Tarea 3. Estadística

Katherin A. Murcia S. y Juan Carlos Rojas V.

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
import sympy as sym
```

Propagting the error through a multi-variable function

Three variables are measured to be $A = 12.3 \pm 0.4$, $B = 5.6 \pm 0.8$ and $C = 89.0 \pm 0.2$. Calculate the mean and ucertatinties of Z when it is related to A, B and C via the relations:

In [3]:
```
# Dado que soy un perezoso, dejaré que SymPy haga las derivadas.
a,b,c,un_a,un_b,un_c = sym.symbols("A B C sigma_A sigma_B sigma_C")

A = 12.3 # Medida promedio de A
un_A = 0.4 # Incertidumbre de A
B = 5.6 # Medida promedio de B
un_B = 0.8 # Incertidumbre de B
C = 89.0 # Medida promedio de C
un_C = 0.2 # Incertidumbre de C
```

(i) $Z = A + B$

Calculando la incertidumbre de Z, se tiene que

$$\sigma_Z = \sqrt{\left(\frac{\partial Z}{\partial A}\right)^2 \sigma_A^2 + \left(\frac{\partial Z}{\partial B}\right)^2 \sigma_B^2}$$

Para la derivada parcial de Z con respecto de A está dada por $\frac{\partial Z}{\partial A} = 1$. Análogamente para B, se tiene que $\frac{\partial Z}{\partial B} = 1$ por tanto:

$$\sigma_Z = \sqrt{\sigma_A^2 + \sigma_B^2}.$$

Se obtiene entonces que para $Z = A + B$ el calculo de Z es:

In [4]:
```
un_Z = np.sqrt(un_A**2 + un_B**2) # Incertidumbre de Z
Z = A + B # Medida promedio de Z
Z,un_Z
```

Out[4]:
```
(17.9, np.float64(0.894427190999916))
```

Se tiene entonces que $\bar{Z} = 17.9$ y $\sigma_Z = 0.9$.

(ii) $Z = A - B$

Similarmente que para el punto anterior, se tomando las derivadas parciales con respecto a cada variable. Sin embargo, la derivada parcial de Z con respecto de B va a ser -1. Por lo que se tiene que σ_Z está dado por:

$$\sigma_Z = \sqrt{\sigma_A^2 + (-1)^2\sigma_B^2} = \sqrt{\sigma_A^2 + \sigma_B^2}$$

Por tanto, la incertidumbre va a ser la misma que para el punto anterior. Ahora, para el promedio, se tiene que:

In [5]:
```
Z = A - B
un_Z = np.sqrt(un_A**2 + un_B**2)
Z,un_Z
```

Out[5]:
```
(6.700000000000001, np.float64(0.894427190999916))
```

Se tiene entonces que $\bar{Z} = 6.7$ y $\sigma_Z = 0.9$.

(iii) $Z = \frac{A-B}{A+B}$

Definiendo la expresión en SymPy:

In [6]:
```
z = (a-b)/(a+b)
z
```

Out[6]:

$$\frac{A - B}{A + B}$$

Tomando la derivada parcial con respecto a A:

In [7]:
```
dzda = sym.diff(z,a)
dzda
```

Out[7]:

$$-\frac{A - B}{(A + B)^2} + \frac{1}{A + B}$$

Tomando la derivada parcial con respecto a B:

In [8]:
```
dzdb = sym.diff(z,b)
dzdb
```

Out[8]:

$$-\frac{A - B}{(A + B)^2} - \frac{1}{A + B}$$

La incertidumbre σ_Z es, entonces:

In [9]:
```
un_z = sym.sqrt(dzda**2 * un_a**2 + dzdb**2 * un_b**2)
un_z
```

Out[9]:

$$\sqrt{\sigma_A^2 \left(-\frac{A - B}{(A + B)^2} + \frac{1}{A + B}\right)^2 + \sigma_B^2 \left(-\frac{A - B}{(A + B)^2} - \frac{1}{A + B}\right)^2}$$

Al calcularla se tiene, entonces que:

In [10]:
```
un_Z_f = sym.lambdify([a,b,un_a,un_b],un_z)
un_Z = un_Z_f(A,B,un_A,un_B)
un_Z
```

Out[10]:
```
np.float64(0.06299266062854407)
```

Por otro lado, para $\bar{Z}$ se tiene que:

In [11]:
```
Z = (A-B)/(A+B)
Z
```

Out[11]:
```
0.37430167597765374
```

Por tanto la medida es $\bar{Z} = 0.37$ y $\sigma_Z = 0.06$.

(iv) $Z = \frac{AB}{C}$

Se define la expresión simbólica en SymPy:

In [89]:
```
z = a*b/c
z
```

Out[89]:

$$\frac{AB}{C}$$

Tomando las derivadas parciales:

In [13]:
```
dzda = sym.diff(z,a)
dzda
```

Out[13]:

$$\frac{B}{C}$$

In [14]:
```
dzdb = sym.diff(z,b)
dzdb
```

Out[14]:

$$\frac{A}{C}$$

In [15]:
```
dzdc = sym.diff(z,c)
dzdc
```

Out[15]:

$$-\frac{AB}{C^2}$$

In [16]:
```
un_z = sym.sqrt(dzda**2 * un_a**2 + dzdb**2 * un_b**2 + dzdc**2 * un_c**2)
un_z
```

Out[16]:

$$\sqrt{\frac{A^2B^2\sigma_C^2}{C^4} + \frac{A^2\sigma_B^2}{C^2} + \frac{B^2\sigma_A^2}{C^2}}$$

In [17]:
```
un_Z_f = sym.lambdify([a,b,c,un_a,un_b,un_c],un_z)
un_Z = un_Z_f(A,B,C,un_A,un_B,un_C)
un_Z
```

Out[17]:
```
np.float64(0.11340366493873848)
```

In [18]:
```
Z = A*B/C
Z
```

Out[18]:
```
0.7739325842696628
```

Se tiene entonces, que $\bar{Z} = 0.7$ y $\sigma_Z = 0.1$.

(v) $Z = \arcsin\left(\frac{B}{A}\right)$

In [19]:
```
z = sym.asin(b/a)
z
```

Out[19]:

$$\operatorname{asin}\left(\frac{B}{A}\right)$$

Tomando la derivada parcial con respecto a A:

In [20]:
```
dzda = sym.diff(z,a)
dzda
```

Out[20]:

$$-\frac{B}{A^2\sqrt{1-\frac{B^2}{A^2}}}$$

Tomando la derivada parcial con respecto a B:

In [21]:
```
dzdb = sym.diff(z,b)
dzdb
```

Out[21]:

$$\frac{1}{A\sqrt{1-\frac{B^2}{A^2}}}$$

La incertidumbre σ_Z es, entonces:

In [22]:
```
un_z = sym.sqrt(dzda**2 * un_a**2 + dzdb**2 * un_b**2)
un_z
```

Out[22]:

$$\sqrt{\frac{\sigma_B^2}{A^2\cdot\left(1-\frac{B^2}{A^2}\right)} + \frac{B^2\sigma_A^2}{A^4\cdot\left(1-\frac{B^2}{A^2}\right)}}$$

Al calcularla se tiene, entonces que:

In [23]:
```
un_Z_f = sym.lambdify([a,b,un_a,un_b],un_z)
un_Z = un_Z_f(A,B,un_A,un_B)
un_Z
```

Out[23]:
```
np.float64(0.0749198656837616)
```

Por otro lado, para $\bar{Z}$ se tiene que:

In [24]:
```
Z = np.arcsin(B/A)
Z
```

Out[24]:
```
np.float64(0.47269178607772744)
```

Por tanto la medida es $\bar{Z} = 0.47$ y $\sigma_Z = 0.07$.

(vi) $Z = AB^2C^3$

Se define la expresión simbólica en SymPy:

In [25]:
```
z = a*b**2 * c**3
z
```

Out[25]: AB^2C^3

Tomando las derivadas parciales:

In [26]:
```
dzda = sym.diff(z,a)
dzda
```

Out[26]: B^2C^3

In [27]:
```
dzdb = sym.diff(z,b)
dzdb
```

Out[27]: $2ABC^3$

In [28]:
```
dzdc = sym.diff(z,c)
dzdc
```

Out[28]: $3AB^2C^2$

In [29]:
```
un_z = sym.sqrt(dzda**2 * un_a**2 + dzdb**2 * un_b**2 + dzdc**2 * un_c**2)
un_z
```

Out[29]: $\sqrt{9A^2B^4C^4\sigma_C^2 + 4A^2B^2C^6\sigma_B^2 + B^4C^6\sigma_A^2}$

In [30]:
```
un_Z_f = sym.lambdify([a,b,c,un_a,un_b,un_c],un_z)
un_Z = un_Z_f(A,B,C,un_A,un_B,un_C)
un_Z
```

Out[30]:
```
np.float64(78216357.72884737)
```

In [33]:
```
Z = A*B**2*C**3
Z
```

Out[33]:
```
271926282.432
```

Se tiene entonces, que $\bar{Z} = 2.7 \times 10^9$ y $\sigma_Z = 0.8 \times 10^9$.

(vii) $Z = \ln(ABC)$

Se define la expresión simbólica en SymPy:

In [34]:
```
z = sym.ln(a*b*c)
z
```

Out[34]: $\log(ABC)$

Tomando las derivadas parciales:

In [35]:
```
dzda = sym.diff(z,a)
dzda
```

Out[35]: $\dfrac{1}{A}$

In [36]:
```
dzdb = sym.diff(z,b)
dzdb
```

Out[36]: $\dfrac{1}{B}$

In [37]:
```
dzdc = sym.diff(z,c)
dzdc
```

Out[37]: $\dfrac{1}{C}$

In [38]:
```
un_z = sym.sqrt(dzda**2 * un_a**2 + dzdb**2 * un_b**2 + dzdc**2 * un_c**2)
un_z
```

Out[38]: $$\sqrt{\frac{\sigma_C^2}{C^2}+\frac{\sigma_B^2}{B^2}+\frac{\sigma_A^2}{A^2}}$$

In [39]:
```
un_Z_f = sym.lambdify([a,b,c,un_a,un_b,un_c],un_z)
un_Z = un_Z_f(A,B,C,un_A,un_B,un_C)
un_Z
```

Out[39]:
```
np.float64(0.14652912571933396)
```

In [40]:
```
Z = np.log(A*B*C)
Z
```

Out[40]:
```
np.float64(8.721002229851615)
```

Se tiene entonces, que $\bar{Z} = 8.7$ y $\sigma_Z = 0.1$.

(viii) $Z = \exp(ABC)$

Se define la expresión simbólica en SymPy:

In [48]:
```
z = sym.exp(a*b*c)
z
```

Out[48]: e^{ABC}

Tomando las derivadas parciales:

In [49]:
```
dzda = sym.diff(z,a)
dzda
```

Out[49]: BCe^{ABC}

In [50]:
```
dzdb = sym.diff(z,b)
dzdb
```

Out[50]: ACe^{ABC}

In [51]:
```
dzdc = sym.diff(z,c)
dzdc
```

Out[51]: ABe^{ABC}

In [54]:
```
un_z = sym.sqrt(dzda**2 * un_a**2 + dzdb**2 * un_b**2 + dzdc**2 * un_c**2)
un_z
```

Out[54]:

$$\sqrt{A^2B^2\sigma_C^2e^{2ABC} + A^2C^2\sigma_B^2e^{2ABC} + B^2C^2\sigma_A^2e^{2ABC}}$$

Calculando en otro lado (dado que es algo re grande).

Se tiene entonces, que $\bar{Z} = 0.002 \times 10^{2665}$ y $\sigma_Z = 2 \times 10^{2665}$.

(ix) $Z = A + \tan\left(\frac{B}{C}\right)$

Se define la expresión simbólica en SymPy:

In [112...
```
z = a + sym.tan(b/c)
z
```

Out[112]:

$$A + \tan\left(\frac{B}{C}\right)$$

Tomando las derivadas parciales:

In [113...
```
dzda = sym.diff(z,a)
dzda
```

Out[113]: 1

In [114...
```
dzdb = sym.diff(z,b)
dzdb
```

Out[114]:

$$\frac{\tan^2\left(\frac{B}{C}\right) + 1}{C}$$

In [115...
```
dzdc = sym.diff(z,c)
dzdc
```

Out[115]:

$$-\frac{B\left(\tan^2\left(\frac{B}{C}\right) + 1\right)}{C^2}$$

In [116...
```
un_z = sym.sqrt(dzda**2 * un_a**2 + dzdb**2 * un_b**2 + dzdc**2 * un_c**2)
un_z
```

Out[116]:

$$\sqrt{\frac{B^2\sigma_C^2\left(\tan^2\left(\frac{B}{C}\right) + 1\right)^2}{C^4} + \sigma_A^2 + \frac{\sigma_B^2\left(\tan^2\left(\frac{B}{C}\right) + 1\right)^2}{C^2}}$$

In [117...
```
un_Z_f = sym.lambdify([a,b,c,un_a,un_b,un_c],un_z)
un_Z = un_Z_f(A,B,C,un_A,un_B,un_C)
un_Z
```

Out[117]:
```
np.float64(0.40010181300479225)
```

In [118...
```
Z = A + np.tan(B/C)
```

```
z
```

Out[118]:
```
np.float64(12.363004517247631)
```

Se tiene entonces, que $\bar{Z} = 12.4$ y $\sigma_Z = 0.4$.

(x) $Z = 10^{ABC}$

Se define la expresión simbólica en SymPy:

In [58]:
```
z = 10**(a*b*c)
z
```

Out[58]: 10^{ABC}

Tomando las derivadas parciales:

In [59]:
```
dzda = sym.diff(z,a)
dzda
```

Out[59]: $10^{ABC} BC \log(10)$

In [60]:
```
dzdb = sym.diff(z,b)
dzdb
```

Out[60]: $10^{ABC} AC \log(10)$

In [61]:
```
dzdc = sym.diff(z,c)
dzdc
```

Out[61]: $10^{ABC} AB \log(10)$

In [62]:
```
un_z = sym.sqrt(dzda**2 * un_a**2 + dzdb**2 * un_b**2 + dzdc**2 * un_c**2)
un_z
```

Out[62]: $\sqrt{10^{2ABC} A^2 B^2 \sigma_C^2 \log(10)^2 + 10^{2ABC} A^2 C^2 \sigma_B^2 \log(10)^2 + 10^{2ABC} B^2 C^2 \sigma_A^2 \log(10)^2}$

Calculando en otro lado (dado que es algo re grande).

Se tiene entonces, que $\bar{Z} = 0.002 \times 10^{6133}$ y $\sigma_Z = 4 \times 10^{6133}$.

Angular dependence of the reflection coefficient of light

The intensity reflection coefficient, R, for the component of the field parallel to the plane of incidence is

$$R = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}$$

where θ_i and θ_t are the angles of incidence and transmission, respectively. Calculate R and its associated error if $\theta_i = (45.0 \pm 0.1)^\circ$ and $\theta_t = (34.5 \pm 0.2)^\circ$.

Primero, pasamos todo a radianes:

In [64]:
```
theta_i = 45.0 *np.pi/180
theta_t = 34.5 *np.pi/180
un_theta_i = 0.1*np.pi/180
un_theta_t = 0.2*np.pi/180
theta_i,un_theta_i,theta_t,un_theta_t
```

Out[64]:
```
(0.7853981633974483,
 0.0017453292519943296,
 0.6021385919380436,
 0.003490658503988659)
```

In [72]:
```
theta_i_,theta_t_,un_theta_i_,un_theta_t_ = sym.symbols(r"theta_i theta_t \sigma_{\theta
```

In [75]:
```
R = sym.tan(theta_i_ - theta_t_)**2/(sym.tan(theta_i_ + theta_t_)**2)
R
```

Out[75]:

$$\frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}$$

In [78]:
```
dRdthetai = sym.diff(R,theta_i_)
dRdthetai
```

Out[78]:

$$\frac{\left(2\tan^2(\theta_i - \theta_t) + 2\right)\tan(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} + \frac{\left(-2\tan^2(\theta_i + \theta_t) - 2\right)\tan^2(\theta_i - \theta_t)}{\tan^3(\theta_i + \theta_t)}$$

In [79]:
```
dRdthetat = sym.diff(R,theta_t_)
dRdthetat
```

Out[79]:

$$\frac{\left(-2\tan^2(\theta_i - \theta_t) - 2\right)\tan(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} + \frac{\left(-2\tan^2(\theta_i + \theta_t) - 2\right)\tan^2(\theta_i - \theta_t)}{\tan^3(\theta_i + \theta_t)}$$

In [80]:
```
un_R = sym.sqrt(dRdthetai**2*un_theta_i_**2 + dRdthetat**2 * un_theta_t_**2)
un_R
```

Out[80]:

$$\sqrt{\sigma_{\theta_i}^2\left(\frac{\left(2\tan^2(\theta_i - \theta_t) + 2\right)\tan(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} + \frac{\left(-2\tan^2(\theta_i + \theta_t) - 2\right)\tan^2(\theta_i - \theta_t)}{\tan^3(\theta_i + \theta_t)}\right)^2 + \sigma_{\theta_t}^2\left(\frac{\left(-2\tan^2(\theta_i - \theta_t) - 2\right)\tan(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} + \frac{\left(-2\tan^2(\theta_i + \theta_t) - 2\right)\tan^2(\theta_i - \theta_t)}{\tan^3(\theta_i + \theta_t)}\right)^2}$$

In [81]:
```
un_R_f = sym.lambdify([theta_i_,theta_t_,un_theta_i_,un_theta_t_],un_R)
```

In [82]:
```
un_R_f(theta_i,theta_t,un_theta_i,un_theta_t)
```

Out[82]:
```
np.float64(9.194664167000746e-05)
```

In [83]:
```
R_f = sym.lambdify([theta_i_,theta_t_],R)
```

In [84]:
```
R_f(theta_i,theta_t)
```

Out[84]:
```
np.float64(0.0011799610805957297)
```

Se obtuvo entonces que $R = (117 \pm 9) \times 10^{-5}$

Poiseuille's method for determing viscosity

The volume flow rate, $\frac{\mathrm{d}V}{\mathrm{d}t}$, of fluid flowing smoothly through a horizontal tube of lenght L and radius r is given by Poiseuille's equation:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\pi \rho g h r^4}{8 \eta L}$$

where η and ρ are the viscosity and density, respectively, of the fluid, h is the head of pressure across the tube, and g the acceleration due to gravity. In an experiment the graph of the flow rate versus height has a slope measured to 7%, the length is known to 0.5%, and the radius to 8%. What is the fractional preciosion to which the viscosity is known? If more expreimental time is available, should this be devote to **(i)** collecting more flow-rate data, **(ii)** measuring the length, or **(iii)** the radius of the tube?

Calculando la precisión fraccional se tiene que:

(4.8)

- $\frac{dV}{dt} = \Phi = \frac{\pi \rho g h r^4}{8 \eta L}$.
- $\frac{\Phi}{h} = S$.
- $\frac{\sigma_S}{S} = \frac{7}{100}$.
- $\frac{\sigma_L}{L} = \frac{0,5}{100}$.
- $\frac{\sigma_r}{r} = \frac{8}{100}$.

- $\eta = \frac{\pi \rho g r^4}{8 S L}$.
- $\frac{\partial \eta}{\partial S} = \frac{(-1)}{S} \eta$.
- $\frac{\partial \eta}{\partial L} = \frac{(-1)}{L} \eta$.
- $\frac{\partial \eta}{\partial r} = \frac{4}{r} \eta$.
- $\sigma_\eta = \left[\frac{\eta^2}{S^2} \sigma_S^2 + \frac{\eta^2}{L^2} \sigma_L^2 + \frac{16 \eta^2}{r^2} \sigma_r^2 \right]^{1/2}$

Lo cual se traduce numéricamente a:

In [85]:
```
frac_un_slope = 7/100
frac_un_L = 0.5/100
frac_un_r = 8/100
```

In [87]:
```
frac_un_viscosity = np.sqrt(frac_un_slope**2 + frac_un_L**2 +16*frac_un_r**2)
frac_un_viscosity
```

Out[87]:
```
np.float64(0.32760494501762333)
```

Por tanto, se puede calcular la viscosidad con una precisión de 32%. Por otro lado, si se tiene más tiempo de experimental, este debe tiempo extra debería estar dedicado a medir mejor el radio del tubo, pues este tiene un mayor "weight" con respecto a las otras variables.

Error spreadsheet for Van der Waals calculation

Construct a spreadsheet which has the data from the calculation in Section 4.2.2. Include cells for: **(i)** the variables (molar volume and the absolute temperature), **(ii)** the uncertainties, and **(iii)** the universal gas constant as well as the parameters a and b. Verify the numbers obtained in the worked example. Repeat the calculation for **(i)** $V_m = (2.000 \pm 0.003) \times 10^{-3} m3mol^{-1}$ and $T = 400.0 \pm 0.2K$; **(ii)** $V_m = (5.000 \pm 0.001) \times 10^{-4} m^3 mol^{-1}$ and $T = 500.0 \pm 0.2K$. Repeat the calculations with the same variables for **(a)** He with $a = 3.457209 \times 10^{-3} m^6 mol^{-2} \mathrm{Pa}$, and $b = 2.37 \times 10^{-5} m^3 mol^{-1}$; **(b)** CO2 with $a = 3.639594 \times 10^{-1} m^6 mol^{-2} \mathrm{Pa}$, and $b = 4.267 \times 10^{-5} m^3 mol^{-1}$; and **(c)** Ar with $a = 1.36282125 \times 10^{-1} m^6 mol^{-2} \mathrm{Pa}$, and $b = 3.219 \times 10^{-5} m^3 mol^{-1}$.

ESPECIE	V_m [m^3/mol]	σ_V_m [m^3/mol]	T [K]	σ_T [K]	R [J/molK]	a [Pam^6/mol^2]	b [m^3/mol]	P(V, T + σ_T) [Pa]	P(V+σ_V, T) [Pa]	P [Pa]	σ_P [Pa]
N	0,0002	0,0000003	298	0,2	8,3145	1,41E-01	3,91E-05	1,1892E+07	1,1864E+07	1,188E+07	2E+04
N	2,00E-03	3,00E-06	400	0,2	8,3145	1,41E-01	3,91E-05	1,6617E+06	1,6584E+06	1,661E+06	3E+03
N	5,00E-04	1,00E-07	500	0,2	8,3145	1,41E-01	3,91E-05	8,4608E+06	8,4555E+06	8,457E+06	4E+03
He	0,0002	0,0000003	298	0,2	8,3145	3,46E-03	2,37E-05	1,3977E+07	1,3944E+07	1,397E+07	3E+04
He	2,00E-03	3,00E-06	400	0,2	8,3145	3,46E-03	2,37E-05	1,6828E+06	1,6794E+06	1,682E+06	3E+03
He	5,00E-04	1,00E-07	500	0,2	8,3145	3,46E-03	2,37E-05	8,7179E+06	8,7126E+06	8,714E+06	4E+03
CO2	0,0002	0,0000003	298	0,2	8,3145	0,3639594	0,00004267	6,6601E+06	6,6468E+06	6,65E+06	1E+04
CO2	2,00E-03	3,00E-06	400	0,2	8,3145	0,3639594	0,00004267	1,6090E+06	1,6058E+06	1,608E+06	2E+03
CO2	5,00E-04	1,00E-07	500	0,2	8,3145	0,3639594	0,00004267	7,6381E+06	7,6330E+06	7,634E+06	4E+03
Ar	0,0002	0,0000003	298	0,2	8,3145	1,36E-01	3,22E-05	1,1368E+07	1,1342E+07	1,136E+07	2E+04
Ar	2,00E-03	3,00E-06	400	0,2	8,3145	1,36E-01	3,22E-05	1,6569E+06	1,6536E+06	1,656E+06	3E+03
Ar	5,00E-04	1,00E-07	500	0,2	8,3145	1,36E-01	3,22E-05	8,3450E+06	8,3398E+06	8,341E+06	4E+03

Weighted mean

A group of six students make the following measurements of the speed of light (all $\times 10^{-8} ms^{-1}$): 3.03 ± 0.04, 2.99 ± 0.03, 2.99 ± 0.02, 3.00 ± 0.05, 3.05 ± 0.04 and 2.97 ± 0.02. What should the cohort report as their combined result? If another student then reports $c = (3.0 \pm 0.3) \times 10^8 ms^{-1}$, is there any change to the cohort's combined measurement? If a further student resports $c = (4.01 \pm 0.01) \times 10^8 ms^{-1}$, is there any change to the cohort's combined measurment?

#	c	sigma_c	sigma_c^2
1	3,03E+08	4,00E+06	1,60E+13
2	2,99E+08	3,00E+06	9,00E+12
3	2,99E+08	2,00E+06	4,00E+12
4	3,00E+08	5,00E+06	2,50E+13
5	3,05E+08	4,00E+06	1,60E+13
6	2,97E+08	2,00E+06	4,00E+12
7	3,00E+08	3,00E+07	9,00E+14
8	4,01E+08	1,00E+06	1,00E+12
PROMEDIO (N=6)	3,01E+08	SIGMA_PROMEDIO	1E+06
PROMEDIO (N=7)	3,00E+08	SIGMA_PROMEDIO	4E+06
PROMEDIO (N=8)	3,13E+08	SIGMA_PROMEDIO	4E+06

donde $\bar{x} = \frac{\sum_i x_i}{N}$, $\frac{\partial \bar{x}}{\partial x_i} = \frac{1}{N}$ y $\sigma_{\bar{x}} = \frac{1}{N}\sqrt{\sum_i \sigma_{x_i}^2}$. Los cambios provocados por las dos nuevas medidas se pueden ver al final de la tabla.