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Designing a distribution network in a supply chain system: Formulation and efficient solution procedure

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Abstract

This paper addresses the distribution network design problem in a supply chain system that involves locating production plants and distribution warehouses, and determining the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers. The goal is to select the optimum numbers, locations and capacities of plants and warehouses to open so that all customer demand is satisfied at minimum total costs of the distribution network. Unlike most of past research, our study allows for multiple levels of capacities available to the warehouses and plants. The paper presents a computational study to investigate the value of coordinating production and distribution planning. We develop a mixed integer programming model and provide an efficient heuristic solution procedure for this supply chain system problem.

Keywords: Distribution; Facility planning and design; Supply chain system; Heuristics

1. Introduction

It is quite common nowadays to see manufacturers and retailers like Proctor & Gamble and Wal-Mart joining efforts to efficiently handle the flow of products and to closely coordinate the production and supply chain system. An important strategic issue related to the design and operation of a physical distribution network in a supply chain system is the determination of the best sites for intermediate stocking points, or warehouses. The use of warehouses provides a company with flexibility to respond to changes in the marketplace and can result in significant cost savings due to economies of scale in transportation or shipping costs. In this paper, we consider the problem of designing a distribution network that involves determining

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simultaneously the best sites of both plants and warehouses and the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers.

A common objective in designing such a distribution network is to determine the least cost system design such that the demands of all customers are satisfied without exceeding the capacities of the warehouses and plants. This usually involves making tradeoffs inherent among the cost components of the system that include: (1) costs of opening and operating the plants and warehouses, and (2) the inbound and outbound transportation costs.

Many researchers have extensively studied facility and demand allocation problems. Previous research studies are well surveyed by Francis et al. [5], Aikens [1], Brandeau and Chiu [4], and Avella et al. [2]. More recently, Jayaraman [8] studied the capacitated warehouse location problem that involves locating a given number of warehouses to satisfy customer demands for different products. Pirkul and Jayaraman [9] extended the previous problem by considering locating also a given number of plants. They formulated the problem as a mixed integer model and developed a Lagrangean based heuristic solution procedure. The procedure was tested using problem instances with up to 100 customers, 20 potential warehouses and 10 potential plants.

Tragantalerngsak et al. [11] considered a two-echelon facility location problem in which the facilities in the first echelon are uncapacitated and the facilities in the second echelon are capacitated. The goal is to determine the number and locations of facilities in both echelons in order to satisfy customer demand of the product. They developed a Lagrangean relaxation based branch and bound algorithm to solve the problem and reported results of computational tests with up to 100 customers and 15 facilities. Gourdin et al. [7] studied a particular type of the uncapacitated facility location problem where two customers allocated to the same facility are matched. They developed several methods to solve the problem after deriving valid inequalities, and optimality cuts for the problem.

One major drawback in most of past research studies like [7–9,11] is that they limit the number of capacity levels available to each facility to just one. However, as it is the case in practice, there exist usually several capacity levels to choose from for each facility. The use of different capacity levels makes the problem more realistic and, at the same time, more complex to solve. Another major drawback in some previous studies like [8,9] is that they limit the number of facilities to open to a pre-specified value. Moreover, these studies fail to describe how this value can be determined in advance.

Our current study represents a significant improvement over past research by presenting a unified model of the problem that includes the numbers, locations, and capacities of both warehouses and plants as variables to be determined in the model and develops at the same time the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers. An efficient heuristic solution procedure based on Lagrangean relaxation of the problem is developed and extensive computational tests with up to 500 customers, 30 potential warehouses, and 20 potential plants are reported.

The remainder of this paper is organized as follows. In Section 2, a mathematical formulation of the distribution design problem is presented. A Lagrangean relaxation of the problem is proposed in Section 3. An efficient solution algorithm based on this relaxation is developed in Section 4. Computational results are reported in Section 5. A summary of the work presented in this paper is given in Section 6.

2. Model formulation

The following notation is used in the formulation of the model.

N	index set of customers/customer zones
M	index set of potential warehouse sites
L	index set of potential plant sites
R	index set of capacity levels available to the potential warehouses

H	index set of capacity levels available to the potential plants
C_{ij}	cost of supplying one unit of demand to customer zone i from warehouse at site j
$\overline{C}_{jk}$	cost of supplying one unit of demand to warehouse at site j from plant at site k
F_j^r	fixed cost per unit of time for opening and operating warehouse with capacity level r at site j
G_k^h	fixed cost per unit of time for opening and operating plant with capacity level h at site k
a_i	demand per unit of time of customer zone i
b_j^r	capacity with level r for the potential warehouse at site j
e_k^h	capacity with level h for the potential plant at site k

The decision variables are:

X_{ij}	= fraction (regarding a_i) of demand of customer zone i delivered from warehouse at site j
Y_{jk}^r	= fraction (regarding b_j^r) of shipment from plant at site k to warehouse at site j with capacity level r
U_j^r	$= \begin{cases} 1 & \text{if a warehouse with capacity level } r \text{ is located at site } j \\ 0 & \text{otherwise} \end{cases}$
V_k^h	$= \begin{cases} 1 & \text{if a plant with capacity level } h \text{ is located at site } k \\ 0 & \text{otherwise} \end{cases}$

In terms of the above notation, the problem can be formulated as follows.

Problem *DistriNet*:

$$Zp = \text{Min} \sum_{i\in N}\sum_{j\in M} C_{ij}a_iX_{ij} + \sum_{r\in R}\sum_{j\in M}\sum_{k\in L}\overline{C}_{jk}b_j^rY_{jk}^r + \sum_{j\in M}\sum_{r\in R}F_j^rU_j^r + \sum_{k\in L}\sum_{h\in H}G_k^hV_k^h \tag{1}$$

subject to

$$\sum_{j\in M} X_{ij} = 1 \quad \forall i \in N, \tag{2}$$

$$\sum_{i\in N} a_iX_{ij} \leqslant \sum_{r\in R} b_j^rU_j^r \quad \forall j \in M, \tag{3}$$

$$\sum_{r\in R} U_j^r \leqslant 1 \quad \forall j \in M, \tag{4}$$

$$\sum_{i\in N} a_iX_{ij} \leqslant \sum_{k\in L}\sum_{r\in R} b_j^rY_{jk}^r \quad \forall j \in M, \tag{5}$$

$$\sum_{j\in M}\sum_{r\in R} b_j^rY_{jk}^r \leqslant \sum_{h\in H} e_k^hV_k^h \quad \forall k \in L, \tag{6}$$

$$\sum_{h\in H} V_k^h \leqslant 1 \quad \forall k \in L, \tag{7}$$

$$X_{ij} \geqslant 0 \quad \forall i \in N \text{ and } j \in M, \tag{8}$$

$$U_j^r \in (0,1) \quad \forall j \in M \text{ and } r \in R, \tag{9}$$

$$Y_{jk}^r \geqslant 0 \quad \forall k \in L,\ j \in M \text{ and } r \in R, \tag{10}$$

$$V_k^h \in (0,1) \quad \forall k \in L \text{ and } h \in H. \tag{11}$$

The model minimizes total costs made of: the costs to serve the demands of customers from the warehouses, the costs of shipments from the plants to the warehouses, and the costs associated with opening and operating the warehouses and the plants. Constraint set (2) ensures that the demands of all customers are satisfied by open warehouses. Constraint sets (3) and (5) guarantee that the total customer demands satisfied by an open warehouse do not exceed both the capacity of the warehouse and the total shipments to the warehouse from all open plants, respectively. Constraint set (4) and (7) ensures that a warehouse and a plant, respectively, can be assigned at most one capacity level. Constraints in set (6) represent the capacity restrictions of the plants in terms of their total shipments to the warehouses. Finally, constraints in sets (8) and (10) enforce the non-negativity restrictions on the corresponding decision variables and constraints in sets (9) and (11) enforce the integrality restrictions on the binary variables.

Even though the model explicitly consider just one product or one family of products, it can be easily extended to handle multiple products or families of products by adding an index to the decision variables X_{ij} and Y^r_{jk} for the different products and modifying the corresponding constraints accordingly. The Lagrangean relaxation and solution method presented in the following sections can also be easily modified to handle the extended model.

3. A Lagrangean relaxation of problem *DistriNet*

Problem *DistriNet* is a mixed-integer programming problem which includes as a special case the classical uncapacitated facility location problem which is well known to be NP-hard [7]. Commercial general purpose optimization software can solve small instances of problem P; however, computational times with such software become prohibitive for reasonably sized instances. For this reason, we will adopt a heuristic method to solve problem *DistriNet* based on the well-established Lagrangean relaxation technique. The reader is referred to references [3,6,10] for detailed discussion on the Lagrangean relaxation methodology.

We consider the Lagrangean relaxation of the problem obtained by dualizing constraints in sets (2) and (5) using multipliers α_i and β_j for all $i \in N$ and $j \in M$, respectively.

Problem L:

$$\begin{aligned} Z_L = \text{Min} & \sum_{i\in N}\sum_{j\in M}(C_{ij}a_i + \alpha_i + \beta_j a_i)X_{ij} + \sum_{r\in R}\sum_{j\in M}\sum_{k\in L}(\overline{C}_{jk}b^r_j - \beta_j b^r_j)Y^r_{jk} + \sum_{j\in M}\sum_{r\in R}F^r_j U^r_j \\ & + \sum_{k\in L}\sum_{h\in H}G^h_k V^h_k - \sum_{i\in N}\alpha_i \end{aligned} \tag{12}$$

subject to (3), (4), (6)–(11).

Problem L can be further decomposed into two subproblems $L1$ and $L2$:

Problem $L1$:

$$Z_{L1} = \text{Min} \sum_{i\in N}\sum_{j\in M}(C_{ij}a_i + \alpha_i + \beta_j a_i)X_{ij} + \sum_{j\in M}\sum_{r\in R}F^r_j U^r_j \tag{13}$$

subject to (3), (4), (8) and (9),

and

Problem $L2$:

$$Z_{L2} = \text{Min} \sum_{r\in R}\sum_{j\in M}\sum_{k\in L}(\overline{C}_{jk}b^r_j - \beta_j b^r_j)Y^r_{jk} + \sum_{k\in L}\sum_{h\in H}G^h_k V^h_k \tag{14}$$

subject to (6), (7), (10) and (11).

These problems can be solved independently and their solutions can be used to solve the Lagrangean problem L whose objective function value is given by the following proposition whose proof is obvious.

Proposition 1

$$Z_L = Z_{L1} + Z_{L2} - \sum_{i \in N} \alpha_i.$$

Problem $L1$ can be decomposed into $|M|$ subproblems (one for each potential warehouse site). To solve a subproblem for a warehouse site j, we observe that constraint set (4) requires that at most one capacity level be selected for that site. For a given capacity level r, the decision variable U_j^r is equal either to 0 or 1. If U_j^r is equal to 0, then X_{ij} is equal to 0 $\forall i \in N$ because of the constraint set (3). If U_j^r is equal to 1, then the subproblem becomes a continuous knapsack problem whose objective function value includes a constant term equal to F_j^r. If we let r^* be the capacity level with the most negative optimal objective function value of the continuous knapsack problem then $U_j^{r^*}$ is set equal to 1 and the solution to the continuous knapsack problem corresponding to r^* is retained and $U_j^r = 0 \quad \forall r \neq r^*$. If no such r^* exits, then the solution to the subproblem is taken to be the zero vector.

Similarly, problem $L2$ can be decomposed into $|L|$ subproblems (one for each potential plant site). These subproblems are solved in similar way to those of problem $L1$.

4. A heuristic solution procedure

Using the Lagrangean relaxation presented in the previous section, we can generate feasible solutions as well as lower bounds for the optimal solution of problem *DistriNet*. The success of this approach depends heavily on the ability to generate good Lagrangean multipliers. If we let $Z_L(\alpha, \beta)$ be the value of the Lagrangean function with a multiplier vector (α, β), then the best bound using this relaxation is derived by calculating $Z_L(\alpha^*, \beta^*) = \mathrm{Max}_{(\alpha,\beta)}\{Z_L(\alpha, \beta)\}$. In this study we use the subgradient optimization method to search for "good" multipliers [3,6,10]. As an integral part of the subgradient optimization method, a heuristic procedure, called DESIGN, to solve problem *DistriNet* is developed. The procedure consists of two different steps. The first step looks for warehouses to open and their capacities to satisfy customer demands using the solution to problem *L1*. The second step looks for plants to open and their capacities to make necessary shipments to the open warehouses using the solution to problem *L2*.

We note that in the solution to problem *L1* no capacity is violated. But there may be customer zones that are assigned to more than one warehouse and/or customer zones which are not assigned to any warehouse. Furthermore, the set of warehouses that are opened may not provide enough capacity to satisfy total demand of customers. First the procedure determines total capacity of the open warehouses to check if there is enough capacity to satisfy total demand. If not, two things can be done to increase total available capacity: either open a warehouse that was not selected in the solution to problem *L1* or increase the capacity of an open warehouse to the next higher level. The option which will cause the least increase in the objective function value of problem *L1* is chosen. This process is repeated until there is enough capacity to satisfy total demand. Next, the assignment of customers to the set S of the selected (open) warehouses is considered. This assignment problem is solved using a penalty cost approach. If, at any point in the procedure, there is not enough capacity to satisfy a customer demand, total available capacity is increased as described above and the assignment process starts all over again. We describe next the penalty cost based approach to assign customers to the open warehouses. Let

$N' = \{i$/customer zone i is not assigned to an open warehouse$\}$ (initially set to N),
$S = \{j$/a warehouse with some capacity level r^* is installed at site $j\}$, and
$T_j =$ total customer demand satisfied by the warehouse at site j (initially set to zero).

For each $i \in N'$, let (j_{i1}, j_{i2}) be two open warehouses with sufficient available capacities to serve customer i demand and such that $C_{ij_{i2}} \geqslant C_{ij_{i1}} \geqslant C_{ij}\ \forall j \in S$ and define $D_i = (C_{ij_{i2}} - C_{ij_{i1}})/a_i$.

Step 1: For all $i \in N'$, determine j_{i1}, j_{i2} and D_i.
Step 2: Select an index $i^* \in N'$ such that $D_{i^*} = \text{Max}_{i \in N'}\{D_i\}$.
Step 3: Assign customer i^* to warehouse $j_{i^*}1$ and adjust the available capacity of this warehouse. Set $N' = N'\{i^*\}$ and $T_j = T_j + a_{i^*}$. If $N' = \phi$, then stop; otherwise go to step 1.

In step 2, once the warehouses to be opened and their capacities become known, the plants to be opened and their capacities are determined using the solution to the Lagrangean subproblem *L2* in a similar way as in the beginning of step 1; also the assignment of the open warehouses to the open plants are determined using a penalty cost based approach similar to the one used in step 1.

The overall solution algorithm can be summarized as follows.

Procedure COMPLETE
Step 0: Set initial values for:
- The current iteration (q) to 1,
- All α_i and β_j, the Lagrangean multipliers, to 0,
- Δ, the step-size multiplier, initially set to 2,
- *No_Improv*, the parameter specifying the maximum number of iterations allowed when there has been no improvement in the best lower bound found so far (i.e., the objective function value of problem L) to 100,
- *Max_Iter*, The maximum number of iterations to be carried out, to 500.

Step 1: Solve the Lagrangean problems *L1* and *L2*.
Step 2: Find a feasible solution to the original problem *DistriNet* using procedure DESIGN.
Step 3: Adjust the Lagrangean multipliers as follows:

$$(\alpha_i)_{q+1} = (\alpha_i)_q + t_q\left[\sum_{j\in M}(X_{ij})_q - 1\right] \quad \text{for all } i \in N, \text{ and}$$

$$(\beta_j)_{q+1} = (\beta_j)_q + t_q\left[\sum_{i\in N} a_i(X_{ij})_q - \sum_{k\in L}\sum_r b_j^r(Y_{jk}^r)_q\right] \quad \text{for all } j \in M, \text{ and}$$

$$\text{where } t_q = \Delta\frac{(Z_L)_h - \overline{Z_p}}{\left(\sum_{i\in N}\left[\sum_j (X_{ij})_h - 1\right]^2\right) + \left(\sum_j\left[\sum_{i\in N} a_i(X_{ij})_q - \sum_{k\in L}\sum_r b_j^r(Y_{jk}^r)_q\right]^2\right)}.$$

$(X_{ij})_q$ and $(Y_{jk}^r)_q$ are part of the solution to the Lagrangean problem L at iteration q,
$\overline{Z_p}$ = Value of the best (smallest) feasible solution found so far, and
$(Z_L)_q$ = Value of the solution to the Lagrangean problem L at the current iteration (q).

Step 4: If there is no improvement in the best lower bound found so far $(\overline{Z_L})$ in *No-Improv* consecutive iterations, then replace Δ with $\Delta/2$ and reset the multipliers α_i and β_j to the values used to obtain the current best lower bound $(\overline{Z_L})$.
Step 5: If the iteration index $(q) = \textit{Max\_Iter}$, then stop; otherwise set this index to $q+1$ and go to step 1.

5. Computational results

The computational experiments described in this section were designed to evaluate the performance of our overall solution procedure with respect to a series of test problems. It was coded in Borland Delphi and run on a Pentium-300. Twenty-eight problem sets were generated randomly but systematically to capture a wide range of problem structures. Ten problems from each group with the same structure were solved in order to achieve a reasonable level of confidence about the performance of the solution procedure on that problem structure. A total of 280 problem instances were solved. The numbers of customer zones and potential warehouse sites vary from 100 to 500 and from 10 to 30, respectively. The number of potential plants sites was fixed to either 10 or 20.

Customer zones, potential warehouse and plant sites were generated from a uniform distribution over a square with side 100. The demand requirements of the customers were drawn from a uniform distribution between 10 and 100. Five capacity levels were used for the capacities available to both the potential warehouses and plants (i.e., $|R| = |H| = 5$). If we let $TotDem$ represent total demand requirements and $\lfloor A \rfloor$ be the integer part of A, and if we define $\text{Cap} = \lfloor 0.75 * TotDem/(|M|) \rfloor$, then the different capacities available to a potential warehouse at site j are computed as $b_j^1 = 0.5 * \text{Cap}$, $b_j^2 = 0.75 * \text{Cap}$, $b_j^3 = \text{Cap}$, $b_j^4 = 1.25 * \text{Cap}$, and $b_j^5 = 1.5 * \text{Cap}$. Similarly, the capacities available to a potential plant at site k are computed in terms of the warehouse capacities as $e_k^h = 6 * b_k^h$ for $h \in H$.

The cost coefficients C_{ij} and $\overline{C}_{jk}$ have been computed as being proportional to the Euclidean distance among the locations of customers and warehouses, and plants and warehouses, respectively. Fixed setup costs of locating and operating warehouses are assumed, as it is the case in practice, to exhibit economies of scale. Specifically, cost coefficient F_j^3 for a potential warehouse at site j with capacity level 3 is determined as five times the Euclidean distance between node j and the center of the square. The coefficients for the other capacity levels are computed as follows: $F_j^1 = \lfloor 0.6 * F_j^3 \rfloor, F_j^2 = \lfloor 0.85 * F_j^3 \rfloor, F_j^4 = \lfloor 1.15 * F_j^3 \rfloor$, and $F_j^5 = \lfloor 1.35 * F_j^3 \rfloor$. Similarly, cost coefficients G_k^h are assumed to exhibit economies of scale and are computed in terms of the warehouse costs as $G_k^h = 4 * F_k^h$ for $h \in H$.

The results of the first set of experiments testing the solution procedure are reported in Table 1. The results are described by providing the number of user nodes ($|N|$), the number of potential warehouse sites ($|M|$), the number of potential plant sites ($|L|$), the numbers of opened warehouse and plants, the average percentage gap between the feasible solution value and the lower bound. The gap, defined as 100 ∗ (feasible solution value − lower bound)/lower bound, is used to evaluate the quality of the solutions. We also include the average, maximum, and minimum capacity utilizations for the warehouses as well as for the plants. Finally, we report the average CPU times in seconds.

Table 1 shows that the solution procedure does well for a wide range of problem sizes, with an average gap of 2.82% over all 220 test problems. The number of customer zones does not seem to have a significant effect on the quality of the solution procedure. It seems however that, for problems with a given number of customer zones, the gaps are generally smaller when the number of potential warehouse sites is smaller. In all test problems used in Table 1, capacities of warehouses and plants are efficiently utilized as indicated by the capacity utilization measures. The numbers of opened warehouses and plants increase, in general, with the number of potential warehouses and plants to allow for savings in the transportation costs from warehouses to customers and plants to warehouses.

In order to compare the quality (tightness) of the lower bounds obtained by the Lagrangean relaxation (LR) proposed in our paper with that of the lower bounds obtained by the linear programming (LP) relaxation, we conducted additional computational experiments (Table 2). The LP-relaxation lower bounds are generated using the state-of-the-art commercial package CPLEX 7.0. The Lagrangean relaxation produces significantly tighter lower bounds than the LP relaxation. The Lagrangean relaxation bounds are on average 4.63–11.54% higher than those from LP relaxation. Furthermore, the computational times required by

Table 1
Performance of the overall solution procedure

$\|N\|$	$\|M\|$	$\|L\|$	Gap (%)	# of open warehouses	Average	Warehouse utilization (%)		# of open plants	Average	Plant utilization (%)		CPU
						Max	Min			Max	Min	
100	10	10	2.32	5.6	95.1	99.4	88.6	1.4	84.1	89.1	79.1	16
100	15	10	5.74	6.0	96.4	99.3	92.4	1.6	90.9	95.9	86.0	20
100	20	10	6.80	7.8	96.6	99.5	90.1	2.2	86.8	93.8	77.6	36
100	25	10	7.88	9.0	95.4	99.4	85.1	2.2	90.5	98.7	81.9	45
200	10	10	1.06	8.0	94.9	99.6	77.8	1.6	84.1	88.3	80.0	78
200	15	10	2.28	10.2	97.3	99.7	91.2	2.6	82.7	91.6	73.3	105
200	20	10	3.12	10.2	97.7	99.8	92.6	2.4	92.0	95.8	88.7	136
200	25	10	2.58	10.4	96.9	99.8	86.5	2.6	85.9	94.1	74.9	180
300	10	10	1.58	7.4	95.4	99.7	83.2	1.4	84.1	86.6	81.6	102
300	15	10	2.74	11.0	96.3	99.8	84.2	2.4	82.2	93.3	70.0	128
300	20	10	2.62	13.2	97.7	99.7	90.8	2.4	87.8	97.3	80.0	167
300	25	10	3.30	13.6	97.3	99.9	90.2	2.2	91.1	96.8	85.7	287
400	10	10	0.58	8.4	93.8	99.6	76.6	2.0	85.0	91.7	78.3	145
400	15	10	1.84	11.4	96.1	99.8	79.3	2.2	86.2	94.2	80.0	302
400	20	10	1.72	13.0	96.6	99.7	84.5	2.6	91.3	94.7	88.3	431
400	25	10	2.94	15.8	97.7	99.8	88.5	3.4	85.4	96.9	73.1	723
400	30	20	3.08	16.2	97.3	99.9	87.3	3.4	88.2	97.8	73.3	869
500	10	10	0.58	8.4	90.4	99.9	66.7	1.8	82.8	94.7	70.8	360
500	15	10	1.24	11.4	95.4	99.8	74.3	2.2	81.2	90.0	72.5	420
500	20	10	2.46	14.8	97.5	99.9	88.6	3.0	86.4	97.2	73.0	736
500	25	10	2.84	17.0	97.0	99.8	84.5	3.4	86.3	96.1	75.7	1144
500	30	20	2.68	19.2	97.3	99.9	85.3	3.4	90.6	96.7	81.0	1835

$|N|$: number of customer zones, $|M|$: number of potential warehouse sites, $|T|$: number of potential plants, Gap (%) = 100 $*$ (best feasible solution value − Lagrangean lower bound)/Lagrangean lower bound.

Table 2
Comparison of Lagrangean and LP bounds

$\|N\|$	$\|M\|$	$\|L\|$	Number of		L-gap	CPU time	
			Variables	Constraints		LAG	LP
50	10	10	1100	650	4.63	7	8
100	10	10	1600	1200	8.30	16	12
150	10	10	2100	1750	10.51	63	75
200	10	5	2325	2290	11.54	68	95

$|N|$: number of customer zones, $|M|$: number of potential warehouse sites, $|T|$: number of potential plants, L-gap = 100 $*$ (Lagrangean lower bound − LP bound)/LP bound.

the Lagrangean relaxation procedure are in general less than the times required to solve the LP problem. (Note that the CPU time for the LR relaxation is the total time for running the overall solution procedure COMPLETE.) It is also important to note that LP-relaxation bounds for larger-size problems could not be obtained because of the excessive amounts of CPU time and/or memory size required to solve the LP problems. It is therefore fair to conclude that, since the LP bound is far worse than the Lagrangean bound in terms of both tightness and CPU time, any commercial integer programming package based on LP-relax-

Table 3
Comparison of heuristic feasible solutions to CPLEX solutions

$\|N\|$	$\|M\|$	$\|T\|$	Number of		H-gap	CPU time	
			Variables	Constraints		Heuristic	CPLEX
50	10	10	1100	650	0.000*	7	36,000
100	5	5	675	650	0.000	9	45
150	5	5	925	950	0.000	22	120
100	10	10	1600	1200	0.003	16	2344
100	15	10	2375	1740	0.030*	25	36,000
100	20	10	3150	2280	0.050*	70	36,000
100	25	10	3925	2820	−0.500*	81	36,000

$|N|$: number of customer zones, $|M|$: number of potential warehouse sites, $|T|$: number of potential plants, H-gap = 100 ∗ (Heuristic solution value − CPLEX best solution value)/CPLEX best solution value.
* CPLEX failed to produce the guaranteed optimal values.

ation would take unacceptable levels of computing time. Hence the necessity to develop an efficient heuristic to solve the problem of designing the distribution network in a supply chain system.

Similarly, we conducted additional computational tests to evaluate the quality of the feasible solutions obtained by the heuristic solution procedure (DESIGN) compared to the optimal solutions if they are available or the best available feasible solutions obtained after running CPLEX for 10 hours or 36,000 seconds (Table 3). The results of the experiments reported in the table show that the proposed heuristic procedure, DESIGN, produces very good feasible solutions compared to the optimal/best available ones generated by CPLEX in significantly less CPU time. CPLEX failed to produce the guaranteed optimal solutions for four out of the seven instances reported in Table 3 after running for 10 hours. In fact, for the problem instance with $|N| = 100$, $|M| = 25$ and $|T| = 10$, our proposed solution heuristic produced a better solution than the best available solution obtained by CPLEX in significantly less time. Hence the merit of developing the special purpose heuristic, DESIGN, to solve the problem of designing the distribution network in a supply chain system.

The computational results reported in Tables 1–3 indicate that the proposed solution procedure is effective even for large systems with up to 500 customer zones 30 potential warehouse sites, and 20 potential plants. Additionally, computing times are reasonable. As such the proposed model and the solution procedure can be used as an effective and efficient tool for designing the distribution network in a supply chain system.

6. Conclusion

In this paper, we studied the problem of designing a distribution network in a supply chain system that involves determining simultaneously the best sites of both plants and warehouses and the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers. Unlike most of past research, our study allows for multiple levels of capacities available to the warehouses and plants. We develop a mixed integer programming model and present a Lagrangean based solution procedure for the problem. The results of extensive computational tests indicate that the procedure is both effective and efficient for a wide variety of problem sizes and structures.

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