Financial Frictions, Firm Dynamics and the Aggregate Economy:
Insights from Richer Productivity Processes

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Seminar

Date

Motivation

- The allocation of capital is not optimal
  - Firms with low/high levels of MRPK
- Dispersion in the MRPK reduces aggregate productivity
- What can prevent firms from achieving their optimal size?
  - Financial Frictions
  - Small effects in quantitative set ups
- This paper focuses on the exogenous productivity process

Why are productivity dynamics important?

- They determine the Firms Productivity Distribution
- Firms grow and decline according to their productivity dynamics
- Uncertainty in future productivity affects investment decisions
- Productivity dynamics and frictions, i.e. financial frictions, interact
  - Shock distribution determines the probability of having large productivity
  - Persistence determines the speed firms can surpass financial frictions
- Usual Assumption: Productivity dynamics follow a standard AR(1)
  - Productivity persistence and shock variability is the same for all firms
  - productivity shocks come from a Gaussian distribution

What I do - I

- What are the characteristics of the productivity process firms face?
- Use firm level data for the Spanish Economy covering 1999-2014
- A non-parametric estimation of the productivity dynamics to
  - Characterize the productivity process
  - Show in which dimensions it differs from a standard AR(1) representation

Results and Contribution - I

Results on Productivity Dynamics

- The productivity process firms face is non-linear
  - Persistence is low for low and high productivity firms
  - Shock variability is high for low and high productivity firms
- Productivity shocks are non-Gaussian
  - Shock skewness is positive for low productivity firms and negative for high productivity ones
  - Shock kurtosis is high

Contribution

- Apply recent econometric techniques to the estimation of productivity process
- Characterize the productivity process that firms face

What I do - II

- What are the effects of financial frictions on firm dynamics and the aggregate economy?
- A model of firm dynamics
  - Productivity evolves as in the data
  - Financial frictions - borrowing constraint
- The model is consistent with empirical evidence on misallocation and firm financial behavior
- Quantify the effects of financial frictions on
  - the firm life cycle
  - the aggregate economy
- Contrast the results with the standard AR(1) used in the literature

Results and Contribution - II

Results on the Effects of Financial Frictions

- The aggregate effects of financial frictions are large
  - 1/3 of the firms are constrained in their capital decision
  - Productivity losses are 16%
- The aggregate effects are smaller under an AR(1) process
  - 1/4 of the firms are constrained
  - Productivity losses are 8%

Contribution

- Show how financial frictions affect firms and the aggregate economy
- Which characteristics of the productivity process interact the most with financial frictions

Literature Review

- **Allocation of Capital**
  - Financial Frictions: Buera, Kaboski and Shin (2011, AER), Midrigan and Xu (2014, AER), Moll (2014, AER)
  - Uncertainty: Asker, Collard-Wexler and De Loecker (2014, JPE)
  - Taxonomy: David and Venkateswaran (2019, AER)
- **Firm Dynamics**
  - Model: Khan and Thomas (2013, JPE)
  - Borrowing Constraint: Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-Sanchez (2017, QJE)
- **Non-Linear Process**
  - Arellano, Blundell and Bonhomme (2017, ECTA), De Nardi, Fella and Paz-Pardo (2018, JEEA), Guvenen, Karahan, Ozkan and Song (2015, WP)
- **Firm Financial Behavior**
  - Dinlersoz, Kalemli-Ozcan, Hyatt and Penciakova (2019, WP), Graham and Leary (2011, ARFE), Lemmon, Roberts and Zender (2008, JF)

Data

▸ Variables

- Administrative data of Spanish firms, Central de Balances Integrada (CBI) hosted by Bank of Spain, from 1999 to 2014
- Representative sample (quasi universe) of Spanish firms
- 6.5 million firm-year observations from 1 million of different firms
- Complete income statement and balance sheet
- Privately-held firms from 50 sectors (2-digits) ▸ Data - I
- Representative of the whole range of the size distribution ▸ Data - II

Firm-Level Productivity

▸ Labor Share

- Cobb-Douglas production function

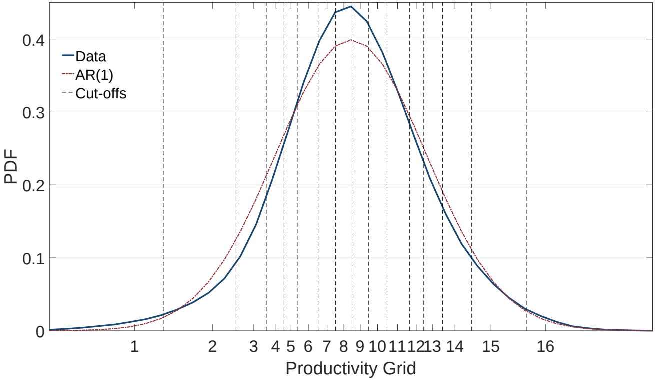
$$py_{si} = exp(a_{si})[k_{si}^{\alpha_s} l_{si}^{1-\alpha_s}]^{\eta} \qquad \alpha_s \in (0,\ 1) \text{ and } \eta \in (0,\ 1)$$

  - Variables
    - py_{si}: value added
    - k_{si}: capital (long-term assets)
    - l_{si}: labor (wage bill)
    - a_{si}: firm-level productivity
  - Parameters
    - α_s: estimated at the sector level
    - η: calibrated

Stationary Distribution

- Million of observations from firms in different sectors and years
- Homogenize sectors and years: Standardize at the sector-year level
- Discretize the distribution in 16 non-equally spaced intervals

Figure 1. Firm Productivity Distribution

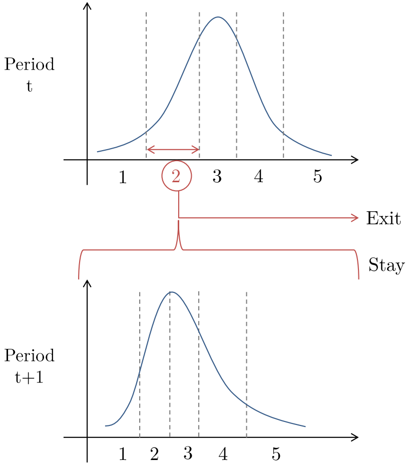


Productivity Process: An Illustration

AR(1)

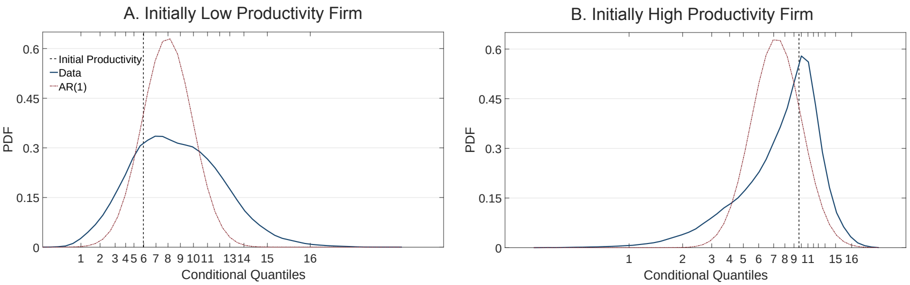
- How does firm productivity evolve?

Figure 2. Productivity Dynamics - An Illustration



Productivity Process: Estimation

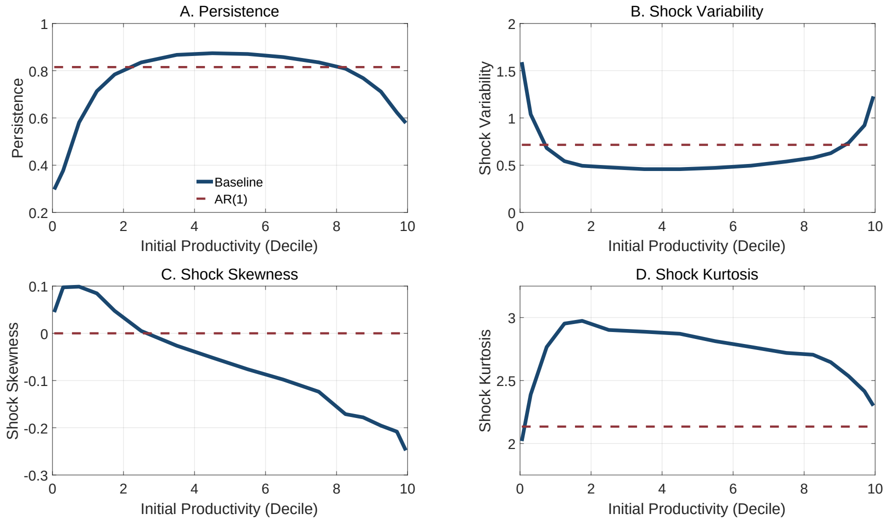
Figure 3. Conditional Firms Productivity Distribution



- Characterize the process ▸ Estimates
    - Productivity Persistence ▸ More on Persistence
    - Shock Variability
    - Shock Skewness - Asymmetry
    - Shock Kurtosis - Concentration and Tail

Estimated Productivity Process

Figure 4. Characteristics of the Productivity Process



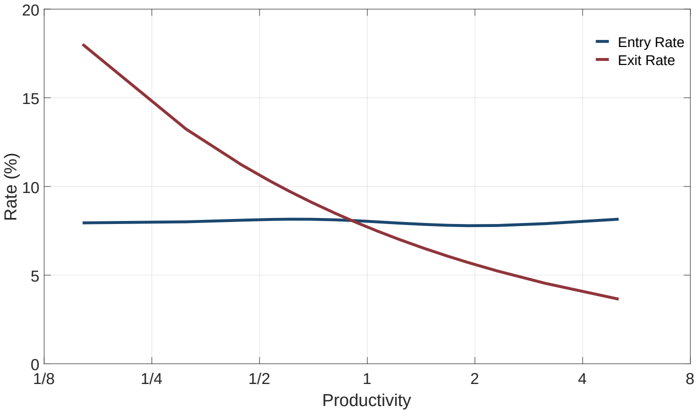
Implications of the Productivity Process

- Low productivity firms: Low persistence, high shock variability, positive skewness and larger kurtosis than AR(1)
    - Probability of having a large positive productivity shock
    - From 1st decile to above the median: 6.7% vs 1.2% in AR(1)
- High productivity firms: Low persistence, high shock variability, negative skewness and larger kurtosis than AR(1)
    - Probability of having a large negative productivity shock
    - From top decile to below the median: 7.0% vs 1.2% in AR(1)
- Good opportunities for low productivity firms arrive more frequently but, they are not long-lasting for all the firms

Estimated Entry and Exit Rates

- Exit probabilities: Fraction of firm that exit
- Entry probabilities: Fraction of firm that enter

Figure 5. Entry and Exit Rates Conditional on Past Productivity



Robustness & Discussion

Robustness

- Characterizing the productivity process ▸ Monte Carlo
- Heterogeneity ▸ Heterogeneity
- Sector heterogeneity ▸ One Sector
- Time Period ▸ Sub-Periods
- Mark Up variation ▸ Mark Ups
- Sensitivity to the DRS parameter ▸ DRS
- Higher Order Markov ▸ 2nd Order Markov

Discussion

- Other productivity process ▸ Other Process

Wrap Up

- The productivity process firms face does not follow an AR(1)
- Productivity dynamics are non-linear
- Productivity shocks are non-Gaussian

Next Questions

- What are the implications for firm dynamics?
- How does it interact with frictions, i.e. financial frictions?

Model - Firms I

- A GE firm dynamics model in steady state
- They are heterogeneous in productivity

  Productivity dynamics are stochastic: Non-Linear and non-Gaussian
- Firm exit

  Exogenous exit rates depending on firm productivity - $\vartheta(a)$
- Firm entry - $\Omega(e_0,\ a_0)$

  Marginal $\Omega(a)$ from the data

  Stochastic initial level of equity, conditional on entry productivity

$$\Omega(e|a) \sim N\Big(\mu_e + \frac{\sigma_e}{\sigma_a}\rho_{a,e}(a-\mu_a);\ (1-\rho_{a,e}^2)\sigma_e^2\Big)$$

Model - Firms II

- Produce a homogeneous good

$$py = F(k,\ l,\ a) = A_g exp(a)[k^{\alpha} l^{1-\alpha}]^{\eta}$$

- Capital is pre-determined

  Chosen before productivity is known

- Finance
  - Borrowing is limited

$$b' \leq \theta(k',\ a)k'$$

  - Borrowing vs savings vs dividend decision

$$d \equiv (1-\tau)\widehat{\pi}(k,\ a) + (1-\delta)k - b - k' + qb' \geq 0$$

- Labor

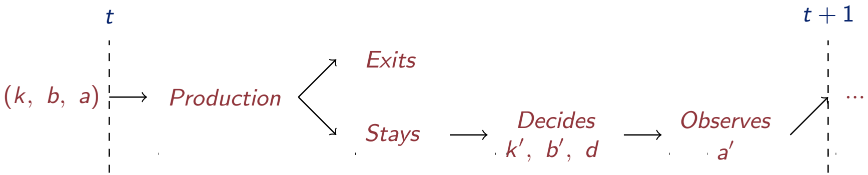
  Not subject to any market friction

- All markets are perfectly competitive

  They take prices as given

Timing - Incumbent Firm

Incumbent Firm



Firm's Problem - Incumbent Firm if Stays

$$
\begin{aligned}
V(k,\ b,\ a) = \max_{\{k',\ b',\ d\}} \quad & d\ + \\
& \beta(1-\vartheta(a))E\left[V(k',\ b',\ a')|a\right]\ + \\
& \beta(1-\vartheta(a))E\left[\tau\widehat{\pi}(k',\ a')|a\right]\ + \\
& \beta\vartheta(a)E\left[\widehat{\pi}(k',\ a') + (1-\delta)k' - b'|a\right]
\end{aligned}
$$

subject to

$$d = (1-\tau)\widehat{\pi}(k,\ a) + (1-\delta)k - b - k' + qb' \geq 0$$

$$b' \leq \theta(k',\ a)k'$$

where

$$\widehat{\pi}(k,\ a) = \max_{\{l\}}\{F(k,\ l,\ a) - wl\}$$

Timing - Entrant Firm

Household Equilibrium

Entrant Firm

$t \longrightarrow$ draw $\Omega(e_0, a_0) \longrightarrow$ decides $k', b' \longrightarrow$ observes $a' \longrightarrow (k', b', a')$ $t+1$

- Initial equity e_0
- Entry distribution: $\Omega(e_0, a_0)$
  - Marginal $\Omega(a)$ from the data
  - Conditional $\Omega(e|a) \sim N\left(\mu_e + \frac{\sigma_e}{\sigma_a}\rho_{a,e}(a-\mu_a);\ (1-\rho_{a,e}^2)\sigma_e^2\right)$
- Free Entry Condition: $EC = \int_{kxbxa} V(0, -e_0, a_0)\,\Omega(e_0, a_0)\, d\,[kxbxa]$

Calibration

▸ Borrowing Constraint ▸ Identification

- There are 11 parameters, which I calibrate to match 11 moments

Table 1. Moments of the calibration

Parameter	Value	Moment	Data	Model
η	0.83	$SD(k)$	1.79	1.76
β	0.97	K/Y	2.0	2.2
α	0.35	K/L	4.0	4.1
δ	0.05	Inv/Y	0.12	0.13
A_g	1.22	L	15.5	15.5
θ	0.81	P_{95}^{lev}	0.19	0.19
Ψ	0.48	$avg(lev)$	0.71	0.71
τ	0.43	$Profits/Y$	0.15	0.15
μ_e	1.95	k_{ent}	0.36	0.36
σ_e	1.92	$SD(k_{ent})$	0.95	0.95
$\rho_{a,e}$	0.02	$\rho(a_{ent}; k_{ent})$	0.05	0.05

Model Fit

- The model matches very well
  - untargeted moments ▸ Other Moments
  - the firm life cycle ▸ Life Cycle
  - the misallocation profiles wrt firm characteristics ▸ Misallocation
  - the financial behavior of firms ▸ Financial Behavior
- What is the effect of financial frictions on the firm life cycle?
- What are their consequences to the aggregate economy?
- How does it compare if productivity dynamics follow a standard AR(1)?

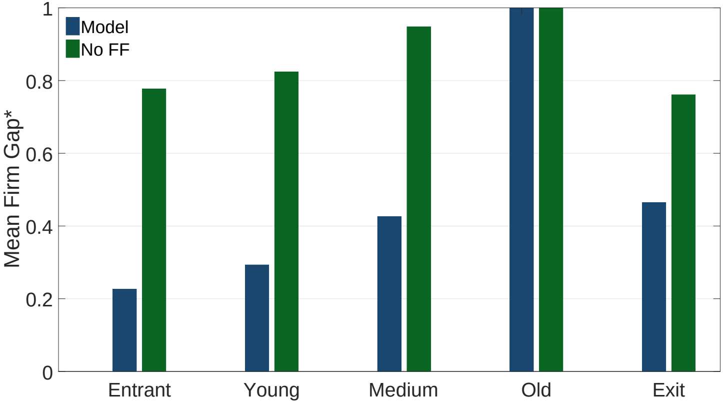
The Effects of Financial Frictions

- Isolate the effects of financial frictions and firm uncertainty
- Capital allocation of a Social Planner that abstracts from FF
- Reallocate capital following her rule

$$k_{SP} \propto E\left[a'^{\frac{1}{1-\eta(1-\alpha)}} \middle| a\right]^{\frac{1-\eta(1-\alpha)}{1-\eta}}$$

Firm Life Cycle

Figure 6. The Effects of Financial Frictions over the Firm Life Cycle



* wrt Old firms

- Entrant firms are highly constrained by financial frictions
- The gap gets reduced over the firm's life cycle, but not closed

Aggregate Consequences of FF - I

▸ Sensitivity

- 34% of the firms are constrained in their capital decision
- The aggregate consequences of FF are large
- Aggregate productivity would increase more than 16%

Table 2. Aggregate Consequences of Financial Frictions

	Benchmark
Constrained (% of firms)	34.4%
SD(log ARPK)	1.065
SD(log ARPK) No FF	0.843
Productivity Loss (%)	31.5%
Productivity Loss FF (%)	16.4%

Aggregate Consequences of FF - II

- I recalibrate the model with AR(1) using the same targets
- I redo the exercise
- The effects on firm's life cycle, fraction of constrained firms and aggregate productivity losses are larger in the Benchmark

Table 3. Aggregate Consequences of Financial Frictions

	Benchmark	AR(1)
Constrained (% of firms)	34.4%	25.4%
SD(log ARPK)	1.065	0.847
SD(log ARPK) No FF	0.843	0.684
Productivity Loss (%)	31.5%	18.6%
Productivity Loss FF (%)	16.4%	8.1%

- Robustness: SBC-Lev SBC-D/Y BC with Profits

Decomposition

- Why are the effects of FF larger under the Benchmark (8.3 p.p.)?
- There are three candidates
  - Differential Persistence (1.5 p.p.)
  - Differential Shock Variability (3.1 p.p.)
  - Non-Gaussian Shocks (4.9 p.p.)

Table 4. Decomposition of the Effects of Financial Frictions

	Constrained	PL*	PL* FF
Benchmark = (1)	34.4%	31.5%	16.4%
(1) + Gaussian Shocks = (2)	35.9%	32.6%	11.5%
(2) + Linear Productivity Persistence	26.7%	25.1%	11.2%
(2) + Linear Shock Variability	42.2%	30.0%	9.6%
AR(1)	25.4%	18.6%	8.1%

* PL: Aggregate Productivity Loss

General Equilibrium

- What are the General Equilibrium Effects of financial frictions?

Table 5. Decomposition of the General Equilibrium Effects

	Benchmark	(1)	(2)	(3)	(4)	(5)	GE
Output	100	119.5	399.9	104.1	146.7	127.1	154.1
Consumption	100	122.0	383.9	99.9	139.6	120.8	146.4
Capital	100	100.0	527.5	137.3	203.1	177.3	214.9
Labor	100	100.0	384.3	100.0	100.0	82.5	100.0
Agg. Productivity	100	119.5	93.2	93.2	114.5	117.9	117.9
Number of Firms	100	100.0	100.0	26.0	100.0	100.0	121.2
Wage	100	100.0	100.0	100.0	141.7	149.0	149.0
Reallocation	-	x	x	x	x	x	x
Capital	-	o	x	x	x	x	x
Number of Firms	-	o	o	x	o	o	x
Wage	-	o	o	o	x	x	x
Free Entry Cond.	-	o	o	o	o	x	x

Conclusion

- I have shown that productivity dynamics differ from the standard AR(1)
  - They are non-linear, and
  - productivity shocks are non-Gaussian
- I build on a model of firm dynamics with financial frictions
  - with the productivity dynamics estimated from the data
  - consistent with the evidence of the presence of financial frictions
  - consistent with firm level behavior on debt usage and leverage
- I have used the model to show
  - the effects of financial frictions over the firm life cycle
  - the large consequences of financial frictions on aggregate productivity
  - how the N-L and N-G productivity dynamics interact with financial frictions

Variable Definition

▸ Back

- Value Added: Revenue minus intermediate goods
- Capital: Long term assets
- Debt: Costly debt (long term liabilities + costly short term liabilities)
- Net Worth: Assets minus liabilities
- Wage Bill: Wages + Social Security paid by the firm
- Employment: Number of workers in full-time basis
- Profit: Profit after taxes
- Dividend: Fraction of profits devoted to the shareholders

Sample Selection - I

▸ Back

- I select no publicly listed firms (column 1)
  - Publicly listed firms represent 0.1% of total firms, and
  - around 5% of total activity in terms of value added and employment
- I select no public firms (column 2)
  - Public firms represent 0.5% of total firms, and
  - around 15% of total activity in terms of value added and 3% in terms of employment
- I select limited liability firms (column 3)
  - No limited liability firms represent 0.8% of total firms, and
  - around 3% of total activity in terms of value added and employment

Sample Selection - II

Back

- The final sample represents 98.6% of total firms
- it accounts for 74% of total value added, and
- 91% of total employment

Table 1. Sample Selection

	(1)	(2)	(3)	Total	Sample Selection
Firms	0.1	0.5	0.8	1.4	98.6
Value Added	5.1	19.9	3.0	26.0	74.0
Capital	11.5	21.8	4.3	34.3	65.7
Wage Bill	4.6	14.5	2.4	20.3	79.7
Employment	4.2	3.3	2.3	8.7	91.3
Total Assets	10.1	18.0	3.6	28.6	71.4
Net Worth	9.3	20.0	4.0	30.1	69.9

Representativeness - I

▸ Back

- Comparing the final database with the Spanish directory (tables 2 and 3)
- The selected sample covers around 50% of all the firms, and the coverage is consistent over the studied period
- In terms of employment the coverage is smaller around 30% of the total. This is due to the focus on private firms
- Regarding the firm size distribution, the coverage is consistent across size groups. It is only slightly lower for very small and large firms
- The coverage is similar in the manufacturing sector

Representativeness - II

Back

Table 2. Sample Representativeness. Aggregate

Year	Employment	Wage Bill	Firms
1999	22.2	31.9	43.1
2000	23.0	27.3	44.0
2001	24.5	44.4	45.7
2002	26.4	29.8	46.8
2003	28.8	31.0	49.2
2004	31.0	30.7	50.3
2005	32.8	32.1	51.5
2006	33.7	33.1	50.6
2007	32.3	31.8	46.2
2008	35.4	32.8	47.4
2009	34.0	30.4	46.5
2010	34.2	31.0	48.6
2011	34.7	31.7	49.0
2012	34.9	32.1	48.3
2013	35.6	32.8	47.5
2014	36.9	34.0	51.1
Average	31.3	32.3	47.9

Representativeness - III

Back

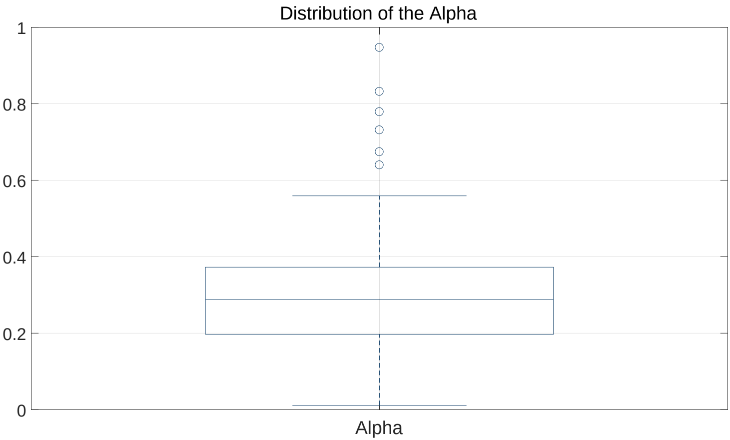
Table 3. Sample Representativeness. Firm Size Distribution

Year	1-5	5-20	20-50	50-200	+200
1999	25.8	46.1	46.2	34.8	32.0
2000	28.2	49.0	47.9	34.5	30.5
2001	31.4	50.4	55.0	35.8	30.6
2002	33.2	52.0	57.3	40.1	31.8
2003	36.1	57.0	64.5	44.6	35.7
2004	38.2	60.2	68.2	48.7	37.5
2005	39.8	64.0	70.4	50.5	40.4
2006	39.6	62.3	70.2	53.7	43.0
2007	36.3	57.8	64.5	47.2	40.4
2008	39.7	63.0	68.1	48.7	41.8
2009	40.0	59.9	64.5	46.9	51.2
2010	41.6	62.8	71.2	52.1	56.5
2011	42.3	62.6	72.1	54.0	58.6
2012	42.1	61.2	70.7	53.8	58.5
2013	40.0	63.0	75.2	57.8	56.0
2014	41.5	71.2	82.8	68.1	60.3
Average	37.2	58.9	65.5	48.2	44.1

Labor Share

▸ Back

Figure 1. Labor Share Estimates



- The unweighted average and median are 0.32 and 0.29, respectively
- The weighted average and median are 0.38 and 0.35, respectively

Estimation AR(1)

▸ Back

- I estimate an AR(1) process

$$a_{it} = \alpha + \rho_a a_{it-1} + \sigma_\varepsilon \varepsilon_{it} \qquad \varepsilon_{it} \sim N(0,\ 1)$$

- As in the N-L productivity process the η is chosen to match the SD of the size distribution in the model ($\eta = 0.78$)

Table 4. Estimation of the coefficients of the AR(1) process

η	ρ_a	σ_ε
0.75	0.8130	0.3324
0.77	0.8127	0.3350
0.78	0.8128	0.3364
0.79	0.8133	0.3378
0.80	0.8137	0.3369
0.81	0.8126	0.3361
0.82	0.8133	0.3376
0.83	0.8135	0.3392
0.85	0.8146	0.3408

Characterizing the Process - I

▸ Back

- Productivity Persistence

$$\rho(log(A_{i,t-1}),\ \tau) = \frac{\partial Q(log(A_{i,t-1});\ \tau)}{\partial log(A)}$$

τ represents the quantile of the productivity in period t, given the productivity in period $t-1$

Integrating over the productivity shock distribution

$$\rho(log(A_{i,t-1})) = E\left[\frac{\partial Q(log(A_{i,t-1});\ \tau)}{\partial log(A)}\right]$$

- Productivity Shock Variability

$$\sigma(log(A_{i,t-1})) = Q(log(A_{i,t-1});\ \tau) - Q(log(A_{i,t-1});\ 1-\tau)$$

The previous measure is valid for any $\tau \in (1/2,\ 1)$
I use $\tau = 0.75$, which corresponds to the interquartile range

Characterizing the Process - II

▸ Back

- Productivity Shock Skewness

$$sk(log(A_{i,t-1})) = \frac{Q(log(A_{i,t-1});\ \tau) + Q(log(A_{i,t-1});\ 1-\tau) - 2Q(log(A_{i,t-1});\ 0.5)}{Q(log(A_{i,t-1});\ \tau) - Q(log(A_{i,t-1});\ 1-\tau)}$$

The previous measure is valid for any $\tau \in (1/2,\ 1)$
I use $\tau = 0.75$

- Productivity Shock Kurtosis

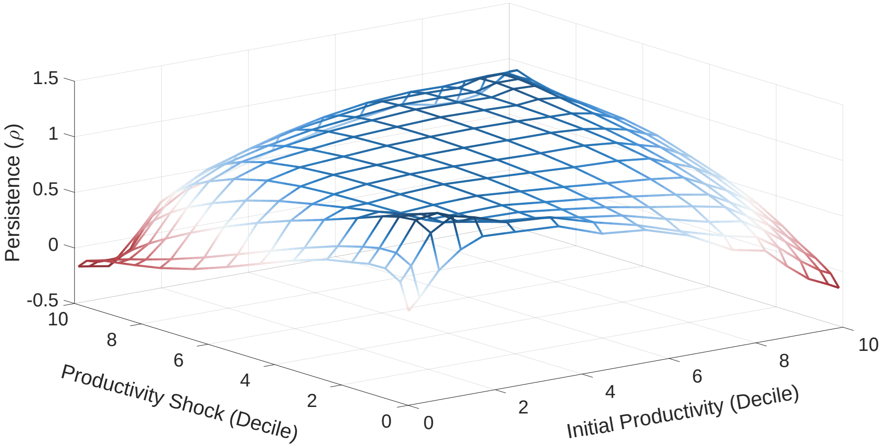
$$kur(log(A_{i,t-1})) = \frac{Q(log(A_{i,t-1});\ 1-\alpha) - Q(log(A_{i,t-1});\ \alpha)}{Q(log(A_{i,t-1});\ \tau) - Q(log(A_{i,t-1});\ 1-\tau)}$$

The previous measure is valid for any $\tau \in (1/2,\ 1)$ and $\alpha < 1 - \tau$
I use $\tau = 0.75$ and $\alpha = 0.075$

Persistence

▸ Back

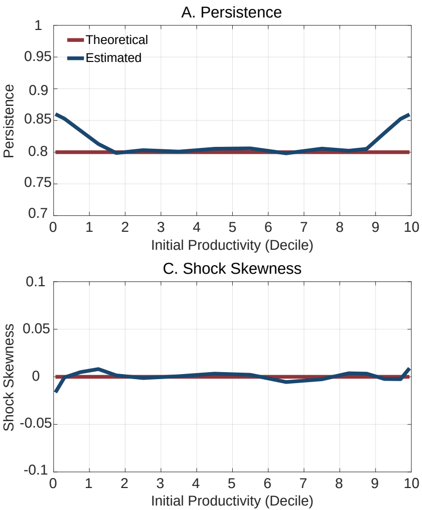
Figure 2. Productivity Persistence



Robustness - Simulation AR(1)

Back

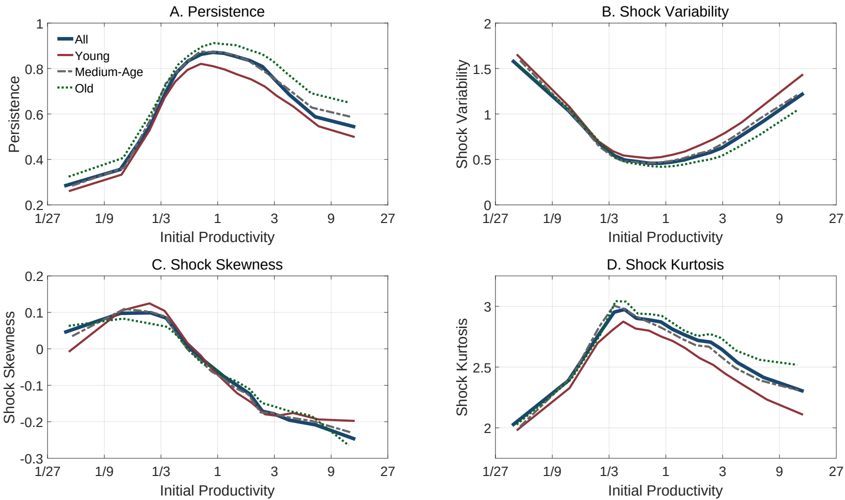
Figure 3. Monte Carlo Simulation - AR(1) Process



Robustness - Age Heterogeneity

▸ Back

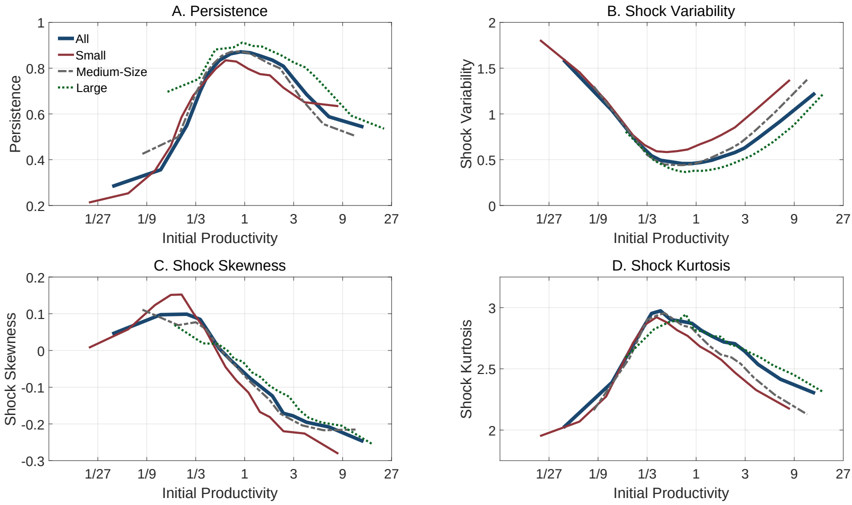
Figure 4. Productivity Process for Firms of Different Age



Robustness - Size Heterogeneity

▸ Back

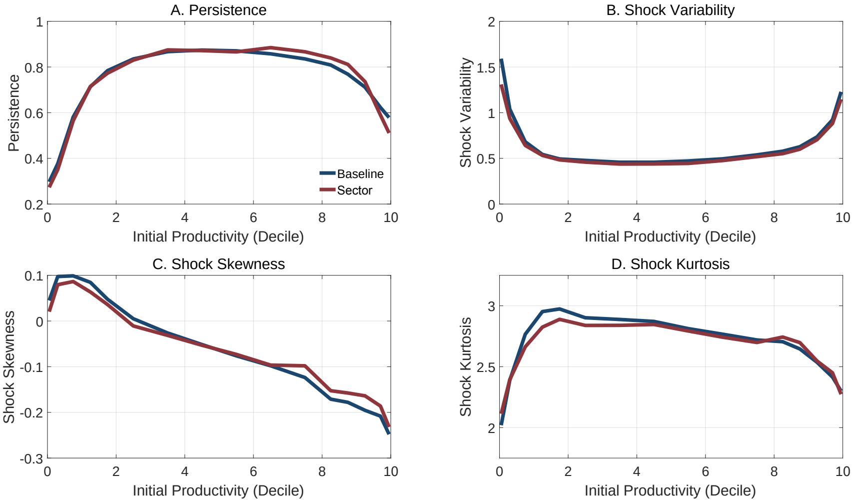
Figure 5. Productivity Process for Firms of Different Size



Robustness - Specification

▸ Back

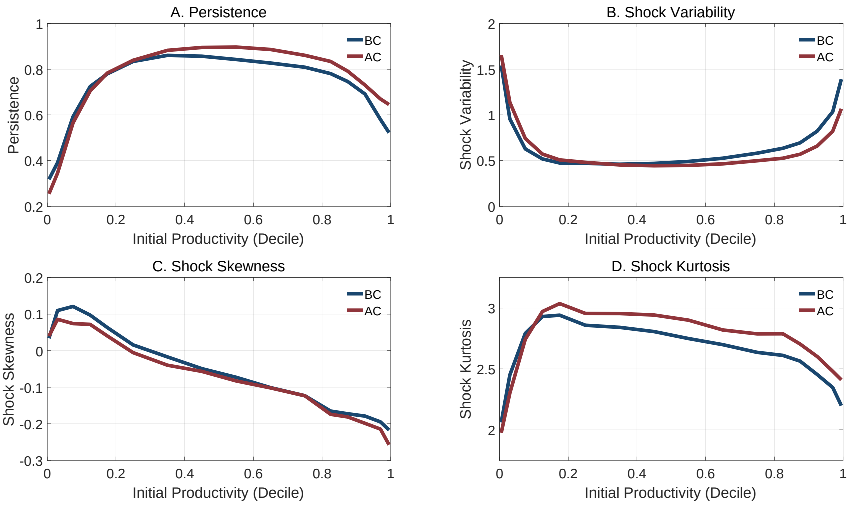
Figure 6. Productivity Process across Sectors Aggregation



Robustness - Different Periods

Back

Figure 7. Productivity Process across Time



Robustness - Mark Up Variation I

▸ Back

- I use the methodology of *De Loecker and Warzynski (2012)*
- Extend the production function to account for materials

$$py_{si} = exp(\widetilde{a_{si}}) k_{si}^{\chi_s} l_{si}^{\xi_s} m_{si}^{\zeta_s} \qquad \chi_s,\ \xi_s \text{ and } \zeta_s \in (0,\ 1)$$

  - py_{si}: revenue
  - k_{si}: capital (long-term assets)
  - l_{si}: labor (wage bill)
  - m_{si}: materials (expenditure)
  - $\widetilde{a_{si}}$: firm-level productivity

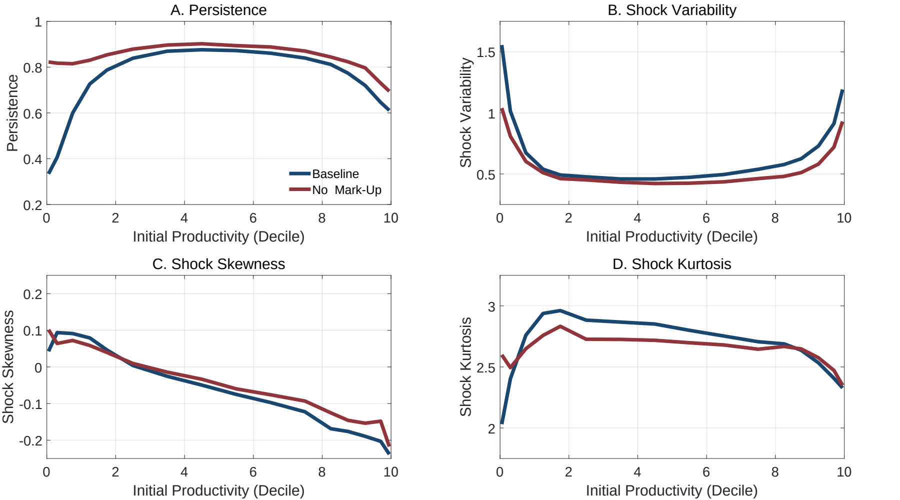
- Cost minimization implies the following mark up

$$\mu_{si} \equiv \frac{p_{si}}{MC_{si}} = \zeta_s \left[\frac{m_{si}}{py_{si}} \right]^{-1}$$

Robustness - Mark Up Variation II

▸ Back

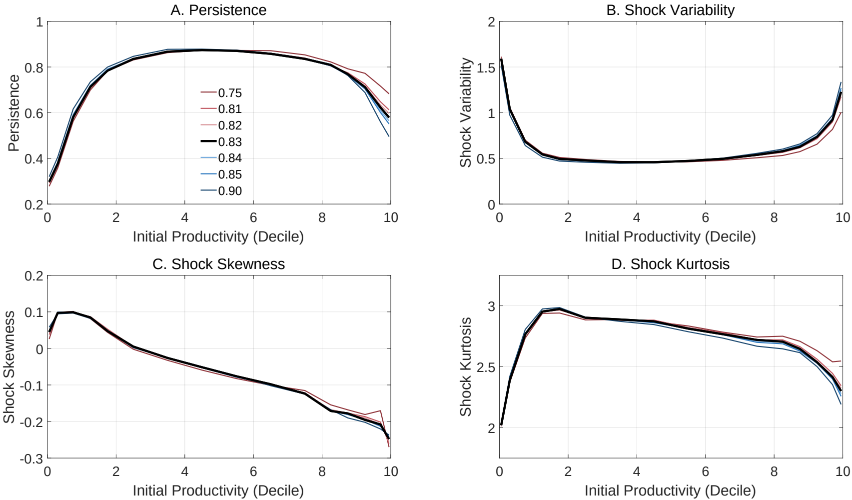
Figure 8. Productivity Process Cleaned of Mark Up Variation



Robustness - Different DRS

Back

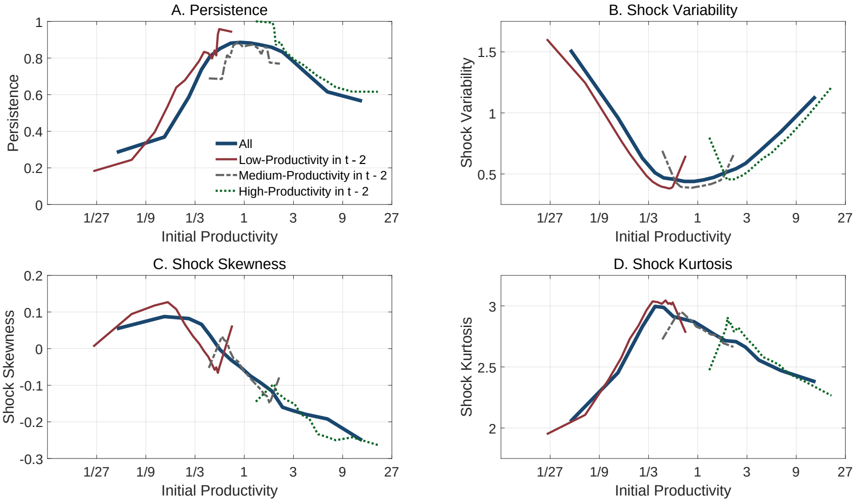
Figure 9. Productivity Process with different DRS



Robustness - 2nd Order Markov

Back

Figure 10. Productivity Process Conditional on 2-periods Productivity



Discussion - Other Process I

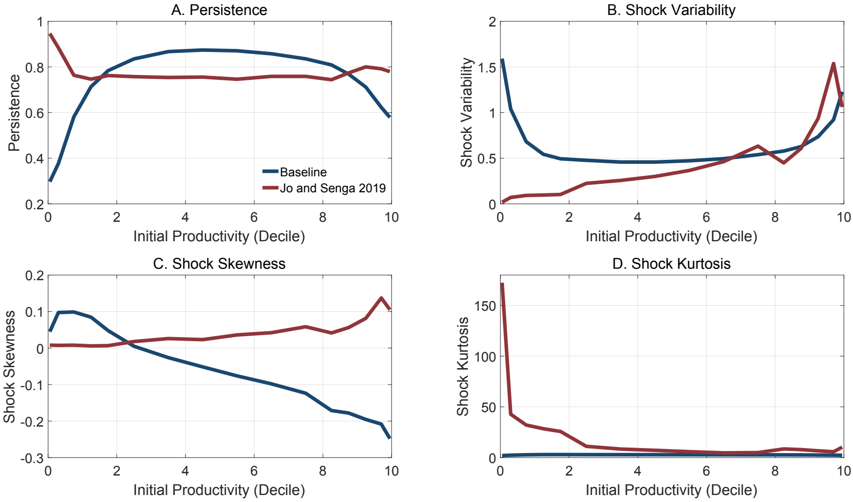
Back

- Productivity Resetting Process, i.e. *Buera, Kaboski and Shin 2011*, *Jo and Senga 2019*
  - with probability p keep your past productivity
  - with probability $1 - p$ new productivity draw
- Do the productivity dynamics generated by this process look like in the data?

Discussion - Other Process II

▸ Back

Figure 11. Characteristics of the Productivity Process in Jo and Senga 2019



Model - Representative Household

Back

- Maximize her current utility + continuation value

  Discounted flow of per period utility

- Own the Firms

  Receive the dividends paid by the firms

- Consumption

  Consume the good produced by the firms

- Finance

  Hold the bonds that provide finance to the firms

- Labor

  Provide 1 unit of labor inelastically

Household's Problem

Back

Household

$$V^h(\Lambda,\ \phi) = \max_{\{C^h,\ \Lambda',\ \phi'\}} \left\{ U\left(C^h\right) + \beta V^h(\Lambda',\ \phi') \right\}$$

subject to

$$C^h + q\phi' + \int_{k'xb'xa'} \rho_1(k',\ b',\ a')\Lambda'(k',\ b',\ a')d\left[k'xb'xa'\right] \leq$$
$$w + \phi + \int_{kxbxa} (\rho_0(k,\ b,\ a) + \tau\widehat{\pi}(k,\ A))\Lambda(k,\ b,\ a)d\left[kxbxa\right]$$

where

- $\rho_1(k',\ b',\ a')$: price (ex-dividend) of firm's shares for current period
- $\rho_0(k,\ b,\ a)$: price (dividend inclusive) of firm's shares for current period

Equilibrium

Back

A stationary recursive competitive equilibrium is a set of functions: prices $(w,\ q,\ \rho_0,\ \rho_1)$, quantities $(n,\ k', b',\ \ d,\ C^h,\ \Lambda)$, a distribution $\mu(k, b,\ a)$, a mass of firms (M) and values $(V,\ V^h)$ that

- V solve the firm's problem and $(n,\ k',\ b',\ d)$ are the associated policy functions
- V^h solves the household's problem and $(C^h,\ \Lambda)$ are the associated policy functions
- Markets clear
  - Labor market
  - Bond market
  - Stock markets
  - Good market
- The distribution of firms $\mu(k,\ b,\ a)$ is a fixed point consistent with the policy functions $(k',\ b')$, the exogenous exit rate $(\vartheta(a))$, the entry distribution $\left(\Omega(e_0,\ a_0)\right)$ and the law of motion for a

Borrowing Constraint

▸ Back

- Financial behavior asks for a size-dependent borrowing constraint as *Gopinath et al. (2017)*

$$b' \leq \theta(k', a)k' \quad \rightarrow \quad b' \leq \theta\left(\frac{k'}{k'_u(a)}\right)^{\Psi} k',$$

where $k'_u(a)$ is the optimal unconstrained level of capital

  - Pledge-ability parameter depends on the size of the firm
  - The elasticity of pledge-ability and firm size is captured with Ψ

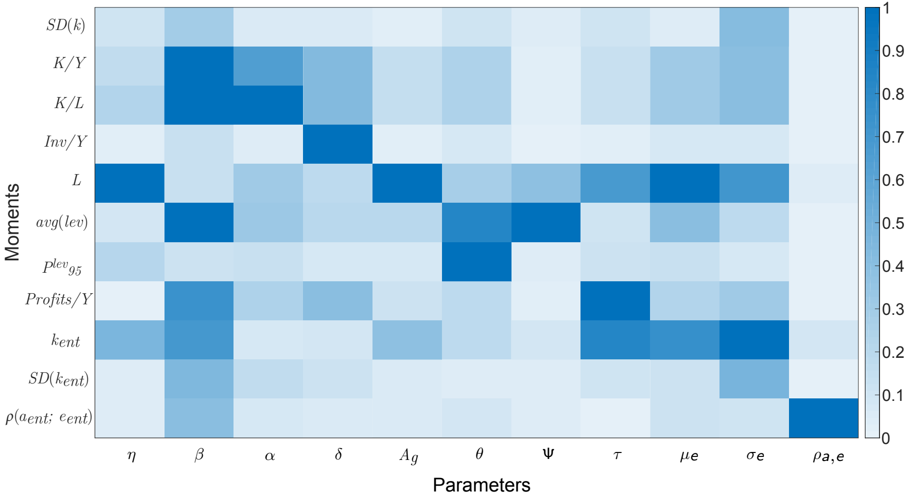
$$\Psi = 0 \quad \rightarrow \quad b' \leq \theta k',$$

  - θ is the maximum fraction of capital a firm can pledge

Identification

Back

Figure 12. Moments to Parameter Identification



Non-Targeted Moments

▸ Back

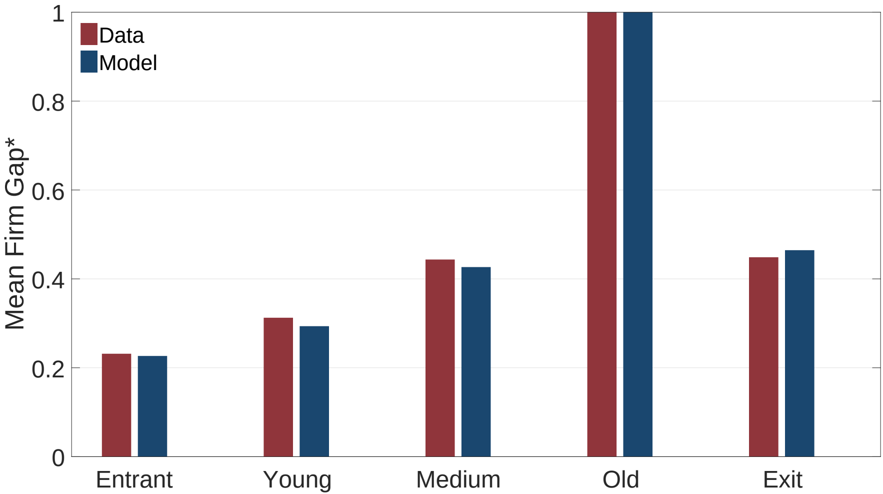
Table 5. Non-Targeted Moments

Moment	Data	Model
$Concentration_{99}(K)$	0.34	0.33
$P_{10}^{lev} \| Debt > 0$	0.03	0.08
$P_{25}^{lev} \| Debt > 0$	0.09	0.15
$P_{50}^{lev} \| Debt > 0$	0.22	0.29
$P_{75}^{lev} \| Debt > 0$	0.42	0.51
$P_{90}^{lev} \| Debt > 0$	0.61	0.67
$Debt/Y$	0.81	0.82
$Debt > 0$	0.71	0.57
$Div > 0$	0.01	0.00
Div/Y	0.14	0.00
$Med(K_{ent})$	0.08	0.08

Firm Life Cycle

Back

Figure 13. Model Fit of the Firm Life Cycle



* wrt Old firms

Misallocation

▸ Back

- Financial frictions restrict the capital decision of constrained firms
- We define Average Revenue Product of Capital (ARPK) as

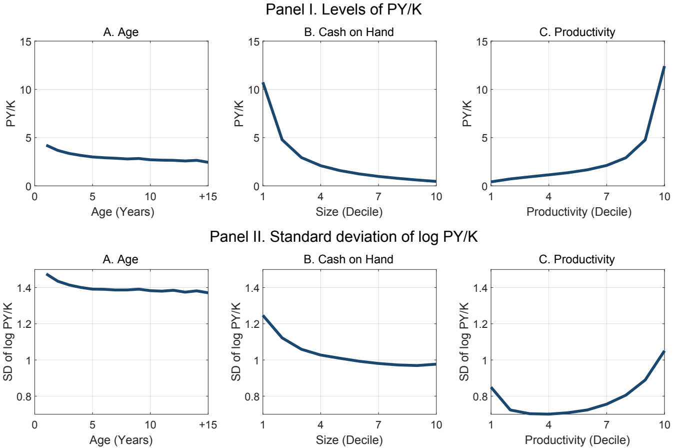
$$ARPK_{i,t} = \frac{py_{i,t}}{k_{i,t}}$$

- ARPK should be equalized across firms, under frictionless input markets
- Constrained firms have large ARPK
- Variance of log ARPK has become the standard measure of misallocation
- I look at the levels of ARPK and variance of log ARPK within sectors and firm characteristics

Misallocation - Results

Back

Figure 14. Facts on the Capital Misallocation

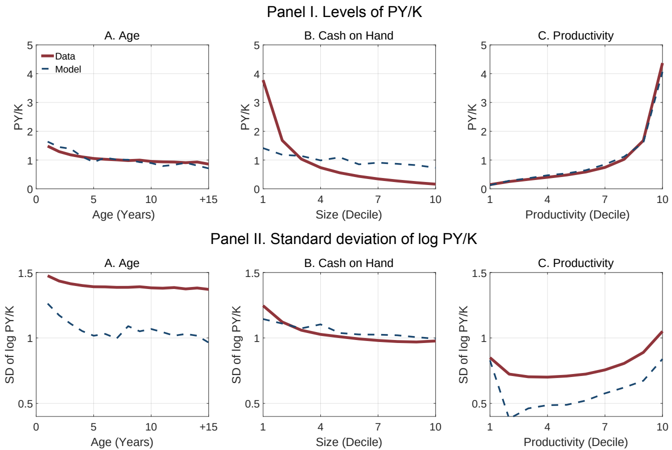


- Profiles are suggestive of the importance financial frictions
- Robustness: Measurement Error Misallocation Trend Sub-Periods

Misallocation: Data vs Model

▸ Back

Figure 15. Model Fit of Capital Misallocation

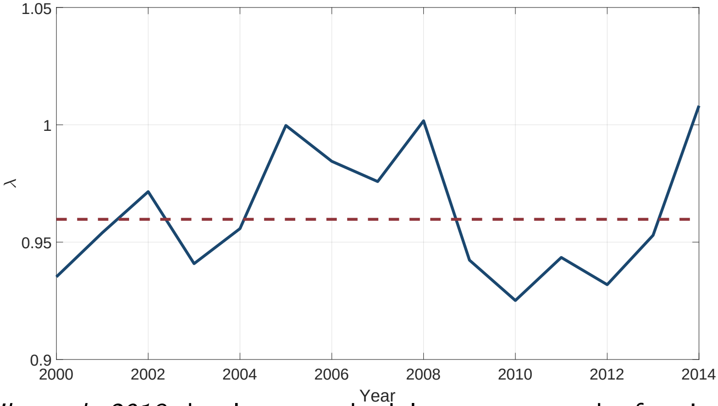


- The model matches the profiles of ARPK wrt firm characteristics
- FF generate ARPK (+) correlated with productivity and (-) with age and size

Misallocation - Measurement Error

Back Back Pres

Figure 16. Presence of Measurement Error in the Data



- *Bils et al. 2018* develop a methodology to assess the fraction of measurement error in the data
- Measurement error accounts for around 5% of total variation in ARPK
- The severity of measurement error is constant over time

Financial Side

▸ Back

- Firm level leverage (debt over total assets) varies widely across firms

Table 6. Leverage Distribution

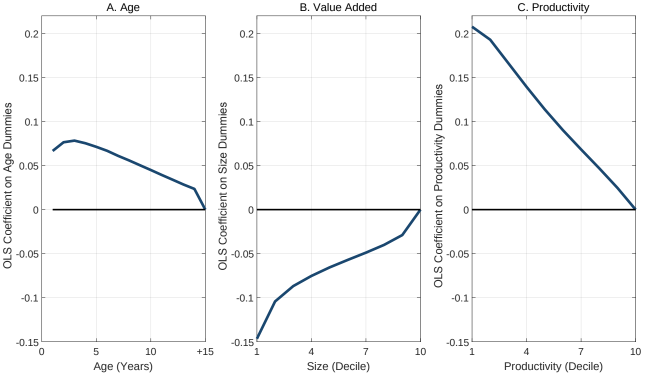
Moment	Data
Fraction with 0	0.29
Percentile $\mid$ $Debt > 0$	
$P_5^{lev} \vert Debt > 0$	0.01
$P_{10}^{lev} \vert Debt > 0$	0.02
$P_{25}^{lev} \vert Debt > 0$	0.08
$P_{50}^{lev} \vert Debt > 0$	0.22
$P_{75}^{lev} \vert Debt > 0$	0.42
$P_{90}^{lev} \vert Debt > 0$	0.61
$P_{95}^{lev} \vert Debt > 0$	0.71

- How does firm level leverage vary with firm characteristics?
- Are the patterns similar for the extensive and intensive margin?

Financial Side: Results

Back

Figure 17. Facts on Financial Behavior

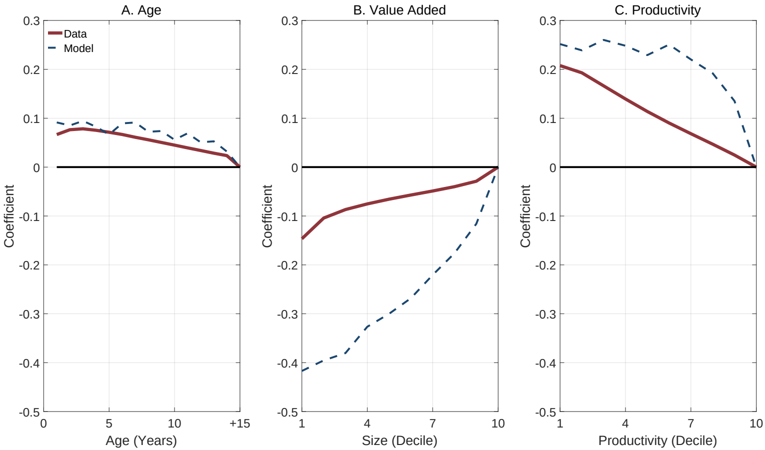


- Firm level leverage is
  - decreasing with firm's age and productivity
  - increasing with firm's size
- The same patterns arise in the extensive and intensive margin
- Robustness: Profitability Dinlersoz et al. 2019 Sub-Periods

Financial Side: Data vs Model

▸ Back

Figure 18. Facts on Financial Behavior - Extensive vs Intensive Margin

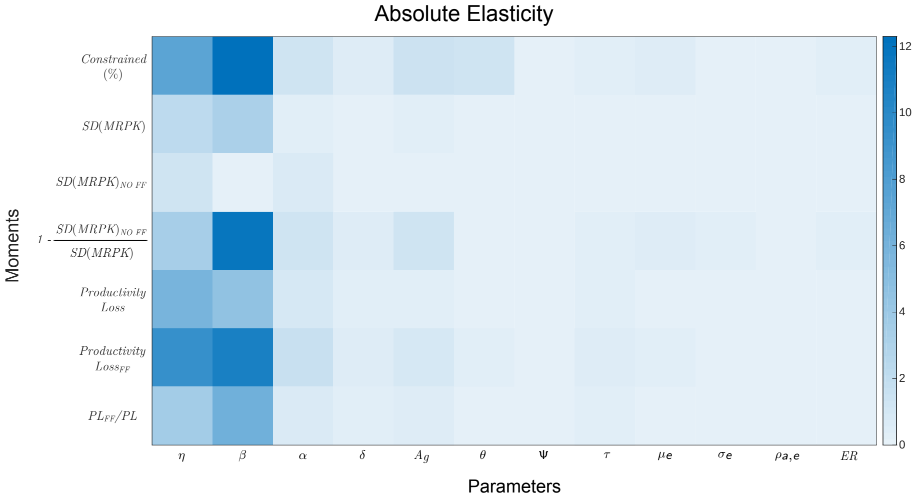


- The model captures the sign of the correlation of leverage with firm characteristics
- The elasticity of leverage wrt firm size and productivity is overstated

Sensitivity

▸ Back

Figure 19. Sensitivity of the Results to Primitive Parameters



Aggregate Consequences - SBC (Lev)

Back

Table 7. Aggregate Consequences of Financial Frictions

Moment	Benchmark	AR(1)
Constrained (% of firms)	56.5%	36.4%
SD(log ARPK)	1.081	0.817
SD(log ARPK) No FF	0.847	0.684
Productivity Loss (%)	30.6%	17.0%
Productivity Loss FF (%)	15.2%	6.3%

Aggregate Consequences - SBC (Debt/Y)

Back

Table 8. Aggregate Consequences of Financial Frictions

Moment	Benchmark	AR(1)
Constrained (% of firms)	46.2%	33.6%
SD(log ARPK)	1.025	0.796
SD(log ARPK) No FF	0.847	0.688
Productivity Loss (%)	27.6%	16.1%
Productivity Loss FF (%)	11.4%	5.2%

Aggregate Consequences - Profits BC

Back

Table 9. Aggregate Consequences of Financial Frictions

Moment	Benchmark	AR(1)
Constrained (% of firms)	58.1%	45.9%
SD(log ARPK)	1.033	0.802
SD(log ARPK) No FF	0.847	0.684
Productivity Loss (%)	26.8%	16.1%
Productivity Loss FF (%)	10.6%	5.3%