

Exercises

- (1) Find the Maclaurin series expansion of the function

$$f(z) = \frac{z}{z^4 + 9}$$

- (2) Show that when $0 < |z| < 4$,

$$\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}$$

(3) Write the two Laurent series in powers of z that represent the function

$$f(z) = \frac{1}{z(1+z^2)}$$

in certain domains, and specify those domains.

Hint 1: For one domain you should get

$$\sum_{n=0}^{\infty} (-1)^{n+1} z^{2n+1} + \frac{1}{z}.$$

For the other domain, you should get

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{z^{2n+1}}.$$

Hint 2: Observe that $(-1)^{n-1} = (-1)^{n-1}(-1)^2 = (-1)^{n+1}$.