Exercises

(1) Evaluate the following integrals, justifying your procedures:

(a) $\displaystyle\int_C \frac{2dz}{z^2-1}$, where C is the circle with radius $1/2$, centre 1, positively oriented;

(b) $\displaystyle\int_0^i ze^{z^2}dz$.

(2) Let D be the annulus $6 < |z| < 8$, and let C be any simple closed contour inside D. Show that:

$$\int_C \frac{dz}{z^2+1} = 0$$

(3) Let C_R denote the upper half of the circle $|z| = R$ $(R > 2)$, taken in the counterclockwise direction. Show that

$$\left| \int_{C_R} \frac{2z^2 - 1}{z^4 + 5z^2 + 4} dz \right| \leq \frac{\pi R \left(2R^2 + 1\right)}{\left(R^2 - 1\right)\left(R^2 - 4\right)}.$$

Then, by dividing the numerator and denominator on the right here by R^4, show that the value of the integral tends to zero as R tends to infinity.