FFR135
Artificial Neural Networks

Examples sheet 1

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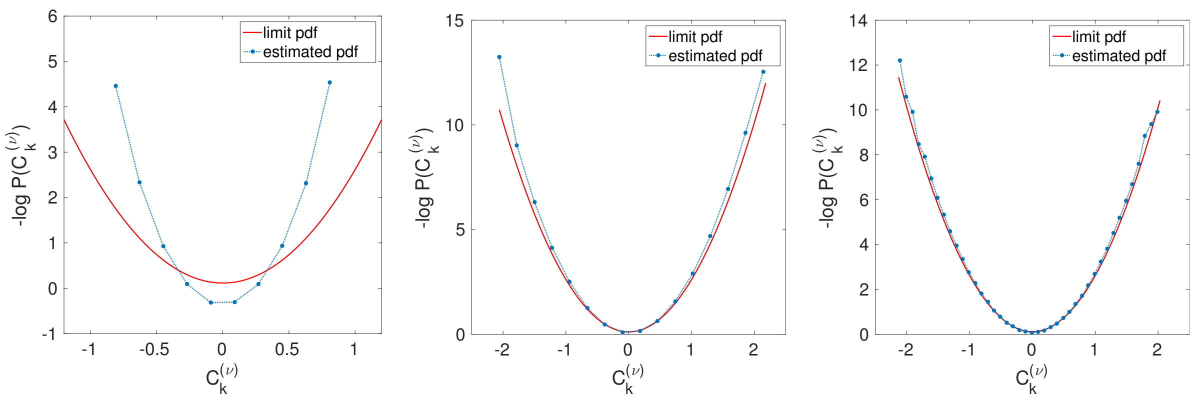
Problem 1(a)



(a) $N = 10,\ p = 2$ (b) $N = 50,\ p = 10$ (c) $N = 100,\ p = 20$

Figure 1: Blue dots and dashed line: numerically estimated pdf of the cross-talk terms in a deterministic synchronous Hopfield network. Red solid line: theoretical limit pdf.

Discussion

A Hopfield network with synchronous deterministic update was implemented and used to numerically evaluate the distribution $P(C_k^{(\nu)})$ of the cross-talk terms $C_k^{(\nu)}$ for different values of N (number of bits) and p (number of patterns).
In Fig. 1, the obtained distributions are plotted and compared with the theoretical limit distribution

$$P(C_k^{(\nu)}) = \sqrt{\frac{N}{2\pi p}}\exp\Big(-\frac{(C_k^{(\nu)})^2}{2p/N}\Big) \ , \qquad \text{valid for} \quad N \to \infty \ . \tag{1}$$

Here, since the ratio p/N is the same in the tree cases, the limit distribution is the same. Clearly, the agreement between the numerically estimated pdf and the limit pdf appears to increase as the number N of bits increases.
For low values of N (see e.g. the case $N = 10$), the distribution is taller and narrower than the limit distribution, suggesting that the single-step error probability, i.e. $P_{\text{error}} = P(C_k^{(\nu)} > 1)$, is actually less than its corresponding theoretical value

$$P_{\text{error}}^{\text{th}} = \frac{1}{2}\,\Big(1 - \text{erf}\big(\sqrt{\frac{N}{2p}}\big)\Big) \tag{2}$$

in a deterministic Hopfield network of finite size, and tends to it as $N \to \infty$.

Reference code

See MATLAB script `Exercise1a.m` and custom functions called in the script itself.

Problem 1(b)

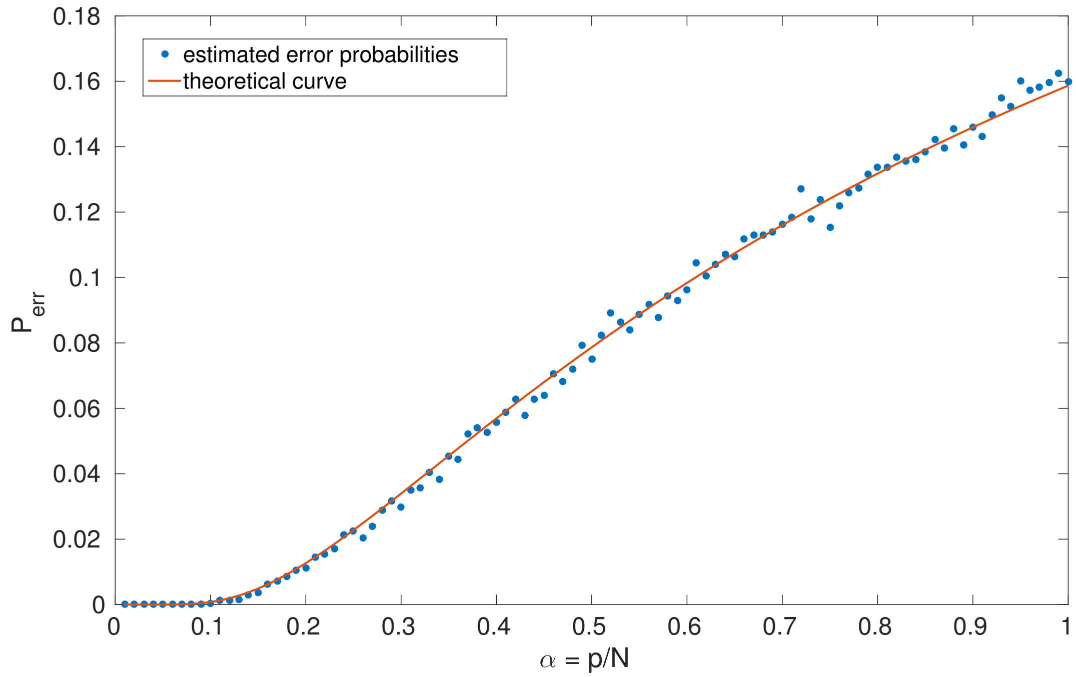


Figure 2: Blue dots: estimated one-step error probabilities in a deterministic Hopfield network with $N = 100$ bits plotted against the ratio p/N. Red solid line: theoretical curve.

Discussion

Here, $P_{\text{error}}(\alpha)$ was estimated by keeping the number N of bits fixed ($N = 100$) and increasing the number of patterns by one, starting from $p = 1$ and going up to $p = 100$. For each value of p, the following procedure was repeated $M = 10000$ times:

- Generate p random patterns and determine network weights using Hebb's rule.
- Set the initial state to be pattern $\boldsymbol{\zeta}^{(1)}$ and perform a **single** network update step.
- Check whether $s_1(t = 1) = s_1(t = 0)$. If not, then register an "error event".

At the end of this simulation, $P_{\text{error}}(\alpha)$ is estimated by

$$P_{\text{error}}^{\text{est}} = \frac{\text{number of error events}}{M} \; .$$

In Fig. 2, the estimated probabilities are compared to the theoretical curve (see Equation 2), displaying a remarkably good agreement.

Reference code

See MATLAB script `Exercise1b.m` and custom functions called in the script itself.

Problem 2(a)

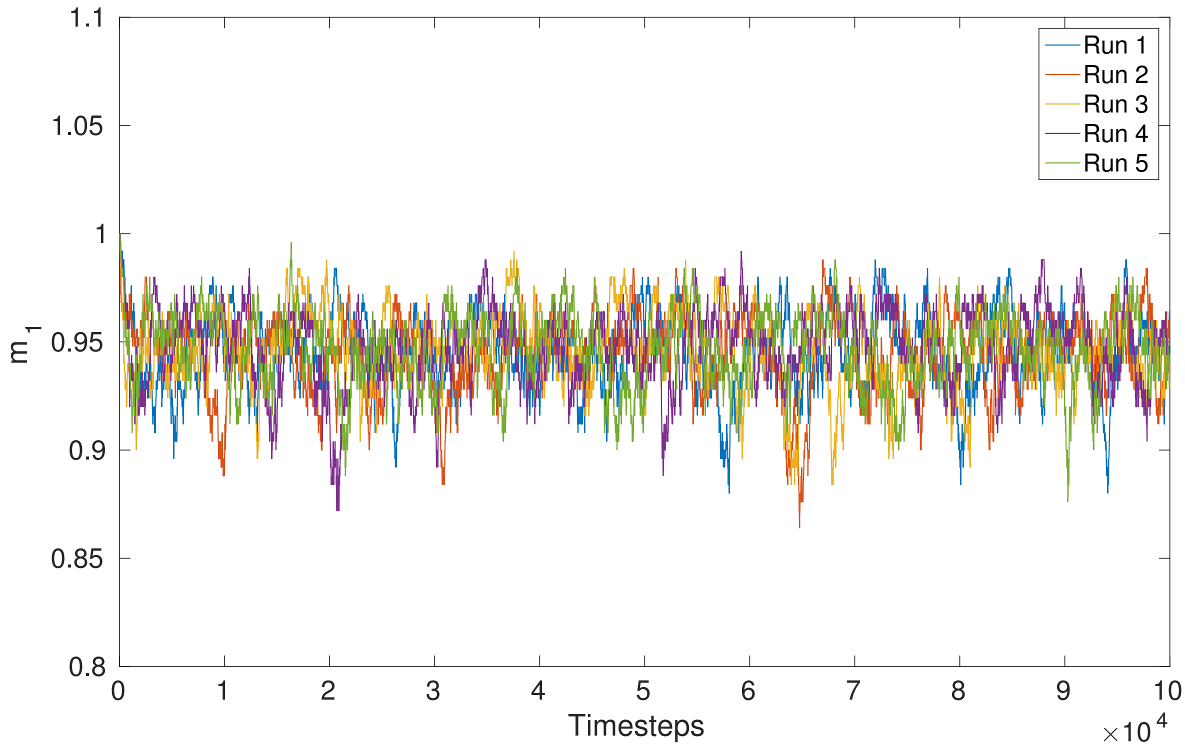


Figure 3: Evolution of a stochastic Hopfield network with $N = 500$ bits and $p = 10$ patterns. In all 5 runs, the initial configuration is the stored random pattern $\boldsymbol{\zeta}^{(1)}$ and the network is updated 10^5 times. Noise level is fixed $\beta^{-1} = 0.5$.

Discussion

A Hopfield neural network with asynchronous stochastic updating was implemented and tested with a fixed noise level of $\beta^{-1} = 0.5$ and a load parameter $\alpha = 0.02$, corresponding to $N = 500$ bits and $p = 10$ patterns. The stored pattern $\boldsymbol{\zeta}^{(1)}$ is fed as initial configuration, and the network is then run for 10^5 asynchronous updates. The whole process is repeated 5 times.
Fig. 3 shows that at this noise and load level the initial pattern $\boldsymbol{\zeta}^{(1)}$ is stable, as the order parameter m_1 stabilises around a value of 0.95. This suggests that in this point (β^{-1}, α) of the parameter space, the neural network can be used as a good memory device.
This simulation, however, does not allow us to give an estimate the burn-in time of the MCMC, as the equilibrium state is too close to the starting configuration.

Reference code

See MATLAB script `Exercise_2a_2b.m` and custom functions called in the script itself.

Ex. 2 (b)

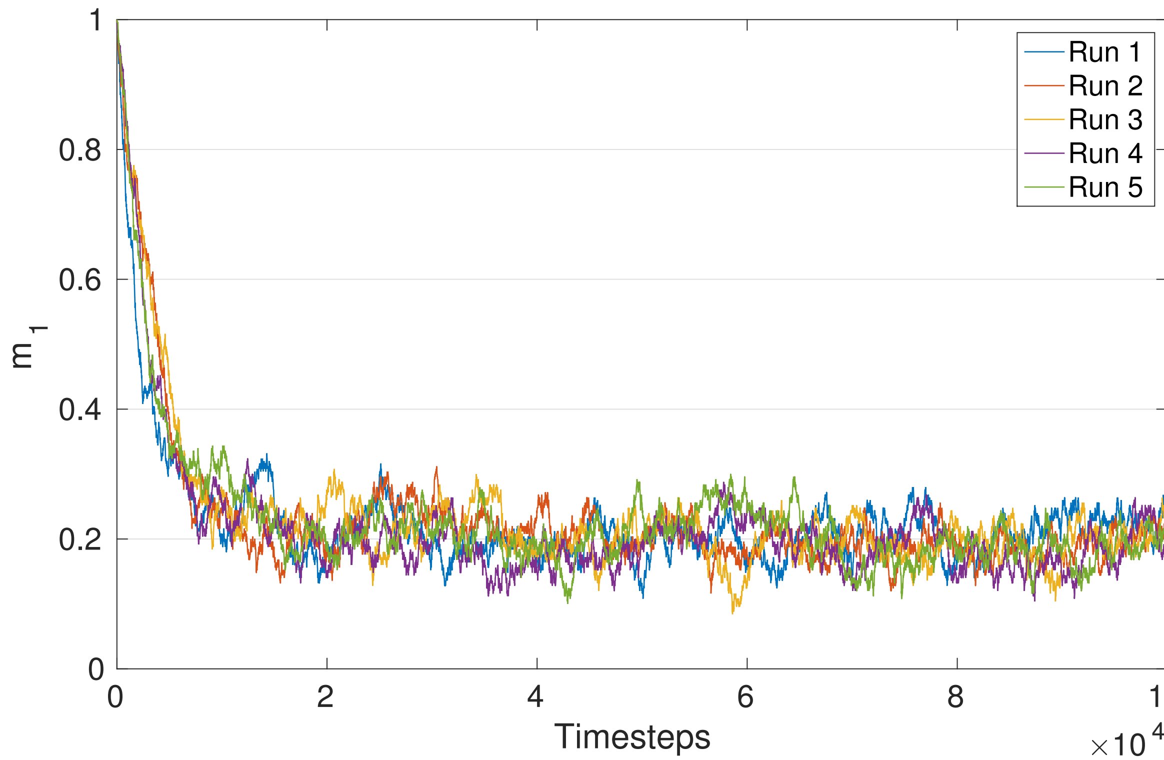


Figure 4: Evolution of a stochastic Hopfield network with $N = 500$ bits and $p = 100$ patterns. In all 5 runs, the initial configuration is the stored random pattern $\boldsymbol{\zeta}^{(1)}$ and the network is updated 10^5 times. Noise level is fixed $\beta^{-1} = 0.5$.

Discussion

In this second case, the number of stored pattern is increased to $p = 100$, corresponding to a network load $\alpha = 0.2$.
As predicted by the theory, the order parameter converges to a lower value than that of the previous case, as the network load is much higher. The theory predicts in fact that, when the noise is fixed, $\langle m_1(\alpha)\rangle$ is a monotonically decreasing function of α.
With these parameters, the network cannot obviously be used as a memory device. However, the equilibrium value of the order parameter appears to be significantly different from zero, as the dimension of the network is finite. Some exploratory simulations with a much larger number of bits (up to $N = 10000$) showed that indeed the order parameter converges to zero for $\alpha = 0.2$.
Furthermore, Fig. 4 suggests that for these values of β, N and p, a burn-in time of around 50000 iterates is sufficient to allow the system to reach the "thermal" equilibrium.

Reference code

See MATLAB script `Exercise_2a_2b.m` and custom functions called in the script itself.

Ex. 3 (a)

$N = 500$	$\alpha = 0.025$	0.05	0.1	0.125	0.15	0.2
$\beta^{-1} = 0.1$	$1 \pm 2 \cdot 10^{-15}$	$1 \pm 3 \cdot 10^{-6}$	$1 \pm 3 \cdot 10^{-4}$	0.994 ± 0.002	0.05 ± 0.02	0.238 ± 0.005
0.2	$1 \pm 4 \cdot 10^{-4}$	0.998 ± 0.001	0.958 ± 0.008	0.986 ± 0.004	0.11 ± 0.11	-0.2 ± 0.02
0.3	0.996 ± 0.001	0.988 ± 0.003	0.971 ± 0.004	0.075 ± 0.019	-0.02 ± 0.03	0.2 ± 0.2
0.4	0.980 ± 0.003	0.953 ± 0.007	-0.003 ± 0.014	0.11 ± 0.07	0.15 ± 0.04	0.12 ± 0.02
0.5	0.943 ± 0.006	0.91 ± 0.01	0.02 ± 0.03	0.19 ± 0.02	0.07 ± 0.02	0.12 ± 0.04
0.6	0.87 ± 0.01	0.06 ± 0.07	0.06 ± 0.08	0.10 ± 0.02	-0.01 ± 0.03	0.11 ± 0.03
0.7	0.03 ± 0.06	0.11 ± 0.03	-0.06 ± 0.13	0.13 ± 0.03	0.0 ± 0.1	0.22 ± 0.03
0.8	0.08 ± 0.04	0.01 ± 0.10	-0.05 ± 0.03	0.07 ± 0.04	0.02 ± 0.14	-0.06 ± 0.10

Table 1: Average value $\langle m_1 \rangle$ of the order parameter in equilibrium. N = 500.

$N = 1000$	$\alpha = 0.025$	0.05	0.1	0.125	0.15	0.2
$\beta^{-1} = 0.1$	$1 \pm 2 \cdot 10^{-7}$	$1 \pm 4 \cdot 10^{-5}$	0.995 ± 0.001	0.979 ± 0.002	0.194 ± 0.007	0.20 ± 0.02
0.2	$1 \pm 2 \cdot 10^{-4}$	0.999 ± 0.001	0.995 ± 0.001	0.979 ± 0.003	0.13 ± 0.02	0.13 ± 0.01
0.3	0.996 ± 0.001	0.992 ± 0.001	0.963 ± 0.006	0.052 ± 0.014	0.075 ± 0.012	0.034 ± 0.006
0.4	0.978 ± 0.002	0.969 ± 0.004	0.19 ± 0.06	0.23 ± 0.01	-0.09 ± 0.05	0.19 ± 0.02
0.5	0.936 ± 0.004	0.90 ± 0.01	-0.07 ± 0.03	0.007 ± 0.01	0.13 ± 0.01	0.00 ± 0.02
0.6	0.87 ± 0.01	0.07 ± 0.02	0.17 ± 0.02	0.01 ± 0.09	0.03 ± 0.01	0.02 ± 0.01
0.7	0.14 ± 0.09	0.13 ± 0.03	0.14 ± 0.04	0.19 ± 0.04	0.01 ± 0.03	-0.01 ± 0.02
0.8	0.19 ± 0.07	-0.01 ± 0.08	0.04 ± 0.02	0.12 ± 0.02	0.07 ± 0.06	0.13 ± 0.03

Table 2: Average value $\langle m_1 \rangle$ of the order parameter in equilibrium. N = 1000.

Discussion

Using the asynchronously stochastic updating network of Problem 2, the parameter space (α, β^{-1}) was explored, computing the average value of the order parameter $\langle m_1 \rangle$ in equilibrium.
Values in Table 1 (corresponding to a network of size $N = 500$) were obtained by performing $5 \cdot 10^6$ updates and acquiring samples for m_1 every 500 steps, after a burn-in time of $5 \cdot 10^5$ steps. These samples were then smoothed by using a moving average with a span of 100 values. Then, the resulting smoothed samples were averaged again to obtain the displayed values, and the 95% quantiles were used as statistical errors.
Values in Table 2 (corresponding to a network of size $N = 1000$) were obtained in the same way, but sampling a value for m_1 every 1000 steps instead.
Here, statistical errors are probably underestimated, because they are calculated on moving averages, instead of raw data.

Reference code

See MATLAB script `Exercise_3a.m` and custom functions called in the script itself.

Ex. 3 (b)

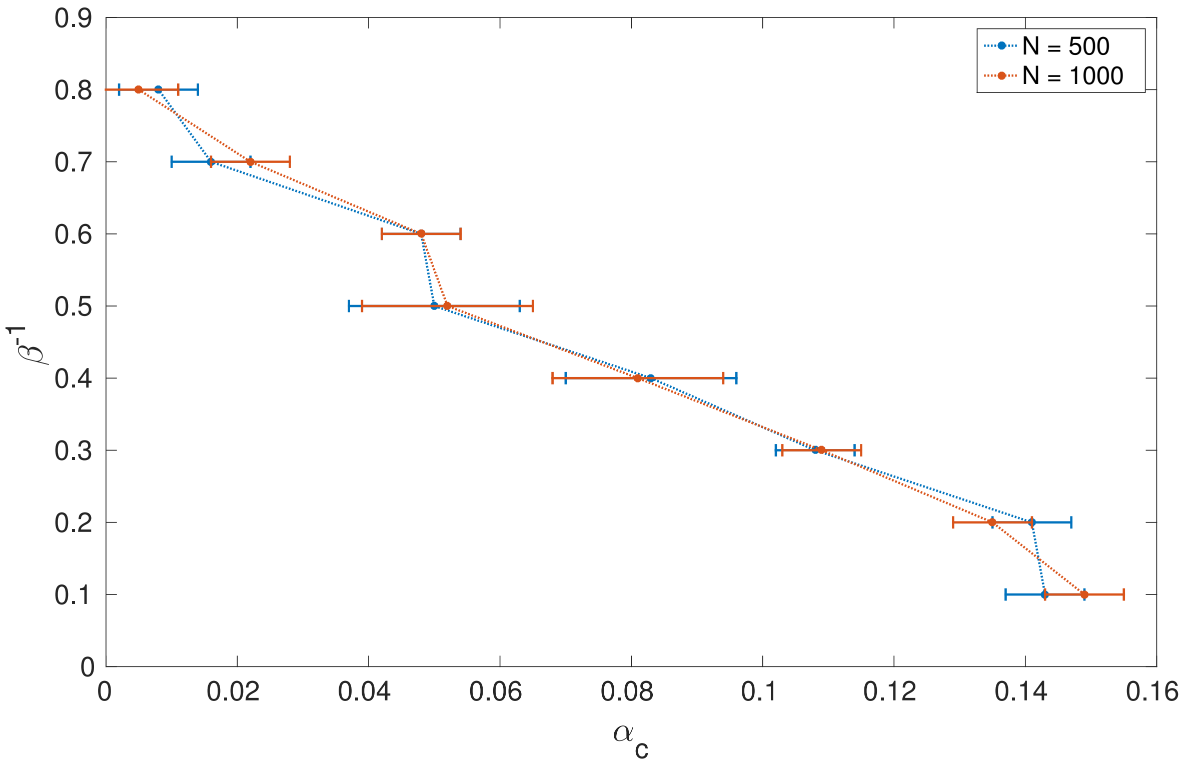


Figure 5: Critical values α_c (x-axis) as a function of β^{-1} (y-axis) in a stochastic Hopfield network with either $N = 500$ bits (blue dots) or $N = 1000$ bits (red dots).

Using the values in Table 1 and 2, for each value of N and β^{-1} the couple of values (α_1,α_2) corresponding to the largest drop of $\langle m_1 \rangle$ were identified (e.g. for $N = 500$ and $\beta^{-1} = 0.1$, the interval of interest is $[\alpha_1,\alpha_2] = [0.125, 0.15]$). Then, for each such interval, additional simulations were run, for 3 more equally spaced values of α inside interval $[\alpha_1,\alpha_2]$, acquiring average values of $\langle m_1 \rangle$ in equilibrium ($1 \cdot 10^6$ updates, burn-in of $5 \cdot 10^5$).
The resulting set of 5 values $\langle m_1(\alpha) \rangle$ were used for a spline interpolation, to find the value α^* corrsponding to to $\langle m_1(\alpha^*) \rangle = 0.5$. This value is our estimate of $\alpha_c(\beta^{-1})$, and the difference between two consecutive sampled (and *not* interpolated) values of α is used as statistical error.
The resulting curves $\alpha_c(\beta^{-1})$ in Fig. 5 roughly reflect the shape of the phase diagram of the stochastic Hopfield model, and this is the reason why α was kept on the x-axis and β on the y-axis in this plot.
However, a much larger network size N and longer simulation runtime (which would allow, for instance, to be able to acquire uncorrelated MCMC samples for the order parameter) are required to give better estimates of α_c.

Reference code

See MATLAB script `Exercise_3b.m` and custom functions called in the script itself.

Appendix: MATLAB code

Exercise1a.m

```
%EXERCISE1A
clear
clc
tic

% ========================================================== %
% Parameters
% ========================================================== %

nBits = 50;
nPatterns = 10;
nSamples = 4000;

% ========================================================== %

% Initializations
crossTalkTerms = zeros(nBits, nPatterns, nSamples);

% Main loop
h = waitbar(0, 'Please wait...');
for sample = 1:nSamples

    randomPatterns = GenerateRandomPatterns( nBits, nPatterns );
    crossTalkTerms(:,:,sample) = ComputeCTT(randomPatterns);

    waitbar(sample/nSamples,h);
end
close(h);

[h1 , f1] = PrettyPlotCTT(crossTalkTerms(:), nPatterns/nBits);
[h2 , f2] = LogPlotCTT(crossTalkTerms(:), nPatterns/nBits);
toc
```

Exercise1b.m

```
%EXERCISE1B
clear
clc
tic

% ========================================================== %
% Parameters
% ========================================================== %

nBits = 100;
nSamples = 10000;
```

```
maxNPatterns = 100;
patternID = 1; % first pattern by default
bitID = 1; % first bit by default

% ========================================================= %

[errorProbs] = CheckBitStability( nBits, maxNPatterns, nSamples,
    patternID, bitID );

% plot estimated probabilities
alpha = (1:maxNPatterns)/maxNPatterns;
plot(alpha,errorProbs,'bo');
hold on

% plot theoretical error probabilities
theorVals = 1/2*(1-erf(sqrt(1./(2.*alpha))));
plot(alpha,theorVals,'r');
hold off

% plot settings
set(gcf,'color','w')
xlabel('\alpha = p/N');
ylabel('P_{err}');
pbaspect([1.618 1 1]);
set(gca,'fontsize', 24);
toc
```

Exercise2a2b.m

```
%EXERCISE_2A_2B
clear
clc
tic

% ========================================================= %
% Parameters
% ========================================================= %

nBits = 500;
nPatterns = 100;
noise = 0.5;
tMax = 50000;
nRuns = 1;
patternID = 1;

% ========================================================= %

% Initializations
orderParams = zeros(nRuns,tMax);
fig = figure;
```

```
% Preliminary operations
beta = 1/noise;
randomPatterns = GenerateRandomPatterns( nBits, nPatterns );
weightMatrix = SetHebbsWeights( randomPatterns );
storedPattern = randomPatterns(:,patternID);

for jj = 1:nRuns

    % Re-initialise network at each run
    networkState = randomPatterns(:, patternID);

    message = ['Please wait... Run ' num2str(jj) ' of ' num2str(nRuns)];
    h = waitbar(0, message);

    for tt = 1:tMax

        % compute order parameter
        orderParams(jj,tt) = ComputeOrderParam(storedPattern, networkState);

        % update network
        networkState = StochasticAsyncUpdate( networkState, weightMatrix, beta );

        waitbar(tt/tMax,h);

    end
    close(h);

    figure(fig);
    plot(orderParams(jj,:),'LineWidth',1);
    hold on;

end

% Plot settings
set(gcf,'color','w');
f1 = plot(nan);
f2 = plot(nan);
f3 = plot(nan);
f4 = plot(nan);
f5 = plot(nan);
legend([f1,f2,f3,f4,f5],'Run 1', 'Run 2', 'Run 3', 'Run 4', 'Run 5')
pbaspect([1.618 1 1])
hold off;
toc
```

Exercise3a.m

```
%EXERCISE3A
clear

% ========================================================= %
% Parameters
% ========================================================= %

noiseMax = 8; % using integer because of parfor loop
nBits = 500; % use 500 and 1000
nSweeps = 10000;
patternID = 1; % pattern to feed
alphaValues = [0.025, 0.05, 0.1, 0.125, 0.15, 0.2];
burnIn = 1000;

% ========================================================= %

% Initializations
orderParamDataset = zeros(noiseMax,length(alphaValues),nSweeps);
alphaID = 1;

h = waitbar(0, 'Please wait...');

for alpha = alphaValues
    tic

    % compute number of patterns according to alpha
    nPatterns = fix(nBits*alpha);

    % initialize local variable to temporary store m values
    orderParams = zeros(noiseMax,nSweeps);

    % parallel loop over noise
    parfor noise = 1:noiseMax

        beta = 10/noise;

        % generate random patterns and set weigths
        randomPatterns = GenerateRandomPatterns( nBits, nPatterns );
        weightMatrix = SetHebbsWeights( randomPatterns );

        % Feed network with pattern # patternID (default 1)
        networkState = randomPatterns(:, patternID);
        % and define pattern to compare with the network state
        storedPattern = randomPatterns(:,patternID);

        for sweep = 1:nSweeps % run MCMC sweeps

            for tt = 1:nBits % and in each sweep update the network nBits times
```

```
                            % (so that on average each bit is
                               updated once)
                networkState = StochasticAsyncUpdate(
                   networkState, weightMatrix, beta );
            end

            % then compute and store order parameter
            orderParams(noise,sweep) = ComputeOrderParam(
               storedPattern, networkState);
        end
    end

    % finally store order parameter samples in an outer 3D array
    orderParamDataset(:,alphaID,:) = orderParams;
    alphaID = alphaID +1;

    waitbar(alphaID/length(alphaValues),h);
    toc
end
close(h);

% smoothing
for ii = 1:noiseMax
    for jj = 1:length(alphaValues)
        orderParamDataset(ii,jj,:) = smooth(squeeze(
           orderParamDataset(ii,jj,:)),100);
    end
end

% remove initial values (burnin)
dataset = dataset(:,:,burnIn+1:end);

meanValues = (mean(dataset,3));
lowerQuantiles = (quantile(dataset,0.05,3));
upperQuantiles = (quantile(dataset,0.95,3));
errors = max(upperQuantiles-meanValues,meanValues-lowerQuantiles
   );
```

Exercise3b.m

```
%EXERCISE3B
clear
tic

% ========================================================= %
% Parameters
% ========================================================= %

noise = 0.1; % depending on this value, set alpha values below
```

```
howManyPoints = 5; % points to sample and then use for interpolation
alphaValues = linspace(0.125,0.15,howManyPoints); % alpha window of interest found in previous exercise (DEPENDS on noise!!)
nBits = 500; % use 500 and 1000
nSweeps = 1000;
patternID = 1; % pattern to feed
burnIn = 500;

% ======================================================= %

% Initializations
orderParamDataset = zeros(length(alphaValues),nSweeps);
beta = 1/noise;

parfor alphaID = 1:howManyPoints

    alpha = alphaValues(alphaID);
    % compute number of patterns according to alpha
    nPatterns = max(1,fix(nBits*alpha));

    % generate random patterns and set weigths
    randomPatterns = GenerateRandomPatterns( nBits, nPatterns );
    weightMatrix = SetHebbsWeights( randomPatterns );

    % Feed network with pattern # patternID (default 1)
    networkState = randomPatterns(:, patternID);
    % and define pattern to compare with the network state
    storedPattern = randomPatterns(:,patternID);

    for sweep = 1:nSweeps % run MCMC sweeps

        for tt = 1:nBits % and in each sweep update the network nBits times
                         % (so that on average each bit is updated once)
            networkState = StochasticAsyncUpdate( networkState, weightMatrix, beta );
        end

        % then compute and store order parameter
        orderParamDataset(alphaID,sweep) = ComputeOrderParam( storedPattern, networkState);
    end

end

% remove initial values
nchorderParamDataset = orderParamDataset(:,burnIn+1:end);

% compute mean values
```

```
meanOrderParam = mean(orderParamDataset,2);

% interpolation
xq = linspace(min(alphaValues),max(alphaValues),5*howManyPoints)
    ;
vq2 = interp1(alphaValues,meanOrderParam,xq,'spline');
tmp = abs(vq2-0.5);
[val , idx] = min(tmp);

alphaCrit = xq(idx)
error = alphaValues(2)-alphaValues(1)

toc
```

Additional subroutines

```
function [ randomPatterns ] = GenerateRandomPatterns( nBits,
    nPatterns )

if nargin < 2
    nPatterns = 1;
end

randomPatterns = 2*round(rand(nBits,nPatterns))-1;

end
```

```
function [ crossTalkTerms ] = ComputeCTT( patterns )
%COMPUTECTT

nBits = size(patterns,1);
nPatterns = size(patterns,2);
crossTalkTerms = zeros(size(patterns));

muVals = 1:nPatterns;

for k= 1:nBits

    %create temp matrix without j-th row (logical indexing for
        performance)
    index = true(1, size(patterns, 1));
    index(k) = false;
    tempMatrix = patterns(index,:);

    for nu = 1:nPatterns

        % allowed values of mu (all but mu=nu)
        allowedMus = muVals(muVals~=nu);
```

```
        for mu = allowedMus
            crossTalkTerms(k,nu) = crossTalkTerms(k,nu) + ...
                patterns(k,mu)*sum(tempMatrix(:,nu).*tempMatrix(:,mu));
        end
    end
end

% do not forget to multiply element-wise and divide by N
crossTalkTerms = -1/nBits*(crossTalkTerms.*patterns);
end
```

```
function [ histHandle, curveHandle ] = PrettyPlotCTT( crossTalkTerms , alpha)
%PRETTYPLOTCTT
%   Remember that alpha = p/N

figure('units','normalized','outerposition',[0 0 1 1])

% plot histogram
histHandle = histogram(crossTalkTerms(:),16,'Normalization','pdf');
%histHandle.NumBins = round(histHandle.NumBins/20);

% create linspaced x values for normal curve
xVals = linspace( min(-2,min(crossTalkTerms(:))), max(2,max(crossTalkTerms(:))), 100);
yVals = normpdf(xVals, 0, sqrt(alpha));

% plot curve
hold on
curveHandle = plot(xVals,yVals,'r','LineWidth',2);
hold off

% additional settings
set(gcf,'color','w');
xlabel('C_k^{(\nu)}');
ylabel('-log P(C_k^{(\nu)})');
axis square;
set(gca,'fontsize', 30);

end
```

```
function [ histHandle, curveHandle ] = LogPlotCTT( crossTalkTerms , alpha)
%LOGPLOTCTT
%   Remember that alpha = p/N
```

```

figure('units','normalized','outerposition',[0 0 1 1])

% create and plothistogram
%[binnedVals] = histcounts(crossTalkTerms(:));
%nBins = round(length(binnedVals)/23);
[estimatedProb,edges] = histcounts(crossTalkTerms(:),16,'Normalization','pdf');
binCenters = edges(1:end-1)+0.5*(edges(2)-edges(1));
histHandle = bar(binCenters,-log(estimatedProb));
hold on

% create normal curve points and plot them
xVals = linspace( min(-2,min(crossTalkTerms(:))), max(2,max(crossTalkTerms(:))), 100);
yVals = normpdf(xVals, 0, sqrt(alpha));
curveHandle = plot(xVals,-log(yVals),'r','LineWidth',2);
hold off

% plot settings
set(histHandle,'FaceColor', [0 0.4470 0.7410]);
set(gcf,'color','w');
xlabel('C_k^{(\nu)}');
ylabel('-log P(C_k^{(\nu)})');
axis square;
set(gca,'fontsize', 30);

end
```

```
function [errorProbs] = CheckBitStability( nBits, maxNPatterns, nSamples, patternID, bitID )
%CHECKBITSTABILITY

% initialize 2D logical array of errors
boolErrors = false(maxNPatterns,nSamples);

h = waitbar(0,'Please wait...');

for nPatterns = 1:maxNPatterns % run over different p/N ratios

    parfor ii = 1:nSamples

        % generate nPatterns new random patterns and train network
        randomPatterns = GenerateRandomPatterns( nBits, nPatterns );
        weightMatrix = SetHebbsWeights(randomPatterns);

        % feed pattern # patternID and check stability of bit # bitID
        outputPattern = DeterministicSyncUpdate(randomPatterns(:,patternID),weightMatrix);
```

```
        if outputPattern(bitID,patternID) ~= randomPatterns(
           bitID,patternID)
            boolErrors(nPatterns,ii) = true;
        end
    end

    waitbar(nPatterns/maxNPatterns,h)
end
close(h);

errorProbs = mean(boolErrors,2); % average over samples

end
```

```
function [ weightMatrix ] = SetHebbsWeights( patterns )
%SETHEBBSWEIGHTS

nBits = size(patterns,1);

% compute W matrix in a smart way
weightMatrix = 1/nBits*(patterns*patterns');

% set diagonal elements to zero
weightMatrix(logical(eye(size(weightMatrix)))) = 0;

end
```

```
function [ orderParam ] = ComputeOrderParam( storedPattern,
   networkState )
%COMPUTEORDERPARAM

orderParam = 1/length(storedPattern)*(storedPattern'*
   networkState);

end
```

```
function [ newNetworkState ] = DeterministicSyncUpdate(
   networkState, weightMatrix )
%DETERMINISTICSYNCUPDATE

newNetworkState = sign(weightMatrix*networkState);
% ...as easy as that!

end
```

```
function [ newNetworkState ] = StochasticAsyncUpdate(
   networkState, weightMatrix, beta )
%STOCHASTICASYNCUPDATE

% Pick bit to update uniformly at random
nBits = length(weightMatrix);
ii = randi(nBits);

% Evaluate Boltzmann factor
b_ii = weightMatrix(ii,:)*networkState;
boltz = 1/(1+exp(-2*beta*b_ii));

% Update bit i
newNetworkState = networkState;
if rand() < boltz
    newNetworkState(ii) = +1;
else
    newNetworkState(ii) = -1;
end

end
```