Chapter 30

Sources of the Magnetic Field

PHYS 2321
Week 9: Sources of Magnetic field

Day 1 Outline

1) Hwk: Ch. 30 P. 3,5,13,15,25,31,38,43,46,47 Due Fri

2) Magnetic Fields and Forces (Ch 29)
   a. Force on current-carrying wire $F_B = I\vec{L} \times \vec{B}$
   b. Square loop of current
   c. Loops feel torques $\tau = I\vec{A} \times \vec{B}$

3) Sources of magnetic fields (Ch 30)

Notes: Quiz Today (Ch. 29) at 11:36
Will post homework scores on Moodle soon.

PHYS 2321
Week 9: Sources of Magnetic field

Day 2 Outline

1) Hwk: Ch. 30 P. 3,5,13,15,25,31,38,43,46,47   Due Fri

2) Review of Quiz 5

3) Sources of magnetic fields (Ch 30)

   a. Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2}$$

      * B-field due to arc of current

      * B-field due to straight current segment

   b. Ampere's law

Notes: Quiz5 and Hwk 5 scores on Moodle now.

   Exam II will be on Monday 10/19.

PHYS 2321
Week 9: Sources of Magnetic field

Day 3 Outline

1) Hwk: Ch. 30 P. 3,5,13,15,25,31,38,43,46,47   Due tonight

2) Sources of magnetic fields (Ch 30)
   a. Force between parallel currents
   b. Ampere's law $\oint \vec{B}\cdot d\vec{r} = \mu_0 I_{enc}$
      * B inside and out of long straight wire
      * B inside toroid and solenoid

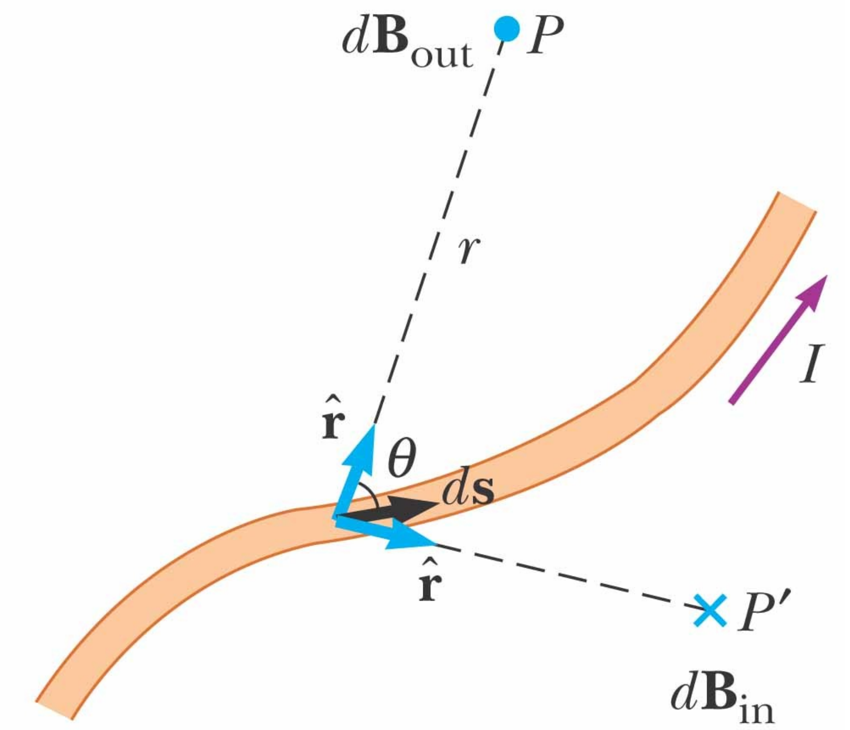
Notes: Quiz5 and Hwk 5 scores on Moodle now.
   Exam II will be on Monday 10/19.

Biot-Savart Law – Introduction

- Biot and Savart conducted experiments on the force exerted by an electric current on a nearby magnet
- They arrived at a mathematical expression that gives the magnetic field at some point in space due to a current

Biot-Savart Law – Set-Up

- The magnetic field is $d\vec{B}$ at some point P
- The length element is $d\vec{s}$
- The wire is carrying a steady current of I



Biot-Savart Law – Observations

- The vector $d\vec{B}$ is perpendicular to both $d\vec{s}$ and to the unit vector $\hat{r}$ directed from $d\vec{s}$ toward P
- The magnitude of $d\vec{B}$ is inversely proportional to r^2, where r is the distance from $d\vec{s}$ to P

Biot-Savart Law – Observations, cont

- The magnitude of $d\vec{B}$ is proportional to the current and to the magnitude *ds* of the length element $d\vec{s}$
- The magnitude of $d\vec{B}$ is proportional to sin θ, where θ is the angle between the vectors $d\vec{s}$ and $\hat{r}$

Biot-Savart Law – Equation

- The observations are summarized in the mathematical equation called the **Biot-Savart law**:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

- The magnetic field described by the law is the field *due to* the current-carrying conductor
  - Don’t confuse this field with a field *external* to the conductor

Permeability of Free Space

- The constant μ_o is called the **permeability of free space**
- $\mu_o = 4\pi \times 10^{-7}$ T· m / A

Total Magnetic Field

- $d\vec{B}$ is the field created by the current in the length segment ds
- To find the total field, sum up the contributions from all the current elements I$d\vec{s}$

$$\vec{B} = \int d\vec{B} = \int_{current} \frac{\mu_0}{4\pi} \frac{I\,d\vec{s} \times \hat{r}}{r^2}$$

  - The integral is over the entire current distribution

$\vec{B}$ Compared to $\vec{E}$

- Distance
  - The magnitude of the magnetic field varies as the inverse square of the distance from the source
  - The electric field due to a point charge also varies as the inverse square of the distance from the charge

$\vec{B}$ Compared to $\vec{E}$, 2

- Direction
  - The electric field created by a point charge is radial in direction
  - The magnetic field created by a current element is perpendicular to both the length element $d\vec{s}$ and the unit vector $\hat{r}$

$\vec{B}$ Compared to $\vec{E}$, 3

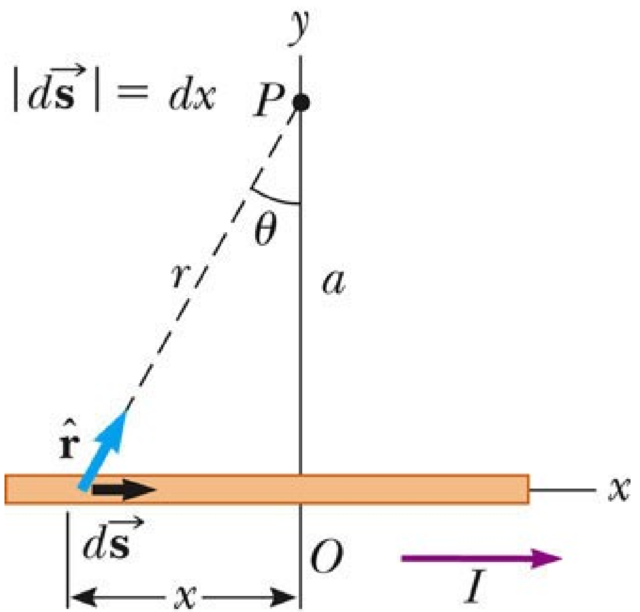
- Source
  - An electric field is established by an isolated electric charge
  - The current element that produces a magnetic field must be part of an extended current distribution
    - Therefore you must integrate over the entire current distribution

$\vec{B}$ for a Long, Straight Conductor

- The thin, straight wire is carrying a constant current, I
- $d\vec{s} \times \hat{r} = (dx \sin\theta)\hat{k}$
- Integrating over all the current elements gives

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta$$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$



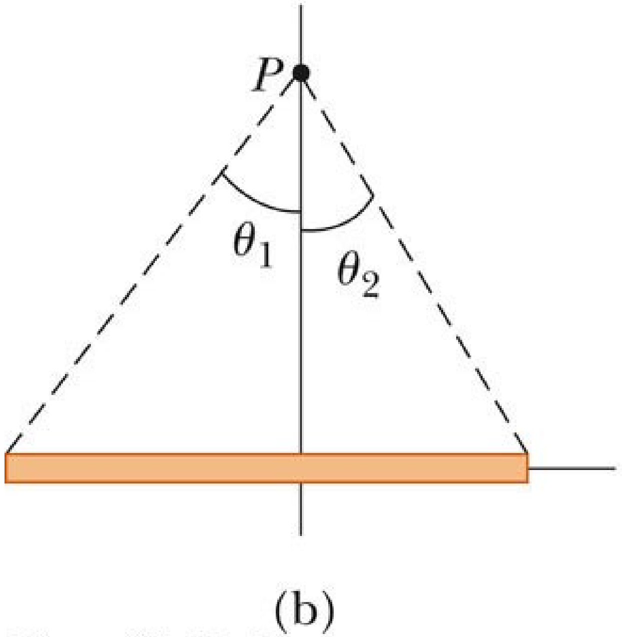
(a)

$\vec{B}$ for a Long, Straight Conductor, Special Case

- If the conductor is an infinitely long, straight wire, $\theta_1 = \pi/2$ and

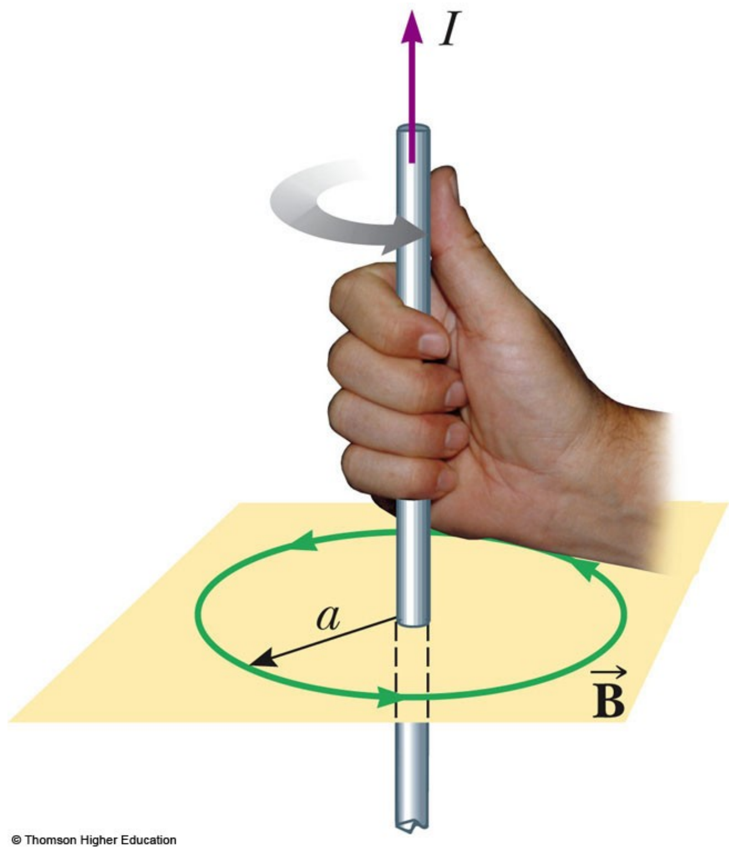
  $\theta_2 = -\pi/2$
- The field becomes

$$B = \frac{\mu_0 I}{2\pi r}$$



B for a Long, Straight Conductor, Direction

- The magnetic field lines are circles concentric with the wire
- The field lines lie in planes perpendicular to to wire
- The right-hand rule for determining the direction of the field is shown

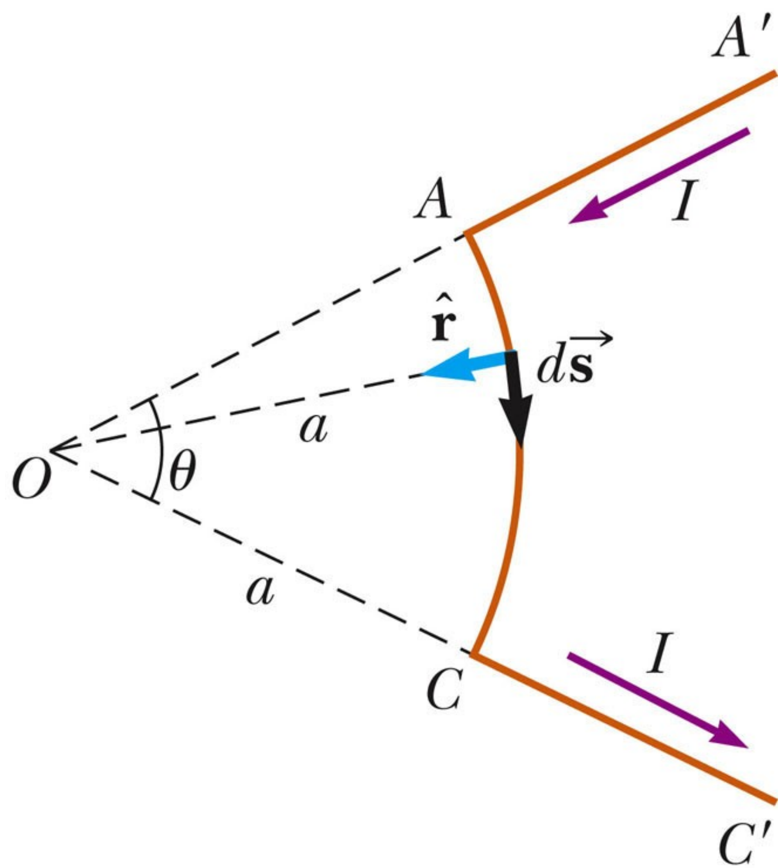


B for a Curved Wire Segment

- Find the field at point *O* due to the wire segment
- I and *R* are constants

  $B = \mu_0 I\theta / 4\pi a$

  θ will be in radians



B for a Circular Loop of Wire

- Consider the previous result, with a full circle
  - $\theta = 2\pi$

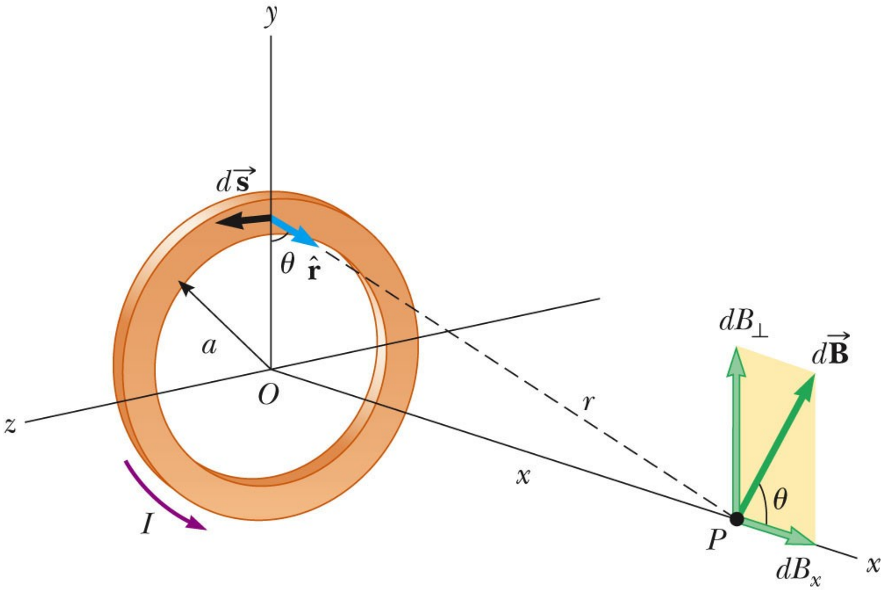
$$B = \frac{\mu_0 I \theta}{4 \pi a} = \frac{\mu_0 I \, 2\pi}{4 \pi a} = \frac{\mu_0 I}{2a}$$

- This is the field at the *center* of the loop

B for a Circular Current Loop

- The loop has a radius of R and carries a steady current of I
- Find the field at point P

$$B_x = \frac{\mu_0 I a^2}{2\left(a^2 + x^2\right)^{3/2}}$$



Comparison of Loops

- Consider the field at the center of the current loop
- At this special point, $x = 0$
- Then,

$$B_x = \frac{\mu_0 I a^2}{2\left(a^2 + x^2\right)^{3/2}} = \frac{\mu_0 I}{2a}$$

  - This is exactly the same result as from the curved wire

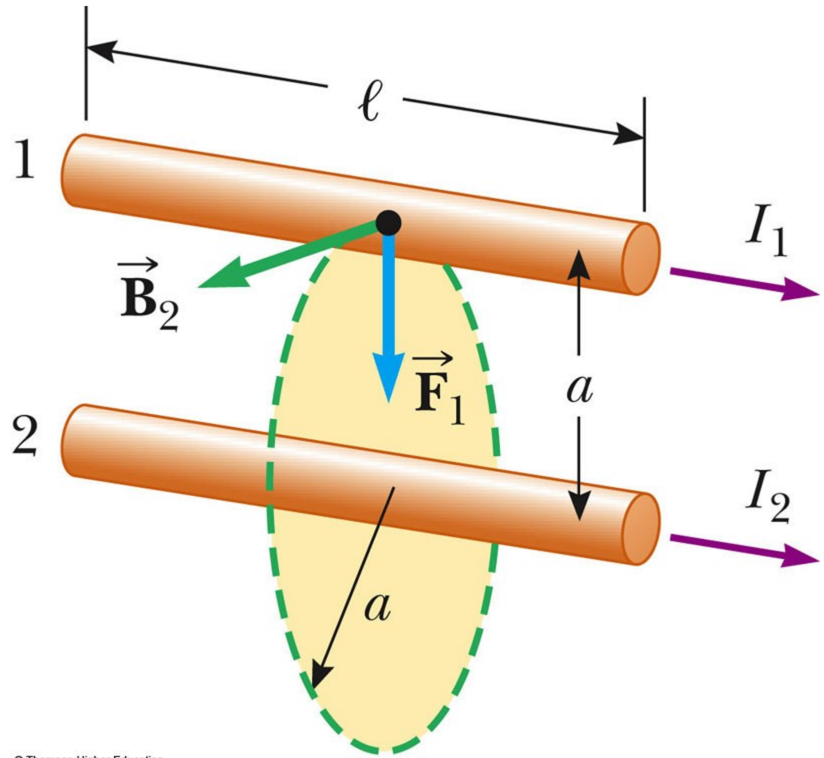
Magnetic Field Lines for a Loop

(a) (b) (c)

- Figure (a) shows the magnetic field lines surrounding a current loop
- Figure (b) shows the field lines in the iron filings
- Figure (c) compares the field lines to that of a bar magnet

Magnetic Force Between Two Parallel Conductors

- Two parallel wires each carry a steady current
- The field $\vec{B}_2$ due to the current in wire 2 exerts a force on wire 1 of $F_1 = I_1 \ell B_2$



PLAY ACTIVE FIGURE

Magnetic Force Between Two Parallel Conductors, cont.

- Substituting $\vec{B}_2 = \frac{\mu_0 I_2}{2\pi a}$ gives

$$F_1 = \frac{\mu_0 I_1 I_2 l}{2\pi a}$$

  - RHR #1 shows us F_1 is down and F_2 is up.
  - Parallel conductors carrying currents in the same direction attract each other
  - Parallel conductors carrying current in opposite directions repel each other

Magnetic Force Between Two Parallel Conductors, final

- The result is often expressed as the magnetic force between the two wires, F_B
- This can also be given as the force per unit length:

$$\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

Definition of the Ampere

- The force between two parallel wires can be used to define the ampere
- When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A

Definition of the Coulomb

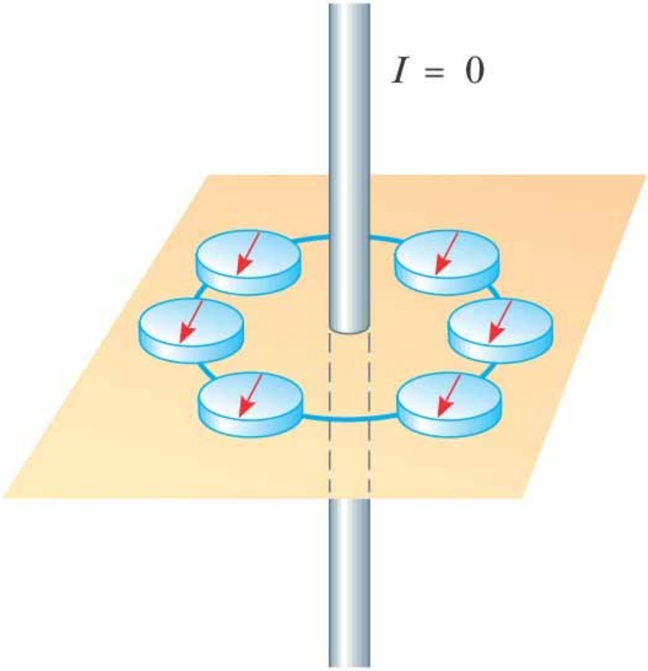
- The SI unit of charge, the coulomb, is defined in terms of the ampere
- When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C

Andre-Marie Ampère

- 1775 – 1836
- French physicist
- Credited with the discovery of electromagnetism
  - The relationship between electric current and magnetic fields
- Also worked in mathematics

Magnetic Field of a Wire

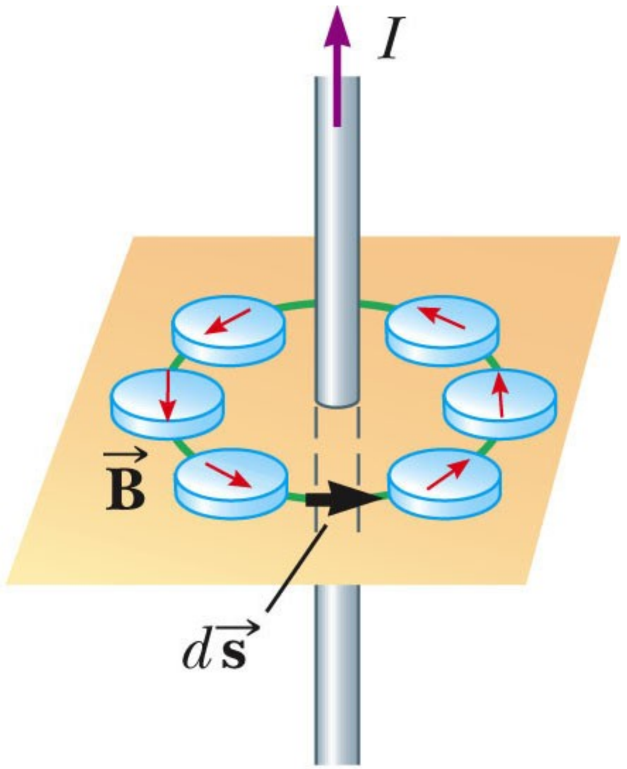
- A compass can be used to detect the magnetic field
- When there is no current in the wire, there is no field due to the current
- The compass needles all point toward the Earth's north pole
  - Due to the Earth's magnetic field



(a)

Magnetic Field of a Wire, 2

- Here the wire carries a strong current
- The compass needles deflect in a direction tangent to the circle
- This shows the direction of the magnetic field produced by the wire
- Use the active figure to vary the current



(b)

PLAY
ACTIVE FIGURE

Magnetic Field of a Wire, 3

- The circular magnetic field around the wire is shown by the iron filings

Ampere's Law

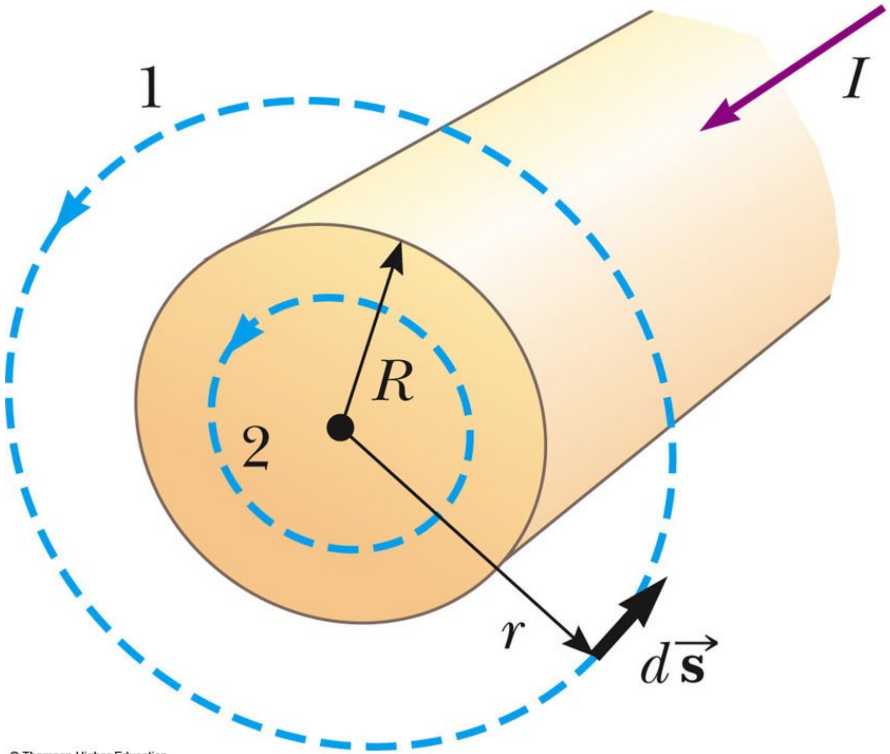
- The product of $\vec{B} \cdot d\vec{s}$ can be evaluated for small length elements $d\vec{s}$ on the circular path defined by the compass needles for the long straight wire
- Ampere's law states that the line integral of $\vec{B} \cdot d\vec{s}$ around any closed path equals $\mu_o I_{enc}$ where I_{enc} is the total steady current passing through any surface bounded by the closed path: $$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

Ampere’s Law, cont.

- Ampere’s law describes the creation of magnetic fields by all continuous current configurations
  - Most useful for this course if the current configuration has a high degree of symmetry
- Put the thumb of your right hand in the direction of the current through the amperian loop and your fingers curl in the direction you should integrate around the loop

Field Due to a Long Straight Wire – From Ampere's Law

- Want to calculate the magnetic field at a distance *r* from the center of a wire carrying a steady current I
- The current is uniformly distributed through the cross section of the wire



Field Due to a Long Straight Wire – Results From Ampere's Law

- Outside of the wire, $r > R$

$$\int \vec{B} \cdot d\vec{s} = B(2\pi r) = \mu_0 I \qquad B = \frac{\mu_0 I}{2\pi r}$$

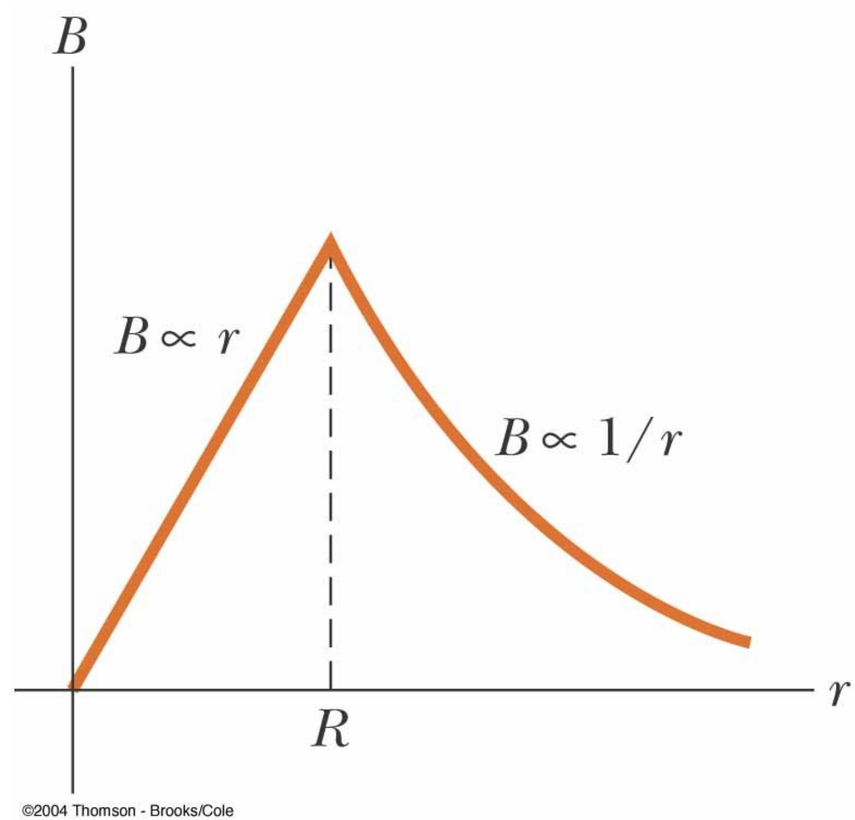
- Inside the wire, we need I', the current inside the amperian circle

$$\int \vec{B} \cdot d\vec{s} = B(2\pi r) = \mu_0 I' \leftarrow I' = \frac{r^2}{R^2} I$$

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

Field Due to a Long Straight Wire – Results Summary

- The field is proportional to *r* inside the wire
- The field varies as 1/*r* outside the wire
- Both equations are equal at *r* = *R*

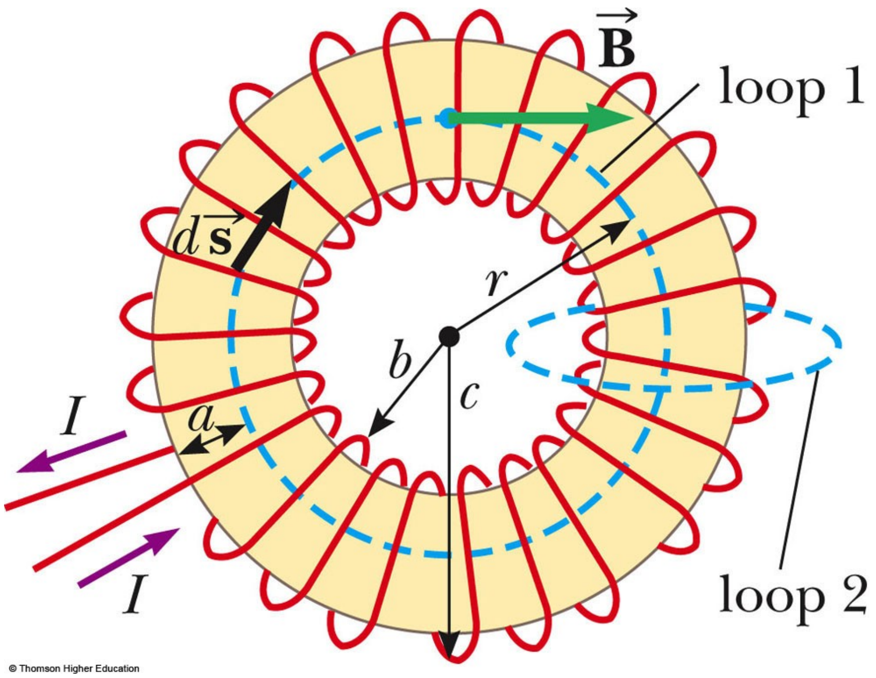


Magnetic Field of a Toroid

- Find the field at a point at distance *r* from the center of the toroid
- The toroid has *N* turns of wire

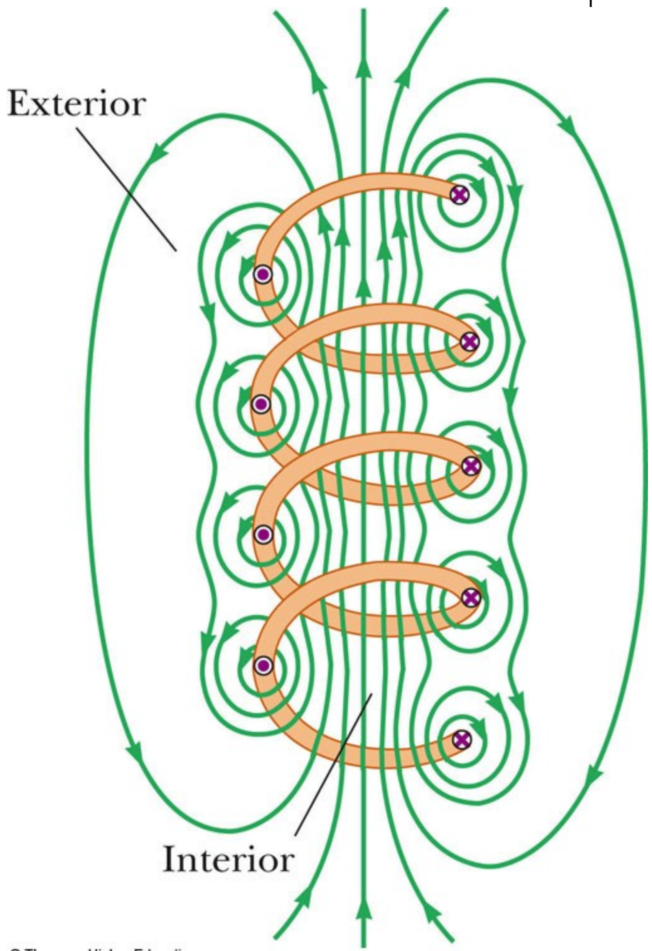
$$\oint \vec{B} \cdot d\vec{r} = B(2\pi r) = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$



Magnetic Field of a Solenoid

- A **solenoid** is a long wire wound in the form of a helix
- A reasonably uniform magnetic field can be produced in the space surrounded by the turns of the wire
    - The ***interior*** of the solenoid

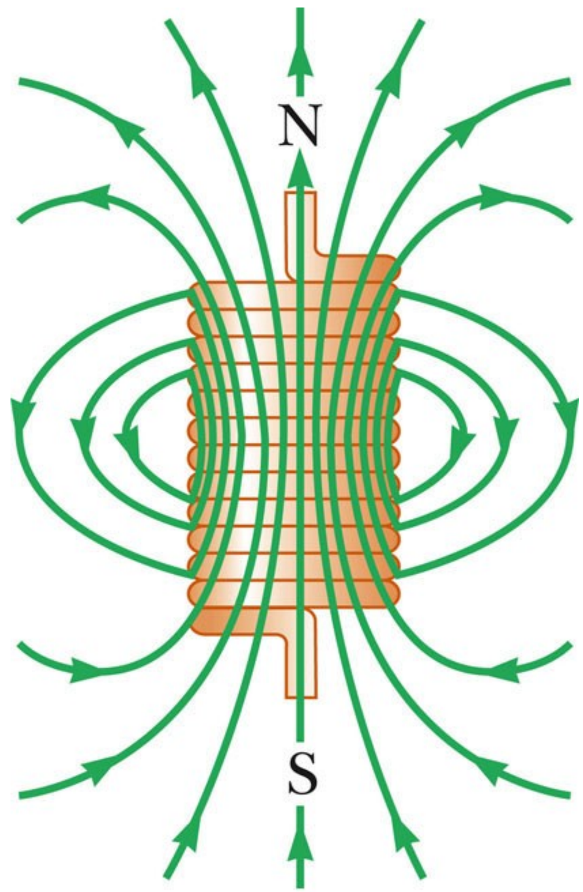


Magnetic Field of a Solenoid, Description

- The field lines in the interior are
  - nearly parallel to each other
  - uniformly distributed
  - close together
- This indicates the field is strong and almost uniform

Magnetic Field of a Tightly Wound Solenoid

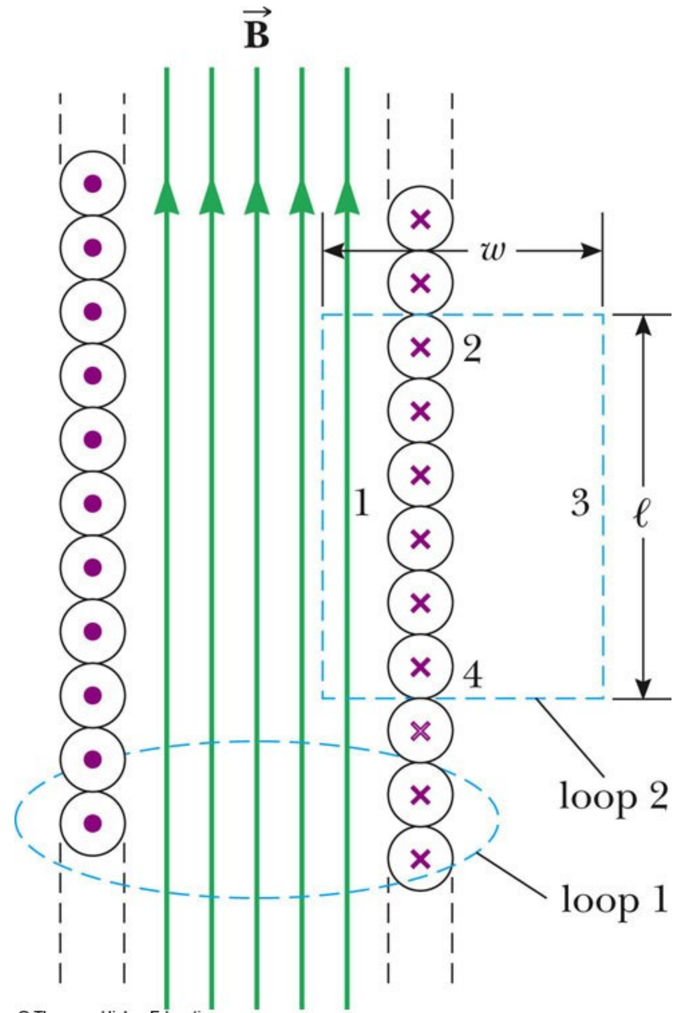
- The field distribution is similar to that of a bar magnet
- As the length of the solenoid increases
  - the interior field becomes more uniform
  - the exterior field becomes weaker



(a)

Ideal Solenoid – Characteristics

- An *ideal solenoid* is approached when:
  - the turns are closely spaced
  - the length is much greater than the radius of the turns



Ampere's Law Applied to a Solenoid

- Ampere's law can be used to find the interior magnetic field of the solenoid
- Consider a rectangle with side ℓ parallel to the interior field and side *w* perpendicular to the field
  - This is loop 2 in the diagram
- The side of length ℓ inside the solenoid contributes to the field
  - This is side 1 in the diagram

Ampere's Law Applied to a Solenoid, cont.

- Applying Ampere's Law gives

$$\oint \vec{B}\cdot d\vec{s} = \int_{path\,1} B\cdot d\vec{s} = B\int_{path\,1} ds = Bl$$

- The total current through the rectangular path equals the current through each turn multiplied by the number of turns

$$Bl = \mu_0 N I \ldots B = \frac{\mu_0 N I}{l}$$

Magnetic Field of a Solenoid, final

- Solving Ampere's law for the magnetic field is

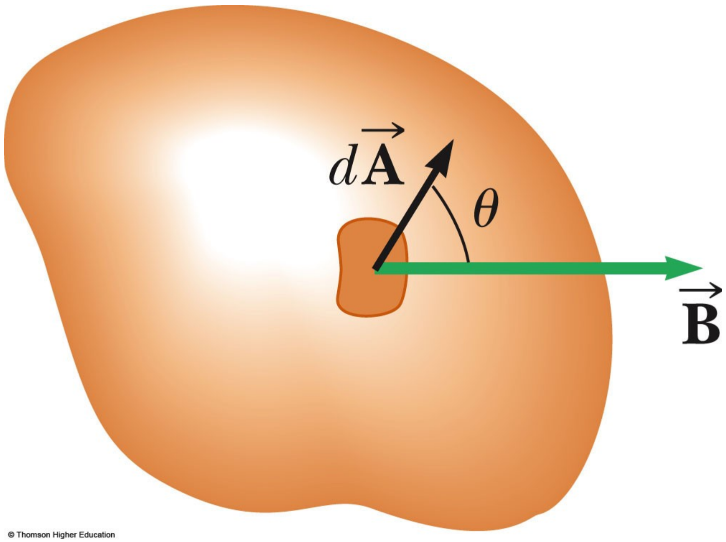
$$B = \mu_o \frac{N}{\ell} I = \mu_o n I$$

  - $n = N / \ell$ is the number of turns per unit length
- This is valid only at points near the center of a very long solenoid

$$B = \frac{\mu_0 N I}{l} = \mu_0 n I$$

Magnetic Flux

- The magnetic flux associated with a magnetic field is defined in a way similar to electric flux
- Consider an area element *dA* on an arbitrarily shaped surface



Magnetic Flux, cont.

- The magnetic field in this element is $\vec{B}$
- $d\vec{A}$ is a vector that is perpendicular to the surface
- $d\vec{A}$ has a magnitude equal to the area *dA*
- The magnetic flux Φ_B is

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

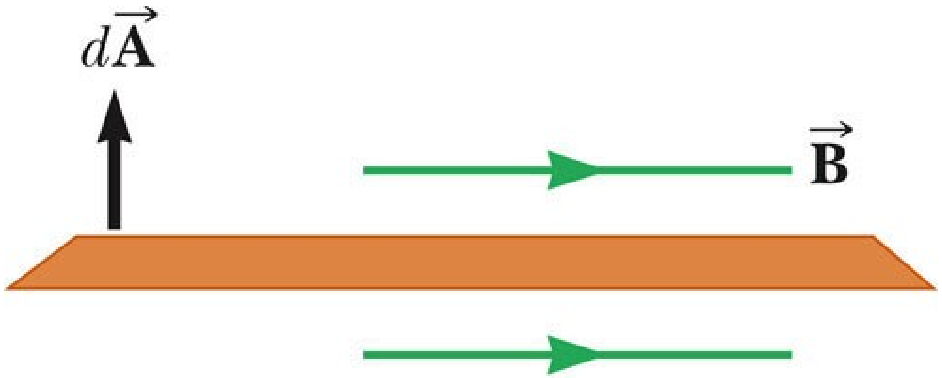
- The unit of magnetic flux is T·m² = Wb
  - Wb is a *weber*

Magnetic Flux Through a Plane, 1

- A special case is when a plane of area *A* makes an angle *θ* with $d\vec{A}$

The magnetic flux is $\Phi_B = BA\cos\theta$

- In this case, the field is parallel to the plane and $\Phi_B = 0$



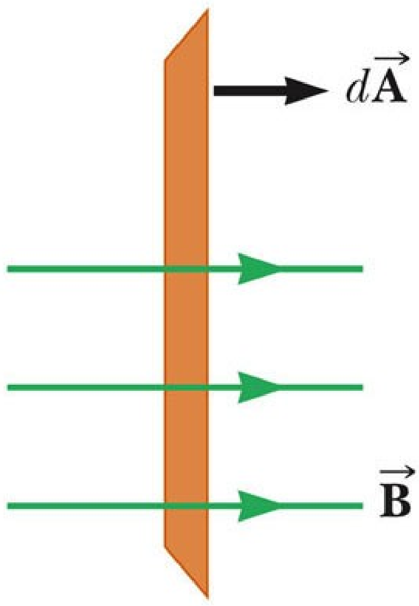
(a)

PLAY ACTIVE FIGURE

Magnetic Flux Through A Plane, 2

- The magnetic flux is $\Phi_B = BA \cos \theta$
- In this case, the field is perpendicular to the plane and

  $\Phi_B = BA$
- This will be the maximum value of the flux
- Use the active figure to investigate different angles



(b)

PLAY ACTIVE FIGURE

Gauss' Law in Magnetism

- Magnetic fields do not begin or end at any point
  - The number of lines entering a surface equals the number of lines leaving the surface
- **Gauss' law in magnetism** says the magnetic flux through any closed surface is always zero:

$$\oint \vec{B} \cdot d\vec{A} = 0$$