

## Magnetostriction and Magnetomechanical Effects

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# MAGNETOSTRICTION AND MAGNETOMECHANICAL EFFECTS

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*Abstract.* The energy of a ferromagnetic substance comprises three parts, the exchange, anisotropy and magnetostatic (demagnetizing) energies. These all vary with the state of strain of the substance and so it will deform spontaneously if the deformation reduces the total energy (equal to the sum of the magnetic energy and the elastic energy). This phenomenon is known as magnetostriction and the three magnetic energies respectively give rise to the volume magnetostriction, the linear magnetostriction and the form effect. The greater part of this report is taken up with the linear magnetostriction since this is the effect most easily and often observed. The formal theory as developed by Becker and others is given in § 2. § 3 reviews the various attempts to calculate the magnetostriction constants in terms of interatomic forces. The fact that the linear magnetostriction can interact with stress to give rise to an additional anisotropy is discussed in § 6 together with some consequences of this fact. § 7 deals with the volume magnetostriction and related topics which arise from the variation of the exchange energy with interatomic distance. The form effect which arises incidentally in the analysis of volume magnetostriction is treated more fully in § 8. The only secondary effect of any importance, the  $\Delta E$  effect, is discussed in § 9. The final section describes such topics as magnetic strain analysis and the magnetostriction of ferrites and antiferromagnetics.

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## § 1. INTRODUCTION

### 1.1. *General*

THE post war position in the field of ferromagnetism has already been well reviewed in these Reports. The first of these dealt with the fundamental aspects of intrinsic magnetization (Stoner 1948) whilst the second covered the field of technical magnetization (Stoner 1950). In the space available it was not found possible to include in those reports any detailed discussion of the numerous secondary phenomena which are exhibited by ferromagnetics as a result of their possession of spontaneous magnetization. Some of these (for example the change of thermoelectric power with magnetization, Hall effect and some aspects of the magneto-resistance effect) are as yet little understood as far as their basic origins are concerned and still require a good deal of further investigation. The most important of the secondary effects, and that most closely related to the fundamental ones, namely magnetostriction, is understood, at least in principle and has an extensive literature. This report, whilst in no way intending to supplement the above mentioned articles, attempts to review this literature and to present the present state of the field as a whole.

There are three main reasons for studying magnetostriction. In the first place such a study may throw light on the nature and behaviour of the special internal forces in a ferromagnetic crystal. Some of this information may not be obtainable by other means. Secondly, the phenomenon is of use in investigating magnetization processes and analysing magnetization curves. The third reason is purely a practical one. The essence of the magnetostriction effect is that an electrical pulse can be converted into a mechanical one and vice versa. This enables it to be used for the production of electromechanical transducers, particularly for obtaining high intensity ultrasonic waves, magnetostriction oscillators and filters of very high stability, storage delay lines etc. One of the less beneficial aspects is that a good deal of the noise produced by power transformers operating at high flux densities is caused by the magnetostrictive vibrations of the core. The information required for these various aspects of the effect are necessarily somewhat different. For the first, the magnetostriction constants alone are usually required whilst for the second the manner of the variation of magnetostriction with magnetization may be needed as well. For the third, the requirements vary with the individual application.

Partly for this reason there seems to be some uncertainty as to what precisely is meant by magnetostriction and which, of a number of manifestations of the effect, are the fundamental ones. In what follows an attempt is made to co-ordinate the various aspects of the effect. Little more than can co-ordination be attempted at this stage since detailed knowledge of the basic origins of the effect is at present lacking.

The term magnetostriction may be said to embrace all those effects involving the interaction between magnetization and mechanical stress. A ferromagnetic substance, when magnetized, is observed to exert a stress against its surroundings and as a result its dimensions will change if they are free to do so. By application of the principle of *le Chatelier* it may be expected that if a ferromagnetic undergoes this effect, then its state of magnetization will change if its dimensions are forcibly altered by external stresses. These two phenomena, the change of dimensions produced by a change in magnetization, and the change in magnetization with stress are both different aspects of the same thing and it is the purpose of both theoretical and experimental investigation to establish the nature of the fundamental phenomenon which gives rise to these two effects.

### 1.2. Early Experimental Work

The fact that a ferromagnetic substance changes its length when magnetized was first established by Joule (1842). Using a bar of iron and a system of mechanical levers he was able to show that iron expands along the direction of magnetization in small fields, the expansion being reduced as the field strength is increased. Joule's experiments in this field are remarkable for the thoroughness and accuracy which distinguishes so much of his work. In addition to the effect already mentioned he was able to show that although the length of an iron bar changes when magnetized, the change in volume is very much smaller in comparison. Thus the longitudinal expansion must be accompanied by a transverse contraction. This he confirmed later by direct observation. Finally, he investigated the effects of external stresses on the magnetostriction of iron and showed that, under tension, it is a contraction for all field strengths. Some of Joule's results have been recalculated and expressed in terms of current quantities; these are shown in figure 1.1. The only observation that need be made at the moment is that the effect is really quite small and corresponds to a strain of the order  $10^{-5}$ .

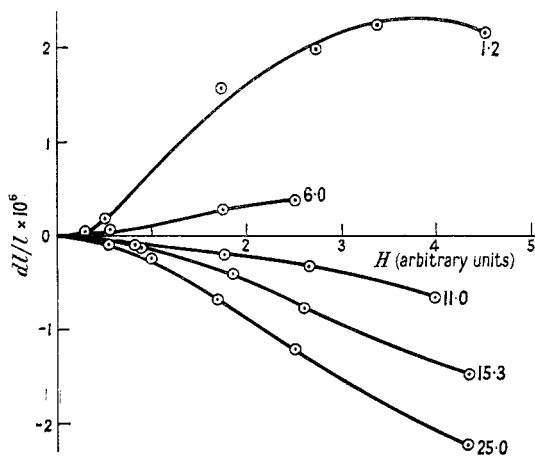


Figure 1.1. Joule's original measurements of the magnetostriction of iron (1842). The figures give the applied tension in  $\text{kg mm}^{-2}$ .

It is to be regretted that the immediately subsequent work on magnetostriction was not carried out with the insight which so characterized Joule's own. Later investigations, up to almost the end of the nineteenth century, were very largely concerned with various subsidiary and reciprocal effects which are related to the ordinary longitudinal effect. We mention the more important of these partly for the sake of completeness and partly because they still occasionally appear in the literature, but chiefly to condemn them since they tend to confuse rather than clarify the main issue. Out of a very large number of effects we may single out the following as being of greatest interest and importance.

(a) *Guillemin (1846) effect.* If a bar of ferromagnetic material is bent, either permanently or elastically, it is observed to straighten when magnetized. Since a bent bar has one half in a state of tension and the other in compression it is evident that this is merely one aspect of the influence of stress on magnetostriction and magnetization. This will be dealt with in § 6, from which it will be evident that a bent bar will always tend to straighten when magnetized, never the reverse. In addition there exists a torsional analogue, namely that if a rod or wire is given a twist then the amount of twist is decreased when it is magnetized.

(b) *Wiedemann (1883) effect.* If a ferromagnetic rod, clamped at one end is circularly magnetized by the passage of an electric current along its axis and is at the same time subjected to a longitudinal magnetic field, a twist of the free end occurs. Evidently the resultant of the two fields is a helix and the effect merely a special case of the longitudinal magnetostriction effect. This provides about the simplest means of demonstrating the existence of magnetostriction for the twist that occurs is of such magnitude that it can be observed by fixing a small mirror to the free end and noticing the deflection of a beam of light reflected from it. The effect has been very completely investigated by Pigeon (1919).

Every magnetostrictive effect has an associated inverse effect. The longitudinal magnetostriction (sometimes known as the Joule effect) has its inverse in the Villari effect (Villari 1865) which is the change of magnetization produced by an external tension. As previously mentioned these inverse effects follow from le Chatelier's principle and the Joule and Villari effects are related by the thermodynamic identity

$$\frac{1}{l} \left( \frac{dl}{dH} \right)_{\sigma} = \left( \frac{dI}{d\sigma} \right)_H \dots \dots (1.1)$$

where  $\sigma$  and  $H$  are the tension and magnetic field strength respectively. For the derivation of this equation and a complete discussion of the thermodynamics of magnetization, including magnetomechanical effects, reference should be made to the papers by Stoner (1935, 1937) and Guggenheim (1936). In this connection mention should also be made of a related paper by Brown (1953). The Villari effect is the most important of all the inverse effects and has been the subject of a vast amount of work (for a good account see Bozorth 1951). It will receive further attention in § 6.

The other inverse effects need not concern us. Each is governed by a thermodynamic equation with appropriate variables similar to equation (1.1). For further details the reader may be referred to an article by McKeehan (1926) and the book by Williams (1931).

Another subsidiary effect of a somewhat different nature is the change in elastic constants with magnetization. This will be treated in some detail in § 9.

### 1.3. Magnetostriction and Magnetization

The salient features of the magnetostriction effect in polycrystalline ferromagnetics may be seen from the magnetostriction-field and magnetostriction-magnetization curves of figures 1.2 and 1.3. The effect is in general quite small and varies in magnitude and sign from one material to another. It varies with the

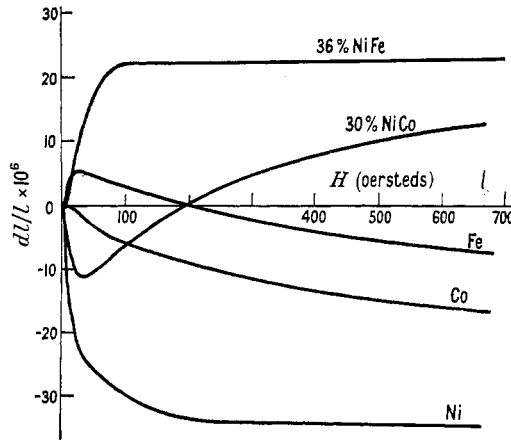


Figure 1.2. The magnetostriction of some common substances as a function of magnetic field strength.

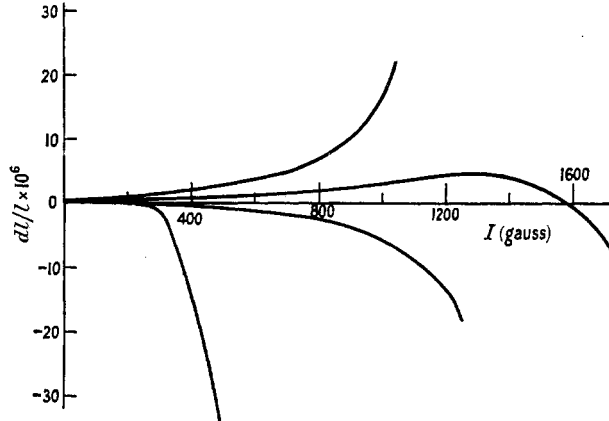


Figure 1.3. The same as in figure 1.2 as a function of the intensity of magnetization.

state of magnetization and in high fields approaches a limiting value known as the saturation magnetostriction (cf. saturation magnetization) hereafter denoted by  $(dl/l)_s$ . The effect exhibits hysteresis when displayed as a function of both magnetic field strength and magnetization (figures 1.4 and 1.5). In order to analyse and understand such curves, in particular those of figures 1.2 and 1.3, it is necessary to review briefly the current views on magnetization processes.

According to present day ideas a ferromagnetic substance below its Curie temperature is spontaneously magnetized to a degree which depends chiefly on its temperature and which decreases with increasing temperature. The effect of an

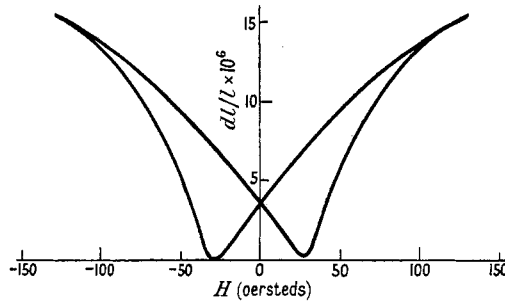


Figure 1.4. The magnetostriction of cold-worked nickel during cyclic magnetization as a function of magnetic field strength.

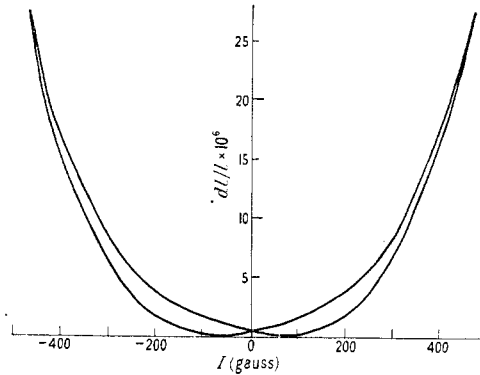


Figure 1.5. The same as in figure 1.4 as a function of the intensity of magnetization.

external magnetic field on the value of the spontaneous magnetization is usually very small. That such a material can exist in an apparently un-magnetized condition was explained by Weiss (1907) who postulated the existence of domains in the material, each of which is magnetized to saturation as stated. The direction of the spontaneous magnetization is however assumed to vary throughout the material in such a way that in the absence of an external field, the resultant component of the magnetization in any given direction is zero. In a ferromagnetic single crystal in zero applied field there is a limited number of directions which the domain magnetization can take up with respect to the crystal axes. This is known as ferromagnetic anisotropy. The number of possible directions at each point in the crystal along which the domain magnetization may lie, is governed by the symmetry of the crystal. The effects of anisotropy are usually expressed by an expression for the free energy in the form of a power series in  $a_1, a_2, a_3$ , the direction cosines of the domain magnetization vector with respect to orthogonal axes. For a cubic crystal, to which crystal class most of the ferromagnetic metals and their alloys belong, the lowest term consistent with cubic symmetry is

$$E_K = K_1(a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2).$$

$K_1$  is the first anisotropy constant, a characteristic of the material, and may have either positive or negative values. If  $K_1$  is positive  $E_K$  has a minimum value when

the domain magnetization lies along the six cube axes—[100]. If  $K_1$  is negative the minimum occurs along the body diagonals—[111] directions. These are the so-called directions of easy magnetization.

When a magnetic field is applied to a crystal in an arbitrary direction its net magnetization is increased in two ways: firstly by movement of the boundaries separating adjacent domains whereby domains with magnetization vectors oriented favourably (energetically) with respect to the field grow in volume at the expense of their less favourably oriented neighbours. In the absence of any perturbing influences this process takes place in an infinitely small field since as long as the spontaneous magnetization—hereafter denoted  $I_s$ —is in an easy direction the anisotropy energy  $E_K$  remains unchanged. In practice this process is usually complete for ordinary materials in fields of a few oersteds. The magnetization can be further increased only by rotation of the domain vectors out of their easy directions towards the field direction. This requires the expenditure of energy, the magnitude of which is determined by the magnitude of  $K_1$  and for most materials the magnetization-field curve flattens out considerably once this stage has been reached. This process is not usually completed until quite large fields (of the order of a thousand oersteds for ordinary materials) have been applied. Subsequent increase in the magnetization can only take place by increasing the value of the spontaneous magnetization within the domain. Unless the substance is maintained at a temperature just below its Curie point or unless huge fields are applied this increase is very small—the differential susceptibility corresponding to an increase of spontaneous magnetization with field well below the Curie temperature is of the same order of magnitude as the paramagnetic susceptibility above the Curie temperature.

If the corresponding magnetostriction-field curves are examined it can be seen that they can be divided up into the same three regions as the magnetization curves (figure 1.6). For example, in polycrystalline iron there is an initial expansion which occurs in the same field region in which the domain boundary displacements occur. This expansion is succeeded in higher fields by a gradual contraction which

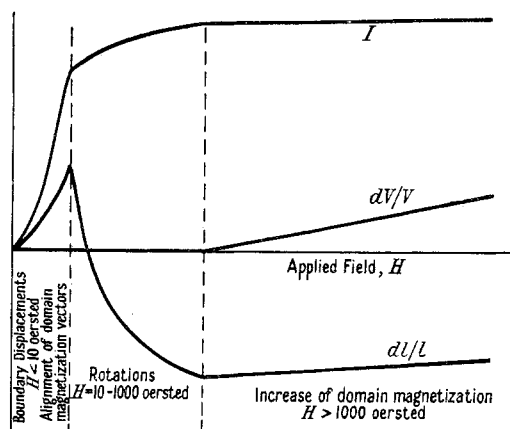


Figure 1.6. Schematic representation of magnetization, linear and volume magnetostriction of iron as a function of magnetic field strength. All ferromagnetics show essentially the same behaviour.



begins at a field strength roughly corresponding to the 'knee' of the magnetization curve, i.e., at the field strength where the boundary displacements are more or less complete and the rotational processes are just beginning. The contraction continues until the rotations are complete and the saturation magnetization and the saturation magnetostriction are reached at the same field strength. Throughout these processes the volume of the substance remains sensibly constant. In very high fields a further expansion is found (in iron). This expansion is proportional to the field strength and differs from the low field magnetostriction in that whereas the latter takes place without change of volume, the expansion in high fields is primarily a volume effect, the expansion being the same in all directions. It is natural to associate this volume strain with the field induced increase in the spontaneous magnetization.

It is evident that we are dealing with two completely different kinds of magnetization process. The first is purely one of alignment of domain magnetization vectors whilst the second involves an increase in the spontaneous magnetization of the domain itself. These two processes give rise to different forms of magnetostriction and we shall assume that they can be treated separately. We shall now concern ourselves with magnetostriction associated with the alignment process and leave the volume magnetostriction to a later section.

In the alignment process the domain magnetization vectors change their direction; the crystallographic orientation remains fixed. It should therefore be possible to explain the magnetostriction curves by assuming that below the Curie temperature all the domains are spontaneously strained by the spontaneous magnetization within each domain. The simplest assumption is that the strain within each domain varies only with the direction of the domain magnetization with respect to a single fixed axis. We may take this to be the direction of an applied field which is coincident with the direction of observation. If the angle between the domain magnetization and this axis is  $\theta$  then  $dl/l = f(\theta)$ . Since  $dl/l$  must remain unchanged when  $\theta$  changes by  $180^\circ$  the simplest form for  $dl/l$  is obtained by putting  $f(\theta) = A \cos^2\theta$ , where  $A$  is a constant of the material. It is customary to measure the magnetostriction from a hypothetical demagnetized state for which  $dl/l$  is arbitrarily put equal to zero. This hypothetical demagnetized state is defined as one in which the domain magnetization vectors are distributed uniformly over all possible directions throughout a sphere. The mean value of  $\cos^2\theta$  for such a distribution is  $\frac{1}{3}$ . Hence  $dl/l = A(\cos^2\theta - \frac{1}{3})$ . The saturation value of the magnetostriction is attained when  $\theta = 0$  for all the domain vectors. Thus we have, for a single domain,

$$dl/l = \frac{3}{2}\lambda_s(\cos^2\theta - \frac{1}{3}) \quad \dots\dots(1.2)$$

in which  $\lambda_s$  is a constant representing the total strain when the substance initially in the hypothetical demagnetized state is magnetized to saturation. This implies that the magnetostriction can be described by a single constant. In particular if the direction of measurement is kept fixed and the material saturated in any direction by the application of a large field, then the variation of the magnetostriction with direction of saturation magnetization should be representable by a simple  $\cos^2\theta$  curve. This seems to hold fairly well for polycrystalline samples containing no preferred orientation of crystal axes but the behaviour of single crystals is much too complicated to be described by equation (1.2) (see for example figure 1.7). Equation (1.2) has a further simple interpretation. In any material below saturation the

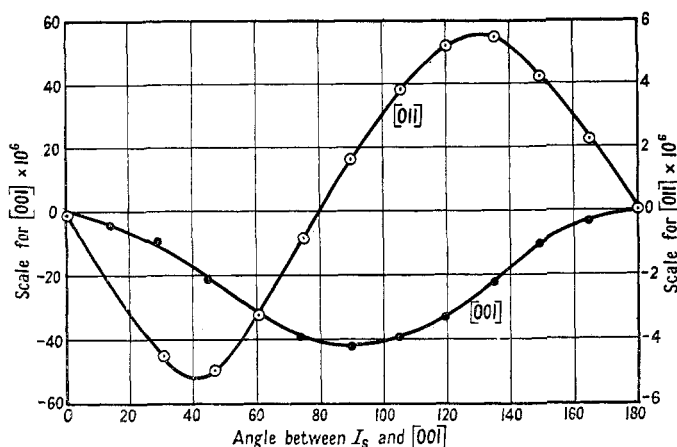


Figure 1.7. The saturation magnetostriction in the (100) plane of a single crystal of 2.52% Si-Fe, measured in the [001] and [011] directions as a function of the angle between the [001] direction and the direction of saturation magnetization (Carr and Smoluchowski 1951).

magnetization can be written  $I = I_s \overline{\cos \theta}$ , where  $\overline{\cos \theta}$  is the mean value of  $\cos \theta$ , the projection of  $I_s$  for each domain along the direction of  $H$  (we ignore complications due to demagnetizing effects). Similarly the mean magnetostriction observed in the material is  $\frac{2}{3} (\overline{\cos^2 \theta} - \frac{1}{3})$ . This would lead us to suppose that the magnetostriction-magnetization curves could likewise be characterized by a single constant. Inspection of figures 1.2 and 1.3 shows that over the same range of magnetization there is considerable diversity of behaviour between the various materials. Thus the likelihood of this supposition being correct is small.

We are thus led to the conclusion that the assumptions on which equation (1.2) is derived are invalid. Moreover, the fact that the behaviour of random polycrystals can be represented by equation (1.2), whereas that of single crystals in general cannot, suggests that the main error lies not in breaking off a series expansion at the first term but in taking no account of the symmetry of the crystal structure of the materials whose behaviour is to be described. This will be rectified in the next section.

## § 2. PHENOMENOLOGICAL THEORY OF MAGNETOSTRICTION IN CRYSTALS

In this section we show the way in which the spontaneous lattice strain must depend on the direction of the spontaneous magnetization in the lattice, and how the dependence is obtained from considerations of lattice symmetry. (N.B. Throughout this section and the next a tensor notation is used. It is to be understood that the occurrence of repeated suffixes indicates summation in the conventional manner.)

### 2.1. General

Consider a point P in a crystal lattice, whose distance from some fixed lattice point is  $r_0$ , the components of  $r_0$  being  $x_0$ ,  $y_0$  and  $z_0$ . If this lattice is subject to a homogeneous deformation the new distance of the point P will be  $r$  with components  $x$ ,  $y$ ,  $z$  such that

$$\begin{aligned} x &= x_0 + A_{11}x_0 + A_{12}y_0 + A_{13}z_0 \\ y &= y_0 + A_{21}x_0 + A_{22}y_0 + A_{23}z_0 \\ z &= z_0 + A_{31}x_0 + A_{32}y_0 + A_{33}z_0 \end{aligned}$$

The strain tensor  $A_{ij}$  is symmetric, hence  $A_{ij}=A_{ji}$ . Introducing the direction cosines of the point P by the relations  $\beta_1=x_0/r_0$ ,  $\beta_2=y_0/r_0$ ,  $\beta_3=z_0/r_0$  it is readily found that  $x=r_0(\beta_i+A_{1j}\beta_j)$  with similar expressions for  $y$  and  $z$ . Hence  $r^2=r_0^2(1+2A_{ij}\beta_i\beta_j)$  neglecting products of the  $A_{ij}$ , and thus  $r-r_0=r_0A_{ij}\beta_i\beta_j$  so that the strain in any direction whose direction cosines are  $\beta_1, \beta_2, \beta_3$  is related to the components of the strain tensor by the expression

$$dl/l=A_{ij}\beta_i\beta_j. \quad \dots\dots(2.1)$$

The corresponding volume strain is  $dV/V=A_{11}+A_{22}+A_{33}$ .

As stated earlier it is possible to explain magnetostriiction by assuming that the crystal lattice is spontaneously strained by the spontaneous magnetization, the strain depending in some way on the direction of the spontaneous magnetization with respect to the crystal axes. Thus  $A_{ij}=f(\alpha_1, \alpha_2, \alpha_3)$  and the problem is to find the form of this function. It turns out that the form which  $f$  must take is completely determined by the requirements of lattice symmetry. To do this the usual procedure is to express the total energy of the crystal and then determine the equilibrium condition from the requirement that this energy should be a minimum.

This energy is made up of three parts which, following Becker and Döring, can be designated the crystal energy  $E_K$ , which is simply the crystal anisotropy energy, the magnetostriiction energy  $E_M$  and the elastic energy  $E_L$ .  $E_K$  depends only on  $\alpha_i$  whilst  $E_L$  depends only on the components of the strain tensor  $A_{ij}$ .  $E_M$  on the other hand is assumed to be a function of both  $A_{ik}$  and  $\alpha_i$ .  $E_M$  therefore represents the interaction between the elastic and anisotropy energies. Thus it is possible to write, purely formally,

$$\begin{aligned} E_K &= a_i \alpha_i + a_{ij} \alpha_i \alpha_j + \dots & + a_{ijkl} \alpha_i \alpha_j \alpha_k \alpha_l \\ E_M &= & b_{ijk} \alpha_i A_{jk} + b_{ijkl} \alpha_i \alpha_j A_{kl} \\ E_L &= & c_{ijkl} A_{ij} A_{kl} \end{aligned}$$

Each of these terms must satisfy the requirements of crystal symmetry since the energy is a scalar and independent of the choice of axes. The quantities  $a_{ij}$ ,  $b_{ijk}$  etc. are constants of the material and must conform to the symmetry of the crystal lattice. Thus these quantities must be invariant when subjected to the operations which characterize the symmetry properties of the crystal under investigation. The most general method for performing these operations is due to Seitz (1934) and is described in detail by Bitter (1937). It is shown that each of the constants must obey transformations of the form

$$a_{ijkl}=P_{ip}P_{jq}P_{kr}P_{ls}a_{pqrs}. \quad \dots\dots(2.2)$$

The quantities  $P_{ip}$  etc. are the symmetry operators for the particular type of crystal under consideration. In general the requirements of symmetry impose severe restrictions on the number of independent constants. Fortunately the ferromagnetic metals nickel, iron and cobalt all belong to crystal classes of high symmetry and in consequence the expressions for  $E_K$ ,  $E_M$  and  $E_L$  can be reduced to fairly simple forms.

## 2.2. Cubic Crystals

Crystals of iron and nickel are body- and face-centred cubic respectively and for these crystals class 0<sup>h</sup> the symmetry operators are

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

These will be referred to as I, II and III. The relations to be satisfied are those of equation (2.2) where the  $P$ 's are any one of the three matrices above.

From III it follows that all terms containing an odd number of subscripts must vanish. For cubic crystals these could have been used from the start for it follows immediately from the fact that they possess a centre of symmetry. II permits cyclic interchange of subscripts  $1 \rightarrow 3$ ,  $2 \rightarrow 1$  and  $3 \rightarrow 2$ , without altering the value of the term. From I it follows that the subscripts 2 and 3 may be interchanged but each interchange is to be accompanied by a change of sign. Furthermore by definition all subscripts in the  $E_K$  terms may be interchanged without altering their value whilst for the  $E_M$  and  $E_L$  terms  $i$  and  $j$  and also  $k$  and  $l$  may be interchanged. The energy terms become

$$E_K = K_1(a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2) \quad \dots\dots(2.3)$$

dropping a constant term and replacing the constant  $a_{1111}$  by  $K_1$ . The elastic energy term contains only three independent constants and is of the form

$$E_L = \frac{1}{2}c_{11}(A_{11}^2 + A_{22}^2 + A_{33}^2) + c_{12}(A_{11}A_{22} + A_{22}A_{33} + A_{33}A_{11}) \\ + 2c_{44}(A_{12}^2 + A_{13}^2 + A_{23}^2). \quad \dots\dots(2.4)$$

The magnetostriction part  $E_M$  involves only two independent constants (see for example Bitter p. 175).  $B_1 = b_{1111}$ ,  $2B_2 = b_{1212}$ . The  $B_1$  and  $B_2$  are then identical with the magnetoelastic coupling constants of Kittel (1949). Thus

$$E_M = B_1(a_1^2 A_{11} + a_2^2 A_{22} + a_3^2 A_{33}) \\ + 2B_2(a_1 a_2 A_{12} + a_2 a_3 A_{23} + a_3 a_1 A_{31}). \quad \dots\dots(2.5)$$

The total energy density depending on strain and on the direction of the spontaneous magnetization is the sum of these three terms. Thus

$$E = K_1(a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2) + \frac{1}{2}c_{11}(A_{11}^2 + A_{22}^2 + A_{33}^2) \\ + c_{12}(A_{11}A_{22} + A_{22}A_{33} + A_{33}A_{11}) + 2c_{44}(A_{12}^2 + A_{13}^2 + A_{23}^2) \\ + B_1(a_1^2 A_{11} + a_2^2 A_{22} + a_3^2 A_{33}) \\ + 2B_2(a_1 a_2 A_{12} + a_2 a_3 A_{23} + a_3 a_1 A_{31}). \quad \dots\dots(2.6)$$

The equilibrium configuration is obtained by setting  $\partial E / \partial A_{ij} = 0$ . This gives six equations, viz.

$$\frac{\partial E}{\partial A_{11}} = c_{11}A_{11} + c_{12}(A_{22} + A_{33}) + B_1 a_1^2 = 0 \\ \frac{\partial E}{\partial A_{22}} = c_{11}A_{22} + c_{12}(A_{33} + A_{11}) + B_1 a_2^2 = 0 \\ \frac{\partial E}{\partial A_{33}} = c_{11}A_{33} + c_{12}(A_{11} + A_{22}) + B_1 a_3^2 = 0 \\ \frac{\partial E}{\partial A_{12}} = 4c_{44}A_{12} + 2B_2 a_1 a_2 = 0 \\ \frac{\partial E}{\partial A_{13}} = 4c_{44}A_{13} + 2B_2 a_1 a_3 = 0 \\ \frac{\partial E}{\partial A_{23}} = 4c_{44}A_{23} + 2B_2 a_2 a_3 = 0. \quad \dots\dots(2.7)$$

The solutions are

$$\begin{aligned} A_{ii} &= \frac{B_1 \{c_{12} - (c_{11} + 2c_{12})\alpha_i^2\}}{(c_{11} - c_{12})(c_{11} + 2c_{12})} \\ A_{ij} &= -\frac{B_2 \alpha_i \alpha_j}{2c_{44}}. \end{aligned} \quad \dots\dots(2.8)$$

Now from equation (2.1)

$$dl/l = \sum A_{ii}\beta_i^2 + 2 \sum_{i>j} A_{ij}\beta_i\beta_j.$$

Substituting the values of  $A_{ii}$  and  $A_{ij}$  given above we have

$$\begin{aligned} \frac{dl}{l} &= -\frac{B_1}{c_{11} - c_{12}} (\alpha_1^2 \beta_1^2 + \alpha_2^2 \beta_2^2 + \alpha_3^2 \beta_3^2) \\ &\quad - \frac{B_2}{c_{44}} (\alpha_1 \alpha_2 \beta_1 \beta_2 + \alpha_2 \alpha_3 \beta_2 \beta_3 + \alpha_3 \alpha_1 \beta_3 \beta_1) \\ &\quad + \frac{3c_{12}B_1}{(c_{11} - c_{12})(c_{11} + 2c_{12})}. \end{aligned} \quad \dots\dots(2.9)$$

By common consent for a demagnetized crystal with a perfectly random distribution of domains  $dl/l=0$ . In this state the mean values  $\overline{\alpha_i^2}$  and  $\overline{\alpha_i \alpha_j}$  are respectively equal to  $\frac{1}{3}$  and 0 both when [100] and [111] are easy directions. Therefore

$$\begin{aligned} dl/l &= h_1 (\alpha_1^2 \beta_1^2 + \alpha_2^2 \beta_2^2 + \alpha_3^2 \beta_3^2 - \frac{1}{3}) \\ &\quad + 2h_2 (\alpha_1 \alpha_2 \beta_1 \beta_2 + \alpha_2 \alpha_3 \beta_2 \beta_3 + \alpha_3 \alpha_1 \beta_3 \beta_1). \end{aligned} \quad \dots\dots(2.10)$$

It may be noticed in addition that the volume change produced by the spontaneous magnetization is

$$\frac{dV}{V} = A_{11} + A_{22} + A_{33} = \frac{B_1}{c_{11} + 2c_{12}} \quad \dots\dots(2.11)$$

so that this volume strain is related to the magnetostriiction constant  $h_1$ . It is however a good deal smaller and to this approximation is independent of  $\alpha_i$ .

If the two-constant equation given by equation (2.10) is not adequate to describe the experimental results, then higher terms must be added. This is done by adding a term  $b_{ijklmn} \alpha_i \alpha_j \alpha_k \alpha_l A_{mn}$  to  $E_M$  and proceeding as before. It turns out (Becker and Döring 1939 p. 132) that in the next approximation five constants are required and the expression is

$$\begin{aligned} dl/l &= h_1 (\alpha_1^2 \beta_1^2 + \alpha_2^2 \beta_2^2 + \alpha_3^2 \beta_3^2 - \frac{1}{3}) \\ &\quad + 2h_2 (\alpha_1 \alpha_2 \beta_1 \beta_2 + \alpha_2 \alpha_3 \beta_2 \beta_3 + \alpha_3 \alpha_1 \beta_3 \beta_1) \\ &\quad + h_4 (\alpha_1^4 \beta_1^2 + \alpha_2^4 \beta_2^2 + \alpha_3^4 \beta_3^2 + \frac{2}{3}s - \frac{1}{3}) \\ &\quad + 2h_5 (\alpha_1 \alpha_2 \alpha_3^2 \beta_1 \beta_2 + \alpha_2 \alpha_3 \alpha_1^2 \beta_2 \beta_3 + \alpha_3 \alpha_1 \alpha_2^2 \beta_3 \beta_1) \\ &\quad + h_3 s \dots \quad \text{when } K_1 > 0, \text{ e.g., iron} \\ &\quad + h_3 (s - \frac{1}{3}) \dots \quad \text{when } K_1 < 0, \text{ e.g., nickel} \end{aligned} \quad \dots\dots(2.12)$$

with

$$\begin{aligned} h_1 &= -\frac{B_1}{c_{11} - c_{12}}, & h_2 &= -\frac{B_2}{c_{44}} \\ h_4 &= -\frac{B_4}{c_{11} - c_{12}}, & h_5 &= -\frac{B_5}{c_{44}} \end{aligned} \quad \dots\dots(2.13)$$

and  $s = \alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_3^2 \alpha_1^2$ .

This equation has been adjusted so that  $dl/l=0$  for the demagnetized state as defined above taking into account that the mean values are as follows:

$$\begin{aligned} K_1 > 0, \quad \overline{\alpha_i^4} = \frac{1}{3}, \quad \overline{\alpha_i \alpha_j \alpha_k^2} = 0, \quad \overline{s} = 0 \\ K_1 < 0, \quad \overline{\alpha_i^4} = \frac{1}{9}, \quad \overline{\alpha_i \alpha_j \alpha_k^2} = 0, \quad \overline{s} = \frac{1}{3}. \end{aligned}$$

The term in  $h_3$  represents the change in volume associated with rotation of the domain magnetization vector (the constant term, given by equation (2.11), having been dropped in the zero fixing procedure). One would expect this to be small since it is zero in the lower approximation and  $h_3$  is often put equal to zero.

For a long while the only material for which sufficient data existed to enable the constants  $h_1$ – $h_5$  to be evaluated was nickel for which Becker and Döring calculated the constants from the experimental results of Masiyama (1928). The agreement between experiment and theory was not very good and it was thought that further terms would be needed to describe the experimental results accurately. However it now seems that the experimental results were at fault rather than the symmetry equations and the most recent work by Bozorth and Hamming (1953) gives the data of table 2.1.

Table 2.1. Magnetostriction Constants

Material	$h_1 \times 10^6$	$h_2 \times 10^6$	$h_3 \times 10^6$	$h_4 \times 10^6$	$h_5 \times 10^6$
Nickel	$-68.8 \pm 3.8$	$-36.5 \pm 1.9$	$-2.8 \pm 3.8$	$-7.5 \pm 5.2$	$+7.7 \pm 3.1$
78% NiFe (quenched)	$13.7 \pm 1.0$	$2.6 \pm 0.5$	$-0.3 \pm 0.8$	$1.1 \pm 1.4$	$-0.1 \pm 0.8$
78% NiFe (slowly cooled)	$20.9 \pm 0.7$	$2.8 \pm 0.3$	$1.7 \pm 0.5$	$-1.4 \pm 1.0$	$-0.2 \pm 0.5$

It had already been stated by Goldman (1950) and by Carr and Smoluchowski (1951) that for the materials which they had studied  $h_4$  and  $h_5$  were negligible when  $h_1$  and  $h_2$  were of opposite sign. It appears from the table that this is also true when  $h_1$  and  $h_2$  are of the same sign. For most materials it seems likely therefore that the magnetostriction can be represented by the two-constant expression, equation (2.10), and the use of equation (2.12) can only be regarded as extravagance.

A rather more general equation than equation (2.12) has been derived in a very elegant manner by Mason (1950) who has also taken morphic effects into account. As a result of the spontaneous lattice distortion the crystal lattice can no longer be considered to obey cubic symmetry. Thus terms appear in the expression for the total energy which are zero for a cubic crystal. Effects of this type are called 'morphic effects' (Mueller 1940) and may be taken into account formally by adding to the existing energy terms, morphic energy terms of the form  $d_{ijklmn} \alpha_i \alpha_j A_{kl} A_{mn}$ . For the ferromagnetic metals these terms are considerably smaller than the others. Such effects should be of importance in some non-metallic ferromagnetic compounds, for example cobalt ferrite ( $\lambda_{100} = 515 \times 10^{-6}$ , Bozorth and Walker 1952) and a few ferromagnetic alloys, e.g., 63% FePt, ( $\lambda_s = 170 \times 10^{-6}$ , Kussmann and v. Rittberg 1950) where the magnetostriction is large and the departure from normal symmetry is great.

It is sometimes convenient to know the value of the saturation magnetostriction measured from the ideal demagnetized state along certain crystallographic axes.

These can be obtained directly from equation (2.12) and for an ideal crystal are as follows:

$$[100], \quad \alpha_1 = \beta_1 = 1, \alpha_2 = \alpha_3 = \beta_2 = \beta_3 = 0$$

$$\lambda_{100} = \frac{2}{3}h_1 + \frac{2}{3}h_4 \quad \dots\dots(2.14)$$

$$[110], \quad \alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1/\sqrt{2}, \alpha_3 = \beta_3 = 0$$

$$\lambda_{110} = \frac{1}{6}h_1 + \frac{1}{2}h_2 + \frac{1}{12}h_4 \quad \dots\dots(2.15)$$

$$[111], \quad \alpha_i = \beta_i = 1/\sqrt{3}$$

$$\lambda_{111} = \frac{2}{3}h_2 + \frac{2}{3}h_5 \quad \dots\dots(2.16)$$

As already mentioned the two-constant equation is usually perfectly adequate to deal with the present experimental results. Putting  $h_3 = h_4 = h_5 = 0$  and making use of equations (2.12), (2.14) and (2.16) we obtain the familiar form for the magnetostriction

$$dl/l = \frac{3}{2}\lambda_{100}(\alpha_1^2\beta_1^2 + \alpha_2^2\beta_2^2 + \alpha_3^2\beta_3^2 - \frac{1}{3}) + 3\lambda_{111}(\alpha_1\alpha_2\beta_1\beta_2 + \alpha_2\alpha_3\beta_2\beta_3 + \alpha_3\alpha_1\beta_3\beta_1)$$

with  $\lambda_{100} = -\frac{2}{3}\frac{B_1}{c_{11}-c_{12}}, \lambda_{111} = -\frac{1}{3}\frac{B_2}{c_{44}} \quad \dots\dots(2.17)$

This is the simplest equation consistent with anisotropic magnetostriction and contains two constants which have the significance given above, namely that  $\lambda_{100}$  and  $\lambda_{111}$  represent the total strain when a crystal is magnetized from the ideal demagnetized state to saturation along the [100] and [111] axes. Recent work (Lee 1955) suggests that this ideal demagnetized state very rarely exists in actual crystals so it is not possible to measure the saturation magnetostriction from the actual demagnetized state and assume that these values are identical with the constants  $\lambda_{100}$  and  $\lambda_{111}$ . Therefore in view of the arbitrariness associated with the above interpretation of  $\lambda_{100}$  and  $\lambda_{111}$  it is best to regard these simply as constants of the material.

If a further approximation is made, namely that  $\lambda_{100} = \lambda_{111} = \lambda_s$  say, then equation (2.10) reduces to

$$dl/l = \frac{3}{2}\lambda_s(\cos^2\theta - \frac{1}{3}) \quad \dots\dots(2.18)$$

where  $\cos\theta = \Sigma\alpha_i\beta_i$  is the angle between the magnetization vector and the direction of observation. This expression contains no reference to the crystal axes and is therefore isotropic. The values usually assigned to  $\lambda_s$  are  $-34 \times 10^{-6}$  for nickel and  $-7 \times 10^{-6}$  for iron though in neither case does equation (2.18) fit the experimental results at all well. The two cases in which equation (2.18) may be used with justification are (i) for a polycrystal at saturation as mentioned in § 1.3, and (ii) for the individual domains—and thence to describe the magnetostriction-magnetization curves by a suitable averaging process—in a polycrystalline sample which has been cold worked so that the easy directions are determined entirely by the local strain anisotropy. These requirements are satisfied by cold-worked materials with low crystal anisotropy and large magnetostriction constants, e.g. nickel. For iron the crystal anisotropy is always greater than the strain anisotropy so that equation (2.18) cannot under any circumstances be used to describe the magnetostriction-magnetization curves.

It may be mentioned in passing that equations (2.10) and (2.12) show that the transverse magnetostriction, which is that obtained when the measuring direction

is perpendicular to the direction of magnetization, is simply a trivial special case of the longitudinal magnetostriction since it merely involves different values of  $\beta_i$ .

### 2.3. Hexagonal Crystals

Some ferromagnetics, e.g. cobalt and cobalt-rich alloys, form hexagonal crystals and it is of interest to know the form that the magnetostriction must take in this case. The problem is approached in exactly the same way as outlined above for cubic crystals. This has been done by Bitter (1937) and independently by Mason and Lewis (1954). Bitter's expression is (including a term in  $k_1$  independent of  $\alpha_1$  and omitted by him)

$$\begin{aligned} dl/l = & k_1(\alpha_1^2 - 1)\beta_1^2 + k_2(\alpha_2^2\beta_2^2 + \alpha_3^2\beta_3^2) \\ & + k_3(\alpha_3^2\beta_2^2 + \alpha_2^2\beta_3^2) + 2(k_2 - k_3)\alpha_2\alpha_3\beta_2\beta_3 \\ & + 2k_4\alpha_1\beta_1(\alpha_3\beta_3 + \alpha_2\beta_2) \end{aligned} \quad \dots\dots(2.19)$$

where the  $\alpha_i$  and the  $\beta_i$  are with respect to orthogonal axes such that  $Oz$  corresponds to the  $c$  axis and  $Ox$  or  $Oy$  to one of the hexagonal axes. The expression obtained by Mason and Lewis can be put in the same form as equation (2.19). This is the first approximation and corresponds to equation (2.10) for cubic crystals. Four constants are thus required to describe the behaviour. Mason and Lewis show that equation (2.19) is the same as that obtained for cylindrical symmetry. Terms involving  $\alpha_i^4$ , i.e. those involving sixth order tensors, have to be employed before a characteristic hexagonal term appears. In this case eleven constants are required unless the saturation magnetization is parallel or perpendicular to the direction of measurement, in which case the number of constants reduces to eight.

The constants  $k_1$ – $k_4$  have been measured by Bozorth and Sherwood (1954) who obtain  $k_1 = -110$ ,  $k_2 = -45$ ,  $k_3 = -95$  and  $k_4 = -235$ , all  $\times 10^{-6}$ . According to these authors equation (2.19) (cylindrical symmetry) is an adequate approximation for cobalt. Further details are given by Bozorth (1955).

The forms of the expressions for the magnetostriction and anisotropy energy for crystals of lower symmetry have been obtained by Mason (1954). In the following sections the discussion will be confined to cubic crystals.

## § 3. ORIGINS OF MAGNETOSTRICTION

### 3.1. Formal Origin

So far the idea of a spontaneous lattice distortion has been introduced in a completely *ad hoc* manner. Moreover, the way in which the magnetocrystalline and elastic energies are supposed to interact to give the magnetostriction energy in the form stated is left unsaid. The origin of this term has been pointed out by Kittel (1949) whose treatment we now follow.

It is reasonable to suppose that the crystal anisotropy energy like the exchange energy will depend on the interatomic distance, i.e. on the state of strain of the crystal lattice. To express the strain dependence of this energy, the latter is expanded in a Taylor series, viz.

$$E_K = E_K^0 + \frac{\partial E_K}{\partial A_{ij}} A_{ij} + \frac{1}{2} \frac{\partial^2 E_K}{\partial A_{ij} \partial A_{kl}} A_{ij} A_{kl} + \dots \quad \dots\dots(3.1)$$

In this expression  $E_K^0$  refers to the undistorted lattice and must satisfy cubic symmetry. The second term represents what we have in § 2.2 called  $E_M$ , represent-



ing the interaction between the magnetic anisotropy energy and strain. The two are equivalent if

$$\frac{\partial E_K}{\partial A_{ij}} A_{ij} = b_{ijkl} A_{kl}$$

which on account of symmetry was reduced to two terms only. Hence

$$\frac{\partial E_K}{\partial A_{ii}} = B_1 a_i^2, \quad \frac{\partial E_K}{\partial A_{ij}} = 2B_2 a_i a_j$$

so that the constants  $B_1$  and  $B_2$  called by Kittel magnetoelastic coupling constants represent the strain gradient of the anisotropy energy. The third term in equation (3.1) represents the morphic energy terms. This gives a physical explanation for the occurrence of magnetostriction which, though implicit in the work of Van Vleck (1937) does not seem to have been stated until Kittel's formulation. The formal origin of magnetostriction lies in the fact that the anisotropy energy varies with strain (i.e. interatomic distance) and if the strain is such as to decrease, numerically, the anisotropy energy, then the lattice will strain spontaneously.

There are two great merits in this realization. In the first place any physical theory of anisotropy in terms of interatomic interactions is, in principle, automatically adequate for the magnetostriction and so the two problems are reduced to one. This is actually rather less useful than may appear at first sight since there is at present no detailed satisfactory theory of ferromagnetic anisotropy. Secondly Kittel shows that the solutions of equation (2.7) depend on the direction cosines in such a way that the energy in the equilibrium state given by equation (2.6) may be expressed in the form

$$E_K = (K^0 + \Delta K) (a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2).$$

Here  $K^0$  refers to the unstrained crystal whilst  $\Delta K$  represents the contribution to the anisotropy caused by magnetostriction. The measured anisotropy constant  $K_1 = K^0 + \Delta K$ . It may be shown from equations (2.6), (2.8), (2.13)–(2.16) that

$$\Delta K = \frac{9}{4} \left\{ (c_{11} - c_{12}) \lambda_{100}^2 - 2c_{44} \lambda_{111}^2 \right\}. \quad \dots\dots(3.2)$$

Some constants for iron and nickel are given in table 3.1.

Table 3.1

	$c_{11}$	$c_{12}$	$c_{44}$	$\lambda_{100}$	$\lambda_{111}$	$K_1$	$K^0$	$\Delta K$
	dyn cm <sup>-2</sup> × 10 <sup>12</sup>			× 10 <sup>6</sup>		ergs cm <sup>-3</sup> × 10 <sup>4</sup>		
Fe	2.41 <sup>1</sup>	1.46 <sup>1</sup>	1.12 <sup>1</sup>	20.7 <sup>1</sup>	−21.2 <sup>2</sup>	+47 <sup>3</sup>	47.14	−0.14
Ni	2.50 <sup>4</sup>	1.60 <sup>4</sup>	1.185 <sup>4</sup>	−45.9 <sup>5</sup>	−24.3 <sup>5</sup>	−5.9 <sup>3</sup>	−6.01	+0.11

<sup>1</sup> Kimura and Ohno (1934)

<sup>2</sup> Carr, W. J., quoted in Carr and Smoluchowski (1951)

<sup>3</sup> Bozorth, private communication

<sup>4</sup> Bozorth, Mason and McSkimmin (1951)

<sup>5</sup> Bozorth and Hamming (1953) (assuming  $h_3 = h_4 = h_5 = 0$ )

Thus  $\Delta K/K_1 \sim 2 \times 10^{-3}$  for iron and  $\sim 10^{-2}$  for nickel.  $\Delta K$  will be large when one of the constants (particularly  $\lambda_{111}$ ) is numerically much greater than the other.

### 3.2. Theoretical Treatments

Having given the formal origin of the spontaneous lattice strain, it is now necessary to consider the fundamental interatomic forces responsible.

The earliest serious attempts to explain the origin of magnetostriction were made by Akulov (1928) and Becker (1930) who calculated the magnetostriction that would exist if the forces were purely magnetic in origin. Each lattice point in a ferromagnetic crystal is regarded as possessing a magnetic dipole moment, all the dipoles being parallel at the absolute zero where the calculations are intended to apply. Each dipole interacts with its neighbours, the interaction being represented classically by a scalar magnetic potential

$$V = \sum_{i,j} \frac{\mu_i \mu_j}{r_{ij}^3} \left\{ 1 - \cos^2 (\mu_i, r_{ij}) \right\} \quad \dots\dots(3.3)$$

where the summation is over the entire crystal. It can be shown (Kittel 1949) that such a potential does not give rise to magnetic anisotropy in the first approximation either for a body or a face-centred cubic lattice. If however the lattice is allowed to deform spontaneously there is a small contribution given by equation (3.2). To this magnetic dipole energy is added the elastic energy and the whole is minimized with respect to the strains to find the equilibrium conditions. It turns out that the state of lowest energy is that in which the crystal is distorted from its otherwise cubic form. On Becker's calculations the magnetostriction constants are

$$\lambda_{100} = \frac{2SI_s^2}{c_{11} - c_{12}}, \quad \lambda_{111} = \frac{4}{3} \frac{SI_s^2}{c_{44}} \quad \dots\dots(3.4)$$

for a body-centred lattice whilst for a face-centred cubic lattice the constants are just one half these. Here  $I_s$  is the spontaneous magnetization equal to  $N\mu$  where  $N$  is the number of interacting dipoles per unit volume, each of moment  $\mu$ , and  $S$  is a constant depending on the geometry of the crystal lattice, being equal to 0.4 for body-centred cubic and 0.6 for face-centred cubic. Akulov's result differs somewhat from that of Becker and this was shown by Powell (1931) to be due to the fact that since  $V$  involves a summation over all lattice points in the crystal the effect of crystal shape must come into play. Powell shows that Becker's calculations apply to a spherical crystal whereas Akulov's results hold for an infinite flat sheet.

The magnetostriction and anisotropy constants due to dipolar forces calculated from equations (3.2) and (3.4) are given in table 3.2 and compared with experiment.

Table 3.2. Dipolar Magnetostriction and Anisotropy of Iron and Nickel

	$\lambda_{100} \times 10^6$	$\lambda_{111} \times 10^6$ calculated	$K_1 (= \Delta K)$ $\times 10^{-4}$	$\lambda_{100} \times 10^6$	$\lambda_{111} \times 10^6$ observed	$K_1 \times 10^{-4}$
Fe	4.9	-1.5	+0.004	20.7	-21.2	+47
Ni	0.15	-0.05	+0.00004	-45.9	-24.3	-5.9

It is evident that dipolar interactions can account for only a very small part of the observed anisotropy and magnetostriction of most ordinary materials. In addition the signs of  $K_1$  and  $\lambda_{100}$  and  $\lambda_{111}$  are completely determined by the expressions (3.2) and (3.4). This is contrary to experience which indicates that these constants can have any sign whatever.

Vonsovsky (1940) has calculated the dipolar magnetostriction on a quantum mechanical basis and obtains an expression for the (isotropic) magnetostriction of the form  $\lambda = \{S_1 + S_2 \cos^2 \theta\}/C$  in which  $C$  is an elastic constant and  $\theta$  has the same significance as in equation (2.18).  $S_1$  and  $S_2$  are constants involving the wave functions corresponding to the atoms in their appropriate quantum state and which may, in principle, have either positive or negative sign.

The greater part of the observed magnetostriction, like the anisotropy, is believed to have its origin in the spin-orbit interaction within the atoms. In ferromagnetic crystals the orbital moments are very largely quenched owing to inhomogeneous crystalline electric fields resulting in a kind of antiferromagnetic alignment of the orbital moments. The energy of orbital momentum depends not only on the orientations of the orbital magnetic vectors relative to each other but also on their orientation with respect to the line joining the two atoms. However the spin moments are coupled to the orbital moments (and hence to the crystal lattice) by the spin-orbit coupling. This viewpoint was suggested by Bloch and Gentile (1931) as a possible explanation of ferromagnetic anisotropy and was subsequently developed by Van Vleck (1937). After making rather large approximations and simplifications the latter was able to show that spin-orbit interactions of this nature simulated dipolar coupling (or quadrupolar coupling if  $j=1$ ) in that the interaction energy is of the same form as equation (3.3) but with  $\mu_i \mu_j r_{ij}^{-3}$  replaced by a constant  $S$ , say, which has its origin in the spin-orbit coupling and is between ten and one hundred times larger than  $\mu_i \mu_j r_{ij}^{-3}$ . Van Vleck shows that  $S$  is of the right order of magnitude to explain ferromagnetic anisotropy and, in principle, magnetostriction as well.

The subject has been treated by Vonsovsky (1940) whose calculations, based on a Heisenberg model, take into account both spin-spin and spin-orbit interaction. His final expressions may be written in the form

$$\lambda_{100} = -\frac{2}{3} \frac{I_s^2/I_\infty^2}{c_{11}-c_{12}} N \left\{ A - B \left[ \frac{1}{C} (1 - e^{-C/kT}) - \frac{1}{4D} (1 - e^{-D/kT}) \right] \right\}$$

$$\lambda_{111} = -\frac{1}{3} \frac{I_s^2/I_\infty^2}{c_{44}} N \left\{ A' - B' \left[ \frac{1}{C} (1 - e^{-C/kT}) - \frac{1}{4D} (1 - e^{-D/kT}) \right] \right\}$$

in which  $N$  is Avogadro's number and  $A, B, A', B', C$  and  $D$  are constants. By ascribing plausible values to these Vonsovsky shows that  $\lambda_{100}$  and  $\lambda_{111}$  are of the correct order of magnitude. The first term in the bracket represents the spin-spin part of the magnetostriction. This always varies with temperature as  $(I_s/I_\infty)^2$ ,  $I_\infty$  being the spontaneous magnetization at absolute zero. This sort of temperature dependence has been found in polycrystalline nickel by Döring (1936). The  $B$  term represents the perturbing influence of spin-orbit interaction. Thus any departure from the law of the temperature dependence of magnetostriction  $\lambda_s \propto I_s^2$  is to be regarded as due to the effects of spin-orbit interaction.

A rather different approach to the problem has been used by Katayama (1951) who takes as a starting point the calculations of Brooks (1940) of ferromagnetic anisotropy. This uses a collective electron approximation with 'tight-binding' approximation wave functions, and spin-orbit coupling introduced as a perturbation. Effectively the only difference between the two calculations is calculating the perturbation energies. For the anisotropy the fourth order perturbation theory has to be employed since the perturbation energy is zero up to the third

order on account of the cubic symmetry. For the magnetostriction however the symmetry is destroyed by the strain and a second order treatment is sufficient. Katayama's final expressions for  $\lambda_{100}$  and  $\lambda_{111}$  are nevertheless quite complicated and for further details reference should be made to the original paper. He is able to show that the calculated magnetostriction constants are of the required order of magnitude and that their observed temperature dependence (complicated in the case of iron, see § 4) is not incompatible with theory.

From the foregoing survey it will be apparent that the difficulty of a proper theoretical treatment of magnetostriction lies not so much in our ignorance of its likely origin but in the mathematical complexities of the formulations that remain even after many drastic approximations have been made in order to make the problem tractable. In view of these difficulties it is perhaps scarcely surprising that no precise numerical estimates can be made. The calculated orders of magnitude are in reasonable agreement with experiment. At this stage it is futile to expect more.

#### § 4. FACTORS AFFECTING THE MAGNETOSTRICTION CONSTANTS

The constants  $\lambda_{100}$  and  $\lambda_{111}$  are characteristics of the material and are changed only if the interactions responsible for magnetostriction are altered. This may be brought about in several ways.

##### 4.1. Temperature

The most important influence on the magnetostriction constants is temperature. The importance of temperature measurements has been stated in § 3.2. Unfortunately measurements of the temperature variation of the magnetostriction constants present very great experimental difficulties and, with one exception to be referred to later, none of the existing measurements can be considered entirely satisfactory. Measurements of the saturation magnetostriction of polycrystalline nickel have been made by Döring (1936), Kirkham (1937), Dyakov (1947) and Sucksmith (1950). These are shown in figure 4.1. It is evident that there are

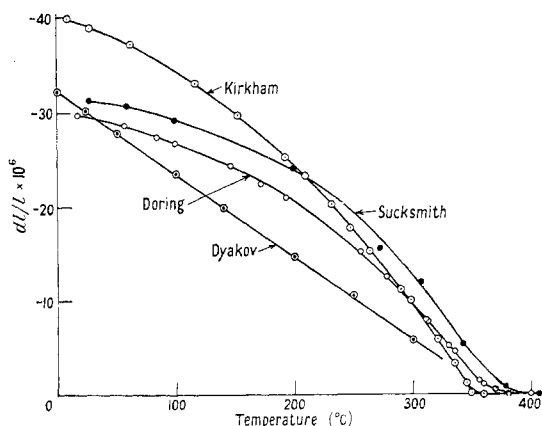


Figure 4.1. The temperature variation of the magnetostriction of polycrystalline nickel according to various observers.

considerable differences in the room temperature values and also in the actual temperature variation. This is undoubtedly caused by differences in the demagnetized state, possibly due to some preferred orientation of crystal grains. The room temperature value of Sucksmith is nearest to the accepted value and for this reason these results are probably the most reliable. The experiments of Dyakov were undertaken to provide an experimental test for the quasi-empirical relations derived by Akulov (1939) for the temperature variation of the magnetostriction, namely

$$\lambda_{100, T} = \lambda_{100, 0}(1 - T/\theta) \quad \dots\dots(4.1)$$

with a corresponding equation for  $\lambda_{111}$  and in which  $\theta$  is a constant. Dyakov's experimental results obey equation (4.1) well but they differ considerably from the results of the other workers. The magnetostriction of nickel can be approximately represented by the relation  $\lambda_s = CI_s^2$  which from § 3.2 presumably means that the effects of spin-orbit interaction are not very great.

For iron, measurements of  $\lambda_{100}$  and  $\lambda_{111}$  at different temperatures have been made by Takaki (1937) using as a reference state, that of the remanent magnetization. Here the variation is complicated and points to a considerable spin-orbit interaction. A 3.5% Si-Fe alloy is the only material for which satisfactory data is available. This is due to Shturkin (1947) who measured the strain at saturation as a function of crystal orientation. His results are shown in figure 4.2 together

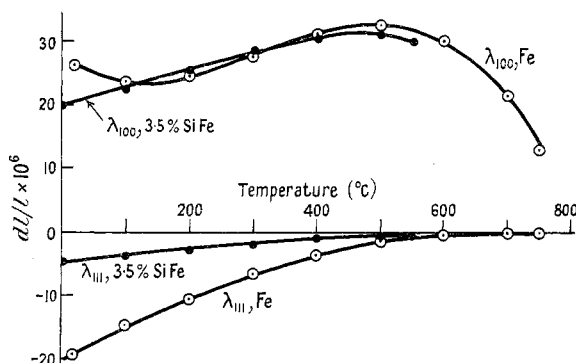


Figure 4.2. The temperature dependence of the magnetostriction constants of iron (Takaki 1936) and 3.5% Si-Fe (Shturkin 1947).

with those of Takaki for pure iron. The form of the curves for the two metals is similar, a fact which is to be expected since silicon acts simply as a diluent.

The only other materials on which any similar data exist are 15% Mo-Ni (Sucksmith 1950) and 'Alcomax' (McCaig 1952).

#### 4.2. Composition of Alloys

Another way of altering the interactions responsible for magnetostriction and thereby the magnetostriction constants is to vary the composition of alloys. The magnetostriction of binary alloys, particularly those of the ferromagnetic metals, has been extensively studied, notably by the Japanese workers, but no conclusions

seem to have been drawn from their results. The effect of alloy composition on the magnetostriction has been treated on general grounds by Vonsovsky (1940) who shows that for a binary alloy consisting of two types of atom A and B and where both A and B are magnetic, the magnetostriction should be of the form

$$\lambda_s = C (N_A^2 E_{AA} + 2N_A N_B E_{AB} + N_B^2 E_{BB}^2) \quad \dots\dots(4.2)$$

in which  $C$  is a constant and  $N_A$  and  $N_B$  are the concentrations of atoms A and B respectively.  $E_{AA}$ ,  $E_{BB}$  and  $E_{AB}$  represent the energies of interaction between like and unlike atoms. An expression exactly similar to equation (4.2) has been derived by Néel (1954a) who points out that for the nickel-iron system the magnetostriction constants can be approximately represented by

$$\begin{aligned} \lambda_{100} \times 10^6 &= -27N_A^2 + 134N_A N_B + 13N_B^2 \\ \lambda_{111} \times 10^6 &= -55N_A^2 + 340N_A N_B - 245N_B^2 \end{aligned} \quad \dots\dots(4.3)$$

over the range 40–100% Ni and in which the subscript A refers to Ni. For the nickel-rich Ni-Co alloys equation (4.2) breaks down since the magnetostriction constants are linearly proportional to the cobalt concentration (Yamamoto and Miyasawa 1953).

The magnetostriction of alloy systems in which only one component is ferromagnetic has been studied, notably by Went (1951) who investigated the magnetostriction of binary alloys of nickel with a non-magnetic metal. The conclusions drawn from the investigation were: (i) In alloys containing Al, Si, Cr, V, Cu, Mo, Sa or W, the magnetostriction constants, extrapolated to 0°K, decrease numerically, linearly as the spontaneous magnetization decreases. The results can be fitted fairly well to the equation  $\lambda_s \times 10^6 = -70\beta + 6$  where  $\beta$  is the spontaneous magnetization expressed in Bohr magnetons per atom. (ii) For alloys of Ni with Co, Mn and Fe in which the saturation moment increases with the amount of added element, a single linear relation, namely  $\lambda_s \times 10^6 = 9\beta - 90$ , is again found although this holds only for a small range of composition near 100% Ni. (iii) For Ni-Pd alloys in which the saturation moment remains approximately constant the magnetostriction becomes more negative. Pd is the only element that increases (numerically) the magnetostriction of nickel. Similar measurements, with iron as the magnetic constituent have been carried out by Messkin, Somin and Nekhamkin (1941), Went (see Snoek 1947) and Alizade (1950). In this case much less co-ordination is possible owing to the very great anisotropy of the magnetostriction of iron. Most of the results obtained in these investigations are so far completely unexplained.

#### 4.3. Crystallographic Order in Alloys

The effect of the degree of crystallographic order on the magnetostriction of binary alloys was first studied by Goldman and Smoluchowski (1949) who showed that for a 50–50 Fe-Co alloy the magnetostriction in the ordered state was considerably greater than in the disordered state. The results were interpreted in terms of a classical dipole-dipole theory. This is unfortunate for the dipolar magnetostriction of the alloy is of the order  $2 \times 10^{-6}$  whereas the measured value is  $92 \times 10^{-6}$  in the ordered state. Thus although the theoretical analysis was confirmed by experiment it is difficult to attach much importance to the confirmation. The effect of order has been treated by Vonsovsky (1940) who points

out that if equation (4.2) represents the magnetostriction of a binary alloy, then the effect of order can be taken into account by replacing equation (4.2) by

$$\lambda_s = C \{ N_A(1-pN_B)E_{AA} + 2N_A N_B p E_{AB} + N_B(1-pN_A)E_{BB} \} \quad \dots\dots(4.4)$$

in which  $p$  is a measure of the degree of order such that  $p=1$  represents complete disorder, in which case equation (4.2) is regained and  $p=2$  corresponds to perfect order. Quite apart from the limited range of application of the parent equation (4.2) however, equation (4.4) does not appear to give the right sort of variation with ordering. In an A-B type alloy, the effect of ordering is to increase the number of A-B interactions, thus to increase the second term in equation (4.2) at the expense of the first and third. From equation (4.3), for the Ni-Fe system the second term is positive so that in the range of composition for which equation (4.3) holds the effect of order is to make the constants more positive, and this is contrary to the experimental results (Bozorth 1953).

It is perhaps hardly surprising that no simple expression is able to cope with changes observed during ordering. The significant fact is that the change in the magnetostriction can be very large, increases of more than 100% being observed in some cases (Goldman 1949). The existence of such changes on ordering is of great importance in the study of high permeability materials.

## § 5. MAGNETOSTRICTION BELOW SATURATION

In this section we discuss the magnetostrictive behaviour of ferromagnetic materials, both single crystals and polycrystals, in the presence of applied fields, that is, the magnetostriction curves in which the literature of the subject abounds. This involves, in effect, the application of the symmetry equation (2.17). It is convenient to deal first with single crystals.

### 5.1. *Single Crystals*

The basic problem is, given a single crystal of known shape and size, to calculate the magnetostriction as a function of its magnetization. Equation (2.17) applies only to a single region in which the magnetization is uniform in magnitude and direction, i.e. to a single domain. Therefore if this equation is to be extended to a complete crystal the domain structure existing in it must be known for all values of the resultant magnetization. Since the domain structure will depend on the shape and size of the crystal it will be realized that this knowledge is not normally available. Even if it were, there would still remain the problem of how to calculate the mean strain in a crystal in which the various domains are deformed differently. In the general case an accurate detailed form of the magnetostriction curves ( $d\ell/\ell$ ,  $H$  or  $d\ell/\ell$ ,  $I$ ) is therefore hardly to be expected. Fortunately a knowledge of the domain structure becomes less important as the magnetization approaches saturation and in this case magnetostriction curves can be calculated more or less exactly.

Calculations of magnetostriction curves for crystals magnetized along the three principal crystallographic directions have been made by Akulov (1931), Heisenberg (1931), Gans and von Harlem (1933), Takagi (1939) and others. As pointed out by Becker and Döring such calculations can only be carried out unambiguously when magnetization proceeds in a known manner; this usually means when it is by domain vector rotation only. The calculations are as follows (it is

assumed that  $K_1 > 0$ ; similar considerations apply for  $K_1 < 0$ . For the time being, the detailed domain structure is ignored.

[100]. Magnetization can at any stage be by  $90^\circ$  or  $180^\circ$  boundary displacements. Hence the magnetostriction cannot be calculated as a function of the magnetization unless some assumption is made concerning the distribution of these two processes as a function of the magnetization or unless the formation can be obtained by other means. This could in principle be obtained from powder pattern investigations but so far this has not been attempted.

[110]. In low fields both  $90^\circ$  and  $180^\circ$  boundary displacements may occur and so no calculation is possible. These however are complete when all the magnetization vectors are distributed symmetrically about the two easy directions lying nearest the applied field when  $I = I_s/\sqrt{2}$ . Thereafter magnetization proceeds by rotations and for  $I_s/\sqrt{2} < I < I_s$  the magnetostriction is

$$dl/l = \frac{1}{4}\lambda_{100} + \frac{3}{4}\lambda_{111}(2I^2/I_s^2 - 1). \quad \dots\dots(5.1)$$

[111]. Again  $90^\circ$  and  $180^\circ$  boundary displacements can occur in low fields although the latter are the more likely. For these  $dl/l = 0$ . Moreover from the symmetry of the direction of observation  $dl/l = 0$  for the  $90^\circ$  boundary movements as well. These are complete when  $I = I_s/\sqrt{3}$  whence the magnetization can only be increased by rotations. For  $I_s/\sqrt{3} < I < I_s$

$$dl/l = \frac{1}{2}\lambda_{111}(3I^2/I_s^2 - 1). \quad \dots\dots(5.2)$$

The equations (5.1) and (5.2) are shown in figure 5.1 together with some experimental points.

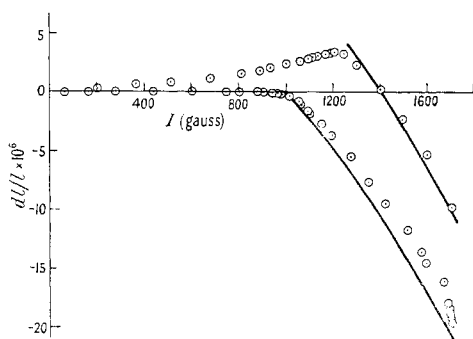


Figure 5.1. The magnetostriction of single crystals of iron in the region of domain rotation as a function of the intensity of magnetization.

Full curves calculated from equations (5.1) and (5.2) with

$$I_s = 1720, \lambda_{100} = 20.7 \times 10^{-6}, \lambda_{111} = -21.2 \times 10^{-6}$$

Experimental points due to Kaya and Takaki (1935).

It is also possible, in principle, to calculate magnetostriction curves in directions other than these three for crystals of suitable shapes since the basic magnetization processes are known from the work of Lawton and Stewart (1948). This has so far not been carried out however and in any case there are no reliable experimental data with which to compare the results.

Somewhat more refined calculations have been made by Lee (1955) for



magnetization along [110] using for the domain structure the model proposed by Néel (1944). For a long strip parallel to the [110] direction it is shown that the magnetostriction depends on the width of the crystal employed.

### 5.2. Polycrystals

For polycrystals the situation is still more complicated since in addition to all the difficulties mentioned in connection with single crystals there is the additional one of interaction between the crystallites. The mean magnetization in each crystal grain will in general be influenced by that of its neighbours and hence the properties of a polycrystalline aggregate will not necessarily be obtained by simply averaging the known behaviour of single crystals over all relevant orientations. In the case of magnetostriction it is by no means certain how to calculate the mean magnetostriction of such an aggregate since the strain in the isolated individual crystallites is not the same. This fact has been stressed by Vladimirsky (1943) who points out that in polycrystalline material the mechanical stresses and the elastic properties must vary from point to point in a thoroughly non-uniform manner. Vladimirsky shows that the problem is capable of an exact solution only if the shape of the crystallites and the elastic properties of the medium are known.

As mentioned in § 1.3 the saturation magnetostriction of a polycrystal varies with the direction of measurement according to  $dl/l = \frac{2}{3}\lambda_s(\cos^2\theta - \frac{1}{3})$ . The first problem is to calculate  $\lambda_s$  in terms of the single crystal constants  $h_1 - h_5$ . The assumption usually made is that the stress is uniform throughout the material. It is then possible to show by averaging equation (2.12) over all possible directions that

$$\lambda_s = \frac{4}{15}h_1 + \frac{2}{5}h_2 + \frac{8}{35}h_4 + \frac{2}{35}h_5. \quad \dots\dots(5.3)$$

Using the two-constant formula, equation (2.17), it is found that

$$\lambda_s = \frac{2}{5}\lambda_{100} + \frac{3}{5}\lambda_{111} \quad \dots\dots(5.4)$$

a relation first derived by Akulov (1930).

Making the assumption that the crystallites are spherical and surrounded by an infinite uniform isotropic solid, characterized by the same values of the elastic constants as the polycrystal as a whole, Vladimirsky obtains the expression

$$\lambda_s = \frac{1}{5} \frac{(c_{11} - c_{12})}{\mu} C_1 \lambda_{100} + \frac{3}{5} \frac{c_{44}}{\mu} C_2 \lambda_{111} \quad \dots\dots(5.5)$$

where  $C_1$  and  $C_2$  are constants relating the strain within and outside the crystal and involve  $c_{11}$ ,  $c_{12}$ ,  $c_{44}$ ,  $\lambda$  and  $\mu$ , the latter two being the isotropic elastic constants (Lamé constants) as defined by Love (1936). In the case of uniform strain (an unlikely state of affairs) this expression reduces to

$$\lambda_s = \frac{1}{5} \frac{c_{11} - c_{12}}{\mu} \lambda_{100} + \frac{3}{5} \frac{c_{44}}{\mu} \lambda_{111} \quad \dots\dots(5.6)$$

whilst for uniform stress equation (5.4) is obtained. In table 5.1 values of  $\lambda_s$  obtained from equations (5.4)–(5.6) are shown and compared with experiment. On the whole the assumption of uniform stress seems to be about the best approximation though it should be stated that  $\lambda_s$  is not accurately known for iron.

Table 5.1

Magnetostriction Constants (all  $\times 10^6$ ) of Polycrystalline Iron and Nickel calculated from the Formula of Vladimírsky (eqn 5.5) and for Uniform Strain (5.6) and Uniform Stress (5.4)

	$\lambda_{100}$	$\lambda_{111}$	$\lambda_s$ (5.5)	$\lambda_s$ (5.6)	$\lambda_s$ (5.4)	$\lambda_s$ (exptl)
Fe	20.7	-21.2	-8.9	-13.0	-4.4	-8.0 <sup>1</sup>
Ni	-45.9	-24.3	-31.1	-46.9	-32.9	-34 <sup>2</sup>

<sup>1</sup> Kornetzki (1934)    <sup>2</sup> Goldman (1947)

Below saturation the problem is even more complicated since the strain in the individual crystallites is not accurately known. Near saturation it is possible to express the magnetostriction as a function of the applied magnetic field strength as

$$\frac{dl}{l} = \frac{1}{5} \left( 2\lambda_{100} + 3\lambda_{111} \right) + \left( \lambda_{100} - \lambda_{111} \right) \left( \frac{8}{35} \frac{K}{I_s H} - \frac{32}{1001} \frac{K^2}{I_s^2 H^2} \right) G - \lambda_{111} \frac{8}{35} \frac{K^2}{I_s^2 H^2} G \quad \dots\dots(5.7)$$

where  $G$  is Néel's factor (1948b) for the effects of crystallite interaction. This expression, without the factor  $G$ , was first derived by Dyakov (1951) and independently by Lee (1954). The effect of magnetic interactions between the crystal grains was first correctly taken into account by Néel (1954b). Since the magnetization can be expressed in the form

$$I = I_s \left( 1 - \frac{8}{105} \frac{K^2}{I_s^2 H^2} G \right) \quad \dots\dots(5.8)$$

it is possible to obtain  $dl/l$  as a function of  $I$  but the range over which equation (5.7) is valid—approximately  $0.97I_s < I < I_s$  is too small to be of much use.

If the approximation of isotropic magnetostriction is acceptable then it is possible to obtain the magnetostriction as a function of magnetization in the following manner. Assume that at any stage of magnetization, the domain magnetization vectors are distributed uniformly within a cone of semi-angle  $\theta_m$  and whose axis coincides with the direction of the applied field. Then for any domain whose magnetization vector makes an angle  $\theta$  with this direction  $dl/l = \frac{2}{3}\lambda_s(\cos^2 \theta - \frac{1}{3})$  and  $I = I_s \cos \theta$ . For the whole material  $\cos^2 \theta$  and  $\cos \theta$  must be replaced by mean values. By virtue of the assumptions made these are

$$\overline{\cos^2 \theta} = \frac{\int_0^{\theta_m} \sin \theta \cos^2 \theta d\theta}{\int_0^{\theta_m} \sin \theta d\theta}$$

and

$$\overline{\cos \theta} = \frac{\int_0^{\theta_m} \sin \theta \cos \theta d\theta}{\int_0^{\theta_m} \sin \theta d\theta}.$$

Carrying out these integrations one obtains

$$dl/l = \lambda_s \eta (2\eta - 1) \quad \dots\dots(5.9)$$

with  $\eta = I/I_s$ . Equation (5.19) may be expected to hold for  $\eta > 0.5$  when the anisotropy is uniaxial and the magnetostriction isotropic. These conditions are

fulfilled in cold worked nickel and in hexagonal cobalt. Figure 5.2 shows that the former obeys equation (5.9) fairly well. For other materials it is found that a

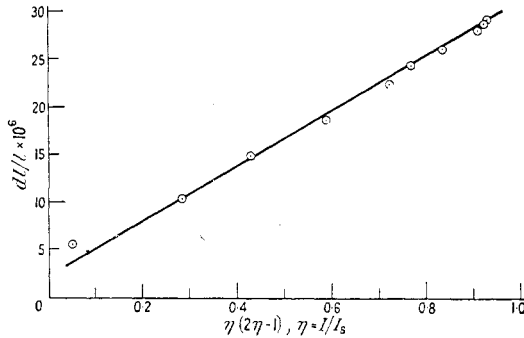


Figure 5.2. Magnetostriction of cold-worked nickel as a function of  $\eta(2\eta-1)$  (see equation (5.11)).

graph of  $dl/l$  against  $\eta(2\eta-1)$  gives two straight lines intersecting at a point corresponding to  $\eta \sim 0.9$ , i.e. roughly where the magnetization process changes from boundary displacements to domain vector rotation (see figure 5.3). However,

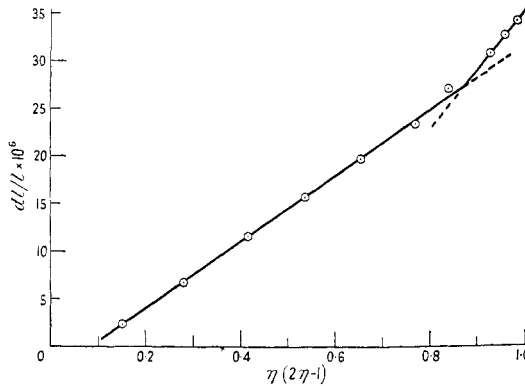


Figure 5.3. Magnetostriction of annealed nickel as a function of  $\eta(2\eta-1)$ .

for some substances, e.g. iron, it is not always possible to fit the  $(dl/l, I)$  curves to equation (5.9) at all.

In passing we may note that if equations (5.8) and (5.9) are combined there results the equation

$$\frac{dl}{l} = \lambda_s \left( 1 - \frac{8}{35} \frac{K^2}{I_s^2 H^2} G \right) \quad \dots\dots(5.10)$$

which is the isotropic form of (5.7).

If the assumption of isotropic magnetostriction is inadmissible, e.g. in the case of iron, then some fairly drastic assumptions have to be made to calculate  $dl/l$  as a function of  $I$ . In small fields magnetization proceeds by  $90^\circ$  and  $180^\circ$  boundary movements. (In nickel and for substances with  $K_1 < 0$  the  $90^\circ$  boundaries are replaced by boundaries separating domains whose magnetization

vectors make angles of  $71^\circ$  and  $109^\circ$  with each other but this does not affect the argument and it is convenient to refer to them all as  $90^\circ$  boundary movements). It is therefore impossible to calculate the magnetostriction for the reasons given in § 5.1. However in all cases the magnetization vectors lie in an easy direction and the usual assumption made (Gans 1932) is that such boundary movements continue until all the magnetization vectors lie in that easy direction lying nearest the applied field. When this state is reached the following conditions hold:

(1)  $K_1 > 0$ , easy direction [100]. Since each domain vector lies in a [100] direction, for each domain two of the  $a_i$  in equation (2.17) are zero and consequently the second bracket vanishes. Therefore  $dl/l = \frac{2}{3}\lambda_{100}(\cos^2\theta - \frac{1}{3})$ . For a polycrystal with no preferred orientation of crystal axes  $\overline{\cos^2\theta} = 0.71$  (Becker and Döring 1939). The corresponding value of  $\overline{\cos\theta} = 0.83$ . Hence

$$dl/l = 0.55\lambda_{100} \text{ when } I = 0.83I_s. \quad \dots\dots(5.11)$$

(2)  $K_1 < 0$ , easy direction [111]. Similar considerations lead to the conclusion that  $dl/l = \frac{2}{3}\lambda_{111}(\cos^2\theta - \frac{1}{3})$  with  $\overline{\cos^2\theta} = 0.758$  (Yamamoto and Miyasawa 1953) and  $\overline{\cos\theta} = 0.87$ . Thus

$$dl/l = 0.637\lambda_{111} \text{ when } I = 0.87I_s. \quad \dots\dots(5.12)$$

For values of  $I$  greater than these, magnetization must take place by rotations. ( $dl/l, I$ ) curves have been calculated by the author (unpublished) by assuming that for all values of  $I$  each domain vector lies in the plane defined by the applied field and the easy axis lying nearest it. The results show that in the rotation region both  $\lambda_{100}$  and  $\lambda_{111}$  contribute to the magnetostriction but in different proportion depending on the magnetization. Therefore one would not expect equation (5.9) to apply exactly even for rotation since  $\lambda_s$  is not strictly constant. For the case in which  $K_1 < 0$ , this variation is, however, quite small.

It is nevertheless difficult to justify the assumptions made in the previous paragraph except on the grounds of convenience. In the first place it is not possible to split up the boundary movements and the rotations as two separate occurrences, particularly if the crystal anisotropy is small. Rotations will always be present even though the rotational susceptibility may be much smaller than that due to boundary displacement. The other objection is of a rather different nature. The state of affairs assumed by Gans in which the domain vectors in each crystallite all lie in a single easy direction is such as to give a non-uniform magnetization which would give rise to free magnetic poles at the surface of each crystal grain. It is most unlikely that this is allowable except near saturation. The opposite assumption, namely that the tendency to avoid free poles is so great that the mean magnetization of each crystal grain is constant in magnitude and direction, and equal to the mean magnetization of the whole material, has been developed by Lawton and Stewart (1950). Unfortunately it does not seem possible to apply their approach to the calculation of magnetostriction, unless further information is available.

## § 6. MAGNETOSTRICTION AND THE EFFECTS OF STRESS

Since the strain in a ferromagnetic crystal is determined partly by the direction of the spontaneous magnetization, it is to be expected that if such a crystal is strained by an external force, this additional strain will in general alter the direction of the spontaneous magnetization. Usually the external force is a

simple compression or tension and it will now be shown that the effect of such a stress is to introduce an additional term to the anisotropy energy superimposed on the normal crystal energy  $E_K$ .

Suppose a uniform tension of magnitude  $\sigma$  to be applied to a crystal such that  $\gamma_1, \gamma_2, \gamma_3$  are the direction cosines of  $\sigma$  with respect to the crystal axes. The stress components are  $T_{ij} = \sigma \gamma_i \gamma_j$ . The resulting strains are:

$$A_{ii} = -\sigma \{ (s_{11} - s_{12}) \gamma_i^2 + s_{12} \}, \quad A_{ij} = -\frac{1}{2} \sigma s_{44} \gamma_i \gamma_j$$

where the  $s$ 's are elastic moduli. If these expressions are substituted in the second term of equation (3.1) there results an expression for that part of the energy depending on the direction of magnetization

$$\begin{aligned} E = & K_1 (a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2) \\ & - B_1 \sigma \{ (s_{11} - s_{12}) (a_1^2 \gamma_1^2 + a_2^2 \gamma_2^2 + a_3^2 \gamma_3^2) + s_{12} \} \\ & - B_2 \sigma s_{44} (a_1 a_2 \gamma_1 \gamma_2 + a_2 a_3 \gamma_2 \gamma_3 + a_3 a_1 \gamma_3 \gamma_1). \end{aligned}$$

Using the known relations between the elastic moduli and the elastic constants and expressing  $B_1$  and  $B_2$  in terms of  $\lambda_{100}$  and  $\lambda_{111}$  one obtains

$$\begin{aligned} E = & K_1 (a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2) \\ & - \frac{3}{2} \lambda_{100} \sigma (a_1^2 \gamma_1^2 + a_2^2 \gamma_2^2 + a_3^2 \gamma_3^2) \\ & - 3 \lambda_{111} \sigma (a_1 a_2 \gamma_1 \gamma_2 + a_2 a_3 \gamma_2 \gamma_3 + a_3 a_1 \gamma_3 \gamma_1) \end{aligned} \quad \dots \dots (6.1)$$

omitting a term independent of  $a_i$ . Thus to the crystal energy is added a term  $-\sigma dl/l$  usually known as the magnetoelastic energy. The values of  $a_i$  required to make  $E$  a minimum thus depend in a complicated manner on the magnitude and signs of  $K_1, \lambda_{100}, \lambda_{111}$  and  $\sigma$ . It may be noted that from equation (6.1) the energy depends on  $\lambda_{100}\sigma$  and  $\lambda_{111}\sigma$  so that a substance with positive magnetostriction constants, subject to tension, behaves exactly as one whose magnetostriction constants are negative, and submitted to compression. The effect of stress therefore is to create new energy minima for the domain vectors in addition to those given by the crystal anisotropy.

The general case, represented by equation (6.1) is extremely complicated for arbitrary values of the constants and no general solution has been put forth. The equations resulting by expressing  $a_i$  and  $\gamma_i$  in terms of polar coordinates  $\theta$  and  $\phi$  and setting  $\partial E / \partial \theta$  and  $\partial E / \partial \phi$  equal to zero are very complicated and cannot be solved explicitly for  $\theta$  and  $\phi$ . It is necessary therefore to consider approximate solutions in the two cases when the crystal energy is much greater than or much less than the magnetoelastic energy.

The simplest case occurs when the crystal energy is the dominant term in equation (6.1). It is then reasonable to assume that the domain vectors remain in one or other of the easy directions caused by the crystal energy alone; that is, that the magnetoelastic energy merely makes some of the energy minima of the crystal energy deeper than others. There are then two cases to be considered:

(1)  $K_1 > 0$ , easy direction [100]. In this case two of the  $a_i$  are zero for each domain and so the third bracket in equation (6.1) vanishes. If  $\lambda_{100}\sigma$  is positive then the second term has a minimum value when each domain vector lies parallel or antiparallel to that easy direction making the smallest angle with the direction of  $\sigma$ . This is the case for iron under tension. If  $\lambda_{100}\sigma$  is negative then it will have an (algebraic) minimum when the vectors make the largest possible angle with  $\sigma$ , still remaining in an easy direction.

(2)  $K_1 < 0$ , easy direction [111]. In this case  $\alpha_i^2 = \frac{1}{3}$  and so the term involving  $\lambda_{100}$  is constant, the magnetoelastic energy being determined solely by the term in  $\lambda_{111}$ . Here again this has an (algebraic) minimum or maximum when each domain vector lies parallel or antiparallel to the easy direction making the least or greatest angle with  $\sigma$ , according as  $\lambda_{111}\sigma$  is positive or negative. This would occur in nickel under very small compression or tension respectively.

In each case provided the magnetoelastic energy is smaller than the crystal energy the effect of a stress  $\sigma$  is to make the easy axis lying nearest  $\sigma$ , the deepest or shallowest minimum according as  $\lambda_e\sigma$  is positive or negative, where  $\lambda_e$  is the magnetostriction constant in the easy direction. Some consequences of this are as follows. For iron  $\lambda_{100}$  is positive, so that under tension, boundary movements occur in polycrystalline iron which cease when all the domain vectors are parallel or antiparallel to the easy direction lying nearest the tension. Then  $I=0$  by symmetry, but  $dl/l=0.55\lambda_{100}$  with respect to the demagnetized state. For the magnetostriction is unchanged if the magnetization is reversed and so this value of  $dl/l$  is the same as that obtained in § 5.2, equation (5.11). It follows that the magnetostriction at any given field strength, under tension, is equal to the magnetostriction for the same specimen without tension, less  $0.55\lambda_{100}$  (assuming that the strain does not greatly affect the 'ease' of magnetization). This is actually observed in practice. For nickel for a similar reason the saturation magnetostriction increases in magnitude with tension but this case is complicated by the fact that for quite small stresses the magnetoelastic and crystal energies are comparable in magnitude.

When  $K_1$  is small it is permissible to neglect the crystal energy and so the domain vector orientation is determined by the magnetoelastic energy alone. In the case in which  $\lambda_{100}$  is much greater or less than  $\lambda_{111}$  inspection of the last two terms in equation (6.1) shows that under tension this energy will be a minimum when the vectors lie parallel and antiparallel to the direction of greatest algebraic magnetostriction making the smallest angle with  $\sigma$ . The opposite is true for compression.

One of the consequences of the reorientation of domain vectors that occurs as a result of an external stress is that the magnetostrictive strain produced by the reorientation may actually be greater than the polycrystalline saturation magnetostriction. It follows that even though  $\lambda_{100}$  and  $\lambda_{111}$  are both positive, the saturation magnetostriction of a polycrystalline specimen under stress may be negative. This is actually observed in certain Ni-Fe alloys and was originally explained by Braunewell and Vogt (1949) in terms of a change in the magnitude and signs of  $\lambda_{100}$  and  $\lambda_{111}$  with stress. Further investigation by Elsas and Vogt (1951) showed this to be unnecessary and that all the results could be explained in terms of a reorientation process due to the applied stress.

When  $\lambda_{100}$  and  $\lambda_{111}$  are equal the magnetoelastic energy reduces to the very simple form

$$E_\sigma = -\frac{3}{2}\lambda_s \sigma \cos^2\phi \quad \dots\dots(6.2)$$

in which  $\phi$  is the angle between  $I_s$  and  $\sigma$ . This statement together with the assumption that  $K_1=0$  is the one most often used to analyse the behaviour of ferromagnetic substances under stress. If  $\lambda_s\sigma$  is positive as in 68 Permalloy under tension or nickel in compression the domain vectors align themselves parallel and antiparallel to the stress axis and the material eventually behaves like

a uniaxial crystal. The magnetization-field curve becomes rectangular and the saturation magnetostriction becomes zero. If  $\lambda_s\sigma$  is negative, the stress axis becomes a difficult one and the domain vectors orientate themselves in the plane perpendicular to this. In this state, for a magnetizing-field along the stress axis,  $\cos^2\theta = \cos^2\theta = 0$ . At saturation  $\cos^2\theta = 1$  and so the saturation magnetostriction which is the difference in strain between these two states is

$$\left(\frac{dl}{l}\right)_s = \frac{2}{3}\lambda_s(1 - \frac{1}{3}) - \frac{2}{3}\lambda_s(0 - \frac{1}{3}) = \frac{2}{3}\lambda_s$$

or one and a half times the polycrystalline magnetostriction constant. This has been verified by Kirchner (1936) for nickel.

The total energy of the system in the presence of a magnetic field is  $E = -I_s H \cos\phi - \frac{2}{3}\lambda_s\sigma \cos^2\phi$ . For equilibrium  $\partial E/\partial\phi = 0$  whence

$$I = \frac{I_s^2}{3\lambda_s\sigma} H \quad \text{and} \quad \frac{dl}{l} = \frac{2}{3}\lambda_s \frac{I^2}{I_s^2} \quad \dots\dots(6.3)$$

so that the magnetic susceptibility is constant and the magnetization curve degenerates into a straight line (Becker and Kersten 1930). The magnetostriction is proportional to the square of the magnetization (Kirchner 1936). From equation (6.3) it follows that the quantity  $\chi_a\sigma$ ,  $\chi_a$  being the initial susceptibility under tension large enough to cause complete reorientation, should be proportional to  $I_s^2/\lambda_s$  at all temperatures. From Döring's measurements  $\lambda_s \propto I_s^2$  and so  $\chi_a\sigma$  should be a constant independent of temperature. The experimental results of Scharff (1935) and Brockhouse (1953) are not in agreement with this prediction. The reason for this is not known.

The discussion so far has been limited to polycrystals since for these only do the experimental data exist. In the case of single crystals the situation is somewhat simplified by the fact that  $\sigma$  is in the same direction with respect to the crystal axes throughout. On the other hand it is now essential to take demagnetizing effects into account. A first attempt at this problem has been made by Stewart (1949) who calculated magnetization curves of single crystal Si-Fe rods subject to tension. The experimental results confirm the theoretical predictions as well as can be expected.

It has already been mentioned that, on account of symmetry, an external stress cannot, in the absence of a magnetic field, bring about the magnetization of a demagnetized material. In the presence of a field, or when the sample possesses residual magnetization, a stress can change the magnitude of the magnetization. This change is related to the magnetostriction by the thermodynamic identity

$$\left(\frac{dI}{d\sigma}\right)_H = \frac{1}{l} \left(\frac{dl}{dH}\right)_\sigma \quad \dots\dots(6.4)$$

which relates the change in magnetization produced by stress to the slope of the magnetostriction-field curve. Attempts to verify equation (6.4) have, in the past, met with little success because it holds only for reversible processes and these are not easily separated from the irreversible ones. Auwers (1944) has shown that under conditions of strict reversibility and when allowance is made for demagnetizing effects equation (6.4) is obeyed within the limits of experimental error.

For a full account of the reversible and irreversible effects of stress reference should be made to the excellent account by Bozorth (1951).

The conclusions reached in this section apply only in the case of an externally applied tension. With plastically strained materials stresses may be set up inside the metal and these will vary in magnitude and direction throughout it. Thus in order to account for the observed effects it is necessary to apply equation (6.1) or (6.2) to every region where the stress may be considered uniform and to average the results. This can be done only if the stress distribution is known.

Since an external tension is not thought to affect the magnetostriction constants it is likely that these are not greatly affected by plastic strain. Thus cold-worked materials may be used in the study of magnetostriction provided that the added strain anisotropy does not make measurements too difficult.

## § 7. VOLUME MAGNETOSTRICTION

### 7.1. General

As mentioned in the introduction, it is found that if a ferromagnetic is magnetized beyond technical saturation, the magnetostriction observed is primarily a volume change, the strain being the same in all directions. It is this volume magnetostriction to which we now turn. It is customary to study the volume change as a function of the applied field strength. The observed behaviour is usually rather complicated and will be analysed following closely the treatment of Becker (1934).

From thermodynamics it is shown that

$$\left(\frac{\partial V}{\partial H_e}\right)_p = -\left(\frac{\partial M}{\partial p}\right)_{H_e} \quad \dots\dots(7.1)$$

where  $M$  is the magnetic moment,  $V$  the instantaneous volume,  $p$  the pressure and  $H_e$  the external field strength. Thus  $M=VI$ . If the demagnetizing factor of the specimen is  $N$ , the internal field is  $H=H_e-NI$  so that  $H_e=H+NI$  and  $dH_e=dH+NdI$ . The volume strain  $dV/V_0$  is usually denoted by  $\omega$  whence, from equation (7.1),

$$\begin{aligned} \omega &= -\frac{1}{V_0} \frac{\partial}{\partial p} \int_0^{H_e} VI dH_e \\ &= -\frac{1}{V_0} \frac{\partial}{\partial p} \left\{ \int_0^H VI dH + \frac{1}{2} NVI^2 \right\}. \end{aligned}$$

The integration and differentiation are independent and so

$$\omega = -\frac{1}{V_0} \int_0^H \frac{\partial}{\partial p} (VI) dH + \frac{1}{2} \frac{NI^2}{k} \quad \dots\dots(7.2)$$

where  $k$  is the bulk modulus. It is assumed that the saturation spontaneous magnetic moment is volume dependent, the variation being expressed in terms of a quantity  $\nu$  such that

$$M_s = M_s^0(1 + \nu\omega) \quad \text{or} \quad VI_s = V_0 I_s^0(1 + \nu\omega).$$

Thus

$$\frac{\partial}{\partial p} (VI_s) = -\frac{V_0 I_s^0 \nu}{k}.$$



The integral in equation (7.2) is now split up into two parts, one from 0 to  $H_s$ ,  $H_s$  being the least field to produce saturation, and the second from  $H_s$  to  $H$ . So that

$$\omega = \frac{1}{2} \frac{NI^2}{k} - \frac{1}{V_0} \int_0^{H_s} \frac{\partial}{\partial p} (VI) dH - \frac{I_s^{0v}}{k} H_s + \frac{I_s^{0v}}{k} (H_e - NI_s).$$

Thus the volume magnetostriction is made up of three parts. The first depends only on the demagnetizing factor, i.e. on the shape of the sample. It is proportional to the mean magnetization and reaches a limiting value when  $I=I_s$ . This is usually the dominant term in low fields. The second two terms together depend only on the saturating field  $H_s$  and not on the applied field. This part is caused by rotation of the magnetic domain vectors within the sample and is identical with the term  $h_3$  in equation (2.12). The third part is the volume effect proper; it is proportional to the external field and is independent of the orientation of the crystal axes with respect to it (Auwers 1933). It has its origin in the volume dependence of the spontaneous magnetization. This is the fundamental effect which requires attention.

It follows that if the spontaneous saturation moment is volume dependent, then it must change with external pressure. This is related to the volume magnetostriction through equation (7.1). Before discussing the volume magnetostriction and pressure experiments it is necessary to add a word about experimental difficulties. The volume magnetostriction is very small in most ferromagnetics, the quantity  $\partial\omega/\partial H$  being of the order of  $10^{-10}$  per oersted at temperatures well below the Curie temperature. Moreover it is the isothermal value of  $\partial\omega/\partial H$  which is required and this is difficult to measure since with the fields employed—of the order of a few thousand oersteds—the heating due to the magnetocaloric effect is quite large. If measurements are carried out dilatometrically some of the heat liberated from the sample may be communicated to the surrounding fluid and a spurious expansion will result. Döring (1936) sought to overcome this by using liquids of known thermal expansion and making the necessary correction. However the correction is much larger than the quantity to be measured. Moreover there still remains the thermal expansion in the sample itself which may not be allowed for in this way. This also is of the same order of magnitude as the quantity to be measured. It is evident therefore that such measurements cannot be considered satisfactory and it is significant that whereas the dilatometer measurements give  $\partial\omega/\partial H = 1 \times 10^{-10}$  per oersted (Döring) and  $0.95 \times 10^{-10}$  per oersted (Snoek 1937) for nickel, the most recent direct measurements of the isothermal  $\partial\omega/\partial H$  for nickel, by Azumi and Goldman (1954) give  $\partial\omega/\partial H = -0.55 \times 10^{-10}$  per oersted, with a possible error of 10%.

Measurements of the pressure dependence of the spontaneous magnetization present even greater difficulties and the values of  $\partial M_s/\partial p$  obtained by various workers disagree violently with each other. For nickel the most recent experiments are those of Jones and Stacey (1953) which show a very large increase in the saturation moment with pressure, of the order of 10% at 10 000 atmospheres. From equation (7.1) this implies that  $\partial\omega/\partial H \sim 50 \times 10^{-10}$  per oersted, in very poor agreement with the value obtained by direct measurement.

For iron the experiments of Ebert and Kussmann (1937) on the change of saturation moment with pressure lead to a value of  $\partial\omega/\partial H$  equal to  $10 \times 10^{-10}$ . This is in reasonable agreement with the value  $5 \times 10^{-10}$  obtained directly by Kornetzki (1935).

### 7.2. The Thermal Expansion Anomaly

The volume dependence of the saturation moment also gives rise to an anomaly in the coefficient of thermal expansion near the Curie temperature, since as the temperature is increased towards the Curie point there is an additional volume change caused by the destruction of the spontaneous magnetization. The linear expansion coefficient arising from this mechanism alone is

$$\alpha_M = \frac{1}{3} \frac{\partial \omega}{\partial T} = \frac{1}{3} \frac{\partial \omega}{\partial H} \frac{\partial I_s}{\partial T} \frac{\partial I_s}{\partial H}.$$

Since  $\partial I_s / \partial T$  is always negative and  $\partial I_s / \partial H$  positive it follows that the sign of the expansion coefficient anomaly is always the opposite of that of  $\partial \omega / \partial H$ . In addition the anomaly is greatest when  $\partial I_s / \partial T \div \partial I_s / \partial H$  is greatest, i.e. just below the Curie point. In nickel the anomaly is very definitely positive and this lends considerable support to the negative value of  $\partial \omega / \partial H$  obtained by Azumi and Goldman.

It is of considerable importance to know the total volume change  $\omega_s$  associated with the appearance of the spontaneous magnetization. From theoretical considerations Fowler and Kapitza (1929) obtained the expression

$$\omega_s = k \frac{\partial J}{\partial V} (AI_s^2 + BI_s^4) \quad \dots\dots(7.3)$$

where  $k$ ,  $A$  and  $B$  are constants and  $J$  is a measure of the exchange interaction. Essentially the same formula has been derived by Néel (1937). Unfortunately it is not possible to estimate  $\omega_s$  from this formula since the quantity  $\partial J / \partial V$ , which represents the variation of exchange interaction with interatomic distance, is completely unknown. Alternatively it is possible to calculate this quantity if  $\omega_s$  is known.

There are at least three ways of doing this. The most obvious is to measure the coefficient of thermal expansion over a wide enough temperature range that the 'magnetic expansion' can be seen superimposed on the normal thermal expansion. The two can then be separated in the way shown in figure 7.1. For nickel the experimental measurements of Williams (1934) interpreted in this way give  $\omega_s = -3.24 \times 10^{-4}$  at room temperature. Of course this is a dubious procedure but one would hardly expect the estimate to be in error by more than about 50%.

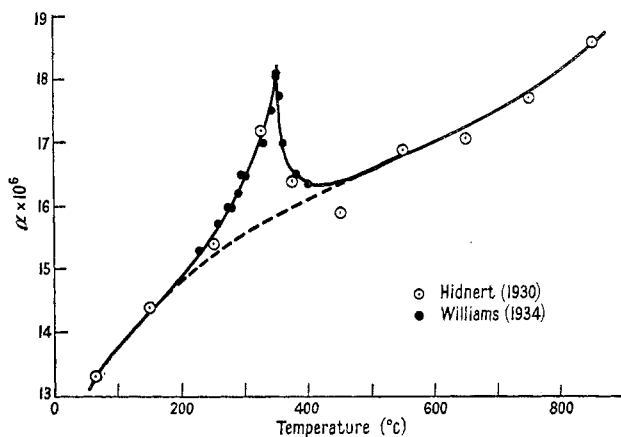


Figure 7.1. Coefficient of thermal expansion of nickel. The total volume change associated with the spontaneous magnetization is equal to the area between the full and broken curves.

The method cannot be used except for positive anomalies. For negative ones the dip in the  $(\alpha, T)$  curve is much broader than the peak obtained for a positive anomaly and the uncertainty much greater. From x-ray measurements of the lattice constants over a wide range of temperatures by Chevenard (1927) it can be deduced that  $\omega_s = +4 \times 10^{-4}$  for nickel at absolute zero (Néel 1939). This may indicate that the volume magnetostriction changes its sign at low temperatures. This could only result from a change of sign in  $\partial J / \partial V$  at low temperatures, a not unlikely occurrence in view of the position usually accorded to nickel on the exchange interaction curve, namely just to the right of the maximum (see figure (7.2)). A more elegant method is due to Köster and Schmidt (1934). This involves measurements of the lattice constants of binary alloys of one of the ferromagnetic metals with a non-ferromagnetic one. For example if Mo is added to Ni, the resulting alloy system is homogeneous up to 18% Mo. Addition of Mo to Ni lowers the Curie point until it is below room temperature at about 10% Mo and the measured lattice constants show a marked change at that composition at which the alloy is no longer magnetic at room temperature. The lattice constants above this composition in the paramagnetic range vary linearly with composition and can be extrapolated back to 100% Ni to obtain the lattice constant that Ni would have if it were non-magnetic. In this way Köster and Schmidt found  $\omega_s = +0.01$  for nickel, the lattice constants in the ferromagnetic and (extrapolated) paramagnetic states being  $3.515\text{\AA}$  and  $3.505\text{\AA}$  respectively at room temperature. The third method is to use the volume magnetostriction data. According to Néel (1939) the expression  $\omega_s = kI_s^2$  is a good approximation to equation (7.3). Thus  $\partial \omega / \partial H = 2kI_s \partial I_s / \partial H$ . Elimination of  $k$  gives

$$\omega_s = \frac{I_s}{2} \frac{\partial \omega}{\partial H} \bigg/ \frac{\partial I_s}{\partial H}. \quad \dots\dots(7.4)$$

The chief difficulty here lies in our ignorance of  $\partial I_s / \partial H$ . According to Kapitza (1931) this is less than  $10^{-7}$  for nickel. The value obtained by Polley (1939) is  $1.3 \times 10^{-4}$  and this is in good agreement with the value  $1.2 \times 10^{-4}$  estimated from theoretical considerations by Holstein and Primakoff (1940). Using this with the value  $-0.55 \times 10^{-10}$  for  $\partial \omega / \partial H$ , equation (7.4) gives  $\omega_s = -1 \times 10^{-4}$  in fair agreement with the estimate from thermal expansion. To summarize, one can say that for nickel the volume magnetostriction and expansion anomaly data are at least in qualitative agreement. The value of  $\omega_s$  deduced by Köster and Schmidt is incompatible with these both in magnitude and sign. Part of the disagreement is no doubt due to the smallness of the effects which itself may be due to the fact that the position of nickel on the exchange interaction curve is just to the right of the maximum at room temperature. Data for other metals are less complete but the agreement for nickel does not inspire confidence. There is considerable need for further experimental work on this rather untidy topic. It should be mentioned that the greater part of the theoretical work on this subject is due to Néel (1937, 1939), who has used the available data to construct curves showing the variation of exchange interaction with interatomic distance. For further information reference should be made to these two papers.

### 7.3. Change of Curie Temperature with Pressure

It is somewhat easier to investigate experimentally the pressure dependence of the Curie temperature than that of the saturation moment, and as a result the data

for the former are much more complete than for the latter. The Curie temperature is a measure of the exchange interaction and since this is known to depend on interatomic distance, it is bound to be indirectly pressure dependent. The present status of the theory of ferromagnetism is such that any direct calculation of the change of Curie temperature with pressure is out of the question. Instead the theoretical treatments have been to use thermodynamic arguments to derive relations between measured quantities and also to obtain information concerning quantities which cannot be measured directly.

The earliest attempt at a correlation process of this kind was made by Kornetzki (1935) who used a molecular field treatment, and, by thermodynamic reasoning, related the pressure dependence of the Curie temperature to the volume magnetostriction by means of the equation

$$\frac{1}{\theta} \frac{\partial \theta}{\partial p} = \frac{1}{T} \frac{\partial \omega / \partial H}{(\partial M_s / \partial T)_p - 3ak \partial \omega / \partial H} \quad \dots\dots(7.5)$$

in which  $\theta$  is the Curie temperature,  $a$  the coefficient of thermal expansion, and  $k$  the bulk modulus. Usually the second term in the denominator is much smaller than the first and since  $(\partial M_s / \partial T)_p$  is always negative, a positive  $\partial \theta / \partial p$  is associated with a negative  $\partial \omega / \partial H$  and vice versa.

The exchange interaction, or molecular field coefficient, in ferromagnetics is usually represented by the well-known curve due to Slater (1930) and Bethe (1933) shown in figure 7.2. Now the Curie temperature is proportional to the magnitude

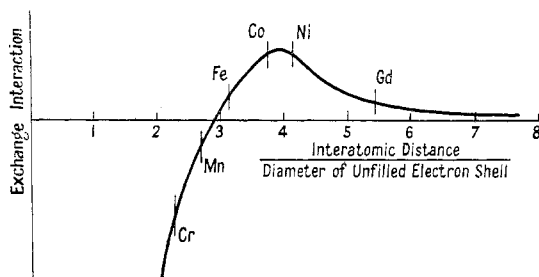


Figure 7.2. Bethe-Slater type interaction curve.

of the exchange interaction whatever particular model is used, and hence one would expect at least a qualitative correlation between the Curie temperature and the value of the ordinate for the given metals on the curve. For nickel we have seen that the most reliable measurements indicate that  $\partial \omega / \partial H$  is negative and so  $\partial \theta / \partial p$  should be positive, i.e. nickel should be placed to the right of the maximum in figure 7.2. This is in qualitative agreement with the most recent measurements of Patrick (1954). For other metals the predictions of figure 7.2 are in complete contradiction to the results obtained by Patrick who finds  $\partial \theta / \partial p$  to be zero for iron and cobalt and what is worse, to be negative for gadolinium. Patrick suggests that the interaction curve of figure 7.2 is valid only for a single element. That the form of the curve is essentially correct seems to be confirmed by the fact that while the experimental results show quite large decreases of Curie point with pressure (e.g. for 30% Ni-Fe) there have been no large increases found. Similar conclusions can be drawn from the volume magnetostriction and expansion anomaly data.

Equation (7.5) was derived by Kornetzki on the assumption that the spontaneous

magnetization is volume independent. The particular advantage of this assumption is that it enabled him to avoid a specification of the exact form of the temperature dependence of the spontaneous magnetization. This restriction has been removed by Smoluchowski (1941 a, b) who, still using a molecular field treatment, assumed that at constant volume this dependence could be represented by a Brillouin function. Smoluchowski finds that the experimental data can be interpreted in this way only if  $j=1$  for both iron and nickel. For  $j=\frac{1}{2}$  the agreement between theory and experiment is poor, in some cases even disagreeing as to sign. This is interpreted as evidence for coupled spins in nickel in agreement with the conclusions of Mott and Jones (1936). Unfortunately the very fact that the theoretical results depend so sensitively on the form of the  $(I_s, T)$  curve is in itself a warning not to place too much reliance on the results obtained by using a special form of the relation  $I_s=f(T)$ . The spontaneous magnetization (even at constant volume) of iron and nickel cannot in all probability be accurately described by a Brillouin function and it may be suggested that the predictions of the theory would be completely different if the correct form of  $I_s=f(T)$  were used. Nevertheless Smoluchowski is certainly right in removing the restriction of a volume independent magnetization. On the simplest picture of overlapping bands this clearly cannot be valid. It is unfortunate that the more general treatment cannot be viewed with complete confidence.

Some of the effects produced by hydrostatic pressure are quite appreciable. One consequence of this is that one would expect similar though smaller effects and of the opposite sign to be produced by plastic strain. There is increasing evidence (Averbach and Warren 1952) that a heavily strained material contains a high density of dislocations. In the immediate vicinity of a dislocation one would expect both the Curie temperature and the spontaneous magnetization to vary considerably on account of the large local variations in lattice constant. Experiments to show the existence of such changes have so far proved inconclusive.

### § 8. THE FORM EFFECT

Even if the spontaneous lattice deformations produced by both the exchange and anisotropy energies were zero, ferromagnetic materials would still exhibit a magnetostriction effect owing to the fact that their demagnetizing energy is dependent on strain. For example a sphere of ferromagnetic material will tend to decrease in volume and increase in length along the direction of magnetization since both changes decrease the demagnetizing energy, the latter by lowering the demagnetization factor. This effect gives rise to an additional magnetostriction known as the form effect. It has been previously encountered in connection with the volume magnetostriction. The magnitude of the effect can be calculated classically as follows.

Consider a ferromagnetic ellipsoid of revolution whose demagnetizing factor along the long axis is  $N$ . Choose orthogonal axes such that  $Ox$  coincides with this long axis. The demagnetizing energy is  $\frac{1}{2}NI^2$  per unit volume in the unstrained state. If allowed to deform this energy will be

$$E_D = \frac{1}{2}NI^2(1-\omega) = \frac{1}{2}NI^2(1-A_{11}-A_{22}-A_{33})$$

using the same notation as in previous sections. The elastic energy of an isotropic solid is

$$E_L = \frac{1}{2}c_{11}(A_{11}^2 + A_{22}^2 + A_{33}^2) + c_{12}(A_{11}A_{22} + A_{22}A_{33} + A_{33}A_{11}) \\ + (c_{11} - c_{12})(A_{12}^2 + A_{13}^2 + A_{23}^2).$$

The total energy  $E = E_D + E_L$  and the conditions of equilibrium are  $\partial E / \partial A_{ij} = 0$ . This gives (neglecting second order terms)

$$\frac{\partial E}{\partial A_{11}} = \frac{1}{2}NI^2 + \frac{1}{2}I^2 \frac{\partial N}{\partial A_{11}} - c_{11}A_{11} - c_{12}(A_{22} + A_{33}) = 0 \quad \dots\dots(8.1)$$

$$\frac{\partial E}{\partial A_{22}} = \frac{1}{2}NI^2 + \frac{1}{2}I^2 \frac{\partial N}{\partial A_{22}} - c_{11}A_{22} - c_{12}(A_{33} + A_{11}) = 0 \quad \dots\dots(8.2)$$

$$\frac{\partial E}{\partial A_{33}} = \frac{1}{2}NI^2 + \frac{1}{2}I^2 \frac{\partial N}{\partial A_{33}} - c_{11}A_{33} - c_{12}(A_{11} + A_{33}) = 0. \quad \dots\dots(8.3)$$

For an ellipsoid of revolution it is permissible to write

$$N = N_0[1 - a\{A_{11} - \frac{1}{2}(A_{22} + A_{33})\}]$$

for  $N$  must remain unchanged by a uniform dilation  $A_{11} = A_{22} = A_{33}$ . Thus

$$-\frac{\partial N}{\partial A_{11}} = -N_0 a, \quad \frac{\partial N}{\partial A_{22}} = \frac{\partial N}{\partial A_{33}} = \frac{1}{2}N_0 a.$$

Adding equations (8.1)–(8.3) and expressing the elastic constants  $c_{11}$  and  $c_{12}$  in terms of the more usual bulk and shear moduli  $k$  and  $n$ , one obtains for the total volume strain caused by the form effect:

$$\omega_f = \frac{1}{2}NI^2/k.$$

From equation (8.1) the longitudinal magnetostriction due to the form effect is

$$\left(\frac{dl}{l}\right)_f = A_{11} = \frac{1}{2}NI^2 \left\{ \frac{1}{3k} + \frac{a}{2n} \right\} \quad \dots\dots(8.4)$$

and from equation (8.2) the accompanying transverse contraction is

$$\left(\frac{dl}{l}\right)_{f, \text{trans.}} = A_{22} = A_{33} = \frac{1}{2}NI^2 \left\{ \frac{1}{3k} - \frac{a}{4n} \right\}.$$

These results were first obtained by Becker (1934). Equation (8.4) has been verified experimentally by Kornetzki (1934). For an iron sphere  $(dl/l)_f \sim 4 \times 10^{-6}$ .

The analysis given here holds only for isotropic solids. The calculation of the effect in single crystals has been carried out by Carr and Smoluchowski (1951). In general the contribution due to the form effect is small. However, with the recent tendency to use discs or oblate spheroids for experimental work, a knowledge of the part played by the shape of the sample is essential for accurate evaluation of the magnetostriction constants.

## § 9. THE $\Delta E$ -EFFECT

It has been shown in § 6 that when a ferromagnetic substance is subjected to a longitudinal tension, a redistribution of the domain magnetization vectors occurs, even in the absence of a magnetic field. This redistribution will in general be accompanied by a magnetostrictive strain  $\epsilon_m$ , of magnitude  $\frac{3}{2}\lambda_s(\cos^2\theta)_\sigma - \frac{3}{2}\lambda_s(\cos^2\theta)_{\sigma=0}$  in an obvious notation. The total strain produced by the tension is thus the sum of the ordinary elastic strain  $\epsilon_0$  and the magnetostrictive strain  $\epsilon_m$ . Now  $\epsilon_m$  will be a function of the applied stress and as the latter is increased  $\epsilon_m$  will approach a limiting value  $\epsilon_{ms}$  when the domain vectors are completely aligned by the tension in a manner given in § 6. It follows that for a ferromagnetic substance with non-

zero magnetostriction the measured stress-strain relation will be non-linear until the stress attains a value sufficiently great to cause the redistribution process to be complete. It is evident that if this redistribution is prevented from taking place, for example, by the internal magnetic anisotropy or by application of a magnetic field sufficiently great to produce saturation, the ratio stress to strain, i.e. Young's modulus, will be different from that obtained when the redistribution is allowed. This means that the measured Young's modulus of a ferromagnetic will be a function of its magnetization. This phenomenon first investigated in detail by Honda and Terada (1907) is usually known as the  $\Delta E$ -effect. The usual measure of the magnitude of the effect is the difference in the values of Young's modulus in the demagnetized and magnetically saturated conditions. For annealed nickel this difference is of the order of several per cent; for iron, rather less than 1%.

Attention has been drawn in § 6 to the similarity between the effects produced by a small tension and a small magnetic field in promoting boundary movements. In both cases these will be only partly reversible. It follows that the stress-strain curve of a ferromagnetic substance will exhibit hysteresis even for stresses which are so small that the ordinary elastic hysteresis is negligible. This has been observed by Becker and Kornetzki (1934) in nickel wires subject to torsion.

It is well known that in weak magnetic fields the magnetization obeys Rayleigh's law, viz.

$$I = aH + bH^2$$

where  $a$  and  $b$  are constants. Arguing that a tension and a magnetic field act in a very similar manner Kornetzki (1938) has suggested that the magnetic strain depends on the applied stress according to a quadratic law of the same form as Rayleigh's, i.e.

$$\epsilon_m = \frac{\sigma}{E'} + \frac{\sigma^2}{E''}. \quad \dots\dots(9.1)$$

The magnetic strain  $\epsilon_m$  is superimposed on the normal elastic strain  $\epsilon_0$  equal to  $\sigma/E_0$ . Thus the total strain

$$\epsilon = \epsilon_0 + \epsilon_m = \sigma \left( \frac{1}{E_0} + \frac{1}{E'} + \frac{\sigma}{E''} \right). \quad \dots\dots(9.2)$$

By analogy with Rayleigh's law the remanent strain when the stress is removed is

$$\epsilon_{mr} = \sigma^2/2E''.$$

The value of Young's modulus in the demagnetized state is given by

$$\frac{1}{E_a} = \left( \frac{\epsilon}{\sigma} \right)_{\sigma \rightarrow 0} = \frac{1}{E_0} + \frac{1}{E'}.$$

At saturation  $\epsilon_m = 0$  and so the value of Young's modulus is simply  $E_0$ . From equation (9.2) it follows that Young's modulus defined as  $\sigma/\epsilon$  is stress dependent and it is easy to show that

$$E = E_a \left( 1 - \frac{E_a}{E''} \sigma \right)$$

for small stresses. A linear decrease of  $E$  with strain has been observed by Forster and Köster (1937). This decrease is fortunately small ( $E^{-1}dE/d\epsilon \sim 5 \times 10^{-6}$  for Ni) compared with the  $\Delta E$  effect as usually defined; otherwise it would be necessary to specify the strain (or stress) when measuring Young's modulus in the demagnetized state.

If equation (9.2) is valid then the stress-strain curve has the form shown in figure 9.1. From this it follows that the instantaneous value of Young's modulus

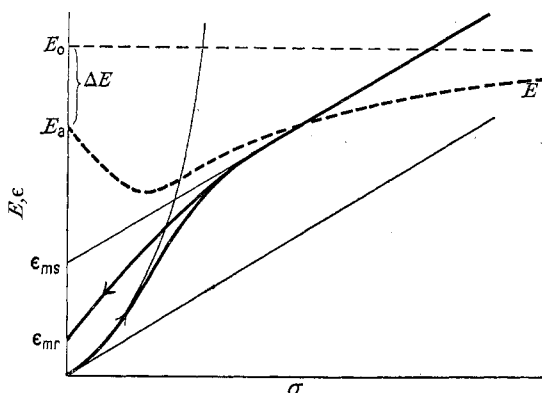


Figure 9.1. Idealized stress-strain diagram of a magnetically soft ferromagnetic substance according to Kornetzki (1938), eqn 8.

$E = \sigma/\epsilon$  decreases at first with increasing  $\sigma$ , goes through a minimum value and finally approaches the value  $E_0$ . This is observed in practice. It thus appears that equation (9.2) is capable of giving a good representation of the experimental facts. Nevertheless it should be borne in mind that equation (9.1) cannot be derived from Rayleigh's law and has no theoretical basis.

The customary measure of the magnitude of the  $\Delta E$ -effect is the difference between the value of Young's modulus in the demagnetized state when the degree of redistribution of domain vectors is determined solely by the internal anisotropy,  $E_a$ , and its value at saturation when no such redistribution is possible,  $E_0$ . This difference will depend on (1) the distribution of domain vectors in the demagnetized state since the influence of the tension is determined by the mean value of  $\cos^2(\sigma, I_s)$ , (2) the magnetostriction constants since these determine the magnitude of the magnetic strain  $\epsilon_m$ , and (3) on the ratio of the magnetoelastic energy to the internal anisotropy energy. If for example the crystal energy or the internal strain energy is large compared with the magneto-elastic energy associated with the external tension, then the equilibrium directions of  $I_s$  will be determined chiefly by the former; no reorientation can take place in the demagnetized state and so  $E_0 - E_a$  is very small. Since the magnetoelastic energy involves the magnetostriction constants it follows that  $E_0 - E_a$  will be of the form  $a\lambda_s^2/C$  where  $a$  is a constant involving the geometry of the demagnetized state and  $C$  is a general internal anisotropy constant. Moreover from the conclusions reached in § 6 it is evident that  $\epsilon_m$  is always positive whatever the sign of  $\lambda_s$  and so  $E_0 - E_a$  is always positive. It should be stressed that  $E_0$ , the value at saturation, is the normal Young's modulus, i.e. the value it would possess if the magnetostriction were zero. The smaller value of  $E$  in the demagnetized state can be regarded as the effect of the magnetostrictive expansion  $\epsilon_m$ , giving rise to an additional negative Young's modulus of magnitude  $\sigma/\epsilon_m$ .

That these predictions are fairly well confirmed by experiment may be seen from figure 9.2. In the first place  $E_0 - E_a$  has a maximum around 200°C. This coincides exactly with the position of the maximum initial permeability found by Kirkham



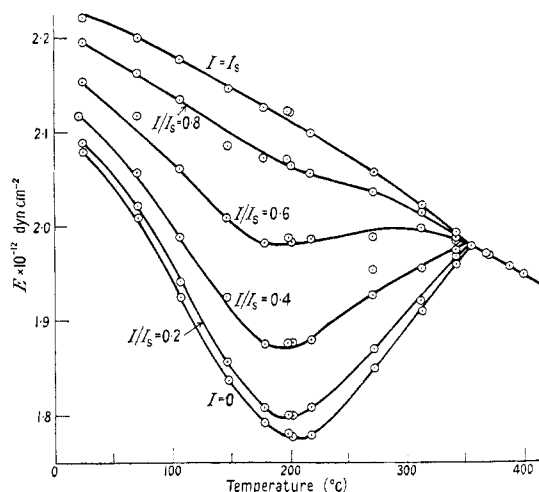


Figure 9.2. Temperature variation of Young's modulus of polycrystalline nickel for different values of magnetization (Siegel and Quimby 1936).

(1937) and is presumably associated with a minimum in  $C$ . Secondly the  $(E, T)$  curve for  $I = I_s$  is more or less continuous with that for paramagnetic nickel above the Curie point. We shall refer to this later. For some materials, notably Ni-Fe alloys containing about 40% Ni, the  $(E, T)$  curve at saturation is very much below the  $(E, T)$  curve extrapolated back from the paramagnetic region (see figure 9.3).

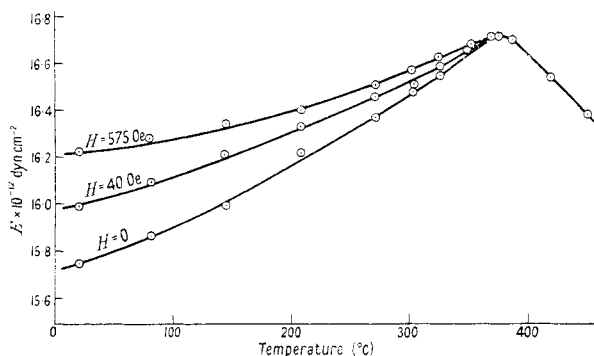


Figure 9.3. Temperature variation of Young's modulus of 42% Ni-Fe for different magnetizations showing marked change in  $dE/dT$  at the Curie temperature (Engler 1938).

This is quite contrary to expectation. Döring (1938) has proposed an explanation along the following lines. Above the Curie point one measures  $E$  at constant magnetization  $E_I$ , below it,  $E$  at constant field strength,  $E_H$ . From thermodynamics it can be shown that

$$\frac{1}{E_H} - \frac{1}{E_I} = \left( \frac{\partial \epsilon}{\partial H} \right)^2 \frac{\partial I}{\partial H}. \quad \dots\dots(9.3)$$

It is suggested that  $\partial \epsilon / \partial H$  arises from the volume magnetostriction; thus  $\partial \epsilon / \partial H = \frac{1}{3} \partial \omega / \partial H$ . If  $\partial \omega / \partial H$  is large the change in  $E$  at the Curie point may be appreciable, and Döring has suggested that the sudden change in  $\partial E / \partial T$  at the Curie point,

observed in 42% Ni-Fe is a result of the very large volume magnetostriction that this alloy is known to possess even at low temperatures (Masiyama 1931). Köster (1943) has drawn attention to the marked correlation between  $E_H$  and  $E_I$  and the volume magnetostriction for the NiFe alloys in the range 30–100% Ni. However this cannot be the complete explanation for equation (9.3) predicts that  $1/E_H - 1/E_I$  should always be positive whereas for Ni it is small but quite definitely negative. This may indicate the existence of a small but genuine change in the elastic constants with magnetization or it may be a morphic effect. The meaning of this term has been explained in § 2.2. As a result of the spontaneous lattice strain the original cubic symmetry is lost. This means that additional elastic constants are required over and above the three (which are altered somewhat by the morphic effect) required for a cubic crystal. The polycrystalline elastic constants, including Young's modulus, will be a function of the single crystal constants and thus may be expected to change on cooling below the Curie point due to the spontaneous deformation and loss of cubic symmetry. This effect will be small unless the magnetostriction constants are very large, and for most substances would be completely masked by the decrease in  $E$  caused by the volume magnetostriction.

A theoretical calculation of the magnitude of the  $\Delta E$ -effect is possible in certain cases. For this purpose the quantity  $\Delta(1/E)$  i.e. the difference in the reciprocal Young's modulus at saturation and in the demagnetized state is the most useful.

$$\Delta\left(\frac{1}{E}\right) = \frac{1}{E_a} - \frac{1}{E_0} = \frac{\epsilon_m}{\sigma} = \frac{1}{l} \left( \frac{dl}{d\sigma} \right)_{\sigma \rightarrow 0}$$

is therefore simply the magneostriptive strain produced by the stress. The effect of an external stress may be to promote either boundary displacements or domain vector rotations. For  $180^\circ$  boundary displacements  $dl/d\sigma$  is zero. For the other processes it is necessary to know the nature of the internal anisotropy. The simplest case, in which the anisotropy is uniaxial and produced by internal stresses of magnitude  $\sigma_i$  has been considered by Kersten (1933) who finds

$$\Delta\left(\frac{1}{E}\right) = \frac{2}{5} \frac{\lambda_s}{\sigma_i} \quad \dots\dots(9.4)$$

for the rotational process. On the same model the rotational initial susceptibility is  $\chi_a = 2I_s^2/9\lambda_s\sigma_i$ . Thus

$$\Delta\left(\frac{1}{E}\right) = \frac{9}{5} \frac{\lambda_s^2 \chi_a}{I_s^2}$$

from which it follows that  $\Delta(1/E) \propto \chi_a$  since  $\lambda_s$  and  $I_s$  are constants. This point has already been noted as an experimental fact for nickel.

Formulae for  $\Delta(1/E)$ , both for single and polycrystals have been given by Becker and Döring (1939) when the anisotropy differs from uniaxial. These are all of the form  $\Delta(1/E) = a\lambda_s^2/C$  as mentioned previously. (Equation (9.4) takes this form when numerator and denominator are multiplied by  $\lambda_s$ .) Each formula for  $\Delta(1/E)$  has its counterpart for  $\chi_a$  and it is possible to show that for each type of anisotropy and domain process the relation  $\Delta(1/E) \propto \chi_a$  holds, i.e. it is possible to eliminate the specific nature of the domain process responsible for the  $\Delta E$ -effect by means of the known formulae for the initial susceptibility.

The fact that the magnitude of  $\Delta(1/E)$  is dependent on the initial domain distribution makes possible the employment of the  $\Delta E$ -effect to study preferred orientation of domains if  $\lambda_s$  and  $C$  are known. This has been done with some success by Williams, Bozorth and Christensen (1941). However the effect appears

to show an even more promising application than this. It has been mentioned that all expressions for  $\Delta(1/E)$  are of the same form. The constants  $a$ ,  $\lambda$  and  $C$  will depend on the particular magnetic process and the type of anisotropy governing it. Thus it is possible, since the expressions for  $\Delta(1/E)$  can be calculated in all cases, to use measurements of the  $\Delta E$ -effect to investigate the nature of the predominant magnetic processes in the demagnetized state. A first attempt to interpret the  $\Delta E$  measurements in this way has been made by Lewis (1954).

The variation of Young's modulus with magnetization below saturation has been treated by Brown (1937), Takagi (1939) and others who show that  $\Delta E \propto I^2$ . This relation is obeyed by quite a large number of materials. For further details reference should be made to these two papers and the book by Bozorth.

### § 10. MISCELLANEOUS TOPICS

In this section we group together a number of topics which, either because they are less fundamental or because our knowledge of them is still fragmentary, hardly justify separate treatment.

In the first place we may mention the use of magnetostriction measurements to investigate the domain structure of the demagnetized state. The quantity most often measured is the saturation magnetostriction, i.e. the total strain between the demagnetized and saturated states. Owing to the fact that the demagnetized state is, from the magnetostriction point of view, indeterminate, such measurements do not give reliable information concerning the magnetostriction constants. However if these are known or can be determined by independent investigations then the saturation magnetostriction data can be used to investigate the nature of the domain vector distribution in the demagnetized state (Lee 1955). In general however they are not known and additional supplementary measurements are required. The whole problem has been treated in detail by Rado (1954) who gives the number of observations required for a complete analysis of the degree of preferred domain orientation. The use of magnetostriction measurements to study the effects of magnetic annealing has been made by Williams, Bozorth and Christensen (1941), Hoselitz and McCaig (1949), Shturkin and Shur (1949, 1950) and others. Much of this work has been criticized by Rado (1954).

The problem of the determination of internal strain in cold-worked materials has been studied by a number of methods, using x-ray diffraction techniques. In ferromagnetic substances the presence of internal strains gives rise to a local internal anisotropy of magnitude  $\frac{3}{2}\lambda_s\sigma_i \sin^2\phi$  (cf. equation (6.2)) where  $\sigma_i$  is the magnitude of the stress acting at any point inside the material. If this energy is much greater than the crystal energy then magnetic processes will be controlled almost exclusively by the former and formulae may be derived for a number of magnetic properties in terms of  $\lambda_s$  and  $\sigma_i$ . If  $\lambda_s$  is known measurements of these quantities can be used to evaluate  $\sigma_i$ . The quantities most often employed are the initial susceptibility,  $\chi_a = \frac{2}{9}I_s^2/\lambda_s\sigma_i$  the change of remanent magnetization with external tension,  $dI_R/d\sigma = I_s/4\sigma_i$ , and the  $\Delta E$ -effect, equation (9.4). These are all derived on the basis of the strain theory developed by Becker and Kersten and suffer from all the defects of that theory. On the other hand the measured quantities give estimates which are self consistent and which seem to be in reasonable agreement with estimates obtained by other methods. There are two great weaknesses in the method however. In the first place it cannot be applied to any material

unless it has a very small crystal anisotropy. The second one stems from one of the weaknesses of the strain theory on which the above equations are derived, namely that  $\sigma_i$  does not have the same meaning in all cases and so the actual significance of the value of  $\sigma_i$  obtained is uncertain. This point has been discussed by Köster (1948). For further information reference should be made to the book by Bozorth (1951) and the article by Becker (1940).

No reference has so far been made to the magnetostrictive properties of ferrites and antiferromagnetics. Ferrites possess an essentially cubic structure and so the equations developed in § 2.2 for cubic metals also apply to ferrites. The available data on ferrites is at the moment scanty, being for the most part published either in brief notes or remaining in the relative obscurity of private research reports. The very large moment associated with each lattice site in ferrites will presumably give rise to a dipolar magnetostriction an order of magnitude greater than that existing in ferromagnetics. However recent measurements by Bozorth and Walker (1952) who find  $\lambda_{100} = -515 \times 10^{-6}$  for cobalt ferrite, indicate that there must be some other contribution in addition to the dipolar magnetostriction just as in ordinary ferromagnetics. It should also be mentioned that the single crystal elastic constants are not known for ferrites so an exact comparison of the internal forces responsible for magnetostriction in ferrites and ferromagnetics is not yet possible.

The situation is somewhat different for antiferromagnetics. On account of the antiferromagnetic alignment of atomic moments which exists in such substances the net moment of any part of it is zero (unlike ferromagnetics where a sufficiently small element will always be found to be spontaneously magnetized). An external magnetic field has therefore virtually no effect on it and the usual methods of investigating magnetostriction applicable to ferromagnetics are of no use. The existence of a volume magnetostriction was demonstrated by Foex (1948) who showed that the thermal expansion coefficients of MnO, FeO, CoO and NiO exhibited sharp maxima at the Néel point. X-ray investigations by Rooksby (1948) on NiO and by Tombs and Rooksby (1950) on MnO, FeO and CoO have revealed the existence of lattice distortions at temperatures below the Néel point where antiferromagnetic alignment is known to set in. In the first three substances the lattice (face-centred cubic above the Néel point) was distorted into rhombohedral but CoO distorts into a tetragonal lattice. It is natural to regard these distortions as analogous to the linear magnetostriction in ferromagnetics, though the distortion is much greater in antiferromagnetics. These results and others have been discussed in some detail by Lidiard (1954).

Néel (1948 a) has put forward several arguments for the existence of antiferromagnetic domains. These differ from ferromagnetic domains in that their net moment is zero so they are not oriented by a magnetic field. Bozorth (1951) has suggested that movements of domain walls might be brought about by external stresses. Street and Lewis (1951) and Fine (1952) have observed large changes in Young's modulus at the Néel point of CoO and NiO. Fine has interpreted this as evidence for the existence of a domain structure and for stress induced domain wall movements, but in view of the very considerable departures from cubic symmetry below the Néel point it is by no means certain that this is the only explanation.

## § 11. CONCLUSION

In this report an attempt has been made to present an account of the basic phenomena underlying the various observed magnetomechanical effects, and to

give, as far as possible, a coherent picture of the present state of the subject. In doing this it was necessary to bear in mind the fact that no other report on magnetostriction, of any magnitude, has appeared since McKeehan's article of 1926. On the other hand the subject is treated fairly fully in the often quoted books by Becker and Döring where the emphasis is rather on the theoretical side, and by Bozorth which deals very largely with the experimental aspects. Since the latter book appeared only quite recently, a good deal of material upon which emphasis was therein laid has been purposely omitted here. This includes, chiefly, the detailed behaviour of ferromagnetics under stress in the presence of applied magnetic fields, and what Bozorth terms 'magnetostriction data'. For a similar reason no mention has been made of magnetomechanical effects at any other than zero frequency. A very complete discussion of dynamic magnetostriction and of systems undergoing magnetostrictive oscillations has been given by Van der Burgt (1953) whose paper contains an extensive bibliography.

No mention has been made of experimental procedure except in cases where this is particularly difficult. In the measurement of the magnetostriction constants this is not so, and the best technique, using electrical resistance strain gauges, is adequately described by Goldman and Smoluchowski (1949). The best procedure for measuring the magnetostriction constants when single crystal specimens are available is described in detail by Bozorth and Hamming (1953).

There is an enormous literature on the subject of magnetostriction and the reference list included below aims to be selective rather than comprehensive. Order was brought into the existing chaos by the appearance of the papers by Akulov (1928) and Becker (1930), and it may safely be said that with the exception of the papers by McKeehan and his school, and certain of the Japanese workers, little of the literature written before 1928 is of any great value. With regard to the work published after this date, although the reference list does not pretend to be complete it is nevertheless hoped that there are no serious omissions.

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