CE1051A Coursework - 3D Rigid Body Dynamics

Jake Morey
BSc(Hons) Computer Games Technology,
School of Arts, Media and Computer Games
University of Abertay Dundee
100424@live.abertay.ac.uk

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Contents

1 Introduction:

The purpose of this project is to create an application that involves 3D rigid body dynamics more specifically an application that simulates rigid body general motion when an impulse force is added to the object at a particular location. Before going into detail about the application this report will explain the physics behind the simulation.

2 Physics:

2.1 Rigid Body:

A rigid body can be described as a system of particles in which each particle remains the same distance from every other particle in that system. This means that the shape retains its shape and size whilst moving or that if there are any changes it is so small it is negligible or can be safely neglected. Also because the particles don't move within the system the objects distribution of mass is kept (Bourg and Bywalec, 2013; MacTaggart, 2013).

2.2 Centre Of Mass:

The centre of mass or the centre of gravity is the point in an object when cut in two will result in two objects with the same weight (Millington, 2007). Millington (2007) also describes how to calculate the centre of mass. The object must be split into particles so that the average position can be calculated.

$$G = \frac{1}{M} \sum_{n} p_i m_i \tag{2.1}$$

Where G is the position of the centre of mass, M is the mass of the entire object, p_i is the position of the particle and m_i is the mass of each particle. The centre of mass for any 3D object with uniform density can be found by performing a triple integral on the each separate coordinate component and then dividing by the mass of the entire object. The triple integrals for the xyz components can be seen in equations 2.2 – 2.4. Equation 2.5 shows the position vector of the centre of mass compiled from the results of equations 2.2 – 2.4.

$$G_x = \frac{1}{M} \int\int\int x \; dxdydz \tag{2.2}$$

$$G_y = \frac{1}{M} \int\int\int y \; dxdydz \tag{2.3}$$

$$G_z = \frac{1}{M} \int\int\int z \; dxdydz \tag{2.4}$$

$$\mathbf{G} = (G_x, G_y, G_z) \tag{2.5}$$

For certain shapes such as cuboids and spheres with uniform density the centre of mass is the centre of the shape (Millington, 2007). Millington (2007) describes the reason why the centre of mass is so important to rigid body physics is because "By selecting the center of mass as our origin position we can completely separate the calculations for the linear motion of the object (which is the same as for particles) and its angular motion."

2.3 Inertia Tensor:

The inertia tensor, which is sometimes called the mass matrix, is a 3x3 matrix which hold the characteristics of a rigid body. This including the moment of inertia for each of its axes stored along the diagonal of the matrix and the products of inertia (Millington, 2007). Millington (2007) describes the products of inertia as the "tendency of an object to rotate in a direction in which torque is being applied." An example of the phenomena is given as a child's top that starts spinning in one direction but will then suddenly jump upside and spin in another direction (Millington, 2007). Below equation 2.6 is the inertia tensor matrix with the previously mention diagonal moments of inertia and the remaining products of inertia. A, B, C or equations 2.7 are the moments of inertia, while F, G, H are the products of inertia or equations 2.8.

$$\begin{bmatrix} A & -H & -G \\ -H & B & -F \\ -G & -F & C \end{bmatrix} \tag{2.6}$$

$$A = \sum \rho \left(y^2 + z^2\right) \qquad B = \sum \rho \left(x^2 + z^2\right) \qquad C = \sum \rho \left(x^2 + y^2\right) \tag{2.7}$$

$$F = \sum \rho \left(yz\right) \qquad G = \sum \rho \left(xz\right) \qquad H = \sum \rho \left(xy\right) \tag{2.8}$$

Equation 2.9-2.11 show the triple integrals needed to calculate the inertia tensor for any object that has uniform density.

$$A = \iiint_V \rho(y^2 + z^2)\ dxdydz \qquad B = \iiint_V \rho(x^2 + z^2)\ dxdydz \tag{2.9}$$

$$C = \iiint_V \rho(x^2 + y^2)\ dxdydz \qquad F = \iiint_V \rho(yz)\ dxdydz \tag{2.10}$$

$$G = \iiint_V \rho(xz)\ dxdydz \qquad H = \iiint_V \rho(xy)\ dxdydz \tag{2.11}$$

2.4 General Motion of a Rigid Body:

General motion of a rigid body is what the application for this product uses to move the object once an impulse force has been applied. As mentioned briefly in section 2.2, motion can be thought of as two parts. The moving of the centre of mass in an inertial frame in the case of the application this would be the world co-ordinates. The next part of general motion would be rotational motion of the object itself about an axis that passes through the centre of mass.

Below there are three equations (2.12 – 2.14) that represent how to calculate the position, velocity, and acceleration of any point of an object at any instant. To start the axes of the body are set so that it aligns with the principle axes at the centre of mass. Also $\tilde{\mathbf{p}}_0$ which is the position vector of the point relative to the centre of mass has to be expressed by the inertial frame co-ordinates or in the application known as world co-ordinates.

$$\mathbf{p} = \mathbf{g} + t\mathbf{V} + R\tilde{\mathbf{p}}_0 \tag{2.12}$$

$$\mathbf{v} = \mathbf{V} + \boldsymbol{\omega} \times R\tilde{\mathbf{p}}_0 \tag{2.13}$$

$$\mathbf{a} = \mathbf{A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times R\tilde{\mathbf{p}}_0) \tag{2.14}$$

Equation 2.12 is the equation for calculating a position of a point at any time. g represents the centre of mass for the object, while t is the time taken, and $\mathbf{V}$ is the velocity of the centre of mass. $\tilde{\mathbf{p}}_0$ is the initial value of $\tilde{\mathbf{p}}$ which as mentioned before is the position vector between the point and the centre of mass. Finally R is a standard rotation matrix by amount θ about an unit vector axis $\hat{\mathbf{v}} = \alpha\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} + \gamma\hat{\mathbf{k}}$ passing through the origin. θ is determined by ωt with the ω being equal to $|\boldsymbol{\omega}|$. The C in the equation is a shorthand for $1 - \cos\theta$.

$$R = \begin{bmatrix} \alpha^2(C) + \cos\theta & \alpha\beta(C) - \gamma\sin\theta & \alpha\gamma(C) + \beta\sin\theta \\ \alpha\beta(C) + \gamma\sin\theta & \beta^2(C) + \cos\theta & \beta\gamma(C) - \alpha\sin\theta \\ \alpha\gamma(C) - \beta\sin\theta & \beta\gamma(C) + \alpha\sin\theta & \gamma^2(C) + \cos\theta \end{bmatrix} \tag{2.15}$$

References

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