SETUP

Households have quasi-linear preferences over consumption c and leisure l. They are indexed by their latent wage $\omega \sim F$, which is continuously distributed over $[\underline{\omega}, \overline{\omega}]$. Depending on which jurisdiction $j \in \{1, \ldots, J\} \equiv [J]$ they reside in, they face a jurisdiction-specific tax-and transfer system consisting of a linear tax τ_j and a transfer T_j. Defining disposable income as $y = wl$ we can write the consumer's choice problem as:

$$\max_{c,y,j \in [J]} c - k(\frac{y}{\omega}) \quad s.t. \ c \leq T_j + (1 - \tau_j)y$$

where $k(\cdot)$ is the differentiable, increasing and convex cost of supplying labor with $k'(0) = 0$. Given a consumer chooses to live in jurisdiction j, denoted $J(\{\tau_j, T_j\}_{j \in [J]}, \omega) = j$, she will choose $y^*(\tau_j, \omega)$ satisfying:

$$(1 - \tau_j) = \frac{1}{\omega} k'(\frac{y^*}{\omega})$$

We know $\frac{\partial y^*}{\partial \tau_j}(\tau_j, \omega) \equiv y^*_\tau(\tau_j, \omega) \leq 0$ and by convexity $\frac{\partial y^*}{\partial \omega}(\tau_j, \omega) \equiv y^*_\omega(\tau_j, \omega) \geq 0$. Households achieve indirect utility $v(\tau_j, T_j, \omega)$ if choosing jurisdiction j.
Household indifference curves in the (τ_j, T_j)-space are upwards sloping with slope of $y^*(\tau_j, \omega)$. From $y_\omega(\tau_j, \omega) \geq 0$ we can conclude that higher types have steeper indifference curves, implying the Spence-Mirrlees single-crossing condition is satisfied.

EQUILIBRIUM

Given a set of taxes and transfers $\{\tau_j, T_j\}_{j \in [J]}$, an equilibrium is an allocation $\{c(\omega), y(\omega), J(\omega)\}$ such that:

1. Households choose optimally, i.e. for each ω the bundle (c, y, J) solves

$$(1 - \tau_J) = \frac{1}{\omega} k'(\frac{y}{\omega}) \tag{1}$$

$$c = T_J + (1 - \tau_J)y \tag{2}$$

$$v(\tau_J, T_J, \omega) \geq v(\tau_j, T_j, \omega) \quad \forall j \in [J] \tag{3}$$

2. Population in each jurisdiction is non-zero and each jurisdiction's budget is balanced:

$$\int \mathbb{I}_{J(\omega)=j} \, dP_F > 0 \quad \forall j \in [J] \tag{4}$$

$$\int (\tau_j y(\omega) - T_j) \mathbb{I}_{J(\omega)=j} \, dP_F \geq 0 \quad \forall j \in [J] \tag{5}$$

This is closely related to equilibrium notions of Epple et al. (1984) and Epple and Romer (1991). However, here we are not restricting the set of tax- and transfer-systems to be in "political equilibrium". I think this has the benefit of accommodating a wide range of potential interactions between jurisdictions and I hope not imposing more structure here does not come with its drawbacks at a later stage. We also exclude the trivial equilibrium where τ_j and T_j are identical for all j.

Towards characterizing such an equilibrium, we first order the equilibrium tax rates as $\tau_1 < \ldots < \tau_J$. Then we can note that if a type ω prefers a lower tax jurisdiction j over a higher tax jurisdiction j', then any type $\omega' > \omega$ also prefers jurisdiction j over j'. This can be shown either with simple algebra[1] or by plotting the indifference curves of type ω and ω' in the (τ_j, T_j)-space through any point. For lower tax rates, the better-than-set of type ω is a subset of the better-than-set of type ω', hence if any tax-transfer bundle to the left of (τ_j, T_j) is preferred by ω, it will also be preferred by ω'.

[1] Suppose $v(\tau_j, T_j, \omega) \geq v(t_{j'}, T_{j'}, \omega)$ and $\omega' > \omega$. Then $v(\tau_j, T_j, \omega') \geq v(t_{j'}, T_{j'}, \omega')$ follows from applying the definition of $v(\cdot)$ and the FOC, and then using $\tau_j < \tau_{j'}$, $y_\omega \geq 0$ and y_ω being decreasing in τ.

If an equilibrium exists, then by the positive population requirement in (4), we can pick two ω's for any two "adjacent" communities j and $j+1$ such that one prefers the higher-tax jurisdiction and the other prefers the lower-tax one. Because the indirect utility function is continuous by Berge's theorem, there must exist a type $\tilde{\omega}_j$ which is exactly indifferent between the two "adjacent" communities. Hence, any $\omega > \tilde{\omega}_j$ prefers jurisdiction j over $j+1$, and similarly lower types prefer $j+1$ over j. Repeat the same exercise for jurisdictions $j+1$ and $j+2$ and we conclude any $\omega \in \left[\tilde{\omega}_j, \tilde{\omega}_{j+1}\right]$ will prefer jurisdiction $j+1$ over all other jurisdictions, since the argument in the previous paragraph did not restrict to "adjacent" communities. Thus in equilibrium households will be fully stratified along their types;

$$J(\omega) = j \iff \omega \in \left[\tilde{\omega}_j, \tilde{\omega}_{j-1}\right]$$

for some $\tilde{\omega}_J < \ldots < \tilde{\omega}_0$ where we define $\tilde{\omega}_0 = \underline{\omega}$ and $\tilde{\omega}_J = \overline{\omega}$.

Combining the stratification result with the FOCs in (1) and (2) we have characterized $\{c(\omega), y(\omega), J(\omega)\}$ as a function of $\{\tau_j, T_j\}_{j\in[J]}$ *if* an equilibrium exists. However, we have yet to characterize the set of (τ, T)-pairs that form an equilibrium. Clearly, the budget constraint in (5) imposes some restrictions on T_j, such as T_j being bounded above. Unfortunately, I don't quite know how to proceed here.

One first observation is that taking $\{t_j, T_j\}_{j\in[J]}$ as fixed and assuming $(\tilde{\omega}_j, \tilde{\omega}_{j-1})$ are differentiable as functions of T_j, we can apply Leibniz' rule to show:

$$\begin{aligned}
&\int (\tau_j y(\omega) - T_j)\, \mathbb{I}_{J(\omega)=j}\, dP_F && \geq 0 \\
\iff &\int_{\tilde{\omega}_j}^{\tilde{\omega}_{j-1}} \tau_j y(\omega) - T_j\, dP_F && \geq 0 \\
\implies &\frac{\partial \tilde{\omega}_{j-1}}{\partial T_j}\left(\tau_j y\left(\tau_j, \tilde{\omega}_{j-1}\right) - T_j\right) - \frac{\partial \tilde{\omega}_j}{\partial T_j}\left(\tau_j y(\tau_j, \tilde{\omega}_j) - T_j\right) && \geq P(J(\omega) = j)
\end{aligned}$$

However, this does not fully characterize the T_j's. First, differentiability of $(\tilde{\omega}_j, \tilde{\omega}_{j-1})$ was assumed. Maybe this is straightforward to show, but I don't see how / don't know where to look for some guidance. Second, using Leibniz' rule this way only generates a necessary condition, not a sufficient one (or am I mistaken?). Third, one needs to show a solution to the equation exists. Fourth, even if the two previous concerns could be alleviated, this only characterizes T_j taking T_{-j} as given. In reality, all T_j's are interdependent through the $\tilde{\omega}_j$'s. This reminds me of a fixed point problem, but I can't quite formalize it.

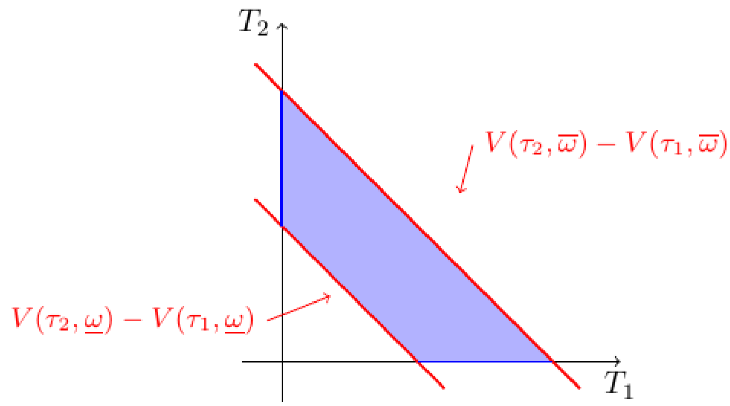
A second attempt is to focus on only two jurisdictions, $J = 2$. Then the positive population requirement simplifies if we define $V(\tau_j, \omega)$ as $v(\tau_j, T_j, \omega) - T_j$:

$$\begin{aligned}
&\int \mathbb{I}_{J(\omega)=j}\, dP_F > 0 \quad \forall j \in [J] \\
\iff &v(\tau_1, T_1, \underline{\omega}) < v(\tau_2, T_2, \underline{\omega}) \text{ and } v(\tau_1, T_1, \overline{\omega}) > v(\tau_2, T_2, \overline{\omega}) \\
\iff &V(\tau_2, \underline{\omega}) - V(\tau_1, \underline{\omega}) < T_2 - T_1 < V(\tau_2, \overline{\omega}) - V(\tau_1, \overline{\omega})
\end{aligned}$$

In words, the transfer compensating households for moving into the higher tax jurisdiction must be high enough to incentivize the lowest-ability household to prefer jurisdiction 2, while low enough to not also attract the highest-ability household. When does budget balance follow? We can rearrange the budget balance constraint to:

$$T_j \leq \frac{\int_{\tilde{\omega}_j}^{\tilde{\omega}_{j-1}} \tau_j y(\tau_j, \omega)\, dP_F}{F(\tilde{\omega}_{j-1}) - F(\tilde{\omega}_j)}$$

However, T_j appears on both sides of the inequality because the $\tilde{\omega}_j$ implicitly depend on T_j, too. We know $\lim_{T_1\to\infty} \tilde{\omega}_1 = \underline{\omega}$ and similarly for T_2 $\tilde{\omega}_1$ converges to $\overline{\omega}$, since at some transfer high enough everybody will be incentivized to move into each respective jurisdiction. Hence $\tau_j \mathbb{E}\left[y(\tau_j, \omega)\right]$ is an upper bound for T_j that need not be sharp. Combining these two thoughts, we know the positive population requirement restricts the admissible pairs (T_1, T_2) to be in a trapezoid but we have struggled to make much of the budget balance constraints.



REFERENCES

Epple, D., Filimon, R., and Romer, T. (1984). Equilibrium among local jurisdictions: toward an integrated treatment of voting and residential choice. *Journal of Public Economics*, 24(3):281–308.

Epple, D. and Romer, T. (1991). Mobility and redistribution. *Journal of Political Economy*, 99(4):828–858.