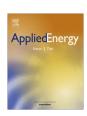


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## Short-term wind speed prediction using an unscented Kalman filter based state-space support vector regression approach



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#### HIGHLIGHTS

- A novel hybrid modeling method is proposed for short-term wind speed forecasting.
- Support vector regression model is constructed to formulate nonlinear state-space framework.
- Unscented Kalman filter is adopted to recursively update states under random uncertainty.
- The new SVR-UKF approach is compared to several conventional methods for short-term wind speed prediction.
- The proposed method demonstrates higher prediction accuracy and reliability.

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#### ABSTRACT

Accurate wind speed forecasting is becoming increasingly important to improve and optimize renewable wind power generation. Particularly, reliable short-term wind speed prediction can enable model predictive control of wind turbines and real-time optimization of wind farm operation. However, this task remains challenging due to the strong stochastic nature and dynamic uncertainty of wind speed. In this study, unscented Kalman filter (UKF) is integrated with support vector regression (SVR) based state-space model in order to precisely update the short-term estimation of wind speed sequence. In the proposed SVR–UKF approach, support vector regression is first employed to formulate a nonlinear state-space model and then unscented Kalman filter is adopted to perform dynamic state estimation recursively on wind sequence with stochastic uncertainty. The novel SVR–UKF method is compared with artificial neural networks (ANNs), SVR, autoregressive (AR) and autoregressive integrated with Kalman filter (AR-Kalman) approaches for predicting short-term wind speed sequences collected from three sites in Massachusetts, USA. The forecasting results indicate that the proposed method has much better performance in both one-step-ahead and multi-step-ahead wind speed predictions than the other approaches across all the locations.

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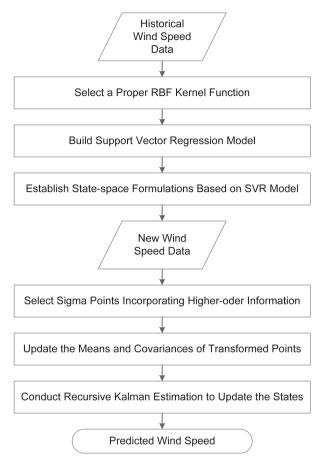
#### 1. Introduction

Green wind power is one of the promising renewable energy sources to substitute traditional coal and fossil fuel based power generation with mitigated carbon footprint and environmental impact. Due to its renewable nature and environmental friendliness, wind energy has received fast growing attention throughout the world and the utilization of wind power has increased dramatically over the past decade. For instance, the construction of new wind power generation capacity in the first three quarters of 2012 was 4728 MW in total and the cumulative wind power capacity in the United States was increased to 51,630 MW [1]. However, significant intermittency and stochastic fluctuation of wind speed

pose great challenges to controlling wind turbines and optimizing wind farm operation towards reliable wind power generation [2]. Therefore, it is crucially important to accurately forecast wind speed so that the model based optimal control of wind turbines can be achieved with stabilized wind power output. Specifically, long-term wind speed forecasting is important for optimizing the site selection and production planning of wind farms, while short-term prediction is vital for controlling wind turbines and improving their power generation efficiency and life span [3–6].

In literature study, the methods developed for wind speed prediction can be divided into two main categories: physical model based approaches and statistical modeling methods. As one type of physical model based approaches, numerical weather prediction (NWP) techniques rely on a class of physical models with numerical parameters characterizing local meteorological and geographical properties such as temperature, atmospheric pressure, surface roughness and obstacles [7–9]. Nevertheless, the prediction

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 $\pmb{\text{Fig. 1.}}$  Schematic diagram of the proposed SVR-UKF approach for wind speed prediction.

**Table 1**Comparison of RMSE values in wind speed prediction using ANN, AR, AR-Kalman, SVR and SVR-UKF methods.

RMSE (m/s)	Blandford		Chester		Falmouth	
	1-Step	5-Step	1-Step	5-Step	1-Step	5-Step
ANN AR AR-Kalman SVR SVR-UKF	0.7534 0.7341 0.2849 0.3336 0.2123	0.8622 1.1846 0.5291 0.4808 0.3337	0.9359 0.9114 0.3427 0.2492 0.1502	1.1295 1.4992 0.6918 0.5180 0.4266	0.7440 0.5839 0.2290 0.2834 0.1843	0.9991 1.0788 0.5211 0.5827 0.4055

**Table 2**Comparison of MAPE values in wind speed prediction using ANN, AR, AR-Kalman, SVR and SVR-UKF methods.

MAPE (%)	Blandford		Chester		Falmouth	
	1-Step	5-Step	1-Step	5-Step	1-Step	5-Step
ANN	13.14	15.61	14.10	17.77	16.69	23.03
AR	12.86	21.79	12.36	21.73	10.58	21.05
AR-Kalman	4.98	9.41	4.84	9.54	4.28	9.72
SVR	5.75	9.72	3.42	10.19	4.10	10.36
SVR-UKF	3.57	6.72	2.07	5.68	2.88	7.51

capability of NWP methods degrade significantly when the random uncertainty of weather conditions is strong. In practice, physical models are often utilized through integration with statistical modeling methods in order to combine the advantages of two different types of techniques while mitigate the restrictions of NWP methods [10–12].

Since physical models alone may not well capture the stochastic nature of wind speed, statistical modeling methods are developed for wind speed prediction with some success. Different from physical models, statistical methods depend on historical wind speed

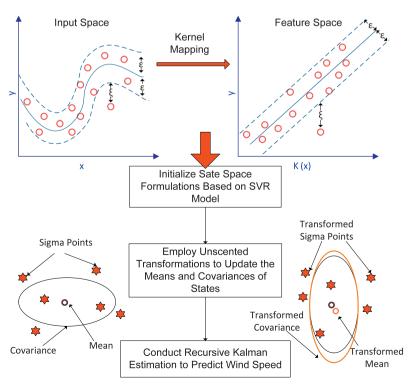


Fig. 2. Graphic illustration of the proposed SVR-UKF approach for wind speed prediction.

Table 3 Comparison of  $\mathbb{R}^2$  values in wind speed prediction using ANN, AR, AR-Kalman, SVR and SVR-LIKE methods

$R^2$	Blandford		Chester		Falmouth	
	1-Step	5-Step	1-Step	5-Step	1-Step	5-Step
ANN	0.7985	0.7360	0.8336	0.7577	0.8350	0.7024
AR	0.8086	0.5017	0.8422	0.5731	0.8984	0.6530
AR-Kalman	0.9712	0.9006	0.9777	0.9074	0.9844	0.9190
SVR	0.9605	0.9178	0.9882	0.9490	0.9760	0.8988
SVR-UKF	0.9840	0.9615	0.9957	0.9654	0.9899	0.9510

data for dynamic predictions without prior knowledge of physical mechanism underlying wind speed pattern. Time-series models and artificial neural network (ANN) models are two main kinds of statistical modeling methods that have been used for wind speed prediction [13]. Among time-series modeling methods, autoregressive moving average (ARMA) models have gained substantial attention in predicting wind speed sequence and wind power output [14,15]. Several variants of ARMA models have been developed for forecasting wind speed and direction, including the simplified autoregressive (AR), autoregressive integrated moving average (ARIMA), ARMA or AR with exogenous input (ARMAX or ARX), fractional-ARIMA with the differencing parameter being a fractional number [16–18]. However, AR, ARMA and ARIMA models require high model orders so as to accommodate stochastic variations of wind speed sequence. Furthermore, these time-series models are

essentially linear so that they may not be well suited for characterizing the stochastic nature and uncertain dynamics of wind speed. Alternatively, ANN is a nonlinear modeling technique for wind speed prediction and it basically maps a random input vector into the corresponding random output scalar or vector through multilayer network structure without presuming any physical relationship [19]. Heuristic algorithms such as genetic algorithm (GA) and particle swarm optimization (PSO) are employed to optimize parameters in ANN [20]. Moreover, geographic parameters can be incorporated into ANN model to predict the monthly average wind speed [21]. Another attempt is to design a generalized feed-forward type of ANN for predicting the probability density of annual wind speed based on the parameters of Weibull function [22]. So far, significant effort has been reported to develop different types of neural networks, improve the network structure, optimize the number of neurons and activation functions, and design network learning algorithms in order to conduct more accurate prediction [23-27]. Though ANN techniques are capable of handling wind speed sequence with nonlinearity, its generalization ability is not guaranteed so that a well trained ANN model may lead to poor prediction performance for new observations. In addition to time-series and ANN models, support vector regression (SVR) method has been applied to predict wind speed through a nonlinear kernel function based predictive model within high-dimensional feature space [28–30]. As opposed to ANN method, SVR approach effective overcomes the drawbacks of model over-fitting and poor generalization. Thus it has desired characteristics such as global optimal

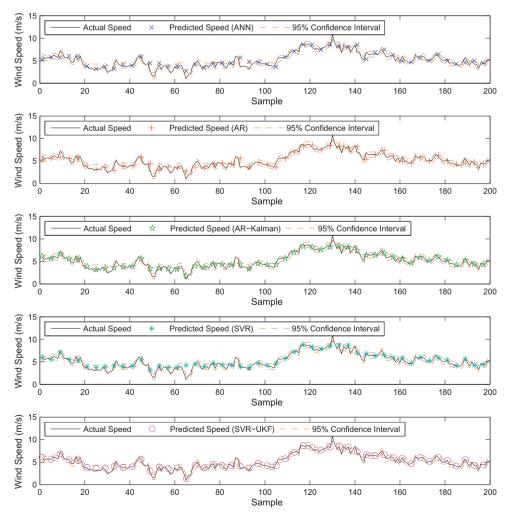


Fig. 3. One-step-ahead wind speed prediction results of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Blandford, MA.

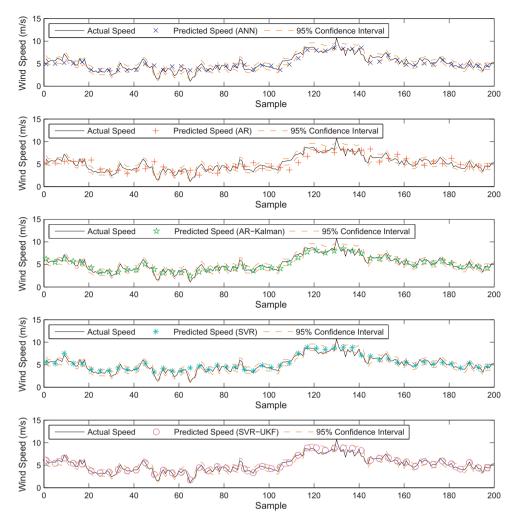


Fig. 4. Five-step-ahead wind speed prediction results of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Blandford, MA.

solution and strong generalization capability. However, SVR technique itself does not have strong capacity to handle system uncertainty and stochastic nature.

More recently, different types of statistical methods are integrated to enable more accurate and reliable wind speed prediction. For instance, wavelet analysis is employed to decompose original time-series of wind speed into different sub-series and then combined with improved time series method (ITSM) to forecast wind speed and wind power generation [31]. Nevertheless, the way of decomposing wind series can be quite empirical so that the optimal prediction performance is obtained by ad-hoc. Meanwhile, AR-IMA model is applied to build the state-space formulations for Kalman filter based wind speed sequence estimation and update [32]. Nevertheless, Kalman filter relies on the basic assumption of Gaussian disturbance, which may not hold for actual wind speed sequence. In addition, Markov chain (MC) is attempted to modify the predicted horizons of ANN method according to long-term patterns of wind speed data so as to avoid the over-fitting issue of ANN model in short-term wind speed forecasting [33]. However, dividing the wind speed variation into different states for MC is arbitrary and lack of systematic feature. Typically, a hybrid model tends to yield more accurate prediction results than a single kind of method. On the other hand, a proper integration of different types of statistical modeling techniques to obtain the optimal prediction performance is not a trivial task.

In addition to physical and statistical models, spatial correlation model provides an alternate way for wind speed prediction. Different from statistical approaches that only take into consideration the historical wind speed measurements, spatial correlation models use the cross-correlation information of wind speed between different geographic locations. Based on the assumption that wind speed measurements in neighboring sites are correlated, the wind speed data at neighboring sites are employed to predict the local wind speed sequence [34,35]. Moreover, a local recurrent multilayer network model is employed to carry out wind speed and power forecasting by considering spatial correlation [36]. Another attempt to predict monthly average wind speed is achieved by a ANN model with resilient propagation, which obtains prediction by using the wind speed in neighboring sites termed as reference stations [37]. Similarly, a spatial correlation model based on prognostic method with exergy analysis is proposed to predict timeseries of wind speed [3]. Spatial correlation models are advantageous in wind speed prediction among multiple wind farms that are geographically close to each other. However, extremely large number of wind speed measurements from multiple related sites are needed for developing the spatial correlation models.

In this article, a novel hybrid predictive modeling approach is proposed by integrating unscented Kalman filter with support vector regression based nonlinear state-space model framework in order to enhance the capacity of handling stochastic and dynamic uncertainty as well as minimize the errors of multi-step-ahead wind speed forecasting. First, support vector regression is employed to formulate a kernel function based nonlinear state-space model structure. Then, unscented Kalman filter instead of regular

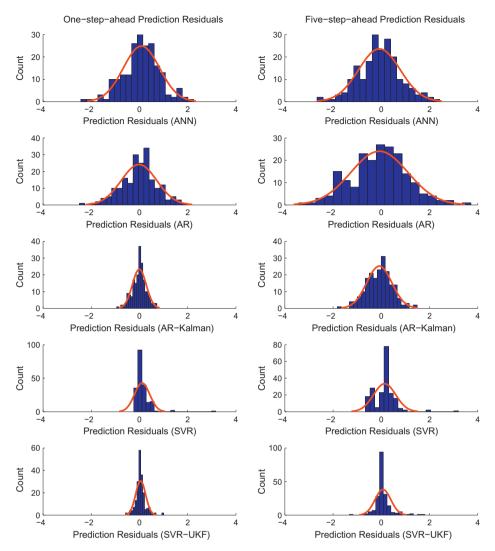


Fig. 5. Probabilistic histograms of prediction residuals of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Blandford, MA.

Kalman filter is adopted to conduct recursive state estimation on the SVR based nonlinear state-space model with strong stochastic uncertainty. Further, the wind speed sequence is predicted from the UKF driven SVR model. In this way, the integrated SVR-UKF approach can mitigate prediction errors by well accounting for the stochastic and dynamic nature of wind speed.

The remainder of the article is organized as follows. Section 2 gives preliminaries on ANN, AR and AR-Kalman based wind speed prediction methods. In Section 3, SVR and UKF techniques are integrated to form the novel SVR-UKF approach for short-term wind speed prediction. The presented SVR-UKF approach is applied to wind speed forecasting of three different locations and its performance is compared with that of the SVR, AR-Kalman, AR and ANN approaches for both one-step-ahead and multi-step-ahead predictions in Section 4. The conclusions of this paper are summarized in Section 5.

#### 2. Preliminaries

#### 2.1. Artificial neural network

ANN is composed of interconnected artificial neurons that are programmed to mimic the properties of biological neurons and can be used to approach an arbitrary nonlinear function. In this study, multi-layer feedforward network with back-propagation

(BP) training is adopted as one of the comparative methods for short-term wind speed prediction. The network consists of three layers including input, hidden and output layers, which can map an input vector to an output scalar or vector through activation function in different neurons. With p input and m hidden neurons, the output of the jth hidden neuron  $z_j$  can be computed as

$$z_j = f_h \left( \sum_{i=1}^p w_{ij} y_{k-i} \right) \tag{1}$$

where  $w_{ij}$  is the connection weight from the ith input node to the jth hidden node,  $y_{k-i}$  is i-step behind past wind speed  $y_k$  and  $f_h(\cdot)$  is a sigmoid activation function in the hidden layer. Then, the future wind speed can be predicted by

$$\hat{y}_k = f_o\left(\sum_{j=1}^m w_j z_j\right) \tag{2}$$

where  $w_j$  is the connection weight from the jth hidden node to the output node,  $\hat{y}_k$  is the predicted wind speed at the kth sampling instant and  $f_o(\cdot)$  is a linear activation function for the output layer. The nonlinear mapping capability of ANN is achieved by minimizing the overall error between the actual wind speed  $y_k$  and the predicted wind speed  $\hat{y}_k$  through Levenberg–Marquardt (LM) algorithm [19].

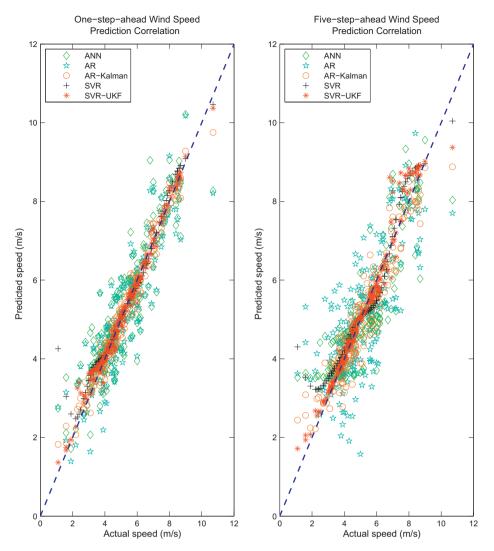


Fig. 6. Correlation plots of predicted versus actual wind speed values of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Blandford, MA.

#### 2.2. Autoregressive model

AR model as well as its variants have been widely applied to predict wind speed and wind power generation. Basically AR model is a linear time-series modeling approach that attempts to predict system output according to the previous output sequence. The general prediction procedure involves model identification, parameter estimation and diagnostic checking. The regular AR model can be expressed as

$$x_k = \varphi_1 x_{k-1} + \varphi_2 x_{k-2} \dots + \varphi_p x_{k-p} + e_k \tag{3}$$

where  $x_k$  represents the measurement at the kth sampling instant,  $\varphi_1, \varphi_2, \ldots, \varphi_p$  are regression coefficients, p denotes the AR model order, and  $e_t$  is white noise. The regression coefficients of AR model can be obtained by least squares method, which is based on the Yule–Walker equations [38]. Various types of AR models including ARMA and ARIMA models have been proposed to capture the dynamic correlations in time-series data and applied to a wide range of forecasting applications including wind speed prediction [17,18].

#### 2.3. Autoregressive-Kalman filter method

An integrated ARIMA and Kalman filter method has been proposed for one-step-ahead and multi-step-ahead wind speed

predictions [32]. In the ARIMA-Kalman approach, the ARIMA model can be simplified as an AR model when the parameters of moving average and integration parts become zero. The Kalman filter is utilized to estimate the state series  $\mathbf{x}_k \in \mathbb{R}^p$  of a stochastic process governed by the following linear state space model

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{w}_k \tag{4}$$

and

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \tag{5}$$

where  $\mathbf{y}_k \in \mathbb{R}$  denotes the measurement,  $\mathbf{w}_k$  and  $\mathbf{v}_k$  represents process and measurement noises, and  $\mathbf{F}$  and  $\mathbf{H}$  are state transition matrix and measurement matrix with proper dimension, respectively. The state equation in AR-Kalman method is formulated as

$$\begin{bmatrix} x_k \\ x_{k-1} \\ x_{k-2} \\ \vdots \\ x_{k-p+2} \\ x_{k-p+1} \end{bmatrix} = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \dots & \varphi_{p-2} & \varphi_{p-1} & \varphi_p \\ 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ x_{k-2} \\ x_{k-3} \\ \vdots \\ x_{k-p+1} \\ x_{k-p} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \mathbf{w}_k$$

$$(6)$$

Meanwhile, the measurement equation is expressed as

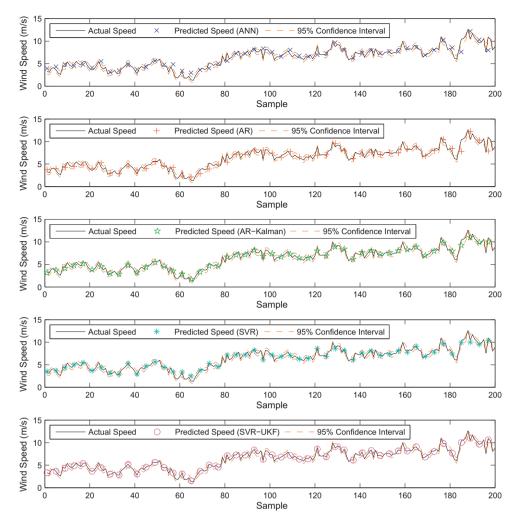


Fig. 7. One-step-ahead wind speed prediction results of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Chester, MA.

$$\mathbf{y}_k = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-p+2} \\ x_{k-p+1} \end{bmatrix} + \mathbf{v}_k$$

Based on the above linear state-space model, Kalman filter can be employed to update state estimation and thus predict wind speed sequence recursively. The detailed recursive estimation steps of Kalman filter can be found in literature paper [39].

#### 3. Support vector regression-unscented Kalman filter approach

Support vector machine (SVM) is a kind of machine learning technique with successful applications in pattern classification and model regression [40,41]. Support vector regression is specifically used to obtain predictive models through nonlinear kernel functions and a number of identified support vectors. Different types of SVR methods have been developed and applied to short-term and long-term wind speed predictions [28,30,42]. Basically SVR algorithm searches for the nonlinear regression function that is linear in high-dimensional feature space through solving a quadratic programming problem. The formulation embodies the structural risk minimization principle, which has been shown to be superior to traditional empirical risk minimization principle employed by conventional ANN method [43]. Therefore, SVR has en-

hanced ability of model prediction and can avoiding over-fitting issue. In SVR, given a set of training samples  $\{(x_1,y_1),(x_2,y_2),\ldots,(x_N,y_N)\}$  with  $x_i\in\mathbb{R}^d$  and  $y_i\in\mathbb{R}$ , the linear regression model can be expressed as

$$f(\mathbf{x}) = \omega_1 \mathbf{x}_1 + \omega_2 \mathbf{x}_2 \dots \omega_N \mathbf{x}_N + b = \langle \omega, \mathbf{x} \rangle + b \tag{8}$$

where  $\omega = [\omega_1 \omega_2 \dots \omega_N]^T$  represents the vector of regression coefficients and b denotes the bias. The regression problem can be solved by the following constrained optimization problem [40]

$$\min \quad \frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^{N} (\xi_i + \xi_i^*)$$
s.t.  $y_i - \langle \omega, \mathbf{x} \rangle - b \leqslant \varepsilon + \xi_i$ 

$$\langle \omega, \mathbf{x} \rangle + b - y_i \leqslant \varepsilon + \xi_i^*$$

$$\xi_i, \xi_i^* \geqslant 0$$
(9)

where  $\xi_i$  and  $\xi_i^*$  represent slack variables to make constraints feasible, C is regularization parameter, and  $\varepsilon$  is the tolerance threshold. By introducing the Lagrange multipliers, the optimization problem is reformulated as [44]

$$\max \ -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\alpha_i-\alpha_i^*)(\alpha_j-\alpha_j^*)\langle x_i,x_j\rangle - \sum_{i=1}^{N}(\alpha_i-\alpha_i^*)y_i - (\alpha_i+\alpha_i^*)\epsilon$$
 (10)

where  $\alpha_i$  and  $\alpha_i^*$  are the Lagrange multipliers that satisfy the conditions  $0 < \alpha_i, \alpha_i^* < C$  and  $\sum_{i=1}^N (\alpha_i - \alpha_i^*) = 0$ . To handle the nonlinear-

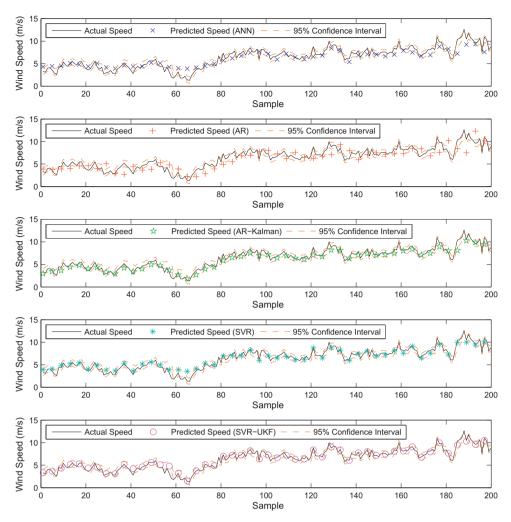


Fig. 8. Five-step-ahead wind speed prediction results of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Chester, MA.

ity in regression problem and avoid the curse of dimensionality, the inner product  $\langle x_i, x_j \rangle$  in the above objective function can be substituted with a kernel function  $K(x_i, x_i)$  as follows [43]

$$\max \ -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\alpha_{i} - \alpha_{i}^{*})(\alpha_{j} - \alpha_{j}^{*}) K(x_{i}, x_{j}) - \sum_{i=1}^{N} (\alpha_{i} - \alpha_{i}^{*}) y_{i} \\ - (\alpha_{i} + \alpha_{i}^{*}) \varepsilon$$
(11)

The Lagrange multipliers  $\alpha_i$ ,  $\alpha_i^*$  can be determined by solving the above equation with constraints and the regression function is given by

$$f(x) = \sum_{i=1}^{N} (\alpha_i - \alpha_i^*) K(x, x_i) + b$$
 (12)

The corresponding regression coefficients and bias are computed as

$$\langle \omega, \mathbf{x} \rangle = \sum_{i=1}^{N} (\alpha_i - \alpha_i^*) K(\mathbf{x}, \mathbf{x}_i)$$
 (13)

$$b = -\frac{1}{2} \sum_{i=1}^{N} (\alpha_i - \alpha_i^*) (K(x_i, x_r) + K(x_i, x_s))$$
 (14)

where  $x_r$  and  $x_s$  denote the identified support vectors. In this study, the commonly used radial basis function (RBF) is selected as kernel function for SVR model and it is defined as

$$K(x,x_i) = \exp\left(-\frac{||x-x_i||^2}{2\sigma^2}\right)$$
 (15)

where  $\sigma$  is an adjustable parameter to determine the RBF kernel width [28]. In this work, a set of values on regularization parameter C at  $\{2^0,2^3,2^7,2^{10}\}$  and RFB kernel width  $\sigma$  at  $\{0.5,1.0,1.5,2.0\}$  are evaluated in grid search. The optimal choice of  $C=2^3$  and  $\sigma=1.0$  is determined by evaluating all possible combinations of parameter values through multi-fold cross-validation. The SVR model is formulated into state-space model framework for unscented Kalman filter based dynamic and recursive state estimation.

UKF is adopted for nonlinear state estimation due to its strong capability of handling random fluctuations and uncertainty in wind speed. The unscented transformation is central in UKF to approximate the probability distribution of a nonlinear function or transformation through a subset of sample points termed as sigma points [45]. The sigma points are chosen so that their means and covariances are known. Each sigma point is then propagated through the nonlinear stochastic system to generate the transformed points along with the transformed means and covariances being estimated.

The state-space formulations derived from SVR for wind speed prediction can be expressed as

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}) + \mathbf{w}_k \tag{16}$$

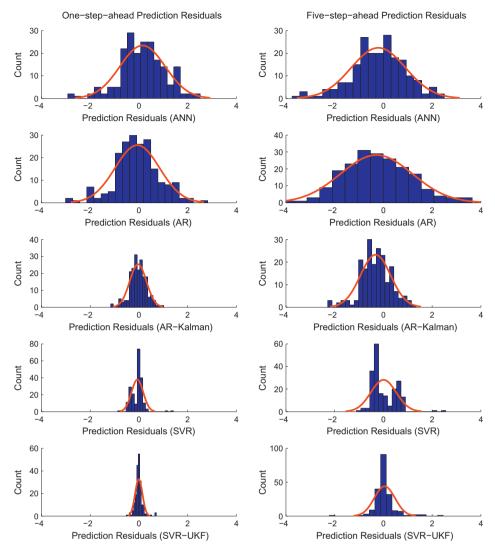


Fig. 9. Probabilistic histograms of prediction residuals of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Chester, MA.

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k \tag{17}$$

where  $f(\cdot)$  represents the nonlinear state transition model based on Eq. (12),  $\mathbf{x}_k$  denotes the state vector containing wind speed sequence,  $\mathbf{w}_k$  is the state noise with covariance  $\mathbf{Q}_k$ ,  $h(\cdot)$  represents the observation model,  $\mathbf{y}_k$  denotes the observation vector containing the predicted wind speed, and  $\mathbf{v}_k$  is the measurement noise with covariance  $\mathbf{R}_k$ . Since both input and output vectors of the observation model are wind speed sequence, the measurement equation can be simplified as

$$\mathbf{y}_k = \mathbf{x}_k + \mathbf{v}_k \tag{18}$$

with  $h(\mathbf{x}_k) = \mathbf{x}_k$ . The initial state vector is assumed to be a random vector with known mean and covariance. Furthermore, let  $\mathbf{X}_{k-1}$  represent a set of sigma points  $\{\mathbf{x}_{k-1}^j, 0 \le j \le 2n\}$  along with the corresponding weights  $\{W^j, 0 \le j \le 2n\}$  as follows

$$\mathbf{X}_{k-1} = {\{\mathbf{x}_{k-1}^0, W^0, \mathbf{x}_{k-1}^1, W^1, \dots, \mathbf{x}_{k-1}^{2n}, W^{2n}\}}$$
(19)

where n is the dimension of state vector. The above sigma points and their corresponding weights are chosen as follows to incorporate higher-order statistics of the stochastic system [46]

$$\mathbf{x}_{k-1}^0 = \mu_{k-1} \tag{20}$$

$$\mathbf{x}_{k-1}^{j} = \mathbf{x}_{k-1}^{0} + \sqrt{\frac{n}{1 - W^{0}}} \mathbf{P}_{k-1} \Big|_{j}, \quad j = 1, \dots, n$$
 (21)

$$\mathbf{x}_{k-1}^{j} = \mathbf{x}_{k-1}^{0} - \sqrt{\frac{n}{1 - W^{0}}} \mathbf{P}_{k-1} \Big|_{i=n}, \quad j = n+1, \dots, 2n$$
 (22)

$$-1 < W^0 < 1$$

$$W^{j} = \frac{1 - W^{0}}{2n}, \quad j = 1, \dots, 2n$$
 (23)

$$\sum_{i=0}^{2n} W^j = 1$$

where  $\mu_{k-1}$  and  $\mathbf{P}_{k-1}$  are the mean and covariance of state vector,  $\sqrt{\frac{n}{1-W^0}}\mathbf{P}_{k-1}|_j$  is the jth column of matrix square root. To approximate the nonlinear state transition function, each sigma point is propagated through the random system to obtain the transformed points as follows

$$\mathbf{x}_{\nu}^{f,j} = f(\mathbf{x}_{\nu-1}^j) \tag{24}$$

where  $\mathbf{x}_k^{f,j}$  is the transformed sigma points. Then the transformed mean  $\mathbf{x}_k^f$  and covariance  $\mathbf{P}_k^f$  of the state vector are updated as

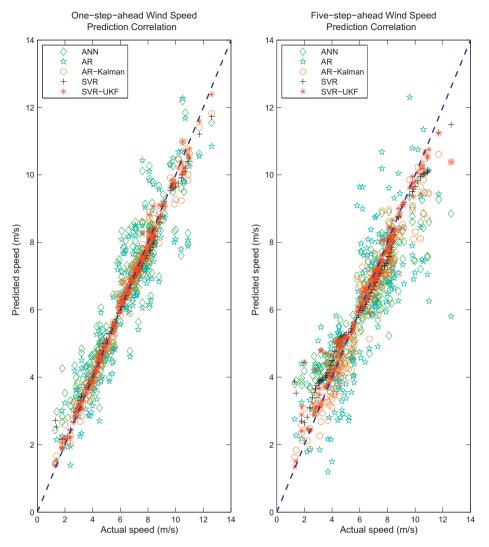


Fig. 10. Correlation plots of predicted versus actual wind speed values of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Chester, MA.

$$\mathbf{x}_k^f = \sum_{j=0}^{2n} W^j \mathbf{x}_k^{fj} \tag{25}$$

$$\mathbf{P}_{k}^{f} = \sum_{j=0}^{2n} W^{j} (\mathbf{x}_{k}^{fj} - \mathbf{x}_{k}^{f}) (\mathbf{x}_{k}^{fj} - \mathbf{x}_{k}^{f})^{T} + \mathbf{Q}_{k}$$
 (26)

Thus, the transformed state vectors can be propagated through the measurement model to get the transformed observation  $\mathbf{y}_k^{fj}$  as follows

$$\mathbf{y}_k^{f,j} = h(\mathbf{x}_k^{f,j}) \tag{27}$$

The mean  $\mathbf{y}_k^f$  and covariance  $\mathbf{\Sigma}_{\mathbf{y}_k^f}$  of the transformed observation are updated as

$$\mathbf{y}_k^f = \sum_{j=0}^{2n} W^j \mathbf{y}_k^{f,j} \tag{28}$$

$$\Sigma_{\mathbf{y}_{k}^{f}} = \sum_{j=0}^{2n} W^{j} (\mathbf{y}_{k}^{f,j} - \mathbf{y}_{k}^{f}) (\mathbf{y}_{k}^{f,j} - \mathbf{y}_{k}^{f})^{T} + \mathbf{R}_{k}$$
(29)

The cross-covariance  $\Sigma_{\mathbf{x}_k^f,\mathbf{y}_k^f}$  between  $\mathbf{x}_k^f$  and  $\mathbf{y}_k^f$  is given by

$$\Sigma_{\mathbf{x}_{k}^{f},\mathbf{y}_{k}^{f}} = \sum_{i=0}^{2n} W^{j} (\mathbf{x}_{k}^{f,j} - \mathbf{x}_{k}^{f}) (\mathbf{y}_{k}^{f,j} - \mathbf{y}_{k}^{f})^{T}$$
(30)

Then the estimation step can be expressed as

$$\hat{\mathbf{x}}_k = \mathbf{x}_{\nu}^f + \mathbf{K}_k(\mathbf{y}_k - \mathbf{y}_{\nu}^f) \tag{31}$$

where  $\hat{\mathbf{x}}_k$  is the estimated state containing future wind speed. The Kalman gain  $\mathbf{K}_k$  is given by

$$\mathbf{K}_{k} = \Sigma_{\mathbf{x}_{i}^{f}, \mathbf{y}_{i}^{f}} \Sigma_{\mathbf{y}_{i}^{f}}^{-1} \tag{32}$$

and the posterior covariance  $P_k$  can be updated as

$$\mathbf{P}_{k} = \mathbf{P}_{k}^{f} - \mathbf{K}_{k} \mathbf{\Sigma}_{\mathbf{y}_{k}^{f}} \mathbf{K}_{k}^{T}$$
(33)

Thus, UKF can be used to recursively estimate the wind speed based on the nonlinear state-space model from SVR. For multi-step-ahead wind speed prediction, the state equation can be written as follows

$$\hat{\mathbf{x}}_k = f(\mathbf{x}_{k-l}) + \mathbf{w}_k \tag{34}$$

where  $\mathbf{x}_{k-l}$  represents the measured wind speed at the k-lth sampling instant,  $\hat{\mathbf{x}}_k$  denotes the predicted wind speed at the kth sampling instant, and the state transition model is obtained from SVR.

The schematic diagram and graphic illustration of the proposed SVR–UKF approach are shown in Figs. 1 and 2, respectively. Meanwhile, the detailed step-by-step procedure of the presented method is summarized below

(i) Collect historical wind speed data set;

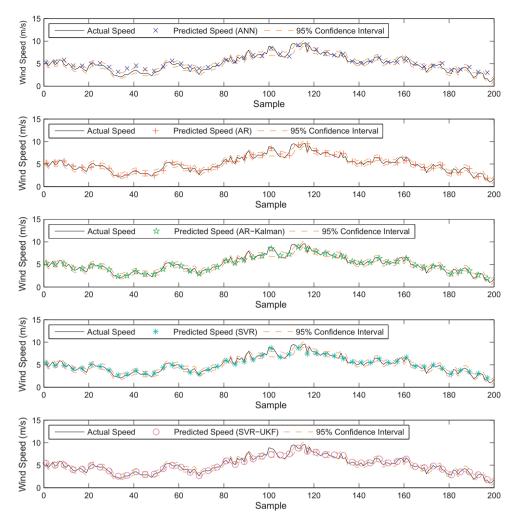


Fig. 11. One-step-ahead wind speed prediction results of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Falmouth, MA.

- (ii) Select RBF kernel function in Eq. (15) with proper parameter values;
- (iii) Build support vector regression model from historical wind speed data;
- (iv) Establish nonlinear state-space formulations based on SVR model given in Eqs. (16) and (17);
- (v) Select a set of sigma points to approximate the stochastic system through unscented transformation technique according to Eqs. (19)–(23);
- (vi) Propagate the above set of sigma points to update the means and covariances of state and observation vectors based on unscented transformation in Eqs. (24)–(30);
- (vii) Conduct unscented Kalman estimation recursively in order to update the states based on Eqs. (31)–(33);
- (viii) Obtain the one-step-ahead and multi-step-ahead predictions of future wind speed.

#### 4. Performance analysis

#### 4.1. Wind speed data set

The state of Massachusetts in USA has significant amount of offshore and onshore wind energy resource potential and the installed wind power capacity in Massachusetts has been growing rapidly in recent years. In this study, wind speed data from three different sites within Massachusetts are used to demonstrate the effectiveness of the proposed SVR–UKF method. The selected locations include Blandford, Chester and Falmouth in Massachusetts with wind speed measured every 10 min at the same position in each site and 25 m above the ground level. The wind speed data are downloaded from the website of the Center for Energy Efficiency and Renewable Energy at University of Massachusetts (http://www.umass.edu/windenergy/resourcedata.php). The model training set includes 500 samples while a test set of 200 samples is used for short-term wind speed prediction. The presented SVR–UKF approach is compared with ANN, AR, AR-Kalman and SVR methods in order to evaluate the forecasting performance. Three different performance indices are employed to evaluate the prediction accuracy in different aspects, including root mean square error (RMSE), mean absolute percentage error (MAPE) and the  $R^2$  coefficient given below

$$RMSE = \sqrt{\frac{1}{N} \sum_{k=1}^{N} (y_k - \hat{y}_k)^2}$$
 (35)

MAPE = 
$$\frac{1}{N} \sum_{k=1}^{N} \left| \frac{y_k - \hat{y}_k}{y_k} \right| \times 100\%$$
 (36)

$$R^{2} = 1 - \frac{\sum_{k=1}^{N} (y_{k} - \hat{y}_{k})^{2}}{\sum_{k=1}^{N} (y_{k} - \bar{y})^{2}}$$
(37)

where  $y_k$  denotes the actual wind speed,  $\hat{y}_k$  represents the predicted wind speed, and  $\bar{y}$  is the mean value of actual wind speed sequence.

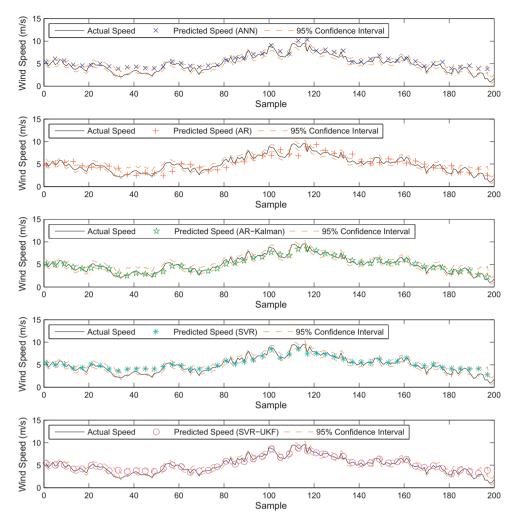


Fig. 12. Five-step-ahead wind speed prediction results of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Falmouth, MA.

#### 4.2. Comparison of wind speed prediction results

The result comparisons of wind speed prediction from ANN, AR, AR-Kalman, SVR and SVR-UKF methods in all three sites are shown in Tables 1–3, respectively. It can be readily seen that the presented SVR-UKF method outperforms ANN, AR, AR-Kalman and SVR approaches in both one-step-ahead and five-step-ahead wind speed predictions. Meanwhile, one can observe that the prediction performance of different approaches does degrade to certain extent in all three locations when the prediction horizon increases. It indicates that longer prediction horizon implies the stronger stochastic uncertainty for forecasting wind speed and power generation. The detailed result analysis of wind speed prediction across different sites are discussed below.

In the first location of Blandford, it can be readily observed that the proposed SVR–UKF method results in the lowest RMSE and MAPE values while the highest  $R^2$  value among all different approaches for both one-step-ahead and five-step-ahead wind speed predictions. More specifically, the proposed method outperforms the other approaches with the lowest RMSE value of 0.2123 m/s as opposed to the higher RMSE values of 0.7534 m/s, 0.7341 m/s, 0.2849 m/s and 0.3336 m/s for ANN, AR, AR-Kalman and SVR approaches in one-step-ahead prediction. In addition, SVR–UKF approach achieves the lowest MAPE value of 3.57% in one-step-ahead prediction while the higher MAPE values of 13.14%, 12.86%, 4.98% and 5.75% are obtained by ANN, AR, AR-Kalman and SVR methods. Similarly, SVR–UKF approach leads to the best prediction results

for five-step-ahead prediction in terms of the minimized RMSE and MAPE values. The time-series trend plots of actual and predicted wind speeds by all different approaches are depicted in Figs. 3 and 4 for one-step-ahead and five-step-ahead predictions, respectively. It should be noted that the 95% confidence interval is derived by using the posterior covariance of the state in the SVR-UKF method and represents the corresponding uncertainty region of the predicted wind speed. One can observe that the uncertainty region in five-step-ahead prediction is larger than that of one-step-ahead prediction because the prediction uncertainty increases as it forecasts further ahead. It is obvious that the proposed SVR-UKF method excels ANN, AR, AR-Kalman and SVR approaches for both prediction horizons because the predicted wind speed values by ANN, AR, AR-Kalman and SVR approaches significantly deviate from the actual wind speed sequence. In contrast, the predicted values by SVR-UKF approach coincide well with the actual wind speed measurements. Moreover, probabilistic histograms of prediction residuals of different methods are shown in Fig. 5 for both one-step-ahead and five-step-ahead predictions. It can be seen that the prediction errors of SVR-UKF method are closer to zero with significantly smaller variations, whereas the prediction errors of the other methods are scattered more widely. This behavior demonstrates the more reliable prediction by the proposed SVR-UKF method. Further, the correlation plots of the predicted versus the actual wind speeds are shown in Fig. 6 for one-step-ahead and five-step-ahead predictions. In contrast to the wide deviations and weak correlations by ANN, AR, AR-Kalman

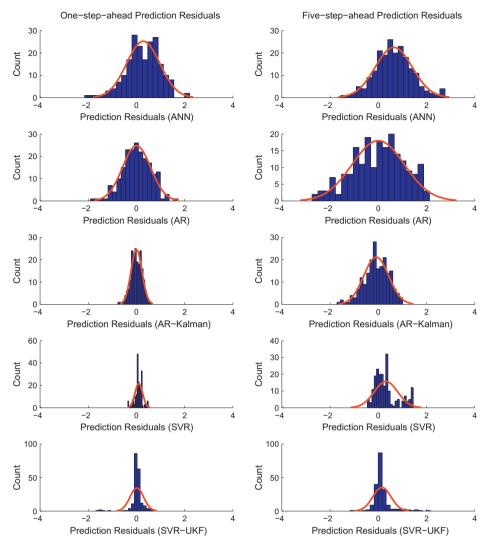


Fig. 13. Probabilistic histograms of prediction residuals of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Falmouth, MA.

and SVR methods, the presented SVR–UKF approach leads to much stronger correlations between the actual and predicted wind speeds, which is consistent with the highest  $R^2$  values of SVR–UKF method as shown in Table 3.

The fact that ANN method does not perform well in one-stepahead and five-step-ahead predictions is because it does not have strong generalization capability but tend to obtain a nonlinear model with significant over-fitting. For AR approach, it is essentially a linear time-series model and the regression coefficients rely on historical wind speed data without the feature of dynamic state update. Thus it lacks strong capacity of handling stochastic uncertainty in wind speed pattern. Such algorithm feature explains why AR-Kalman approach is superior to AR method in prediction performance as Kalman filter can accounts for stochastic uncertainty to some extent during its dynamic state estimation. SVR method results in better forecasting performance than ANN model because it employs an improved structural risk minimization principle and can effectively avoid the model over-fitting issue in ANN method. However, SVR model alone still does not have the feature of dynamic state update to deal with random uncertainty of wind speed. In contrast, SVR-UKF approach combines the merits of SVR and unscented Kalman filter to well account for system uncertainty and thus mitigate prediction errors. Therefore, the proposed SVR-UKF approach provides much better prediction performance in various aspects than the conventional methods.

For the second location of Chester, the wind speed prediction results from ANN, AR, AR-Kalman, SVR and SVR-UKF methods are compared in Tables 1-3. It is obvious that the proposed SVR-UKF method leads to highest prediction accuracy in terms of the lowest RMSE and MAPE values. The RMSE and MAPE values of SVR-UKF are as low as 0.1502 m/s and 2.07% for one-step-ahead prediction, and 0.4266 m/s and 5.68% for five-step-ahead prediction, respectively. The one-step-ahead and five-step-ahead predicted wind speed values are compared with the actual wind speed measurements in Figs. 7 and 8. One can see that the predictions of ANN, AR, AR-Kalman and SVR methods are not in well agreement with the actual wind speed values. In contrast, the predicted values from SVR-UKF approach are highly consistent with the actual wind speed measurements with the smallest deviations. Furthermore, the probabilistic distributions of prediction residuals of all different methods are compared in Fig. 9 for both prediction horizons. It can be observed that the prediction residuals of ANN, AR, AR-Kalman and SVR methods are widely spread while the variability of prediction residuals from SVR-UKF method is substantially smaller. The correlation plots between the actual and predicted wind speed values are shown in Fig. 10 and it can be seen that the predictions of SVR-UKF method have the minimal deviations and the strongest correlation. In contrast, the predicted wind speed values of ANN, AR, AR-Kalman and SVR approaches are not well correlated with the actual wind speed measurements. The

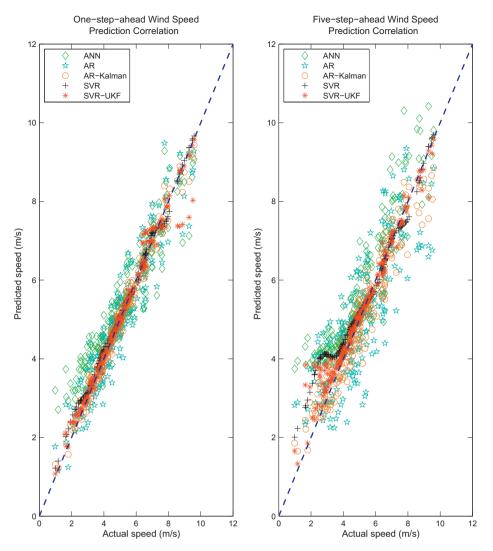


Fig. 14. Correlation plots of predicted versus actual wind speed values of ANN, AR, AR-Kalman, SVR and SVR-UKF methods in Falmouth, MA.

results from this location further confirm that the presented SVR–UKF approach can predict wind speed in short term with high fidelity.

Similar to previous two sites, the prediction results for the location of Falmouth also prove the superiority of the proposed SVR-UKF method over ANN, AR, AR-Kalman and SVR approaches. As shown in Tables 1 and 2, SVR-UKF approach achieves the lowest RMSE value of 0.1843 m/s and MAPE value of 2.88% for one-stepahead prediction and 0.4055 m/s and 7.51% for five-step-ahead prediction, whereas the RMSE and MAPE values of the other methods are significantly higher. In addition, the trend plots of the actual and predicted wind speed values of different methods are compared in Figs. 11 and 12 for both prediction horizons. Apparently, the presented SVR-UKF method outperforms the other methods with the minimal prediction errors. Furthermore, the probabilistic histograms of prediction residuals in Fig. 13 also verify that that the presented SVR-UKF method leads to the smallest deviations. Accordingly, the highest R<sup>2</sup> values of 0.9899 and 0.9510 are accomplished by SVR-UKF method in both one-step-ahead and five-step-ahead predictions. Meanwhile, the predicted wind speeds of SVR-UKF method have the strongest correlations with the actual wind speeds as shown in Fig. 14. In the proposed SVR-UKF approach, UKF recursively estimate the states under stochastic uncertainty within state-space framework constructed from nonlinear kernel function based SVR model. In order to further examine the prediction capability of the proposed SVR–UKF approach, the  $R^2$  values for one-step-ahead until ten-step-ahead predictions are calculated for all three locations and shown in Fig. 15a–c. It can be observed that the proposed SVR–UKF method has fairly reliable predictability of wind speed for even longer prediction horizons. The degraded  $R^2$  value of the SVR–UKF method for continuously increased prediction horizon is due to the weaker correlation between past and future wind speed and stronger stochastic uncertainty in the longer horizon. The above detailed comparisons in three different locations prove the effectiveness of the proposed SVR–UKF method in forecasting wind speed with satisfactory and reliable prediction performance.

#### 5. Conclusion

In this study, a novel hybrid modeling method is proposed for short-term wind speed prediction by integrating nonlinear support vector regression based state-space model with unscented Kalman filter based dynamic state estimation. The presented approach first builds a nonlinear support vector regression model in order to formulate the state-space framework for characterizing wind speed sequence. Then unscented Kalman filter is adopted to dynamically update states with random uncertainty within state-space formulations so that the future wind speed can be more

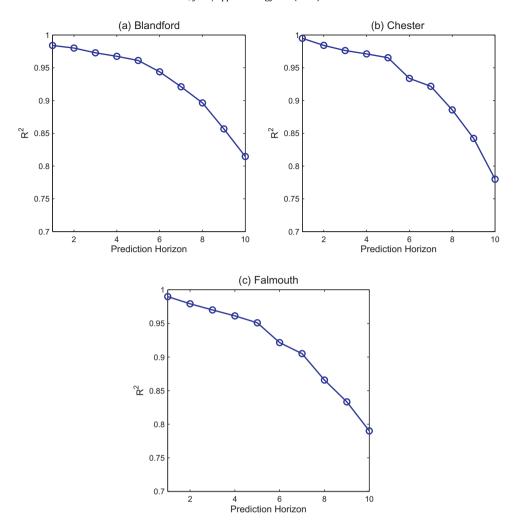


Fig. 15. Trend plots of R<sup>2</sup> value versus prediction horizon of the proposed SVR-UKF approach in the three locations (a) Blandford, MA; (b) Chester, MA; and (c) Falmouth, MA.

precisely predicted. Since the dynamic state is recursively updated in the proposed SVR–UKF approach, the stochastic uncertainty and fluctuations of wind speed can be better accounted for by the presented method than the traditional strategies.

The proposed SVR–UKF method is demonstrated to be capable of predicting short-term wind speed with high accuracy through its comparison with the conventional ANN, AR, AR-Kalman and SVR approaches. The better performance of SVR–UKF method in both one-step-ahead and multi-step-ahead wind speed predictions in different locations indicates its considerably improved reliability and robustness. Future research may further explore the wind power prediction from different kinds of wind turbines and then develop the predictive model based control and optimization strategies for wind farm operation.

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