceleration caused by the



ANS. FIG. P23.54

electric force is constant and downward;
therefore, $\boxed{\text{the proton moves in a parabolic path just like a projectile in a gravitational field.}}$

(d) We may neglect the effect of the acceleration of gravity on the proton because the magnitude of the vertical acceleration caused by the electric force is

$$a_y = \frac{eE}{m_p} = \frac{(1.60\times10^{-19}\ \text{C})(720\ \text{N/C})}{1.67\times10^{-27}\ \text{kg}} = 6.90\times10^{10}\ \text{m/s}^2$$

which is much greater than that of gravity.

Replacing acceleration g in Equation 4.13 with eE/m_p, we have

$$R = \frac{v_i^2 \sin 2\theta}{eE/m_p} = \boxed{\frac{m_p v_i^2 \sin 2\theta}{eE}}$$

(e) $R = \dfrac{m_p v_i^2 \sin 2\theta}{eE} = \dfrac{(1.67\times10^{-27}\ \text{kg})(9.55\times10^3\ \text{m/s})^2 \sin 2\theta}{(1.60\times10^{-19}\ \text{C})(720\ \text{N/C})}$

$= 1.27\times10^{-3}\ \text{m}$

which gives $\sin 2\theta = 0.961$, or

$$\theta = \boxed{36.9°} \quad \text{or} \quad 90.0° - \theta = \boxed{53.1°}$$

(f) $\Delta t = \dfrac{R}{v_{ix}} = \dfrac{R}{v_i \cos\theta}$

If $\theta = 36.9°$, $\Delta t = \boxed{166\ \text{ns}}$. If $\theta = 53.1°$, $\Delta t = \boxed{221\ \text{ns}}$.

P23.55 The work done on the charge is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{d}} = q\vec{\mathbf{E}}\cdot\vec{\mathbf{d}}$ and the kinetic energy changes according to $W = K_f - K_i = 0 - K$.

Assuming $\vec{\mathbf{v}}$ is in the $+x$ direction, we have $(-e)\vec{\mathbf{E}}\cdot d\hat{\mathbf{i}} = -K$.

Then, $e\vec{\mathbf{E}}\cdot\left(d\hat{\mathbf{i}}\right) = K$, and

$$\vec{\mathbf{E}} = \frac{K}{ed}\hat{\mathbf{i}}$$

(a) $E = \boxed{\dfrac{K}{ed}}$

(b) Because a negative charge experiences an electric force opposite to the direction of an electric field, the required electric field will be $\boxed{\text{in the direction of motion}}$.