Physics 231 Mechanics

Equation list for Exam II and Final

Chapter 5 Newtons Laws of Motion

Newton's First law: if $\vec{F}_{net}=0$, $\vec{v}$ is constant in an inertial frame of reference.

Newton's Second Law: $\vec{a}=\frac{\vec{F}}{m}$

Newton's Third Law:. $\vec{F}_{12}=-\vec{F}_{21}$

Gravitational Force (near Earth's surface). $\vec{F}_g=m\vec{g}$

Kinetic Friction $f_k=\mu_k N$

Static Friction $f_s=F_{app}$ if $F_{app}\le f_{s,max}$ where $f_{s,max}=\mu_s N$

Chapter 6 Circular Motion and Other Applications of Newton's Laws

For an object in uniform circular motion, $\Sigma\vec{F}=m\vec{a}_c=\vec{F}_c$

Centripetal force: $\vec{F}_c=m\frac{v^2}{r}(-\hat{r})$

Tension in a pendulum string (non-uniform circular motion): $T=mg\cos\theta+m\frac{v^2}{r}$

Resistive forces

1. force proportional to velocity: $\vec{R}=-b\vec{v}$
2. force proportional to v^2: $R=\frac{1}{2}D\rho A v^2$

Chapter 7 Systems and Environments

Work: $W=F\Delta r\cos\theta=F_{\|}\Delta r=\vec{F}\cdot\vec{r}$ (for a constant force)

Work: $W=\int\vec{F}\cdot d\vec{r}$

Force by a spring (Hooke's Law): $F_s=-kx$

Work done by a spring: $W=\frac{1}{2}kx_i^2-\frac{1}{2}kx_f^2$

Work – kinetic energy theorem: $W_{net}=K_f-K_i=\Delta K$

Gravitational Potential Energy (near surface of Earth): $U_g=mgy$ (y increases upward, g >0)

Potential Energy of a Spring: $U_s=\frac{1}{2}kx^2$

Work by a conservative force: $W_c=U_i-U_f=-\Delta U$

Mechanical energy: $E_{mech}=K+U$

Potential energy due to any conservative force: $U_f-U_i=-\int_{x_i}^{x_f}\vec{F}_x dx$

Obtain a force from a potential energy function: $F_x=-\frac{dU}{dx}$

Chapter 8 Conservation of Energy

For a non-isolated system, $\Delta E_{system} = \Sigma T$

where $\Sigma T = W + Q + T_{MW} + T_{MT} + T_{ET} + T_{ER}$ are transfer energies.

For an isolated system, $\Delta E_{system} = 0$

For an isolated system with only mechanical energy: $\Delta E_{mech} = 0 = \Delta K + \Delta U$

For a non-isolated system with no change in potential energy, and friction and other forces are present: $\Delta K = \Sigma W_{other\ forces} - f_k d$

Internal energy change of a closed system with friction: $\Delta E_{intern} = f_k d$

For an isolated system with changes in potential energy and friction: $\Delta E_{mech} = -f_k d$

For a non-isolated system ...: $\Delta E_{mech} = +\Sigma W_{other\ forces} - f_k d$

Power: $P = \frac{dE}{dt}$

Power expended by a force: $P = \vec{F} \cdot \vec{v}$

Average power by a force that did work W: $P_{avg} = \frac{W}{\Delta t}$

Chapter 9. Linear momentum and collisions

Linear momentum: $\vec{p} = m\vec{v}$

Momentum and force: $\vec{F} = \frac{d\vec{p}}{dt}$

Conservation of momentum: $\vec{p}_{tot} = constant$ or $\Sigma \vec{p}_{j,initial} = \Sigma \vec{p}_{j,final}$

Impulse: $\vec{I} = \Delta \vec{p}$ or $\vec{I} = \int \vec{F}_{net} dt$

Types of collisions (all obey conservation of momentum):

a) elastic: kinetic energy is conserved

b) inelastic: kinetic energy is not conserved

c) perfectly inelastic: kinetic energy is not conserved and particles stick together

Center of mass for discrete masses:

$$x_{com} = \frac{\Sigma m_i x_i}{M_{tot}} \quad \text{and} \quad y_{com} = \frac{\Sigma m_i x_i}{M_{tot}}$$

Center of mass for continuous, extended masses:

$$\vec{r}_{CM} = \frac{1}{M} \int \vec{r}\, dm$$

For a system of particles:

$$\vec{p}_{tot} = M_{tot} \vec{v}_{CM}$$

Chapter 10. Rotation of a Rigid Object

Angular position: $\theta = \frac{s}{r}$ (where s is arclength)

Angular speed: $\omega = \frac{d\theta}{dt}$

Angular acceleration: $\alpha = \frac{d\omega}{dt}$

Relate to translational quantities: $v = r\omega$, $a_t = r\alpha$ and $a_c = \frac{v^2}{r} = r\omega^2$

(Ch. 10 cont.)

Angular kinematic equations for constant angular acceleration:

$$\omega_f = \omega_i + \alpha t$$
$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$
$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$
$$\theta_f = \theta_i + \frac{1}{2}(\omega_i - \omega_f)t$$

Rotational kinetic energy: $K_R = \frac{1}{2} I \omega^2$

Moment of inertia: $I = \Sigma m_i r_i^2$ (for discrete masses)

Moment of inertia: $I = \int r^2 dm$ (for continuous masses)

Mass density: linear mass density, λ , surface mass density, σ , volume mass density ρ

Parallel-axis theorem: $I = I_{CM} + MD^2$

Torque: $\tau = rF \sin\theta$

Torque: $\tau_{net} = I\alpha$

Total kinetic energy: $K_{tot} = K_{trans} + K_{rot}$

For an object that rolls without slipping:

$$\Delta s = R\Delta\theta$$
$$v_{CM} = R\omega$$
$$a_{CM} = R\alpha$$

Chapter 11 Angular Momentum

Torque (as a vector): $\vec{\tau} = \vec{r} \times \vec{F}$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p} = mvr \sin\theta$

Angular momentum and angular quantities: $L = I\omega$

Angular momentum and torque: $\tau_{net} = \frac{d\vec{L}}{dt}$

Conservation of Angular momentum: $\vec{L}_{init} = \vec{L}_{fin}$ (for an isolated system)