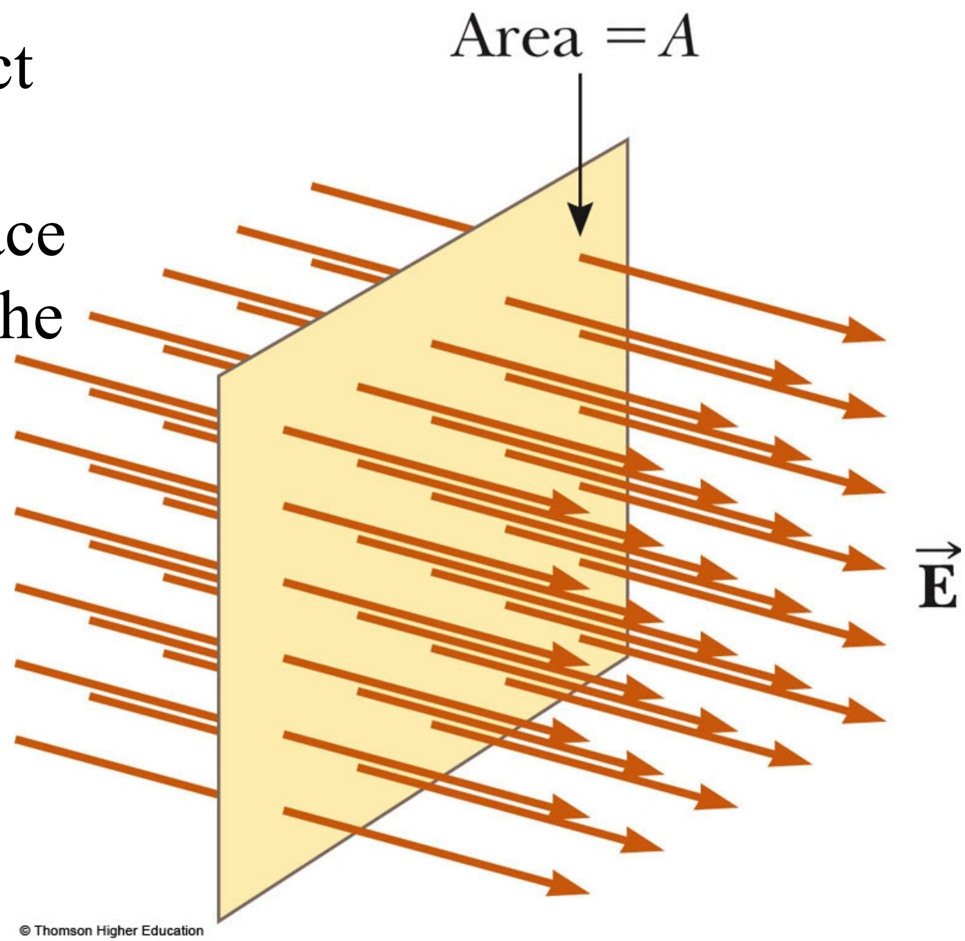
Electric Flux

Electric flux is the product of the magnitude of the electric field and the surface area, *A*, perpendicular to the field

$\mathbf{\Phi}_E = EA$

(Only for the special case where E ⊥ surface and E is uniform over the surface.)



PHYS 2321
Week4: Gauss' Law/Potential

Day 1 Outline

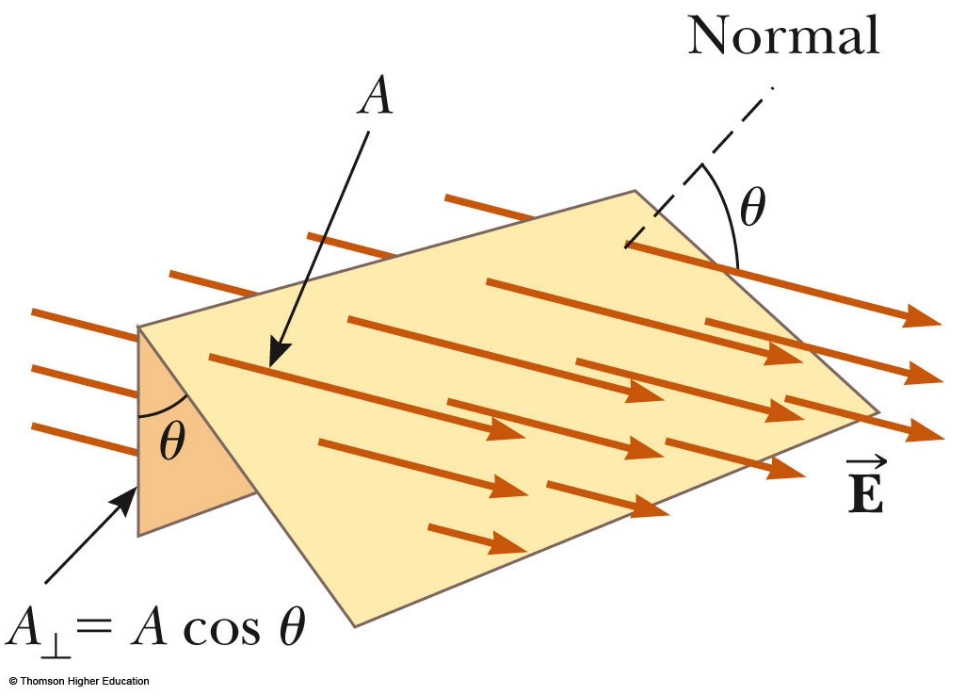
1) Hwk: Ch. 22 P. 1,2,5,6,9,10,13,17,19,20,35 MCQ. 1-9 odd. Due Wed
   Read Ch. 22-1 to 22-3, Ch. 23.1
2) Quiz 2 on E-fields
3) Ch. 22 Gauss' Law – allows new way to find E-fields
   a. Electric flux, Φ, a measure of how many field lines go thru surface
   b. Gauss’ Law: Flux through a closed surface $\propto$ charge enclosed
   c. Apply to point charges
   d. Apply to extended charge distributions.

Notes: Next quiz next week (Wed?) on Flux and Gauss’ Law.
PDF version of this my2321wk3.ppt is under “NEW STUFF”

Electric Flux

Uniform E-field near arbitrary rectangular surfaces

- Need $\Phi_E = \vec{E} \cdot \vec{A}$
- Compare the electric flux through the vertical and slanted rectangles.
- Same # field lines → same flux! Now prove:
- Find $\Phi_E = EA \cos \theta$ for each rectangle ...



Electric Flux, Interpreting the Equation $\Phi_E = \vec{E} \cdot \vec{A}$

- The flux is a maximum when the surface is perpendicular to the field, or when $\vec{A} \| \vec{E}$.
- The flux is zero when the surface is parallel to the field.
- If the field varies over the surface, $\Phi = EA \cos \theta$ is valid for only a small element of the area

Electric Flux, General

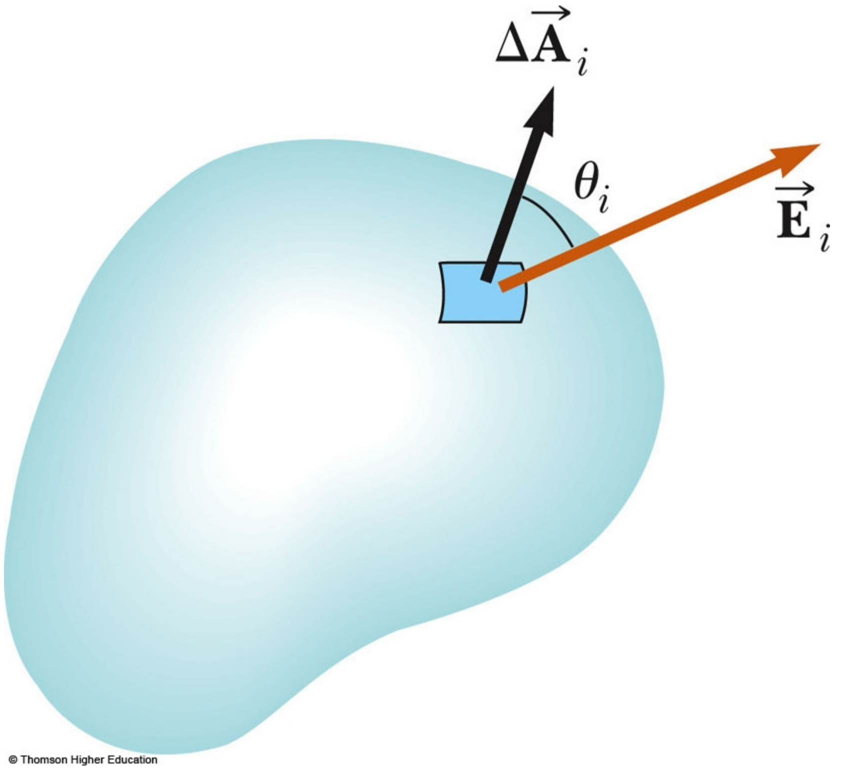
In the most general case, the surface is curved and E is not uniform.

Look at a small area element

$$\Phi_{E,i} = E_i \, \Delta A_i \cos\theta_i$$

The total flux through blue surface becomes:

$$\sum_i \vec{E}_i \cdot \Delta \vec{A}_i \rightarrow \Phi_E = \int_{surface} \vec{E} \cdot d\vec{A}$$

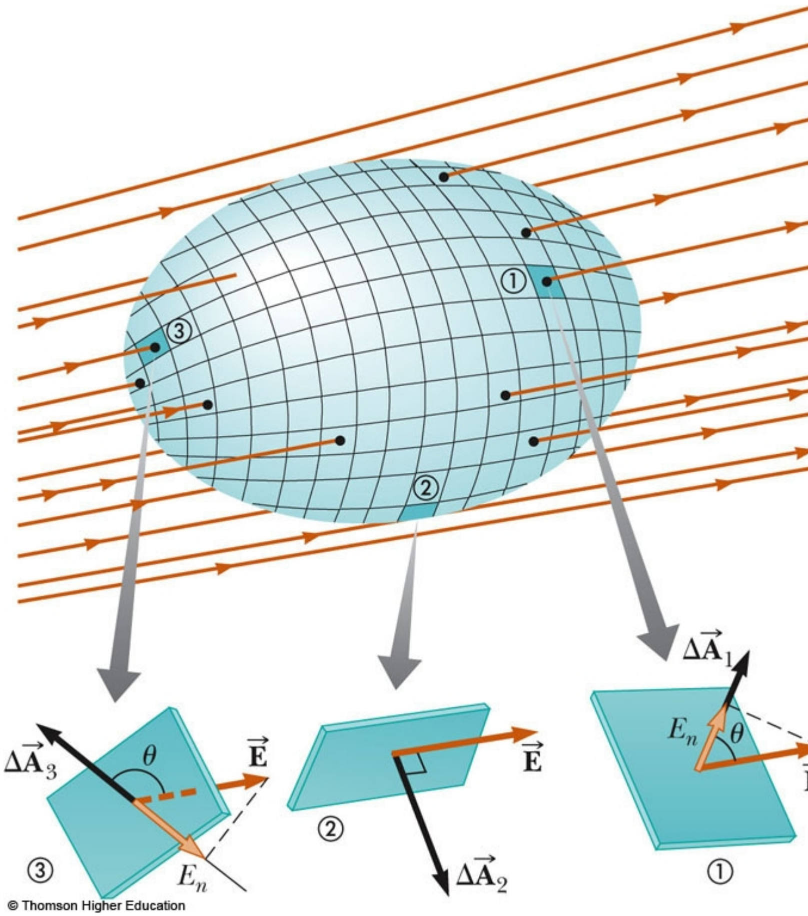


Electric Flux, final

- The surface integral means the integral must be evaluated over the surface in question
- In general, the value of the flux will depend both on the field pattern and on the shape of the surface
- The units of electric flux will be $N \cdot m^2/C$

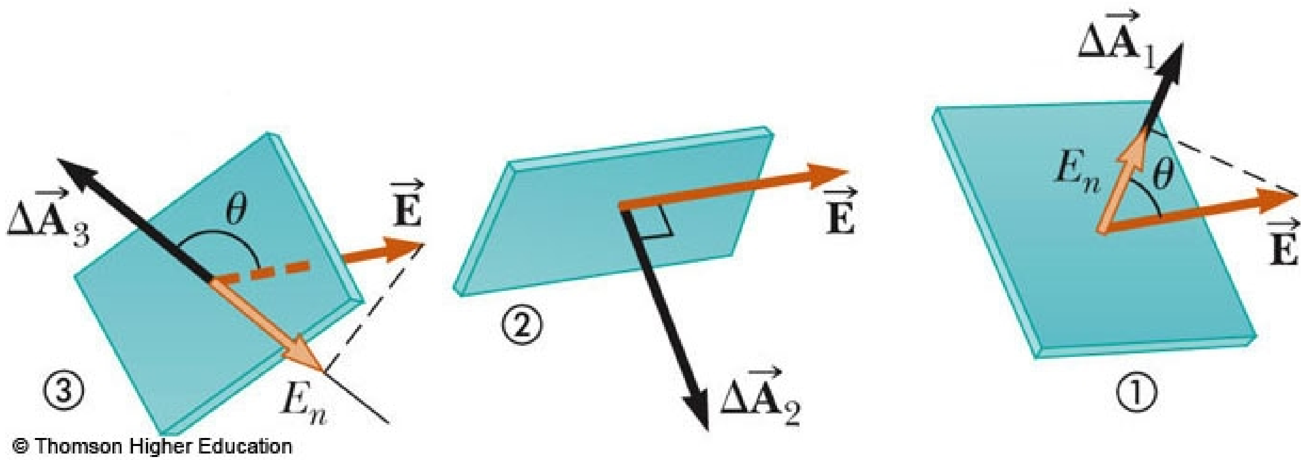
Electric Flux, Closed Surface

- Assume a closed surface
- The vectors $\Delta\vec{A}_i$ point in different directions.
  - At each point, they are perpendicular to the surface.
  - By convention, they point outward.



PLAY
ACTIVE FIGURE

Flux Through Closed Surface, cont.



At (1), the field lines are crossing the surface from the inside to the outside; $\theta < 90^o$, Φ is positive

At (2), the field lines graze surface; $\theta = 90^o$, $\Phi = 0$

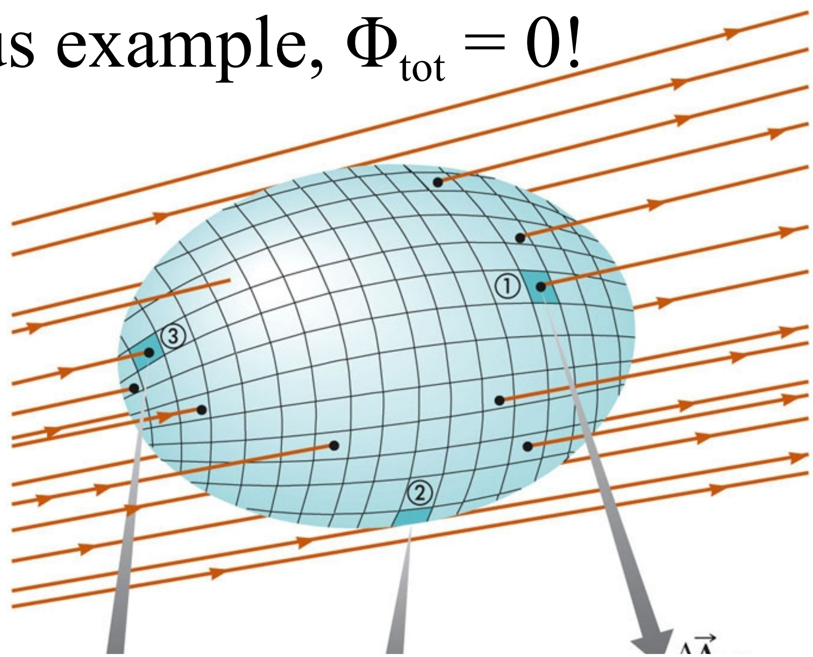
At (3), the field lines are crossing the surface from the outside to the inside; $180^o > \theta > 90^o$, Φ is negative

Flux Through Closed Surface, final

The ***net*** flux through the surface is proportional to the net number of lines leaving the surface

This net number of lines is the number of lines leaving the surface minus the number entering the surface

So, for previous example, $\Phi_{tot} = 0$!



Karl Friedrich Gauss

1777 – 1855

Made contributions in

Electromagnetism

Number theory

Statistics

Non-Euclidean geometry

Cometary orbital mechanics

A founder of the German Magnetic Union

- Studies the Earth's magnetic field

Gauss's Law, Introduction

- Gauss's law is an expression of the general relationship between the net electric flux through a closed surface and the charge enclosed by the surface.
    - The closed surface is often called a *Gaussian surface.*
- Gauss's law says: the total electric flux through a closed surface equals the charge enclosed divided by the constant ε_0:

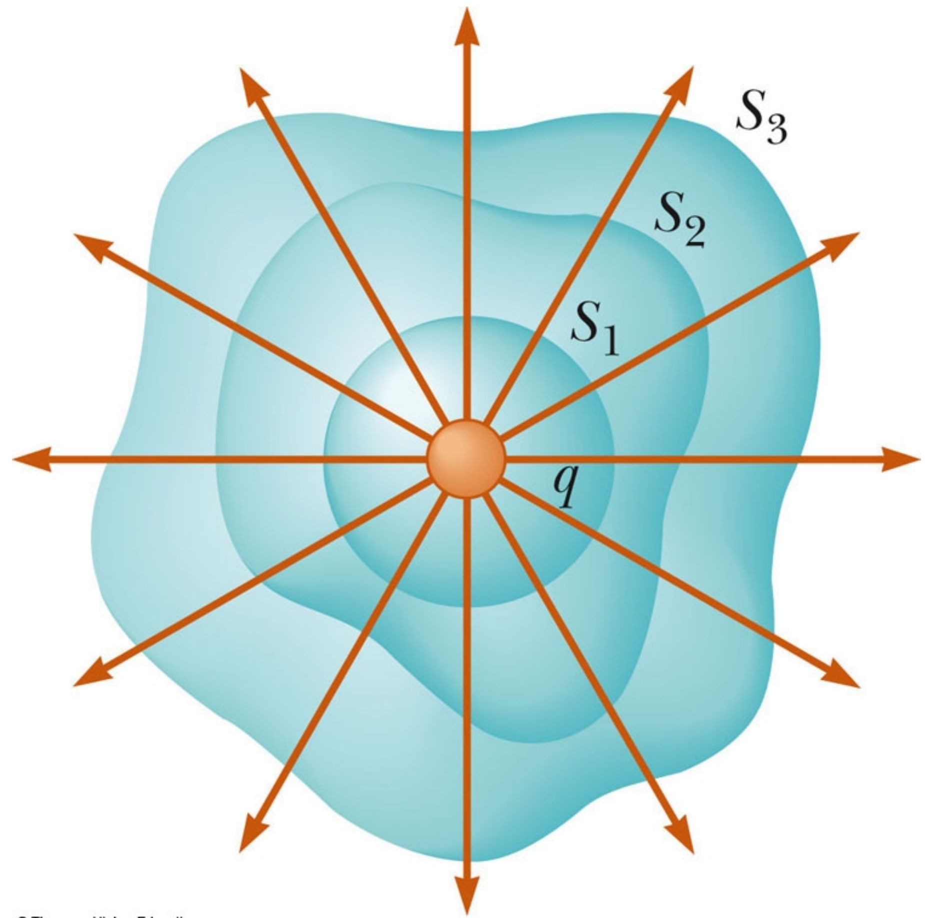
$$\Phi_{tot} = \frac{q_{enc}}{\epsilon_0}$$

Gaussian Surface, Example

Closed surfaces of various shapes can surround the charge

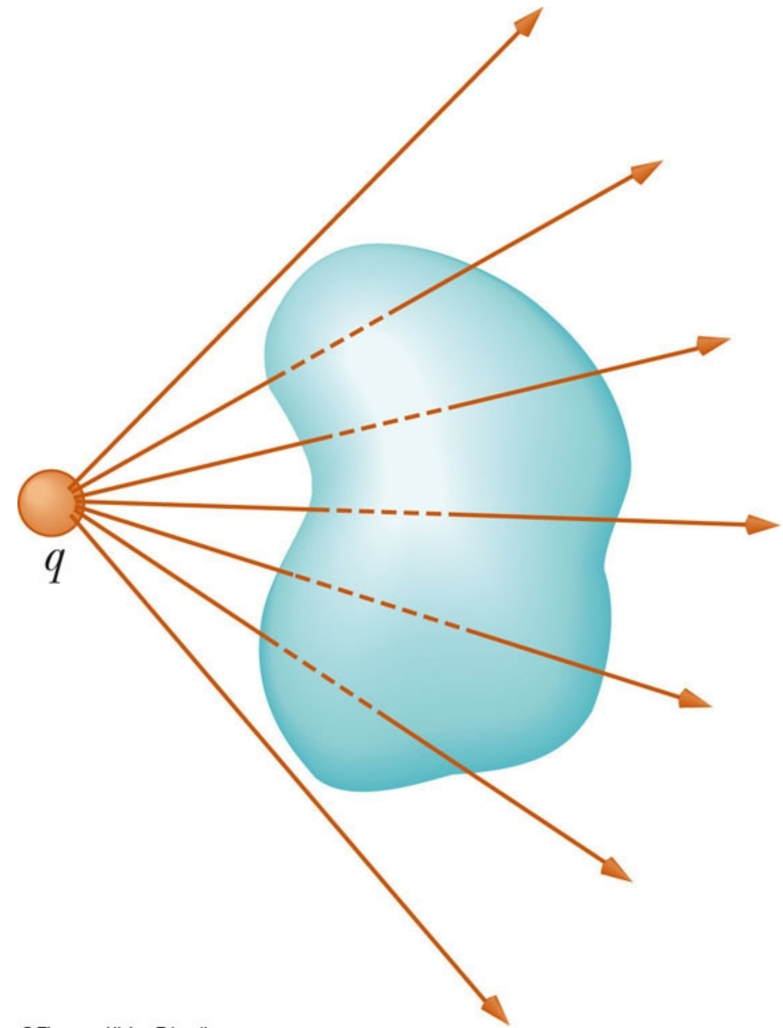
Only S_1 is spherical

All of them have the same flux due to q, namely q/ε_o!



Gaussian Surface, Example 2

- The charge is outside the closed surface with an arbitrary shape
- Any field line entering the surface leaves at another point
- The electric flux through a closed surface that surrounds no charge is zero!



Flux Through a Cube, Example

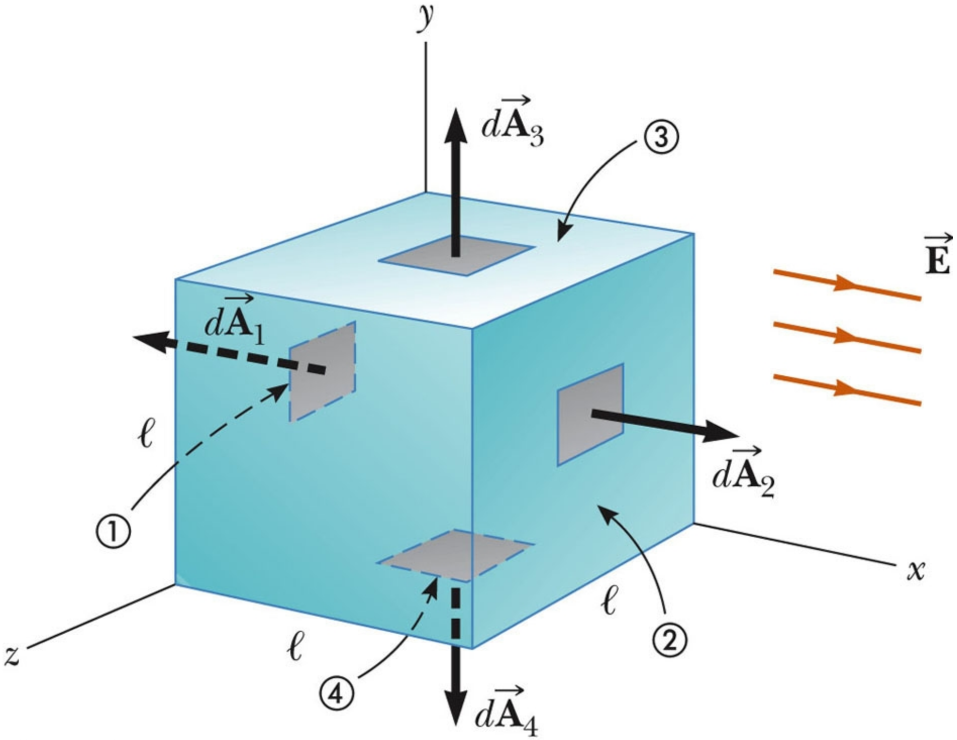
The field lines pass through two surfaces perpendicularly and are parallel to the other four surfaces

For side 1, $\Phi_1 = -El^2$

For side 2, $\Phi_2 = El^2$

For the other sides, $\Phi_{3\text{-}6} = 0$

Therefore, $\Phi_{total} = 0$



Gauss's Law – Final

Gauss's law states: $\Phi_E = \oint\limits_{surface} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$

q_{enc} is the net charge inside the surface.

$\vec{E}$ represents the electric field at points on the surface.

$\vec{E}$ is the *total electric field* and may have contributions from charges both inside and outside of the surface

Although Gauss's law can, in theory, be solved to find $\vec{E}$ for any charge configuration, in practice it is limited to simple, symmetric situations

Applying Gauss's Law

To use Gauss's law to find E, you have to choose a Gaussian surface over which the surface integral can be simplified.

<u>The point P, where you want to find E, is on the surface.</u>

Requires source charge to have simple geometry.

The Gaussian surface is a surface you choose, it does not have to coincide with a real surface

Conditions for a Gaussian Surface

Try to choose a surface that satisfies one or more of these conditions:

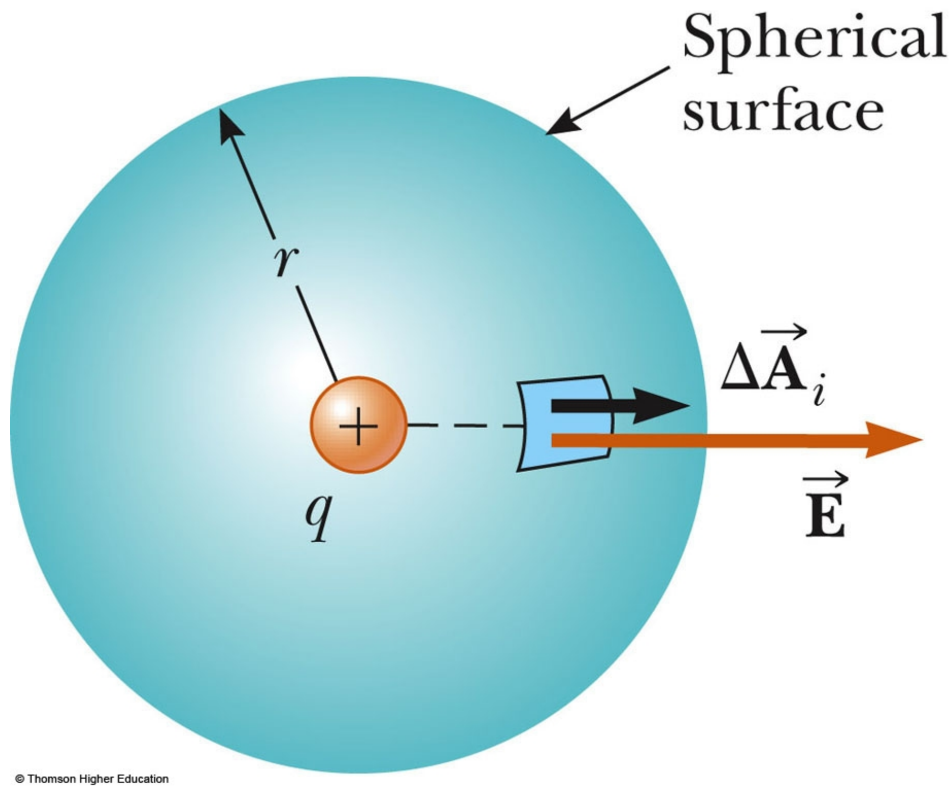
1) The value of the electric field can be argued from symmetry to be constant over the surface
2) The dot product of $\vec{E} \cdot d\vec{A}$ can be expressed as a simple algebraic product EdA because $\vec{E}$ and $d\vec{A}$ are parallel
3) The dot product is 0 because $\vec{E}$ and $d\vec{A}$ are perpendicular
4) The field is zero over some portion of the surface

Gauss's Law – applied to point charge

A positive point charge, q, is located at the center of a sphere of radius r

The magnitude of the electric field everywhere on the surface of the sphere is $E = k_e q / r^2$

On board: we will take the limit of $\Delta A_i \rightarrow dA$ and use Gauss' law to find E at a point P which is a distance r away from q.

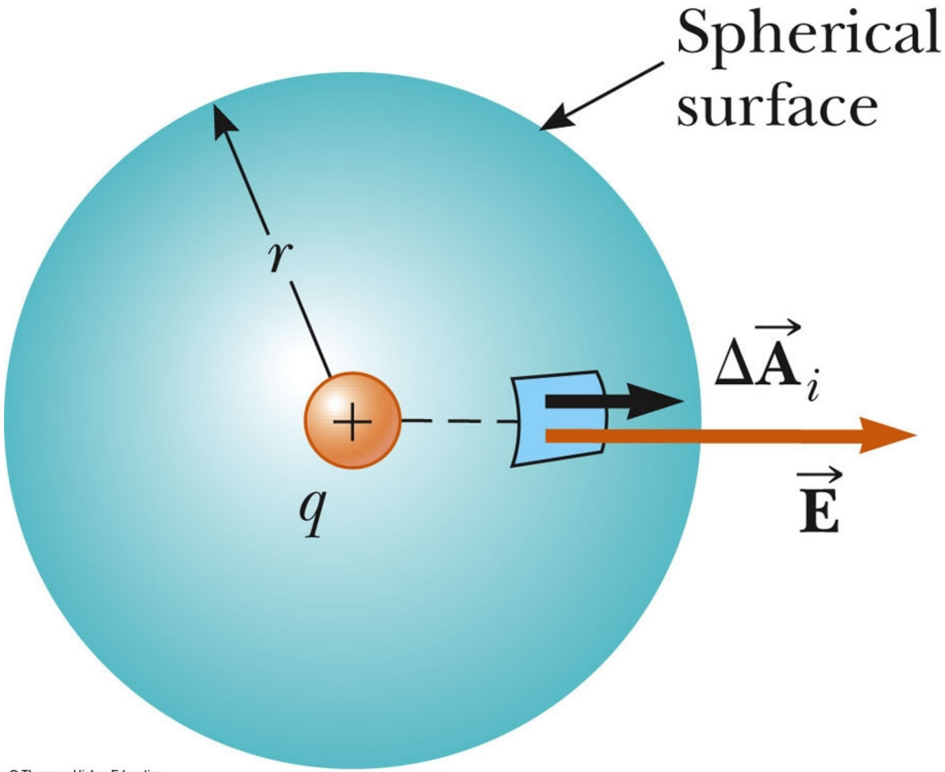


Gauss's Law – applied to point charge

(We're expecting: $E = k_e q / r^2$)

Gauss' law says:

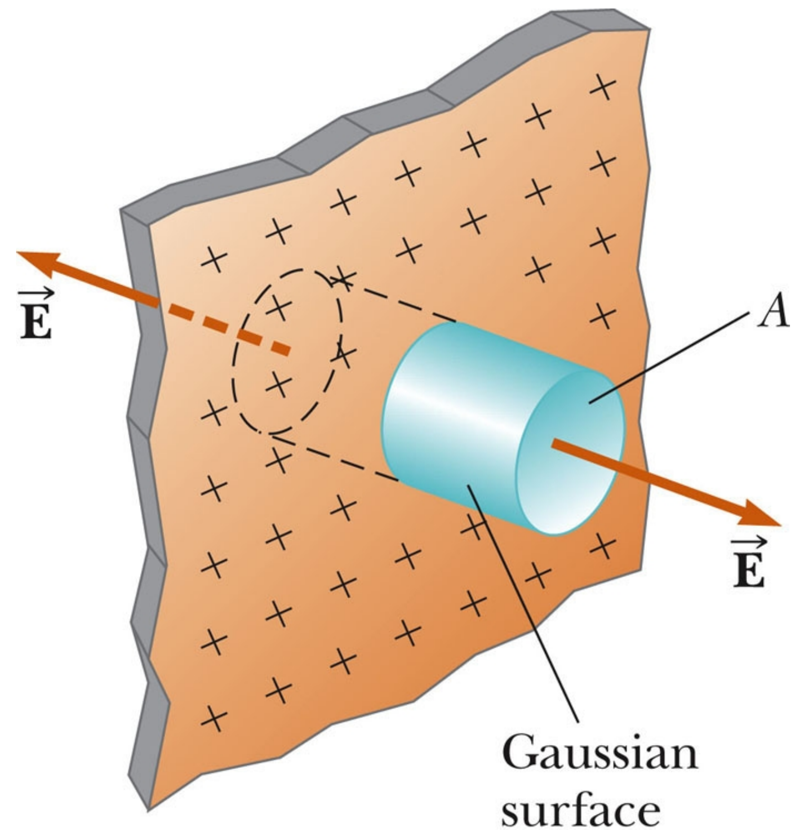
$$\Phi_E = \frac{q_{enc}}{\epsilon_0} = \oint_{sphere} \vec{E} \cdot d\vec{A}$$



$$\oint_{sphere} \vec{E} \cdot d\vec{A} = E \oint_{sphere} dA = E\, 4\pi r^2, \text{ so } E = \frac{q_{enc}}{4\pi\epsilon_0 r^2} = k_e \frac{q}{r^2}$$

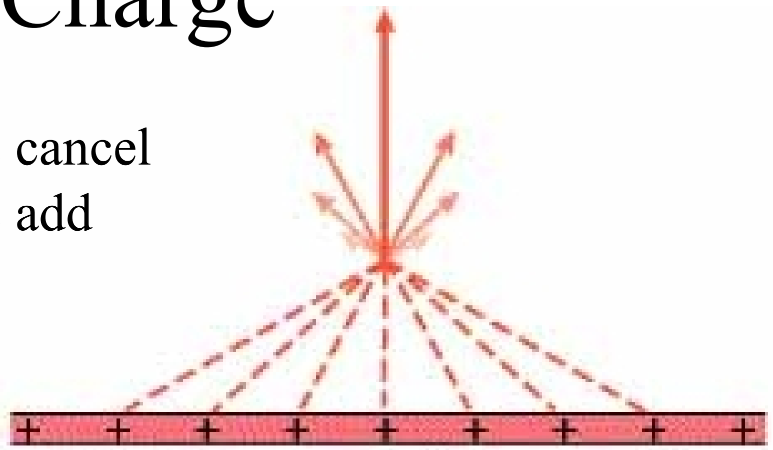
Field Due to a Plane of Charge

- $\vec{E}$ must be perpendicular to the plane and must have the same magnitude at all points equidistant from the plane
- Choose a small cylinder whose axis is perpendicular to the plane for the gaussian surface



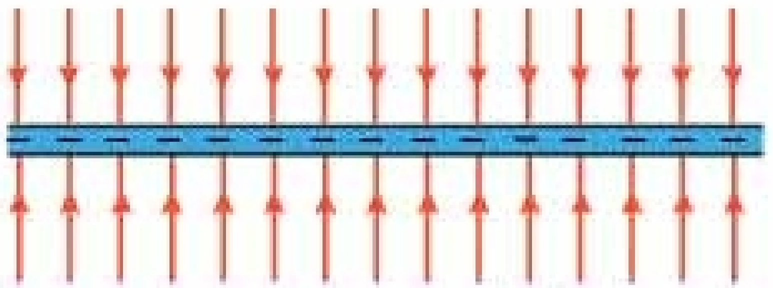
Field Due to a Plane of Charge

x-components cancel
y-components add



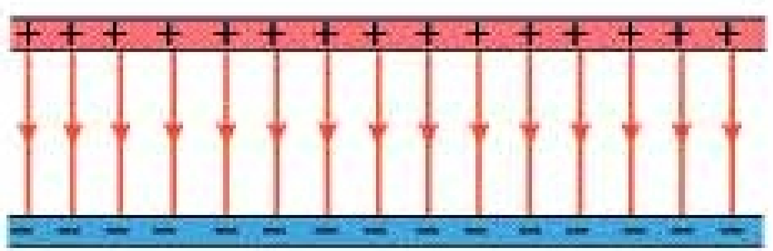
(a)

Positively Charged Plane
(E field away from the plate)



(b)

Negatively Charged Plane
(E field towards the plate)



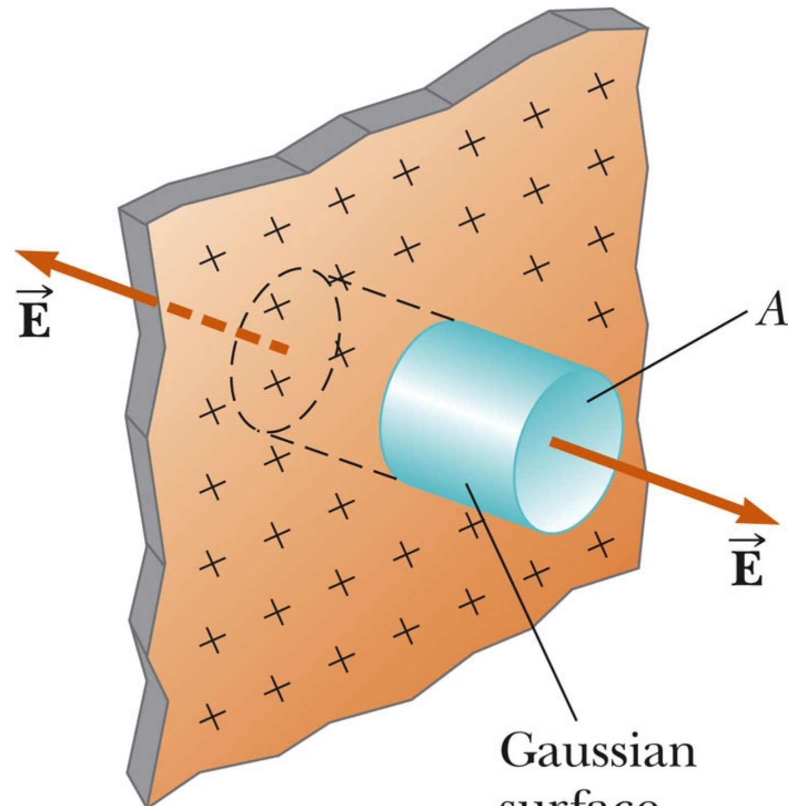
(c)

Superposition or vector addition – fields add like vectors – only a field in the middle

Field Due to a Plane of Charge, cont

$\vec{E}$ is parallel to the curved surface and there is no contribution to the <u>flux</u> from this curved part of the cylinder

The flux through each end of the cylinder is EA and so the total flux is $2EA$



Field Due to a Plane of Charge, final

The total charge in the surface is σA

Applying Gauss's law

$$\Phi_E = \oint_{surface} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$\oint_{surface} \vec{E} \cdot d\vec{A} = \int_{curved} \vec{E} \cdot d\vec{A} + 2\int_{caps} \vec{E} \cdot d\vec{A} = 0 + 2EA$$

$$2EA = \frac{\sigma A}{\epsilon_0}$$

Note, E does not depend on r ! Uniform E-field!

$$E = \frac{\sigma}{2\epsilon_0}$$

A charged conducting plate has 2 layers of charge ...

How would you find this answer using Ch. 21 techniques?

(See Prob 52 in Ch.21 and Examples 21-13 and 21-14.)

Field due to a uniform, spherical volume charge distribution

First consider $r > a$

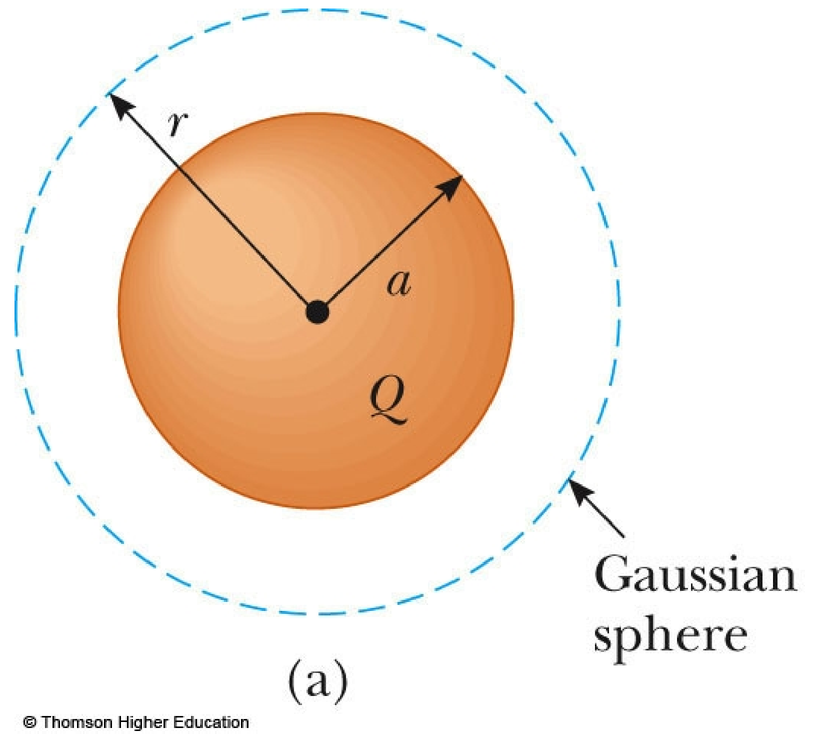
Choose a concentric sphere as the gaussian surface.

$$\Phi_E = \oint_{surface} \vec{E} \cdot d\vec{A} = \oint_{surface} E dA = \frac{q_{enc}}{\epsilon_0}$$

$$E\, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$E = k\frac{Q}{r^2}$$



Spherical charge distribution, cont.

Next consider the inside (*r* < *a*),

Again, select a sphere as the gaussian surface,

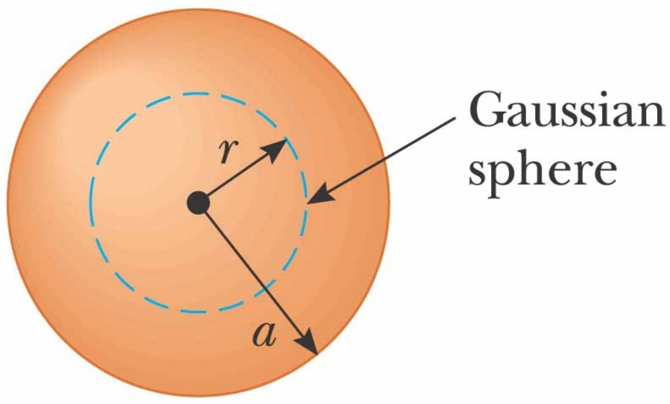
$q_{\mathrm{enc}} < Q$

$q_{\mathrm{enc}} = Q\,(r^3/a^3)$

$$\Phi_E = \oint_{surface} \vec{E} \cdot d\vec{A} = \oint_{surface} E dA = \frac{q_{enc}}{\epsilon_0}$$

$$E = \frac{q_{enc}}{4\pi\epsilon_0 r^2}$$

$$E = k\frac{Q}{a^3}r$$



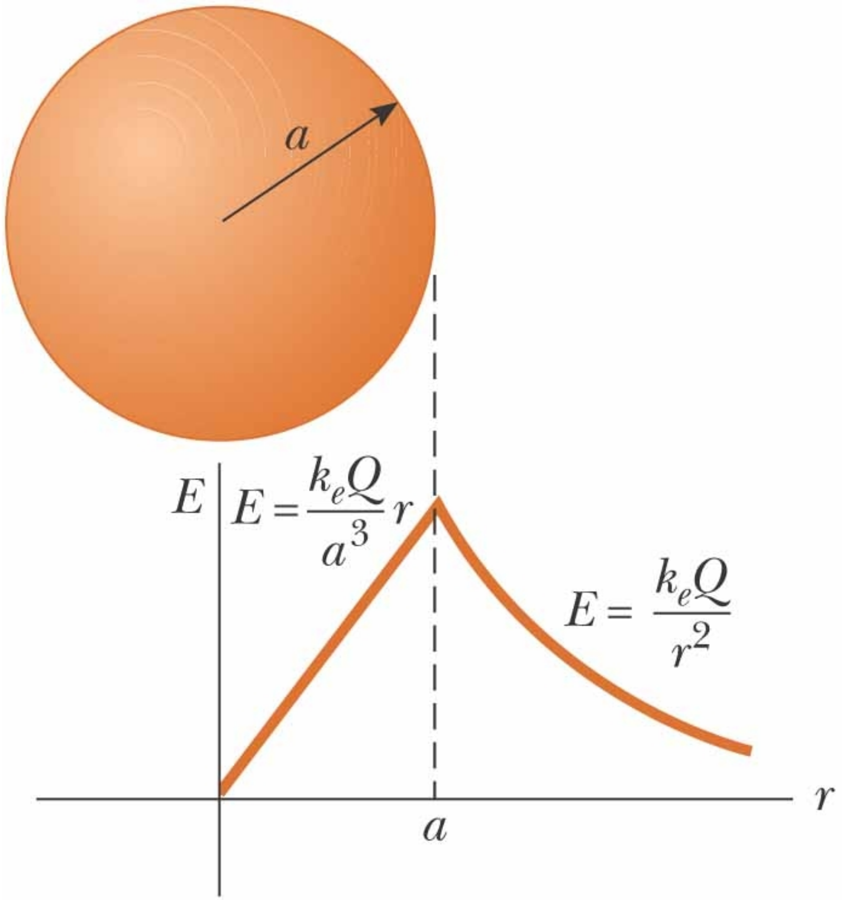
(b)

Spherical charge distribution, final

Inside the sphere, E varies linearly with r

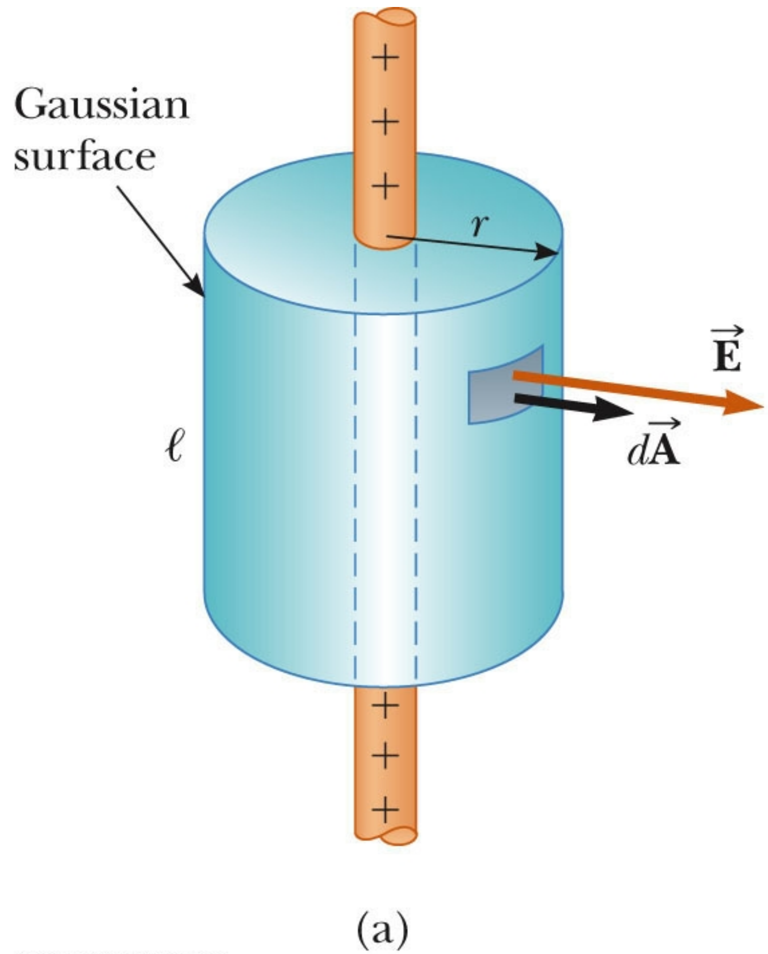
$E \rightarrow 0$ as $r \rightarrow 0$

The field outside the sphere is equivalent to that of a point charge located at the center of the sphere!



Field at a Distance from a Line of Charge

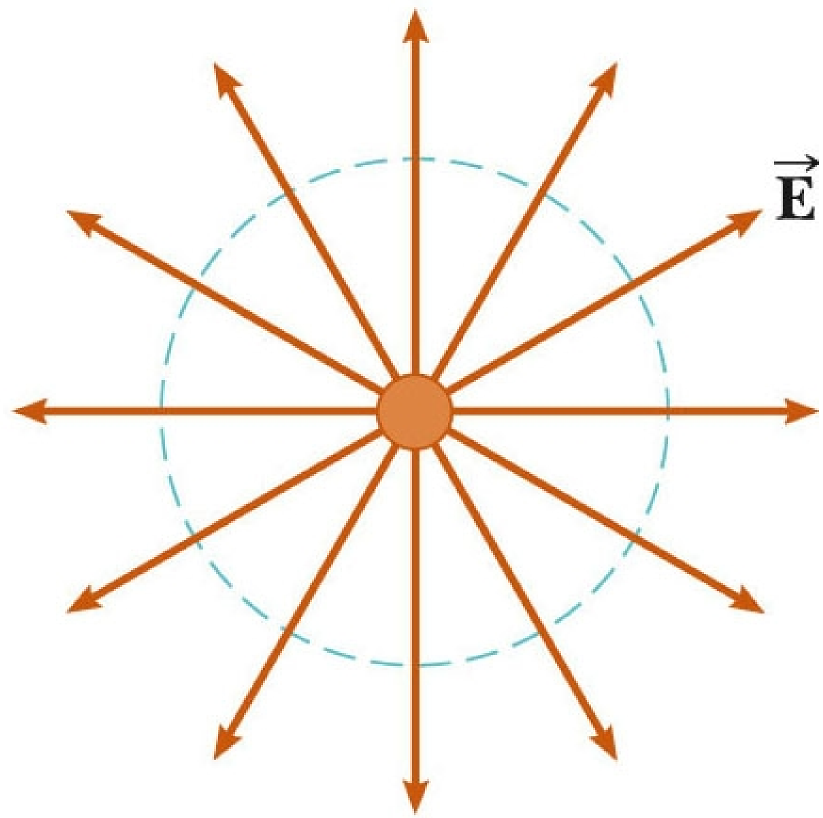
- Note: P. 44(b) of Ch. 23 says that the field of a charged rod of infinite length is E=2kλ/r.
- Select a cylindrical gaussian surface concentric with "line of charge".
  - The cylinder has a radius of *r* and a length of ℓ
- **E** is constant in magnitude and perpendicular to the surface at every point on the curved part of the surface



Field Due to a Line of Charge, cont.

The end view confirms the field is perpendicular to the curved surface.

The field through the ends of the cylinder is 0 since the field is parallel to these surfaces.



(b)

Field Due to a Line of Charge, final

Use Gauss' law to find the field

$$\Phi_E = \oint_{surface} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

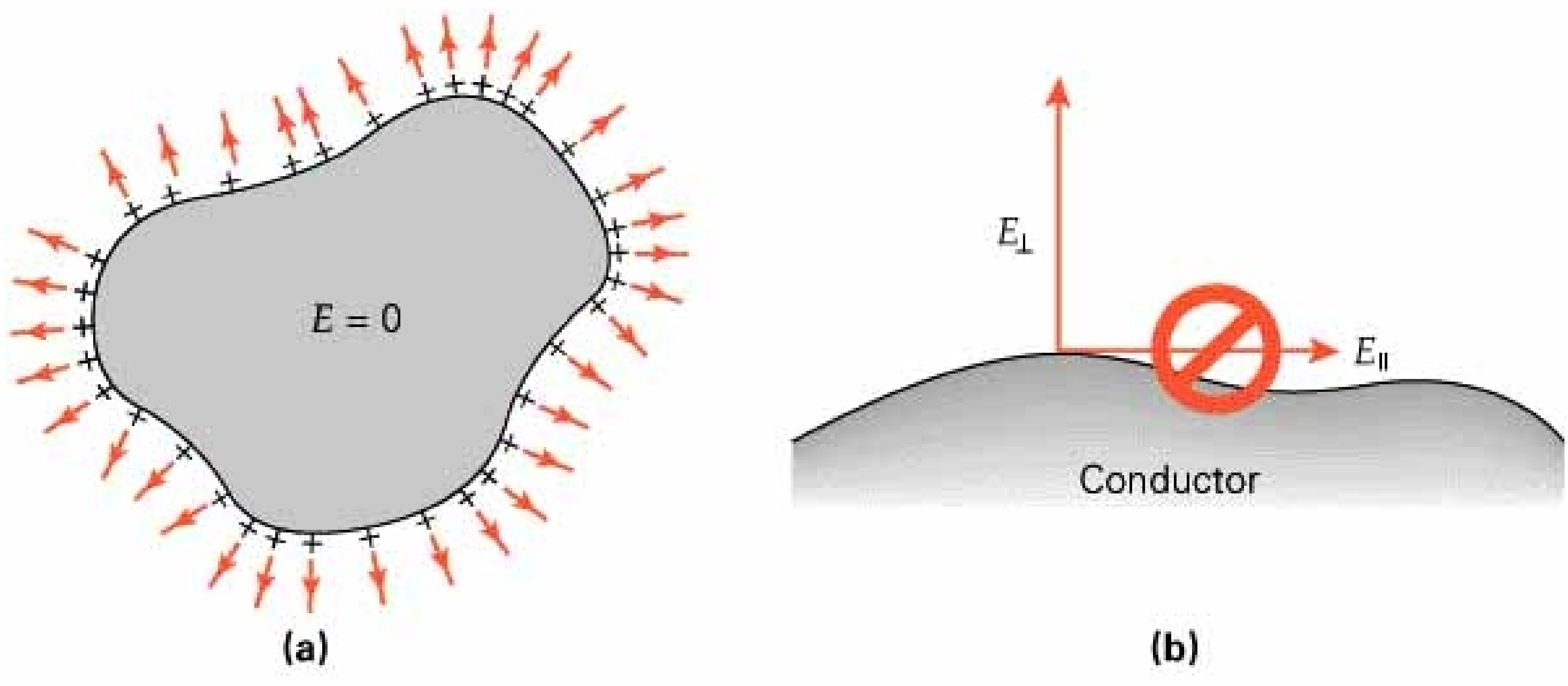
$$\oint_{cylinder} \vec{E} \cdot d\vec{A} = \int_{curved} \vec{E} \cdot d\vec{A} + \int_{caps} \vec{E} \cdot d\vec{A} = E\,2\pi r l$$

But $\frac{q_{enc}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$,

so $E = \frac{\lambda}{2\pi\epsilon_0 r}$

Electrostatics: E fields and conductors

- In electrostatics, E fields always point away (perpendicular) from the conductor and E = 0 inside of a conductor.
  - If they did not, electrical currents would exist, F = q E, and charges would move to an equilibrium configuration.
- Net charge resides on the surface of conductors
- Charge accumulates at sharp points



Example – Conducting Spherical shell

- Metal has zero net charge, $Q = 0 = q_{\text{inside surface}} + q_{\text{outside surface}}$

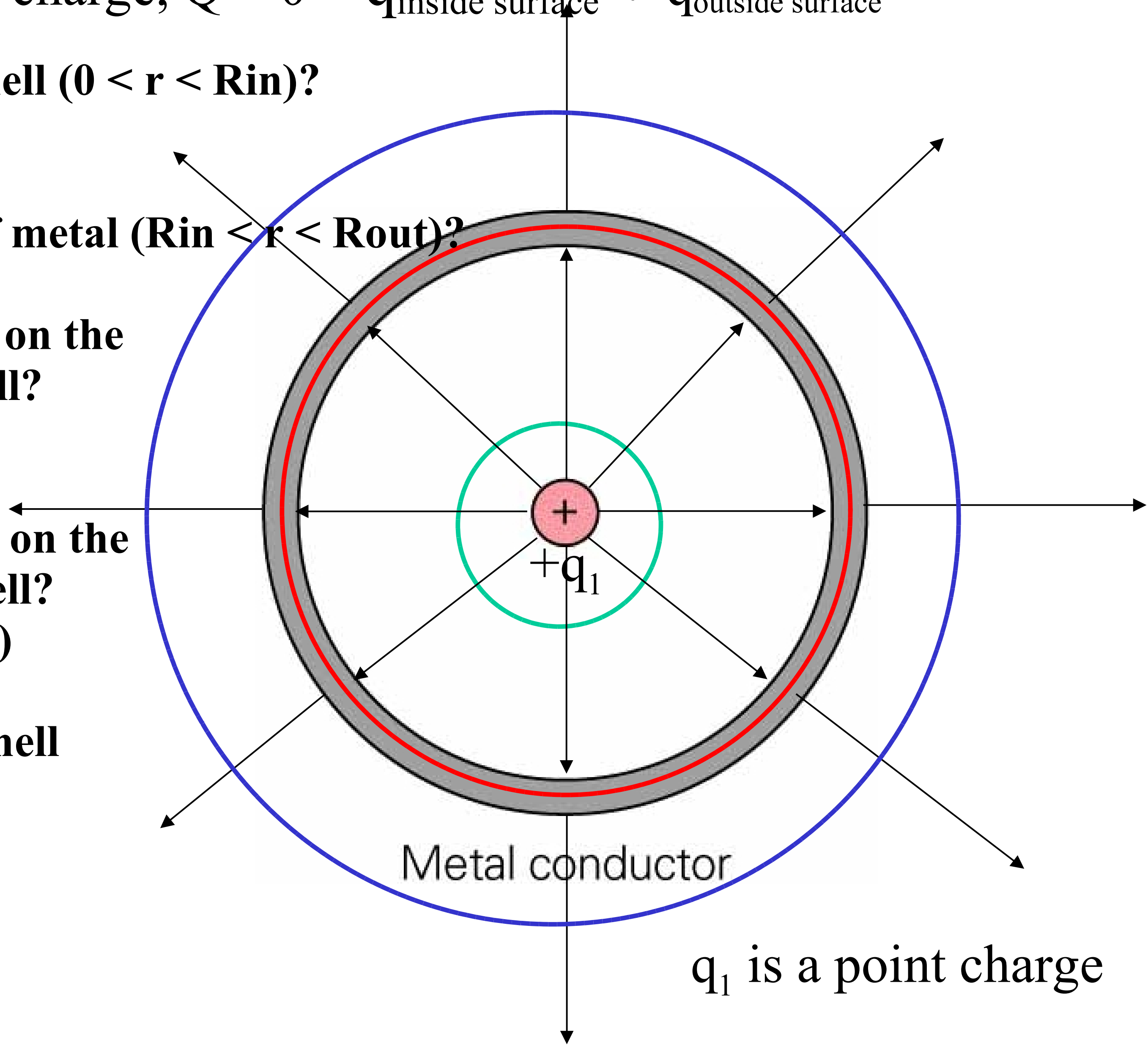
a) What is E inside shell (0 < r < Rin)?
(Use green surface)

b) What is E inside of metal (Rin < r < Rout)?

c) What is the charge on the inner surface of shell?
(Use red surface)

d) What is the charge on the outer surface of shell?
(Use what was given)

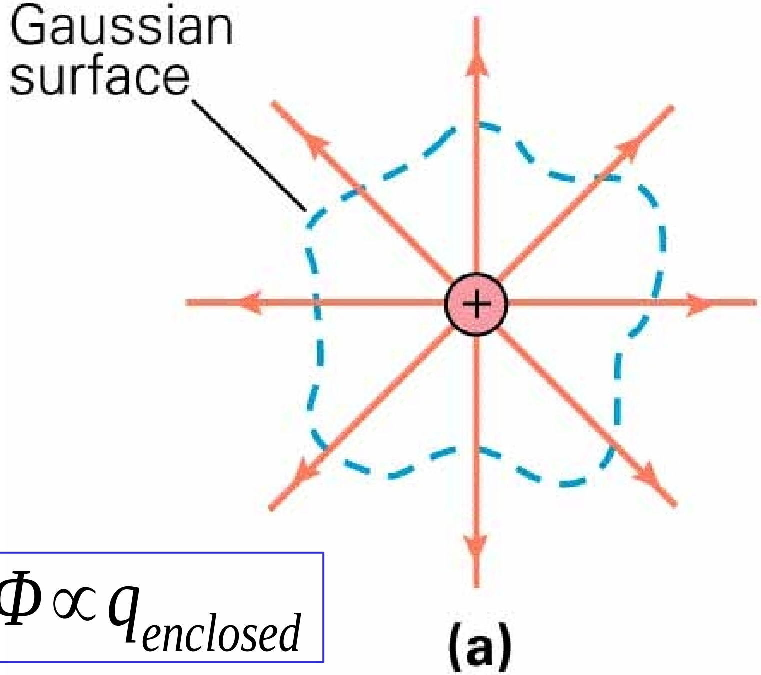
e) What is e outside shell (r > Rout)?
(Use blue surface)



q_1 is a point charge

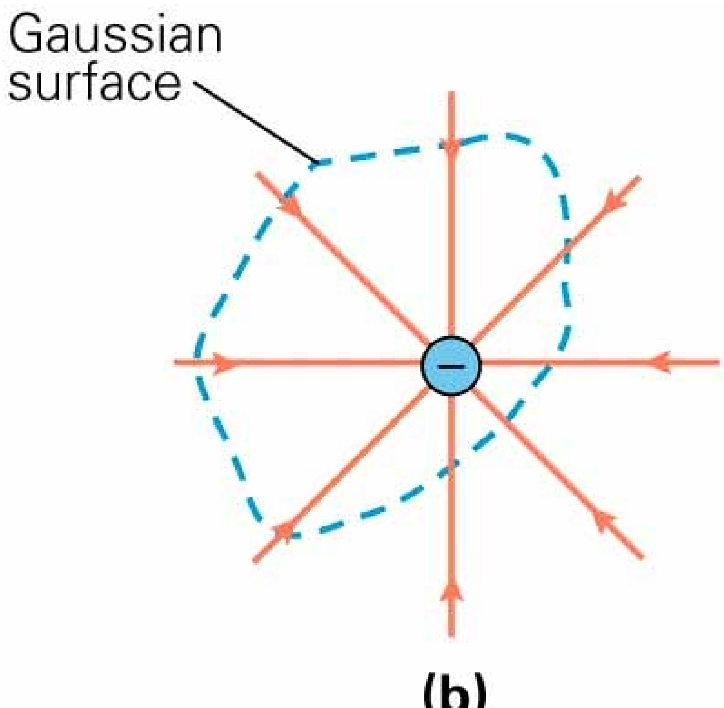
Gauss's Law

- The **net** number of E field lines, Φ, passing through an imaginary closed surface is **proportional** to the amount of net charge enclosed within that surface, $q_{enclosed}$.
- Surface is imaginary and arbitrary, but convenient



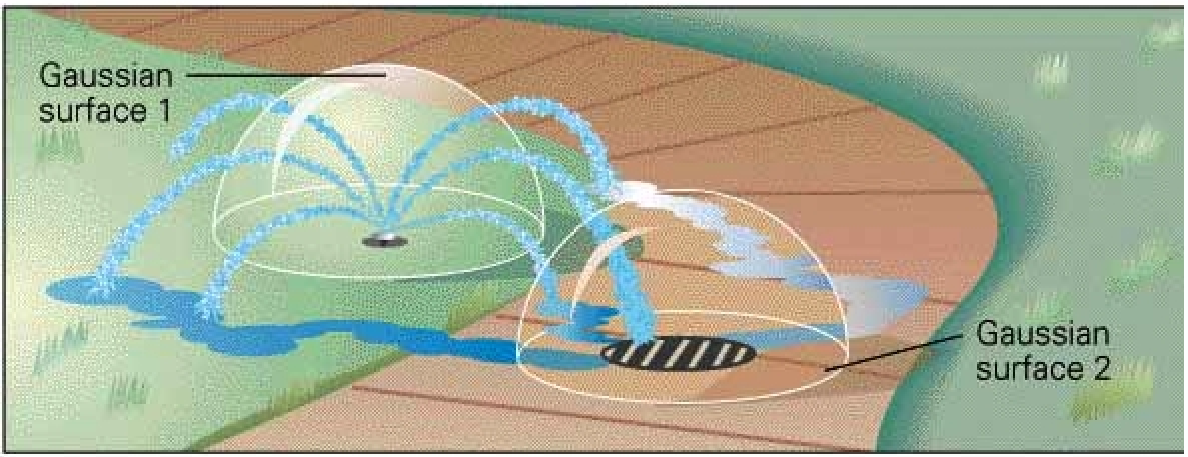
$$\Phi \propto q_{enclosed}$$

(a)

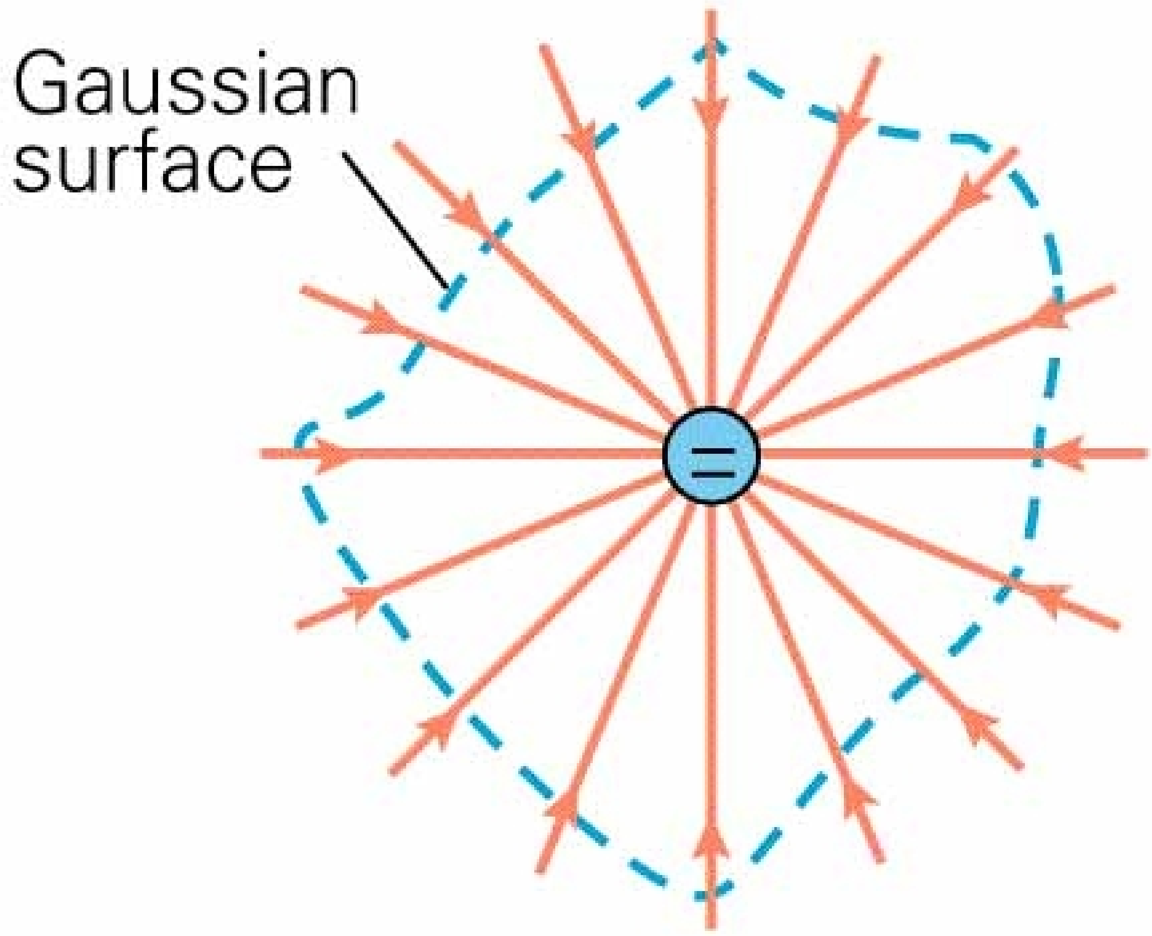


(b)

Analogy with water



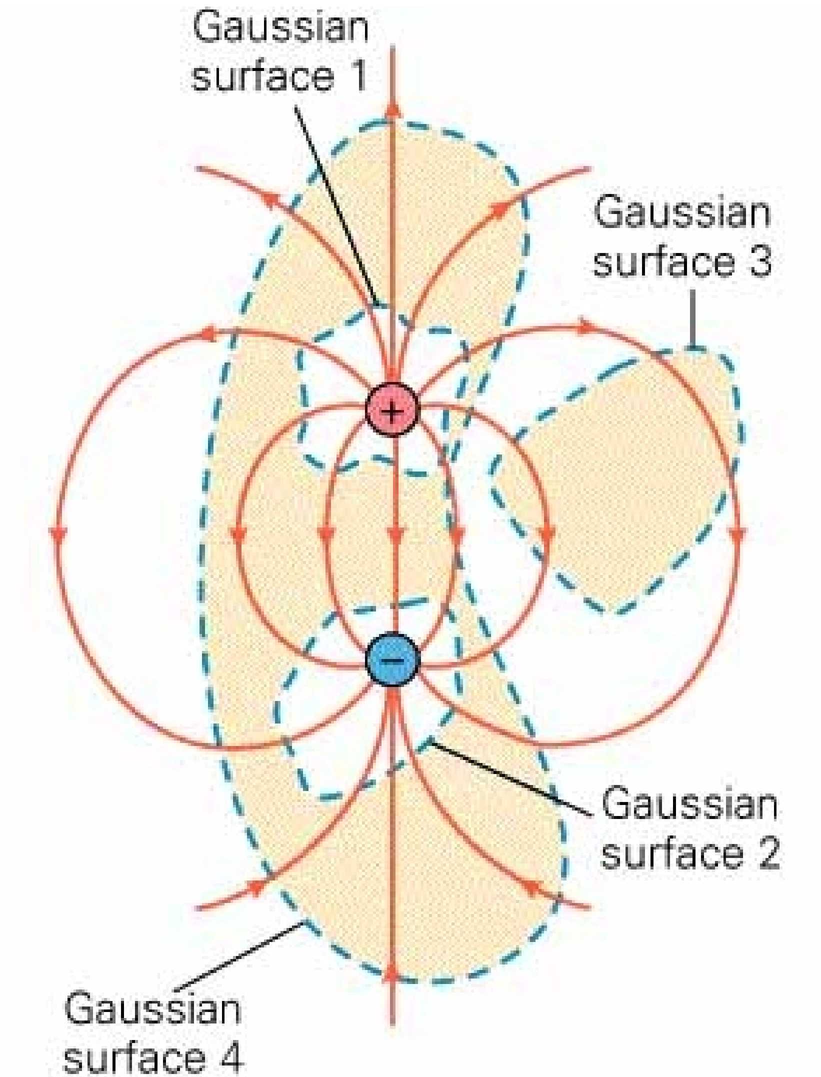
- More E lines enter than leave with a net negative charge
- The larger the net charge, the more lines



(c)

Dipole example

- Surface 1 $\Phi_1 \sim q_{enclosed,1} > 0$
  - net positive charge
- Surface 2 $\Phi_2 \sim q_{enclosed,2} < 0$
  - net negative charge
- Surface 3 $\Phi_3 \sim q_{enclosed,3} = 0$
  - zero net charge
- Surface 4 $\Phi_4 \sim q_{enclosed,4} = 0$
  - zero net charge



(d)

Gauss's Law, Conductors, and Electrostatics

- Very useful in these cases.
- Inside conductors, E = 0, so there will be no flux through a surface inside of a conductor.

$$\Phi \propto q_{enclosed} = 0$$