Exercises

(1) Evaluate the following integrals:

a) $\int_1^2 \left(\frac{1}{t} - i\right)^2 dt$

b) $\int_0^{\pi/6} e^{i2t} dt$

(2) Show that if m and n are integers,

$$\int_0^{2\pi} e^{im\theta} e^{-in\theta} d\theta = \begin{cases} 0, & \text{when } m \neq n, \\ 2\pi, & \text{when } m = n. \end{cases}$$

(3) Evaluate $\int_C f(z)dz$ for $f(z) = (z+2)/z$ and C is

    a) the semicircle $z = 2e^{i\theta}$ $(0 \leq \theta \leq \pi)$;

    b) the semicircle $z = 2e^{i\theta}$ $(\pi \leq \theta \leq 2\pi)$;

    c) the circle $z = 2e^{i\theta}$ $(0 \leq \theta \leq 2\pi)$.

(4) Evaluate

$$\int_{-1}^{1} z^i dz$$

Analyse the function before doing any computations. Is it single valued? Multiple valued?