Financial Frictions, Firm Dynamics and the Aggregate Economy:

Insights from Richer Productivity Processes

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Abstract

How do financial frictions affect firm dynamics, allocation of resources across firms, and aggregate productivity and output? Is the nature of productivity shocks that firms face important for the effects of financial frictions? In order to answer these questions, I first use a comprehensive dataset of Spanish firms from 1999 to 2014 to estimate nonparametrically the firm productivity dynamics. I find that the productivity process is non-linear, as persistence and shock variability depend on past productivity, and productivity shocks are non-Gaussian. These dynamics differ from the ones implied by the standard AR(1) process, commonly used in the firm dynamics literature. I then build a model of firm dynamics with financial frictions in which productivity shocks are non-linear and non-Gaussian. The model is consistent with a host of evidence on firm dynamics, financial frictions, and firms' financial behavior. In the model economy, financial frictions affect the firm life cycle. Without financial frictions, the size of an entrant firm will be three times larger. Furthermore, profit accumulation, which allows firms to overcome financial frictions, is slow, and it only speeds up when firms are mature. As a consequence, the average exiting firm is smaller than it would be without financial frictions. The aggregate consequences of financial frictions are large. They result in misallocation of capital and reduce aggregate productivity by 16%. This figure is only 8% if productivity dynamics evolve according to a standard AR(1) process.

JEL Codes: E22, G32, O16

Keywords: Firm Dynamics, Non-Linear Productivity Process, Financial Frictions, Misallocation

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1 Introduction

Output per capita differs vastly across countries. An extensive literature, see e.g. Klenow and Rodríguez-Clare (1997) and Hall and Jones (1999), shows that differences in output per worker are mainly driven by differences in aggregate productivity. Firm heterogeneity is crucial to understand the differences in aggregate productivity. Firms are heterogeneous in their efficiency to transform inputs, mainly capital and labor, into output. As a result, the aggregate productivity of a country depends on the productivity distribution of firms that operate.

Furthermore, how resources are allocated across firms also matters for aggregate productivity. A growing literature in macroeconomics, starting with Guner, Ventura, and Xu (2008), Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), analyses how resource allocation affects aggregate productivity. The basic idea in this literature is that if the most efficient firms are not the ones using a larger amount of inputs, the total amount of output produced by the country is smaller than in a first-best where that does not happen.

What are the factors behind misallocation? Financial frictions are an obvious culprit. Financial frictions affect the allocation of capital, as they prevent firms with low internal resources from installing their optimal capital level. Yet, generating significant aggregate productivity and output losses from financial frictions in quantitative models of firm dynamics has been a challenging task, see e.g. Buera, Kaboski, and Shin (2011) and Midrigan and Xu (2014).

The effects of financial frictions on the firm dynamics and aggregate productivity depend crucially on the productivity shocks that firms face. On the one hand, as highlighted by Moll (2014), the persistence of productivity shocks determines the speed at which firms can accumulate internal funds and surpass financial frictions. If shocks are very persistent, firms that receive a sequence of favorable shocks will grow, retain profits and be able to finance their investment without borrowing. On the other hand, dispersion (variance), asymmetry (skewness) and tailedness (kurtosis) of shocks also matter. If an initially low productivity firm has a significant probability of having a large productivity shock tomorrow, it is more likely to be financially constrained. This firm would like to invest a copious amount to benefit from the favorable shock, which may be not feasible given its level of internal funds. Finally, the variability of shocks also determines the level of uncertainty the firm faces in its investment decisions. Due to the time-to-build nature of investment decisions, firms decide who much capital to have for the next period based on their expected productivity. A high uncertainty implies that many firms would end up with too little or too much capital with respect to their realized productivity levels, as emphasized by Asker, Collard-Wexler, and De Loecker (2014).

Despite these linkages, almost all existing papers on firm dynamics model firm-level shocks as a simple AR(1) process. Hence, all firms, independently of their current level of productivity face the same persistence and variability of shocks. Furthermore, the innovations to the AR(1) process come from a nice, symmetric normal (Gaussian) distribution.

In this paper, I non-parametrically estimate a non-linear and non-Gaussian firm-level productivity process. I use a comprehensive dataset, with more than 6.5 million firm-year observations, that contains balance sheet data for Spanish firms from 1999 to 2014. I use recently developed techniques that are used to study income dynamics by Guvenen, Karahan, Ozkan, and Song (2015), Arellano, Blundell, and Bonhomme (2017) and De Nardi, Fella, and Paz-Pardo (2019) to show that productivity dynamics are non-linear with non-Gaussian shocks.[1] The estimation allows the persistence, variance, skewness and kurtosis of productivity shocks to depend on where the firms currently are in the productivity distribution.

I find that productivity persistence is hump-shaped, while shock variability is U-shaped with past productivity. Furthermore, skewness is decreasing and shock kurtosis is hump-shaped with past productivity. These features contrast with the AR(1) productivity process usually used in the literature, and they imply very different productivity dynamics. Considering a low productivity firm. I find it has low persistence, so its past low productivity history matter less, and it has more volatile and positively skewed shocks; therefore, it has a large probability of receiving a good productivity realization in the next period. Along those lines, the probability of a firm that is initially in the first decile of the productivity distribution to be above the median in the next period is 6.7% in the

[1] Arellano et al. (2017) use quantile regressions, while Guvenen et al. (2015) and De Nardi et al. (2019) study the earnings distribution conditional on previous earnings. All of them recover an earnings process that looks very different from the canonical AR(1).

estimated productivity process. This contrast with a probability of 1.3% if productivity dynamics are assumed to be follow an AR(1) process. On top of that, the lower persistence and negative skewness of high productivity firms suggests that these high productivity episodes are not long-lasting for some firms. This slows down the speed at which firms can surpass financial frictions through internal profit accumulation, as shown in Moll (2014).

I next build a model of firm dynamics to study how financial frictions affect aggregate productivity by distorting the allocation of capital across firms. The model economy builds on earlier papers on the role of financial frictions and firm dynamics, e.g. Cooley and Quadrini (2001), Gomes (2001), Buera et al. (2011), Khan and Thomas (2013) and Midrigan and Xu (2014). The model economy has three main differences that sets it apart from the existing literature. First, the productivity process is non-linear instead of the AR(1) used in their framework. Second, I model the firm life cycle and tie it to the data. Firms enter the market; and, as they age, they grow and decline depending on how their productivity evolves, their financial conditions, and other frictions, not implicitly modelled in this paper. Finally, they eventually exit the market. Third, financial frictions are based on a size-dependent borrowing constraint, similar to Gopinath, Kalemli-Ozcan, Karabarbounis, and Villegas-Sanchez (2017). Hence, what fraction of its capital a firm can pledge depends on its size.

In order to discipline the quantitative model, I use firm-level data to document several novel facts on misallocation and financial behavior of the firms. Misallocation of capital across firms in the data appears as higher average revenue product of capital (ARPK) for the constrained firms. This contrasts with the predictions in a perfectly competitive world without financial frictions, where the ARPK should be equalized across firms. In that sense, the standard deviation of ARPK has become the standard statistic used to assess the allocation efficiency of capital in the economy. Financial frictions mostly affect the capital level of young, small and high productivity firms. Those firms are the ones less likely to have enough internal funds to sustain their optimal level of capital. In line to this, the data shows that the mean of ARPK and its standard deviation is larger for young, small and high productivity firms.

I also show how the leverage ratio (debt over total assets) varies by firm characteristics. A significant fraction of firms, 29%, do not use costly debt. I also find that on average leverage is decreasing with firm age and firm productivity, but increasing with firm size. Furthermore, these patterns arise both in the extensive margin, probability of using costly debt, and the intensive margin, average leverage conditional on using costly debt.

The simulated economy is consistent with the empirical evidence on financial frictions. The model matches how the average level and dispersion of the ARPK changes with age, size and productivity. The model without financial frictions fails in accounting for those patterns. The model also matches the firm's financial behavior. It generates a leverage distribution very similar to the one in the data. Second, it accounts for the negative relation of firm leverage with firm's age and productivity; and the positive relation with firm's size. As in the data, these regularities are present in both the extensive and intensive margin.

I then use the model to study how financial frictions affect firms, both their initial size and the growth over their life cycle. Finally, I quantify the aggregate consequences of financial frictions.

I obtain two main results. First, financial frictions affect the firm's life cycle. I compare the results from the benchmark model with the solution of a benevolent social planner that maximizes total output taking the structure of the economy as given. The social planner abstracts from financial friction by reallocating capital across firms taking into account only firm productivity. Compared to a world without financial frictions, an average entrant is three times smaller in the benchmark economy with financial frictions. Although, the size-gap between entrants and incumbent is reduced over the firm's life cycle, it is not fully closed. This means that the process to overcome financial frictions is slow. Indeed, it is particularly slow for young (less than 5 years old) firms and only speeds up when firms mature (more than 5 years old).

Second, the aggregate effects of financial frictions are large. Around 1/3 of the firms are constrained in their capital decision. The inefficient allocation of capital translates into productivity losses of 16%. These effects are much smaller in an economy with an AR(1) productivity process: only 1/4 of the firms are constrained and the productivity losses from financial frictions is only 8%.

Finally, I do a decomposition exercise to analyse why the effects of financial frictions are larger in the model with non-linear productivity dynamics than in the standard AR(1). In order to do so, I run several parallel economies modifying the characteristics of the non-linear productivity process, so that it inherits the characteristics of the AR(1) process. Then, I compare the aggregate effects of financial frictions in these parallel economies. I find that around half of the larger productivity losses in the non-linear process are due to the differential persistence and shock variability, while the non-Gaussian shocks (non-constant skewness and kurtosis) contribute another half.

The rest of the paper is organized as follows. Section 2 reviews the related literature and states the contribution of the paper. Section 3 describes the main dataset and variables used in the the rest of the paper. Section 4 covers the empirical part of the paper. It has three subsections that analyse the non-linearities of the productivity dynamics, evidence of the presence of financial frictions and the financial behavior of Spanish firms. Section 5 sets up the model. Section 6 has the main results of the paper. In this section, I show the calibration procedure, the effects of financial frictions over the firm life cycle and their aggregate consequences. Finally, Section 7 concludes.

2 Related Literature

This paper relates and contributes to four strands of the literature: firm dynamics and financial frictions, misallocation, empirical finance and non-linear income processes. This paper relates to the early literature on firm dynamics and financial frictions literature, see e.g. Gomes (2001) and Cooley and Quadrini (2001). Cooley and Quadrini (2001) highlights that persistence in the productivity process and financial frictions are two key elements to obtain realistic firm dynamics.[2] In contrast to them, I introduce a richer productivity process directly estimated from the data. One feature of the productivity process is that persistence is non-linear and it depends on past productivity. I show that the productivity process interacts with financial frictions. The negative effects of financial frictions over the firm life cycle are amplified under the non-linear productivity dynamics.

In a recent paper, Chatterjee and Eyigungor (2019) show that if firms are subject to

[2] They document and rationalize through the lens of the model two empirical regularities. First, size dependence; conditional on firm age, firm growth and exit rates are decreasing with firm size. Second, age dependence; conditional on firm size, firm growth and exit rates are decreasing with firm age.

financial frictions, low interest rates episodes can rationalize the rise in firm concentration, as recently seen for the US. In my paper, I show that the estimated productivity process is essential to generate the firm concentration levels seen in the data. I show that firm concentration is much smaller if productivity dynamics follow a standard AR(1).

I introduce a size-dependent borrowing constraint, similar to Gopinath et al. (2017), to the standard firm dynamics model. The main difference of the functional form regards on the pledge-ability parameter, which is usually assumed to be constant. In the size-dependent borrowing constraint, this pledge-ability parameter is a function of the firm size. This feature allows me to match the firm leverage distribution and yields realistic financial behavior at the firm level. Nevertheless, the aggregate effects of financial frictions are very similar to the case in which a constant pledge-ability parameter is assumed.

Within the misallocation literature, Buera et al. (2011) and Midrigan and Xu (2014) study the effects of financial frictions in developing economies. They model implicitly a dual economy with formal and informal sectors. Both paper find that financial frictions prevent firms from entering into the formal economy. This produces losses in aggregate productivity, as it is not always the case that the most productivity firms operate in the formal sector, which have both productive and technological advantages. But, they disagree on the effects of financial frictions once the firms enter the formal economy. Buera et al. (2011) point out that they can be large; while, Midrigan and Xu (2014) state that they are small. The latter argues that firms can accumulate internal funds pretty fast in the most productive sector; and therefore, overcome the effects of financial frictions. This paper differs from these two papers along several dimensions. First, it focuses in a developed economy, modelling only formal sector. Second, it ties carefully firm entry and exit to the data, so I can match the firm life cycle. Finally, the introduction of the non-linear productivity process affects the assessment of financial frictions. I find that financial frictions have important consequences in the formal sector. The non-linear productivity process is key, as the aggregate productivity losses are twice as large we would find in the standard AR(1) used in the literature. The larger effects of financial frictions under non-linear productivity process goes in line with the work of Asker et al. (2014), and specially Moll (2014). Asker et al. (2014) points out that firm uncertainty affects the investment decision of the firm and it has consequences on aggregate productivity as the ex-ante optimal investment level may not be optimal ex-post, once firm uncertainty has been realized. Moll (2014) highlights the importance of productivity persistence to financial frictions have an effect on aggregate productivity. The non-linear productivity process proposed in this paper has these two features, as persistence and shock variability depend on past productivity.

The paper also relates to the recent work of David and Venkateswaran (2019). They do a taxonomy of frictions that affect the allocation of capital and quantify their importance. They find that correlated distortions are an important source of capital misallocation in China and US.[3] In this paper, I find that average ARPK and its dispersion is higher for young, small and high productivity firms. Although, they do not model financial frictions, they point out that financial frictions can generate those correlated distortions form a simple model; and therefore, they account for a sizeable fraction of total misallocation. In this paper, I confirm that financial frictions can generate correlated distortions with respect to firm productivity. Furthermore, once financial frictions are introduced the model is able to match exactly the magnitude of correlated distortions present in the data. Finally, Jo and Senga (2019) propose a set up to evaluate policies aimed to easy financial frictions faced by firms and evaluate their aggregate effects. This paper differs from Jo and Senga (2019) in two main points. First, the focus is very different. They focus in a policy exercise, while this paper pursues a quantification of financial frictions. Second, They introduce a productivity process with non-Gaussian productivity shocks. In this paper, the estimated productivity process features not only non-Gaussianity productivity shocks, but also non-linear persistence and shock variability.

Regarding the empirical finance literature, this paper relates in two main points. First, financial behavior and capital structure has been extensively studied, both empirically and theoretically, see Lemmon, Roberts, and Zender (2008) and Graham and Leary (2011). But, most of the papers have focused on publicly listed firms. The main reason is due to the lack of comprehensive datasets on private firms. Although publicly listed firms represent a significant fraction of total value added, they are a small fraction of all the firms in the economy. As consequence, their behavior is not representative to the whole economy. Furthermore, the idiosyncrasies of publicly listed firms make their financial behavior not

[3] The term of correlated distortions has been used in the literature to refer to the situation when ARPs are positively correlated with firm characteristics, specially productivity.

being directly extrapolated to private firms. This paper and the contemporaneous work of Dinlersoz, Kalemli-Ozcan, Hyatt, and Penciakova (2018) are the first ones looking at the financial behavior of private firms. Although the focus of the papers are different, we find similar patterns with some differences discussed in Section 4.3. In particular both papers find that larger firms have higher leverage. The current paper is able to generate this fact. Chatterjee and Eyigungor (2019) build a firm dynamics model with default to account for this fact as well.

Finally, this paper relates to the recent literature on non-linear income processes. The focus has been on estimating the income process the households and individuals face. The main result is that the income process differs from a standard AR(1). They show that the non-linear income process has consequence in the saving and consumption behavior of individuals. In this paper, I mainly follow the approach developed in Guvenen et al. (2015) and De Nardi et al. (2019); but, I combine it with features of the work in Arellano et al. (2017) to estimate and characterize the productivity process firms face. As far as I know, this is the first paper that estimates non-parametrically the productivity process of firms. The results show that it differs significantly from the standard AR(1). The main features are that persistence and shock variability are non-linear, as they depend on past productivity; furthermore, productivity shocks are non-Gaussian. This is consistent with recent evidence found by Bachmann, Carstensen, Lautenbacher, and Schneider (2018) using firm survey data. In this paper, I show that the non-linear productivity process affects the investment decision of firms. And, the effect of financial frictions over the firm's life cycle and their aggregate consequences are larger than under the standard AR(1) set up.

3 Data

The main dataset is called *Central de Balances Integrada* (CBI) and it is compiled by Banco de España (BdE). The original source comes from the legal enforcement the Spanish firms have to deposit their annual accounts at the Commercial Registry.[4] At the end of

[4] The Spanish law imposes penalties if the firm does not deposit their annual accounts in form and time. These penalties are from economical, imposed to the firm, to the legal inability of the managers to run other firms or make them respond against the firm liabilities with their own assets in case of bankruptcy.

the economic year, the managers of Spanish firms collect all the information and elaborate the annual accounts. Then, they deposit them at the Commercial Registry during the first half of the year. BdE has an agreement with the Commercial Registry, who gives access to that information. The annual accounts consist of three documents: balance sheets, income statements and annual reports. The balance sheet reflects all the assets and liabilities the firm has at the end of the economic year. The income statement shows all the sources of income and expenses. Finally, the annual report states all the relevant information not considered in the two previous documents, such as dividend payments and employment structure. In the paper, I use the data from 1999 to 2014 covering all economic sectors, which results on more than 12 million firm-year observations.

I focus on privately-held companies that are legally constituted as limited liability firms. There are several reasons for this selection. First, publicly-held companies are a minority in Spain.[5] Furthermore, these companies have access to other sources of funding, such as equity, which are not considered in the proposed framework. Second, I do not include firms in the public sector, since they have access to other sources of funding. Finally, the sample does not include self-employed since they often are not limited liability firms, and hence, do not need to present their accounts at the Commercial Registry. The final sample represents 98.6% of all the firms in the database, and it accounts for 74% of total value added and 91% of total employment.

In order to evaluate the representativeness of the sample for the Spanish economy, I compare it with *el Directorio Central de Empresas* (DIRCE). DIRCE provides aggregate information on the census of Spanish firms. Several points arise. First, the selected sample covers around 50% of all the firms, and the coverage is stable over the studied period. In terms of employment, the coverage is smaller around 30% of the total.[6] This is mainly due to the focus on private firms. Regarding the firm size distribution, the coverage is

[5] According to Spanish Commission of Stock Exchange (CNMV), there are around to 210 listed firms in Spain. This represents a small fraction of the total number of firms, more than 800 thousand firms.

[6] There are several reasons why the CBI does not cover all the firms in the economy. First, the team in charge of data management was not able to compile all the information arriving from the Commercial Registry. This was specially relevant when most of the information was not digital. For this reason, I disregard all the sample before 1999, as it is highly affected by the lack of capacity in the data processing. Second, some firms deposit their accounts after the deadline. Although the BdE receives several updates from the Commercial Registry during the year, if the firm deposit their accounts very late, the information does not arrive to the BdE. Third, some firms do not deposit their annual accounts, a minority due to the legal consequences. Finally, the quality of the information presented by some firms is very poor; and therefore, it is not incorporated in the CBI.

consistent across different size groups. Finally, the coverage is similar if we restrict our attention to the manufacturing sector.

I next construct the main variables used in the analysis. From the information in the balance sheet, I recover capital, debt and net worth. Capital is measured as the book value of long term assets.[7] This measure is deflated at the 2-digits sector level using investment deflators from the Spanish National Accounts. Debt is defined as costly debt, which is the sum of long-term liabilities and costly short term liabilities. These are the funds for which the firm has to pay an interest, and does not include other short term funding, such as working capital. Finally, net worth is computed as the difference between total assets and total liabilities. These measures are deflated using CPI at the province level where the headquarters of the firm are located.

From the information in the income statement, I recover value added, wage bill and profits. Value added is computed as revenue minus intermediate goods. The resulting variable is deflated at the 2-digits sector level using value added deflators from the Spanish National Accounts. The wage bill corresponds to the total cost of employment, which includes wages, bonuses and social security payments by the firm. Finally, profits are measured after taking into account depreciation, fund provisions and taxes. Therefore, it is the available income that the firm can keep as internal funds or pay to the shareholders as dividends. The wage bill and profits are deflated using CPI at the province level where the headquarters of the firm are located.

From the information in the annual report, I recover employment and dividends. Employment is measured in full-time equivalent units. Therefore, it captures the hiring heterogeneity across firms, full vs part-time, and when the firm hires or fires a worker. This makes the employment measure comparable across firms. Finally, I recover the dividend payments from the approval of the profit distribution proposal the managers make to the shareholders. This measure is deflated using CPI at the province level where the headquarters of the firm are located.

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[7] Some papers use perpetual inventory methods to compute the capital, instead of the book value. Both methods have drawbacks. For instance, perpetual inventory method relies on common depreciation rate for all the capital. Not taking into account heterogeneity in capital, buildings, computers, machines ..., introduces measurement error in the capital measure, see for instance Collard-Wexler and De Loecker (2016). This does not happen in the book value as capital is measured after accounting depreciation, which is firm and type of capital specific. The main drawback of the measure of capital as book value is that it is reported at historical cost. This cost may differ from the actual one.

The key variable of interest is firm productivity. In order to estimate it, I first assume a functional form that links the output (value added) and inputs (capital and labor). As it has become standard in the literature, I use wage bill instead of employment to measure labor. The main advantage of wages is that they take into account workers heterogeneity, such as education, experience, that is passed through higher wages. I choose a Cobb-Douglas specification under decreasing returns to scale, governed by a span of control parameter (η).[8] The production function reads as

$$py_{si} = A_{si}[k_{si}^{\alpha_s} l_{si}^{1-\alpha_s}]^{\eta} \qquad \alpha_s \in (0,\ 1) \ and \ \eta \in (0,\ 1), \tag{1}$$

where py_{si} is value added, A_{si} is total factor productivity (TFPQ), k_{si} is capital and l_{si} is labor of a firm i operating in sector s. The model economy in Section 5 displays exactly the same firm-level production function.

I allow for differential output to input elasticity at the 2-digits sector level, which is governed by α_s. I do not allow, however, for differential degree of decreasing returns to scale across sectors, which are assumed to be constant. After parameterization, I invert the production function to infer the firm-level productivity.

Regarding the parameterization, I rely on the static nature of the labor decision to recover the values of α_s at the sector level. In order to do so, I first solve for the labor decision at the firm level and then aggregate at the sector level. The values of α_s are given by the following expression

$$\alpha_s = 1 - \frac{1}{\eta}\frac{wL_s}{Y_s} = 1 - \frac{1}{\eta}\frac{\sum_{i=1}^{Ns} wl_{si}}{\sum_{i=1}^{Ns} py_{si}}, \tag{2}$$

where wL_s is the aggregate wage bill, Y_s is the aggregate value added and N_s is the total number of firms operating in sector s. In order to reduce the scope of measurement error, I rely on aggregate value added and wage bill from the Spanish National Accounts to recover the α_s.

Second, I assign a value to the decreasing returns to scale parameter, η. In order to

[8] This is analogous to a constant returns to scale production function and a constant elasticity of substitution demand system with elasticity parameter, σ. The two models yield to the same decreasing returns to scale in the revenue function when $\eta = \dfrac{\sigma - 1}{\sigma}$.

do so, I follow an iterative process. I consider different values of η and for each value, I estimate the firm level of productivity, A_{si}, and the underlying productivity process. Then, I solve the model economy with the estimated productivity process. In the model, the value of η has a direct influence on the standard deviation of the capital distribution, $SD(k_{si})$. As a result, I choose the value of η for which the model economy gives the best match to this moment. This procedure results in a value of η equal to 0.83.

Lastly, I construct sector weights (ω_s); so that I can aggregate the sector specific measures. In Appendix A, I provide further details on the estimation and the distribution of the recovered parameters.

Finally, I do a last sample restriction and cleaning of the resulting dataset. First, I drop very small firms.[9] I only consider firms with more than 1,000 € in value added, and 500 € in capital in real €2010. Furthermore, I disregard all the firms with less than 0.5 employees in full-time equivalent units. Second, I clean the dataset from outliers and inconsistent observations. Regarding outliers, I do a 1% winsorization of the lower and upper tail of the productivity distribution at the sector level. Regarding inconsistent observations, I drop firms that seem to report the variables with wrong units. In order to do so, I compute average wages as wage bill over number of employees and drop observations with unrealistic figures. Finally, I disregard observations that appear to have huge rank reversals in the output, input and productivity distribution. For instance, firms that are at the top 10 percentile of the sector productivity distribution but at the bottom 1 percentile of the sector employment distribution. In Appendix A, I provide more information of this process. The final dataset consists of 6,500,945 firm-year observations corresponding to 1,024,144 different firms covering the period from 1999 to 2014.

4 Empirics

The productivity process has been pointed out as key in the firm dynamics literature to yield realistic firm behavior. Cooley and Quadrini (2001) show that it is essential for generating age and size dependence, for instance young and small firms grow faster

[9] Firms with zero employment or very small economic activity are very likely to be used as instrumental firms in order to avoid taxes or hide heritage to the fiscal authorities.

than their old and large counter parts. Furthermore, it affects the ability of firms to accumulate enough internal funding to overcome financial frictions, as shown in Moll (2014). Even though its importance, the productivity process has been usually modelled as an AR(1) process, which has several restrictions. First, the productivity persistence and shock variability is assumed to be the same for all the firms. Second, productivity shocks are assumed to come from a Gaussian distribution. In this section, I propose a flexible estimation procedure that overcomes these two drawbacks of the AR(1) process. Finally, I show that the estimated productivity process differs substantially from the standard AR(1) used in the firm dynamics literature.

In order to capture the non-linearities in the productivity dynamics, I estimate non-parametrically the bivariate relation of today's and tomorrow's productivity. There are two important concerns regarding this procedure. First one is that the non-parametric estimation is very data intensive, specially if you want to capture the behavior at the tails of the distribution. Second, as shown in Asker et al. (2014) there is sector heterogeneity in the productivity process, specially in the variance of the productivity shocks. In order to overcome these issues, I first standardize the estimated productivity at the sector-year level; and then, I pool the data across sectors and years. It is important to note that I allow the production function to differ across sectors, as α_s is sector specific.

I first discretize the standardized productivity in 16 non-equally spaced intervals, as shown in Figure 1, paying special attention to the tails of the distribution.[10] This is specially important as it is well known that the size distribution is skewed to the right.[11] Furthermore, output is very concentrated at the top of the distribution.[12] Therefore, it is important to capture not only the productivity behavior of the low and middle productivity firms, but also the high productivity firms, which are responsible of a large fraction of total output. Figure 1 contrasts the empirical productivity distribution with the one implied by a standard AR(1) process. The empirical productivity distribution has a slightly longer tail at the left, i.e. it is negatively skewed. Therefore, there is a larger

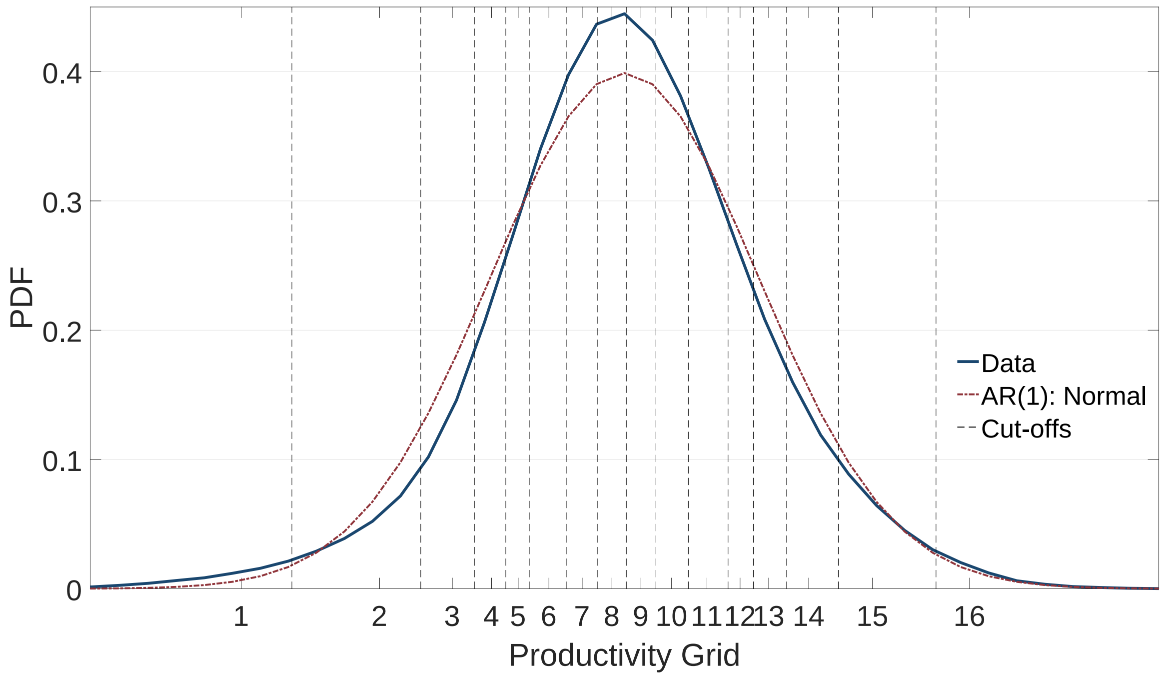
[10] I use the following quantiles as cut-offs: $Q_{0.01}$, $Q_{0.05}$, $Q_{0.10}$, $Q_{0.15}$, $Q_{0.20}$, $Q_{0.30}$, $Q_{0.40}$, $Q_{0.50}$, $Q_{0.60}$, $Q_{0.70}$, $Q_{0.80}$, $Q_{0.85}$, $Q_{0.90}$, $Q_{0.95}$, $Q_{0.99}$. Note that I choose the quantiles carefully, so I can capture the productivity dynamics at the tail of the productivity distribution.

[11] See Decker, Haltiwanger, Jarmin, and Miranda (2015) for a analysis of the skewness in the U.S. over time and its consequences for the economy.

[12] See Autor, Dorn, Katz, Patterson, and Van Reenen (2019) and the note Philippon (2018) for the evolution of concentration in the U.S. and its consequences for investment and growth.

fraction of very low productivity firms than very high productivity ones. Furthermore, the empirical distribution is more concentrated in its center, i.e. high kurtosis, and therefore, having fatter tails. This translates into a larger fraction of very low and high productivity firms than in the standard AR(1).

Figure 1: Productivity Distribution

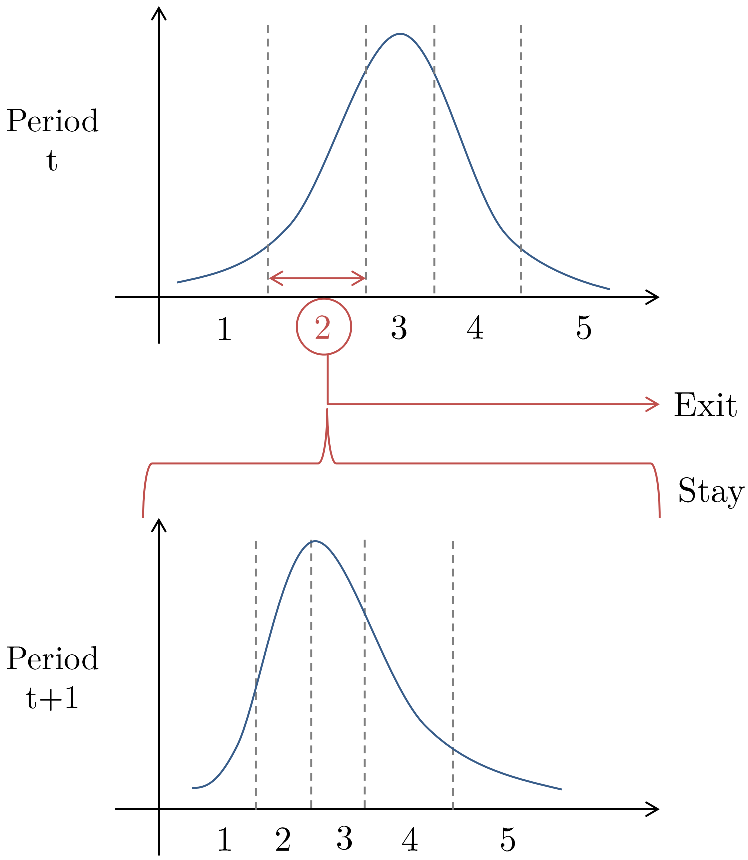


4.1 Productivity Dynamics

The similarity of the empirical firm's productivity distribution with the one implied by the standard AR(1) hides richer productivity dynamics in the data than the ones implied by an AR(1) process. In order to estimate and characterize the productivity dynamics, I use firms that can be tracked for two consecutive years. The procedure is illustrated in Figure 2. Conditional on firms being initially in one region of the productivity distribution, I estimate the same quantiles as in the discretization procedure for the next period productivity distribution. These conditional quantiles allow me to use the definitions of productivity persistence, shock variability, skewness and kurtosis used in Arellano et al. (2017).

In Figure 3, I show the conditional productivity distribution for an initially low productivity firm (left) and an initially high productivity one (right) and compare them with

Figure 2: Estimation of Productivity Dynamics

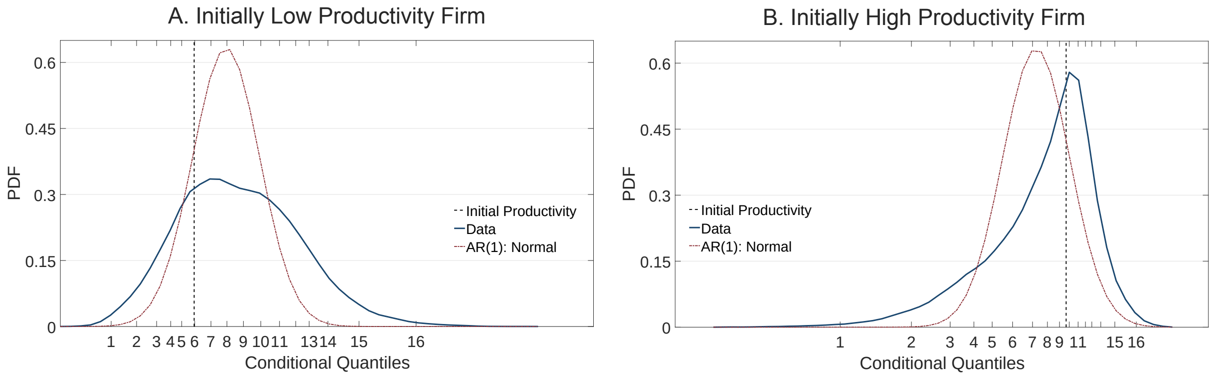


the distributions implied if AR(1) dynamics are assumed. The conditional productivity distributions are far from being Gaussian. For an initially low productivity firm, the empirical distribution is more disperse and it has a longer tail at the right, positive skewness. These features translates in a larger probability of having a large positive shock than in a standard AR(1) case. Regarding an initially high productivity firm, there is a long tail at the left of the distribution, i.e. negative skewness. This contrasts with the symmetric distribution when AR(1) dynamics are assumed. This implies that high productivity firms have a large probability of having a large negative productivity shock. Together with the lower persistence of the high productivity states imply that the good productivity realizations are not long-lasting for a large fraction of firm.

I also compute transitions to exit.[13] The fraction of firms that exit the market, conditional on their initial productivity. This has to be thought as having an absorbing productivity state with zero productivity. Finally, I compute the entry rates of firms for

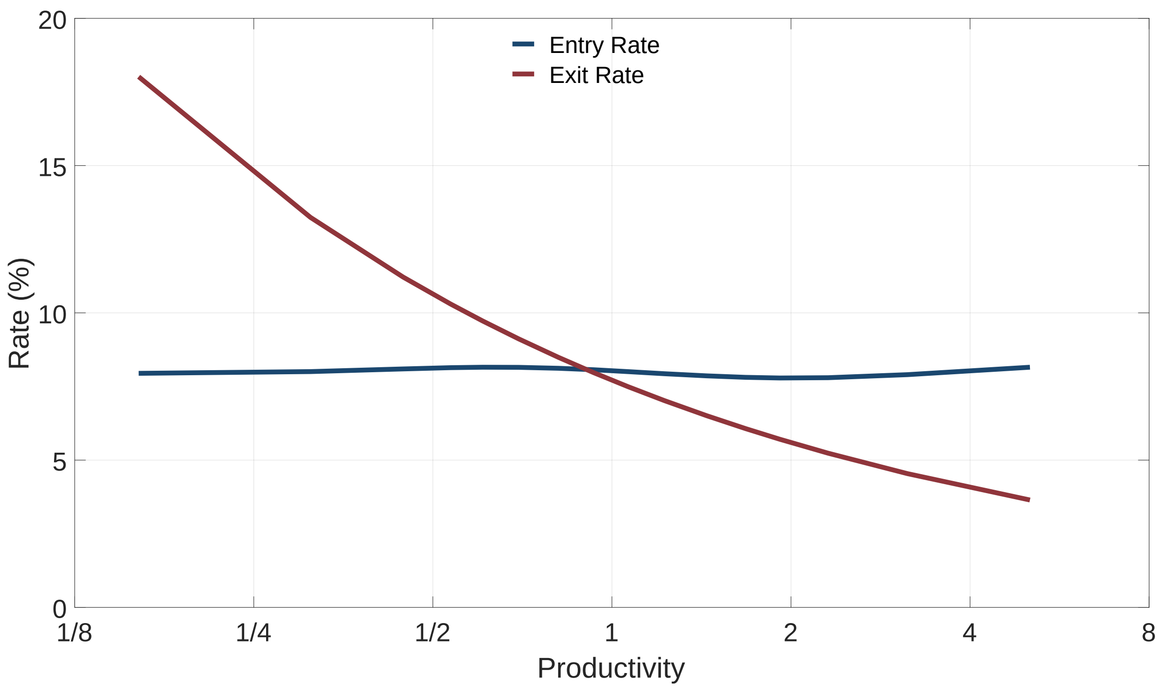
[13] As there is not an exit variable in the dataset, I infer firm exit from continuing firms. Basically, I use panel dimension of the dataset and assess that a firm exits if it does not appear in the following periods.

Figure 3: Conditional Productivity Distribution



each level of productivity. In Figure 4, I show the recovered entry and exit rates. Entry rates are the same for all the productivity levels. This means that entrants draw their productivity from the stationary distribution, a standard assumptions in models of firms dynamics with entry that is confirmed in my data. On the other hand, exit rates are decreasing with firm productivity, so low productivity firms are more likely to exit. The entry and exit rates together with the estimated transition probabilities are the main ingredients that discipline the productivity dynamics in the model.

Figure 4: Entry and Exit Rates



Characteristics of the productivity process In order to compare the estimated non-linear productivity process with the standard AR(1) used in the literature, I estimate four objects. First, productivity persistence, which is defined as the fraction of productivity inherited in the next period conditional on facing the same productivity shock. The expression reads as follows

$$\rho(log(A_{i,t-1}),\ \tau) = \frac{\partial Q(log(A_{i,t-1});\ \tau)}{\partial log(A_{i,t-1})}, \tag{3}$$

where $Q(log(A_{i,t-1});\ \tau)$ represents the quantile function of the productivity distribution in period t conditional on initial productivity, $log(A_{i,t-1})$, and τ is the quantile at which the function $Q(log(A_{i,t-1});\ \tau)$ is evaluated.

This gives a persistence estimate for each level of initial productivity and productivity shock. As I am interested on the persistence conditional on initial productivity regardless of the productivity shock, I integrate over the shock distribution.[14] Therefore, the reported productivity persistence follows from this expression

$$\rho(log(A_{i,t-1})) = E\left[\frac{\partial Q(log(A_{i,t-1});\ \tau)}{\partial log(A_{i,t-1})}\right]. \tag{4}$$

It is important to note that the standard AR(1) features constant productivity persistence equals to the autoregressive parameter independently of the initial level of productivity.

Second, I define shock variability as the difference of two equally spaced quantiles from the median. It measures how disperse is the next period productivity; and therefore, how much uncertainty the firm faces. The expression reads as follows

$$\sigma(log(A_{i,t-1})) = Q(log(A_{i,t-1});\ \tau) - Q(log(A_{i,t-1});\ 1-\tau).\text{[15]} \tag{5}$$

Third, shock skewness describes the asymmetry of the distribution. If the quantiles of the right tail are further away from the median than the left ones, the distribution

[14] The main reason is that the investment decision is made before the productivity shock is realized. Therefore, conditional on initial productivity.

[15] The previous expression is only valid for any $\tau \in (1/2,\ 1)$. In this case, I use $\tau = 0.75$, which corresponds to the interquartile range.

exhibits positive or right skewness. If the contrary happens, it exhibits negative or left skewness. The expression reads as follows

$$sk(log(A_{i,t-1})) = \frac{Q(log(A_{i,t-1});\ \tau) + Q(log(A_{i,t-1});\ 1-\tau) - 2Q(log(A_{i,t-1});\ 0.5)}{Q(log(A_{i,t-1});\ \tau) - Q(log(A_{i,t-1});\ 1-\tau)}.^{16} \tag{6}$$

Finally, shock kurtosis or tailedness captures the concentration of probability in the central part of the distribution; and therefore, the probability of having a low or large productivity shock. It has the following expression

$$kur(log(A_{i,t-1})) = \frac{Q(log(A_{i,t-1});\ 1-\alpha) - Q(log(A_{i,t-1});\ \alpha)}{Q(log(A_{i,t-1});\ \tau) - Q(log(A_{i,t-1});\ 1-\tau)}.^{17} \tag{7}$$

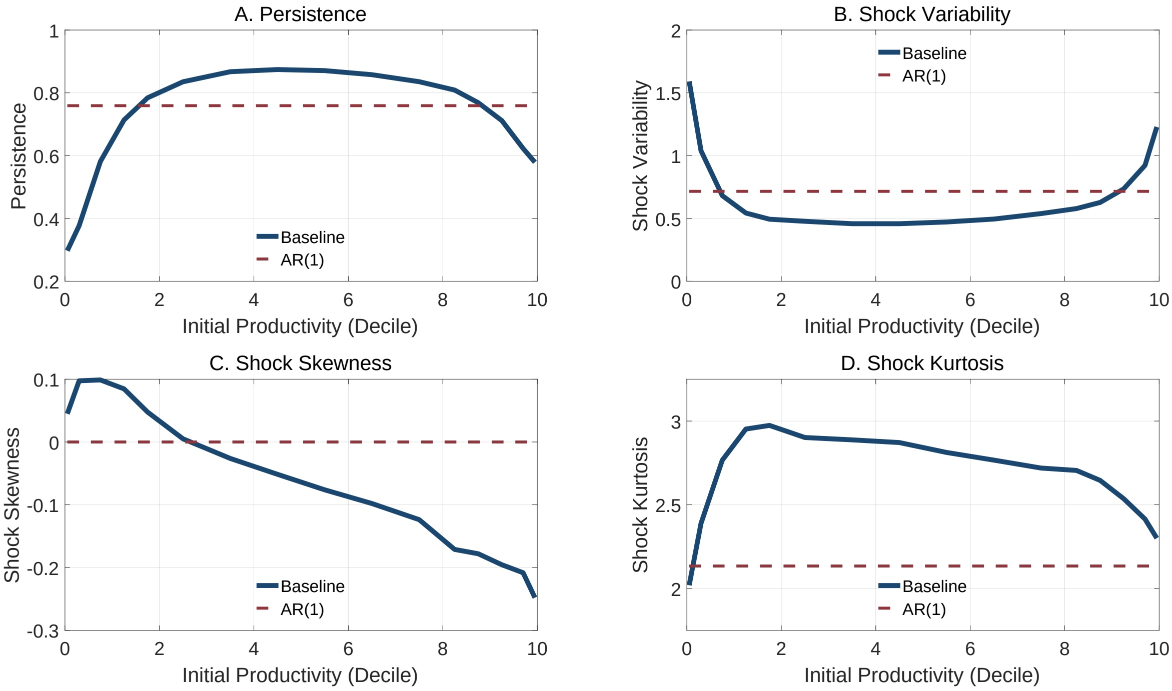
In Figure 5, I plot the four main characteristics of the productivity process estimated for Spanish firms. First, the estimated productivity process is highly non-linear. Productivity persistence is hump-shaped while shock variability is U-shaped with respect to initial productivity. Second, productivity shocks are non-Gaussian. Shock skewness is decreasing, while shock kurtosis is hump-shaped with respect to initial productivity. It is important to note that the standard AR(1) productivity process features constant productivity persistence and shock variability, zero shock skewness and shock kurtosis close to 2.2, as defined here.

The estimated productivity process differs from the standard AR(1). What are the implications of these findings for firm behavior and financial frictions? Under the estimated productivity process, the probability of an initially low productivity firm to have a large positive productivity shock is larger than in a standard AR(1) process. The transition probability from the first decile to the top decile is 0.8% in the estimated productivity process, while it is 0.0% in the AR(1). Similarly, the transition from the first decile to being above the median contrasts from the 6.7% in the estimated process to the 1.2% implied by the standard AR(1). Some of these initially low productivity firms will not have enough internal funds to finance the optimal capital level; and therefore, they will be financially constrained. This is going to be more prevalent under the estimated productivity process as there is a larger fraction of initially low productivity firms having

[16] The previous expression is only valid for any $\tau \in (1/2,\ 1)$. As in the previous case, I use $\tau = 0.75$.

[17] The previous expression is only valid for any $\tau \in (1/2,\ 1)$ and $\alpha < 1-\tau$. I use $\tau = 0.75$ and $\alpha = 0.075$.

Figure 5: Characteristics of the Productivity Process



these good productivity realizations. Furthermore, those good productivity realizations may not be long-lasting. I find that the transition probability of a firm from the top decile to below the median is 7.0% in the estimated productivity process versus a 1.2% if AR(1) productivity dynamics are assumed. Those bad realizations of productivity will slow down the internal profit accumulation of those firms.

Robustness A natural question is whether the proposed approach is able to characterize the productivity dynamics properly. In order to tackle it, I do a Monte-Carlo simulation from an AR(1) productivity process with $\rho_a = 0.8$ and $\sigma_\varepsilon = 0.3$. I simulate 1 million observations from the stationary distribution for two periods. Then, I implement the previous methodology to recover the parameters imposed in the simulation. The persistence parameter from the simulation is accurately estimated to 0.8 in all the range of the productivity distribution, except for the tails. Both at the very top and bottom of the productivity distribution, 1 percentile, the estimate of the persistence parameter jumps to 0.85. Regarding shock variability, a similar pattern arises. The estimation is very accurate in all the range of the productivity distribution, except at the tails; where it is overestimated. Regarding shock skewness and shock kurtosis, the estimated parameters are very close to their theoretical counterparts even at the tails of the distribution. These results can be found in Appendix B.1.2.

Another potential concern is that I treat the whole economy as one sector economy standardizing the productivity data at the sector-year level. The main reason is that the proposed procedure is very demanding, as I want to capture the dynamics at the tail of the productivity distribution. Therefore, pulling the data of all the sectors gives more power to the estimation strategy. As alternative, I estimate the non-linear productivity process at the sector level and then aggregate it using 2-digits sector weights. I show the results in Appendix B.1.3. The main conclusion is that the sector by sector estimation yields very similar estimates.

In the estimation, I set $\eta = 0.83$, so that the model economy is able to match $SD(k_{si})$. A potential concern is the robustness of the characteristics of the productivity process to different values of the η parameter. I estimate the productivity dynamics setting a wide range of η, from 0.75 to 0.90, which fall in the ranged usually used in the firm dynamics literature. Results are in Appendix B.1.4. I conclude that the main characteristics of the productivity process are robust to different levels of the decreasing returns to scale parameter, η.

Finally, the studied period from 1999 to 2014 covers a long time-span, and it includes the Great Recession of 2007 in the middle. In order to check the robustness of the results over time, I split the studied period into two, before the Great Recession, 1999 to 2007, and during and after the Great Recession, 2007-2014. Results are summarized in Appendix B.1.5. The characteristics of the productivity process are very similar in the two periods showing the stability of the results.

4.2 Misallocation

Financial frictions affect firms by restricting their capital level below their optimal one. The standard approach to assess the existence of financial frictions in the literature has been through the following specification

$$inv_{i,s,t} = \alpha \ + \ \beta cf_{i,s,t-1} \ + \ \tilde{\beta}' X_{i,s,t} \ + \ \varepsilon_{i,s,t}, \tag{8}$$

where $inv_{i,s,t}$ is the investment of firm i, in sector s in period t, $cf_{i,s,t-1}$ is the cash flow of firm i, in sector s in period $t-1$, and $X_{i,s,t}$ are controls. A positive estimated β coefficient has been pointed out as evidence on the existence of financial frictions. The reason is simple, if the firm is financially constrained and have a high cash flow in the past period, it can use those funds to self-finance itself.

The usage of Equation 9 to show that firms experience financial constraints can be problematic. Gomes (2001) shows that a model with persistent productivity dynamics and time-to-build in the capital decision is enough to generate a positive coefficient. He proposes a model with productivity persistence, time-to-build and financial frictions. After simulating it, he estimates a positive coefficient as expected in a model with financial frictions. The puzzle is that the positive coefficient appears even in the specification without financial frictions. The main idea is as follows. If the firm has a high cash flow in the past, it is likely to have experienced a high productivity shock. If productivity is persistent, then the firm expects to have higher productivity in the future. As capital takes time-to-build, it starts to invest today in order to take advantage of the expected higher productivity in the next period.

In this section, I propose a different methodology to show indirect evidence on the existence of financial frictions based on the misallocation literature. Hsieh and Klenow (2009) shows that with a Cobb-Douglas production function the Average Revenue Product (ARP) should be equalized across firms in a perfectly competitive economy. Under frictionless input markets, firms would buy or invest in inputs up to the point the return of the last unit equalizes its cost. As the cost of the inputs is the same across firms operating in the same sector due to perfect competition, the ratio of output over input should be equalized. Regarding capital, we define ARPK as

$$ARPK_{i,t} = \frac{py_{i,t}}{k_{i,t}}. \tag{9}$$

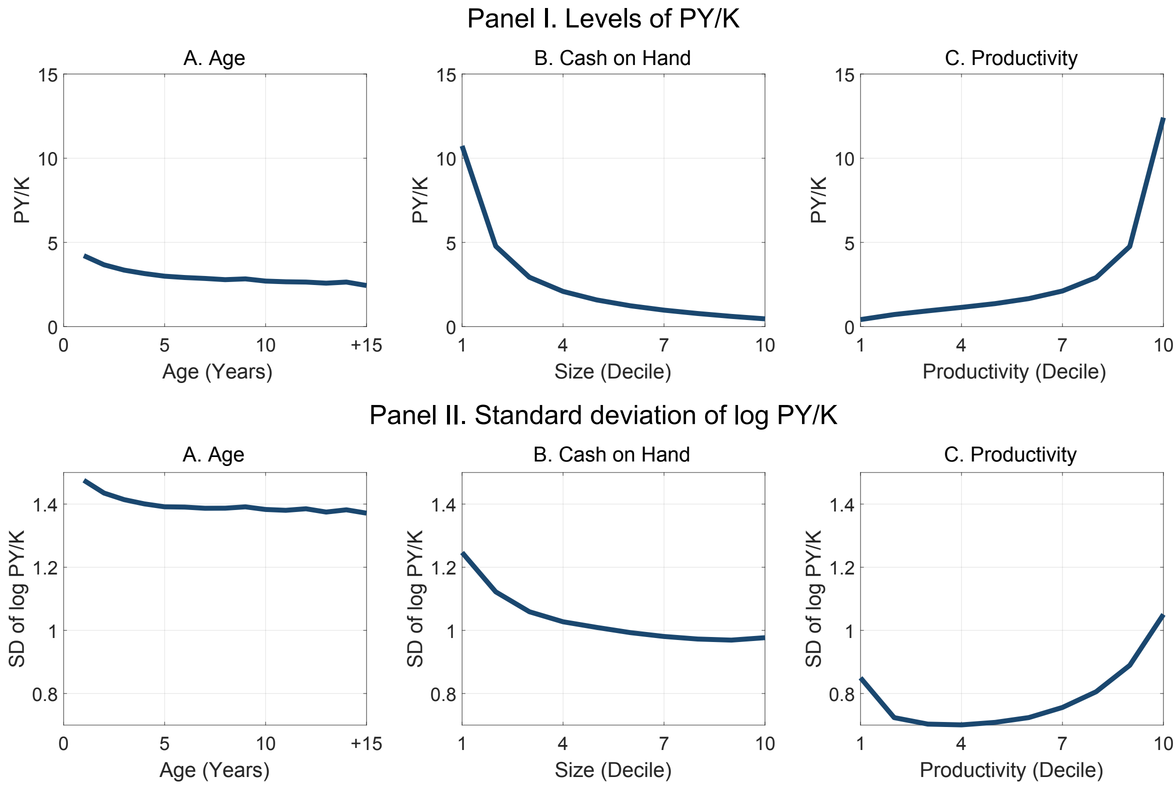
The difference of $ARPK$ among firms operating in the same sector can be due to capital misallocation. Therefore, the variance of $log(ARPK)$ has become the standard to measure allocation efficiency of capital at the sector level, see for instance David and Venkateswaran (2019).

I compute the mean of the $ARPK$ and standard deviation of $log(ARPK)$ at the sector level, and then aggregate it, conditional on firm characteristics. First, I condition on firm age, measured by years since the firm has entered the market. Second, I condition on firm size, measured as net worth. Third, I condition on firm productivity, as measured in Section 3. The mean of the $ARPK$ conditional on firm characteristics captures distortions that are correlated with firm characteristics. On the other hand, the standard deviation of $log(ARPK)$ conditional on firm characteristics captures the variation within each group.

The results are shown in Figure 6. Panel I presents the profiles of the mean $ARPK$ across firm characteristics, while panel II presents the profiles of the standard deviation of $log(ARPK)$. The results indicates the presence of financial frictions. Financial frictions should affect disproportionally to young, small and high productivity firms. Young firms are unlikely to have enough internal funds to surpass the financial frictions. In line with this prediction, I find that young firms have on average larger $ARPK$, which gets slowly lower as firms age, as profit accumulation takes place. Furthermore, the standard deviation of $log(ARPK)$ is larger for young firms, as well. The reason is that not all the young firms are financially constrained. The standard deviation of $log(ARPK)$ reduces as firms age, as they accumulate internal funds to overcome financial frictions. Small firms are also limited by their current net worth to invest in capital. Finally, regarding firm productivity, high productivity firms have a high optimal level of capital, which they may not be able to finance. Accordingly, I find an upwards sloping profile of mean of $ARPK$ and standard deviation of $log(ARPK)$ with respect to firm productivity.

Robustness There are two concerns that might affect the previous analysis. First, during the studied period, the allocation of capital has been gradually deteriorating in Spain, as shown in Gopinath et al. (2017). In order to take into account the increase in capital misallocation over time, I standardize the data on ARPK and log ARPK at the sector-year level. With such standardization, there is no trend in the allocation of capital during the studied period. The results are shown in Appendix B.2.1. The profiles look very similar under the two specifications. The only difference is smaller correlated distortions in the standardized specification. Second and related with the previous concern, the studied period from 1999 to 2014 covers the Great Recession of 2007 in the middle. In order to check the robustness of the results across time, I split the studied period into

Figure 6: Profiles of PY/K



two, before the Great Recession, 1999 to 2007, and during and after the Great Recession, 2007-2014. Results are summarized in Appendix B.2.2. I conclude that the results are very similar in the two periods.

4.3 Financial Behavior

Empirical finance literature has focused on studying the financial behavior of publicly-listed firms. The main reason is the lack of comprehensive dataset of privately-held companies. Furthermore, there are several reason to believe that financial behavior affects differently these two groups, public vs private firms. First, private firms have a widely range of fund raising. They not only have access to the traditional bank lending channel, but also they can raise equity in stock markets and issue debt in bond markets. Second, publicly-listed firms are usually larger than privately-held firms, which may facilitate their access to credit. In fact, Dinlersoz et al. (2018) shows that the financial behavior of these two groups differs. On the other hand, without a consistent set of facts on the financial behaviour of private firms, it is very hard to evaluate models of firm dynamics with financial frictions. In this section, I fill this gap by providing evidence on how the debt structure of privately-held firms differs with firm characteristics.

In Table 1, I show the fraction of firms that do not use any costly debt and the leverage distribution, measured as costly debt over total assets. The usage of debt varies widely across firms. As we can see, the fraction of firms that do not use any debt is large, 29%. Furthermore, among the debt users, there is a 5% of firms with a leverage lower than 0.01, while there is another 5% of firms with a leverage larger than 0.71.

Table 1: Leverage Distribution

Percentile	Data
P_5^{Lev}	0.01
P_{10}^{Lev}	0.02
P_{25}^{Lev}	0.08
P_{50}^{Lev}	0.22
P_{75}^{Lev}	0.42
P_{90}^{Lev}	0.61
P_{95}^{Lev}	0.71
Fraction with 0	0.29

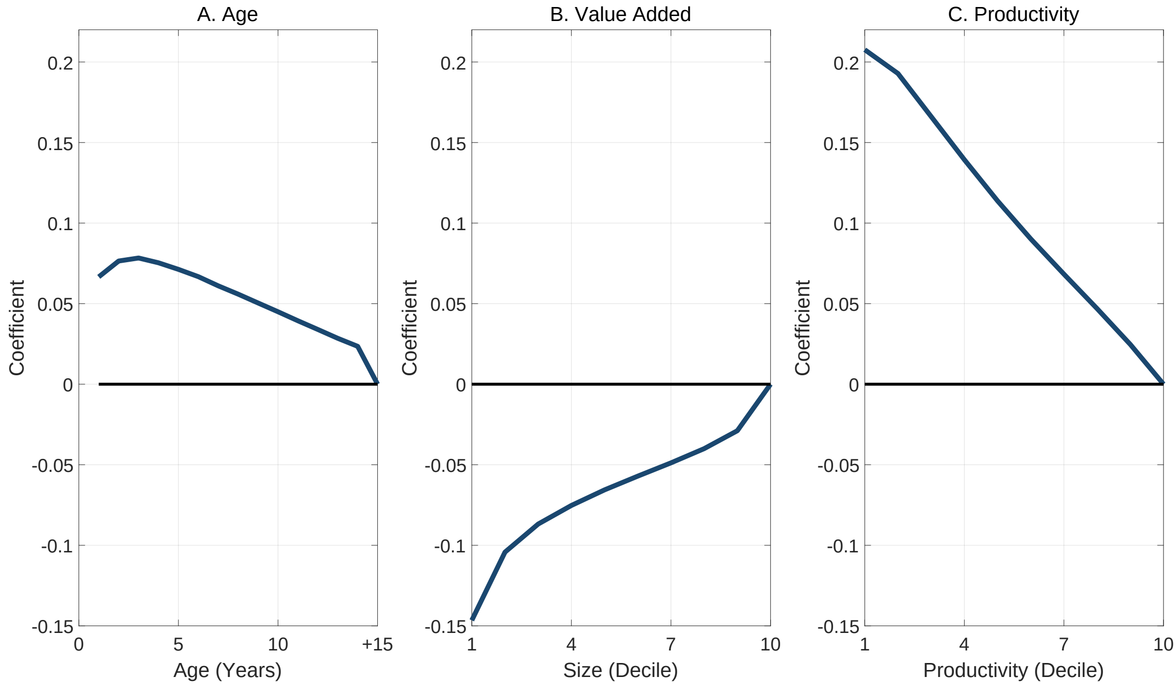
The large variation of firm leverage across firms raises two questions. First, how does it vary with firm characteristics? Second, are the patterns similar for the extensive and intensive margin? In order to answer the first question, I propose a non-parametric model to capture the correlation of leverage on firm characteristics (age, size and productivity). The specification reads as follows

$$Leverage_{i,s,t} = f(age_i) + g(size_i) + h(A_i) + \beta^{'} X_{i,s,t} + \varepsilon_{i,s,t}, \tag{10}$$

where $f(age_i)$ is a function on firm age, which will be approximate by estimating the coefficients on age dummies. The $g(size_i)$ function is approximated with 10 dummies, corresponding to the deciles of the value added distribution. The $h(A_i)$ function is also approximated with 10 dummies, corresponding to the deciles of the productivity distribution. Finally, $X_{i,s,t}$ are the controls, a full set of sector-year fixed effects. They are aimed to capture differential trends on the average financial behavior across sectors over time.

I show the estimated coefficients in Figure 7. Leverage is decreasing with respect to

Figure 7: Financial Behavior



Note: The omitted categories are +15 years old, top size decile and top productivity decile.

firm age and firm productivity. On the contrary, it is increasing with firm size. As we have seen in Table 1, there is a non negligible fraction of firms that do not use debt. Therefore, the relation shown in Figure 6 can come either from the extensive or intensive margin of debt usage. In order to explore the extensive margin, I propose a probit model where the relation with firm age, size and productivity is estimated non-paramatrically analogous to Equation 11. The estimated model is given by

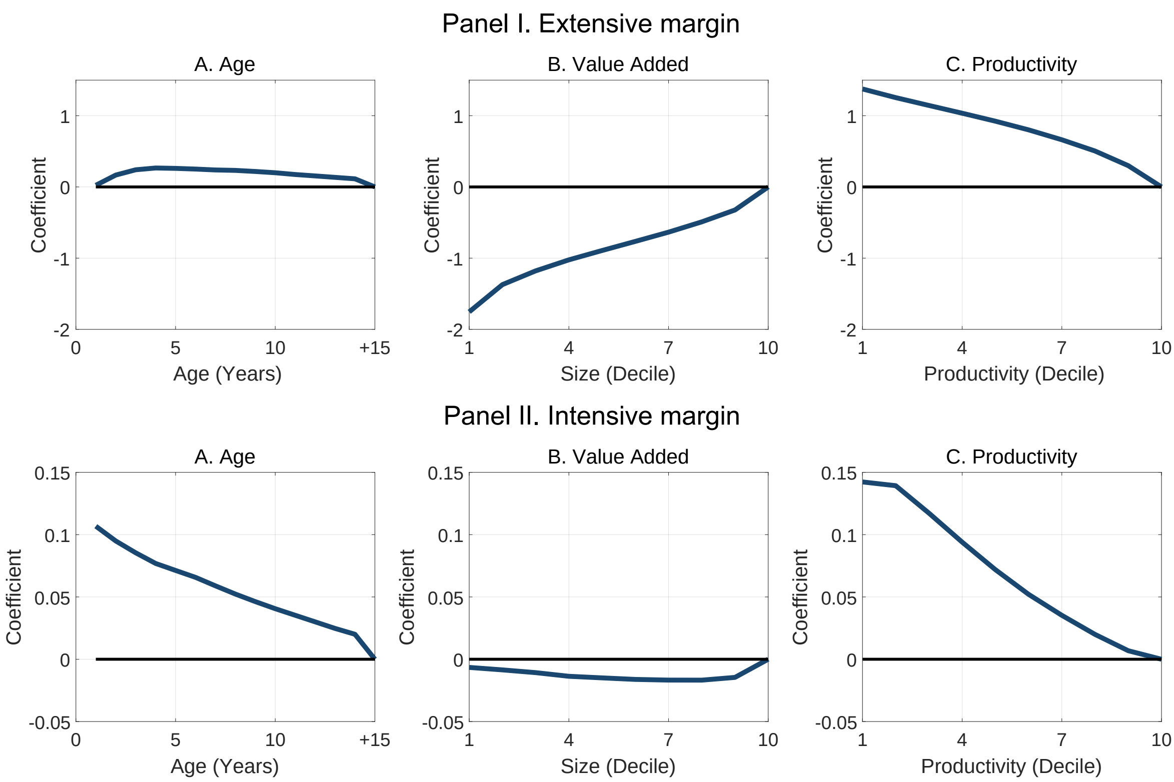
$$P(Debt_{i,s,t} = 1 \mid age_i,\ size_i,\ A_i,\ X_{i,s,t}) = \Phi\left(f(age_i) + g(size_i) + h(A_i) + \beta' X_{i,s,t}\right). \tag{11}$$

Regarding the intensive margin, I estimate Equation 11 conditional on firms having positive debt. Formally

$$Leverage_{i,s,t} \mid Debt_{i,s,t} \geq 0 = f(age_i) + g(size_i) + h(A_i) + \beta' X_{i,s,t} + \varepsilon_{i,s,t}. \tag{12}$$

The estimated functions are shown in Figure 8. In panel I, I show the estimates for the extensive margin, while in panel II the ones regarding the intensive margin. The results show that the negative correlation of leverage with firm age is mostly due to the intensive margin, as the probability of using debt is almost flat with respect to firm age. The results differ markedly for firm size. Conditional of using debt, there is not much difference in the average leverage of firms of different size. But, smaller firms are much less likely than larger ones to use debt to finance their investment. Finally, regarding firm productivity, the negative relation of leverage with firm productivity appears in both the intensive and extensive margin. Low productivity firms are more likely to use debt to finance their investment; and when they use it, they finance a larger fraction of their total assets.

Figure 8: Financial Behavior - Extensive and Intensive Margin



Note: The omitted categories are +15 years old, top size decile and top productivity decile.

Robustness There are three concerns that might affect the previous analysis. First, the finance literature has focused on profitability, measured as profits over total assets, instead of productivity. In fact, the negative relation of firm leverage and profitability has been a puzzle in the literature, see Frank and Goyal (2009) for instance. In order to see the robustness of the results, I extend the previous models controlling for firm profitability. The results and further details are exposed in Appendix B.2.1. The main conclusion is that the relations presented here are very similar even when I control by firm profitability.

Second, in a very similar framework to the one proposed here, Dinlersoz et al. (2018) find a positive relation between leverage and productivity. The main difference between the two frameworks is the definition of firm productivity. In Dinlersoz et al. (2018), they rely on labor productivity, defined as value added over labor, while I rely on total factor productivity. In Appendix B.2.2, I show that if I estimate their specification with labor productivity, I find a positive coefficient on firm productivity as well. In this paper, measuring firm productivity as TFP is more appropriate for two reason. First, labor is treated as a static decision in a perfectly competitive framework. Therefore, as shown in Section 4.2, firms will hire labor up to the point the $ARPL$ (labor productivity) is equalized across firms. In that sense, labor productivity is capturing distortions in the labor market that prevents firms to hire the optimal level of employment. Second, even if the $ARPL$ is positively correlated with firm productivity, as more productivity firms may face larger frictions that prevents them to hire the optimal amount of labor, the measure of productivity used here in more comprehensive. It uses the two main production factors in its calculation, labor and capital.

Finally, I check whether the results change over time. As I did in previous sections, I divide the studied period into two, before the Great Recession, 1999 to 2007, and during and after the Great Recession, 2007-2014. Results are summarized in Appendix B.2.2. The results are very similar in the two periods. The main difference appears in the relation of leverage with firm size, which gets steeper in the after the Great Recession period. This suggest that the financial crisis of 2007 has affected disproportionally to small firms, which are the ones more likely to be constrained.

4.4 A Recap

In this section, I have provided three new sets of facts. First, I show that Spanish firms face a highly non-linear productivity process with non-Gaussian shocks. I show that productivity persistence is hump-shaped with respect to past productivity, while shock variability is U-shaped. I also show that shock skewness is decreasing with past productivity, while shock kurtosis is hump-shaped. The productivity process uncovered in the estimation procedure is very different from a standard AR(1) process, the workhorse in the firm dynamics literature. Under the estimated process, a low productivity firm has a larger probability of becoming highly productive in the next periods. On top of that, those high productivity episodes are not long-lasting. These features of the estimated process are crucial to understand the effects of financial frictions on the firm life cycle and the aggregate economy.

Second, I show that the ARPK and the standard deviation of log ARPK are decreasing both with firm age and size, while they are increasing with firm productivity. This is suggestive evidence on the presence of financially constrained firms, specially among the young, small and high productivity ones.

Finally, I have studied the financial behavior of Spanish firms exploiting variation on the leverage ratio. I first show that there is a large fraction of firms that do not use costly debt, 29%, and the leverage distribution is very disperse. I also show that the average leverage correlates with firm characteristics. It decreases with firm age and productivity, while it increases with firm size. These patterns are present in both the extensive margin, probability of using costly debt, and the intensive margin, average leverage conditional on using costly debt.

5 Model

In this section, I present a model of firm dynamics with financial frictions and the non-linear productivity dynamics. Firms are heterogeneous in their productivity levels, which evolves stochastically according to the Markov process presented in Section 4.1. They produce an homogeneous good combining capital and labor in a Cobb-Douglas production technology under decreasing returns to scale,

$$py = F(k,\ l,\ A) = A_{shift}A[k^{\alpha}l^{1-\alpha}]^{\eta} \qquad \alpha \in (0,\ 1)\ and\ \eta \in (0,\ 1), \tag{13}$$

where py is value added, A_{shift} is the aggregate component of total factor productivity, A is the idiosyncratic component of productivity, k is capital and l is labor of a firm i. This is the same expression as Equation 1, which was used to compute firm-level productivity in the data.

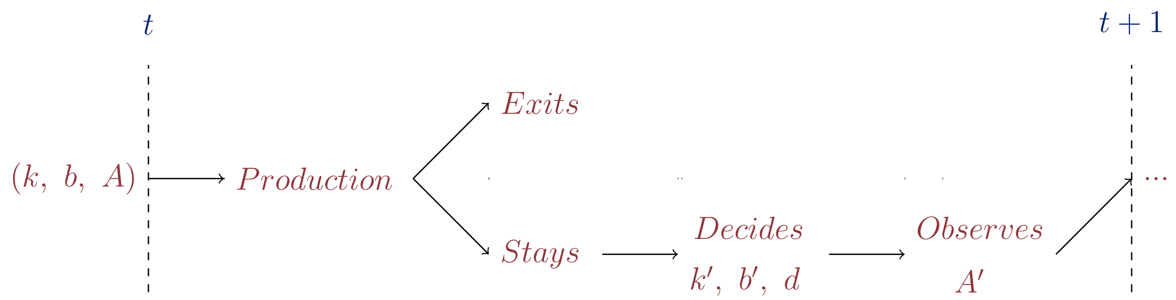
The objective of a firm is to maximize their current value plus continuation value if it does not exit the market. In order to fulfil the objective, the firm chooses how much to invest, how much to borrow to finance its investment, how much labor to hire and how much dividends to pay to the households. The choice of capital takes place before the productivity of the current period is realized. This is the usual information frictions used in the firm dynamics literature, capturing the time-to-build nature of capital. Furthermore, investment is limited by a borrowing capacity that depends on internal funds and its productivity. The choice of labor is static and it is not subject to any friction. Dividends are the residual amount left after the production takes place, the firm adjust its capital and borrowing level. Finally, all the markets are perfectly competitive and the firms take prices as given.

In Figure 9, I summarize the decision tree of an incumbent firm. A firm enters in the period with a level of capital k, borrowing b, which can be positive or negative depending if the firm is a net borrower or saver, and productivity A. At this stage, the firm decides how much labor to hire to maximize its per-period profits

$$\widehat{\pi}(k,\ A) = \underset{\{l\}}{max}\left\{F(k,\ l,\ A) - wl\right\}, \tag{14}$$

where l is the amount of labor, w is the wage rate.

Figure 9: Timing - Incumbent Firm



After the production takes place, firms receive an exit shock $\vartheta(A)$, which depends on the firm productivity. If the firm exits, it is liquidated, and the surplus is returned to the household. Formally, the firm value is given by the following expression

$$V^{exit}(k,\ b,\ A) = \widehat{\pi}(k,\ A) + (1-\delta)k - b. \tag{15}$$

If the firm stays, it decides how much invest in capital and how to finance it, depending on its internal funds and productivity level. Borrowing is limited by the installed capital and a size-dependent pledge-ability parameter. Formally,

$$b' \leq \theta\Big(\frac{k'}{k'_u(A)}\Big)^{\Psi} k'. \tag{16}$$

This borrowing constraint follows Gopinath et al. (2017), and has two components. First, the level of capital the firm will install for the next period, k'. And the pledge-ability component, $\theta\Big(\frac{k'}{k'_u(A)}\Big)^{\Psi}$, which captures the fraction of the installed capital subject to collateralization. I assume it is a non-linear function of the installed capital, k' and the optimal level of capital the firm would like to install, $k'_u(A)$. If the firm enough internal funds, such that $k' = k'_u(A)$, the borrowing constrained turns the standard one used in the firm dynamics literature, $b' \leq \theta k'$. Therefore, the parameter θ governs the maximum amount of capital a firm can pledge. The parameter Ψ governs the difference in pledge-ability among firms that differ in their level of internal funds. Therefore, it is the penalty that the financial markets impose to firms with low internal resources.

Finally, dividends are the remaining funds after the investment and borrowing decisions are made. They are constrained to be non-negative, as firms are not allowed to raise equity. Formally,

$$d \equiv (1-\tau)\widehat{\pi}(k,\ A) + (1-\delta)k - b - k' + qb', \tag{17}$$

where δ is the depreciation rate of capital, q is the price of the debt issued by the firm in order to obtain funding. The value of q is a general equilibrium object that determines the cost of funding of firms. The parameter τ disciplines the wedge between the value added and the after taxes profits. It captures any friction or conditions in the environment not taken into account in the model, such as taxes. This wedge is returned to the household as lump-sum, so that it does not distort firm decisions. Finally, the firm observes the next period productivity and the process restarts.

The problem of the incumbent firm that stays, in recursive formulation, reads as follows

$$
\begin{aligned}
V(k,\ b,\ A) = \underset{\{k',\ b',\ d\}}{max} \quad & d + \\
& \beta(1-\vartheta(A))E\left[V(k',\ b',\ A')|A\right] + \\
& \beta(1-\vartheta(A))E\left[\tau\widehat{\pi}(k',\ A')|A\right] + \\
& \beta\vartheta(A)E\left[\widehat{\pi}(k',\ A') + (1-\delta)k' - b'|A\right],
\end{aligned} \tag{18}
$$

subject to

$$d = (1-\tau)\widehat{\pi}(k,\ A) + (1-\delta)k - b - k' + qb' \geq 0,\ and \tag{19}$$

$$b' \leq \theta\Big(\frac{k'}{k'_u(A)}\Big)^{\Psi} k', \tag{20}$$

where β is the subjective discount factor, $E\,[\ .\ |A]$ is the expectation conditional on today's productivity A. It contains the dynamics of the productivity process and it is the main source of uncertainty firms face. Finally, Equation 19 is the non-equity issuance constraint, and Equation 20 reflects the borrowing constraint.

Exiting firms are replaced by new entrant firms. The timing of the entry problem is summarized in Figure 10. First, entrants observe the distribution of equity (initial internal funds) and firm productivity $\Omega(e_0,\ A_0)$.[18] The initial level of internal funds conditional on firm productivity is assumed to follow a log normal distribution. Formally,

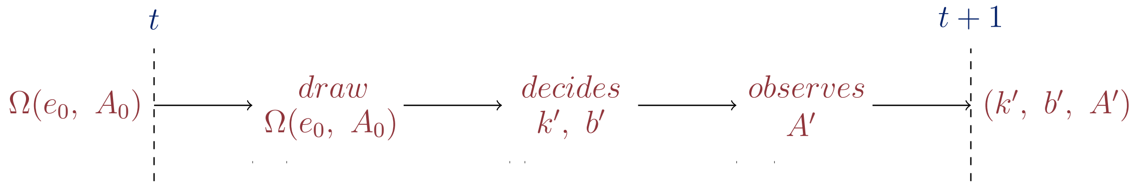
$$\Omega(e|a) \ \sim \ N\Big(\mu_e + \frac{\sigma_e}{\sigma_a}\rho_{a,e}(a-\mu_a);\ (1-\rho^2_{a,e})\sigma^2_e\Big), \tag{21}$$

where a stands for $log(A)$. While the marginal distribution with respect to productivity $\Omega(a)$ is directly estimated from the data, as shown in Figure 4.

The entrant firm gets a draw $(e_0,\ A_0)$ from the distribution. Given their equity e_0 and initial productivity A_0, the firm decides the capital investment, k' and how much to finance b'. Finally, the firm observes the next period productivity and starts production according to the state k', b', A'. At this stage, the firm becomes incumbent and the sequence of events is described according to Figure 8.

[18] Note that this is equivalent to $k_0 = 0$ and $b_0 = -e_0$.

Figure 10: Timing - Entrant Firm



Households There is a representative household that owns the firms. The household maximizes the discounted flow of per period utility. The household provides 1 unit of labor inelastically and it decides how much to consume of the homogeneous good produced by the firms. It also owns the firms and the bonds firms use to finance investment. Finally, she receives the dividends flow paid by the firms. The household problem, in recursive formulation reads as follows

$$V^h(\Lambda,\ \Phi) = \underset{\{C^h,\ \Lambda',\ \Phi'\}}{max} \left\{ U\left(C^h\right) + \beta V^h(\Lambda',\ \Phi') \right\}, \tag{22}$$

subject to

$$\begin{aligned} C^h + q\Phi' + \int_{k'xb'xA'} \rho_1(k',\ b',\ A')\Lambda'(k',\ b',\ A')d\left[k'xb'xA'\right] \leq \\ w + \Phi + \int_{kxbxA} \rho_0(k,\ b,\ A)\Lambda(k,\ b,\ A)d\left[kxbxA\right] + \tau \int_{kxbxA} \widehat{\pi}(k,\ A)\Lambda(k,\ b,\ A)d\left[kxbxA\right], \end{aligned} \tag{23}$$

where Λ is the measure of firms and Φ is amount of bonds the household holds. $\rho_1(k',\ b',\ A')$ is the price (ex-dividend) of firm's shares with state $(k',\ b',\ A')$ for current period, while $\rho_0(k,\ b,\ A)$ is the price (dividend inclusive) of firm's shares with state $(k,\ b,\ A)$ for current period.

Equilibrium A stationary recursive competitive equilibrium consists of prices $(w,\ q,\ \rho_0,\ \rho_1)$, quantities $(l,\ k', b',\ \ d,\ C^h,\ \Lambda,\ \Phi)$, a distribution $\mu(k, b,\ A)$, a mass of firms (M) and values $(V,\ V^h)$ such that: First, V solve the firm's problem and $(l,\ k',\ b',\ d)$ are the associated policy functions. Second, V^h solves the household's problem and $(C^h,\ \Lambda,\ \Phi)$ are the associated policy functions. Third, all the markets clear: labor market, bond market, stock market and good market, which does due to Walras' law. Finally, the distribution of firms $\mu(k,\ b,\ A)$ is a fixed point consistent with the policy functions $(k',\ b')$, the exogenous exit rate ($\vartheta(A)$), the entry distribution $\left(\Omega(e_0,\ A_0)\right)$ and the law of motion for A.

5.1 Aggregation

From the firm level behavior and using the distribution of firms, $\mu(k,\ b,\ A)$, we can aggregate the economy in order to obtain the main economic variables. The total output is given by

$$Y = \int_{kxbxA} F(k,\ l,\ A)\ \mu(k,\ b,\ A)\ d\left[kxbxA\right]. \tag{24}$$

Similarly, total capital and labor are given by

$$K = \int_{kxbxA} k\ \mu(k,\ b,\ A)\ d\left[kxbxA\right] \quad \textit{and} \quad L = \int_{kxbxA} l\ \mu(k,\ b,\ A)\ d\left[kxbxA\right] = 1. \tag{25}$$

I define aggregate productivity as

$$A_g = \frac{Y}{K^{\alpha} L^{1-\alpha}}, \tag{26}$$

where $\alpha \in (0,\ 1)$ is a parameter governing the K/L ratio in the economy. It can be shown that aggregate productivity is an expression with three main elements: the average firm level productivity, the allocation of resources across firms, and the number of firms. This last component arises from the decreasing returns to scale of the production function at the firm level.

Following the same procedure, other variables can be aggregated as well, like total debt, profits and dividends.

5.2 Solution of the Model

The model set up is similar to the one developed in Khan and Thomas (2013). Therefore, I follow their strategy in order to solve the model. In this section, I describe the main points of the solution method and I provide further details in the Appendix C.1.

First, let me define the cash-on-hand variable. Cash-on-hand is the amount of available resources the firm has after undertaken production, selling the undepreciated capital and paying its debts. From the accounting point of view, the closest counterpart is the firm net worth. Formally, it is defined as

$$e(k,\ b,\ A) = \tau\widehat{\pi}(k,\ A) + (1-\delta)k - b. \tag{27}$$

Depending on their level of cash-on-hand, we can classify the firms in three categories. The first group of firms are the unconstrained ones. A firm that currently can implement the optimal level of capital as well as in the future, regardless of its productivity path. They invest up to the optimal unconstrained capital level, $k'_u(A)$, and have a debt level, or savings, such that they will be unconstrained in the future, $b'_u(A)$. In Appendix C.1, I provide the derivation for $k'_u(A')$, which has a closed-form solution, and the algorithm to find $b'_u(A)$. These firms are the only ones that pay positive dividends, as they have accumulated enough internal funds that prevents them for being constrained in the future. Dividends are determined as the residual of the available cash-on-hand after the capital and borrowing decision are made, as shown in Equation 17.

The second group of firms are labelled as constrained type I. A firm that currently can implement the optimal unconstrained level of capital $k'_u(A)$, but not the borrowing $b'_u(A)$. These firms are currently unconstrained, but they can be constrained in the future depending on their productivity shocks. The non-equity issuance constraint, Equation 19, is binding for them. This gives us the threshold that divides constrained from unconstrained firms. Formally,

$$e(k,\ b,\ A) - k'_u(A) + qb'_u(A) = 0 \qquad \rightarrow \qquad \hat{e}(A) = k'_u(A) - qb'_u(A). \tag{28}$$

These firms do not pay dividends. They find optimal to retain all the profits, as internal funding, up to the points they become unconstrained.

Finally, there is a third group of firms labelled as constrained type II. A firm that currently cannot implement the optimal unconstrained level of capital $k'_u(A)$. For these type of firms both the non-equity issuance constraint, Equation 19, and the borrowing constraint, Equation 20, are binding. Formally,

$$\left.\begin{array}{l} e(k,\ b,\ A) - k_u'(A) + qb' = 0 \\ b' = \theta k_u'(A) \end{array}\right\} \quad \rightarrow \quad \hat{\hat{e}}(A) = (1 - q\theta)k_u'(A). \tag{29}$$

These firms do not pay dividends, as they accumulated accumulate all the profits up to the points they become unconstrained.

6 Benchmark Economy

In this section, I calibrate the model and evaluate its performance along several dimensions: firm life cycle, capital misallocation and firm financial behavior.

6.1 Calibration

There are 11 parameters in the model that I calibrate to match 11 moments in the data. Table 2 shows the estimated parameters and their values. It also shows the targeted moments, its value in the data and the model counterpart. In the calibration of the decreasing returns to scale parameter, η I apply a discrete search grid method to match the standard deviation in the capital distribution. For each value of η, I estimate the productivity process and calibrate the remaining 10 parameters of the model using simulated method of moments. I minimize the sum of the squared residuals between a set of moments computed in the model and the data. Although all the moments are jointly determined through the internal mechanisms of the model, some parameters are specially relevant for matching certain moments.

The η parameter is estimated to be 0.83 matching very well the $SD(k)$. The estimated values of the subjective discount factor, β, the output to capital elasticity, α, and the depreciation rate, δ, fall in the usual range consider in the firm dynamics literature. The average productivity of firms, A_{shift}, sets the average firm size in the model as in the data, 15.5 employees. The two parameters governing the borrowing constrained, θ and Ψ, are set to match two moments of the leverage distribution: average leverage, $avg(Lev)$, and the percentile 95, P_{95}^{Lev}. They imply that the maximum fraction of capital that a

Table 2: Calibration

Parameter	Value	Moment	Data	Model
η	0.83	$SD(k)$	1.79	1.76
β	0.97	K/Y	2.0	2.2
α	0.35	K/L	4.0	4.1
δ	0.05	Inv/Y	0.12	0.13
A_{shift}	1.22	L	15.5	15.5
θ	0.81	$avg(Lev)$	0.19	0.19
Ψ	0.48	P_{95}^{Lev}	0.71	0.71
τ	0.43	$Profits/Y$	0.15	0.15
μ_e	1.95	k_{ent}	0.36	0.36
σ_e	1.92	$SD(k_{ent})$	0.95	0.95
$\rho_{a,e}$	0.02	$\rho(a_{ent}; e_{ent})$	0.05	0.05

firm can use as collateral is 0.81. The value of Ψ is different from 0, rejecting a constant pledge-ability parameter in the borrowing constraint. The wedge, τ, is set to match the after taxes profits over total output in the economy. Finally, the parameters governing the initial level of equity of entering firms are set to match moments of the firm entry distribution: average size of entrants with respect to incumbents, $avg(k_{ent})$, the standard deviation capital distribution, $SD(k_{ent})$, and the correlation between initial productivity and equity, $\rho(a_{ent}; e_{ent})$.

I also calibrate a version of the model where productivity dynamics evolve according to a standard AR(1) process. Formally,

$$a_t = \rho a_{t-1} + X_t + \sigma\varepsilon_t \quad \varepsilon_t \sim N(0,\ 1), \tag{30}$$

where a_t stands for $log(A_t)$ and X_t is a full collection of sector-year fixed effects. The autoregressive parameter, ρ is estimated to 0.83, and the shock variability, σ, to 0.34. The values of the remaining parameters and the value of their moment counterparts are shown in Appendix D.1.

6.2 Model Validation

The model performs well in the dimensions targeted in the calibration strategy. But, How does the model behave among other dimensions? And, more importantly, are the mechanisms of the model consistent with firm behavior?

Non-Targeted Moments I check the consistency of the model contrasting a set of non-targeted moments with the data. The results are summarized in Table 3. First, regarding the firm size distribution, the model captures well firm concentration. The top 1% of the firm accumulate around 1/3 of total resources both in the model and in the data.

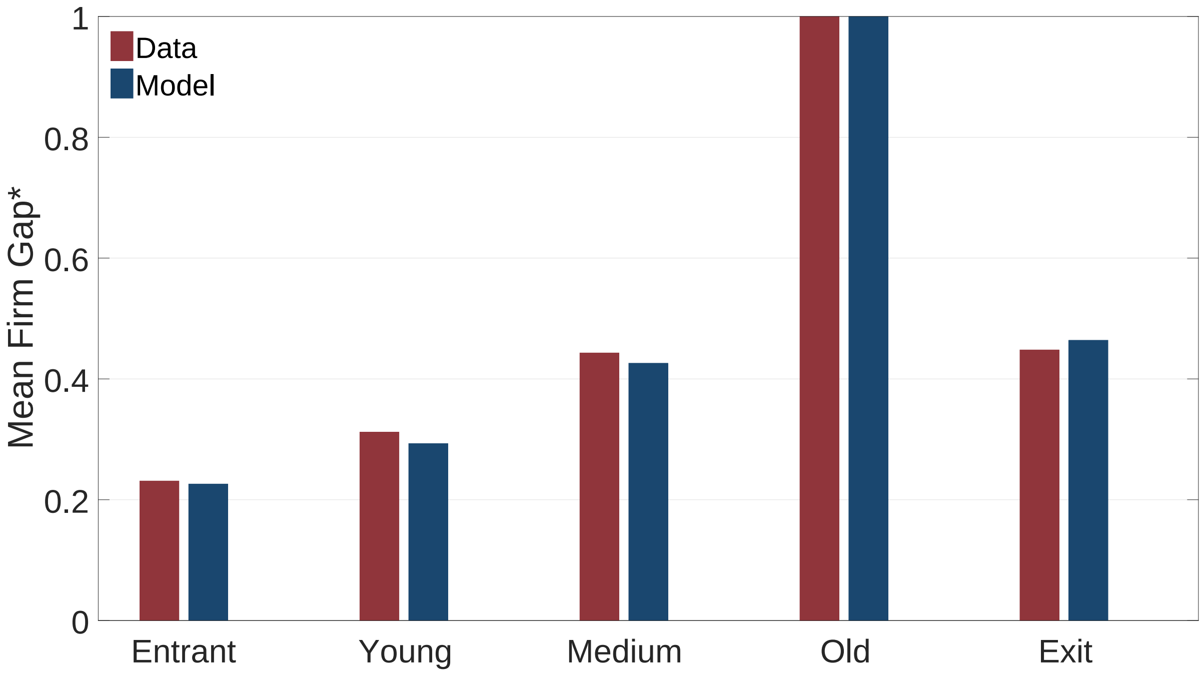
Table 3: Non-Targeted Moments Size Dependent Borrowing Constraint

Moment	Data	Baseline
$Concentration_{99}(K)$	0.34	0.33
$Debt/Y$	0.81	0.82
$Debt > 0$	0.71	0.57
$Div > 0$	0.01	0.00
Div/Y	0.14	0.00
P_{10}^{Lev}	0.03	0.08
P_{25}^{Lev}	0.09	0.15
P_{50}^{Lev}	0.22	0.29
P_{75}^{Lev}	0.42	0.51
P_{90}^{Lev}	0.61	0.67
$Med(K_{ent})$	0.08	0.08

On the financial side, the model matches the debt to output and the leverage distribution quite well. However, the fraction of firms with positive debt is smaller in the model than in the data. This is consistent with a more precautionary dividend paying behavior. The fraction of firms paying dividends and the dividend to output ratio is smaller in the model than in the data. The main reason is that the firms are too precautionary in the model. They save retaining all the profits up to the point they ensure to be unconstrained regardless of any productivity path, even if this is very unlikely. This reduces the fraction of firms using debt to finance their investment, as well as the fraction of firms paying dividend and the total amount of dividends paid. The model also matches other moment of the firm entry distribution, such as the median size of the entrant firm, $Med(K_{ent})$.

Firm Life Cycle In Figure 11, I show the firm life cycle in the data and the model. The model is able to match very well the firm life cycle. As in the data, firms enter very small in the economy, 25% the size of an old firm, more than 10 years old. They gradually grow over the firm life cycle. Although the process is very slow, a medium-age firm, 6 to 10 years old, is half the size of an old firm. Finally, firms eventually exit the market. The average size of an exiting firm is half the one of an old firm.

Figure 11: Firm Life Cycle

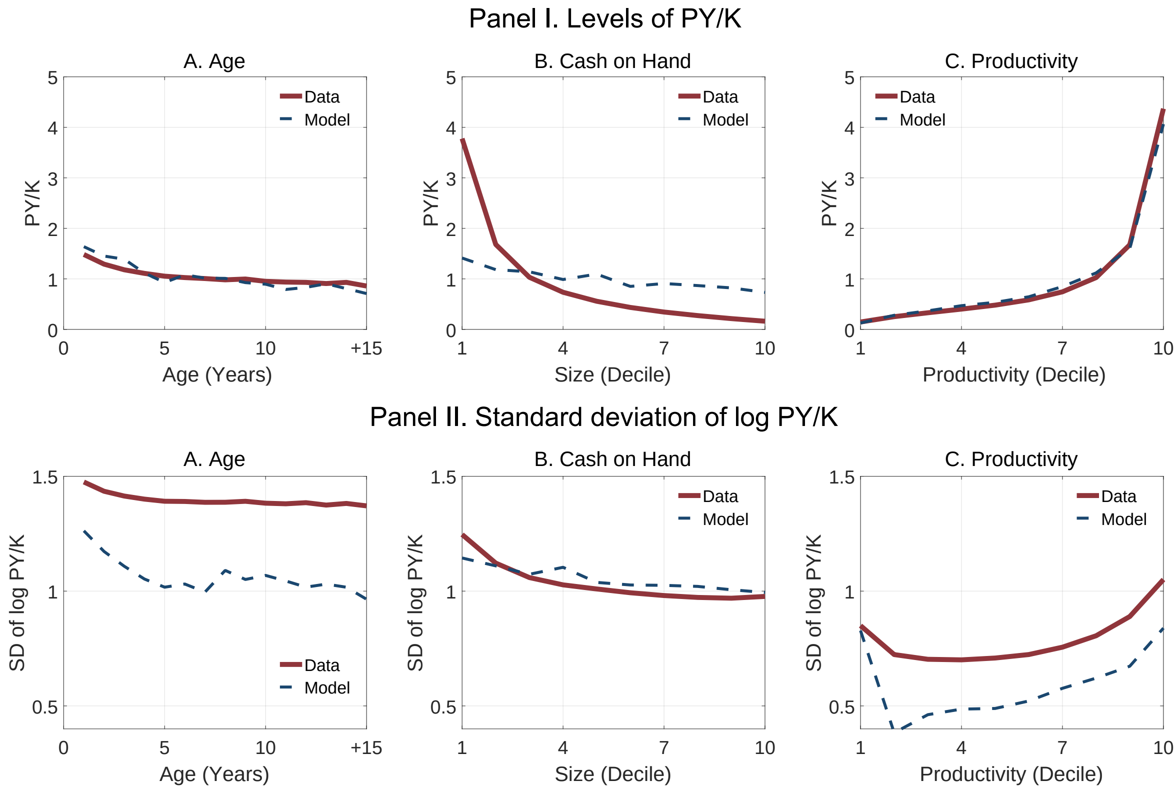


Notes: Young: 1-5 years old, Medium: 6-10 years old, Old: more than 10 years old. *Mean Firm Gap with respect to an old firm.

Misallocation Figure 12 show how level and dispersion of ARPK behaves in the model and the data. Panel I shows the level of ARPK. The model does an excellent job generating the patterns by age and productivity. Both in the data and in the model, young and high productivity firms have higher levels of ARPK. In the model ecomomy, these are exactly the firms that are more likely to be financially constrained. The average ARPK is also larger for smaller firms. While the model is able to generate the same pattern, this is more muted. Yet, a model without financial frictions is not able to generate a negative relation between firm size and the level of ARPK. The flatter profile of ARPK with firm size suggests the presence of other distortions, apart from financial frictions, affecting small firms in the Spanish economy.

Panel II of Figure 12 shows how dispersion of ARPK varies with size, age and produc-

Figure 12: Profiles of PY/K

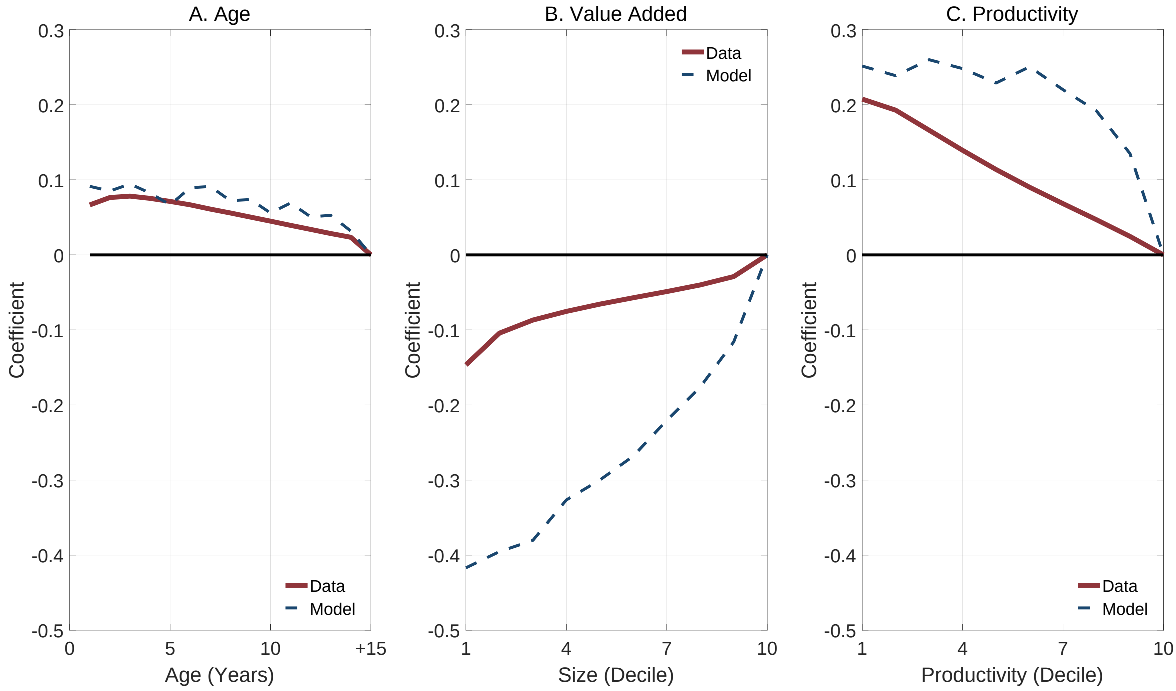


tivity. Both in the model and in the data dispersion of ARPK is declining in age and size, while it is U-shaped with respect to productivity. The level of the dispersion in ARPK in the model, on the other hand, is lower than it is in the data. Overall, variation of ARPK is 1.33 in the data and 1.07 in the model. The level of the dispersion in ARPK, however, can be made arbitrarily large if I allow for idiosynratic distortions in firms capital decisions, as in David and Venkateswaran (2019).

Firm Financial Behavior I also evaluate how the model captures the financial behavior of firms by running the same regression in Section 4.3 with the simulated data from the model. In Figure 13, I compare the data and model counterparts of Equation 11. The model is able to capture the relation of average leverage with respect to firm age, size and productivity. The main discrepancy is with respect to firm size, as the model overstates the estimated elasticity.

Panel I and II of Figure 14 shows the results for the extensive margin (Equation 12) and intensive margin (Equation 13), respectively. The model slightly overstates the elasticity with respect to firm age; while, it understates it with respect to firm size and productivity

Figure 13: Financial Behavior



in the extensive margin. Regarding the intensive margin, the opposite pattern arises. The model slightly understates the elasticity with respect to firm age; while, it overstates it with respect to firm size and productivity.

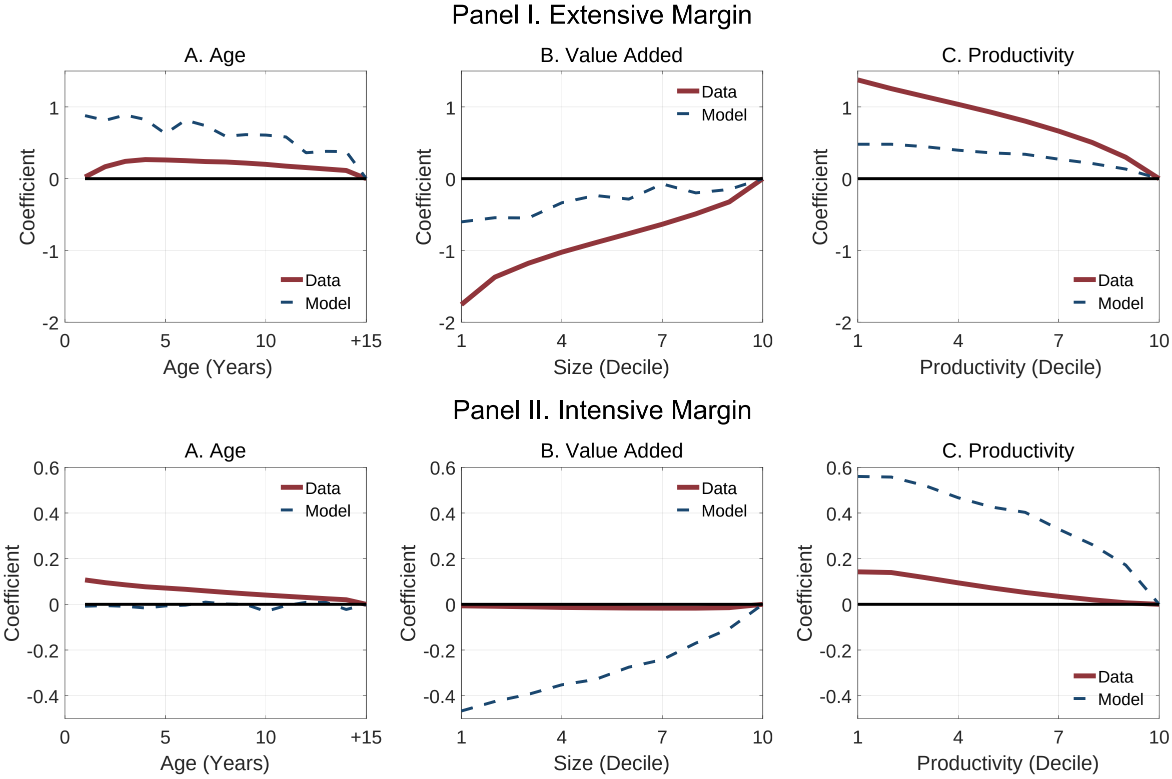
Overall, the model captures pretty well the firm financial behavior, even if these were not targeted explicitly in the calibration. It matches reasonably well the variation in firm leverage and its relation with firm characteristics. Furthermore, the model is able to distinguish the variation in firm leverage between the extensive and intensive margin, as it is in the data.

7 The Effects of Financial Frictions

The model has two main mechanisms that affect the allocation of capital: financial frictions and uncertainty in the capital decision in the form of time-to-build. In order to disentangle these two mechanisms, I solve the problem of a benevolent social planner.[19] The planner can allocate available capital in the economy optimally without any financial

[19] Solving for the social planner problem to quantify the effects of financial frictions has been used in the misallocation literature, e.g. Buera et al. (2011).

Figure 14: Financial Behavior - Extensive and Intensive Margin



frictions. The planner faces, however, the same informational frictions as in the benchmark economy, i.e. she has to decide on investment before she observes the productivity shocks due to time-to-build nature of capital. The social planner takes the total amount of capital and labor from the benchmark economy as given and allocate it to maximize aggregate output.[20] Furthermore, the social planner takes the total number of firms and their productivity level as given. The problem is

$$\max_{\{k^{SP}(A_i)\}_{i=1}^{N}} \sum_{i=1}^{N} E\big(\hat{F}(k^{SP}(A_i),\ A')|A_i\big), \tag{31}$$

subject to

$$K = \int_{kxbxA} k\ \mu(k,\ b,\ A)\ d\,[kxbxA] = \sum_{i=1}^{N} k^{SP}(A_i), \tag{32}$$

where

$$\left\{\hat{F}(k^{SP}(A_i),\ A_i') = \max_{\{l\}} \left\{F(k^{SP}(A_i),\ l,\ A_i')\right\}\right\}_{i=1}^{N}, \tag{33}$$

[20] Labor is not subject to any friction. Therefore, the labor policy function is the same in both problems, de-centralized and social planner.

and

$$L = \int_{kxbxA} l \ \mu(k, \ b, \ A) \ d\,[kxbxA] = \sum_{i=1}^{N} l^{SP}(k^{SP}(A_i), \ A_i'), \tag{34}$$

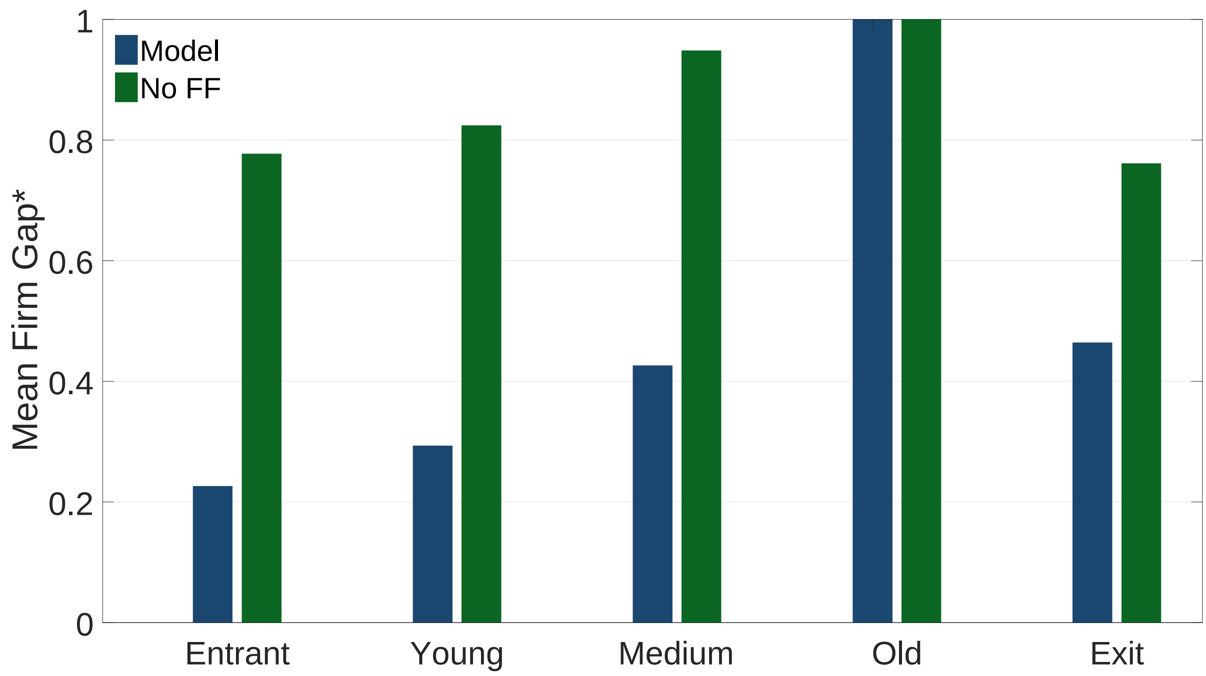
and N is the total number of firms.

The solution to the social planner problems yields an allocation of capital that satisfies

$$k^{SP}(A) \propto E\big(A'^{\frac{1}{1-\eta(1-\alpha)}}|A\big)^{\frac{1-\eta(1-\alpha)}{1-\eta}}. \tag{35}$$

Firm Life Cycle I evaluate how financial frictions affect the firm life cycle by comparing the results from the benchmark model and the ones from the social planner problem. Importantly, the productivity dynamics are unaffected, as the social planner takes the number of firms as given. Figure 15 compares firm life cycle in the benchmark economy and the planner solution.

Figure 15: Effects of Financial Frictions: Firm Life Cycle



Notes: Young: 1-5 years old, Medium: 6-10 years old, Old: more than 10 years old. *Mean Firm Gap with respect to an old firm.

Financial frictions have a very significant effect on firm life-cycle. In a world without financial frictions entrants are much larger. They are only only 20-25% smaller than an average old firm. Entrant firms are much smaller in the data, implying a large effect of financial frictions on entering firms. The existing firms also look very different without financial frictions. They are as large as entrants; their size is about 80% of old firms. In the benchmark economy, on the other hand, they were much smaller. The gap between firm sizes in the benchmark economy and social planner problem gets smaller over the firm life-cycle. Although, the process is very slow, as the gap between the two economies closes faster when firms are mature. The results when the productivity dynamics follow an AR(1) process are in Appendix D.3.1. The main takeaway is that the effects of financial frictions are much smaller under the standard AR(1) dynamics, as the gap between the model and the social planner problem are closer in this case.

7.1 Aggregate Effects of Financial Frictions

In this section, I quantify the aggregate consequences of financial frictions. In Table 4, I summarize the main results. I evaluate the aggregate effects of financial frictions looking at three statistics. First, the fractions of firms which capital decision is affected by financial frictions is 1/3 in the benchmark economy. Second, financial frictions prevent firms from investing the optimal level of capital, which translates into variation in the $ARPK$. The model generates a $SD(logARPK)$ of 1.07 versus the 1.33 present in the data. The remaining variation is due to other frictions that affect the allocation of capital not modelled in this paper, e.g. idiosyncratic distortions. But, not all the variation in $ARPK$ is due to financial frictions, as firms face uncertainty in the capital decision. Using the solution of the social planner problem, we can see that 20% of the variation in $ARPK$ can be attributed to financial frictions, (1.07-0.84)/1.07. Finally, I compute the productivity losses from the inefficient allocation of capital. Productivity losses are large, 32%, and half of them, 16%, are consequence of the misallocation generated by of financial frictions.

The aggregate effects of financial frictions are more muted is productivity dynamics follow a standard AR(1) process. The fraction of firms that are financially constraint is 1/3, the variation in $ARPK$ is smaller, and the aggregate productivity losses from financial frictions are reduced by half: 8%.[21]

[21] In Appendix D, I explore the robustness of the results under different specifications of the borrowing constraint proposed in the literature. First, I solve the model using the standard borrowing constraint, $b' \leq \theta k'$. Second, I also solve the model using a profit based borrowing constraint, $E\left[\widehat{\pi}(k',\ A')|A\right]$, as recently used in Drechsel (2019). In all the cases the aggregate productivity losses are twice as large

Table 4: Aggregate Consequences of Financial Frictions

	Baseline	AR(1)
No Constrained (% of firms)	65.6%	74.6%
Constrained (% of firms)	34.4%	25.4%
SD(log ARPK)	1.065	0.847
SD(log ARPK) No FF	0.843	0.684
Productivity Loss (%)	31.5%	18.6%
Productivity Loss FF (%)	16.4%	8.1%

An interesting question is why the benchmark economy produces larger effects of financial frictions in the aggregate economy compared to the standard AR(1) case. In order to answer this question, I do a decomposition exercise where I shut down one by one the differential characteristics of the estimated productivity process with respect to the AR(1). The results of the exercise are shown in Table 5.

Table 5: Decomposition of the Aggregate Effects

	(1)	(2)	(3)	(4)	(5)
No Constrained (% of firms)	65.6%	64.2%	57.8%	73.3%	74.6%
Constrained (% of firms)	34.4%	35.9%	42.2%	26.7%	25.4%
SD(log ARPK)	1.065	1.150	1.125	0.999	0.847
SD(log ARPK) No FF	0.843	1.023	0.933	0.823	0.684
Productivity Loss (%)	31.5%	32.6%	30.0%	25.1%	18.6%
Productivity Loss FF (%)	16.4%	11.5%	9.6%	11.2%	8.1%

Notes: (1) Benchmark, (2) Benchmark + Gaussian Shocks, (3) Benchmark + Gaussian Shocks + Constant Shock Variability, (4) Benchmark + Gaussian Shocks + Constant Productivity Persistence and (5) AR(1).

Column 1 has the results of the benchmark economy. Column 2 solves the model with non-Gaussian productivity shocks. The difference in aggregate productivity losses are 4.9 p.p.. This is slightly more than 50% the gap between the benchmark economy and the AR(1) case, column 5. Therefore, half of the larger aggregate productivity losses are due to the non-Gaussian nature of the productivity shocks. The other half is due to the non-linear productivity persistence and shock variability. To set these two elements apart, column 3 adds constant shock variability to column 2, while column 4 adds constant

when productivity dynamics follow the estimated non-linear and non-Gaussian dynamics instead of a standard AR(1) process.

productivity persistence. I find that differential shock variability accounts for around 30% of the difference between the benchmark economy and the AR(1) case, while differential persistence is responsible of slightly less than 20%.

8 Conclusion

In this paper, using a comprehensive dataset of Spanish firms, I first show that the productivity process that firms face is highly non-linear with non-Gaussian shocks. Low productivity firms have low productivity persistence, high shock variability and positive skewness. This implies that they have a larger probability of having a good productivity realization than in a standard AR(1) process, the common modelling strategy in the firm dynamics literature. These firms may not have enough internal funds to finance their investment needs. Furthermore, these periods of high productivity are not long-lasting, since high productivity firms have lower productivity persistence than the implied under an AR(1) process. Therefore, some firms will not be able to accumulate enough internal funds, through profit accumulation, to surpass financial frictions. These two elements are fundamental to quantify the effects of financial frictions on the economy.

I then build a firm dynamics model with financial frictions where firm productivity evolves as estimated in the data. I discipline the model with a host of evidence on firm dynamics, misallocation, and firms' financial behavior. Under the lens of the model, the effects of financial frictions are large. It affects the firm life cycle, as firms enter the economy three times smaller than they do in a world without financial frictions. Furthermore, the process of profit accumulation to overcome financial frictions is slow and incomplete for much of the firms. I find that exiting firms are on average 60% larger in an economy without financial frictions.

The effects of financial frictions over the firm life cycle translate into substantial aggregate productivity losses through resource misallocation. In the benchmark economy about 1/3 of all firms are financially constrained and financial frictions lower the aggregate productivity by 16%. These figures are much smaller if productivity dynamics evolve according to the standard AR(1) used in the literature, 1/4 and 8%, respectively.

In the framework presented in this paper, productivity dynamics are exogenous and financial frictions do not affect their evolution. Financial frictions, however, may distort the incentives of the firms to undertake investment opportunities to increase their productivity. Therefore, the effects of financial frictions may be even larger if this channel is important. In Petit and Ruiz-Garcia (2019), we extend the standard firm dynamics model to incorporate endogenous productivity dynamics.

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Appendices

A Data

In this section, I provide further details about the dataset. I first show the sample selection and cleaning procedure. Then, I compare the resulting dataset with the census of Spanish firms to check the sample representativeness. Finally, I show the parameters of the production function.

A.1 Sample Selection

In Table A1, I show the sample selection step by step. First, I select non publicly-listed firms (column 1). Publicly-listed firms represent 0.1% of total firms, and around 5% of total activity in terms of value added and employment. Second, I select non public firms (column 2). Public firms represent 0.5% of total firms, and around 15% of total activity in terms of value added and 3% in terms of employment. Third, I select limited liability firms (column 3). Non limited liability firms represent 0.8% of total firms, and around 3% of total activity in terms of value added and employment. The final sample represents 98.6% of total firms. It accounts for 74% of total value added, and 91% of total employment.

A.2 Cleaning

I summarize the steps I take in order to arrive to the final dataset Iuse in the paper.

1.- I drop all the observation with a real wage (nominal wage bill over CPI over the number of employees) lower than the 1st percentile and larger than the 99nd percentile (no applied to missing wage, public and public sector firms). I drop 111,992 observations in this step.

2.- I drop observations with more than 100,00 workers. The largest Spanish firm has a bit more than 80,000 employees (no applied to public and public sector firms). I drop 46,320 observations in this step.

3.- I apply filters on sector (firm is classified in an economic sector of the CNAE classification), age (reliable age), province (the headquarters are classified in one of Spanish provinces), value added (has positive value added), capital (positive value for capital), wage bill (positive value for the wage bill). I drop 7,289,899 observations in this step.

4.- I restrict the analysis for the years from 1999 to 2014, both included. I drop 2,426,715 observations in this step.

5.- The number of observations left after the previous cleaning are 7,767,289.

6.- I drop economic sectors with capital share lower than 0 (5 economic sectors out of 59). I drop 378,191 observations in this step.

7.- I keep firms with a value added in real terms of more than 1,000 euros in 2010, capital of more than 500 euros in 2010, wage bill of more than 3,000 euros in 2010. I drop 214,815 observations in this step.

8.- I drop weird observations.

8.1.- Firms in the top 90th percentile of the productivity, output, capital, wage bill, labor, revenue productivity, average revenue product of capital, average revenue product of wages, average revenue product of employment distribution that are in the bottom 1st percentile of any of the other distribution.

8.2.- Firms in the bottom 10th percentile of the productivity, output, capital, wage bill, labor, revenue productivity, average revenue product of capital, average revenue product of wages, average revenue product of employment distribution that are in the top 99nd percentile of any of the other distribution.

8.3.- (no applied to public and public sector firms). I drop 287.928 observations in this step.

9.- I drop outliers. Firms in the bottom 1st percentile and the top 99th percentile of the productivity, revenue productivity, average revenue product of capital, average revenue product of wages, and average revenue product of employment distribution (no applied to public and public sector firms). I drop 504,038 observations in this step.

10.- From the combination of the two previous steps, I drop 602,597 observations.

11.- I drop sectors with less than 5,000 firms (6 out of 54 economic sectors). I drop 17,535 observations in this step. As a result all sectors have at least 100 firms in a given year.

12.- The number of firm-year observations that cannot be followed in two consecutive years are 1,505,436 out of 6,500,945.

13.- The final sample corresponds to 6,500,945 firm-year observations from 1999 to

2014 and corresponding to 1,024,144 different firms.

13.1.- In the before crisis period (1999-2007), there are 3,371,530 firm-year observations corresponding to 745,296 different firms.

13.2.- In the after crisis period (2007-2014), there are 3,553,697 firm-year observations corresponding to 822,242 different firms.

A.3 Sample Representativeness

Comparing the final database with the Spanish directory (Table A2 and Table A3). The selected sample covers around 50% of all the firms, and the coverage is consistent over the studied period. In terms of employment the coverage is smaller around 30% of the total. This is due to the focus on private firms. Regarding the firm size distribution, the coverage is consistent across size groups. It is only slightly lower for very small and large firms. The coverage is similar in the manufacturing sector (Table A4 and Table A5).

A.4 Parameterization

I recover the parameters governing the elasticity of output with respect to capital at the sector level. The estimated parameters are shown in Figure A1, Panel A, the unweighed average and median are 0.32 and 0.29, respectively. The weighted average and median are 0.38 and 0.35, respectively. I compute sector specific weights ω_s to aggregate the economy. There are 50 sectors at the 2-digits level. In Figure A1, Panel B, I plot the sector-weight distribution. The average and median sector weight are 2.0% and 1.1%, respectively.

Table A1: Sample Selection

	(1)	(2)	(3)	Total	Sample Selection
Firms	0.1	0.5	0.8	1.4	98.6
Value Added	5.1	19.9	3.0	26.0	74.0
Capital	11.5	21.8	4.3	34.3	65.7
Wage Bill	4.6	14.5	2.4	20.3	79.7
Employment	4.2	3.3	2.3	8.7	91.3
Total Assets	10.1	18.0	3.6	28.6	71.4
Equity	9.3	20.0	4.0	30.1	69.9

Notes: (1) Public listed firms, (2) No public firms and (3) No limited liability firms.

Table A2: Sample Representativeness. Aggregate

Year	Employment	Wage Bill	Firms
1999	22.2	31.9	43.1
2000	23.0	27.3	44.0
2001	24.5	44.4	45.7
2002	26.4	29.8	46.8
2003	28.8	31.0	49.2
2004	31.0	30.7	50.3
2005	32.8	32.1	51.5
2006	33.7	33.1	50.6
2007	32.3	31.8	46.2
2008	35.4	32.8	47.4
2009	34.0	30.4	46.5
2010	34.2	31.0	48.6
2011	34.7	31.7	49.0
2012	34.9	32.1	48.3
2013	35.6	32.8	47.5
2014	36.9	34.0	51.1
Average	31.3	32.3	47.9

Table A3: Sample Representativeness. Firm Size Distribution

Year	1-5	5-20	20-50	50-200	+200
1999	25.8	46.1	46.2	34.8	32.0
2000	28.2	49.0	47.9	34.5	30.5
2001	31.4	50.4	55.0	35.8	30.6
2002	33.2	52.0	57.3	40.1	31.8
2003	36.1	57.0	64.5	44.6	35.7
2004	38.2	60.2	68.2	48.7	37.5
2005	39.8	64.0	70.4	50.5	40.4
2006	39.6	62.3	70.2	53.7	43.0
2007	36.3	57.8	64.5	47.2	40.4
2008	39.7	63.0	68.1	48.7	41.8
2009	40.0	59.9	64.5	46.9	51.2
2010	41.6	62.8	71.2	52.1	56.5
2011	42.3	62.6	72.1	54.0	58.6
2012	42.1	61.2	70.7	53.8	58.5
2013	40.0	63.0	75.2	57.8	56.0
2014	41.5	71.2	82.8	68.1	60.3
Average	37.2	58.9	65.5	48.2	44.1

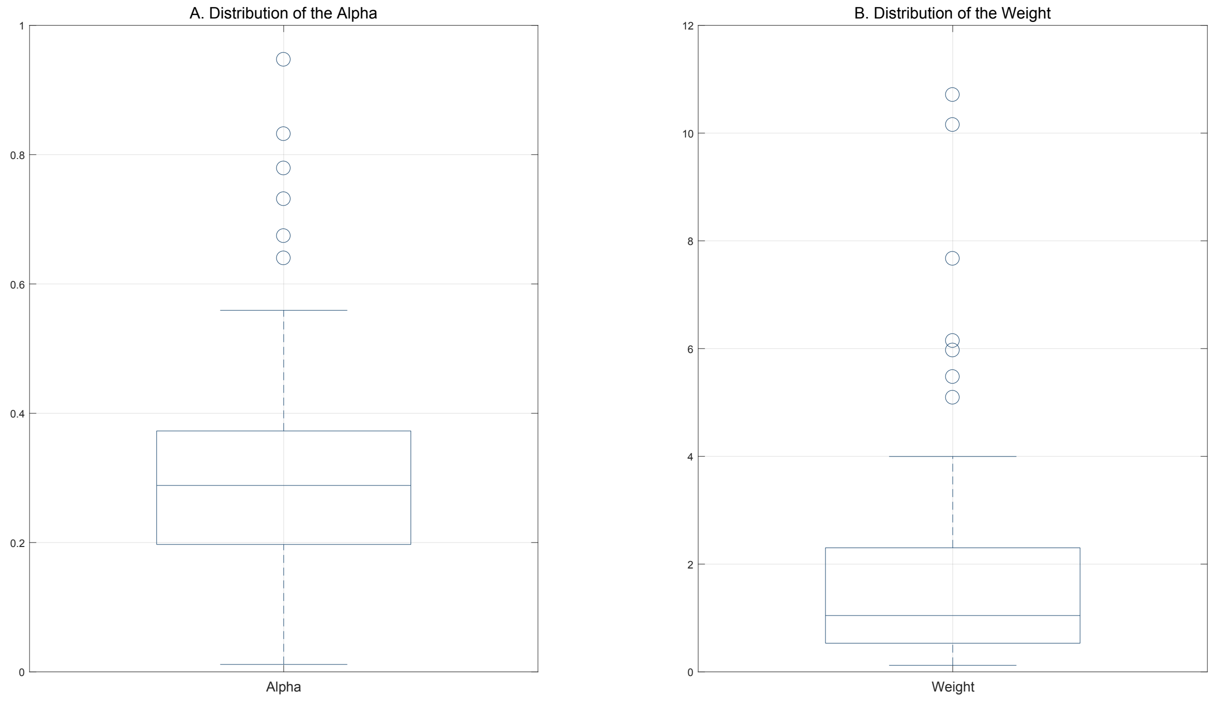
Table A4: Sample Representativeness. Aggregate (Manufacturing)

Year	Employment	Wage Bill	Firms
1999	29.6	51.1	43.8
2000	29.8	39.7	44.8
2001	31.9	82.8	46.9
2002	34.7	38.5	48.1
2003	37.3	42.7	51.6
2004	40.1	42.3	53.2
2005	42.0	43.8	56.3
2006	43.2	44.6	56.4
2007	42.9	44.2	53.3
2008	46.3	44.8	46.4
2009	46.3	43.4	45.2
2010	47.6	44.7	47.0
2011	47.9	45.9	47.1
2012	49.6	47.2	47.6
2013	50.9	48.6	48.9
2014	54.8	52.0	54.9
Average	42.2	47.3	49.5

Table A5: Sample Representativeness. Firm Size Distribution (Manufacturing)

Year	1-5	5-20	20-50	50-200	+200
1999	25.2	44.1	46.2	35.1	35.7
2000	27.5	47.0	48.0	34.7	32.9
2001	31.3	49.0	52.7	36.6	33.2
2002	33.1	50.9	54.8	39.9	33.3
2003	37.1	56.7	60.6	43.4	36.0
2004	39.6	60.9	61.1	47.4	39.4
2005	42.4	66.5	65.6	49.8	41.4
2006	43.7	65.2	64.4	50.6	43.7
2007	41.7	62.3	60.8	45.8	43.0
2008	36.2	62.0	73.2	52.7	44.9
2009	37.2	58.5	67.6	49.3	52.0
2010	38.2	62.9	74.3	54.0	58.7
2011	38.4	62.9	75.8	56.5	61.4
2012	39.8	62.8	72.8	56.8	63.7
2013	39.7	67.0	78.1	62.2	62.3
2014	43.7	74.6	87.2	76.4	69.3
Average	37.2	59.6	65.2	49.5	46.9

Figure A1: Alpha and Weight distribution



B Empirics

In this section, I provide additional details and robustness exercises on the empirical analysis of the main paper. There are three sections covering the three empirical sections of the paper.

B.1 Productivity Dynamics

In this section, I provide further details and robustness exercises on the estimated productivity process.

B.1.1 Persistence

Persistence depends on initial productivity and the productivity shock, as shown in equation xx. In the main paper, I integrate over the productivity shock, as shown in equation xx. The persistence of the shock process conditional on initial productivity and the productivity shock is shown in figure B1, B2 and B3.

B.1.2 Estimation

A concern is that the procedure describe in section 4.1 to characterize the productivity process is not able to capture its characteristics. In order to show the reliability of the estimation procedure, I do a Monte-Carlo simulation of 1 million firms from a AR(1) productivity process with parameters, $\rho = 0.8$ and $\sigma = 0.3$. In this case, we know that persistence should be flat on initial productivity and with a value of 0.8. Shock variability should be flat conditional on initial productivity and with a value close to 0.4. Shock skewness should be flat conditional on initial productivity and with a value of 0. Finally, shock kurtosis should be flat conditional on initial productivity and with a value close to 2.1. The results of this exercise are shown in figure B4 and B5. As we can see, the procedure used to characterize the productivity process captures well the dynamics of the AR(1) process. It suffers an upwards bias in the tails of the distribution in the estimation of persistence and shock variability. Importantly, the upper bias will go against; and therefore, dampens the results found in the empirical section of the paper.

B.1.3 Data as One Sector Economy

I estimate the productivity process sector by sector, instead of pooling the data of all the sectors. Then, I aggregate using the sector weights ω_s. The results are show in figure B6, where the sector by sector estimation is labelled version 2. The results look very similar to the baseline described in the main paper.

B.1.4 Decreasing Returns to Scale

The productivity estimation is sensitive to the decreasing returns to scale (DRS), governed by the parameter η. I repeat the estimation of the productivity process for different values of η. The results are shown in figure B7. The main takeaway is the robustness of the characteristics of the productivity process to the range of DRS used in the literature.

B.1.5 Studied Period Heterogeneity

The time period used, from 1999 to 2014, has the Great Recession of 2007 in the middle. In order to evaluate the consistency of the characteristics of the productivity process across time and specially in the period of recession and recovery, I split the sample in two sub-periods. The first one, before the Great Recession, from 1999 to 2007; and the second one during and after the Great Recession, from 2007 to 2014. The results are shown in figure B8. As we can see, the characteristics of the productivity process has been pretty stable during the whole period.

B.1.6 AR(1) Productivity Process

I estimate the standard AR(1) process for comparison. The specification is as follows

$$log(A_{it}) = \alpha \;+\; \rho_a log(A_{it}) \;+\; \sigma_\varepsilon \varepsilon_{it} \qquad \varepsilon_{it} \sim N(0,\ 1). \tag{36}$$

As in the non-linear productivity process, I choose η such that the model matches the variation of the firm size distribution. The results are summarized in Table B.1. The value of η that yields the best fit is 0.78. The estimation of the AR(1) productivity process results on a persistence parameter (ρ_a) of 0.813 and shock variability (σ_ε) of 0.336. These values fall in the standard range used in the firm dynamics literature. Another interesting point is the stability of the estimated ρ_a and σ_ε parameters to different values of the span of control parameter (η).

Figure B1: Persistence of the Productivity Process Conditional on Past Productivity and Productivity Shock

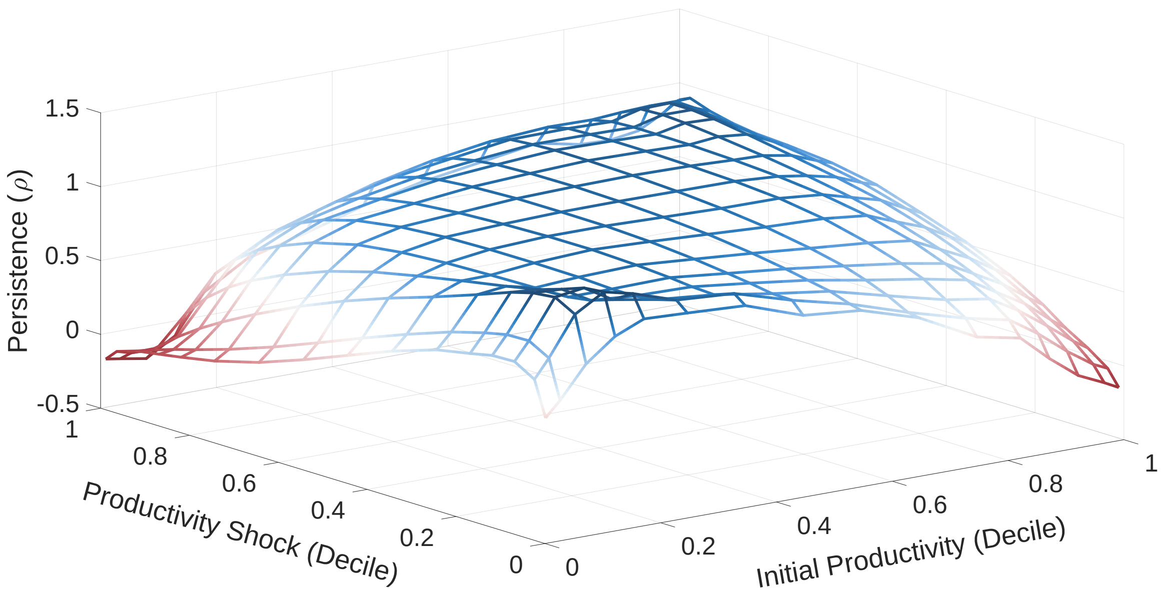


Table B1: Estimation of the coefficients of the AR(1) process

η	ρ_a	σ_ε
0.75	0.8130	0.3324
0.77	0.8127	0.3350
0.78	0.8128	0.3364
0.79	0.8133	0.3378
0.80	0.8137	0.3369
0.81	0.8126	0.3361
0.82	0.8133	0.3376
0.83	0.8135	0.3392
0.85	0.8146	0.3408

Figure B2: Persistence of the Productivity Process Conditional on Past Productivity and Productivity Shock

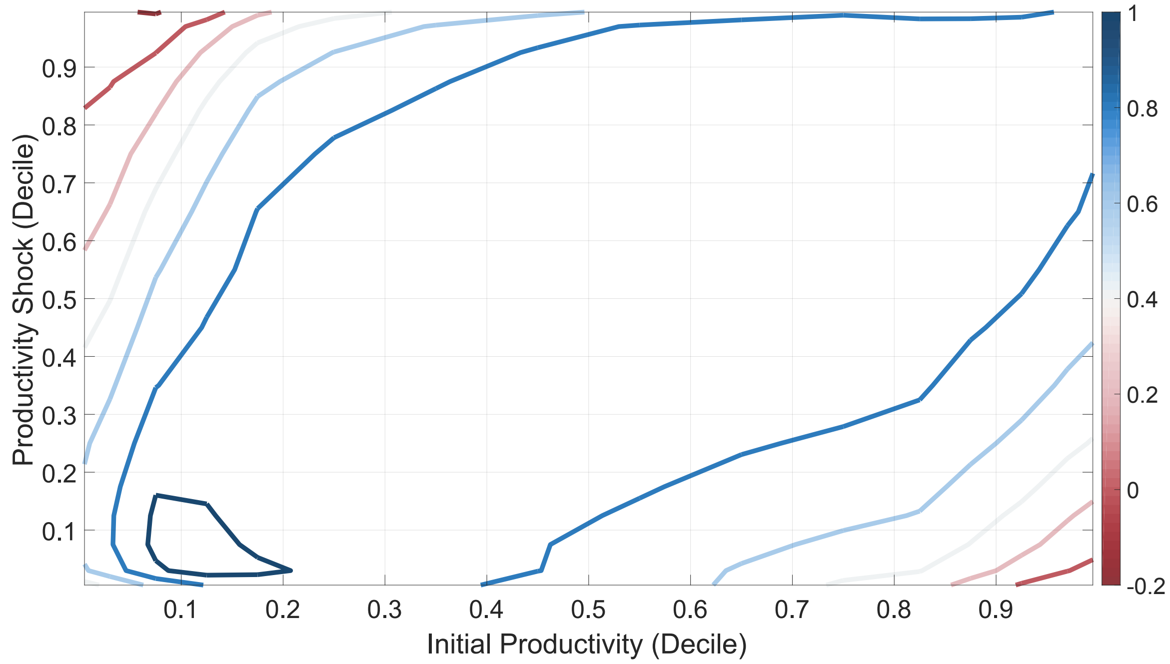


Figure B3: Conditional Persistence

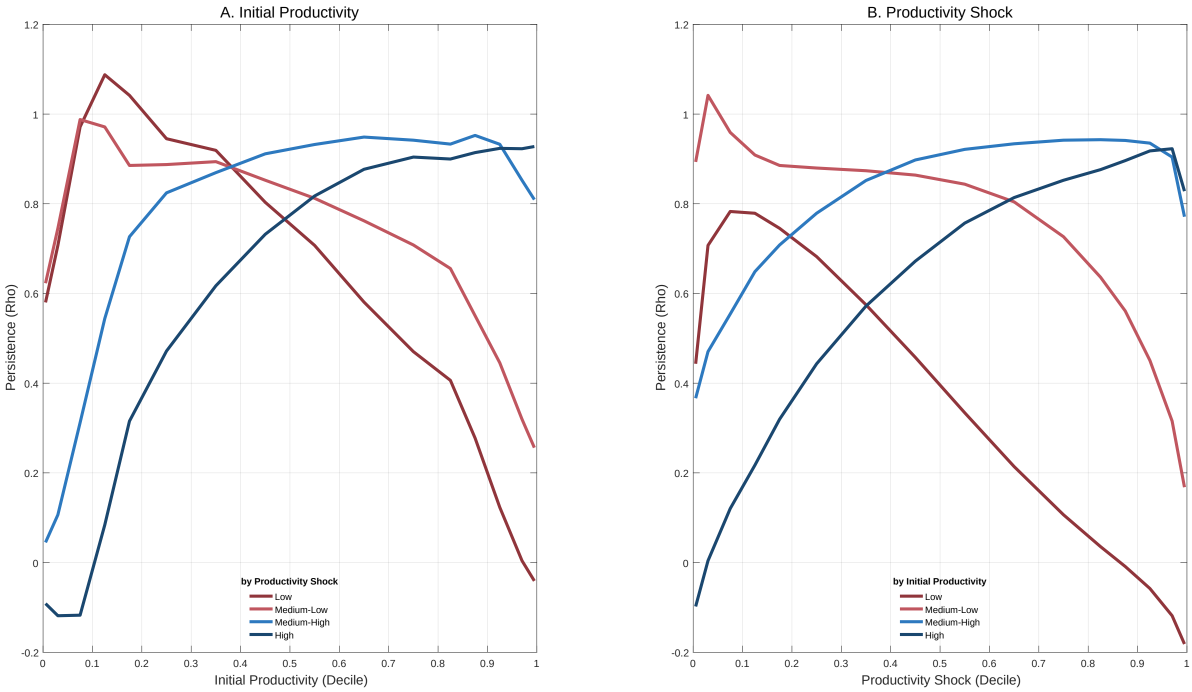


Figure B4: Characteristics of the Productivity Process - Simulation

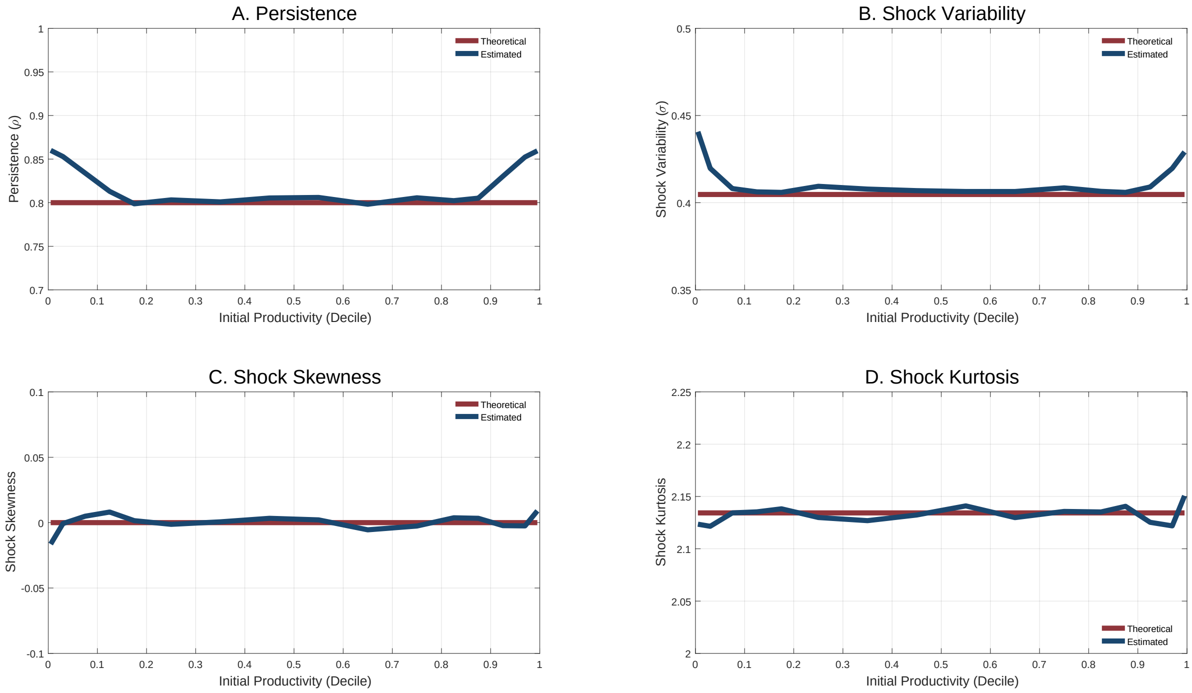


Figure B5: Persistence of the Productivity Process Conditional on Past Productivity and Productivity Shock

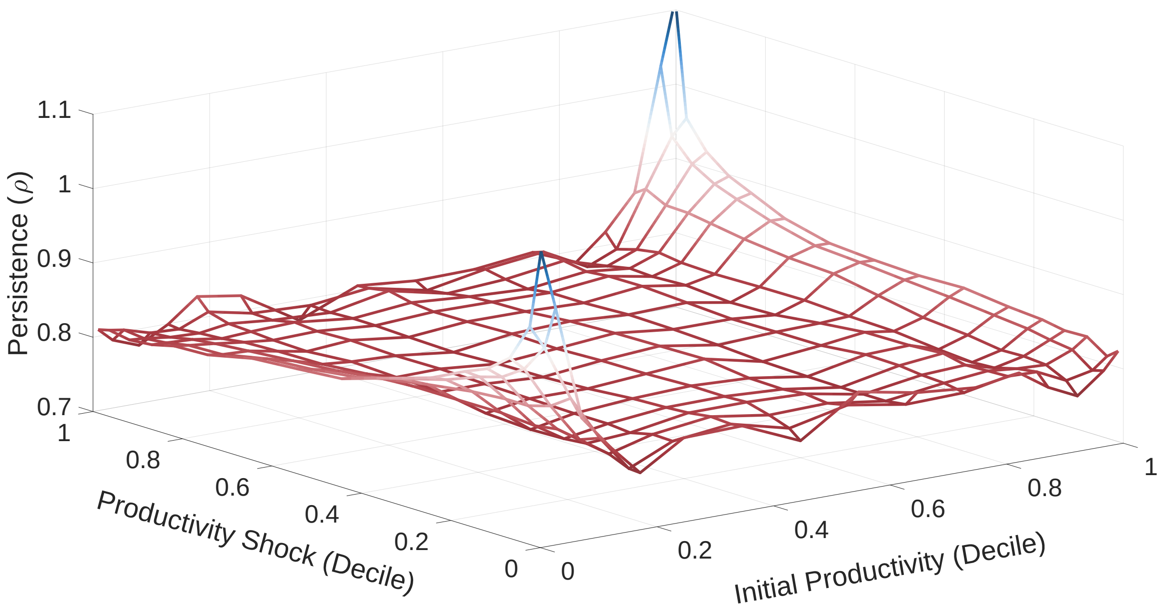


Figure B6: Characteristics of the Productivity Process - Different Specification

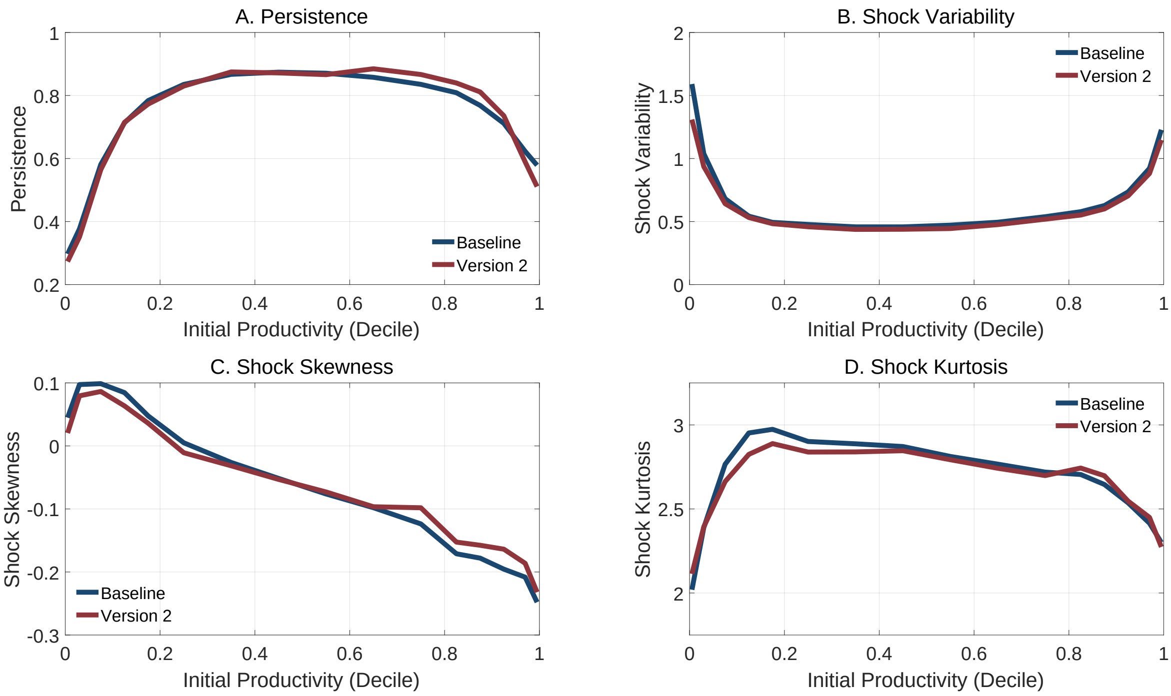


Figure B7: Characteristics of the Productivity Process - Different DRS

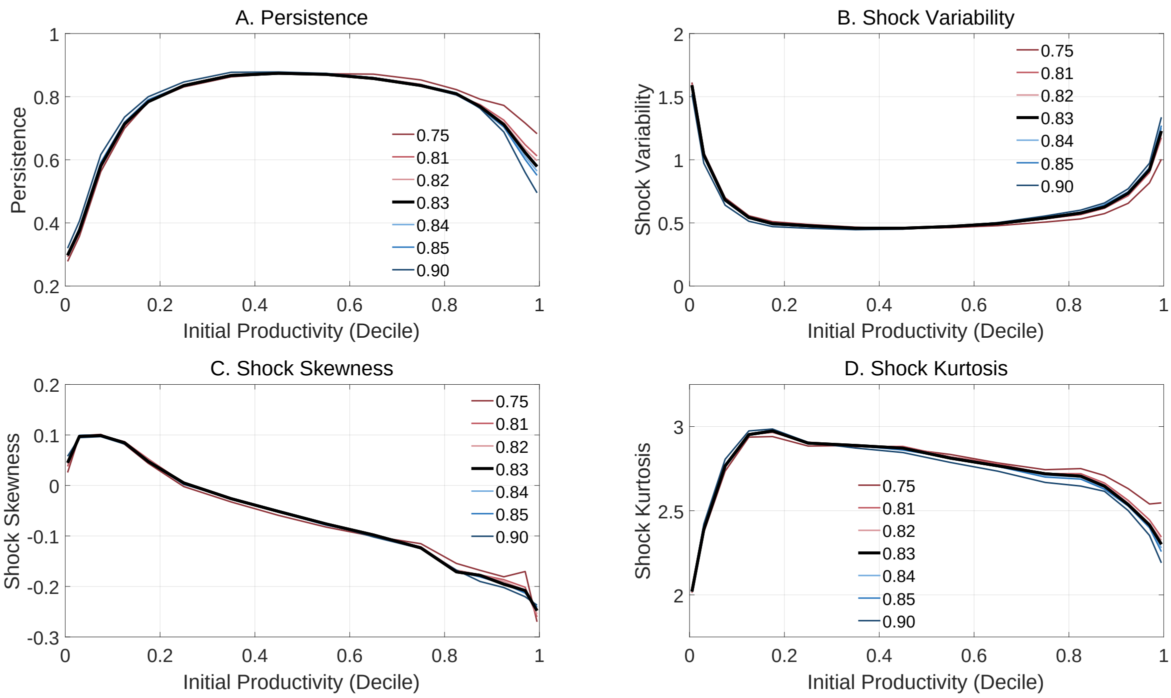
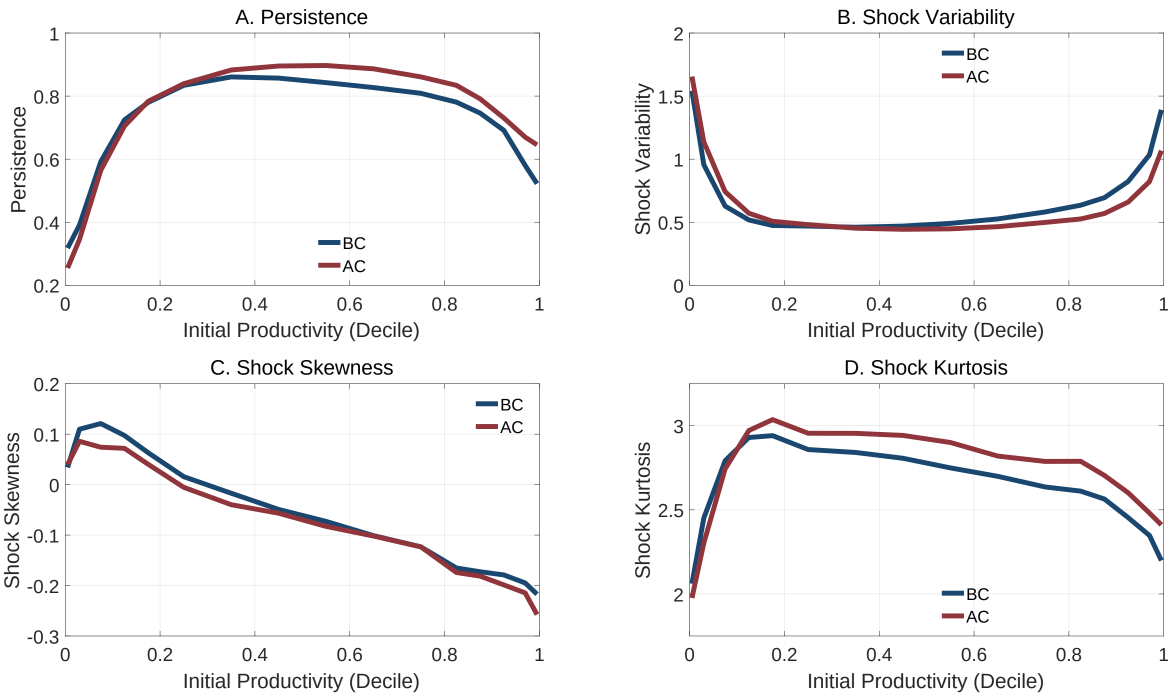


Figure B8: Characteristics of the Productivity Process - Different Periods



B.2 Misallocation

In this section, I provide the robustness checks on the results on misallocation conditional on firm characteristics.

B.2.1 Trend in Misallocation

During the studied period, there is an increasing trend in misallocation in Spain, see for instance Gopinath et al. (2017). In order to show that the results on misallocation shown in the main paper is not due to the increase in misallocation, I standardize the ARPK at the sector-year level. Therefore, the time series of variance of log ARPK does not have any trend on time. The results are shown in figure B9. The standardize profiles are labelled version 2. As we can see, the results are similar in the two versions. Of course, the level of the standard deviation of log ARPK is lower in version 2 due to the standardization procedure.

B.2.2 Studied Period Heterogeneity

The time period used, from 1999 to 2014, has the Great Recession of 2007 in the middle. In order to evaluate the consistency of the misallocation facts across time and specially in the period of recession and recovery, I split the sample in two sub periods. The first one, before the Great Recession, from 1999 to 2007; and the second one during and after the Great Recession, from 2007 to 2014. The results are shown in figure B10. As we can see, the misallocation profiles has been pretty stable during the whole period.

Figure B9: Profiles of PY/K - Specification

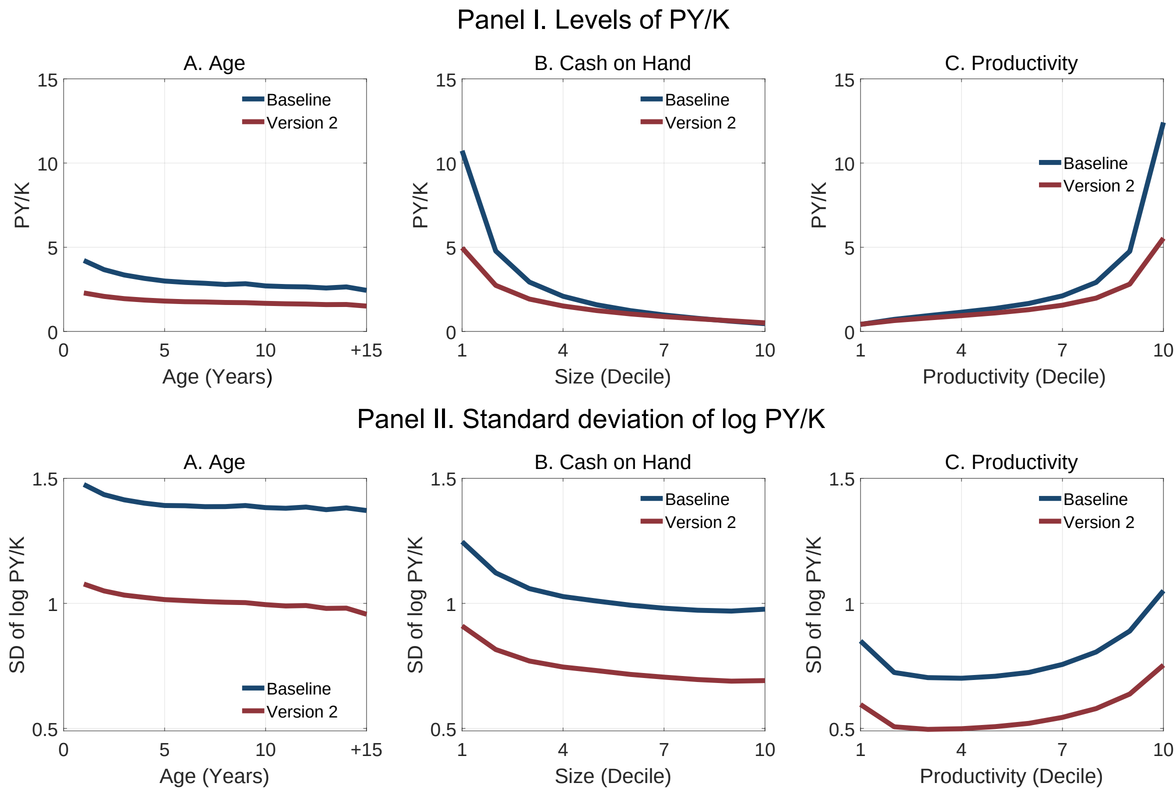
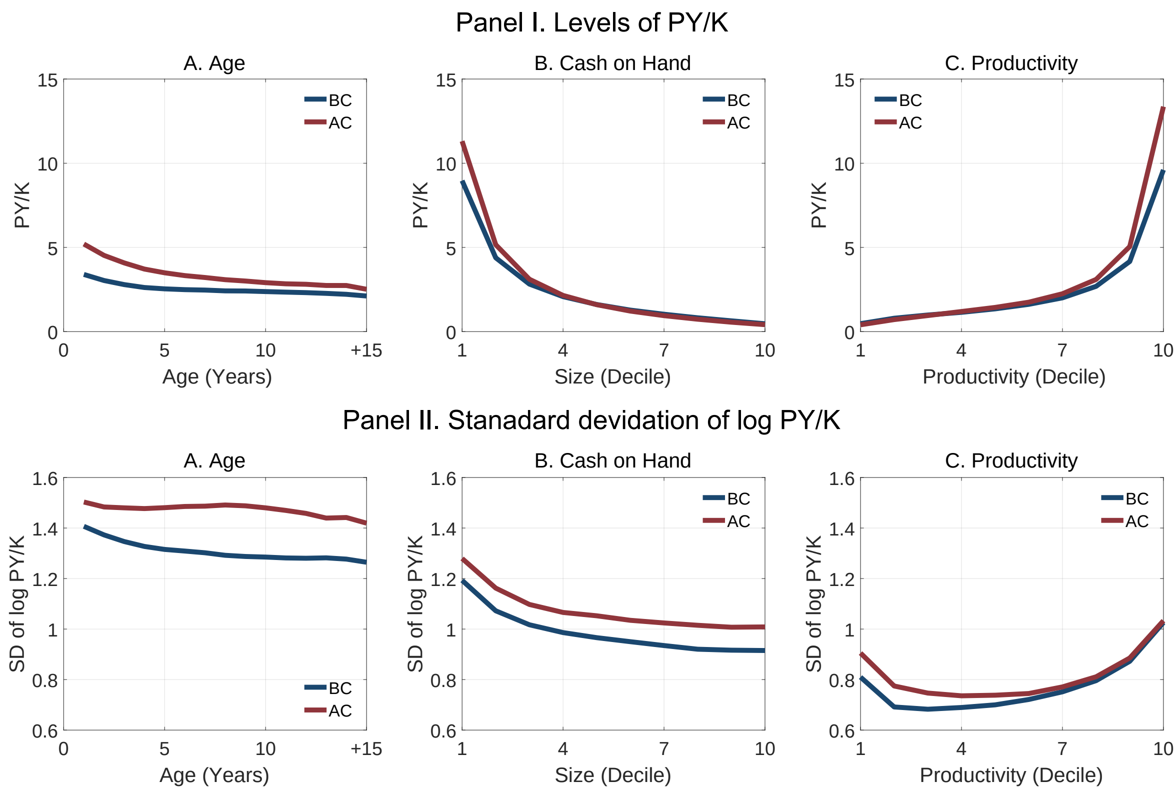


Figure B10: Profiles of PY/K - Periods



B.3 Financial Behavior

In this section, I provide the robustness checks on the financial behavior of Spanish firms.

B.3.1 Controlling by Firm Profitability

The corporate finance literature has looked at the financial behavior of firms with special attention to the relation between leverage and profitability, measured as profits over total assets. The literature usually finds a negative relation, which has been labelled as leverage-profitability puzzle, see Graham and Leary (2011). In this paper, the focus is on firm productivity, and I find a negative relation as well. I want to see if the negative relation of leverage and productivity survives once I control for firm profitability. Figure B11 abd B12 show the results once I control on firm profitability, version 2. As we can see, the profiles are very similar in the baseline and version 2.

B.3.2 labor Productivity Dinlersoz et al. (2018)

To be completed.

B.3.3 Studied Period Heterogeneity

The time period used, from 1999 to 2014, has the Great Recession of 2007 in the middle. In order to evaluate the consistency of financial behavior across time and specially in the period of recession and recovery, I split the sample in two sub periods. This is important as the main characteristic of the Great Recession is that it affected disproportionally the financial sector; and therefore, the level of credit in the economy. The first one, before the Great Recession, from 1999 to 2007; and the second one during and after the Great Recession, from 2007 to 2014. The results are shown in figure B13 and B14. As we can see, the financial behavior has been pretty stable during the whole period. If anything,, it seems that the Great recession affected the credit of small and medium size firms, which are less leverage with respect to large firms in the during and after Great Recession period.

Figure B11: Financial Behavior - Specification

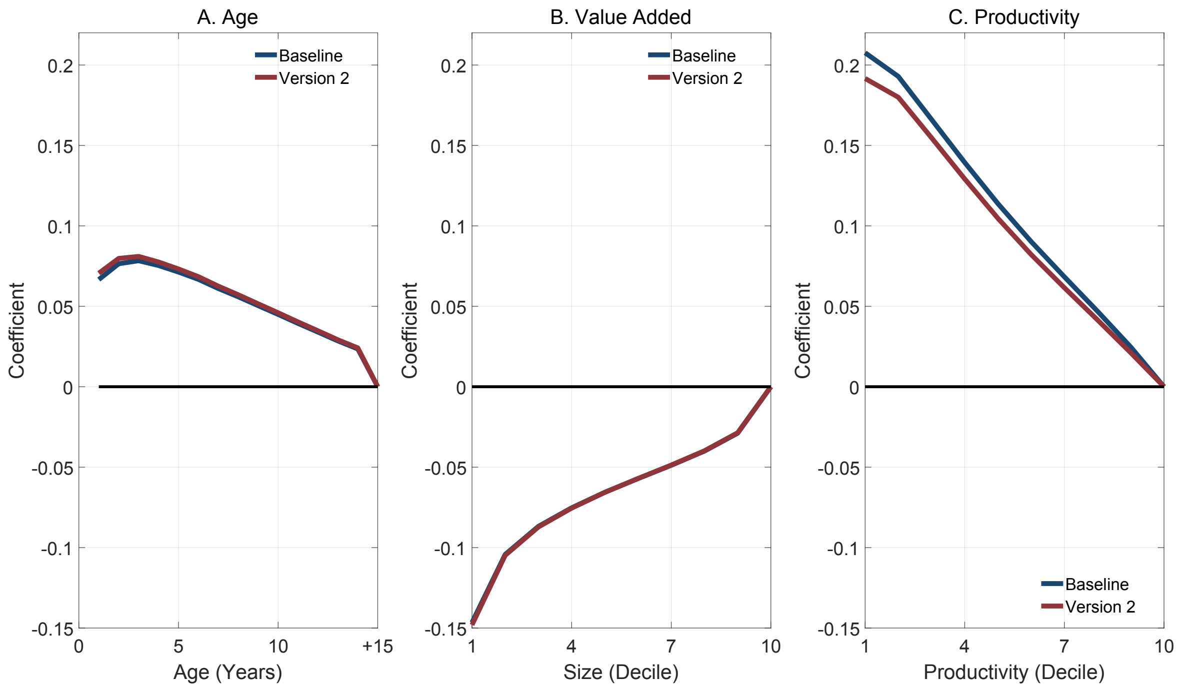


Figure B12: Financial Behavior - Extensive and Intensive Margin - Specification

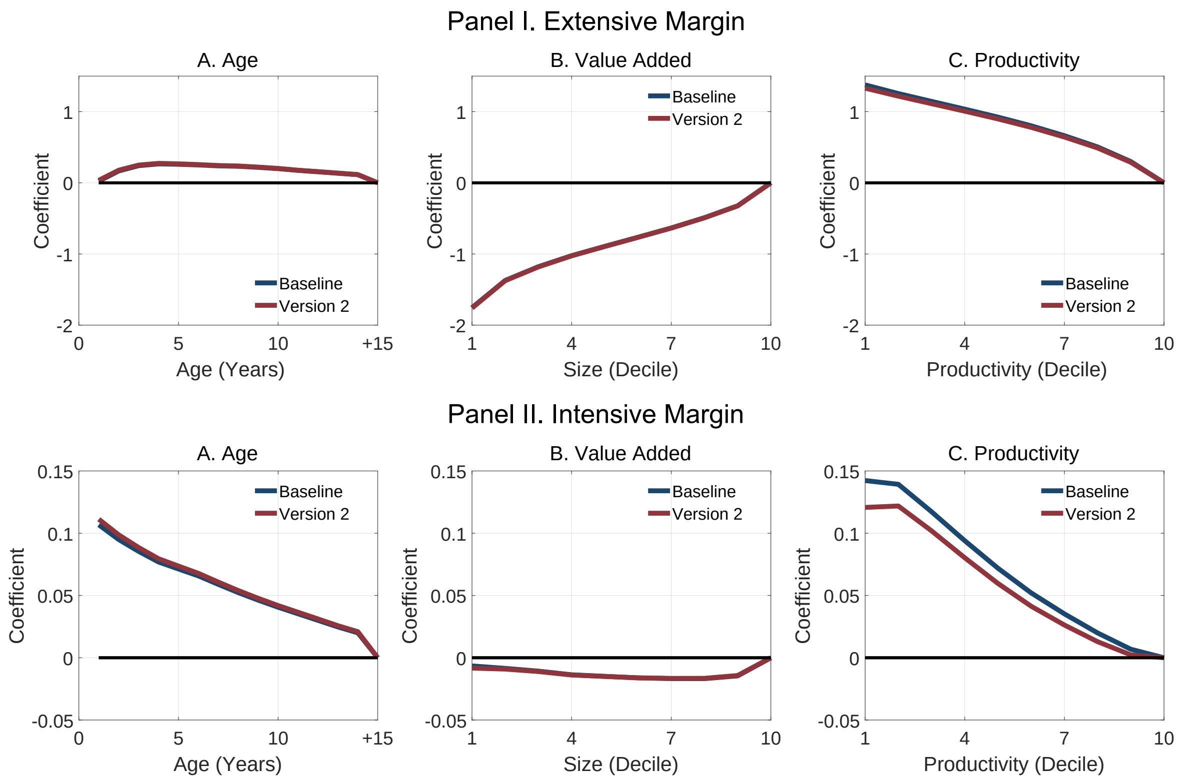


Figure B13: Financial Behavior - Periods

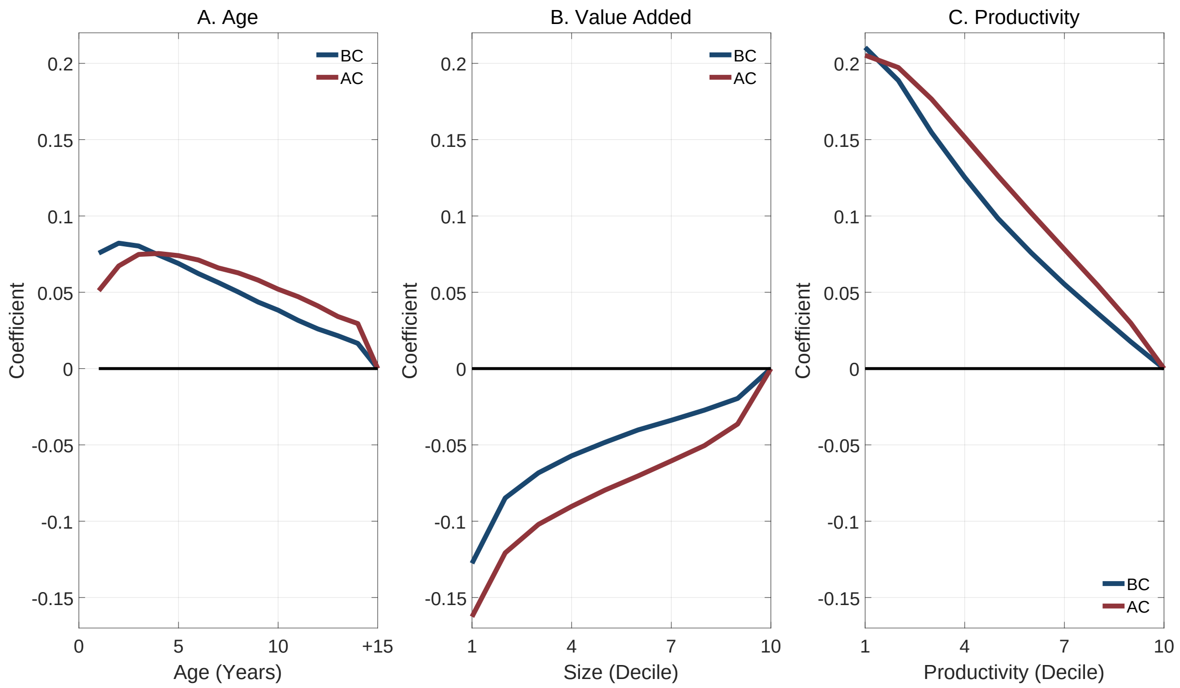
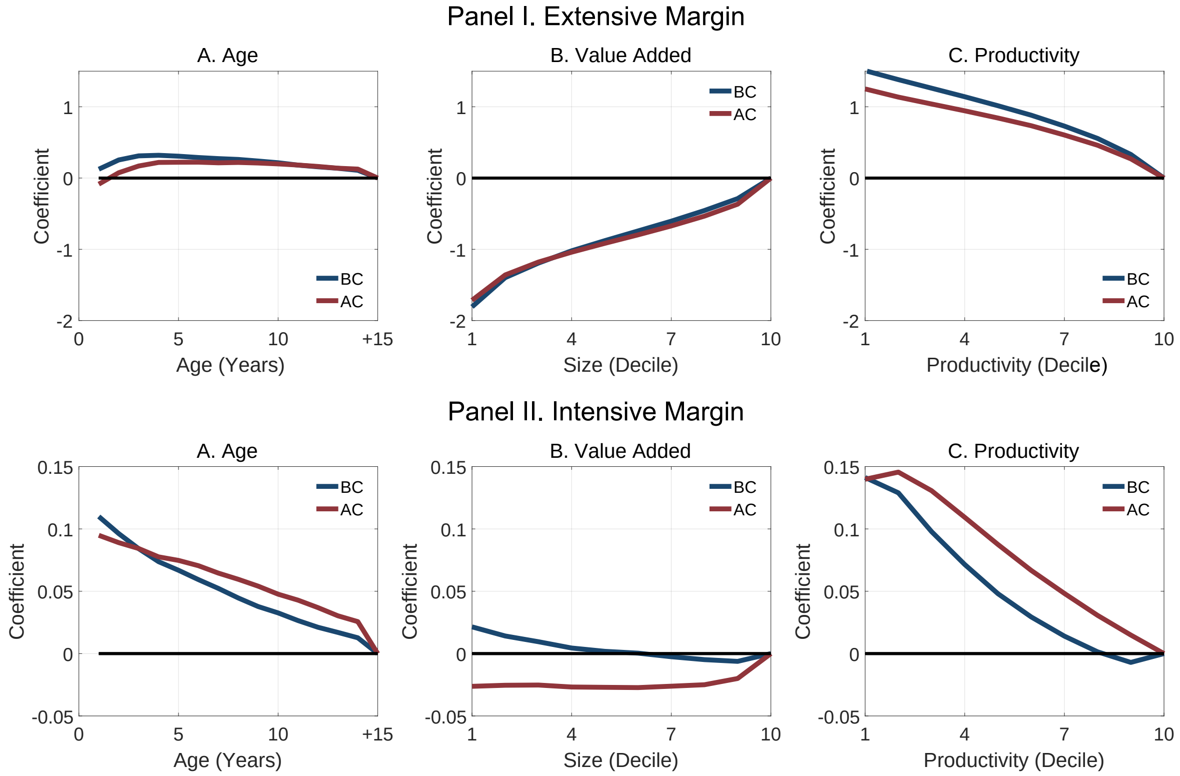


Figure B14: Financial Behavior - Extensive and Intensive Margin - Periods



C Model

In this section, I provide further details on the model, the solution method and results from the model.

C.1 Solution Algorithm

I follow the algorithm developed in Khan and Thomas (2013). First, I solve the model without financial frictions to obtain the optimal unconstrained policy function of capital $k'_u(A)$. Then, I solve for the optimal policy function for borrowing. That is the maximum borrowing (or minimum saving if it is positive) that allows the firm to implement the optimal policy function for borrowing and capital regardless of the productivity realization. This borrowing (or saving) level guarantees that the firm will not be constrained in the future. The current and future multipliers of the borrowing constraint are zero. Next, I characterize the type of firms depending on their state. First, I find the states that allow the firm to achieve the optimal policy functions (capital and borrowing). These are unconstrained firms. Second, I find the states that allow the firm to achieve the optimal capital function but not the borrowing function (the non-equity issuance constraint is binding). These are constrained type-I firms. Then, I find the capital policy function of capital and borrowing of firms that cannot implement the optimal capital (borrowing and non-equity issuance constraint are binding). These are constrained type-II firms. Finally, I find the optimal dividend policy function. Note that it will be only positive for the unconstrained firms.

I simulate a sample of 10,000 firms over 100 periods and take the last two periods to evaluate the performance. The simulation converges in the main variables after the 100 periods. This can be seen in figure C1.

C.2 Figures from the Model

In this section, I provide the figures that summarize the solution of the model. Figure C2 shows the optimal unconstrained policy function for capital. As we can see, the NL productivity process produces a non-linear policy function. The higher level of capital for each level of productivity in the NL productivity process arises from the higher value of η inn the calibration.

Figure C3 shows the 3 type of firms depending on their finance health, with respect to firm productivity and cash-on-hand.

Finally, figure C4 shows how financial frictions translates into lower firm value with respect to the unconstrained case.

Figure C1: Convergence of the Model

Figure C2: Optimal Unconstrained Policy Functions for Capital

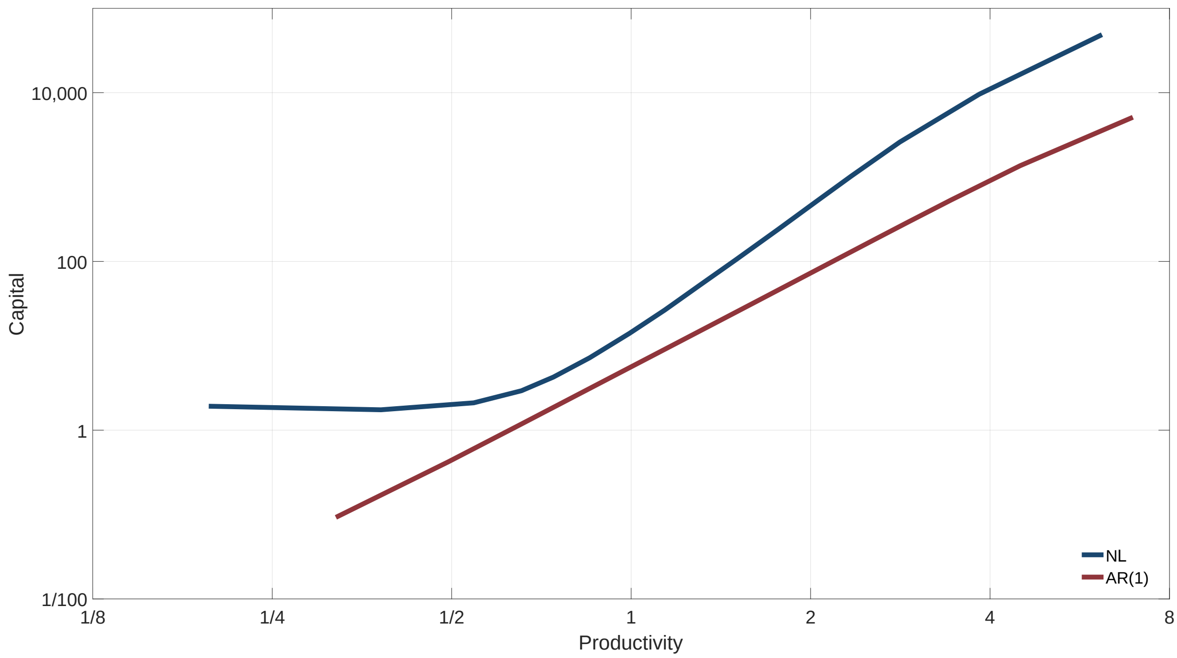


Figure C3: Firm Type by Cash-on-Hand and Productivity Levels

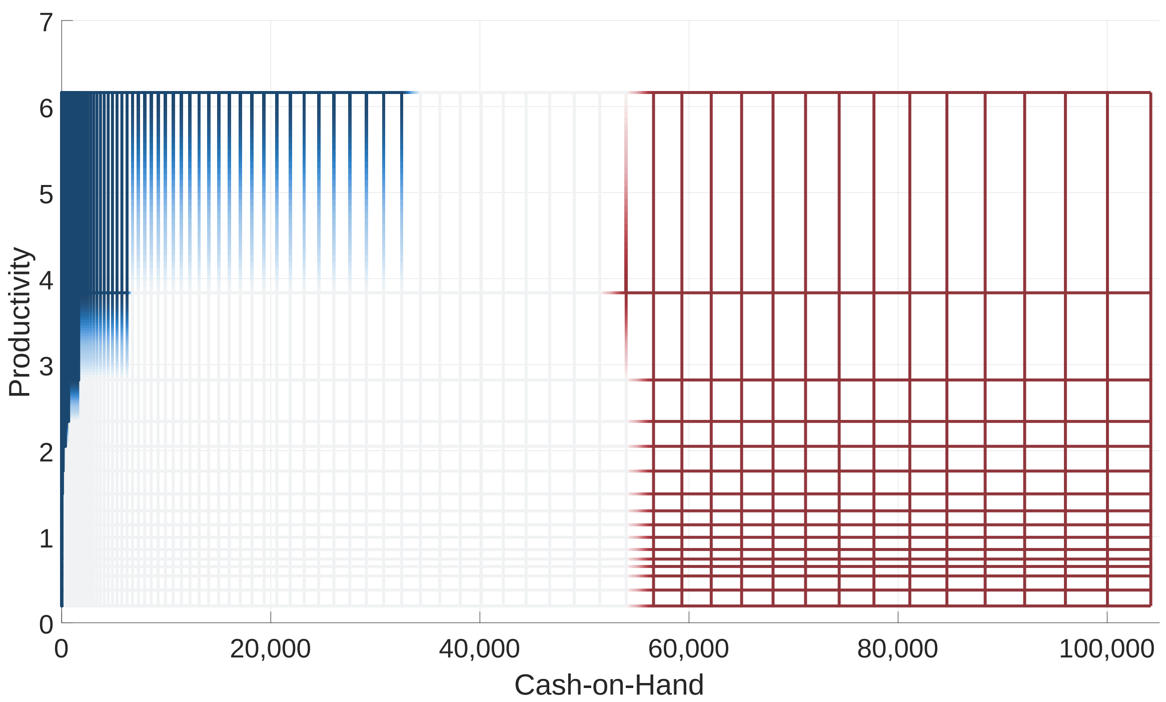
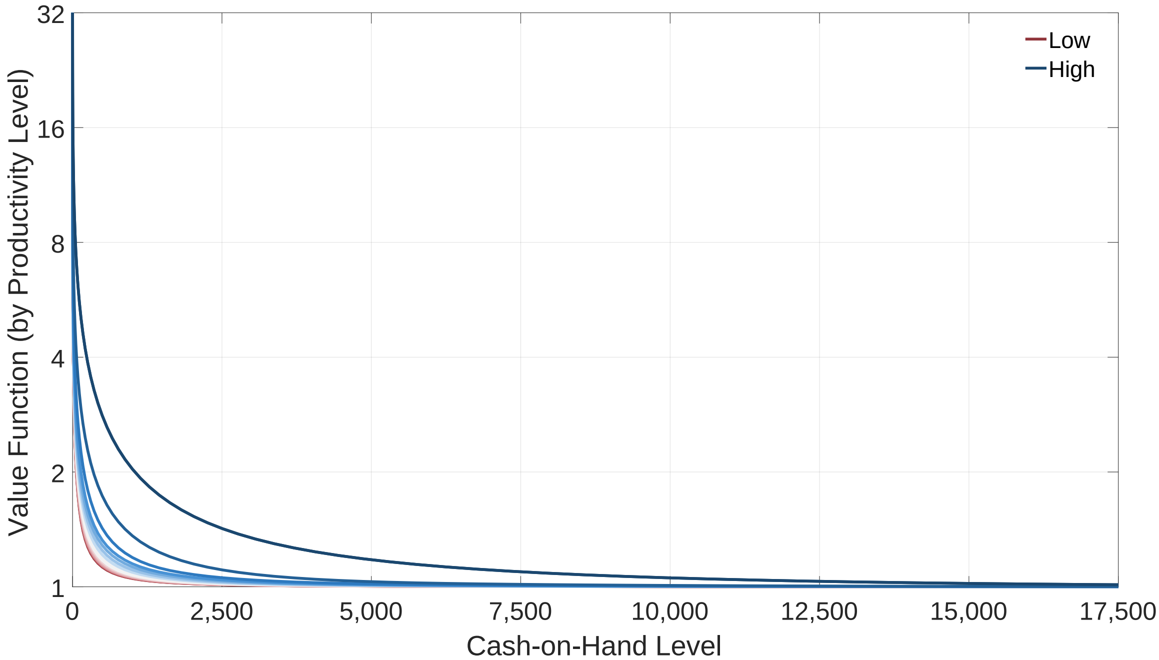


Figure C4: Value Function (NFF/FF) by Cash-on-Hand and Productivity Levels



D Results

In this section, I provide the results when the productivity process follows the standard AR(1) used in the literature. Finally, I review other forms of borrowing constraint used in the literature.

D.1 AR(1)

In this section, I show the results on the model where productivity dynamics follow an AR(1) process. Table D1 shows the calibration of the parameters.

D.1.1 Firm Life Cycle

Figure D1 and D2 show the firm life cycle in terms of entry and exit, and firm ageing, respectively.

D.1.2 Misallocation

Figure D3 shows the results on misallocation across firm characteristics.

D.1.3 Financial Behavior

Figure D4 and D5 show the results on financial behavior.

Table D1: Moments of the calibration - Size Dependent Borrowing Constraint

Parameter	Value	Moment	Data	Model
η	0.83	$SD(k)$	1.79	1.76
β	0.97	K/Y	2.0	2.2
α	0.35	K/L	4.0	4.1
δ	0.05	Inv/Y	0.12	0.13
A_{shift}	1.22	L	15.5	15.5
θ	0.81	$Leverage$	0.19	0.19
Ψ	0.50	$P_{95}^{Leverage}$	0.71	0.71
τ	0.43	$Profits/Y$	0.15	0.15
μ_e	1.95	k_{ent}	0.36	0.36
σ_e	1.92	$SD(k_{ent})$	0.95	0.95
$\rho_{a,e}$	0.02	$\rho(a_{ent}; e_{ent})$	0.05	0.05

Figure D1: Firm Life Cycle - Entry and Exit

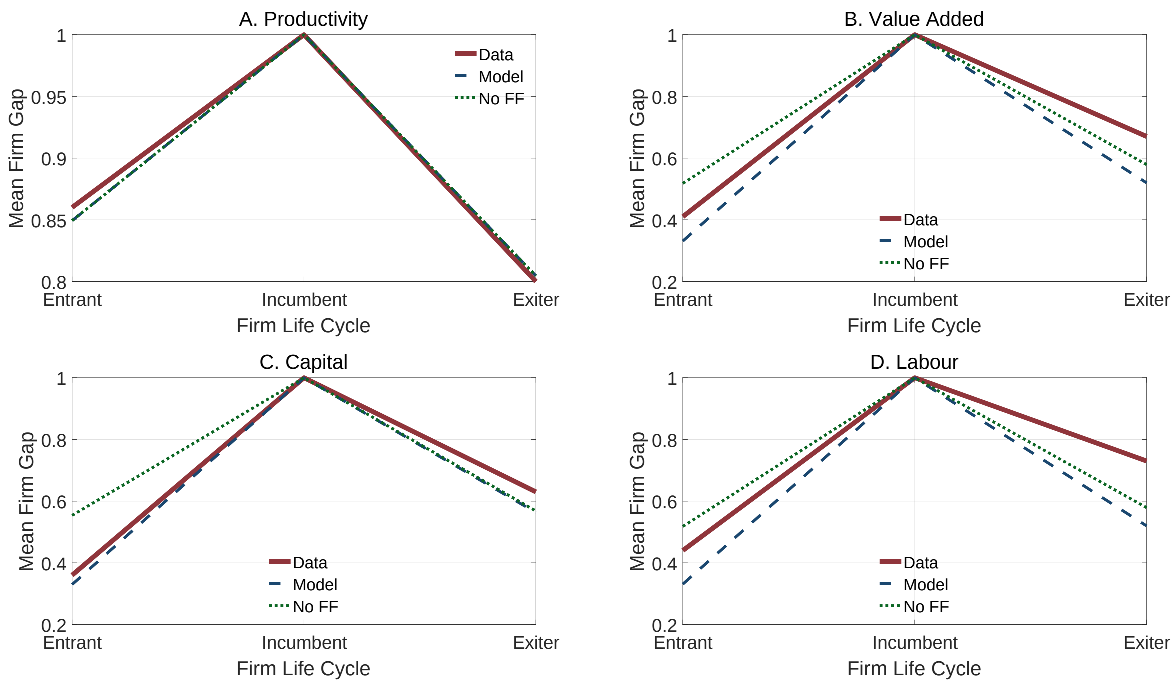


Figure D2: Firm Life Cycle - Firm Ageing

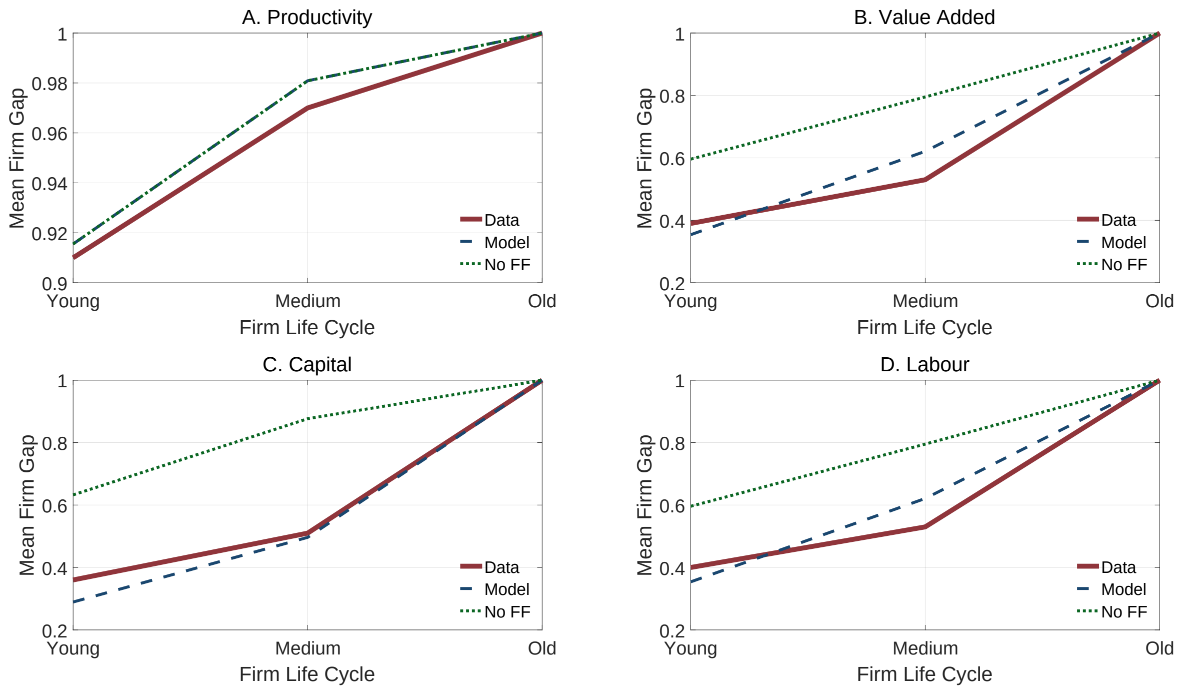


Figure D3: Profiles of PY/K

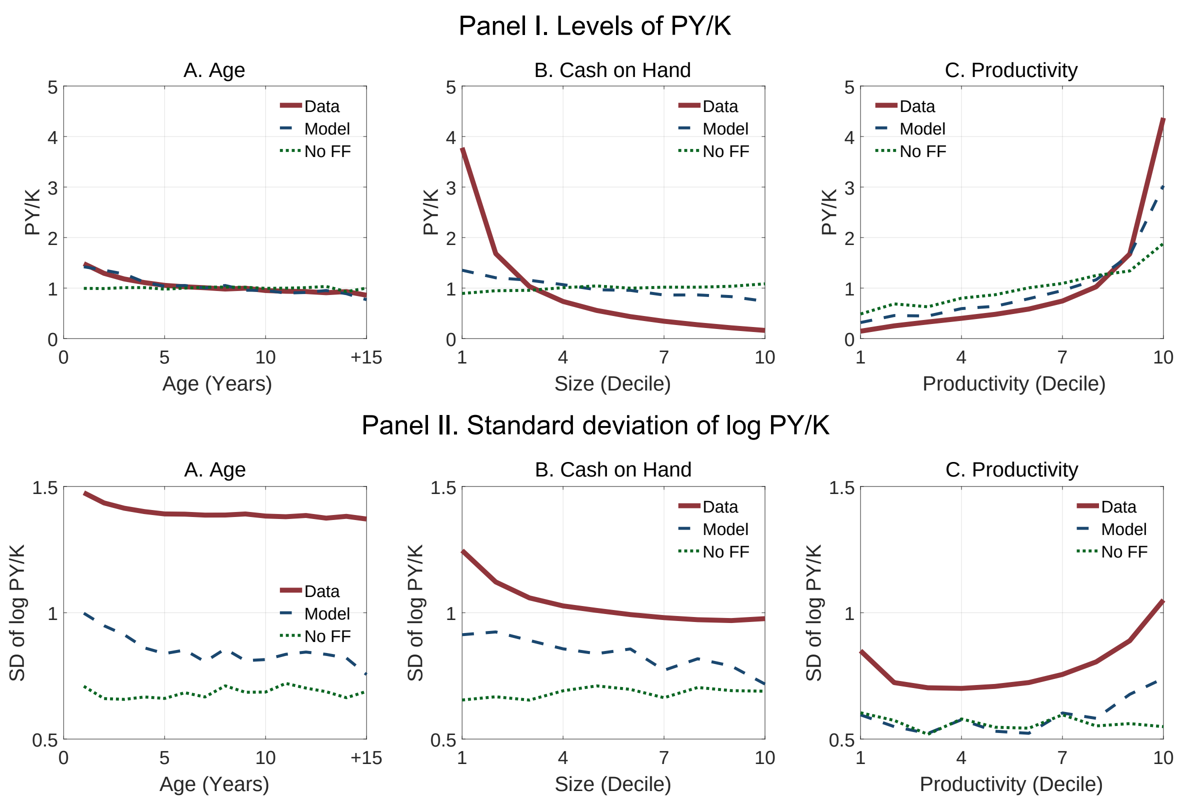


Figure D4: Financial Behavior

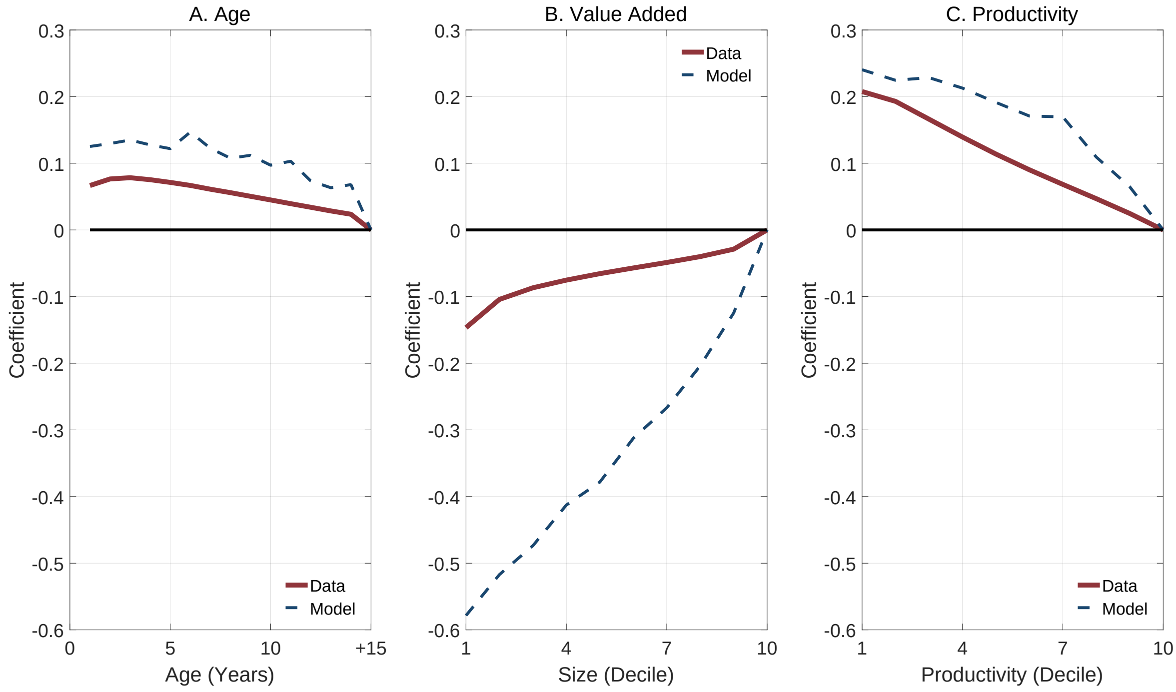
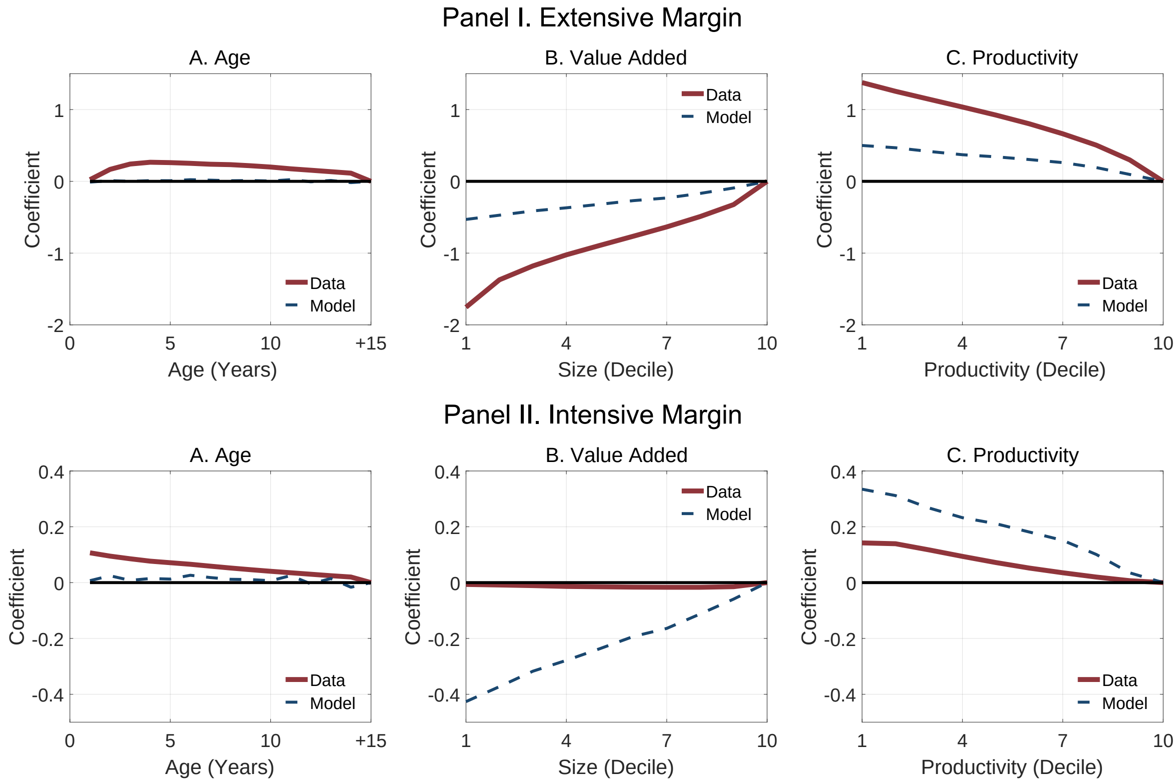


Figure D5: Financial Behavior - Extensive and Intensive Margin



D.2 Other Functional Forms of Borrowing Constraint

In this section, I explore other forms of borrowing constraint that has been broadly used in the literature.

D.2.1 Standard Borrowing Constraint: Target Average Leverage

I start by analysing the standard borrowing constraint used in the literature and the θ parameter being calibrated to match the average leverage.

Table D2: Moments of the calibration - Standard Borrowing Constraint Target Leverage

Moment	Data	N-L	AR(1)	Target
l	15.5	15.48	15.49	A
$SD(k)$	1.79	1.774	1.770	η
K/Y	2.0	2.04	2.06	β
K/L	4.0	3.79	4.06	α
Inv/Y	0.12	0.119	0.124	δ
$Leverage$	0.19	0.190	0.190	θ
$Profits/Y$	0.15	0.150	0.150	ϕ
k_{ent}	0.36	0.360	0.360	μ_e
$SD(k_{ent})$	0.95	0.950	0.950	σ_e
$\rho(a_{ent}; e_{ent})$	0.05	0.050	0.050	$\rho_{a,e}$

Table D3: Calibration Standard Borrowing Constraint Target Leverage

Parameter	N-L	AR(1)
A_{shift}	1.222	1.49
η	0.83	0.78
β	0.97	0.95
α	0.35	0.35
δ	0.04	0.04
θ	0.319	0.443
ϕ	0.503	0.471
μ_e	1.82	2.44
σ_e	1.89	1.77
$\rho_{a,e}$	0.023	0.031

Figure D6: Firm Life Cycle - Entry and Exit

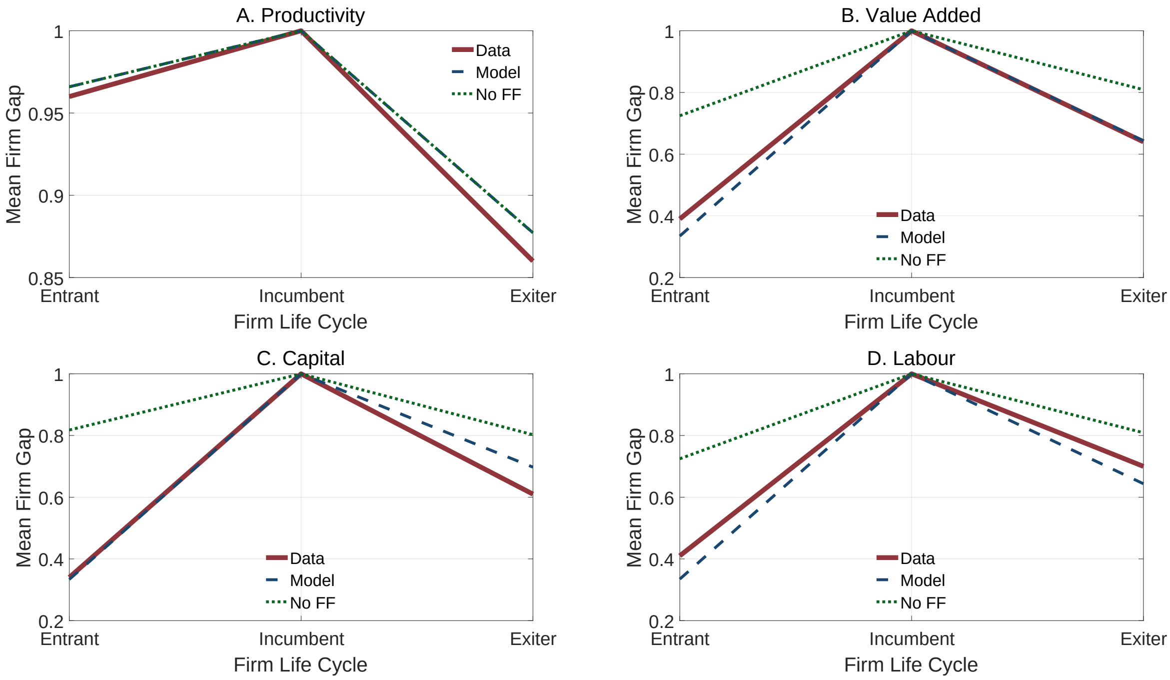


Table D4: Aggregate Consequences - Standard Borrowing Constraint Target Leverage

	N-L	AR(1)
No Constrained Firms	0.0001	0.0064
Constrained Type I Firms	0.4351	0.6298
Constrained Type II Firms	0.5648	0.3638
SD(log MRPK)	1.0809	0.8165
SD(log MRPK) No FF	0.8474	0.6838
Productivity Loss	0.3059	0.1700
Productivity Loss FF	0.1515	0.0626

Figure D7: Firm Life Cycle - Firm Ageing

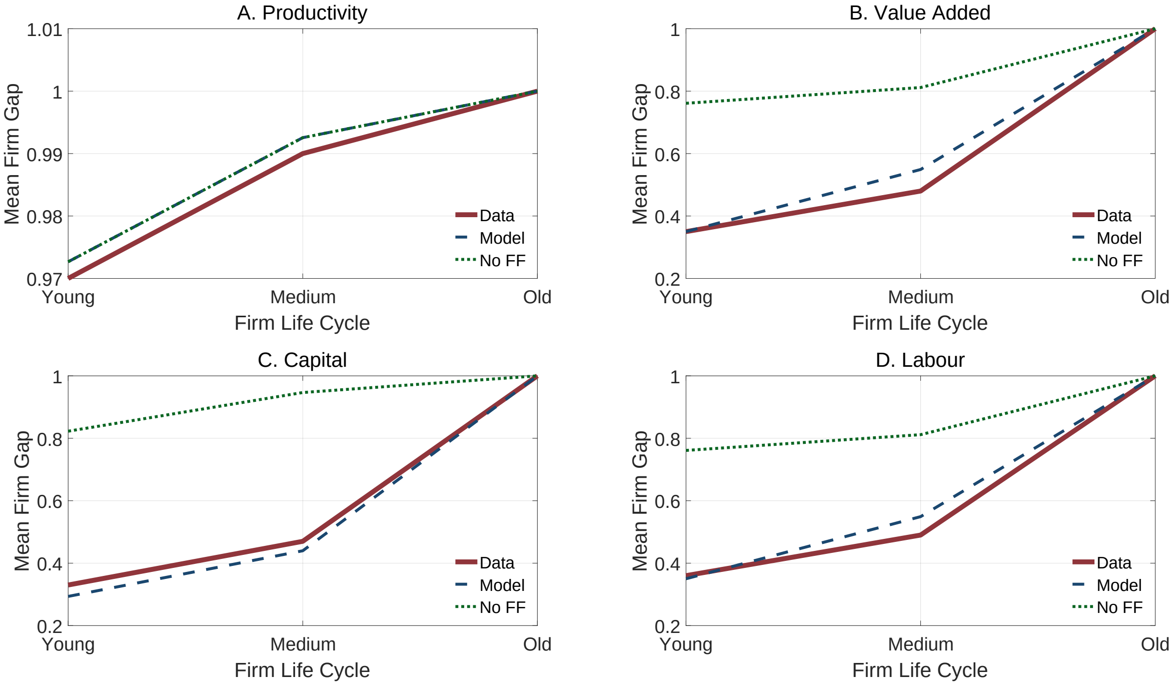


Figure D8: Profiles of PY/K

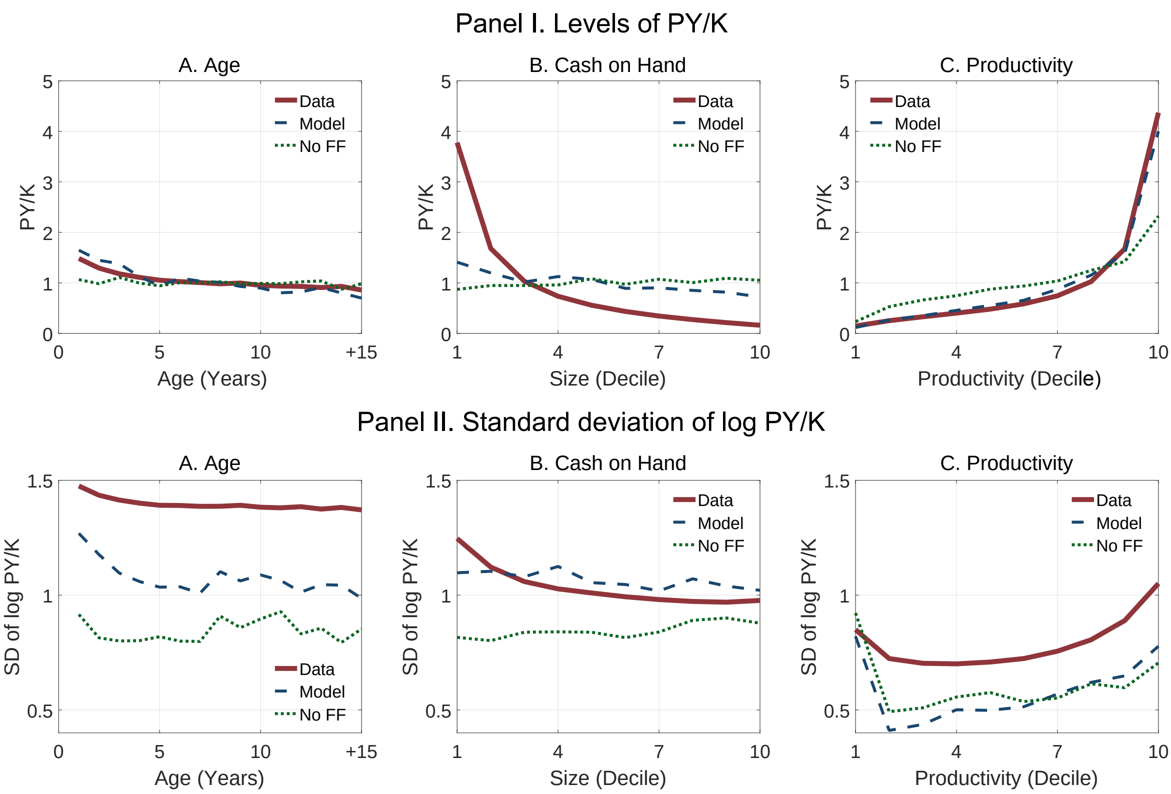


Figure D9: Financial Behavior

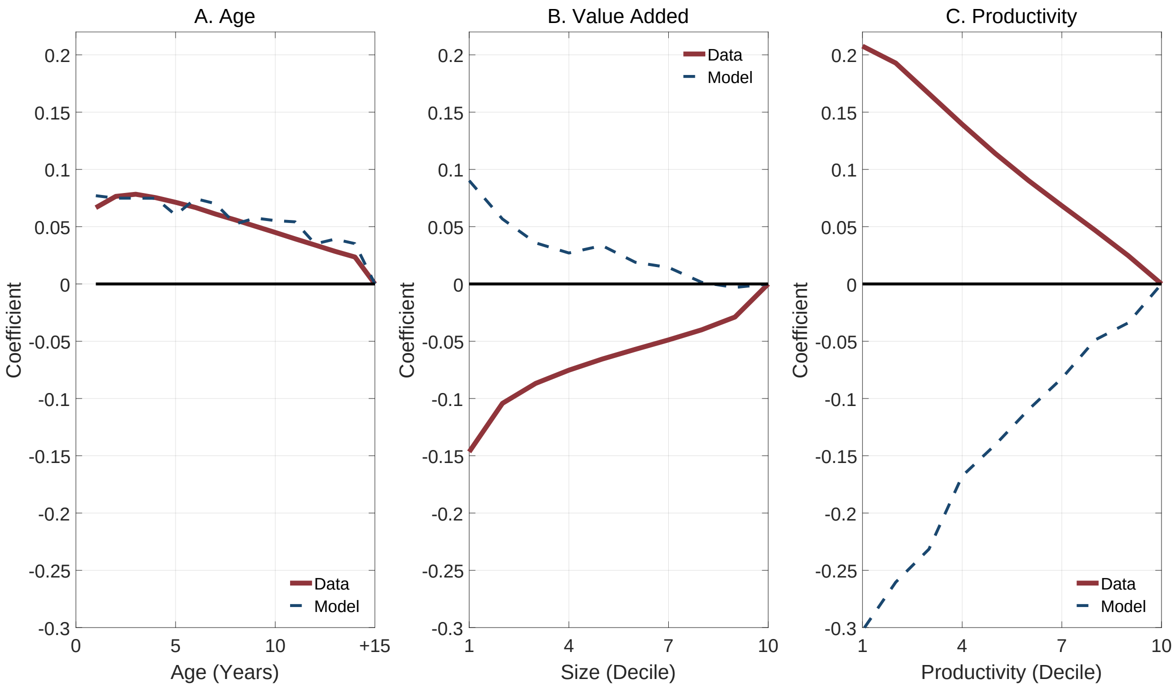
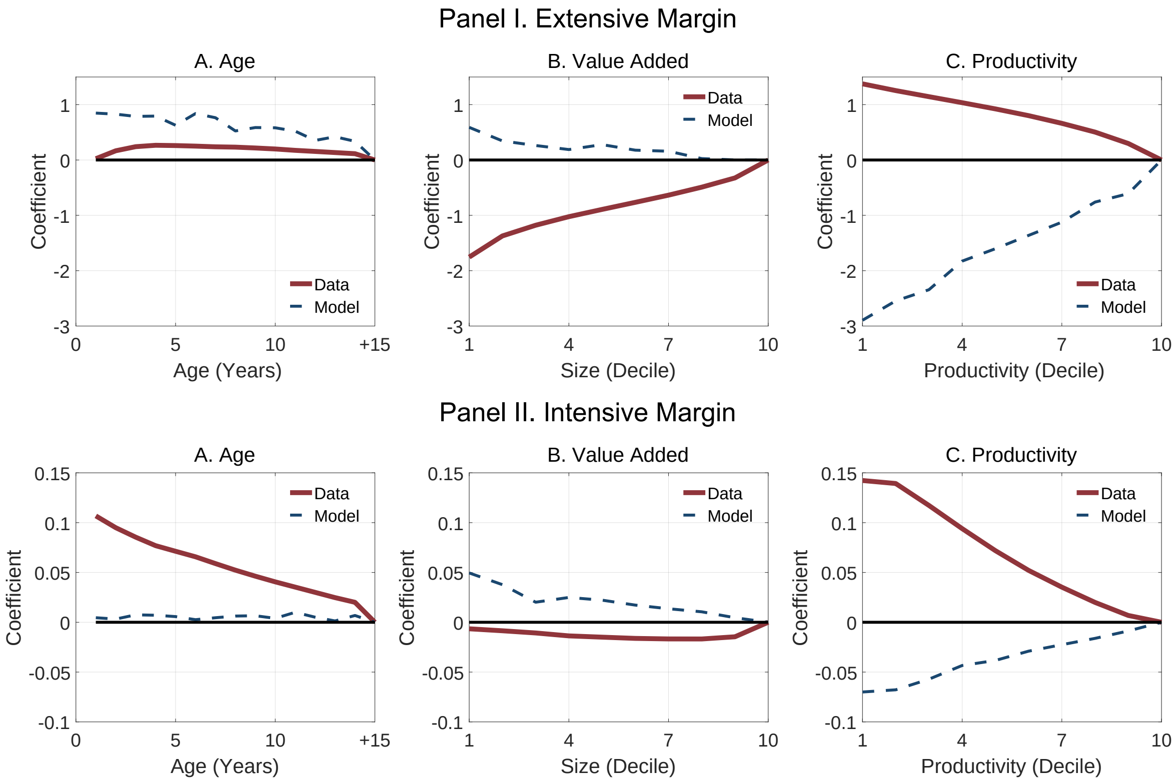


Figure D10: Financial Behavior - Extensive and Intensive Margin



D.2.2 Standard Borrowing Constraint: Target Debt to Output Ratio

In this section, I analyse the standard borrowing constraint used in the literature and the θ parameter being calibrated to match the debt to output ratio.

Table D5: Moments of the calibration - Standard Borrowing Constraint Target Debt to Output Ratio

Moment	Data	N-L	AR(1)	Target
l	15.5	15.47	15.47	A
$SD(k)$	1.79	1.930	1.811	η
K/Y	2.0	2.08	2.16	β
K/L	4.0	3.85	4.25	α
Inv/Y	0.12	0.109	0.116	δ
$Debt/Y$	0.19	0.811	0.810	θ
$Profits/Y$	0.15	0.150	0.150	ϕ
k_{ent}	0.36	0.361	0.360	μ_e
$SD(k_{ent})$	0.95	0.949	0.950	σ_e
$\rho(a_{ent}; e_{ent})$	0.05	0.050	0.049	$\rho_{a,e}$

Table D6: Calibration Standard Borrowing Constraint Target Debt to Output Ratio

Parameter	N-L	AR(1)
A_{shift}	1.165	1.455
η	0.83	0.78
β	0.97	0.95
α	0.35	0.35
δ	0.05	0.04
θ	0.513	0.572
ϕ	0.551	0.479
μ_e	1.52	2.317
σ_e	2.149	1.823
$\rho_{a,e}$	0.025	0.034

Figure D11: Firm Life Cycle - Entry and Exit

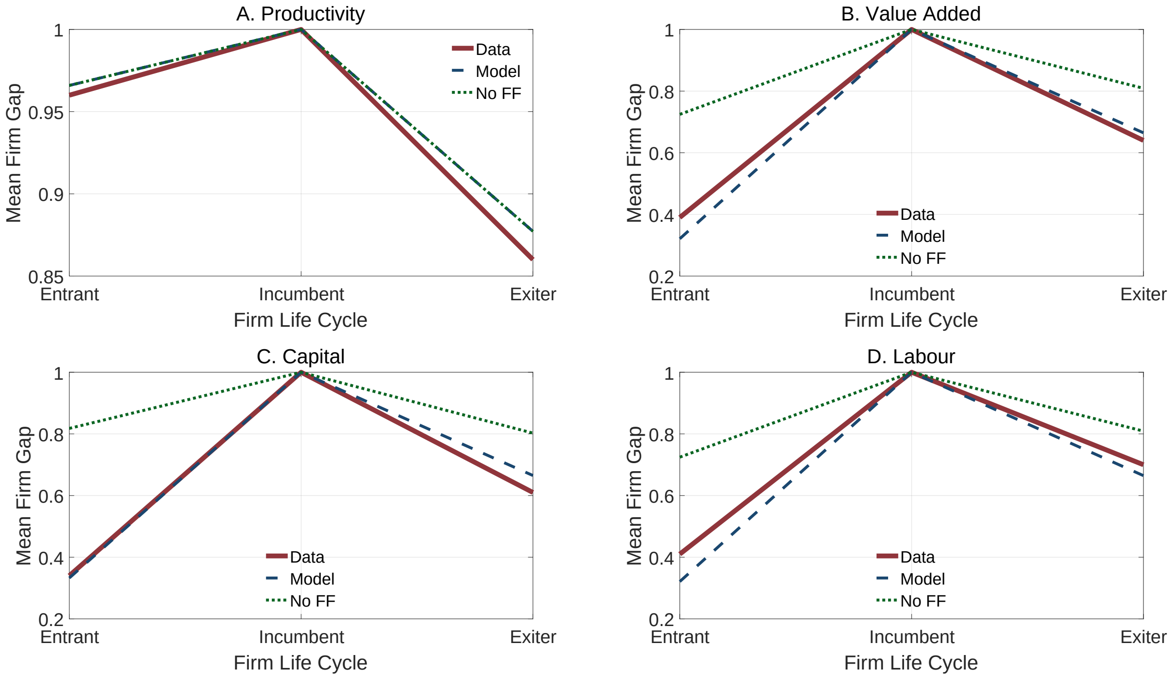


Figure D12: Firm Life Cycle - Firm Ageing

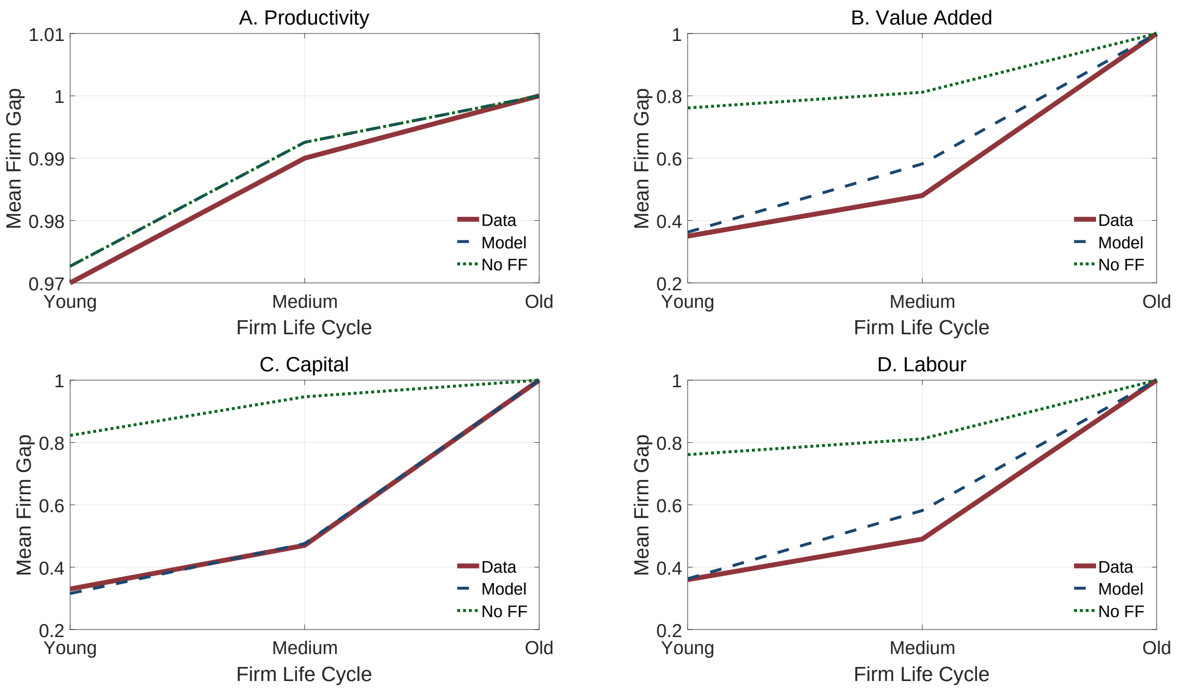


Figure D13: Profiles of PY/K

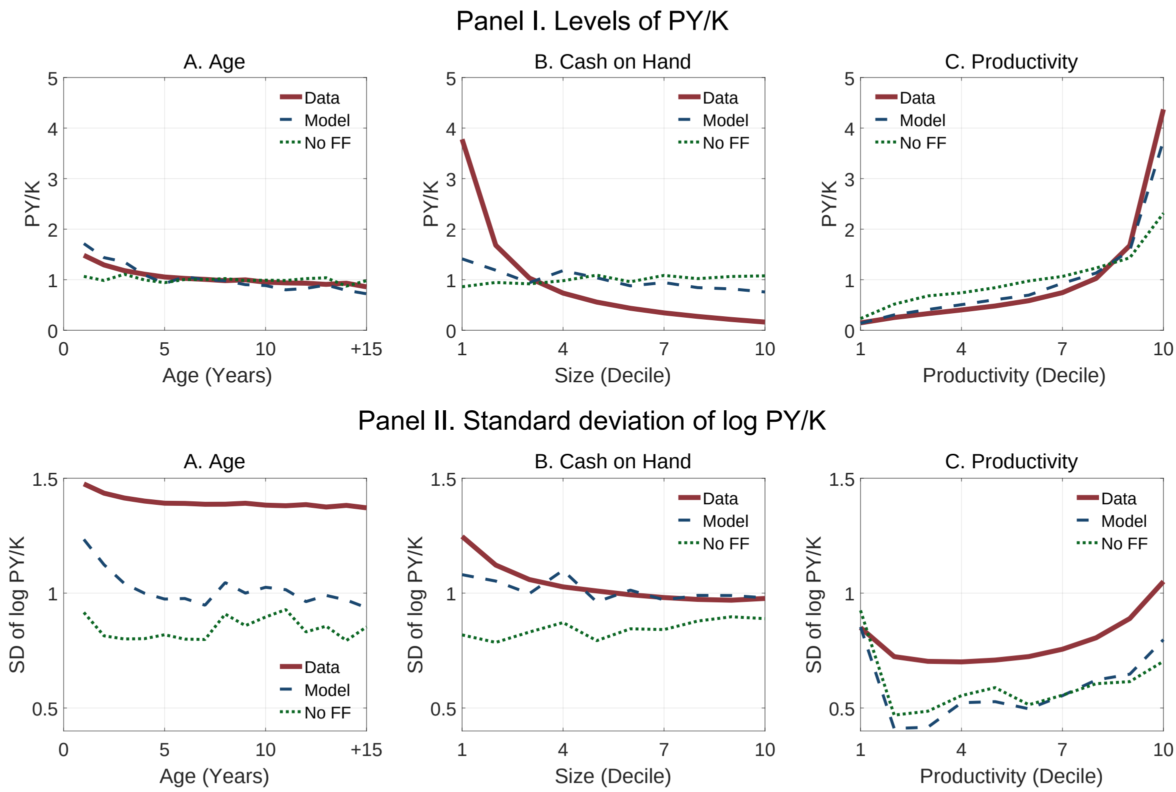


Figure D14: Financial Behavior

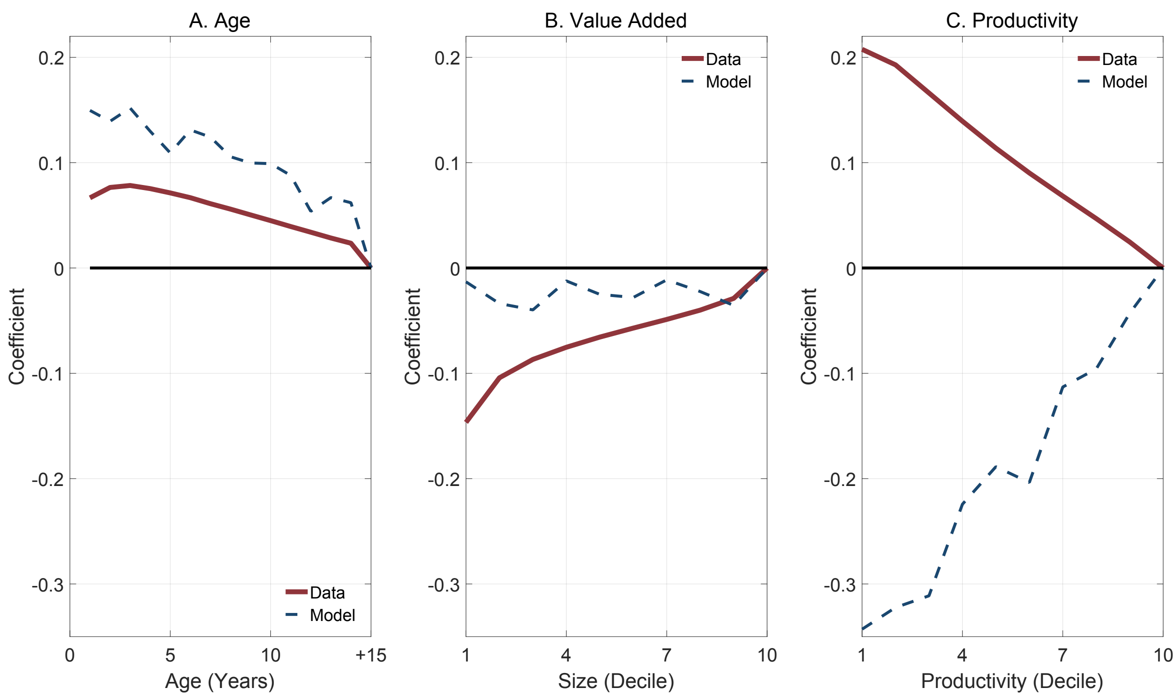


Figure D15: Financial Behavior - Extensive and Intensive Margin

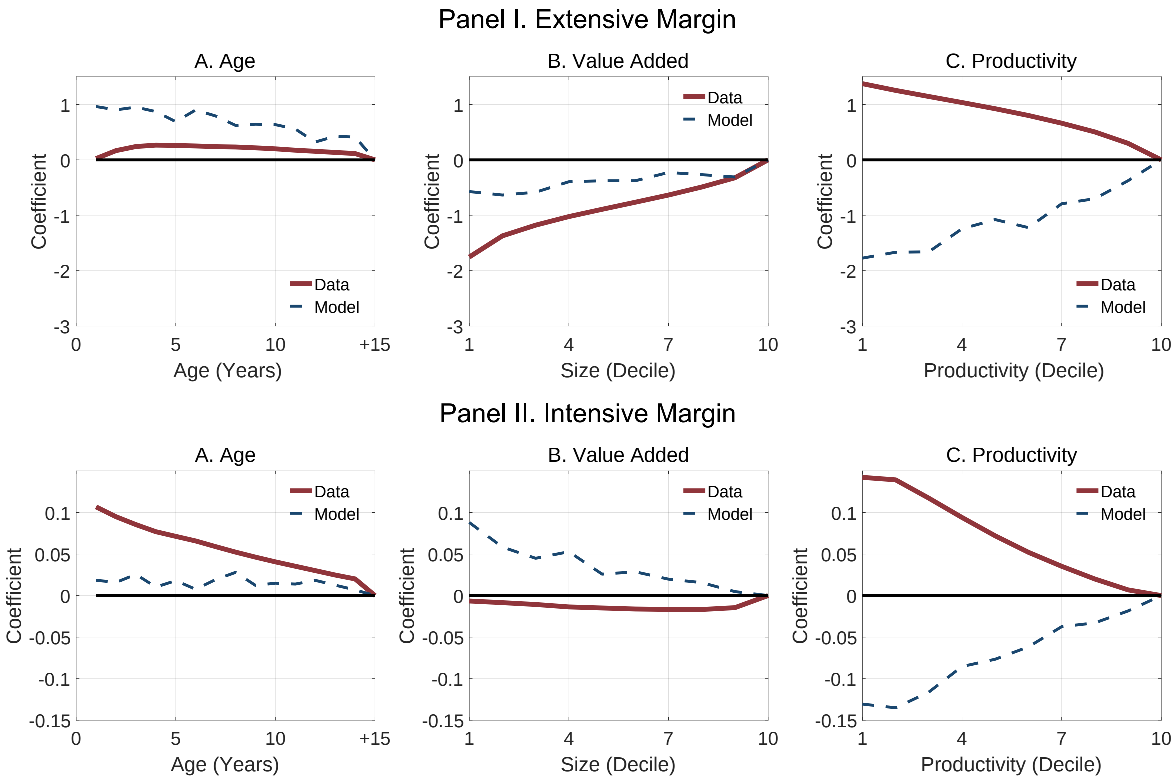


Table D7: Aggregate Consequences - Standard Borrowing Constraint Target Debt to Output Ratio

	N-L	AR(1)
No Constrained	0.0001	0.0070
Constrained Type I	0.5384	0.6566
Constrained Type II	0.4615	0.3364
SD(log MRPK)	1.0254	0.7959
SD(log MRPK) No FF	0.8474	0.6879
Productivity Loss	0.2756	0.1607
Productivity Loss FF	0.1144	0.0522

D.2.3 Borrowing Constraint with Profits: Target Average Leverage

Finally, I analyse the case where the borrowing constraint is earnings based, instead of collateral based. This formulation of borrowing constraint is growing up and it is motivated by the existence of earnings covenants in the debt contracts, as shown in Drechsel (2019).

Table D8: Moments of the calibration - Borrowing Constraint with Expected Profits

Moment	Data	NP	AR(1)	Target
l	15.5	15.47	15.47	A
$SD(k)$	1.79	1.820	1.770	η
K/Y	2.0	1.78	1.93	β
K/L	4.0	3.75	3.80	α
Inv/Y	0.12	0.113	0.119	δ
$Leverage$	0.19	0.190	0.190	θ
$Profits/Y$	0.15	0.150	0.150	ϕ
k_{ent}	0.36	0.360	0.360	μ_e
$SD(k_{ent})$	0.95	0.951	0.939	σ_e
$\rho(a_{ent}; e_{ent})$	0.05	0.050	0.050	$\rho_{a,e}$

Table D9: Calibration Borrowing Constraint with Expected Profits

Parameter	N-L	AR(1)
A_{shift}	1.207	1.501
η	0.83	0.78
β	0.97	0.95
α	0.35	0.35
δ	0.05	0.04
θ	0.953	1.140
ϕ	0.518	0.461
μ_e	1.217	2.034
σ_e	2.300	2.017
$\rho_{a,e}$	0.043	0.059

Figure D16: Firm Life Cycle - Entry and Exit

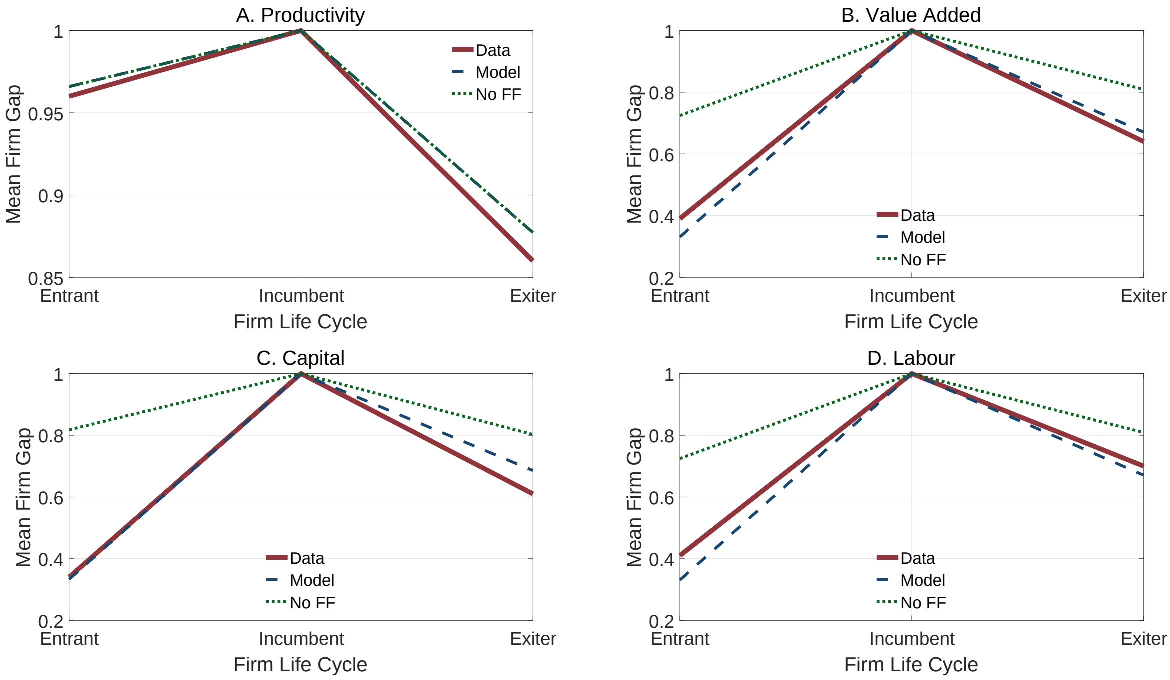


Table D10: Aggregate Consequences - Borrowing Constraint with Expected Profits

	N-L	AR(1)
No Constrained	0.0001	0.0075
Constrained Type I	0.4186	0.5336
Constrained Type II	0.5813	0.4589
SD(log MRPK)	1.0326	0.8024
SD(log MRPK) No FF	0.8474	0.6838
Productivity Loss	0.2684	0.1611
Productivity Loss FF	0.1056	0.0526

Figure D17: Firm Life Cycle - Firm Ageing

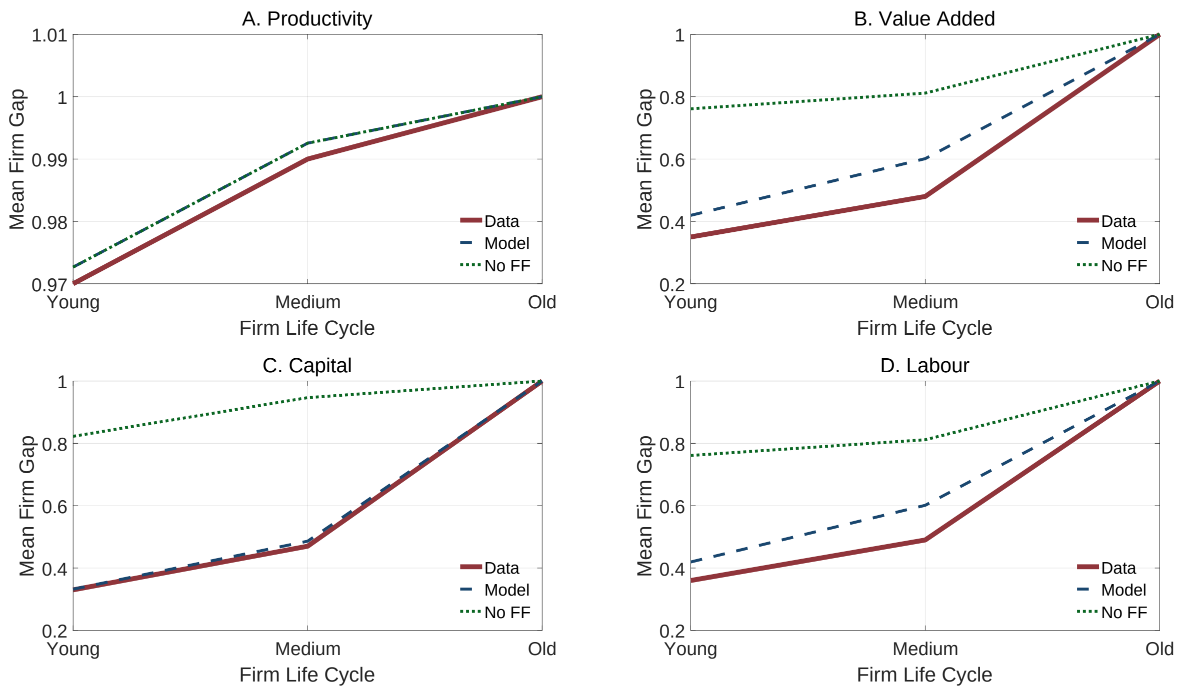


Figure D18: Profiles of PY/K

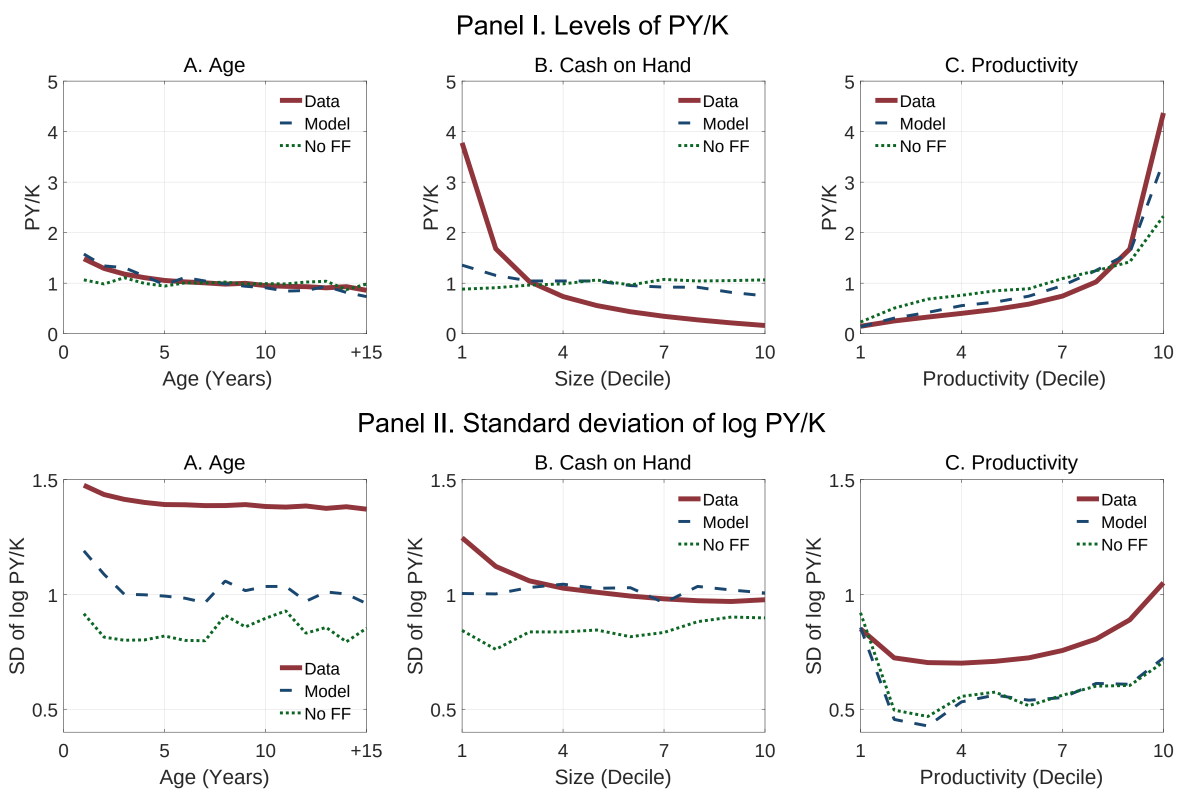


Figure D19: Financial Behavior

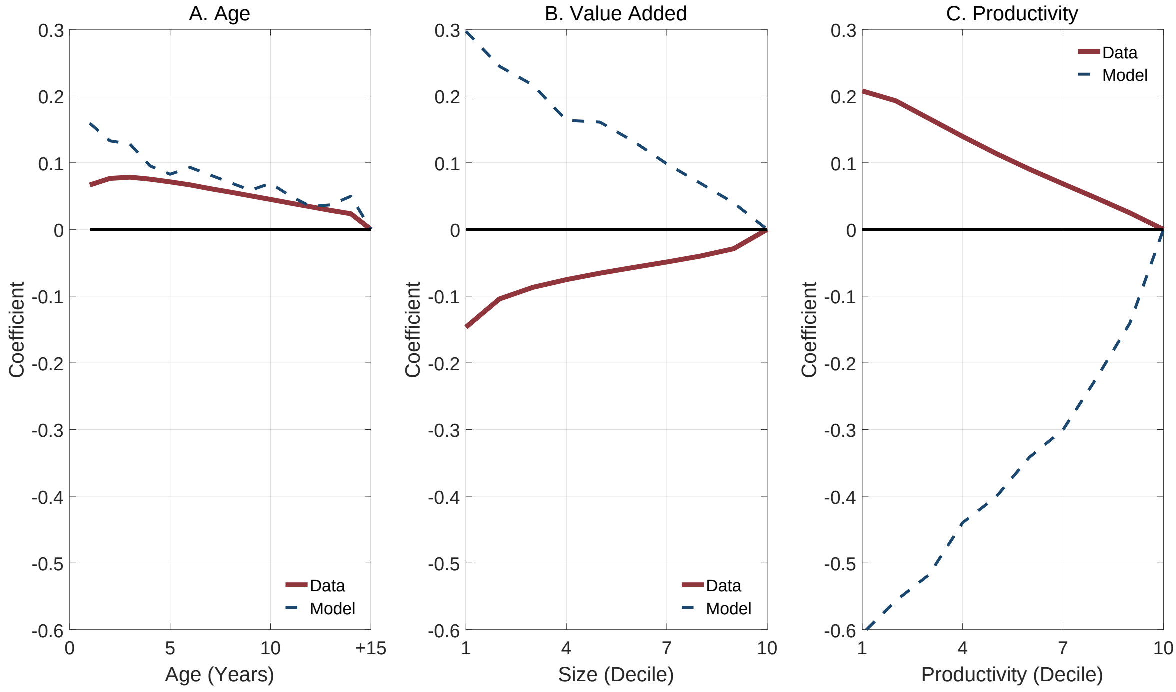


Figure D20: Financial Behavior - Extensive and Intensive Margin