Exercises

(1) Let $f(z) = (\overline{z})^3 + 3\overline{z}$.

a) Find all points $z \in \mathbb{C}$ at which f is differentiable. Make sure you justify your answer.

b) Show that f is nowhere analytic in $\mathbb{C}$.

c) Explain why there is no contradiction between your answers to (a) and (b).

(2) Use the Cauchy-Riemann equations to show that the function

$$f(z) = \exp \overline{z}$$

is not analytic anywhere.

(3) Calculate $\dfrac{d}{dz}(1+2i)^z$. Explain any restrictions you need to make for your answer to be valid.

(4) (*Bonus*) Differentiate $f(z) = \sqrt{e^z + 1}$, giving the appropriate region on which $f(z)$ is analytic.