Non-Linear Productivity and Investment Dynamics

Giulio Fella (QMUL), Julio Gálvez (BdE), Beatriz González (BdE), Juan Carlos Ruiz-Garcia (UIB) and Tatsuro Senga (QMUL)

November 11, 2021

Motivation

- Firms' productivity dynamics define
  1. expectation on the productivity evolution
  2. uncertainty on the productivity realization
- Key determinants of firm-level investment
- The resulting investment behavior contains information on
  1. the nature of economic frictions, e.g., adjustment costs
  2. the allocation of resources among firms, and
  3. the performance of the aggregate economy

Motivation (cnt'd)

- Most dynamics models of investment behavior assume "canonical" AR(1) process for productivity
- The canonical process: AR(1) implies
  1. linear productivity dynamics, with
  2. normally distributed shocks
- Limitations that may bias existing qualitative and quantitative work

This paper

- Uses the dataset *Central de Balances Integrada* from 1999 to 2017
- Estimates sector-specific production functions to extract **revenue** productivity
- Estimates two persistent + transitory productivity processes:
  1. *Canonical process*
     - Persistent: AR(1) with normally distributed shocks
  2. *Non-linear process*
     - Persistent: non-linear with flexible shock distributions
- Studies the implications on investment behavior through
  - Estimation of empirical policy functions
  - Structural investment model with adjustment costs

Findings

- Firm-level productivity series from PF estimation display
  1. non-linearity
  2. non-normality
- The estimated non-linear (NL) productivity process captures those features well
- Estimated empirical policy functions under the NL process imply
  1. better fit of investment dynamics
  2. larger investment responses to persistent productivity shocks
  3. no investment responses to transitory productivity shock
- Structural model under the NL process
  1. fits data moments better
  2. different nature of adjustment costs $\rightarrow$ larger role for convex vs fixed

Related literature

Non-linear processes

Arellano et al. (2017); De Nardi et al. (2020); Guvenen et al. (2021); Ruiz-Garcia (2021)

- Depart from linearity in estimation of productivity process

Investment at the firm level

Caballero and Engel (1999); Cooper and Haltiwanger (2006); Bazdresch et al. (2018); Gala et al. (2020)

- Importance of NL productivity for investment dynamics

Productivity dynamics in macro models

Jaimovich et al. (2021); Ruiz-Garcia (2021); Pugsley et al. (2020); Asker et al. (2014); Bloom et al. (2007)

- Implications of NL productivity for the nature of adjustment costs and aggregate quantification

Roadmap

1. Data and production function estimation
2. Firm productivity dynamics
3. Empirical investment policy functions
4. Structural firm dynamics model with capital adjustment costs

Data

Production Function Estimation

Data

- *Central de Balances Integrada* hosted by *Banco de España*
- Firm-level data
  1. Balance sheet
  2. Earnings statement
  3. Annual report
- Quasi-universe of Spanish firms
- Time period: 1999 - 2017
- Focus on limited liability firms not controlled by the public sector
- Main variables: value-added, capital, labor, and materials
- After cleaning: 1.1 million firms, 7.5 million firm-year observations

Production function estimation

- Consider the following production function specification in logs:

$$y_{jht} = \beta_{h,t} + \beta_{k,h} k_{jht} + \beta_{l,h} l_{jht} + z_{jht} + \varepsilon_{jht}$$

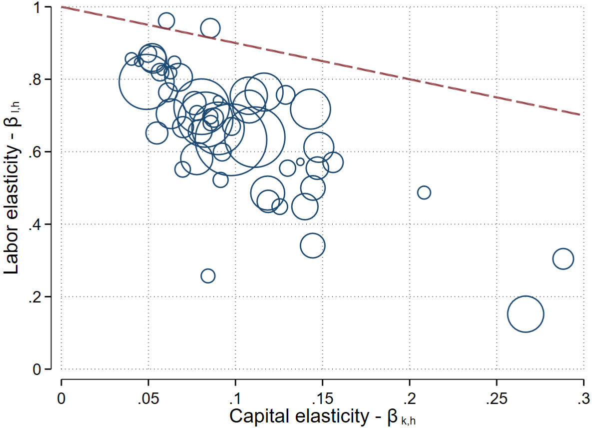
  - y_{jht}: log of value-added
  - k_{jht}: log of physical capital
  - l_{jht}: log of labor – wage bill
  - $\beta_{h,t}$: aggregate evolution of TFP in sector h
  - z_{jht}: firm productivity
  - ε_{jht}: error term

- Modify Ackerberg et al. (2015) estimation procedure to capture aggregate shocks as Hahn et al. (2020) More

- CUGMM for robustness to starting points as Kim et al. (2019)

Production function estimates

Sector-specific value-added elasticities



- Large heterogeneity in the production process across sectors
- Only two sectors display increasing returns to scale
- Large heterogeneity on firm-level productivity ▸ More

Production function estimates

Aggregate value-added elasticities

Aggregation	Capital (90% CI)	Labor (90% CI)	RtS (90% CI)
Unweighted	0.101	0.658	0.759
	(0.073, 0.131)	(0.508, 0.802)	(0.627, 0.884)
Weighted	0.101	0.658	0.759
	(0.085, 0.124)	(0.565, 0.744)	(0.676, 0.834)

- Aggregate diminishing returns to scale in line with the literature
- Low elasticity wrt capital
- Tighter confidence intervals in the weighted aggregation
  —more precise estimates in sectors with high weight in the Economy—

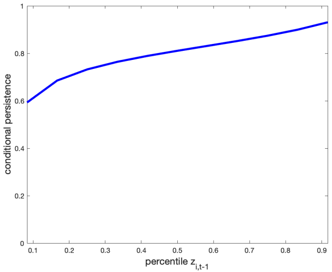
Firm Productivity Dynamics

Features of productivity data

Persistence and skewness ▸ More

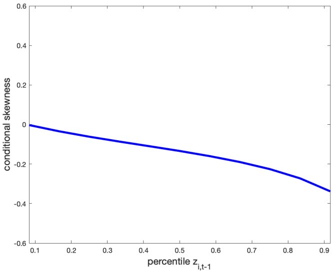
Persistence

First order AR coefficient



Skewness

$$S_K = \frac{(P90-P50)-(P50-P10)}{P90-P10}$$



- Productivity persistence is larger for high-productivity firms
- Conditional productivity distribution displays negative skewness
- High-productivity firms face larger negative skewness

Modelling productivity dynamics

- Typical AR(1) model of productivity dynamics:

$$z_{jt} = \rho z_{j,t-1} + v_{jt}, \quad v_{jt} \sim N(0, \sigma_v)$$

- Implies:
  1. Linearity
  2. Normality

- In the paper, we allow for:
  - measurement error/different shock persistence (e.g., Gourio,2008 ; Roys, 2016; Sterk et al., 2021)

    $$z_{jt} = \eta_{jt} + \varepsilon_{jt}$$

    with η : persistent component
  - non-linearity in η_{jt} and non-normality

Flexible productivity process ▸ More

$$\eta_{it}(q^\eta) = Q_\eta(q^\eta|\eta_{i,t-1}, age_{it}), \quad q^\eta \sim U(0,1)$$
$$\epsilon_{it}(q^\epsilon) = Q(q^\epsilon|age_{it}), \qquad q^\epsilon \sim U(0,1)$$

Flexible, non-linear process:

- Both η and ϵ:
  - Flexible + age-dependent distribution of shocks Q
- Persistent component η :
  - Non-linear dependence between η_{jt} and η_{jt-1}
    —different distributions of shocks for different type of firms—
  - Non-linear dependence between η_{jt} and q^η
    —current shocks can wipe out memory of past shocks—

Flexible productivity process

More

Canonical model

$$\eta_{it}(q^\eta) = Q_\eta(q^\eta|\eta_{i,t-1}, age_{it}), \quad q^\eta \sim U(0,1) \quad = \quad \rho\eta_{i,t-1} + \sigma_v \phi^{-1}(q^\eta)$$

$$\epsilon_{it}(q^\epsilon) = Q(q^\epsilon|age_{it}), \quad q^\epsilon \sim U(0,1) \quad = \quad \sigma_\epsilon \phi^{-1}(q^\epsilon)$$

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Flexible productivity process ▸ More

Canonical model

$$\begin{aligned}
\eta_{it}(q^\eta) = Q_\eta(q^\eta|\eta_{i,t-1}, age_{it}), \quad q^\eta \sim U(0,1) &= \rho\eta_{i,t-1} + \sigma_v\phi^{-1}(q^\eta) \\
\epsilon_{it}(q^\epsilon) = Q(q^\epsilon|age_{it}), \quad q^\epsilon \sim U(0,1) &= \sigma_\epsilon\phi^{-1}(q^\epsilon)
\end{aligned}$$

Flexible, non-linear process:

- Both η and ϵ:
  - Flexible + age-dependent distribution of shocks Q
- Persistent component η :
  - Non-linear dependence between η_{jt} and η_{jt-1}
    —different distributions of shocks for different type of firms—
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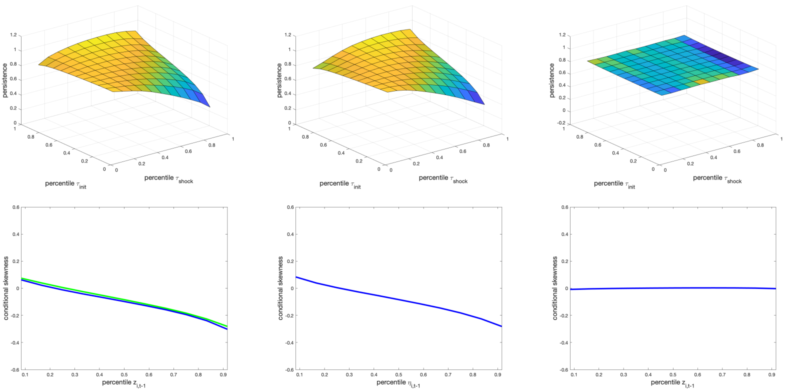
Data vs non-linear vs canonical productivity dynamics SE

Data | Non-linear | Canonical



- Non-linear productivity process captures the data
  1. Non-linearity
  2. Non-normality
- The canonical process cannot capture these, by construction Results

But do they matter for investment?

1. Empirical exercise
   - Estimate flexible investment policy function under the two processes
   - Are the investment reactions –marginal propensities to invest– equivalent?
2. Structural model
   - Calibrate the model with adjustment costs under the two processes
   - Do they imply similar firm-level investment dynamics?
   - What is the nature of capital adjustment costs?
   - What are the implications for the aggregate Economy? - [not today]

Empirical Investment Policy Functions

Empirical investment policy functions

- Empirical investment policy function:

$$i_{jt+1} = g_t(k_{jt}, \eta_{jt}, \varepsilon_{jt}, u_{jt+1}), \quad t = 1, \ldots, T$$

  - i_{jt} is firm net investment rate
  - u_{jt} are unobserved stochastic determinants of firm investment costs

- η_{jt} and ε_{jt} as opposed to z_{jt} in Bazdresch et al. (2018) and Gala et al. (2020) $\rightarrow$ latent variables

- Simulated EM based algorithm following ABB More

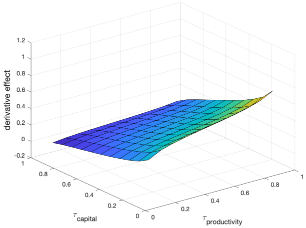
- EIPF under both productivity process fit the investment distribution More

- We define marginal propensities to invest

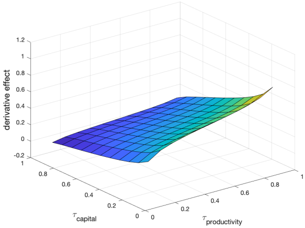
$$\phi_{\chi,t} = \mathbb{E}\left[\frac{\partial g_t(k_{jt}, \eta_{jt}, \varepsilon_{jt}, u_{jt+1})}{\partial \chi_{jt}}\right], \quad \chi_{jt} = \eta_{jt}, \varepsilon_{jt}, k_{jt}$$

Marginal propensities to invest Capital

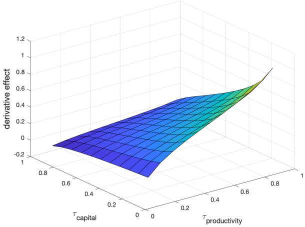
Productivity in real data vs. simulated data



Data



NL

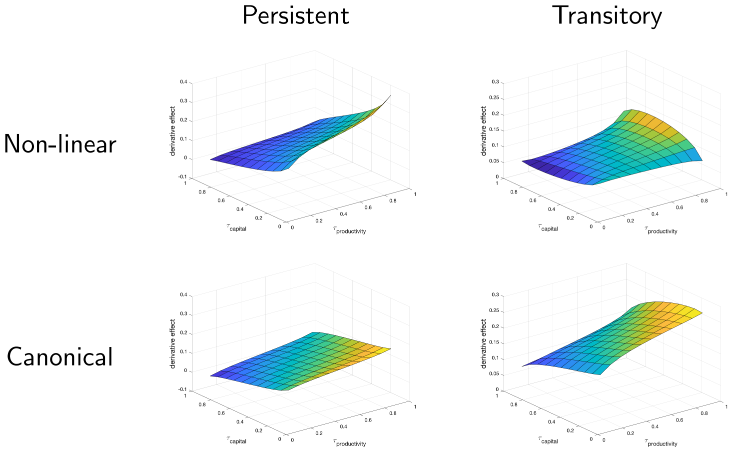


Canonical

- MPI is increasing with productivity and decreasing with capital
- MPI under NL productivity dynamics are closer to the data SE

Marginal propensities to invest ▸ SE

Persistent vs. transitory productivity



- MPI under NL are mostly driven by the persistent component
- MPI under canonical are mostly driven by the transitory component

Structural Firm Dynamics Model with Capital Adjustment Costs

Model – Production

- Time is discrete with infinite horizon
- Production technology is Cobb-Douglas with DRS

$$f(z_{jt}, K_{jt}, L_{jt}) = \exp(z_{jt}) K_{jt}^{\beta_K} L_{jt}^{\beta_L} \quad \beta_K + \beta_L < 1.$$

- Idiosyncratic productivity evolves according to either
  1. estimated NL process
  2. canonical AR(1)
- Labor decision is
  1. static
  2. not subject to frictions
- Capital is
  1. predetermined by previous period investment

$$K_{jt+1} = I_{jt} + (1 - \delta) K_{jt}$$

  2. Investment is subject to adjustment costs

Model – Capital adjustment costs

- Fixed costs governed by FC_K

$$FC_K 1\left\{\left|\frac{I_{jt}}{K_{jt}}\right| > 0.01\right\} \qquad \text{with } FC_K \geq 0,$$

- Convex adjustment costs governed by γ

$$\frac{\gamma}{2}\left(\frac{I_{jt}}{K_{jt}}\right)^2 K_{jt} \qquad \text{with } \gamma \geq 0,$$

- Transaction costs governed by p_K
  —price-gap between the buying and selling price for capital—

$$I_{jt}\, 1\{I_{jt} \geq 0\} + p_k\, I_{jt}\, 1\{I_{jt} < 0\} \qquad \text{with } p_K \in [0,\, 1)$$

Model – Estimation

- Parameters not governing adjustment costs are set externally ▸ More
- Estimate the adjustment costs using SMM and targeting
    1. SD of the investment rate – 0.631
    2. Serial correlation of investment rates – 0.150
    3. Spike rate of positive investment – 0.159
- Compare the results under the two productivity processes
    1. Nature of adjustment costs
    2. Firm investment dynamics

Nature of adjustment costs

		Targeting NL		Targeting AR(1)	
	Data	NL	AR(1)	NL	AR(1)
Parameters					
FC_k		**0.085**	0.085	0.134	**0.134**
γ		**0.123**	0.123	0.057	**0.057**
p_k		**0.798**	0.798	0.733	**0.733**
Loss function		**1.142**	899.673	2001.341	**434.087**
Targeted moments					
SD investment rate	0.631	**0.626**	0.533	1.001	**0.682**
Serial correlation of investment rates	0.150	**0.150**	0.430	-0.099	**0.351**
Spike rate: positive investment	0.159	**0.150**	0.202	0.127	**0.159**

- Model under NL process captures better targeted moments
  —loss function more than 2-orders of magnitude smaller—
- Larger role of convex cost vs fixed cost under the NL process
  —canonical process needs to reduce serial correlation of investment—

Non-targeted firm investment moments

		Targeting NL		Targeting AR(1)	
	Data	**NL**	**AR(1)**	**NL**	**AR(1)**
Parameters					
γ		**0.123**	0.123	0.057	**0.057**
FC_k		**0.085**	0.085	0.134	**0.134**
p_k		**0.798**	0.798	0.733	**0.733**
Non-targeted moments					
Spike rate: negative investment	0.204	0.088	0.068	0.087	0.045
Frac of obs with neg investment	0.645	0.742	0.774	0.770	0.828
Inaction rate: investment	0.057	0.026	0.002	0.009	0.001
Average investment rate	0.073	0.099	0.088	0.168	0.114
Correlation profit shocks and investment	0.100	0.364	0.418	0.314	0.418
Percentiles of investment distribution					
P_{10}	-0.301	-0.187	-0.119	-0.184	-0.117
P_{25}	-0.179	-0.118	-0.113	-0.118	-0.112
P_{50}	-0.065	-0.103	-0.107	-0.105	-0.108
P_{75}	0.061	0.006	-0.100	-0.022	-0.102
P_{90}	0.490	0.704	0.839	0.538	0.933

- Closer-to-data investment dynamics along all non-targeted moments

Conclusion

- Productivity data features non-linearity and non-normality
  - Estimated NL process captures accurately these features
  - Canonical AR(1) fails by construction
- Important implications for understanding the investment behavior
  - NL process captures
    1. the marginal propensities to invest
    2. the investment distribution
  - Canonical process does not do as well
  - NL implies different structure of adjustment costs
    - lower fixed costs and higher convex adjustment costs
  - and more accurate investment dynamics than canonical AR(1)
- Implications for the aggregate economy (next iteration of the paper)

Thank you!

Modification of ACF I

1. *First stage.* Regress y_{jht} on $\tilde{\Phi}_t(k_{jht}, l_{jht}, m_{jht})$, which is the non-parametric function $\Phi_t(k_{jht}, l_{jht}, m_{jht})$ interacted with time dummies, following Hahn et al. (2020), via OLS:

$$y_{jht} = \tilde{\Phi}_t(k_{jht}, l_{jht}, m_{jht}) + \varepsilon_{jht}.$$

2. *Modified second stage.*
   - *Obtaining residuals $\chi_t(\beta_{l,h}, \beta_{k,h})$.* Obtain an estimate of $\beta_{0,h} + \omega_{jht}$ from a given guess of $\beta_{l,h}$ and $\beta_{k,h}$ via the following equation

$$\beta_{0,h} + \widehat{\omega_{jt}(\beta_{l,h}}, \beta_{k,h}) = \tilde{\Phi}_t(k_{jt}, l_{jt}, m_{jt}) - \beta_{l,h} l_{jt} - \beta_{k,h} k_{jt}.$$

   and regress $\beta_{0,h} + \widehat{\omega_{jht}(\beta_{l,h}}, \beta_{k,h})$ on a third-order polynomial of $\beta_{0,h} + \widehat{\omega_{jht-1}(\beta_{l,h}}, \beta_{k,h})$. Get residual $\chi_t(\beta_{l,h}, \beta_{k,h})$.

Modification of ACF II

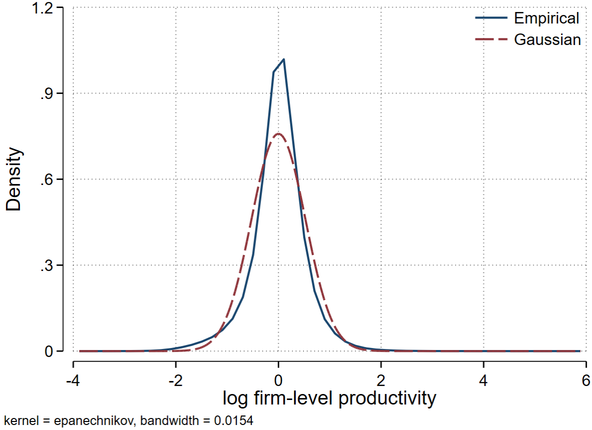
- *CUGMM.* Estimate $\beta_{l,h}$ and $\beta_{k,h}$ with the following over-identified moment conditions via CUGMM:

$$\mathbb{E}\left(\chi_t(\beta_{0,h},\beta_{l,h},\beta_{k,h}) \otimes \begin{pmatrix} 1 \\ k_{jt} \\ l_{jt-1} \\ l_{jt-2} \\ k_{jt-1} \end{pmatrix}\right) = 0. \quad (1)$$

Back

Firm-level productivity estimates

Back



- Large heterogeneity on firm-level productivity
- Subtle deviation of firm productivity distribution from log-normality

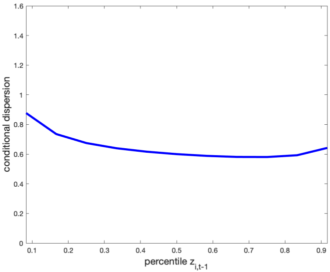
Features of productivity data

Back

Dispersion and kurtosis

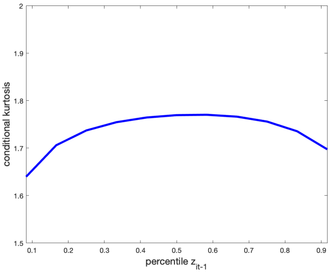
Dispersion

$$\Delta = P90 - P10$$



Kurtosis

$$\kappa = \frac{P92 - P8}{P83 - P17}$$



Model specification ▸ Back

Nonlinear productivity dynamics

Following ABB, we specify the quantile functions for the persistent and transitory components as linear combinations of bivariate (Hermite) polynomial basis functions f_k, for $k = 0, 1, \ldots,$:

$$Q_\eta(\tau | \eta_{it-1}, age_{it}) = \sum_{k=1}^{K} a_k^\eta(\tau) f_k(\eta_{it-1}, age_{it})$$

$$Q_\varepsilon(\tau, age_{it}) = \sum_{k=1}^{K} a_k^\varepsilon(\tau) f_k(age_{it})$$

$$Q_{\eta_{i1}}(\tau) = \sum_{k=1}^{K} a_k^{\eta_1}(\tau) f_k(age_{i1})$$

where age_{it} is the age of the firm at time t.

Canonical productivity process

Back

Estimation results

Table: Parameters of the linear AR(1) process

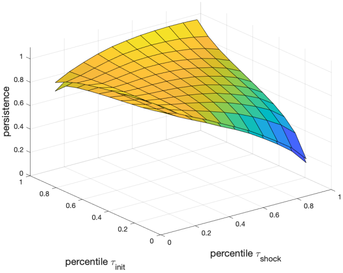
Parameter	Estimate (Std. Err.)
Autoregressive coeff. of persistent component	0.924
	(0.086)
Std. dev. of innovation to persistent component	0.231
	(0.065)
Std. dev. of the transitory component	0.223
	(0.091)
Std. dev. of the initial condition	0.608
	(0.121)

Note: We report the parameter estimates of the linear AR(1) process for productivity. Standard errors are computed via the asymptotic variance calculation. Data from the Spanish Central de Balances, from 1999 to 2017.

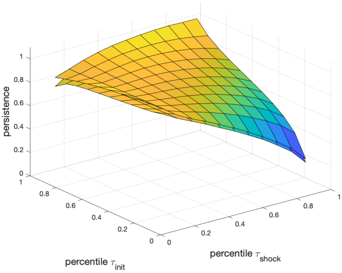
Productivity dynamics

Standard errors – persistence

Data



Nonlinear model

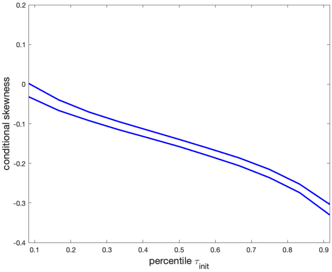


Note: These graphs are 95% confidence bands estimated from a nonparametric bootstrap with 500 bootstrap replications.

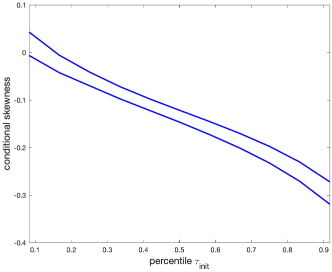
Productivity dynamics ▸ Back

Standard errors – skewness

Data



Nonlinear model



Note: These graphs are 95% confidence bands estimated from a nonparametric bootstrap with 500 bootstrap replications.

Model specification

Empirical investment policy functions

- We specify the conditional distribution of investment rates given capital and the components of productivity as the following quantile model for a given quantile τ_I:

$$i_{jt} = \sum_{k=0}^{K} a_k^I(\tau_I) f_k(k_{jt}, \eta_{jt}, \varepsilon_{jt}, age_{jt}) + \mathbf{X}_{jt}^{'} \beta(\tau_I) \qquad (2)$$

  where $f_k(\cdot)$ is a dictionary of functions.

- In practice, we use tensor products of lower-order Hermite polynomials with order (2,2,1,1).
- Controls $\mathbf{X}_{jt}$: time dummies at the sector level (allowed to be quantile specific)

Model

▸ Back

Starting with an initial guess of the investment function $\widehat{\mu}^{(0)}$, iterate from $s = 1, 2, \ldots$ until convergence of the $\widehat{\mu}^{(s)}$ process:

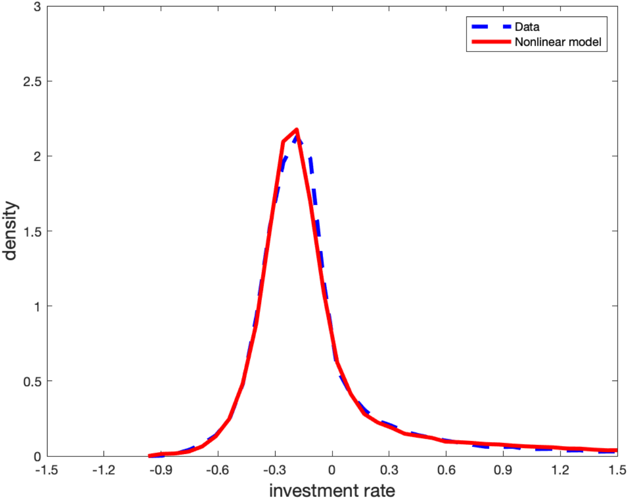
1. *Stochastic E step:* For each firm, draw observations of the persistent component $\eta_j^{(m)} = (\eta_{j1}^{(m)}, \ldots, \eta_{jT}^{(m)})$ from the posterior distribution $f(\eta_{jt} | k_{jt}, i_{jt+1}, \varepsilon_{jt}; \widehat{\mu}^s, \widehat{\theta})$. [In practice, MCMC.]
2. *M-step:* Compute:

$$\min_{a_0^l(\tau_l),\ldots,a_K^l(\tau_l)} \sum_{j=1}^{N} \sum_{t=1}^{T} \sum_{m=1}^{M} \rho_{\tau_l} \left(i_{jt+1} - \sum_{k=0}^{K} a_k^l(\tau_l) f_k(k_{jt}, l_{jt}, \eta_{jt}^{(m)}, \varepsilon_{jt}^{(m)}, age_{jt+1}) - \mathbf{X}_{jt}' \beta(\tau_l) \right)$$

   for $l = 1, \ldots, L$, where $\rho_{\tau_l}(\cdot)$ is the usual check function of quantile regression.

Investment distribution

Back

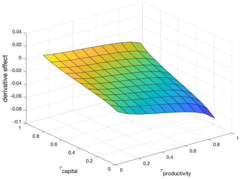


- EIPF under NL productivity process fit the investment distribution

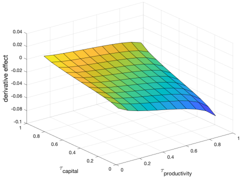
Marginal propensities to invest ▸ Back

Capital

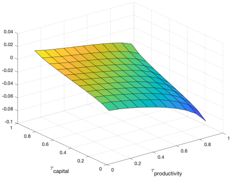
Data



Non-linear



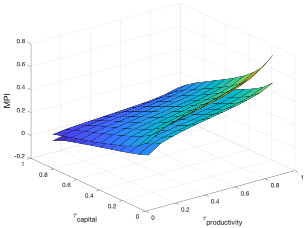
Canonical



- MPI is increasing with productivity and capital (MPI ranges from -0.08 to 0.00)
- MPI under NL productivity dynamics (-0.07 to 0.00) vs. canonical productivity dynamics (-0.09 to 0.02)

Empirical investment functions ▸ Back

Standard errors



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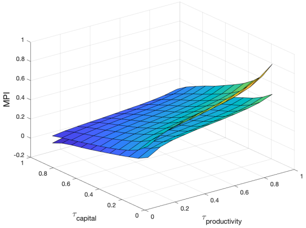
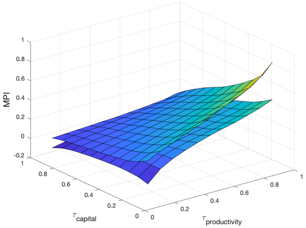


NL



Canonical

Note: 95% confidence bands from a nonparametric bootstrap with 500 replications.

Empirical investment functions

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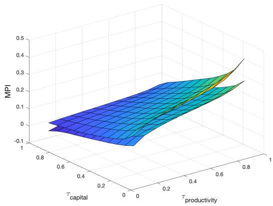
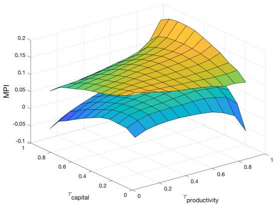
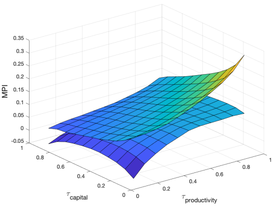
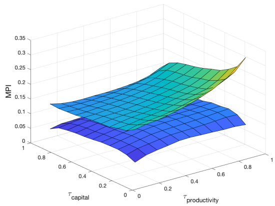
Standard errors

	Persistent	Transitory
Non-linear		
Canonical		

Note: 95% confidence bands from a nonparametric bootstrap with 500 replications.

More on estimation process ▸ Back

Table: External parameters

Parameter		Value	Target/Source
Annual discount factor	β	0.95	Cooper & Haltiwanger (2006)
Annual depreciation	δ	0.11	CBI (own calculations)
Capital share	α_k	0.3*0.85	Span-of-control parameter 0.85
Labor share	α_l	0.7*0.85	Labor share 70%

- Given these parameters, use minimization algorithm to minimize the sum of the square differences between data and model moments.
- Set hicks-neutral productivity term in each economy such that average firm size is constant across all of them.