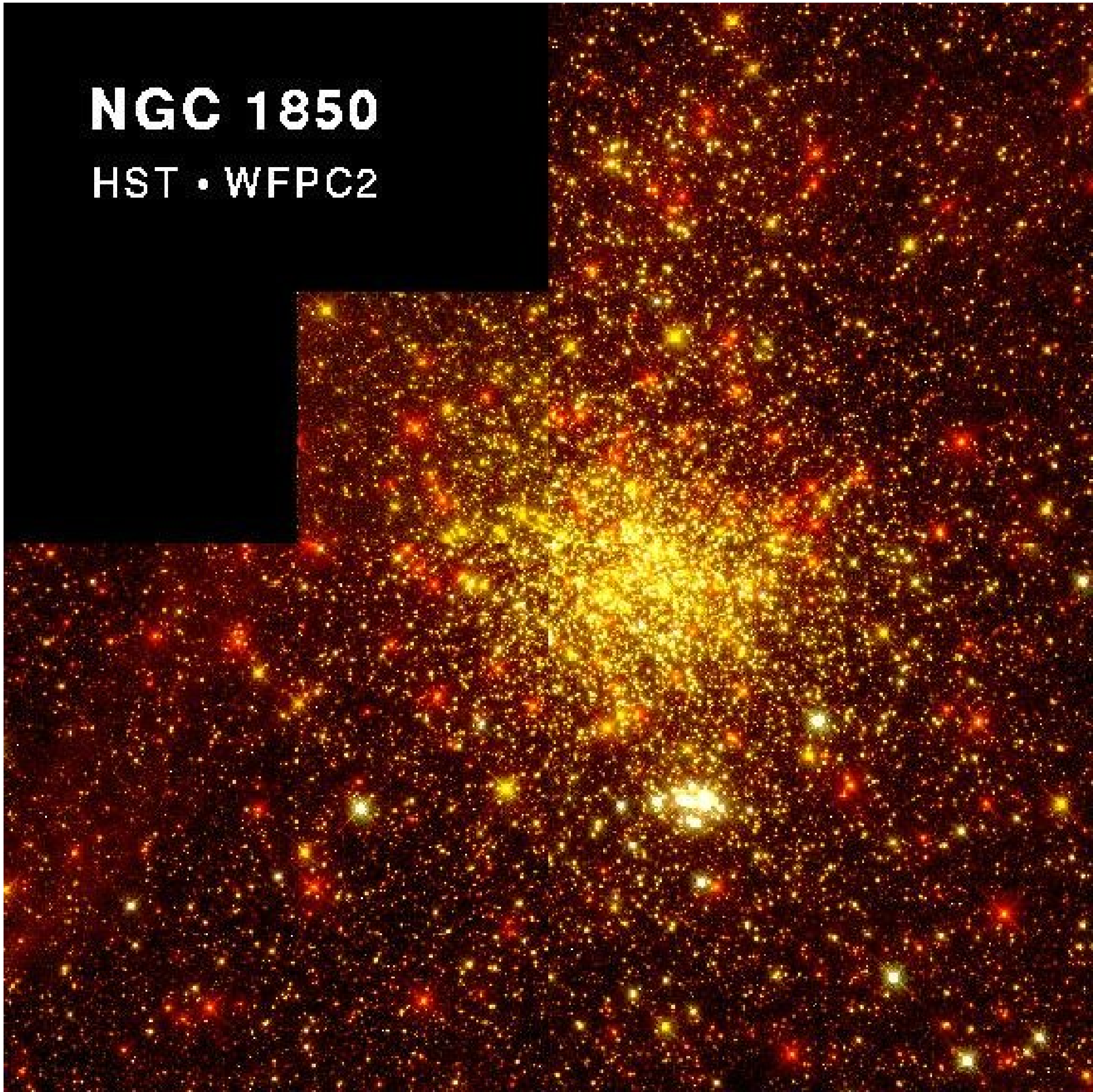
The Classification of Stellar Spectra

Chapter 8

Star Clusters in the Large Magellanic Cloud



http://www.seds.org/hst/NGC1850.html

The Classification of Stellar Spectra

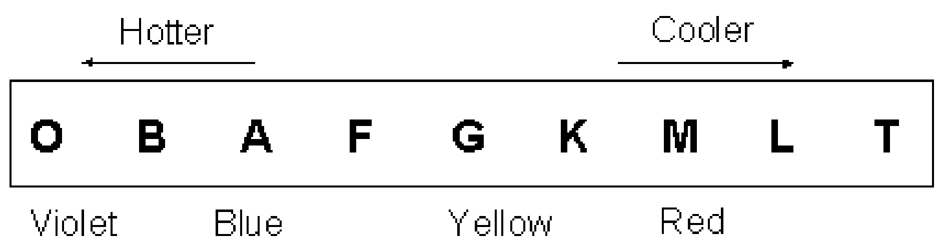
- Classification scheme developed before the physics
- Parameters that can be used to classify stars
  - Luminosity (Brightness)
  - Temperature (Color)
  - Spectra (Composition)
  - Mass
  - Age
- The Henry Draper Catalogue

The Harvard Computers of the Harvard College Observatory

http://cannon.sfsu.edu/%7Egmarcy/cswa/history/pick.html

The Classification of Stellar Spectra

- The Henry Draper Catalogue …
  - HD numbers
  - Originally based on brightness, but switched to temperature
  - Spectral Types:



Early type → late type

L and T are more modern additions – in the infrared

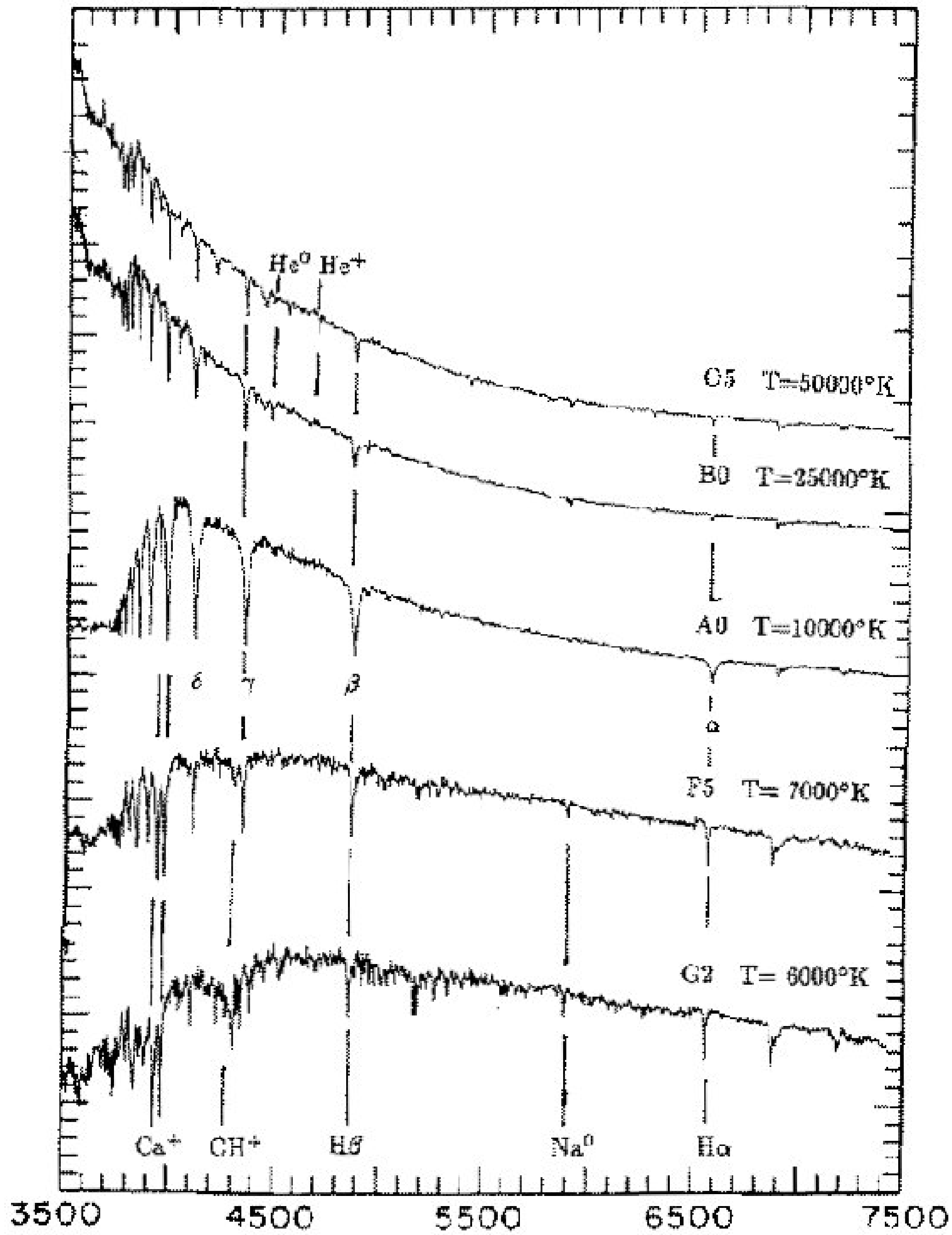
  - Subdivisions in tenths: 0 → 9 (early → late, hot → cool) within a Spectral Type)
  - The Sun is a G2 – an early G-type star
    - G – yellow start
      - Solar type spectra
      - Ca II (singly ionized) lines continue becoming stronger
      - Fe I, other *neutrals* metal lines become stronger

The Classification of Stellar Spectra

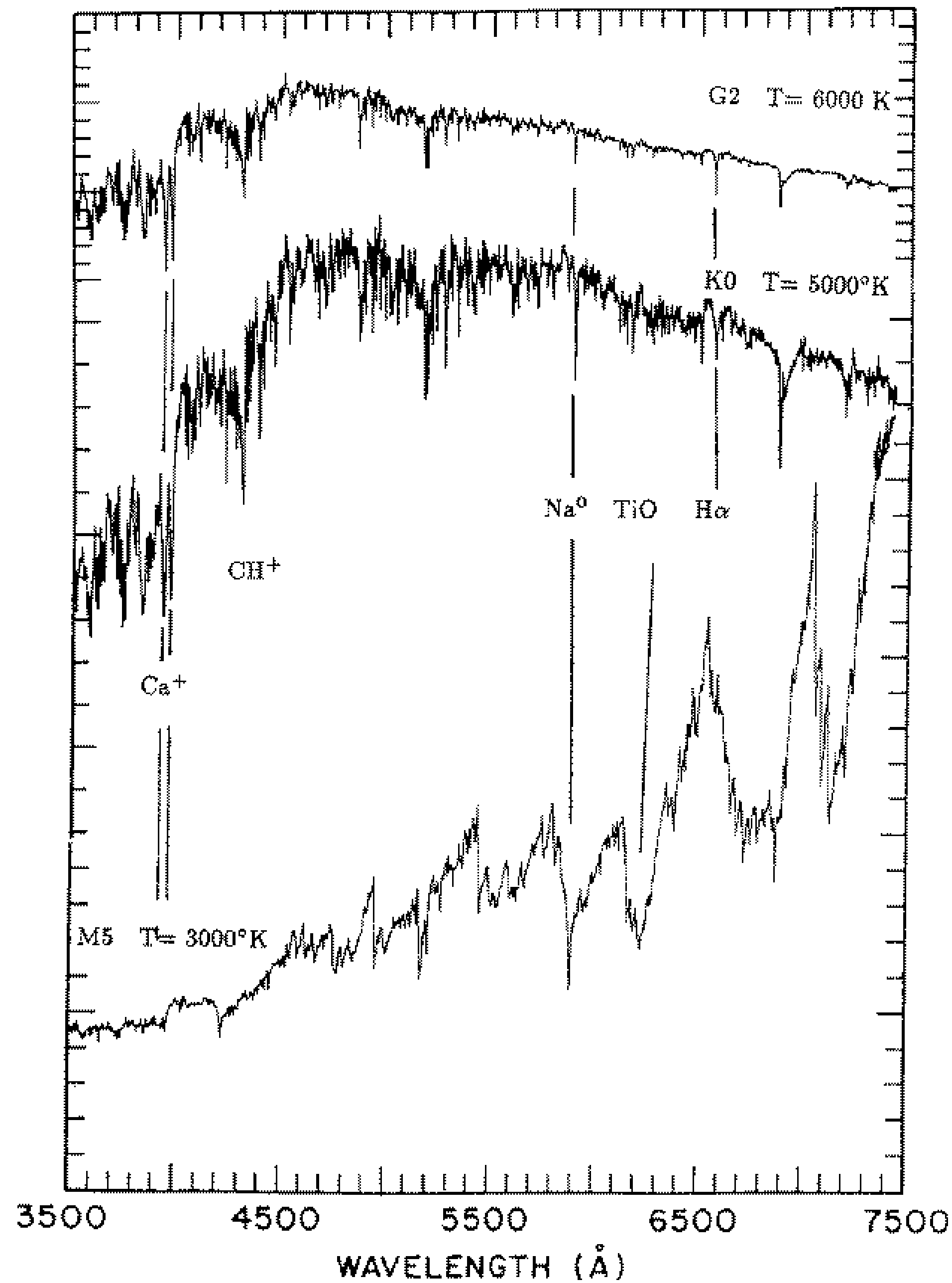
- http://casswww.ucsd.edu/physics/ph7/Stars.html

- Just in case

O to G example

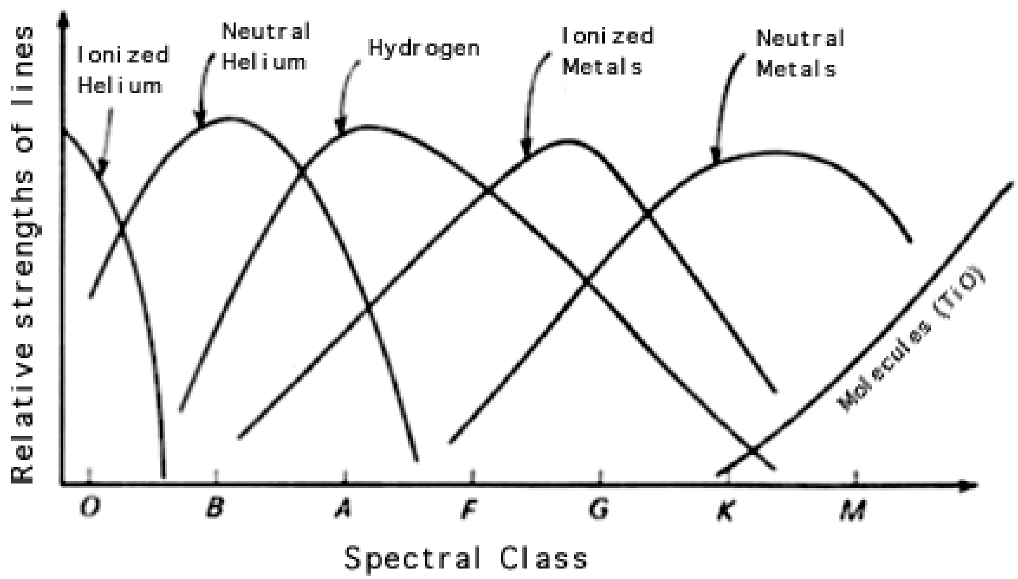


G to M example



The Formation of Spectral Lines

- Fundamental Question:
  - What causes the differences in the observed spectra?
    - Composition?
    - Temperature?
    - Both?
- Answer:
  - Temperature is the main factor



The Formation of Spectral Lines

- Distribution of electrons in different atomic orbitals depends on temperature
- Electrons can jump up in energy by absorption of a photon OR collision with a particle! So KE of surrounding particles important.

- What is the probability of finding an electron in a particular orbital?
  - Answer with Statistical Mechanics…
  - Maxwell-Boltzmann (velocity) Distribution
    - Assumes thermal equilibrium
    - Number of gas particles per unit volume have a speed between v and v+dv

$$n_v \, dv = n \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} e^{-\frac{1}{2} m v^2 kT} 4\pi \, v^2 \, dv$$

Maxwell-Bolzmann Distribution

$$n_v\,dv = n\left(\frac{m}{2\pi\,kT}\right)^{\frac{3}{2}} e^{-\frac{1}{2}mv^2\,kT}\,4\pi\,v^2\,dv$$

- Most probable speed

$$v_{mp} = \sqrt{\frac{2\,kT}{m}} = 1.4\sqrt{kT/m}$$

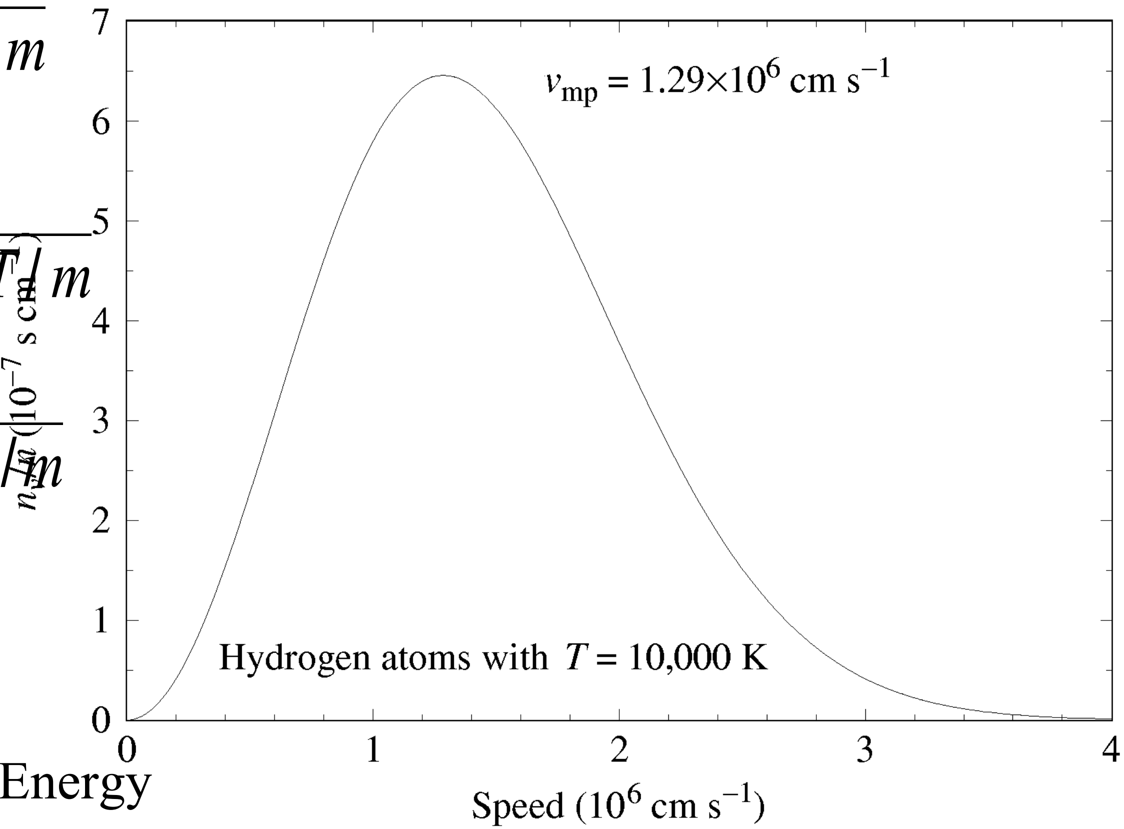
- Root-mean-square

$$v_{rms} = \sqrt{\frac{3\,kT}{m}} = 1.73\sqrt{kT/m}$$

- Average

$$v_{avg} = \sqrt{\frac{8\,kT}{\pi\,m}} = 1.6\sqrt{kT/m}$$

- Collisional energy causes a distribution of electrons among the atomic orbitals (Kinetic Energy → Potential Energy



Boltzmann Factor

- Fundamental idea used in all branches of physics
- The higher the energy of a state, the less likely it will be occupied

$$P_a \propto e^{\frac{-E_a}{kT}}$$

  - For the Maxwell-Boltzmann distribution, the energy is Kinetic Energy

$$P_v \propto e^{-\frac{1}{2}mv^2\, kT}$$

- The "kT" term is associated with the thermal energy of the "gas" as a whole
- Ratio of Probabilities for two different states (and energies)

$$\frac{P_b}{P_a} = \frac{e^{\frac{-E_b}{kT}}}{e^{\frac{-E_a}{kT}}} = e^{\frac{-\left(E_b - E_a\right)}{kT}}$$

Degeneracy Factor

- An energy (eigenvalue) is associated with each set of quantum number (eigenstate or eigenfunction)
- *Degenerate States* have different quantum numbers but the same energy

- Modify the Boltzmann factor

$$P_a \propto g_a e^{\frac{-E_a}{kT}}$$

  – The probability of being in any of the g_a degenerate states with energy E_a
    - g_a is the *degeneracy* or *statistical weight* of state *a*

- Ratio of probabilities between states with two different energies

$$\frac{P_b}{P_a} = \frac{g_b}{g_a} e^{\frac{-(E_b - E_a)}{kT}}$$

Degeneracy Factor

- Details of quantum mechanics determines the energies and quantum numbers…
- Visit the following site on the next page and browse…
- Quantum numbers for Hydrogen $\{n, l, m_l, m_s\}$
  - Table 8.2

State	n Principal quantum number n	l Orbital quantum number	m_l Magnetic quantum number	m_s Spin quantum number	Maximum number of electrons	
1s	1	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2	
2s	2	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2	8
2p	2	1	-1,0,+1	$+\frac{1}{2}, -\frac{1}{2}$	6	
3s	3	0	0	$+\frac{1}{2}, -\frac{1}{2}$	2	18
3p	3	1	-1,0,+1	$+\frac{1}{2}, -\frac{1}{2}$	6	
3d	3	2	-2,-1,0,1,2	$+\frac{1}{2}, -\frac{1}{2}$	10	$=2n^2$

Boltzmann Equation

- Number of atoms in a particular state a

$$N_a = NP_a$$

N = total number of atoms

N_a = number of atoms in state a

P_a = probability of being in state a

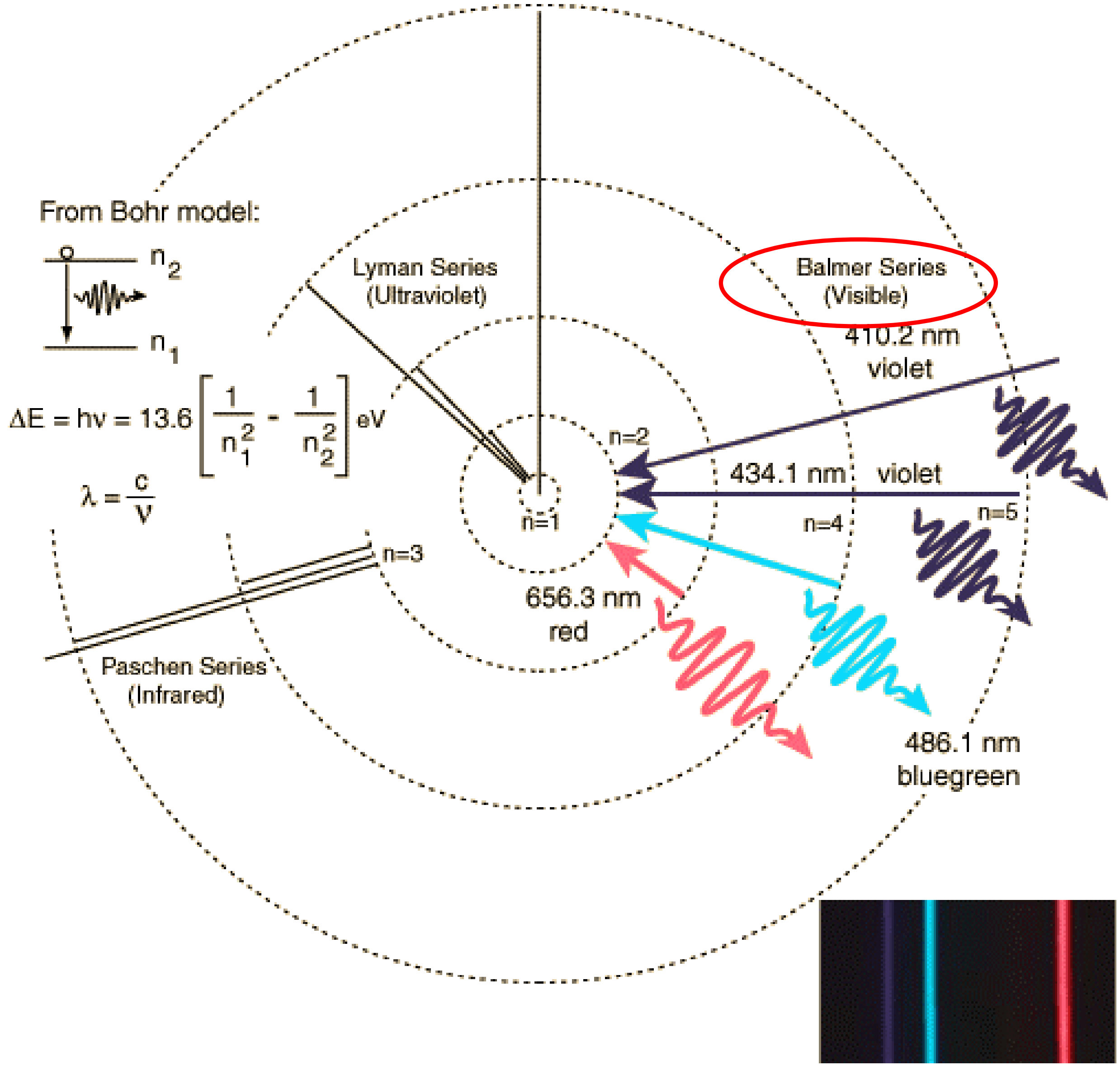
$$\Rightarrow \frac{N_b}{N_a} = \frac{g_b}{g_a} e^{\frac{-\left(E_b - E_a\right)}{kT}}$$

Hydrogen Atom Examples

Hydrogen Atom

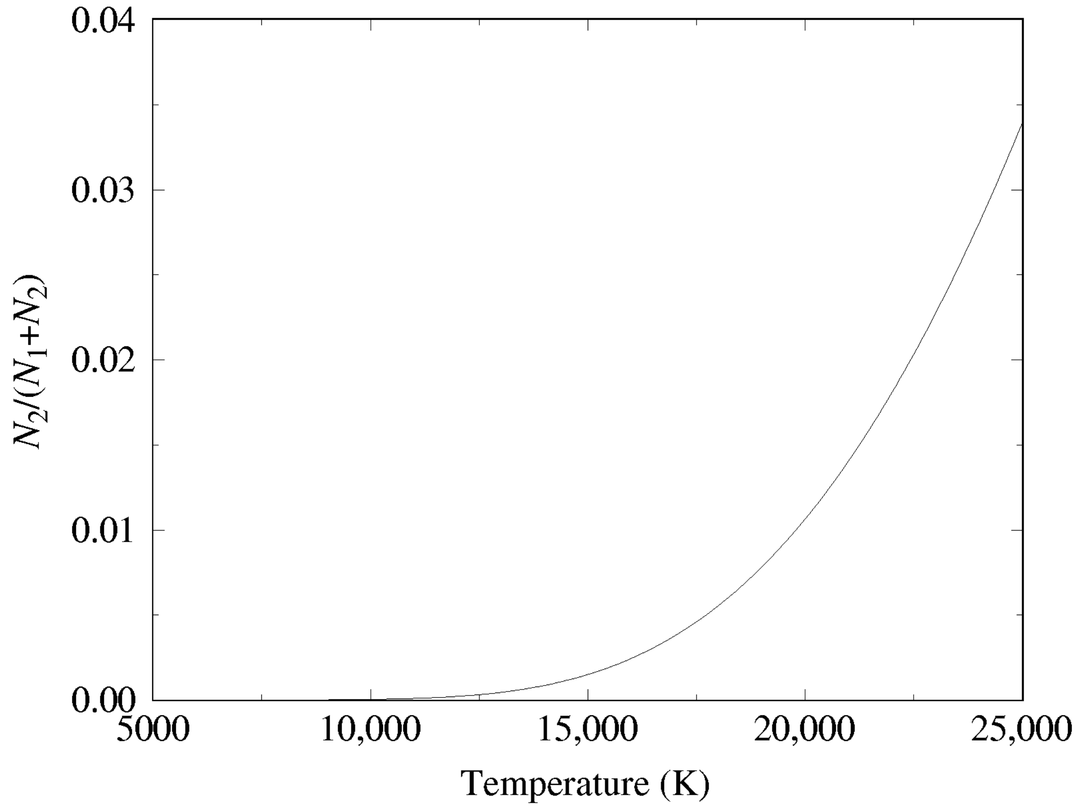
- Balmer series absorption spectra is an upward transition from $n = 2$

- Observation: this series has a peak absorption spectrum at ~9520 K.



Hydrogen Atom Populations

- We just saw that not many Hydrogen atoms are in the *n=1* state at 9520 K!
  - Shouldn't the intensity keep growing as the temperature increases since there is a higher probability for an H atom to be in the *n=2* state?!?!



Partition Function

- We also have to figure in all states that have a significant population
- For one state we have: $$P_1 \propto g_1 e^{\frac{-E_1}{kT}}$$
- Ratio between two states: $$\frac{P_2}{P_1} = \frac{g_2 e^{\frac{-E_2}{kT}}}{g_1 e^{\frac{-E_1}{kT}}} = \frac{g_2}{g_1} e^{\frac{-\left(E_2 - E_1\right)}{kT}}$$
- Ratio of state *2* to *all* other states with reference to the ground state:

$$\frac{P_2}{P_{all}} = \frac{g_b e^{\frac{-(E_2 - E_1)}{kT}}}{g_1 e^{\frac{-(E_1 - E_1)}{kT}} + g_2 e^{\frac{-(E_2 - E_1)}{kT}} + g_3 e^{\frac{-(E_3 - E_1)}{kT}} + \cdots} = \frac{g_2 e^{\frac{-(E_2 - E_1)}{kT}}}{Z}$$

Partition Function

- This tell us how many states are accessible or available at a given temperature (thermal energy)

$$Z = g_1 e^{\frac{-(E_1 - E_1)}{kT}} + g_2 e^{\frac{-(E_2 - E_1)}{kT}} + g_3 e^{\frac{-(E_3 - E_1)}{kT}} + \cdots$$

$$= g_1 + \sum_i g_i e^{\frac{-(E_i - E_1)}{kT}}$$

- The higher the temperature, the more states that are available
- At zero K, everything will be in the ground state
  - Bose-Einstein Condensates

Partition Function and Atoms

- We also have to handle ionization!
- Nomenclature: H I – neutral hydrogen

  H II – singly ionized hydrogen

  He I – neutral Helium

  He II – singly ionized Helium

  He III – doubly ionized Helium

- Ionization Energy for H I to H II

$$\chi_I = 13.6\,eV$$

  - Rather than $n \rightarrow \infty$, the atom will ionize before this happens

Saha Equation

- Determines the ratio of ionized atoms
- Need partition functions since all the atoms are not in the same state
  - Z_i is the initial stage of ionization
  - Z_{i+1} is the final stage of ionization
- Ratio of the number of atoms in each of these stages

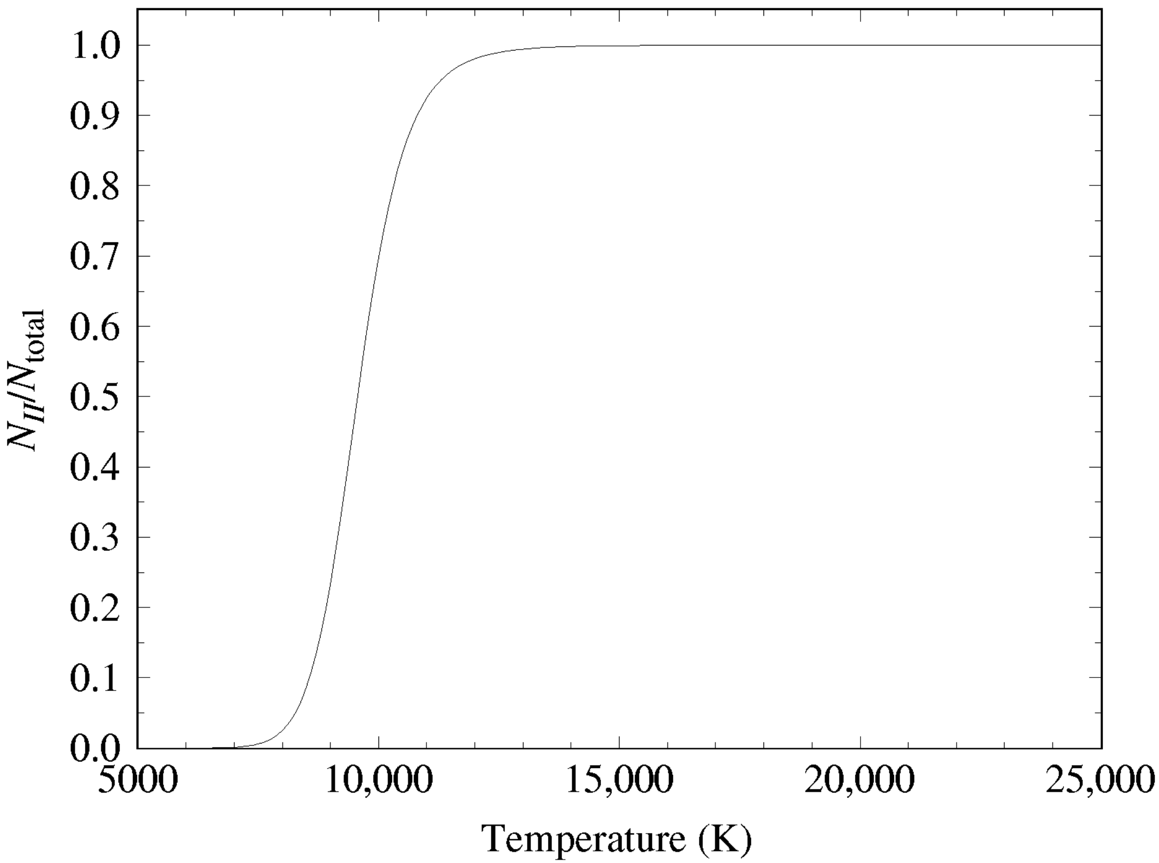
$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{\frac{3}{2}} e^{-\chi_i kT}$$

$$P_e = n_e kT$$

  - n_e is the electron density (an ideal gas of electrons)
    - Electron pressure
    - Electrons recombine with H II to give H I

Ionized Hydrogen Atoms

- Fraction of hydrogen atoms that are ionized
- If we have H II, we can't have the Balmer series!



H I $n = 2$ population

$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_I}\right)\left(\frac{N_I}{N_I + N_{II}}\right)$$

Fraction of non-ionized hydrogen Atoms in the $n = 2$ state

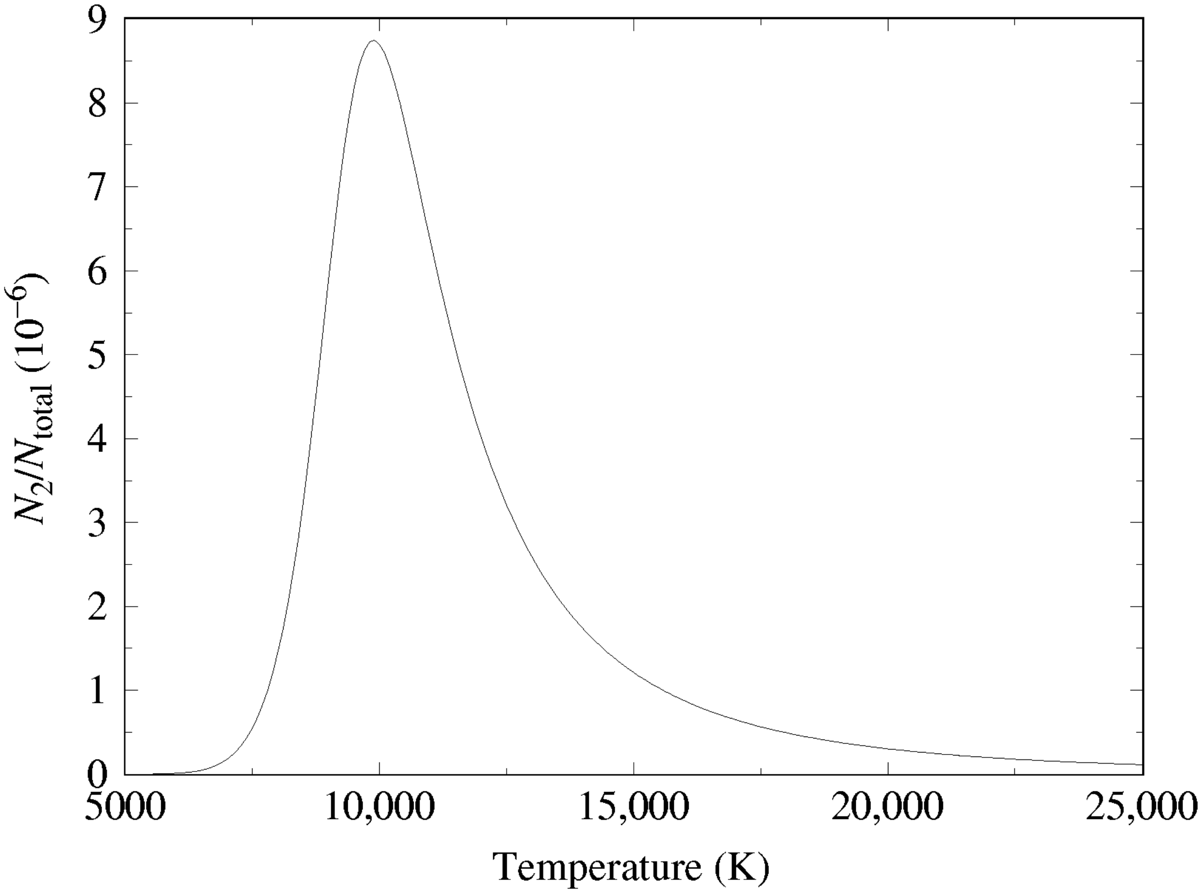
Fraction of non-ionized hydrogen atoms

$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_I}\right)\left(\frac{1}{1 + N_{II}/N_I}\right) \xrightarrow[N_I \approx N_1 + N_2]{} \left(\frac{N_2}{N_1 + N_2}\right)\left(\frac{1}{1 + N_{II}/N_I}\right)$$

$$\frac{N_2}{N_{total}} = \left(\frac{N_2/N_1}{1 + N_2/N_1}\right)\left(\frac{1}{1 + N_{II}/N_I}\right)$$

H I $n = 2$ population

- Includes the Boltzmann factor, partition function and ionization
- Population peak at 9520 K, in agreement with observation of the Balmer series



H I $n = 2$ population

$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_I}\right)\left(\frac{N_I}{N_I + N_{II}}\right)$$

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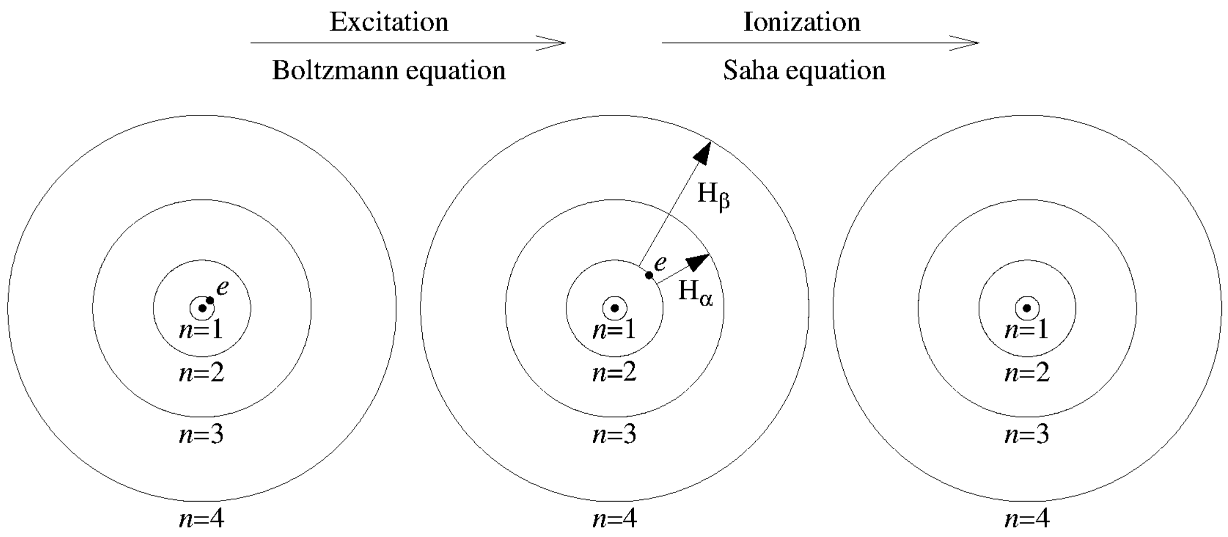
Fraction of non-ionized hydrogen atoms



Example 8.3

- Degree of ionization in a stellar atmosphere of pure hydrogen for the temperature range of 5000-25000 K $\frac{N_{II}}{N_{Total}}$
- Given electron pressure $P_e = 200 \frac{dyne}{cm^2}$
- Saha equation $$\frac{N_{II}}{N_I} = \frac{2kTZ_{II}}{P_e Z_I}\left(\frac{2\pi\, m_e kT}{h^2}\right)^{\frac{3}{2}} e^{-\chi_i kT}$$
- Must determine the partition functions
  - Hydrogen ion is a proton, so $Z_{II} = 1$
  - Neutral hydrogen over this temp range

$\Delta E = E_2 - E_1 = 10.2$ eV

$\Delta E >> kT$, then $e^{-\Delta E / kT} << 1$

$$\Rightarrow Z_I = g_1 + \sum_i g_i e^{\frac{-(E_i - E_1)}{kT}} \quad g_1 = 2$$

$T := 5000K$

$k \cdot T = 0.43\,eV$

$T := 25000K$

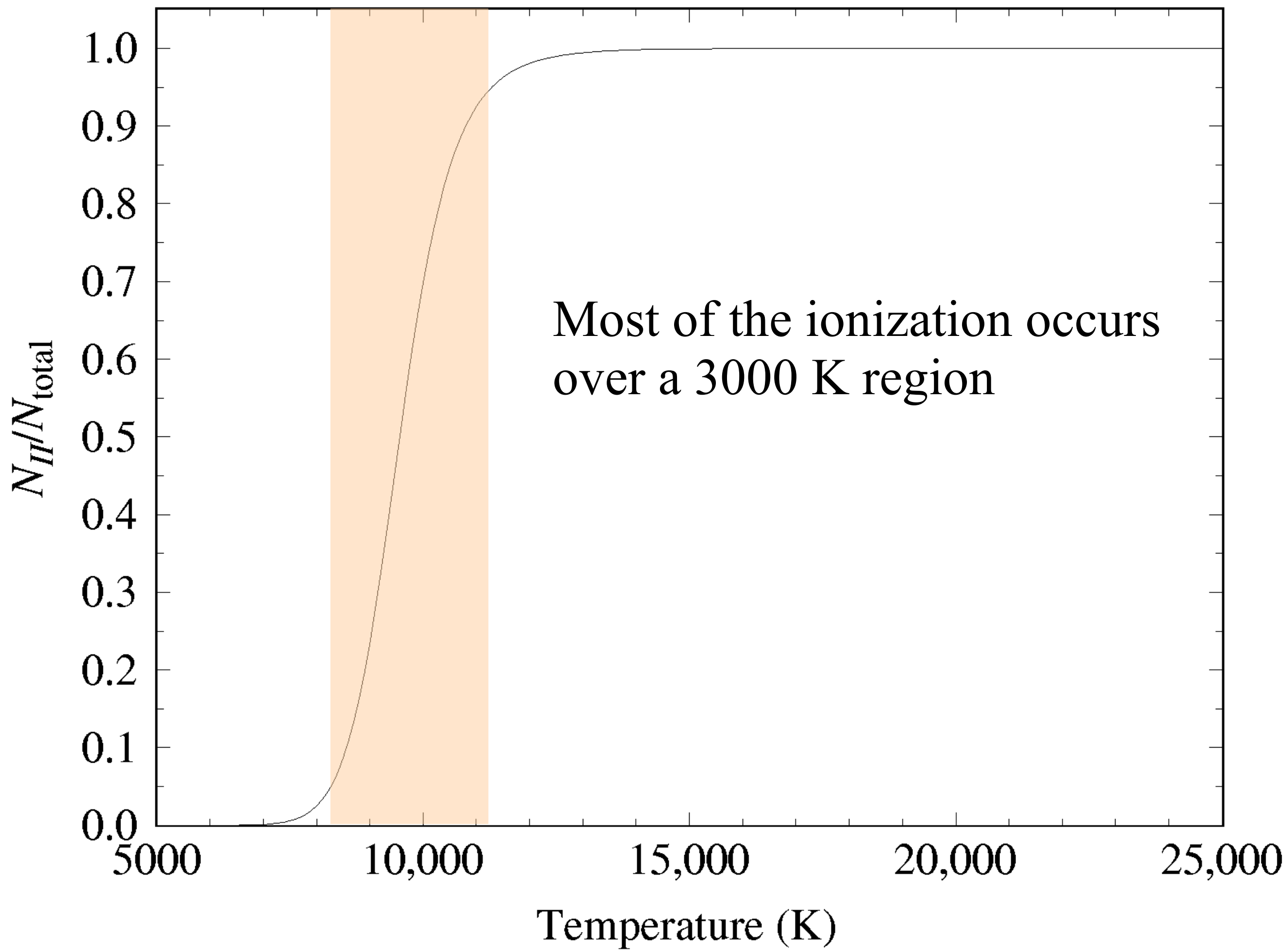
$k \cdot T = 2.15\,eV$

Example 8.3

- Degree of Ionization

$$\frac{N_{II}}{N_I} = \frac{2kT(1)}{P_e(2)}\left(\frac{2\pi\, m_e kT}{h^2}\right)^{\frac{3}{2}} e^{-\chi_i kT}$$

$$\frac{N_{II}}{N_I + N_{II}} = \frac{N_{II}/N_I}{1 + N_{II}/N_I}$$



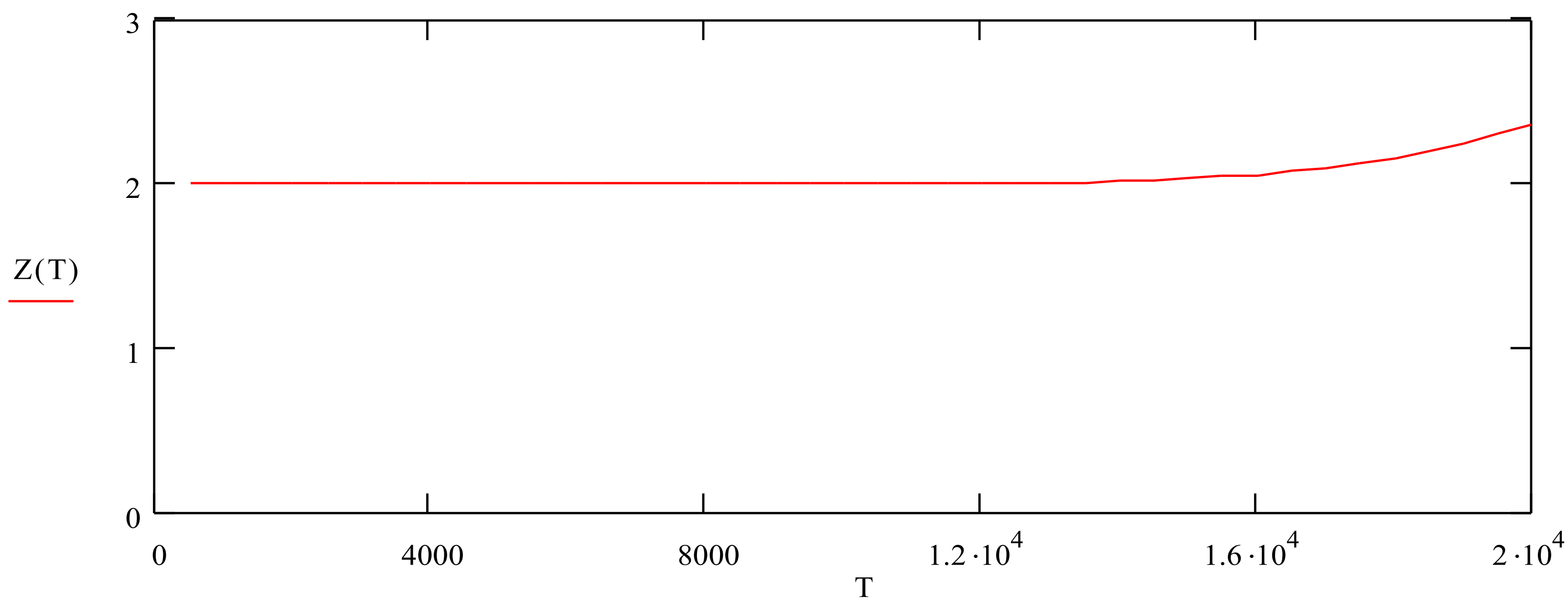
Problem 8.7

- Evaluate the first three terms of the partition function for 10000K

Partition Function: Counting the first ten states... Energy: $E(n) := \frac{-13.6\text{eV}}{n^2}$ Degeneracy: $g(n) := 2 \cdot n^2$

$$f_B(n, T) := \exp\left[\frac{-(E(n) - E(1))}{k \cdot T}\right] \qquad Z(T) := \sum_{n=1}^{10} \left(g(n) \cdot f_B(n, T)\right) \qquad T := 0, 500 .. 20000$$

$Z(6000K) = 2.0000$ $Z(10000K) = 2.0002$ $Z(15000K) = 2.0292$



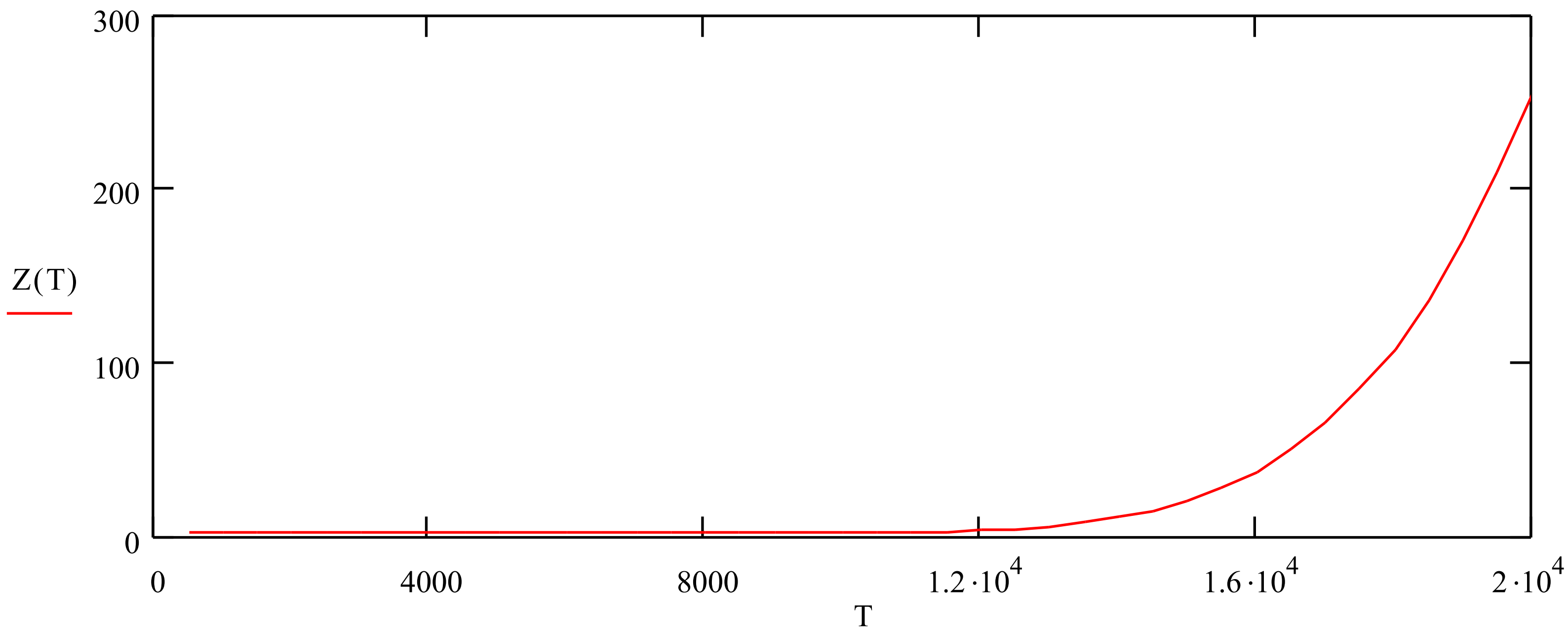
Problem 8.8

- The partition function diverges at $n \rightarrow \infty$
  - Why do we ignore large n?

Partition Function: Counting the first 100 states... Energy: $E(n) := \frac{-13.6\text{eV}}{n^2}$ Degeneracy: $g(n) := 2 \cdot n^2$

$$f_B(n,T) := \exp\left[\frac{-(E(n) - E(1))}{k \cdot T}\right] \qquad Z(T) := \sum_{n=1}^{100} \left(g(n) \cdot f_B(n,T)\right) \qquad T := 0, 500..20000$$

$Z(6000\text{K}) = 2.0000$ $Z(10000\text{K}) = 2.0952$ $Z(15000\text{K}) = 20.2988$

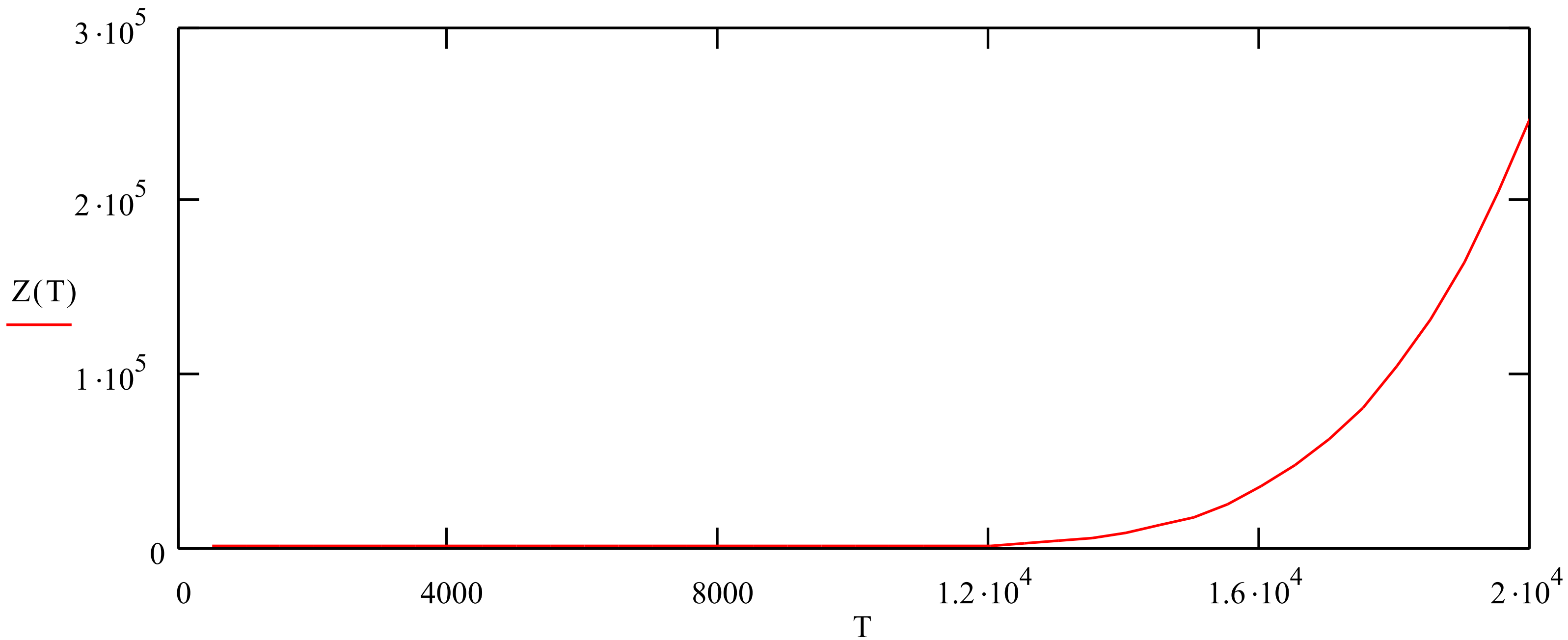


Problem 8.8

Partition Function: Counting the first 1000 states... Energy: $E(n) := \frac{-13.6\text{eV}}{n^2}$ Degeneracy: $g(n) := 2 \cdot n^2$

$$f_B(n,T) := \exp\left[\frac{-(E(n) - E(1))}{k \cdot T}\right] \qquad Z(T) := \sum_{n=1}^{1000} \left(g(n) \cdot f_B(n,T)\right) \qquad T := 0, 500 .. 20000$$

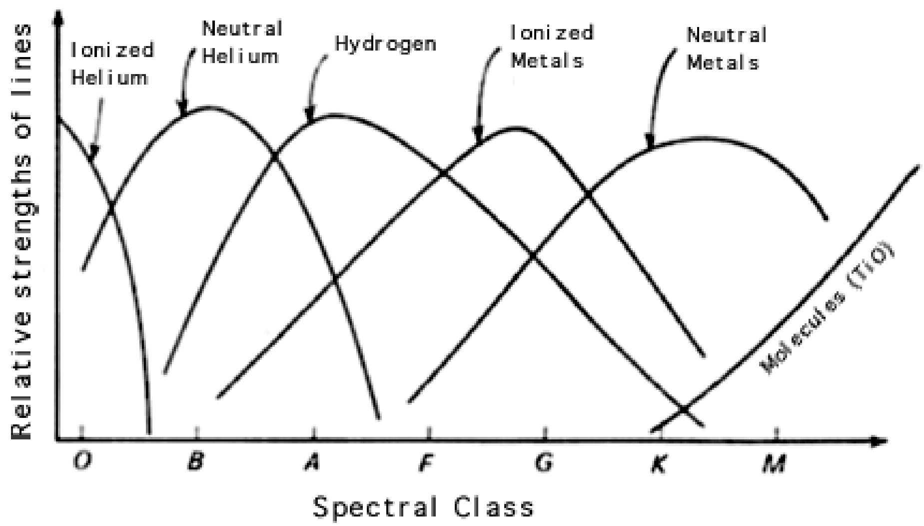
$Z(6000K) = 2.0025$ $\quad Z(10000K) = 95.4311$ $\quad Z(15000K) = 1.7998 \times 10^4$



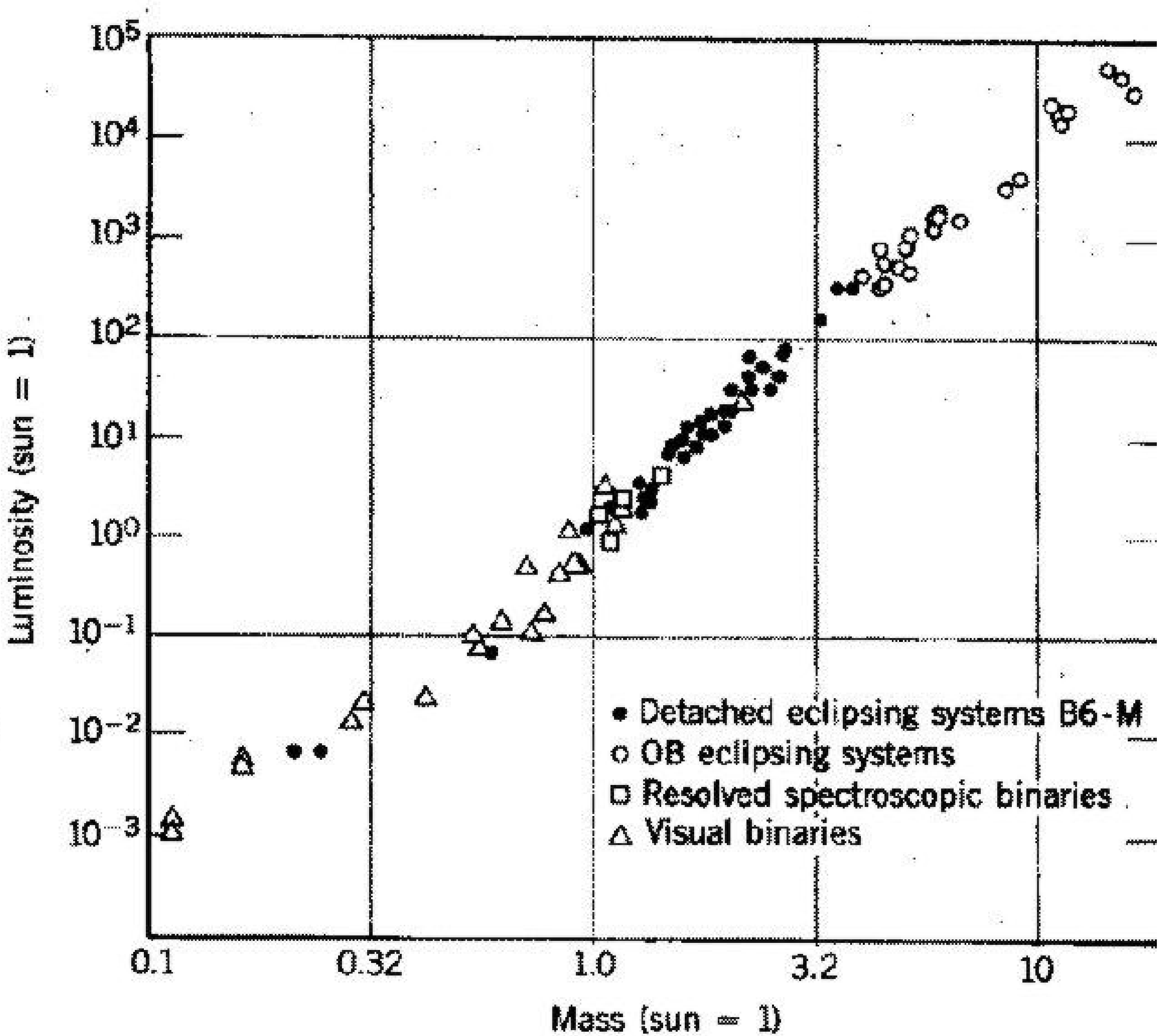
- Ionization
- Unphysical orbital size $\quad r_n = a_o n^2$

Example 8.4

- Surface of the Sun has 500,000 hydrogen atoms per calcium atom, but calcium absorption lines are much stronger than the Balmer series lines.
- The Boltzmann and Saha equations reveal that there are 400× more Ca atoms in the ground electronic state than in the *n=2* hydrogen state.
- Calcium is not more abundant
- Differences are due to sensitive temperature dependence



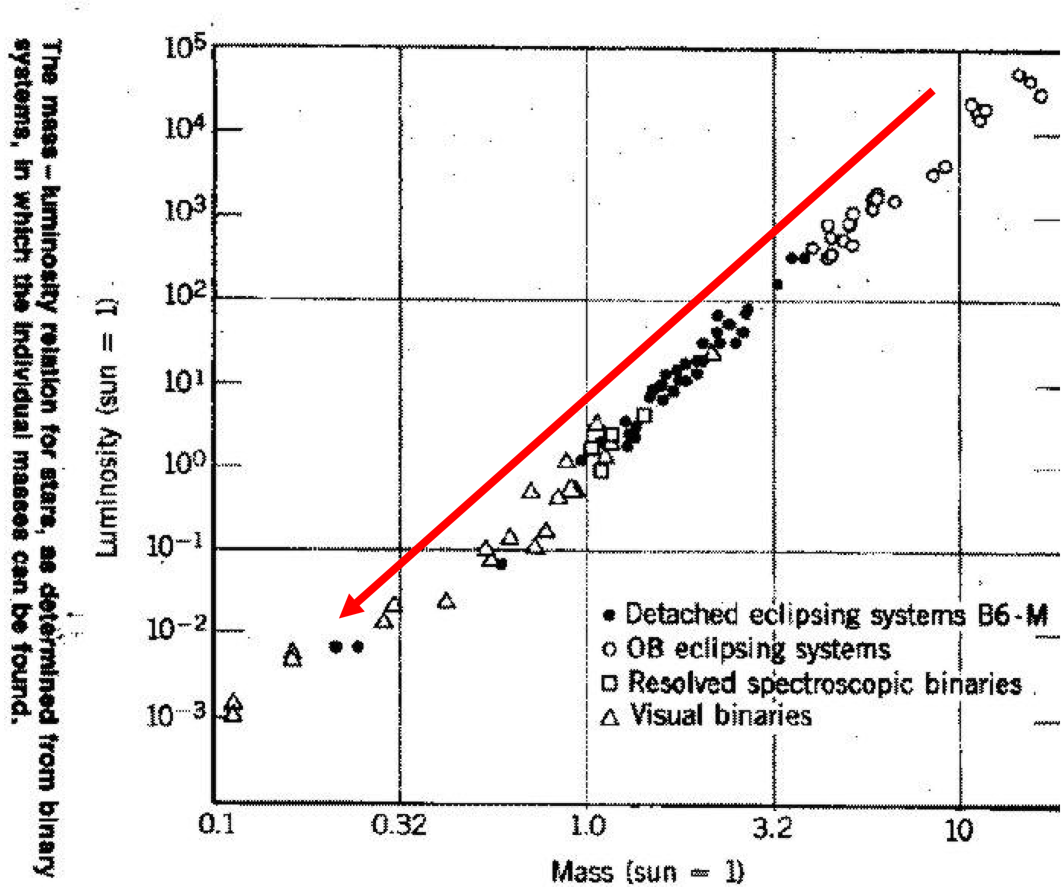
Mass-Luminosity Relation from Binary Systems



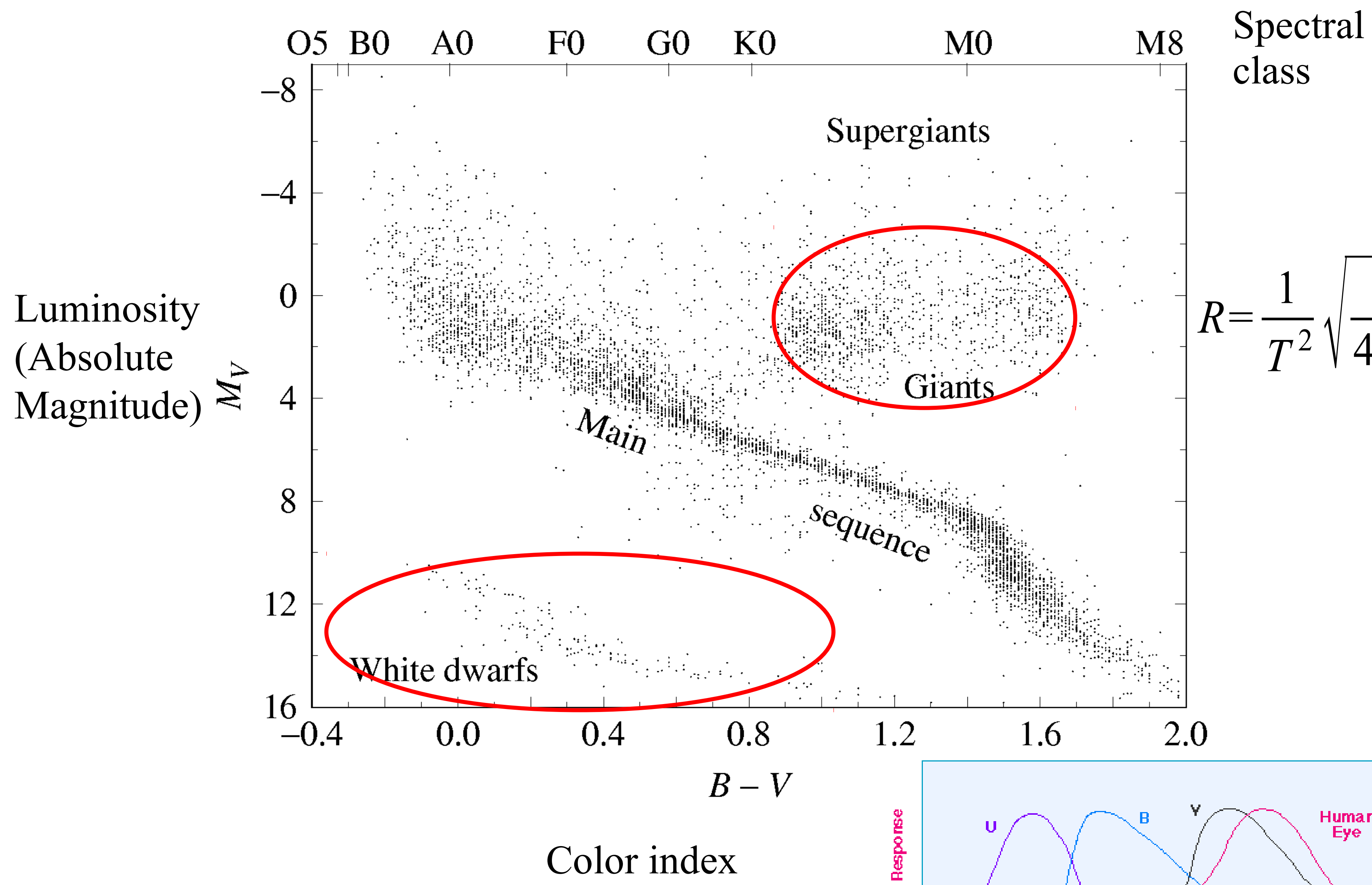
The mass – luminosity relation for stars, as determined from binary systems, in which the individual masses can be found.

Mass-Luminosity Relation

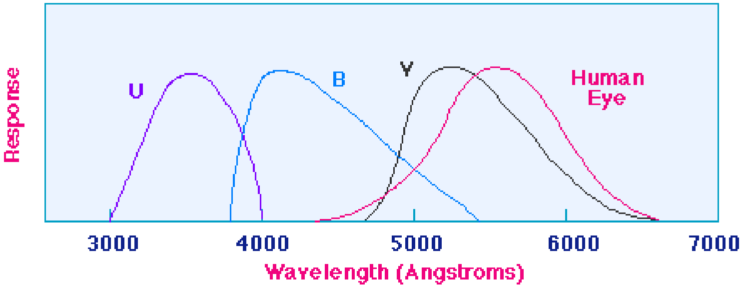
- Early theories had "early" O-type (bright, hot, massive) stars evolving to "old" M-type stars (dim, cool, less massive)



Hertzsprung-Russell (H-R) Diagram



$$R = \frac{1}{T^2}\sqrt{\frac{L}{4\pi\sigma}}$$



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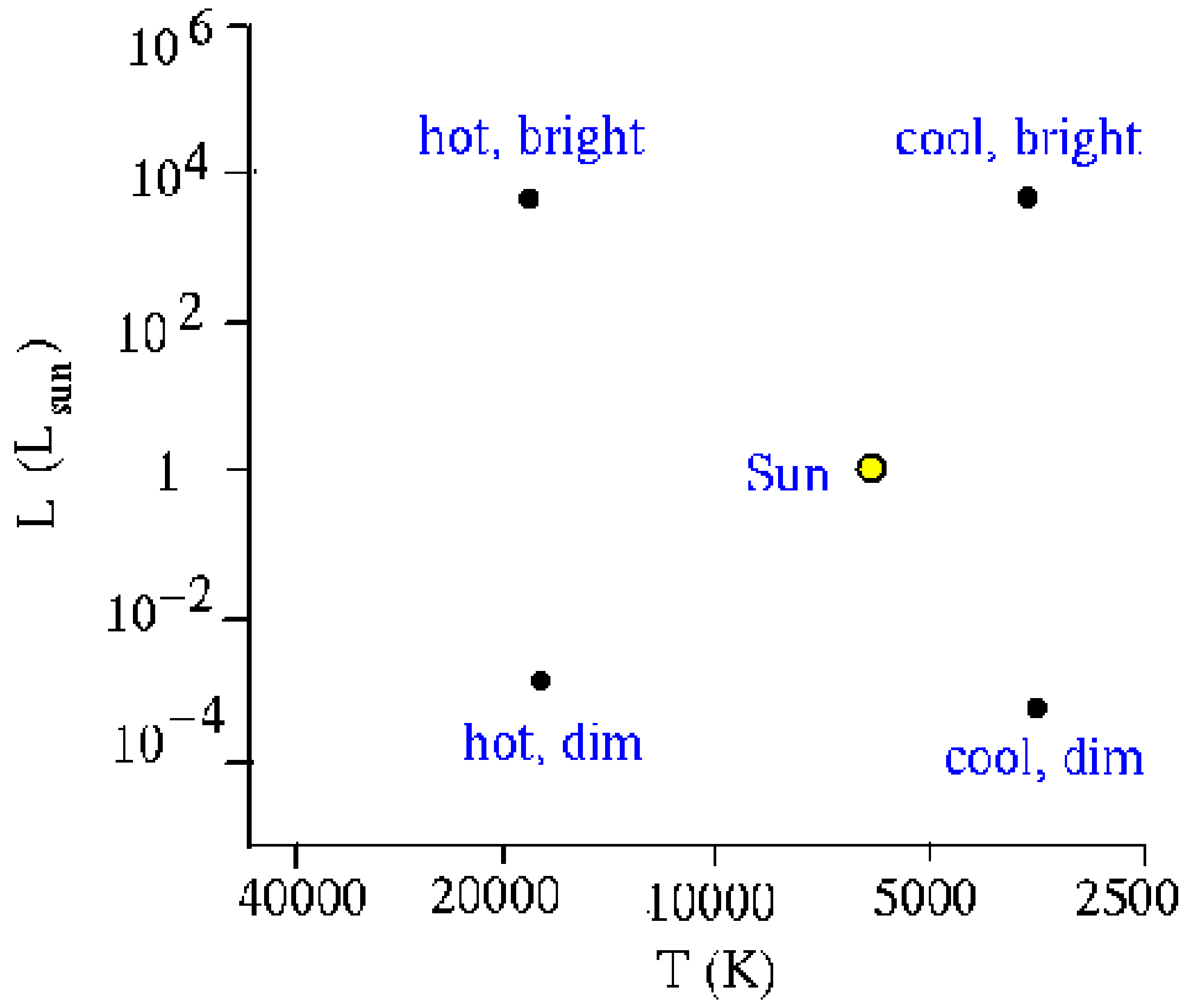
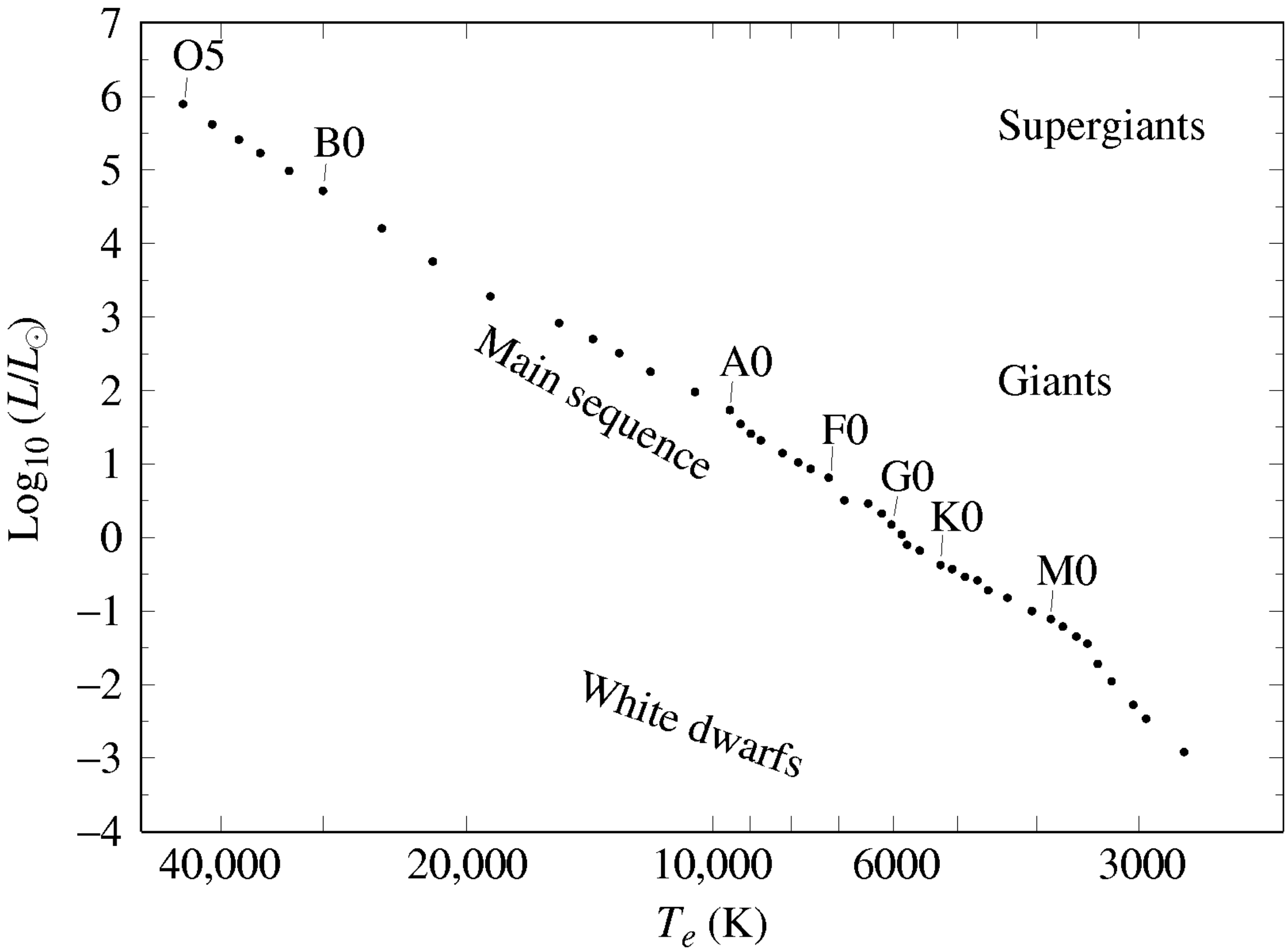


Hertzsprung-Russell (H-R) Diagram

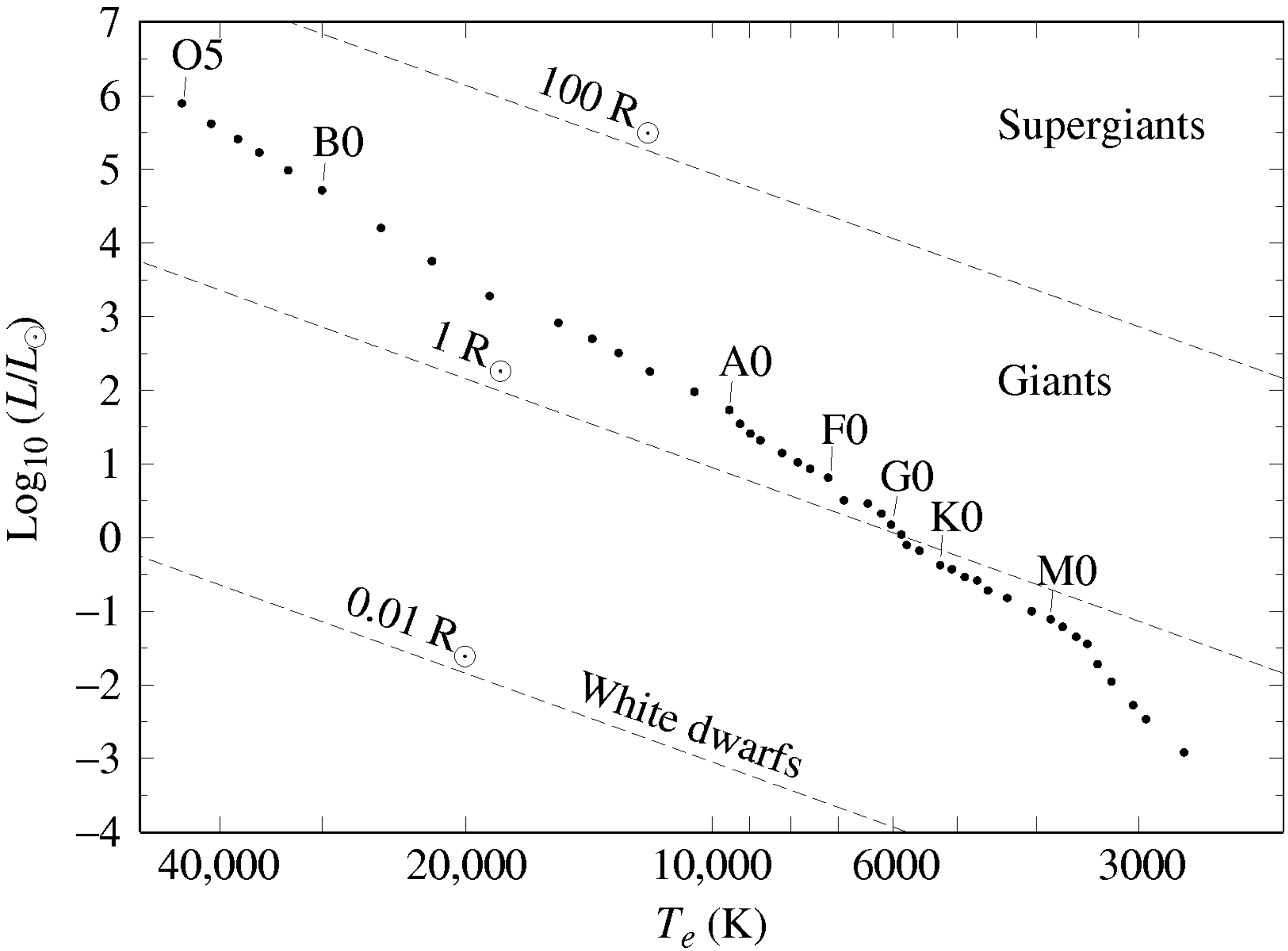
- Luminosity and Temperature rather than Magnitude and Color Index



Hertzsprung-Russell (H-R) Diagram

- Star Radius

$$R = \frac{1}{T^2}\sqrt{\frac{L}{4\pi\sigma}}$$



Spectroscopic “Parallax”

- Method to determine a stars distance
  - Determine the star’s spectral class
  - Read the absolute magnitude from the H-R diagram
  - Compare to apparent magnitude to determine distance
  - Accurate to a factor of ± 1 magnitude
    - $10^{1/5} = 1.6$

Hertzsprung-Russell (H-R) Diagram

- Stellar Evolution
  - Determined by mass
- http://instruct1.cit.cornell.edu/courses/astro101/java/evolve/evolve.htm
- http://cspar181.uah.edu/PHY106/QZ21-movie.html

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- http://cspar181.uah.edu/PHY106/QZ21-movie.html

- http://www.shep.net/resources/curricular/physics/Conf99/parallax.htm
- http://www.skyviewcafe.com/skyview.shtml

- http://jersey.uoregon.edu/vlab/
- http://jersey.uoregon.edu/vlab/elements/Elements.html
- http://jersey.uoregon.edu/vlab/EW2/EW.html
- http://jersey.uoregon.edu/vlab/prf/PRF_plugin.html