Electric Potential of a Point Charge

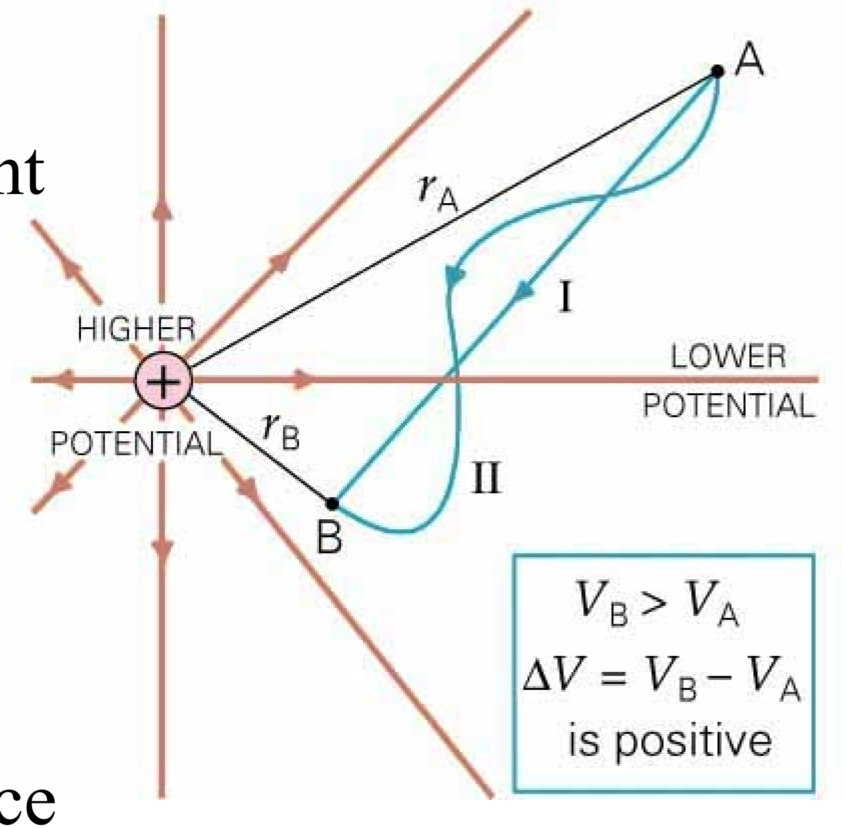
- Recall E field

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

- Electric Potential of a point charge

$$V = \frac{kq}{r}$$

$$V = 0 \quad \text{when} \quad r \rightarrow \infty$$

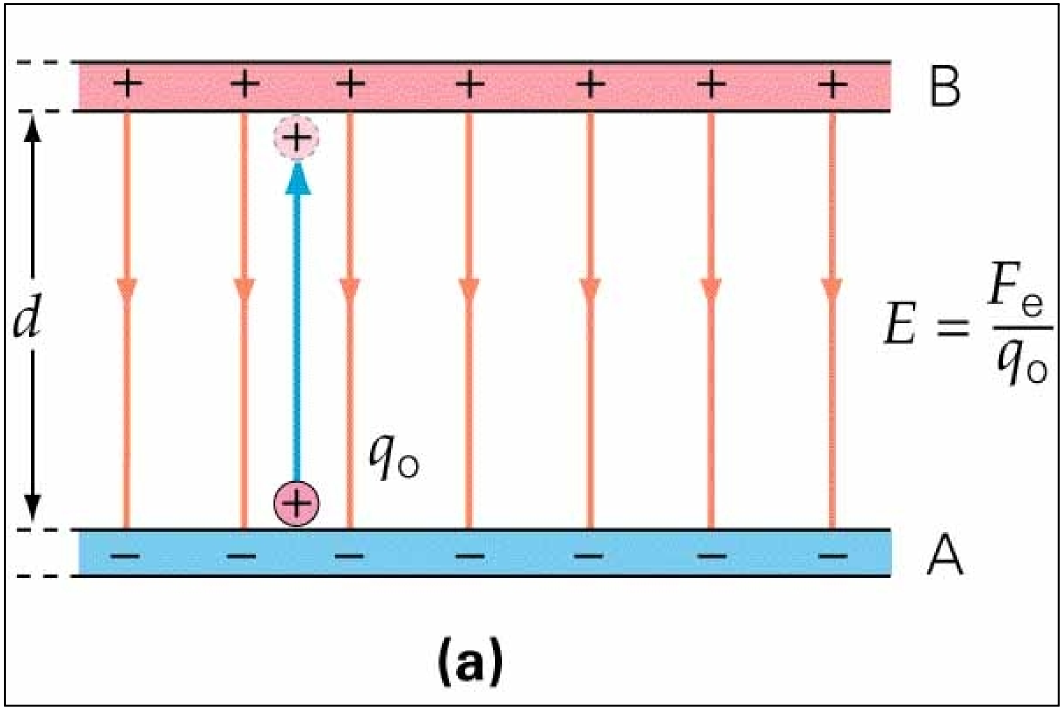


- Electric potential difference

$$\Delta V_{ab} = \frac{kq}{r_b} - \frac{kq}{r_a}$$

Electric Potential

- Electric Potential of a point charge (last slide)
- Electric potential closely related to *potential energy*
  - $\Delta U = q\Delta V$
  - And to *work*: $W_{byfield} = -q\Delta V = -\Delta U$
  - Convention: both U and V = 0 at r=infinity
- Electric potential closely related to electric force
  - $F_E\Delta \mathbf{r} = W_{byfield} = -q\Delta V$
- Electric potential closely related to electric field
  - $\delta V = -E\delta \mathbf{r}$ so that potential difference is: $\Delta V = -\int \vec{E}\cdot d\,\vec{l}$
- Electric potential is easier to find than the E-field because it is not a vector



Analogy with gravity

F_E on q_0 is down

$W_{Efield} = -|F_E|d \quad (F_E = q_0 E)$

$\Delta U = -W_{Efield} = |F_E|d$

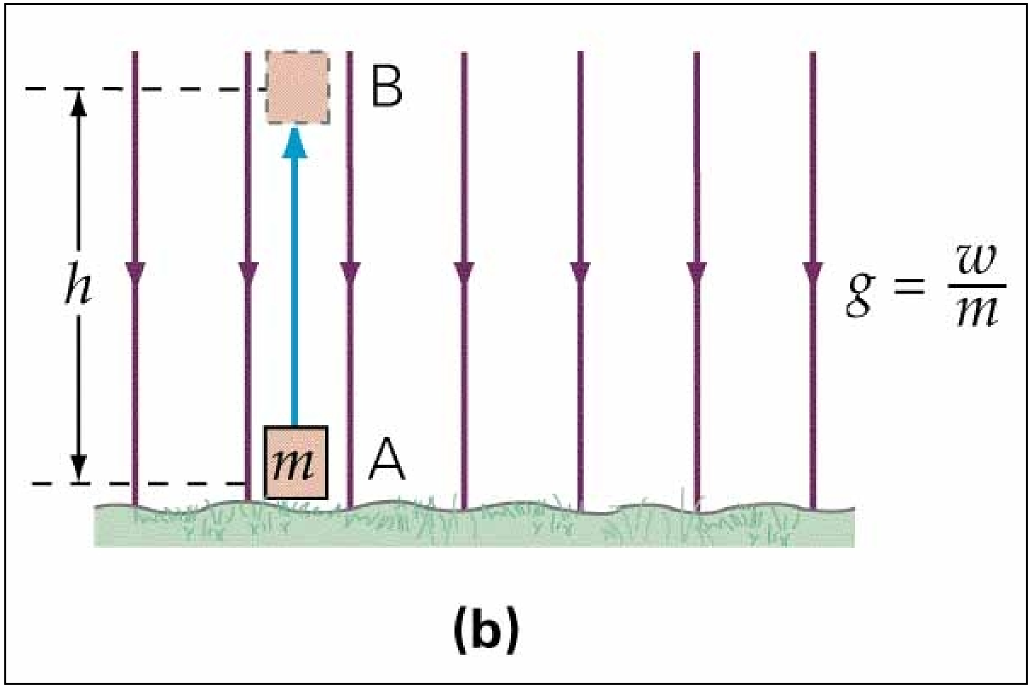
$\Delta V = \Delta U / q_0$

F_g on m is down

$W_{Gfield} = -|F_g|h \quad (F_g = mg)$

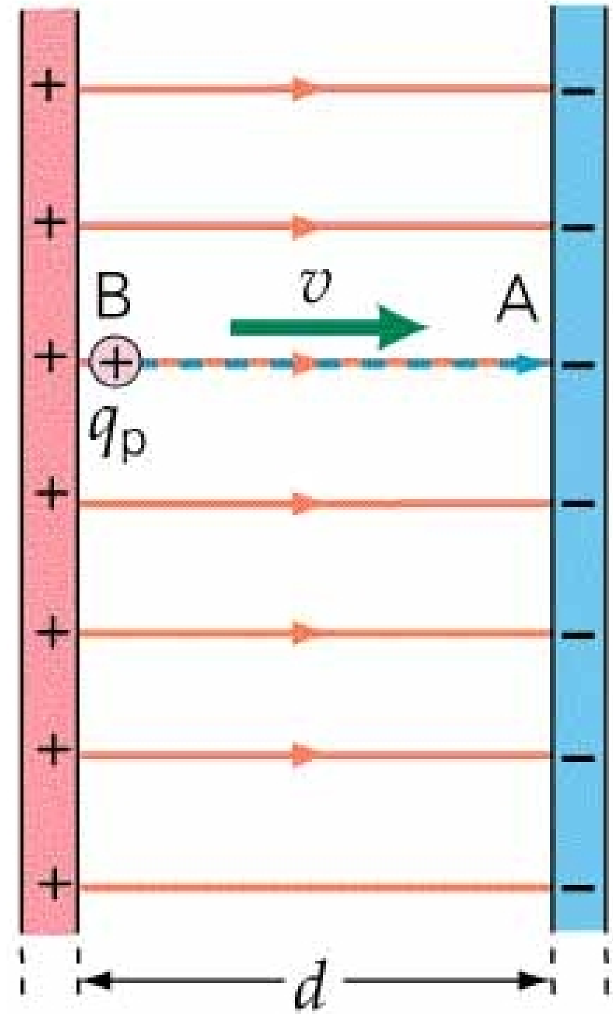
$\Delta U_G = -W_{Gfield} = |F_g|h$

$\Delta V_G = \Delta U_G / m$



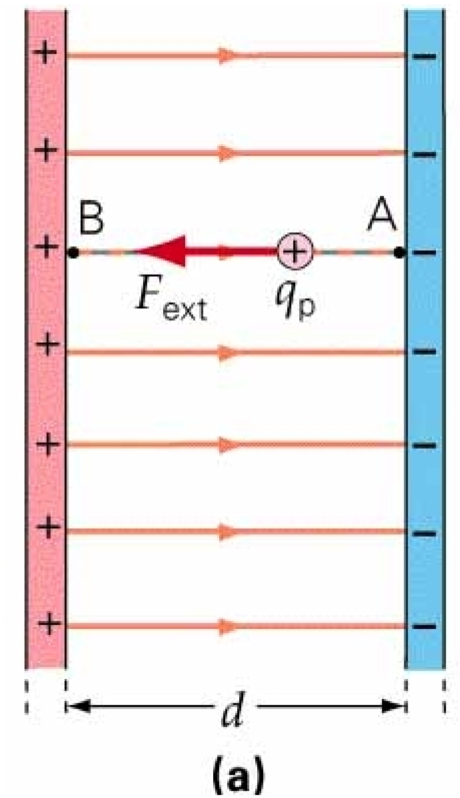
Parallel Plates

- Releasing a positive test charge from rest at point B...

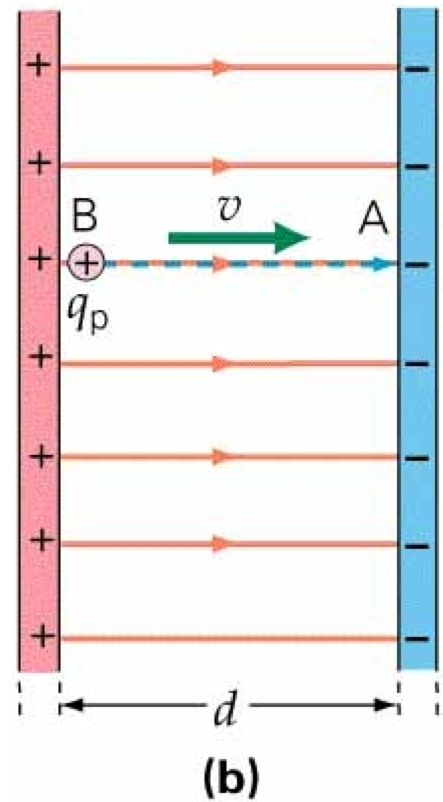


(b)

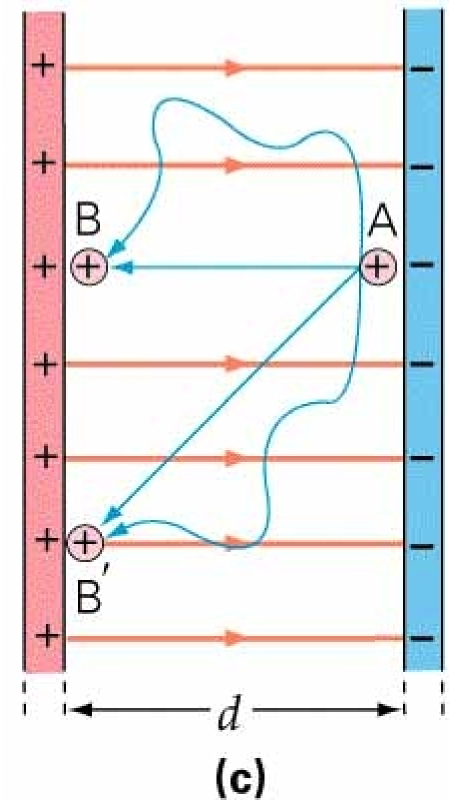
Electric Potential Energy (conservation of energy ideas)



Work is done to move the charge, so we store potential energy, U_E



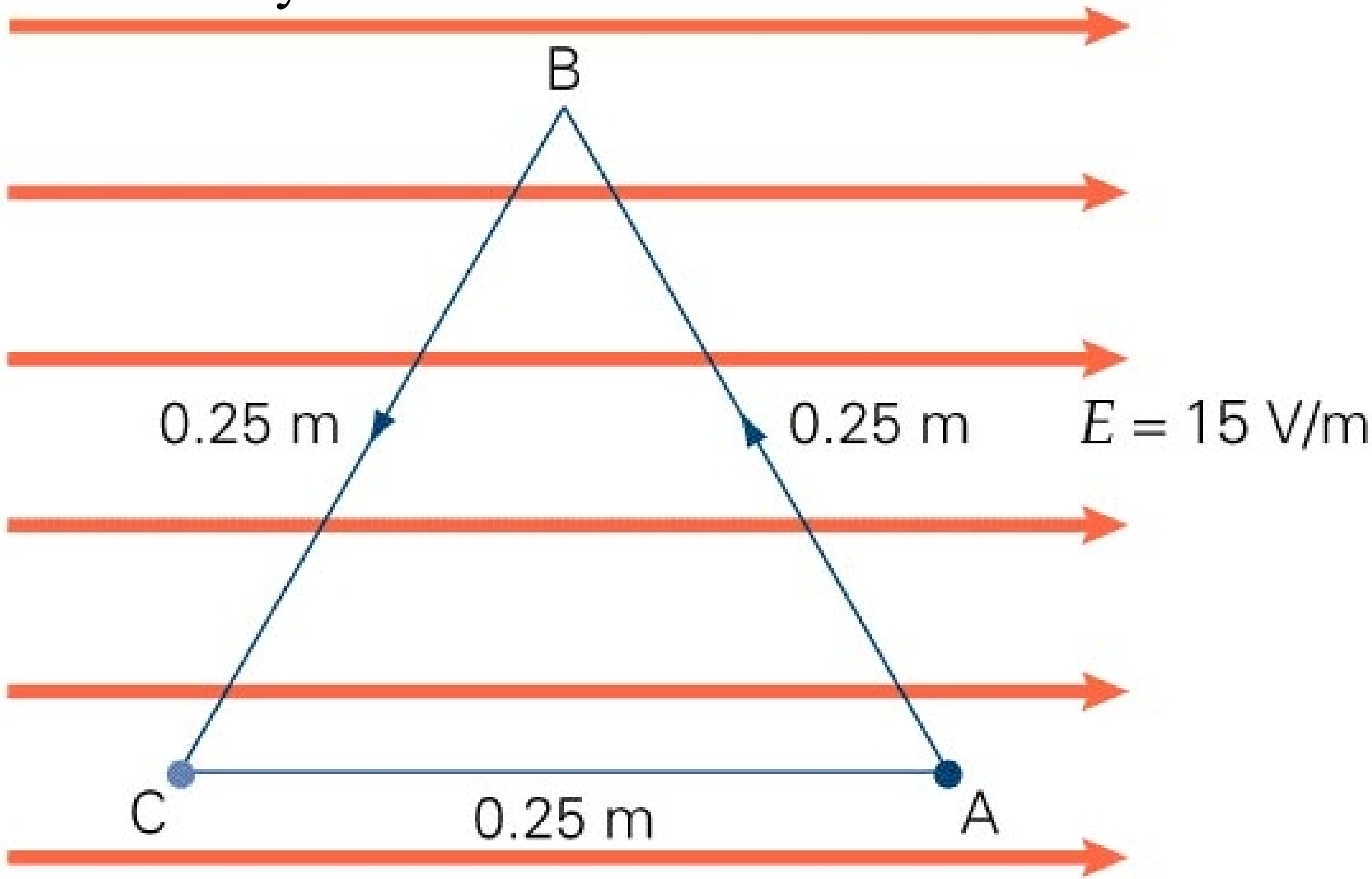
Charge is released and energy is converted from U_E to KE



Only the displacement in the direction of the E field matters
(ΔU_E independent of path)

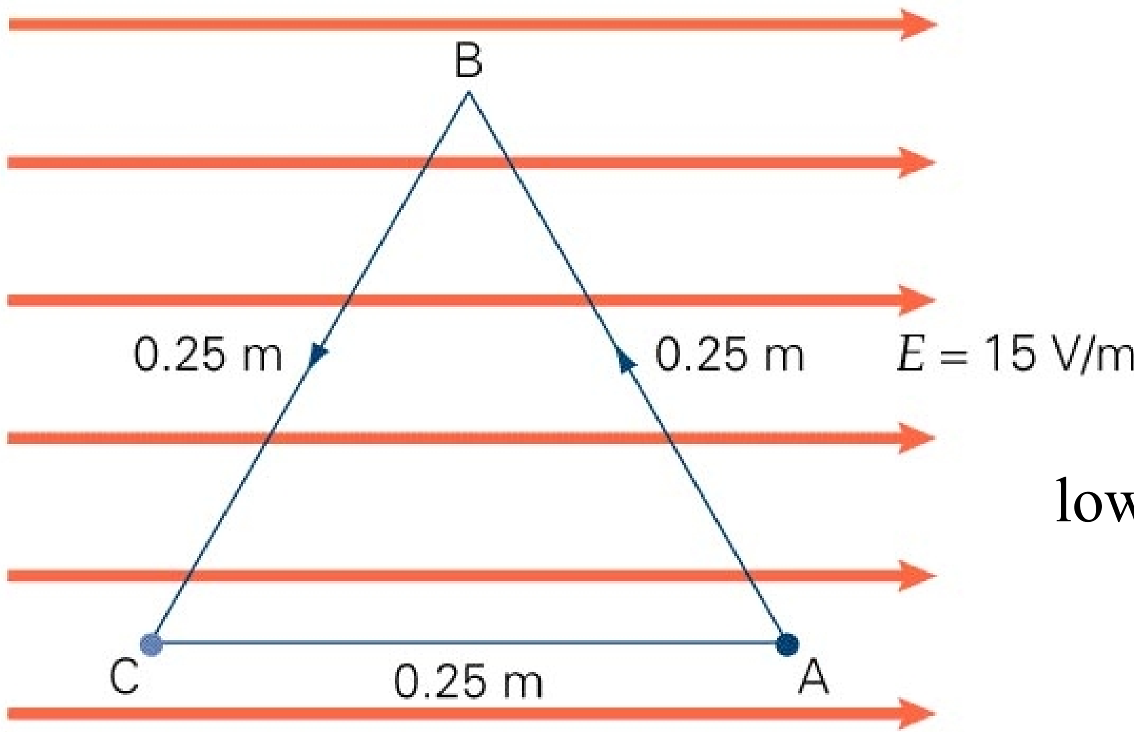
Problem: closed loop path, ABCA

- Work done is path independent
  - Only the initial and final position matter
  - Look for an easy solution!



Problem: find V's and ΔV's

Higher V



lower V

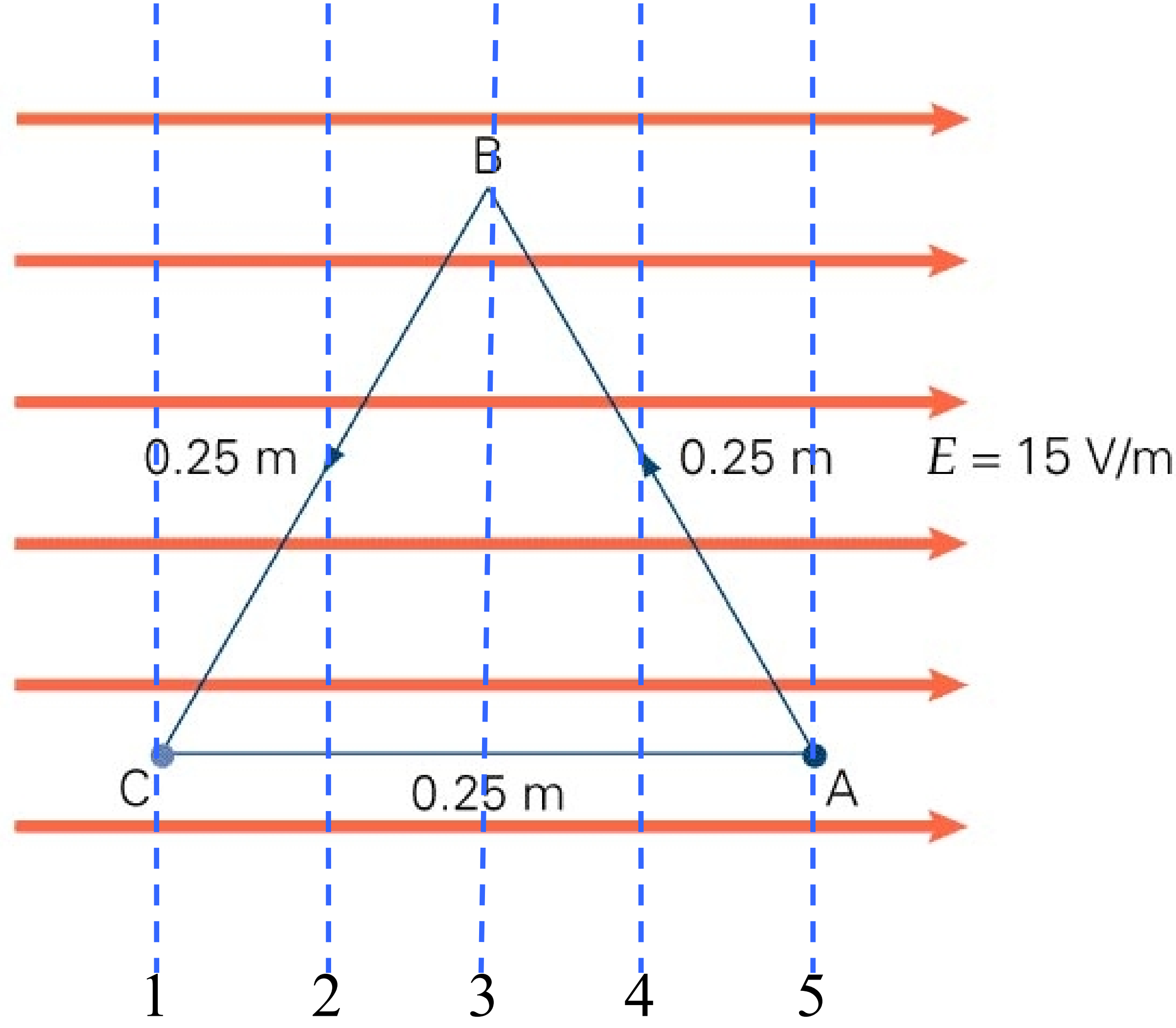
$$|\Delta V_{AC}| = Ed = \left(15\frac{V}{m}\right)(0.25\,m) = 3.75\,V$$

$$\Delta U = q\Delta V = \left(-1.6\times10^{-19}\,C\right)\left(3.75\,V\right) = -6\times10^{-19}\,J$$

Problem: find V and ΔV

$V_1 - V_5 = 3.75$ V

$V_1 = 3.75$ V
$V_2 = 2.8125$ V
$V_3 = 1.875$ V
$V_4 = 0.9375$ V
$V_5 = 0$ V



Electric Potential Energy U_E

- Building up arrangements of charge
  - Energy required to "build" = ΔU
- Bring a point charge in from infinity
  - like charges requires energy
    - repulsive forces
  - unlike charges give up energy
    - attractive forces

$$W = Fd = qEd$$

and $E = \dfrac{kq}{r^2}$

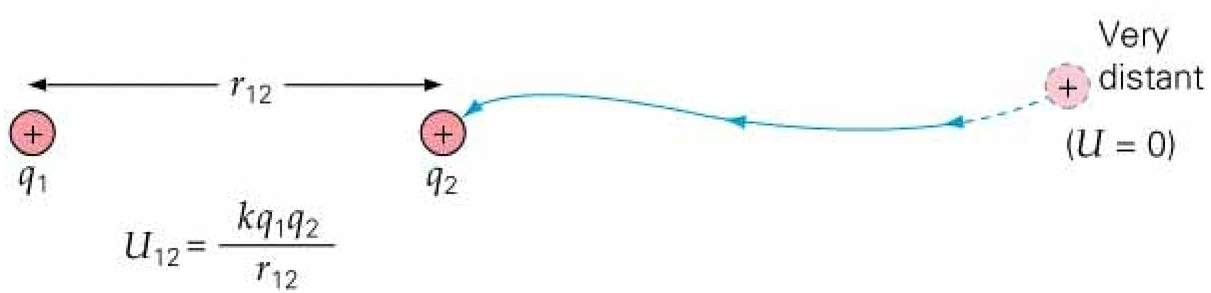
...are difficult to use since E is not a constant.

Can use:

$$U_{12} = \Delta U_{12} = q_2 \Delta V_{\infty 1}$$

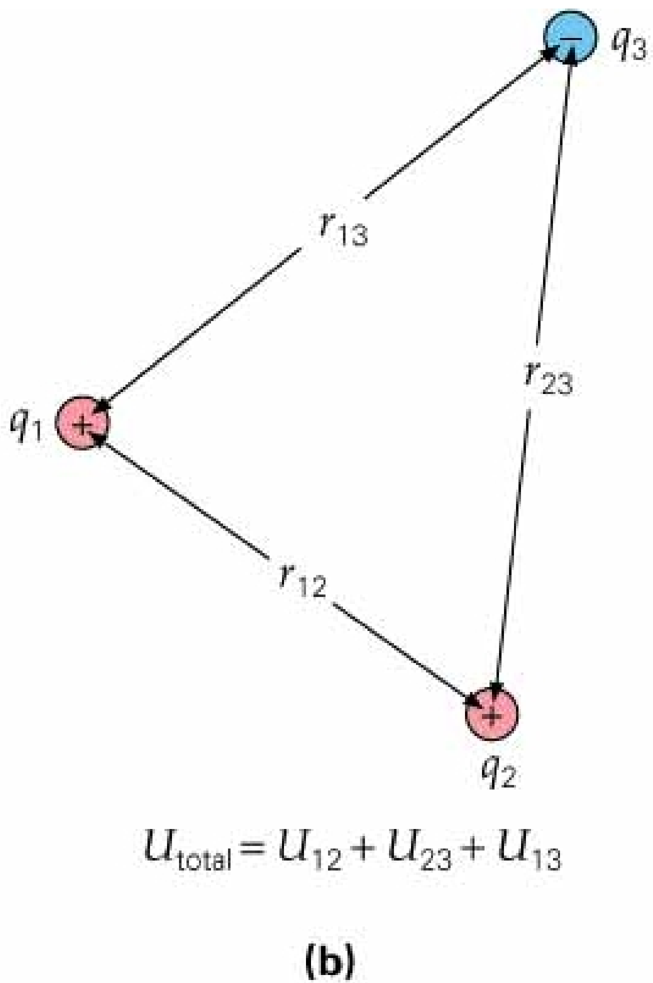
$$V_\infty = 0$$

$$V_1 = k\frac{q_1}{r_{12}}$$



U_E for more than two charges

- Don't double count
- Bring each one in from "infinity"



- Bringing together like charges requires energy (force them together)
- Bringing together un-like charges gives up energy (fall together naturally)

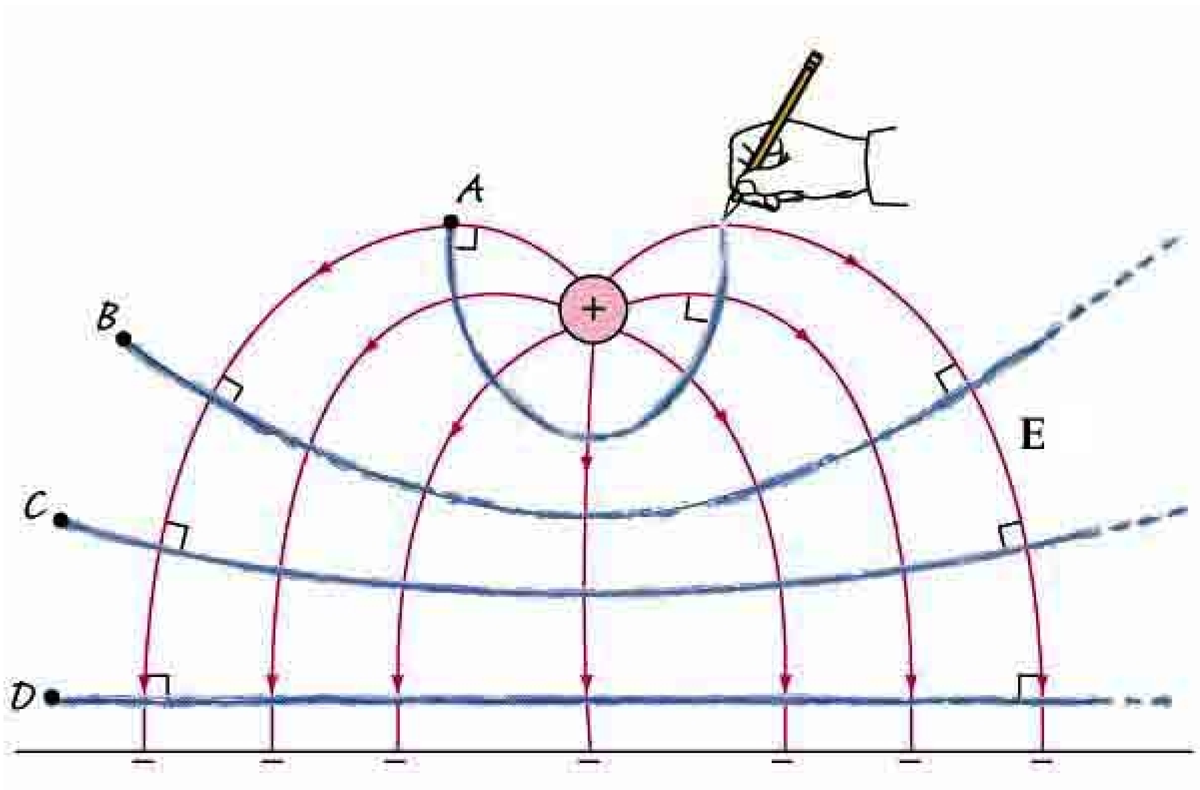
$$U_{12} = k\frac{q_1 q_2}{r_{12}}$$

$$U_{23} = k\frac{q_2 q_3}{r_{23}}$$

$$U_{13} = k\frac{q_1 q_3}{r_{13}}$$

Equipotential Surfaces

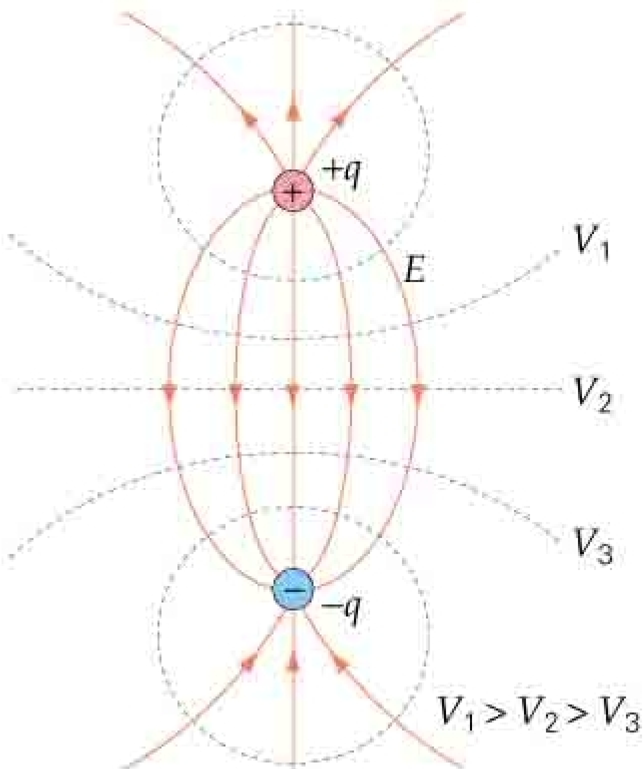
- E field is perpendicular to the equipotential surfaces
- The surface of a conductor is an equipotential surface
  - no E field parallel to the surface in *Electrostatics*
  - gradually "match" the boundaries

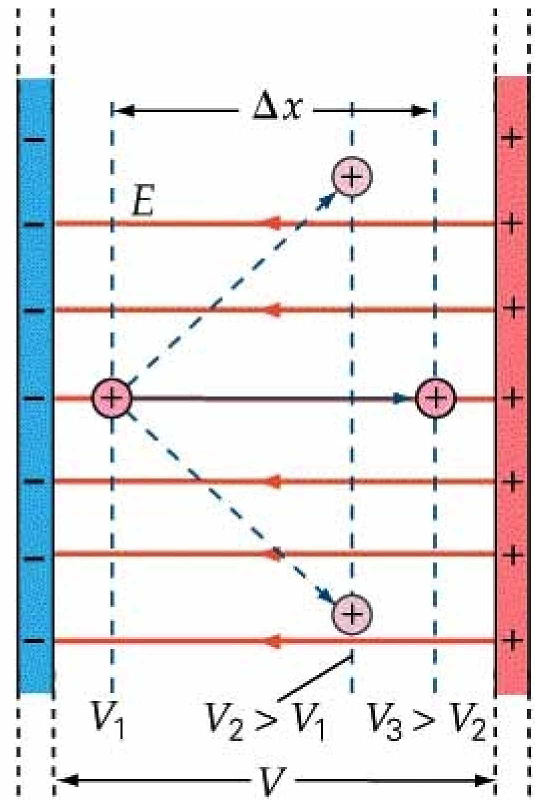


Equipotential Surfaces

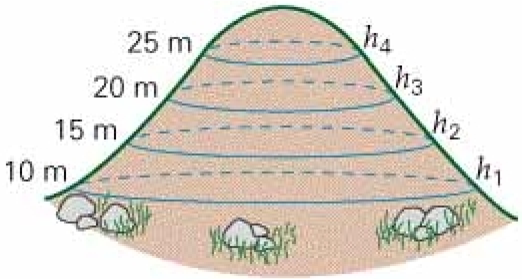
Equipotentials are perpendicular to the E-field lines.

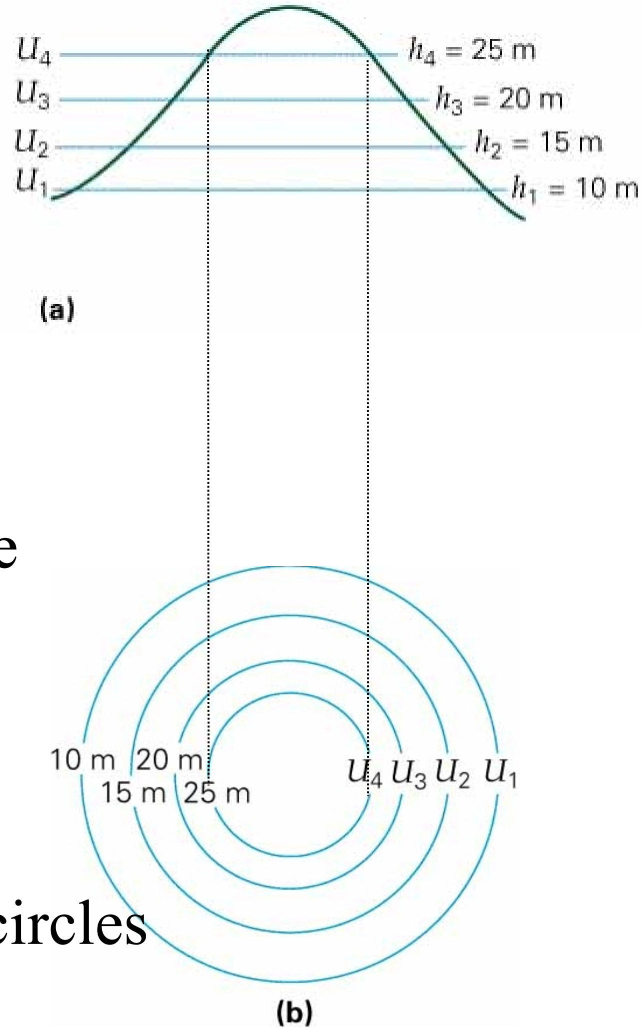
E field points “down hill”





Contours of a map analogy





Lines of equal altitude are like
Lines of equal potential.

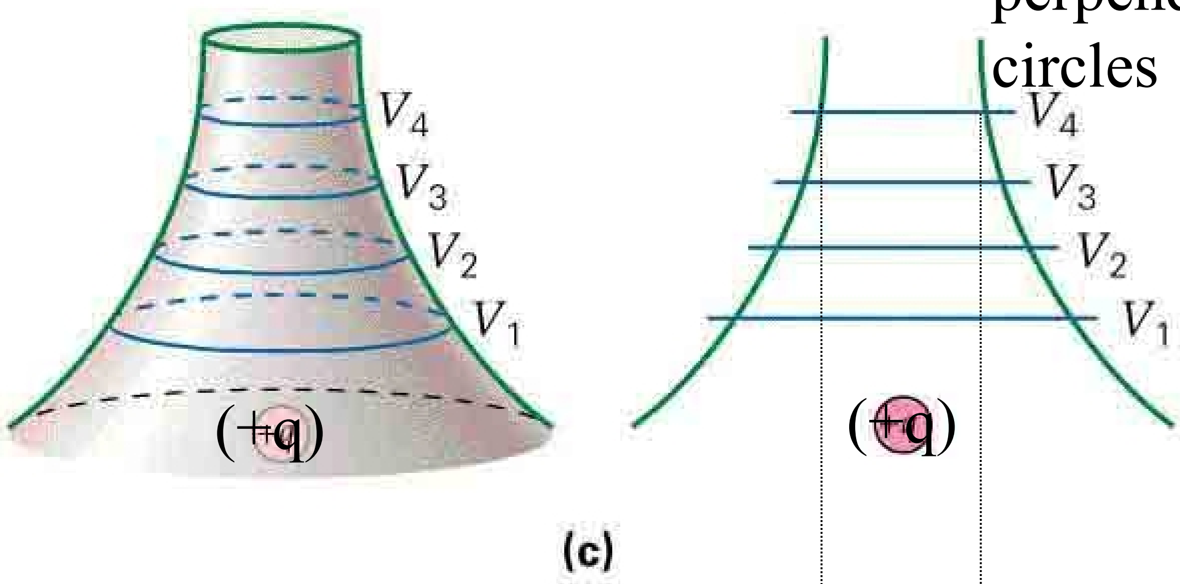
Net force on a positive test charge will point "down hill" just like net force on a boulder will point down hill

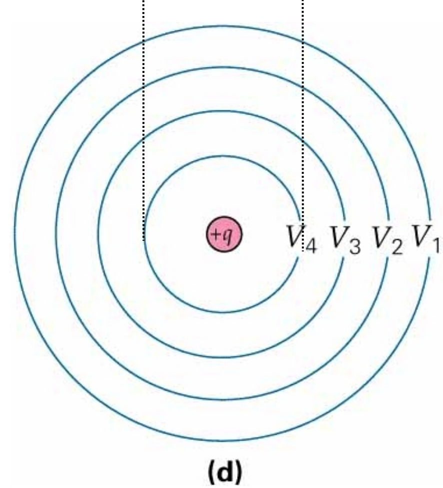
F and E are perpendicular to the circles

$$\Delta V_{ab} = \frac{kq}{r_b} - \frac{kq}{r_a}$$

Analogy with Gravity and hills

E field points "down hill" perpendicular to the circles





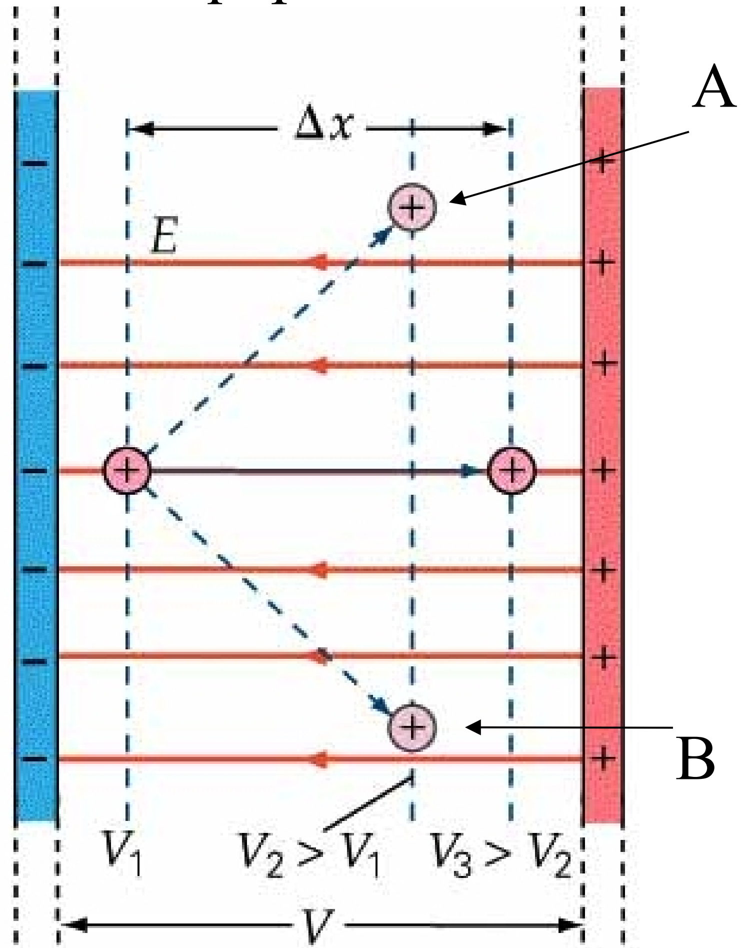
Field gets stronger closer to the point charge. Don't have to go as far to have the same change in electric potential

$$E_r = -\left(\frac{dV}{dr}\right)$$

Slightly misleading (circles would not be evenly spaced for V~1/r)

Equipotential Surfaces

- Imaginary or real surfaces of constant voltage
  - The surfaces of a conductor are equipotential surfaces
- E field and equipotential surfaces are perpendicular to each other



If a charge moves from A to B along an equipotential surface, then

$$\Delta V_{AB} = 0$$

$$\Delta U_{AB} = q\Delta V_{AB} = 0$$