Artificial neural networks FFR135
Examples sheet 1

1. Deterministic Hopfield model. Write a computer program implementing the Hopfield model with synchronous updating according to

$$S_i \leftarrow \operatorname{sgn}\Big(\sum_j w_{ij} S_j\Big)$$

and w_{ij} given by the Hebb rule for $i \neq j$. Take the diagonal elements to be zero ($w_{ii} = 0$). Test your program by storing a number p of random patterns with bits $\zeta_i{}^{(\nu)}$ and evaluating their initial stability.

(**a**). Numerically compute the distribution $P(C_k^{(\nu)})$ of the cross-talk term $C_k^{(\nu)} = -N^{-1}\sum_{\mu\neq\nu}\sum_{j\neq k}\zeta_k^{(\nu)}\zeta_k^{(\mu)}\zeta_j^{(\mu)}\zeta_j^{(\nu)}$ for the three parameter sets $p = 2, N = 10$, $p = 10, N = 50$, and $p = 20, N = 100$. Plot the result in comparison with the theoretical prediction. Choose the y-axis to be $-\log P(C_k^{(\nu)})$, and the x-axis to be $C_k^{(\nu)}$. (**1**p)

(**b**). Find how the probability $P_{\rm Error}$ that a bit $\zeta_i^{(\nu)}$ of a given pattern ν is unstable depends on the values of p/N. Note that $P_{\rm Error}$ is a one-step error probability, as explained in the lectures. Plot your results together with the corresponding theoretical curve as a function of p/N. (**1**p)

2. Stochastic Hopfield model. Write a computer program implementing the Hopfield model (take $w_{ii} = 0$) with asynchronous stochastic updating as discussed in the lecture (pp. 38, 43 in the lecture notes).

(**a**). For $N = 500$ and $p = 10$ and noise level $\beta^{-1} = 0.5$ feed one of the stored random patterns ($\boldsymbol{\zeta}^{(1)}$) and monitor the corresponding order parameter (m_1) as a function of time. Repeat this experiment several times. Plot and describe the results. (**1**p)

(**b**). Repeat the above for $p = 100$ (all other parameters the same). Plot and describe the results. Compare to the previous case. Explain in which case of the two the order parameter is expected to be closer to unity. (**1**p)

3. Stochastic Hopfield model continued. Using the computer program from the previous task (Hopfield model with $w_{ii} = 0$ and with stochastic updating) check the phase diagram drawn on p. 63 of the lecture notes.

(**a**). Choose the following values of β^{-1}: from 0.1 to 0.8 in steps of 0.1. For each value of β^{-1} make several runs for the following values of $\alpha = p/N = 0.025, 0.05, 0.1, 0.125, 0.15, 0.2$ keeping N fixed and large: $N = 500$, and $N = 1000$. For each value of β^{-1} and α, compute the average of the order parameter m_1 in equilibrium. Use a travelling mean over the stochastic time evolution. Make sure to discard the data corresponding to those early stages of the evolution where equilibrium has not yet been reached (refer to your results from the previous task). Quote the values obtained in a Table, together with their statistical errors (95% quantiles). (**1**p)

(**b**). For each value of β^{-1} from the above list estimate the critical value $\alpha_{\rm c}$ where m_1 has dropped to 50% of its value at $\alpha = 0.025$. To get a reasonably precise value you need to interpolate, and you may have to simulate for more values of α than those listed above. Plot your estimate of $\alpha_{\rm c}$ as a function of β^{-1}, for $N = 500$ and 1000. (**1**p).