利用選擇權市場價格校正 GARCH 訂價模型—S&P 100 指數選擇權的實證研究

稟厥庸[**]　洪偉育

長庚大學 工商管理學系

摘要

本研究提出一個簡單且精準的美式選擇 GARCH 訂價方法。此方法源自於 Barone-Adesi, Engle, and Mancini (2008)對歐式選擇權提出的歷史濾嘴模擬法，我們首先修改此方法中的一個重要環節，並進一步將其應用在美式選擇權的訂價。此方法的特色之一是仰賴選擇權市場資料對模型參數進行校正，這樣的優點是納入選擇權市場資訊以獲得精準的評價表現。本研究以 S&P 100 選擇權做為實證標的，研究發現本文提出的修正方法可有效增加樣本內的配適表現與樣本外的預測能力。本研究進一步以實證方法探討選擇權價性(moneyness)對訂價模型的影響，這樣的分析對於不存在恆等式的買賣權平價說(put-call parity)的選擇權(如美式選擇權)格外有意義，研究發現價內與價外美式選擇權對其訂價模型中的參數估計以及訂價表現皆有明顯影響。

關鍵詞：歷史濾嘴模擬法; 美式選擇權; 一般化條件異質變異模型; 蒙地卡羅模擬

JEL classification: G13, C15

[**] 通訊作者。地址：桃園市龜山區文化一路 259 號。電話：03-2118800 轉 3248。傳真：03-2118500。

Market Calibration of GARCH Option Pricing Models: Evidence from S&P 100 Index Options

Chueh-Yung Tsao* and Wei-Yu Hung

Dept. of Industrial and Business Management, Chang Gung University

Abstract

This paper analyzes the issue of GARCH option pricing models as applied to American options. In this study, we do not try to propose alternative theoretical model or a fancy pricing method for American options, in contrary; we propose a simple but accurate market calibration method employing the Monte Carlo simulation framework. This method not only allows us to estimate parameters using options market data, but also provides accurate forecasting power for future prices. This method is a modification and extension of the Filtered Historical Simulation (FHS), as proposed by Barone-Adesi, Engle, and Mancini (2008) for pricing European options. An empirical investigation of S&P 100 options demonstrates that even such a simple modification can significantly improve pricing accuracy in both in-sample and out-of-sample periods. By employing a reliable pricing method, we further study the information content of option prices empirically. We provide evidence indicating that both out-of-the-money and in-the-money option prices contain important information when calibrating parameters of American options under GARCH processes.

Keywords: Filtered Historical Simulation; American Options; GARCH; Monte Carlo Simulation

JEL classification: G13, C15

* Corresponding Author: 259 Wen-Hwa 1st Rd., Kwei-Shan, Tao-Yuan, Taiwan 333. TEL: +886-3-2118800 ext.3248, Fax: 886-3-2118500. Email: cytsao@mail.cgu.edu.tw. An early version of this article was presented at the 15th International Conference on Computing in Economics and Finance (CEF2009), Sydney, Australia, Jul. 13–17, 2009.

1. Introduction

The Black-Scholes (1973) model has been widely employed in European option pricing problems. In the model, the volatility of a stock return is assumed to be constant. However, numerous studies have attempted to modify the Black-Scholes model to account for stochastic volatility. Under the assumption that volatility is uncorrelated with stock price, Hull and White (1987) propose the stochastic volatility model (SV model) and find that the Black and Scholes (1973) model tends to overprice at-the-money options and underprice deep-in-the-money and deep-out-of-the-money options in the presence of stochastic volatility. Johnson and Shanno (1987) use the time-changing stochastic volatility model to value option prices. Heston (1993) releases Hull and White’s (1987) assumption regarding the lack of correlation and derives a closed-form solution for a European call price of an asset with stochastic volatility. Merton (1976) studies the case in which the price follows a mixed jump-diffusion process. Kou (2002) proposes a double exponential jump-diffusion model to capture the leptokurtic feature and the volatility smile phenomenon. Madan, Carr and Chang (1998) add two additional parameters to the Brownian motion and develop the variance gamma model to control for the skewness and kurtosis of the return distribution.

There are many successful applications of GARCH-type models to financial time series (Bollerslev, Chou, and Kroner, 1992; Bollerslev, Engle, and Nelson, 1994; Poon and Granger, 2003), and numerous studies have been published that employ a GARCH model for option pricing. Most such studies consider pricing models based on GARCH-type stochastic volatility under the assumption of Gaussian innovation (e.g., Duan, 1995; Bollerslev and Mikkelsen, 1996; Heston and Nandi, 2000; Duan and Zhang, 2001; Lehar, Scheicher, and Schittenkoph, 2002; Christoffersen and Jacobs, 2004; Hsieh and Ritchken, 2005; Stentoft, 2005). One means of extending the GARCH pricing model is considering an alternative conditional distribution. The advantage of such an extension is that the pricing model can capture the higher moment property of the conditional distribution. Duan (1999) considers pricing models based on the conditional leptokurtic distribution. Christoffersen, Heston and Jacobs (2006) develop a pricing model that allows for a leverage effect, time-varying volatility, and skewness by considering an asymmetric GARCH combined with an Inverse Gaussian innovation. Stentoft (2008) extends Duan's (1999) model by allowing the conditional distribution to be skewed and leptokurtic and uses the Normal Inverse Gaussian distribution as the conditional distribution.

Whereas all of the studies mentioned above are based on parametric models, Barone-Adesi, Engle, and Mancini (2008) (BEM (2008) hereafter) employ a nonparametric method to represent the distribution of the underlying asset. They term this method the Filtered Historical Simulation (FHS). The essence of the FHS is similar to the bootstrapping method. First, it applies a suitable econometric model to historical data to filter out certain stylized facts, such as the leverage, heavy tail and volatility clustering that are commonly observed in real financial time series. The remaining residual thus forms an empirical innovation of the asset; then, the empirical innovation is used to generate a future price path under the risk-neutral measure, and the options traded on that asset can be priced by using the Monte Carlo simulation method. One important implication of the FHS in GARCH option pricing is that when the empirical innovation is not Gaussian, the risk-neutral return could follow a GARCH process with parameters differing from those in the historical return series (BEM, 2008). The GARCH parameters in the risk-neutral world can be estimated by calibrating the model using option prices observed in the market. Based on an extensive empirical analysis of S&P 500 index options, BEM (2008) demonstrate that the FHS, when combined with GJR-GARCH (Glosten, Jagannathan, and Runkle, 1993), can efficiently improve pricing accuracy compared with other GARCH option pricing models, such as Dumas, Fleming, and Whaley (1998), Heston and Nandi (2000), and Christoffersen, Heston and Jacobs (2006).

Most empirical applications of the GARCH option pricing models focus on European-style options; these applications include those of Heston and Nandi (2000), Christoffersen and Jacobs (2004), Bollerslev and Mikkelsen (1996), Hsieh and Ritchken (2005), Duan and Zhang (2001), Lehar, Scheicher and Schittenkoph (2002), and Myers and Hanson (1993). Exceptions include Stentoft (2005) and Weber and Prokopczuk (2011). Stentoft (2005) first uses historical data on the underlying assets to estimate the parameters in a GARCH model. He then uses the Least-Squares Monte Carlo (LSM) method of Longstaff and Schwartz (2001) to price the American options. One disadvantage of this method is that only the market information from the underlying assets is used. Some studies have found that pricing accuracy can be improved by calibrating the model using option prices observed in the market, for example, Bakshi, Cao, and Chen (1997), BEM (2008) and Weber and Prokopczuk (2011). Weber and Prokopczuk (2011) price American options using a lattice framework. They empirically evaluate the performance of GARCH option pricing models by using Rubinstein's Edgeworth binomial tree (Rubinstein, 1998). Moreover, Weber and Prokopczuk (2011) find that the calibrated model can significantly improve pricing performance. However, lattice methods have certain disadvantages, such as the curse of dimensionality and the difficulty of pricing path-dependent options[1], which restrict their usage applicability under real conditions.

The contribution of this study is threefold. First, we propose a modified calibration method using a Monte Carlo simulation framework. Our method is similar to that of BEM (2008), except that it regards the initial conditional volatility of the risk-neutral world as a parameter to be estimated. This statement means that initial conditional volatility should be determined by the options market, instead of basing the historical GARCH estimate on empirical asset returns, as in the case of BEM (2008). This modification is intuitive because of the discrepancy between the volatility of historical and risk-neutral processes; additionally, it implies that the two volatility processes are not identical. Thus, there is no reason that the initial conditional volatility of the risk-neutral world should come from the historical estimate. This study finds that even such a simple modification can significantly improve pricing accuracy.

The second contribution of this study is to extend the BEM (2008) method from European option pricing to American option pricing. In particular, we use options on the S&P 100 index (OEX) in our empirical study. The OEX options are American-style and cash settled. They are one of the most active index options in the world. However, the first issue we face is to select an appropriate American option-pricing method, among many offered in the literature, for the calibration process. On the one hand, as we mentioned above, BEM (2008) use the FHS method for the calibration process. The FHS is essentially a simulation-based method. Therefore, it would be preferable to choose the pricing method in the same manner. On the other hand, as market calibration involves several iterations, the pricing method must be efficient enough to ensure that the implementation remains feasible. As a result, this study uses the LSM method of Longstaff and Schwartz (2001) because it is not only more efficient than other methods based on the Monte Carlo framework but also more flexible in its application to broad asset dynamics.

The third contribution of this study is that, to the best of our knowledge, we are the first to carefully and empirically study the informational content of different moneyness for

[1] Hull and White (1993) demonstrate how to price path-dependent options within a lattice framework.

American options.[2] Out-of-money European options have been used in most empirical studies in the literature. The reason is that, under the no-arbitrage requirement, we can easily replicate the put (call) from their call (put) counterparts, i.e., put-call parity. Moreover, the out-of-money options typically have higher liquidity and are more likely to reflect the true values of the options. For the American-style contracts, however, put-call parity does not exist, and the prices of both out-of-money and in-the-money options may contain different information. Neither should be ignored. We will study how the in-the-money options affect the calibration of the pricing model and how serious the pricing errors are as a consequence of excluding in-the-money options.

The remainder of this paper is organized as follows. Section 2 provides a brief introduction to the calibration method proposed by BEM (2008) and our modified method. The empirical analysis is presented in Section 3. The model performance is described in Section 4. We conclude the paper in Section 5.

2. Methodologies

2.1. Asset price dynamics

2.1.1. Return dynamics under the $\mathbb{P}$ measure

It is well known that constant volatility and the normal return innovation do not conform to real market phenomena. Numerous researchers have addressed how financial return data are asymmetric, with fat tails. To capture the phenomena mentioned above, stochastic volatility has been developed within the framework of the autoregressive conditional heteroskedastic (ARCH) process suggested by Engle (1982) and generalized by Bollerslev (1986).

The GARCH model proposed by Bollerslev (1986) accounts for stochastic, mean-reverting volatility dynamics, and it assumes that the same absolute return shocks should have identical influences on future conditional volatilities. However, some studies have shown that past negative and positive return shocks result in asymmetrical distributions and negative returns along with a tendency towards larger future volatilities. This phenomenon is generally referred to as the “leverage” effect. Three GARCH models are commonly used to describe

[2] Bakshi, Cao and Chen (1997), Dumas, Fleming and Whaley (1998), and Christoffersen and Jacobs (2004) use both out-of-money and in-the-money options in there empirical studies. Bakshi, Cao and Chen (1997) also analyze the relationship between pricing errors and moneyness. However, those studies are based on the S&P 500 options which are European-style options. As we will mention in Section 2.3, the issue of the informational content of different moneyness is more relevant for American options.

this phenomenon of asymmetry. Those approaches include the Exponential GARCH (EGARCH) model of Nelson (1991), the Asymmetric GARCH (AGARCH) model of Engle and Ng (1993), and the GJR or Threshold GARCH (TGARCH) models of Glosten, Jagannathan and Runkle (1993) and Zakoïan (1994).

Under the historical measure $\mathbb{P}$, we follow BEM (2008) in modeling the equity index return. BEM (2008) consider an asymmetric GARCH specification with an empirical innovation density to fit the equity index return. The asymmetric property of a GARCH model is based on Glosten, Jagannathan and Runkle (1993) (GJR-GARCH). Under the historical measure $\mathbb{P}$, the GJR-GARCH (1, 1) model is

$$\begin{aligned} r_t &= \log(S_t / S_{t-1}) = \mu + \varepsilon_t \\ \varepsilon_t &= \sigma_t Z_t \\ \sigma_t^2 &= \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2, \end{aligned} \tag{1}$$

where S_t is the equity index, r_t is the log-return of the equity index, Z_t is a sequence of standard normal random variables, $\theta = \{\omega, \beta, \alpha, \gamma\}$ is the set of parameters of the volatility equation, μ is the constant expected return of the log-return, and $I_{t-1} = 1$ when $\varepsilon_{t-1} < 0$ and $I_{t-1} = 0$ otherwise. It is clear that when $\gamma > 0$, past negative return shocks $(\varepsilon_{t-1} < 0)$ will have more impact on future volatility. Empirically, a sample of Z_t can be determined by dividing each estimated innovation $\hat{\varepsilon}_t$ by its corresponding conditional standard deviation $\hat{\sigma}_t$. The empirical innovation density is constituted by a set of scaled innovations to capture potential non-normality in the real equity market (i.e., the excess skewness, kurtosis, and other extreme return behavior that the normal density cannot describe). The procedure for sampling from the empirical innovation density to simulate the asset dynamics is referred to as the FHS method. The FHS is briefly introduced in the next subsection.

2.1.2. Return dynamics under the $\mathbb{Q}$ measure

Under the risk-neutral measure $\mathbb{Q}$, BEM (2008) consider the return dynamic as follows:

$$\begin{aligned} r_t &= \mu^* + \varepsilon_t \\ \varepsilon_t &= \upsilon_t Z_t \end{aligned} \tag{2}$$

$$\upsilon_t^2 = \omega^* + \beta^* \upsilon_{t-1}^2 + \alpha^* \varepsilon_{t-1}^2 + \gamma^* I_{t-1} \varepsilon_{t-1}^2 .$$

The risk-neutral drift μ^* is set to ensure that the expected asset return equals the risk-free rate minus the dividend yield, i.e., $E_Q\left[S_t / S_{t-1} \middle| F_{t-1}\right] = \exp\left\{r_f - d\right\}$, where r_f and d denote the one-period risk-free rate and one-period dividend yield, respectively. υ_t is the conditional volatility under $\mathbb{Q}$, and $\theta^* = \left\{\omega^*,\ \beta^*,\ \alpha^*,\ \gamma^*\right\}$ is the set of parameters of the volatility equation. Here, we use different notations for variance equations under $\mathbb{Q}$ and $\mathbb{P}$ to demonstrate that they might not be the same series. Some studies, including Chernov and Ghysels (2000) and Christoffen and Jacobs (2004), demonstrate that pricing performance is poor when the volatility equation is directly formulated under the historical measure $\mathbb{P}$.

Therefore, BEM (2008) approximate the risk-neutral return dynamics by calibrating a new set of pricing GJR-GARCH parameters $\theta^* = \left\{\omega^*,\ \beta^*,\ \alpha^*,\ \gamma^*\right\}$ directly using option market prices. The approach used to calibrate the pricing model is the FHS. The FHS algorithm contains the following steps.

Step 1. At time t_0, n historical log-returns of the underlying asset, $\left\{\log(S_t / S_{t-1}),\ t = t_0 - n + 1,\ t_0 - n + 2,\ ...,\ t_0\right\}$, are used to estimate the set of parameters $\theta = \left\{\omega,\ \beta,\ \alpha,\ \gamma\right\}$ in the GARCH model in Eq. (1). Then, we can obtain n empirical scaled or filtered innovations $z_t = \hat{\varepsilon}_t / \hat{\sigma}_t$.

Step 2. Given one set of estimates of the GARCH pricing parameter θ^*, the return process under the $\mathbb{Q}$ measure from t_0 to $t_0 + \tau$ is simulated using the GARCH pricing model in Eq. (2). The simulated period τ is determined by the maturity of the options traded in the market. The random innovation in this simulation comes from the empirical scaled or filtered innovations $\{z_t,\ t = t_0 - n + 1,\ ...,\ t_0\}$. We simulate a set of innovations by randomly drawing a sample $\{z_t^{\#}, t = t_0 + 1, ..., t_0 + \tau\}$ from $\{z_t\}$ with replacement. We then obtain a simulated price path as follows:[3]

$$S_t = S_{t_0} \exp\left((t - t_0)\mu^* + \sum\nolimits_{i=t_0+1}^{t} \upsilon_i z_i^{\#}\right), \tag{3}$$

where $t = t_0 + 1, ..., t_0 + \tau$. The simulation continues N times to obtain N simulated

[3] In this study, a discrete-time model is considered. Therefore, there is no quadratic variation term in Eq. (3).

paths. Notice that at time t_0, the values of the initial innovation z_{t_0} and the initial conditional volatility υ_{t_0} must be determined first. In BEM (2008), not only z_{t_0} but also υ_{t_0} is directly borrowed from the $\mathbb{P}$ measure, i.e., $\upsilon_{t_0} \equiv \hat{\sigma}_{t_0}$. Note that there is a potential disadvantage of the model in BEM (2008) because the conditional volatility under the $\mathbb{P}$ measure may differ from that under the $\mathbb{Q}$ measure.

Step 3. Based on the *N* simulated paths in Step 2, the option prices are computed by using the Monte Carlo simulation method. For example, BEM (2008) consider European options in their study, and the model call and put prices can easily be obtained by:

$$\hat{C}_E = \frac{e^{-r_f \tau} \sum_{i=1}^{N} \max\left(S_{\tau,i} - K, 0\right)}{N} \tag{4}$$

and

$$\hat{P}_E = \frac{e^{-r_f \tau} \sum_{i=1}^{N} \max\left(K - S_{\tau,i}, 0\right)}{N}, \tag{5}$$

respectively. $S_{t,i}$ in the above equations denotes the simulated price at time τ for the *i*th path, and *K* is the strike price. This study investigates American options; the Least-Squares Monte Carlo technique of Longstaff and Schwartz (2001) is applied to obtain the option prices. We discuss how to apply the Least-Squares Monte Carlo technique in next section.

Step 4. Repeat steps 2 and 3 by varying the GARCH pricing parameter θ^* to appropriately fit the cross-section of the option prices. The objective could be set to minimize the mean squared error (MSE) between the model prices and the market prices. Once the MSE cannot be reduced by varying the pricing GARCH parameter θ^* or the MSE falls below some level, the calibration is achieved.[4]

In this study, we modify the pricing model of BEM (2008) by regarding the initial volatility υ_{t_0} under $\mathbb{Q}$ as another parameter to be estimated. The primary reason that the conditional volatilities under $\mathbb{P}$ and $\mathbb{Q}$ may not be the same is intuitive. If we assume that

[4] One can use other objective functions, for example, a mean absolute error, a mean squared percentage error, or a utility-based function, when calibrating. In addition, one can also consider a weighted mean in the objective function and take other market information, such as trading volumes or open interests, as the weights. The variety of choice help researches to empirically test various informational hypotheses in the option markets. In order to have a direct comparison with the original calibration method, we use the same objective function as in BEM (2008) in this study. We leave the issue of objective functions as a future research.

the GARCH parameters under $\mathbb{P}$ and $\mathbb{Q}$ could have their own values to more closely reflect the real derivative market, the two volatility processes would evolve individually according to their dynamic equations. Under our modified pricing model, the pricing GARCH parameters in the set $\theta^* = \{\omega^*, \beta^*, \alpha^*, \gamma^*, \upsilon_{t_0}\}$ are calibrated using the same procedure as previously described. To reduce the Monte Carlo variance, we use the empirical martingale technique of Duan and Simonato (1998) to ensure that the risk-neutral expectation of the underlying asset conforms to its forward price.

The inclusion of the initial volatility υ_{t_0} as another parameter to be estimated is not a novel approach from the econometrician's perspective. When estimating a moving-average model, an estimation that is based on maximizing the conditional likelihood method is referred to as the conditional likelihood method. The alternative estimation approach, which treats the initial shock $\varepsilon_{t_0} = \upsilon_{t_0} Z_{t_0}$ as an additional parameter of the model and estimates it jointly with other parameters, is referred to as the exact likelihood method. Research has found that the exact likelihood estimates are preferred to the conditional estimates (Tsay, 2005). Our modified calibration method is similar in spirit to the exact likelihood method. As this study indicates, the modified calibration method can significantly outperform the original calibration method.

2.2. The Least-Squares Monte Carlo method

American options have an important characteristic distinguishing them from European options, i.e., they can be exercised at any time before maturity, whereas European options can only be exercised at maturity. No closed-form solution has been obtained for the prices of American options in general[5]. As the result, numerical methods have been proposed in the literature. There are three major numerical methods for pricing American options: the lattice method, the finite difference method, and the Monte Carlo method. The lattice method and the finite difference method offer computational efficiency, but they experience difficulties in addressing problems in which multiple assets or stochastic volatilities are involved. The Monte Carlo method is more flexible and intuitive for solving those problems. The difficulty, however, is that the Monte Carlo method cannot easily estimate the continuation values and

[5] Exceptions include the American call without dividend paying and the perpetual American put within a Black-Scholes model (Björk, 2009).

determine the optimal early exercise strategy, as the Monte Carlo method simulates stock prices forward in time but evaluates continuation values successively in a backwards manner and makes the decision of whether the option should be exercised early.

There are numerous modifications of or improvements to the Monte Carlo method to solve the problem of pricing American options, e.g., Tilley (1993), Barraquand and Martineau (1995), Raymar and Zwecher (1997), Grant, Vora, and Weeks (1996), and Longstaff and Schwartz (2001). In this study, we adopt the LSM method proposed by Longstaff and Schwartz (2001) because it is easy to implement and has greater flexibility when applied to stochastic volatility dynamics. The key idea of LSM is that the conditional expectation from continuation can be efficiently estimated from the cross-sectional information revealed by simulation under a Least-Squares method. Then, the holder of an American option can make exercising decisions by comparing the payoff from immediate exercise with the conditional expectation from continuation.

The procedures of the LSM method that apply to our pricing model are summarized as follows. First, we assume that the option expires in τ periods and specify the early exercise points as $t_0 < t_0 + 1 < \cdots < t_0 + \tau$. We then use Eq. (2) to simulate N paths of the underlying asset under the risk-neutral measure. On the expiration date $t_0 + \tau$, the option holder exercises the option if it is in-the-money or chooses not to exercise it if it is out-of-the-money. At exercise time $t_0 + \tau - 1$, the cash flow from immediate exercise is known to the option holder. However, the cash flow from continuation is unknown and needs to be estimated. If the present value of the cash flow from continuation is larger than the value associated with immediately making an exercise decision, the option holder will continue the life of the option.

To estimate the present value of the cash flow from continuation, we simply implement the cross-sectional regression on a set of basis functions of the underlying asset value. Let X be a vector of simulated asset prices at time $t_0 + \tau - 1$ and Y be a vector of the corresponding discounted cash flows received at time $t_0 + \tau$, provided that the option is in-the-money at time $t_0 + \tau - 1$. We have

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i g_i(X) + \varepsilon_i \tag{6}$$

where the β_i coefficients are constants, g_i is consent with the basic functions, and ε_i is the error term. The basis functions of a regression model can be the set of weighted Laguerre, simple ordinary, Hermite, Legendre, Chebyshev, Gegenbauer, and Jacobi polynomials (Longstaff and Schwartz, 2001). After calculating the estimated value of Y, we can determine whether to exercise the option immediately or expand the life of the option at time $t_0+\tau-1$. Then, return to time $t_0+\tau-2$ and repeat the procedure until the exercising decisions at each exercise point have been made.

The final step of LSM is to discount the payoffs for all paths to time t_0 and simply average them to obtain the risk-neutral price of a put

$$\hat{P}_A = \frac{\sum_{i=1}^{N} e^{-r_f\left(t_i^*-t_0\right)} \max\left(K-S_{t_i^*,i},0\right)}{N} \tag{7}$$

where t_i^* is the optimal exercise time along the ith path and r_f is the corresponding risk-free rate. The American call option prices are computed in the same manner. For additional details on the LSM method, see Longstaff and Schwartz (2001).

2.3. The contracts

In most of the literature, only the out-of-the-money (OTM) option prices have been used to calibrate the pricing dynamics in pricing European options. There are some reasons that only the OTM option prices are covered. The first is the existence of put-call parity. Put-call parity states the relationship between European options in an arbitrage-free economy,

$$P_E + S_{t_0}e^{-d\tau} = C_E + Ke^{-r_f\tau}, \tag{8}$$

where P_E and C_E are the European put and call options, both having identical strike prices, expiries, dividend yields, risk-free rates, and underlying assets. This means that, when the market is free of arbitrage, there is a balance among the above terms. This implies that we can price P_E or C_E using Eq. (8) when the other terms are known. Therefore, we can calibrate pricing dynamics simply using either the OTM option prices or the ITM option prices with respect to pricing European options. Moreover, the OTM options often attract greater attention in the market and have higher liquidity than ITM options. As the result, most studies only use the OTM option prices to calibrate pricing dynamics for European options.

Unfortunately, there is no such equality for the American option. The put-call parity condition for American options on dividend-paying stocks is

$$S_{t_0}e^{-d\tau} - K \le C_A - P_A \le S_{t_0} - Ke^{-r_f\tau}, \tag{9}$$

where P_A and C_A are American put and call options, respectively. As depicted in the inequality in Eq. (9), the stock, the American put, the American call, and the risk-free rate are not closely related. The American put and the American call might exhibit unique information, and it is possible that neither is redundant. Therefore, excluding either OTM or ITM options from the empirical calibration might generate a serious modeling error. This study uses both the OTM and ITM option prices to calibrate the pricing dynamics of American options.[6]

3. Empirical Analysis

3.1. Data

In this study, we use options on the S&P 100 index (symbol: OEX) and corresponding necessary data provided by OptionMetrics. The reason for selecting the OEX options rather than individual stock options is that we intend to avoid certain expectation problems associated with individual stocks, particularly that individual stocks often pay discrete dividends. Furthermore, the OEX options are one of the most actively traded index options in the market. They are American-style and cash settled. The option data include trading date, expiration month, strike price, trading volume, open interest, high price, low price, and closing price. Expiration dates are the three near-term months and three additional months from March, June, September, and December, as a quarterly cycle.

We consider closing prices of the OTM and the ITM OEX options including call and put options on each Wednesday from 2 January 2003 to 30 December 2005. If the Wednesday is a no-trade day, we select the Thursday immediately after the Wednesday. The midpoints between bid and ask quotes are taken as option prices. To reduce the illiquidity problem, the qualified sample must be, including the sum of open interest and trading volume, no less than 100. Moreover, we follow the sample selection procedure used in BEM (2008). The option contracts with prices below 0.05, with implied volatility greater than 70%, and with times to maturity of fewer than 10 days or more than 360 days are discarded. Finally, this yields 19,282 observations.

When implementing the pricing formulae, it is necessary to account for the dividends paid by the stocks in the S&P 100 index. Conventionally, the dividend paid by the S&P 100 index

[6] This paper empirically studies the effect of moneyness on option pricing. However, theoretically examination of this issue is required. We leave this as a topic for future research.

is regarded as the continuous payments. For each year, we calculate the mean of daily dividend yields and use it as the constant dividend yields over the annual period while simulating the asset return paths. The riskless rate for each option contract is derived using the linearly interpolating method, using the term structure of zero-coupon default-free interest rates. Table 1 summarizes the 19,282 option prices in terms of time to maturity and moneyness. The range of average put (call) prices is from \$2.04 (\$2.32) for short (< 60 days) maturity OTM options to \$33.69 (\$76.50) for long (> 160 days) maturity ITM options. The OTM put and call options account for 49% and 29% of the total sample, while ITM put and call options account for 8% and 14% of the total sample. Short, middle and long time to maturity options account for 57%, 30%, and 12% of the total sample, respectively.

Figure 1 depicts the movement of the S&P 100 index during the sample period. The S&P 100 index ranges from a minimum of a \$406.74 to a maximum of \$584.43 with an average of \$533.68. Table 2 displays the summary statistics for the S&P 100 index daily log-returns. The mean of daily log-returns is 0.03%, the standard deviation is 0.83%, the skewness is -0.26%, and kurtosis is 4.47. The result of the Jarque-Bera test indicates that there is insufficient evidence to prove that the daily log-return is normal. The first order autocorrelation function (ACF(1)) of the S&P 100 index log-returns is negative and significant. The ARCH LM test indicates that the S&P 100 index daily log-returns clearly exhibit heteroscedasticity phenomena. Based on the above analysis, we conclude that the S&P 100 index daily log-returns have GARCH effects.

3.2. Calibration Results

We use the cross-sectional OTM and ITM OEX option prices to calibrate the pricing models. We obtain 157 sets of pricing parameter estimates for each model by repeating the calibration on the sample period. The parameters in the GJR-GARCH model stated in Eq. (1) are estimated using 3,500 historical S&P 100 log-returns on Wednesdays. We can obtain the scaled innovation z_t by dividing each estimated innovation $\hat{\varepsilon}_t$ by its corresponding conditional volatility $\hat{\sigma}_t$ such that we have one corresponding set of pricing parameter estimates and 3,500 scaled innovations on each Wednesday. The scaled innovations exhibit some non-normal characteristics such as excess skewness and kurtosis. For instance, Figure 2 depicts the S&P 100 log-returns, the conditional GARCH volatility σ_t on an annual basis, and the scaled innovations from 15 June 1989 to 30 April 2003. The Jarque-Bera test indicates that the scaled innovation is significantly non-normal at the 99% confidence level. The pricing parameters in BEM (2008) and our modified model are estimated using the FHS method described in Section 2.1.2 based on 20,000 simulated paths. When calibrating the pricing models, we add a stationary restriction, i.e., $\beta^* + \alpha^* + D\gamma^* < 1$, to the GJR-GARCH model, where D is the probability of the event when $\varepsilon_t < 0$ occurs over the historical 3,500 return series.[7]

Longstaff and Schwartz (2001) contend that there is nearly no difference in pricing performance when choosing Fourier, trigonometric series or simple ordinary polynomials as the basis function. Moreover, according to their tests, using three nonlinear functions of the stock price as the basis function is sufficient to obtain effective convergence for an American option. Stentoft (2004) considers the trade-off between time efficiency and calculation accuracy and argues that using 2nd- or 3rd-order simple ordinary polynomials in LSM is optimal. Therefore, we use the 2nd-order simple ordinary polynomials as the basis function in the LSM method to calculate American option prices.[8]

Table 3 summarizes the statistics for the GJR-GARCH pricing parameter estimates and the historical GJR-GARCH parameter estimates. The leverage parameter estimates in the FHS model and our modified model are both higher than those in GJR-GARCH models under the $\mathbb{P}$ measure, which implies that the investors in the option markets expect greater compensation when the underlying asset declines. As the result, if we directly use the GJR model under the $\mathbb{P}$ measure to evaluate option prices, we are likely to underestimate the volatility and make serious pricing errors. These findings suggest the use of the calibrated model to estimate the pricing parameters, although it is a relatively time-consuming method, instead of simply obtaining parameters under the $\mathbb{P}$ measure. However, the estimates of the β parameter in the original FHS model and our modified model are both only somewhat smaller than that in GJR-GARCH model under the $\mathbb{P}$ measure. Based on the above results, the volatility clustering phenomenon in the risk-neutral world appears to be less pronounced than

[7] Since the pricing method we proposed in this study largely relies on a market calibration to the pricing parameters, the benchmark method considered here should be in the same manner. In addition, this method is modified from BEM (2008). Therefore, we only take the original BEM (2008) method as one to be compared with.

[8] Third Laguerre polynomials and the 3rd-order simple ordinary polynomials are used to perform robustness checks; they lead to similar results.

that in the real world.

When comparing estimation results obtained from the original calibration method and the modified calibration method, the major difference lies in the initial conditional volatility (Table 3). Recall that the initial conditional volatility υ_{t_0} in Eq. (2) is set to be the last conditional volatility (denoted σ_{t_0}) under the $\mathbb{P}$ measure (i.e., the 3,500th conditional volatility estimated by the historical GJR-GARCH model) in BEM (2008) and is estimated in the calibration process in this study. Table 3 indicates that the average $\upsilon_{t_0}^2$ is substantially higher than $\sigma_{t_0}^2$ in 2003 and is much lower than $\sigma_{t_0}^2$ in 2005. There is a smaller difference relative to 2004. However, the paired-sample *t*-test indicates that the three-year differences between $\sigma_{t_0}^2$ and $\upsilon_{t_0}^2$ are all significant at the 1% significance level.

To conclude the above analysis, the initial conditional volatility on each Wednesday in the risk-neutral world was different from that found in the real world. Failing to distinguish $\sigma_{t_0}^2$ from $\upsilon_{t_0}^2$, and incorrectly using $\sigma_{t_0}^2$ as a proxy for $\upsilon_{t_0}^2$ may exacerbate modeled risks as a result of material pricing errors. In the next subsection, we will demonstrate that this is the case.

4. Model Performance

4.1. In-Sample Comparison

We report the in-sample dollar Root-Mean-Square Error (RMSE) in Table 4. The RMSE is the square root of the average squared difference between the model and market prices. Panel A in Table 4 indicates that the overall performance of our modified FHS model is superior to the original FHS model for both the OTM and ITM options. The matched sample *t*-tests further confirm that the superiority is statistically significant at the 1% significance level. Panel B in Table 4 is the RMSE in years. The superiority of the modified FHS model varies over the period considered. The modified FHS model has an absolute advantage relative to the original FHS model for 2003 and 2005. The two methods yield similar results for 2004.

In this paper, we emphasize that the initial conditional volatility in the risk-neutral world υ_{t_0} should not be replaced with the initial conditional volatility in the real world σ_{t_0}. Consequently, a larger difference between υ_{t_0} and σ_{t_0} might induce more serious pricing errors. In accordance with our findings in the last section, there is a substantial difference between $\sigma_{t_0}^2$ and $\upsilon_{t_0}^2$ in the years 2003 and 2005. There is a reasonable explanation for why the modified model has an absolute advantage over the original FHS model for those two years.

To closely examine how the difference between $\sigma_{t_0}^2$ and $\upsilon_{t_0}^2$ can cause a serious pricing error, we randomly take two Wednesdays (29 January 2003 and 28 January 2004) as illustrations. Specifically, 29 January 2003 presents a substantial difference between $\sigma_{t_0}^2$ and $\upsilon_{t_0}^2$, and 28 January 2004 is a day with small differences between $\sigma_{t_0}^2$ and $\upsilon_{t_0}^2$. It is obvious that future volatility is a primary factor determining the current option price. Therefore, we compare the forecasted volatilities of the GJR-GARCH model for the two selected cases. In a GJR-GARCH model under the $\mathbb{Q}$ measure (Eq. (2)), the next-period forecasted variance conditional on time t_0 is

$$\upsilon_{t_0+1|t_0}^2 = \omega^* + \beta^* \upsilon_{t_0}^2 + \alpha^* \varepsilon_{t_0}^2 + \gamma^* I_{t_0} \varepsilon_{t_0}^2 . \tag{10}$$

The h-period (h = 2, 3 …) forecasted variance conditional on time t_0 is

$$, \upsilon_{t_0+h|t_0}^2 = \upsilon^2 + (\alpha^* + D\gamma^* + \beta^*)^{h-1}(\upsilon_{t_0+1|t_0}^2 - \upsilon^2) \tag{11}$$

where

$$\upsilon^2 = \omega^*(1 - \alpha^* - D\gamma^* - \beta^*)^{-1} \tag{12}$$

in the long-run, or unconditional variance. We utilize the forecasting formulae (eqs. (10)-(12)) to obtain the forecasted conditional variance from day 1 to day 500 (h = 1, 2, ..., 500).

Figures 3 and 4 present the results for 29 January 2003 (with a larger difference between $\sigma_{t_0}^2$ and $\upsilon_{t_0}^2$) and 28 January 2004 (with a smaller difference between $\sigma_{t_0}^2$ and $\upsilon_{t_0}^2$), respectively. Figures 3 and 4 indicate that initial volatility is a crucial value for forecasting future volatility. The difference in future volatility between the original FHS and the modified FHS is higher if there is a greater difference between $\sigma_{t_0}^2$ and $\upsilon_{t_0}^2$ (Figure 3). Figure 3 indicates that the difference grows as the forecasting time increases. An inaccurate initial value could result in inaccurate future volatilities, which would lead to mispriced options. This conjecture can further be confirmed by presenting the calibration results of both models on the two Wednesdays in Figures 5 and 6. In Figure 5, it is clear that the modified model overwhelmingly outperforms the original FHS model on 29 January 2003 because the RMSE of the modified FHS model is only 0.47, which is far smaller than that in the original FHS model, 3.07. Figure 6 demonstrates that there is almost no difference between pricing performances across models on 28 January 2004 because the RMSE of the modified FHS model is 0.33, which is slightly smaller than that of the original FHS model, 0.34.

Therefore, we argue that restricting the initial volatility to an improper value under the $\mathbb{Q}$ measure would lead to a serious mispricing of the options; however, our modified FHS model can easily solve this problem.

4.2. Out-of-Sample Comparison

To illustrate how the calibrated pricing model can be used for future forecasting, the in-sample model estimates for each Wednesday are used to price OEX options one week after by using the asset prices, time to maturities, and corresponding interest rate on the next Wednesday. In BEM (2008), the initial volatility on the next Wednesday still comes from the empirical estimate under the $\mathbb{P}$ measure. In the modified FHS model, we adopt a one-week period (the 5th) forecasted conditional volatility as the initial conditional volatility in the forecasting procedure. Formally, the initial variance on the next Wednesday is set to

$$\upsilon^2_{t_0+5|t_0} = \upsilon^2 + (\alpha^* + D\gamma^* + \beta^*)^4 (\upsilon^2_{t_0+1|t_0} - \upsilon^2)\,. \tag{13}$$

Based on the observed asset price on the next Wednesday and the forecasted initial conditional volatility $\upsilon_{t_0+5|t_0}$, we can generate future asset price paths using the GJR-GARCH model obtained this Wednesday. The model prices of the options on the following Wednesday can be obtained via the Monte Carlo simulation method.

Table 4 also presents the out-of-sample results based on RMSE under both pricing models. The advantage of the modified FHS model is pronounced. For example, the overall out-of-sample RMSE of the modified FHS model is 0.86, which is lower than that in the original FHS model, 1.28. The matched sample *t*-test reveals that this advantage is significant at the 5% level. We further study whether model performance varies across different contracts. We find that the modified FHS model outperforms the original FHS model for both OTM and ITM options. Panel B in Table 4 also reports the out-of-sample RMSE by year. Our general findings are similar to those of the in-sample analysis. The modified FHS easily outperforms the original FHS in 2003 and 2005. The two methods lead to similar results in 2004.

From a modeling perspective, including additional parameters in a model may result in problems of excessive in-sample fitting and poor out-of-sample forecasting. Based on the empirical results in our study, we can reasonably claim that our model achieves a high level of pricing performance without the problem of over-fitting.

4.3. The OTM and ITM Contracts

In the previous analysis, we use both the OTM and ITM option prices to calibrate pricing dynamics. The set of pricing parameters calibrated by using all of the contracts is denoted θ^*. In this section, we study the effects of modeling errors when the ITM options are excluded from the calibration of American contracts. Our approach is to repeat our modified pricing models while only considering the OTM option prices when calibrating the models. The set of pricing parameters calibrated using only the OTM contracts is denoted θ^*_{OTM}. The estimation results are presented in Table 5. Table 5 indicates that θ^*_{OTM} is similar to θ^* except for α^*. The α^* in the model that exclusively considers the OTM options is higher than that in the model using all of the contracts. Based on the above results, the volatility clustering phenomenon in the risk-neutral world appears to be overemphasized if only the OTM options are considered.

Table 6 reports the in-sample and out-of-sample RMSE for both θ^* and θ^*_{OTM} under the modified FHS model. We also measure the performance of θ^*_{OTM} with respect to ITM contracts for comparison. Table 6 reports that θ^*_{OTM} exhibits better in-sample performance for the OTM contracts but poorer in-sample performance for the ITM contracts. This finding suggests that neither the OTM nor ITM contract is redundant for pricing American-style options. Ignoring one of them may induce serious pricing errors. A possible explanation for this result is that the ITM option prices contain information that is not reflected in the OTM option prices. This result is consistent with our discussion in Section 2.3. The overall in-sample performance of θ^* for both the OTM and ITM contracts remains superior to that of θ^*_{OTM} at the 10% significance level.

The out-of-sample results further confirm the necessity of incorporating the ITM contracts into the calibration process. The θ^* parameter exhibits better forecasting ability than the θ^*_{OTM} for the ITM contracts, and the *t*-test reveals the significance over our sample period. The θ^*_{OTM} appears to offer better forecasting ability than θ^* for the OTM contracts, however, the *t*-test reveals that there is nearly no difference in forecasting performance for OTM option prices between θ^*_{OTM} and θ^*. Although θ^* is dropped for the in-sample modeling for the OTM contracts (to allow us to consider both OTM and ITM contracts), θ^* delivered sufficient information for forecasting both the OTM and ITM contracts.

In light of the above analysis, we conclude that both the OTM and the ITM contracts should be included in the calibration process to obtain an accurate and robust result.

5. Conclusion

Because practitioners and academics frequently employ GARCH option pricing models, this study analyzes the market calibration problem of American option pricing when the underlying asset follows a GARCH process. We present a pricing method based on the FHS approach proposed by BEM (2008). We modify the FHS to reflect the possibility that the stochastic processes of an asset in the real world and in the risk-neutral world may differ. This implies that the volatility processes may differ. However, the assumption that the two processes have the same initial conditional volatility, as imposed by BEM (2008), appears inconsistent with and in violation of the above inference. In this study, the initial conditional volatility in the modified FHS is determined by the options market. This modification can solve the problem posed by BEM (2008). An extensive empirical analysis based on S&P 100 index options reveals that the modified FHS outperforms the original FHS in both in-sample and out-of-sample data. We also suggest that both the OTM and ITM options are necessary when establishing a pricing model for American-style options.

Acknowledgements

Tsao is grateful for the research support from NSC grant No. NSC95-2416-H-182-005-MY3.

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Appendix A. Tables

Table 1. Database description[1]

Moneyness K/S		Maturity: Less than 60 Mean	std. dev.	60 to 160 Mean	std. dev.	More than 160 Mean	std. dev.
> 1 (ITM)	Put price $	19.69	12.23	26.28	10.09	33.69	7.66
	Observations	1063		392		162	
< 1 (OTM)	Put price $	2.04	2.86	4.47	4.98	7.69	7.40
	Observations	5252		3031		1136	
< 1 (ITM)	Call price $	29.35	22.58	44.75	34.97	76.50	49.78
	Observations	1706		694		330	
> 1 (OTM)	Call price $	2.32	3.11	4.93	5.27	8.58	8.33
	Observations	3028		1763		725	

[1]Mean, standard deviation (Std. dev.), and number of observations for each moneyness/maturity category of OEX options observed on Wednesdays from 2 January 2003 to 30 December 2005; the market price is the average of the bid and ask prices. Moneyness is the strike price divided by the underlying asset price, *K/S*. A put option is said to be out-of-the-money (OTM) if *K/S* > 1 or in-the-money (ITM) if *K/S* < 1. A call option is said to be out-of-the-money (OTM) if *K/S* < 1 or in-the-money (ITM) if *K/S* > 1.

Table 2. Summary Statistics of the S&P 100[1]

Mean	Median	Std. Dev.	Skewness
0.03%	0.04%	0.83%	-0.26%
Kurtosis	Jarque-Bera/*p*-value	ACF(1)/*p*-value	ARCH Test/*p*-value
4.47	68.41/0.00	-0.12/0.00	16.10/0.00

[1] Mean, median, standard deviation (Std. dev.), skewness, and kurtosis for S&P 100 returns from 2 January 2003 to 30 December 2005. Jarque-Bera is a test for normality. ACF(1) refers to the first order auto-correlation. ARCH test is the LM test for heteroscedasticity with 2 lag periods.

Table 3. Estimation and calibration of the GJR-GARCH model

GJR[1]	$\omega \times 10^6$		β		$\alpha \times 10^3$		γ			
Year	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.		
2003	1.18	0.16	0.94	0.01	4.34	1.83	0.10	0.01		
2004	1.09	0.05	0.93	0.00	6.21	1.60	0.10	0.00		
2005	1.00	0.05	0.94	0.00	2.34	2.05	0.10	0.00		
FHS[2]	$\omega^* \times 10^6$		β^*		$\alpha^* \times 10^3$		γ^*		$\sigma_{t_0}^2 \times 10^4$	
Year	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
2003	1.68	1.31	0.92	0.06	3.89	2.77	0.13	0.11	1.14	0.04
2004	0.86	0.23	0.93	0.01	6.45	1.99	0.12	0.02	0.60	0.18
2005	0.68	0.31	0.92	0.02	2.51	2.34	0.13	0.04	0.50	0.13
FHS	$\omega^* \times 10^6$		β^*		$\alpha^* \times 10^3$		γ^*		$\upsilon_{t_0}^2 \times 10^4$	
Year	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
2003	1.24	0.59	0.93	0.01	3.88	2.38	0.11	0.02	1.51 (0.0044)[4]	0.99
2004	0.86	0.22	0.92	0.01	5.88	2.29	0.13	0.02	0.54 (0.0019)	0.15
2005	0.80	0.26	0.91	0.01	2.47	2.51	0.15	0.03	0.30 (0.0000)	0.11

[1] The parameter estimates of the GJR-GARCH model on each Wednesday from January 2003 to December 2004 derived using 3500 historical S&P100 log-returns.

[2] The pricing parameter estimates of the model developed by BEM (2008) (i.e., the FHS), calibrated using the cross-section of OTM and ITM options on each Wednesday from January 2003 to December 2004, where we use the last conditional volatility under the $\mathbb{P}$ measure $\sigma_{t_0}^2$ as the initial conditional volatility in the simulation.

[3] The pricing parameter estimates of our new model (i.e., the modified FHS), calibrated using the cross-section of OTM and ITM options on each Wednesday from January 2003 to December 2004, where we introduce the new parameter $\upsilon_{t_0}^2$ as the initial conditional volatility in the simulation.

[4] The values in parentheses are the *p*-values of paired *t*-tests: H_0: $\sigma_{t_0}^2 = \upsilon_{t_0}^2$ vs. H_a: $\sigma_{t_0}^2 \neq \upsilon_{t_0}^2$.

Table 4. In-sample & out-of-sample pricing errors between the FHS & modified FHS models

	RMSE[1]					
	Total		ITM		OTM	
	in-sample	out-of-sample	in-sample	out-of-sample	in-sample	out-of-sample
Panel A: Aggregate valuation error across all year						
FHS	0.99	1.28	1.35	1.63	0.85	1.15
FHS (modified)	0.55	0.86	0.86	1.15	0.41	0.75
	(0.00) [2]	(0.03)	(0.00)	(0.02)	(0.00)	(0.04)
Panel B: Valuation errors by years						
2003						
FHS	1.37	1.89	1.89	2.38	1.18	1.72
FHS (modified)	0.53	0.90	0.88	1.16	0.36	0.81
2004						
FHS	0.72	0.84	1.04	1.15	0.59	0.72
FHS (modified)	0.61	0.94	0.94	1.29	0.47	0.80
2005						
FHS	0.75	0.84	0.95	1.06	0.68	0.77
FHS (modified)	0.48	0.69	0.72	0.95	0.39	0.60

[1] RMSE is the square root of the average squared difference between the model price and the market price.

[2] The values in parentheses are the *p*-values of the paired *t*-tests to determine whether our modified model outperforms the original FHS model.

Table 5. Summary of pricing parameter estimates calibrated using different sets of option contracts

θ^*_{OTM} [1]	$\omega^* \times 10^6$		β^*		$\alpha^* \times 10^3$		γ^*		$\upsilon^2_{t_0} \times 10^4$	
Year	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
2003	1.23	0.53	0.93	0.01	4.03	2.36	0.11	0.02	1.51	0.99
2004	0.76	0.22	0.92	0.01	6.44	2.54	0.13	0.02	0.54	0.15
2005	0.69	0.24	0.91	0.01	2.54	2.44	0.16	0.03	0.30	0.11
θ^*	$\omega^* \times 10^6$		β^*		$\alpha^* \times 10^3$		γ^*		$\upsilon^2_{t_0} \times 10^4$	
Year	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
2003	1.24	0.59	0.93	0.01	3.88	2.38	0.11	0.02	1.51	0.99
2004	0.86	0.22	0.92	0.01	5.88	2.29	0.13	0.02	0.54	0.15
2005	0.80	0.26	0.91	0.01	2.47	2.51	0.15	0.03	0.30	0.11

[1] θ^*_{OTM} is the set of the pricing parameter estimates calibrated using only OTM option prices, and θ^* is calibrated using both OTM & ITM option prices.

Table 6. Cross test of using the different pricing parameter estimates

RMSE

	Total		ITM		OTM	
	in-sample	out-of-sample	in-sample	out-of-sample	in-sample	out-of-sample
Panel A: Aggregate valuation error across all year						
θ^*_{OTM} [1]	0.56	0.86	0.96	1.20	0.36	0.73
θ^*	0.55	0.86	0.86	1.15	0.41	0.75
	(0.08) [2]	(0.46)	(0.00)	(0.00)	(0.00)	(0.16)
observations	19282	18862	4347	4242	14935	14620
Panel B: Valuation errors by years						
2003						
θ^*_{OTM}	0.55	0.89	0.98	1.18	0.35	0.78
θ^*	0.53	0.90	0.88	1.16	0.36	0.81
observations	6206	6084	1386	1358	4820	4726
2004						
θ^*_{OTM}	0.62	0.95	1.05	1.35	0.41	0.80
θ^*	0.61	0.94	0.94	1.29	0.47	0.80
observations	7071	6921	1658	1613	5413	5308
2005						
θ^*_{OTM}	0.47	0.70	0.82	1.01	0.31	0.59
θ^*	0.48	0.69	0.72	0.95	0.39	0.60
observations	6005	5857	1303	1271	4702	4586

[1] θ^*_{OTM} is the set of pricing parameter estimates calibrated using only OTM option prices, and θ^* is calibrated using both OTM & ITM option prices.
[2] The values in parentheses are the *p*-values of paired *t*-tests to determine whether the parameters θ^* outperform θ^*_{OTM}.

Appendix B. Figures

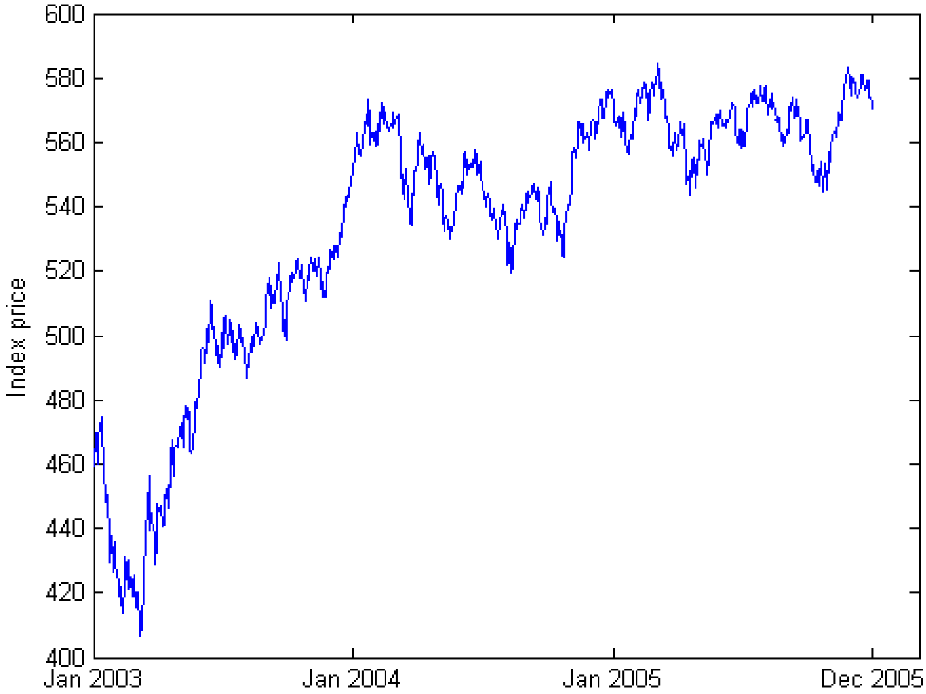

Figure 1. The S&P 100 index from 2 January 2003 to 30 December 2005

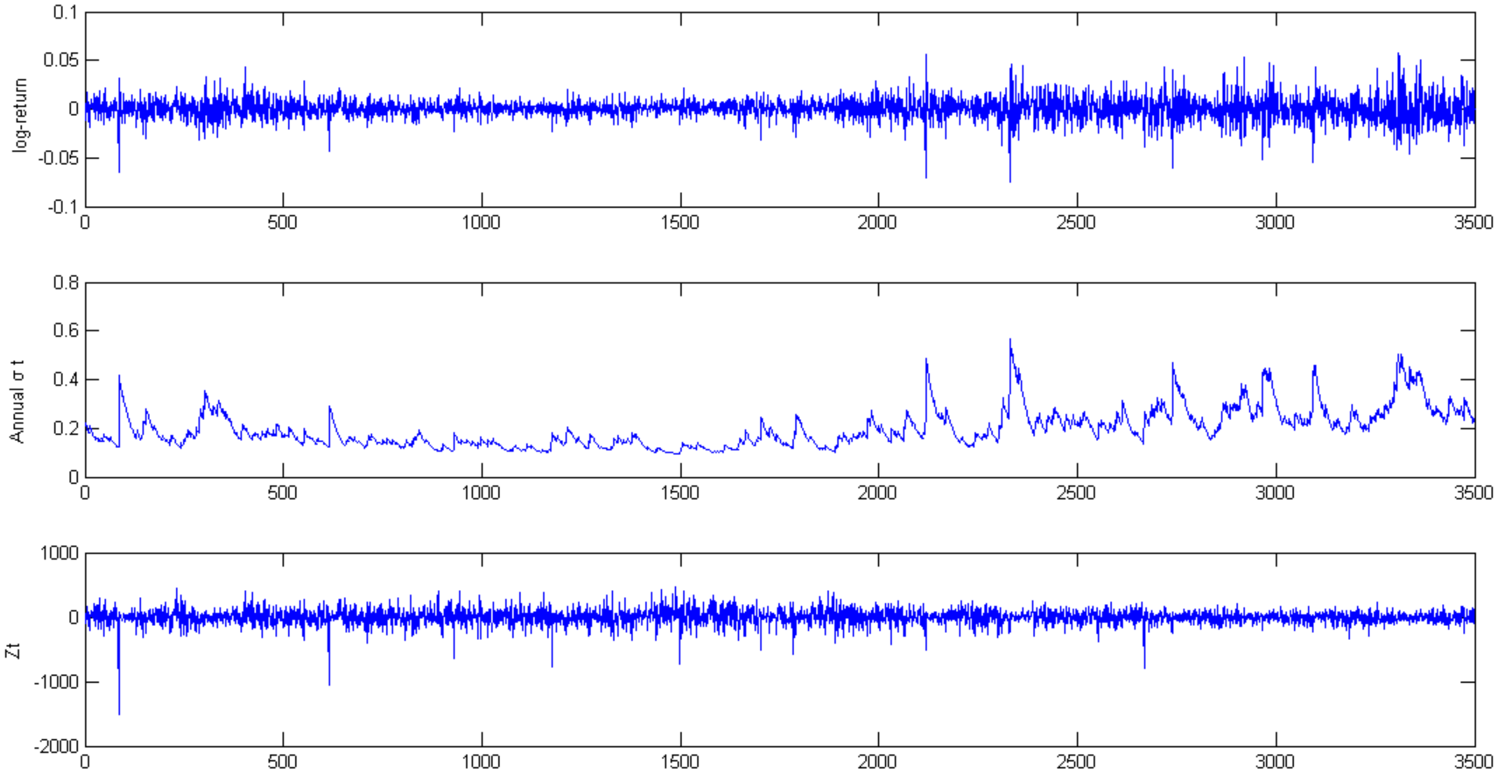

Figure 2. Daily log-returns of the S&P 100 index from 15 June 1989 to 30 April 2003 (3500 log-returns), annual conditional volatility σ_t, and scaled innovations Z_t.

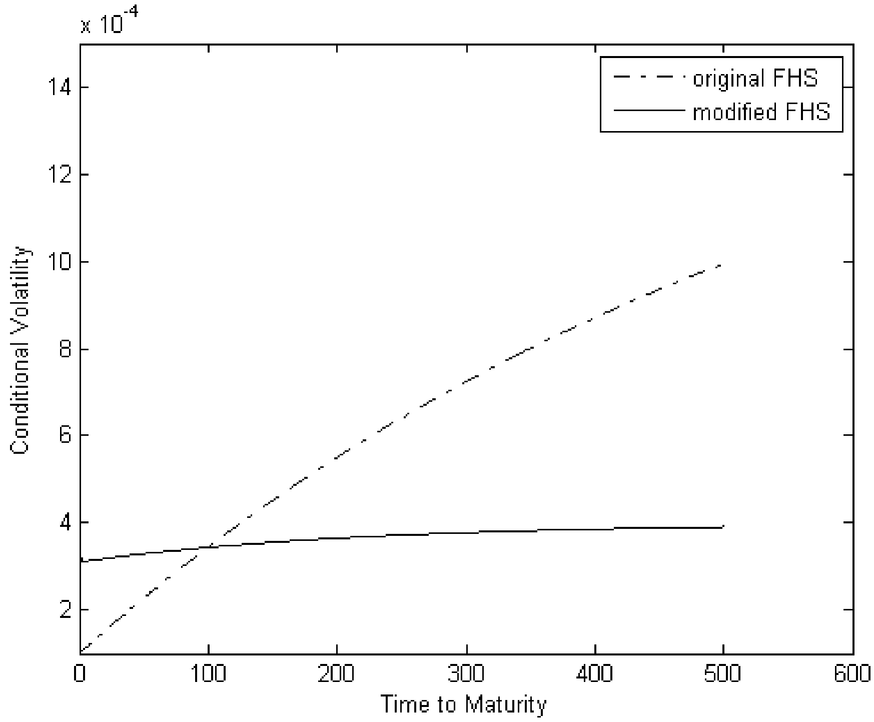

Figure 3. Forecasted volatility based on the pricing parameter estimates of both models on 29 January 2003. The initial volatilities are 0.000113 in the original FHS model and 0.000335 in the modified FHS model.

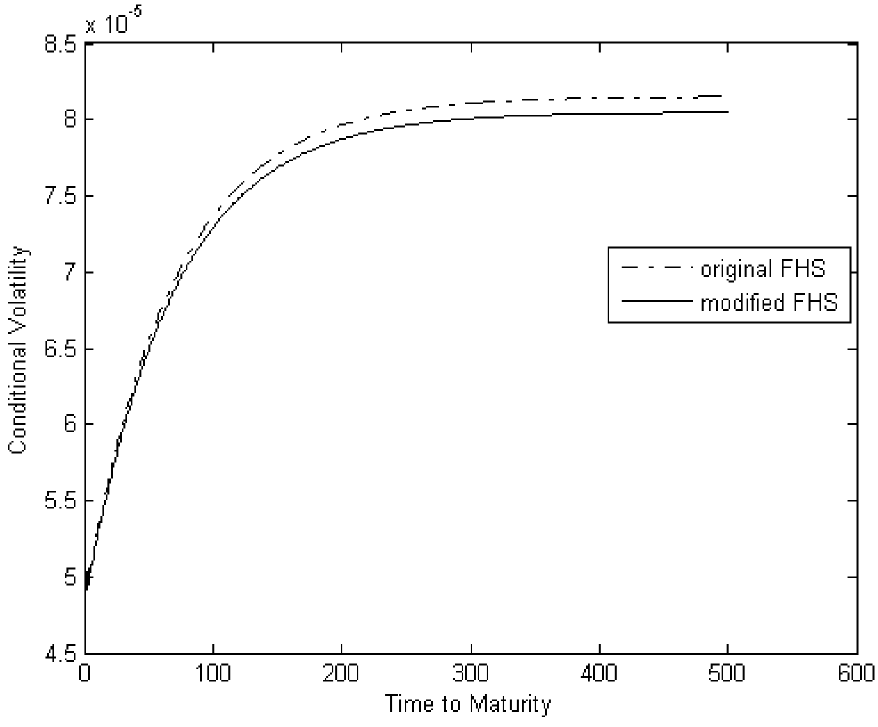

Figure 4. Forecasted volatility based on the pricing parameter estimates of both models on 28 January 2004. The initial volatilities are 0.00005 in both the original FHS model and the modified FHS model.

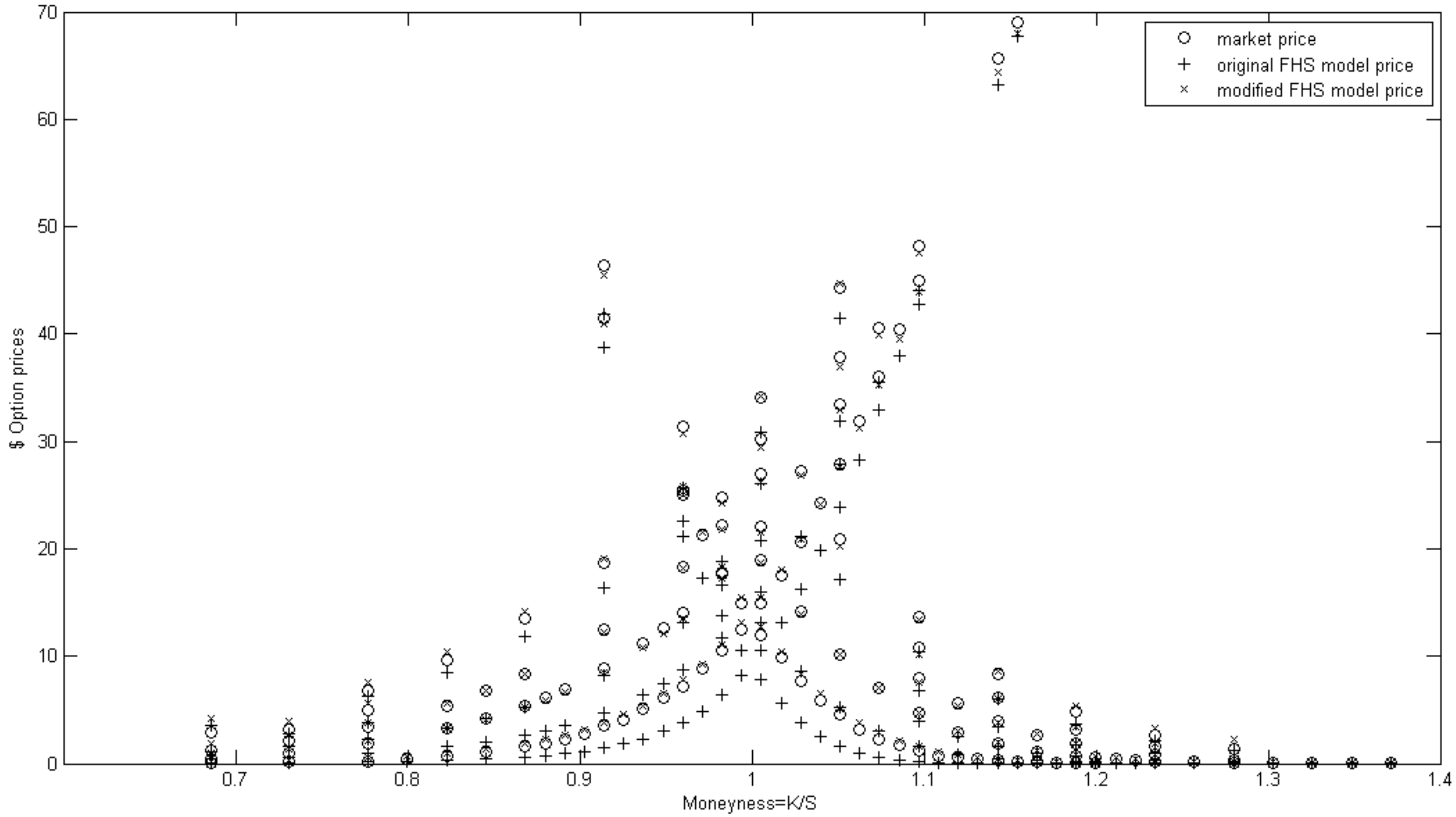

Figure 5. Calibration results of the original FHS and the modified FHS models on the cross-section of OEX options on 29 January 2003. Moneyness is the strike price *K* divided by the underlying asset price *S*.

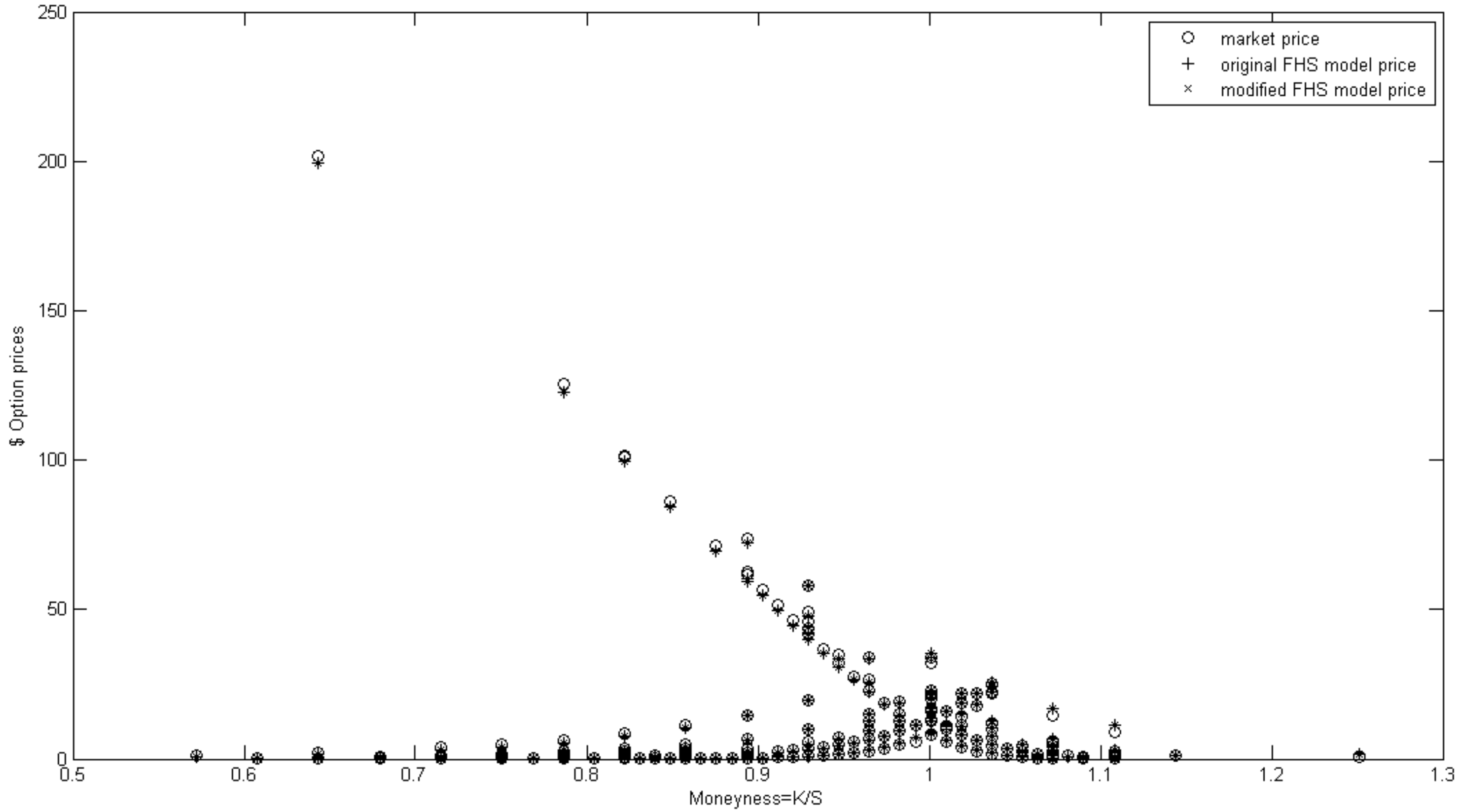

Figure 6. Calibration results of the original FHS and the modified FHS models on the cross-section of OEX options on 28 January 2004. Moneyness is the strike price *K* divided by the underlying asset price *S*.