Arbitrage with bounded liquidity

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Abstract

The arbitrage gains or, equivalently, Loss Versus Rebalacing (LVR) for arbitrage between two imperfectly liquid markets is derived. To derive the LVR, I assume a quadratic trading cost to model the cost of trading on the more liquid exchange and discuss to which situations my model arguably applies well (long tail CEX-DEX arbitrage, DEX-DEX arbitrage) and to which not so well (CEX-DEX arbitrage for major pairs). I discuss extension to other cost functions and directions for future research.

Contrary to popular belief, arbitrageurs cannot trade against infinite liquidity at Binance mid price. Arbitrageurs face costs and ignoring this cost would lead to systematic overestimation of their profits. The first sentence is of course a straw man: Loss versus Rebalancing (LVR) as a metric is supposed to measure the value leakage of passive LPs on DEXes to other actors. Whether these actors are arbitrageurs, market makers, CEXes or block proposers and how the leaked value is distributed among these other actors is usually not the main question. But we should care: for one, knowing how value is distributed among DEX LPs, CEX market makers, arbitrageurs and platforms gives a more holistic picture. Second, there are equilibrium effects: in the long run the liquidity distribution across exchanges (both CEXes and DEXes) should reflect the relative gains to liquidity provision across different exchanges. If LPs on one venue lose relative to LPs on another venue, e.g. if one group internalizes the LVR that the other leaks, this should be reflected in the market evenutally. Third, the standard analysis usually mixes two effects for the reference market which are in reality co-founded but separate: on the reference market liquidity provision is active, whereas DEX LPs are largely passive *and* there is different liquidity in the two markets. It would be nice to disentangle the effect of these two assumptions.

In this note, I derive expressions for LVR (originally defined in Milionis et al. (2022)) between two imperfectly liquid markets.[1] The theory applies, in principle, to the various different cross-domain arbitrage scenarios that we care about, the case of two different AMMs, CEX-DEX arbitrage or spot-futures arbitrage. In practice, the results apply for example to CEX-DEX arbitrage (except possibly for very large market cap tickers, as I argue below) and to (non-atomic) DEX-DEX arbitrage. As we will see, the LVR is determined by the relative liquidity provision *at the margin* around the equilibrium exchange rate. This will answer the above raised points in at least two ways: First, there is a liquidity effect - everything else being equal, a more liquid exchange is subject to less LVR

[1] The theory would extend to the case of more than two venues, by treating the different external markets as one aggregate external market.

per unit of liquidity and there is re-distribution of profits between LPs on the two venue. Second, if a liquidity provider is active, he can try to anticipate the changes in equilibrium exchange rates and determine how much liquidity to provide at the margin to reduce LVR for him and increase LVR for the passive LP.

I close this note with possible extensions of the model that may or may not work and open research questions.

1 Model and Results

There are two venues where a pair of tokens A and B is traded. Let's assume B is the numéraire. Trading on the first, less liquid market follows the setting of the LVR paper, where $x^*(Q)$ denotes the equilibrium reserves of the risky-asset at exchange rate Q.

However, in contrast to the standard model, I assume that the second venue also has bounded liquidity $\tilde{x}^*(Q)$ and that trading on the second venue has a quadratic trading cost that depends on the liquidity as well as the price. More specifically, the trading cost for Δx amount of token A is

$$C(Q, \Delta x) = \frac{(\Delta x)^2}{2|\tilde{x}^{*\prime}(Q)|},$$

i.e. selling Δx amount of token A yields $Q\Delta x - C(Q, \Delta x)$ amount of token B (rather than $Q\Delta x$ if there wasn't a trading cost).

To solve the model, I need to make assumptions on the price dynamic. I assume that the equilibrium market exchange rate after the market has been arbitraged to efficiency follows a GBM

$$\frac{dQ_t}{Q_t} \equiv \sigma dB_t$$

for a Brownian motion B_t. The process, Q_t can be interpreted as the price (prediction) signal that arbitrageurs receive when making their trading decision at time t. This is the same price dynamics assumption as in the standard model, however, with additional subtleties: In a model with perfectly liquid reference market, the exchange rate on the reference market before arbitrage and after arbitrage remains unchanged. The first market is out of equilibrium and brought to equilibrium through arbitrage, the second market is in (partial) equilibrium pre and post arbitrage. In my model, both markets are out of equilibrium, pre arbitrage and arbitrageurs bring it to the correct (correctly reflecting all available information) exchange rate.

Why quadratic trading cost?

I have in mind the situation where small trades have a price impact in the second market. Cost is slippage cost, and I abstract away, as in the LVR paper from other costs (fees, spreads etc.). More precisely, we can justify the quadratic trading cost formula as follows: taking a linear approximation,

$$\tilde{x}(\tilde{Q}) \approx |\tilde{x}'(Q)|(\tilde{Q} - Q),$$

so that a trader trading Δx pays approximately

$$Q\Delta x + \int_0^{\Delta x} \tfrac{1}{|\tilde{x}'(Q)|} x dx = Q\Delta x + \frac{1}{2|\tilde{x}'(Q)|}(\Delta x)^2$$

Thus the cost of trading is

$$C(Q, \Delta x) \approx \frac{1}{2|\tilde{x}'(Q)|}(\Delta x)^2.$$

Two interpretations come to mind:

CEX with bounded liquidity

The model approximates CEXes with bounded liquidity. Practically, we can infer the slope $|\tilde{x}'(Q)|$ from order book data, by running a regression around market mid price. Linear marginal cost is a good approximation for small trade sizes in a limit order book, in some situations. Empirically the approximation tends to work better for smaller market cap pairs, partially for somehow idiosycratic reasons: Min tick sizes tend to be more granular for these pairs, whereas for large cap pairs, such as ETH/USDT and BTC/USDT on Binance, tick sizes are relatively large (1 cent). This leads to the effect that a lot of liquidity is concentrated at the top of book and liquidity is sparser elsewhere (for adverse selection effects). Thus, for these token pairs, a more reasonable model of cost is to assume constant rather than linear marginal cost up to a certain trade size and non-constant cost above that. I will briefly discuss why the results would fail in this case, in Section 2.

Price discovery on a DEX

Suppose market 2 is a DEX which operates through a CPMM. Then the trading cost if the marginal price is given by Q is given by

$$C = Q\Delta x - \Delta y$$

where

$$(\tilde{x}^*(Q) + \Delta x)(\tilde{y}(Q) - \Delta y) = \tilde{x}^*(Q)\tilde{y}(Q) \Rightarrow \Delta y = \frac{\tilde{y}(Q)\Delta x}{\tilde{x}(Q) + \Delta x}.$$

Thus

$$C = Q\Delta x - \frac{\tilde{y}(Q)\Delta x}{\tilde{x}(Q) + \Delta x} = \frac{Q(\Delta x)^2 + Q\tilde{x}(Q)\Delta x - \tilde{y}(Q)\Delta x}{\tilde{x}(Q) + \Delta x} = \frac{Q(\Delta x)^2}{\tilde{x}(Q) + \Delta x} \approx \frac{Q(\Delta x)^2}{\tilde{x}(Q)}.$$

The last approximation is good, as long as the trade size Δx is small relative to the pool reserves $\tilde{x}^*(Q)$. Put differently, for $\tilde{K} := \sqrt{\tilde{x}(Q) * \tilde{y}(Q)}$

$$|\tilde{x}'(Q)| = \frac{\tilde{K}}{2Q^{3/2}} = \frac{\tilde{x}(Q)}{2Q} \Rightarrow C \approx \frac{1}{2|\tilde{x}'(Q)|}(\Delta x)^2 = \frac{Q}{\tilde{x}(Q)}(\Delta x)^2.$$

Calculating LVR

We define LVR, as in the standard framework, as the difference in value between the portfolio value and the rebalancing strategy:

$$LVR_T := R_T - V_T.$$

The rebalancing strategy is now however subject to trading costs, so that

$$R_T = V_0 + \int_0^T x^*(Q_s)dQ_s - \int_0^T C(dx^*(Q_t)).$$

Let's replicate the LVR calculation from the proof of Theorem 1 in .

With N arbitrageurs the arbitrage gains on the time interval $[0, T]$ is given by

$$ARB_T^{(N)} = \sum_{i=1}^{N} Q_{t_{i-1}}(x^*_{t_{i-1}} - x_{t_i}) - (y^*_{t_{i-1}} - y^*_{t_{i-1}}) - \frac{1}{2\tilde{x}'_{t_{i-1}}}\left(x^*_{t_{i-1}} - x^*_{t_i}\right)^2,$$

where the expression is the same as in except for the last term which is the quadratic trading cost. When going to the limit, the first two terms become (as in the calculation in)

$$\lim_{N\to\infty}\sum_{i=1}^{N} Q_{t_{i-1}}(x^*_{t_{i-1}} - x^*_{t_i}) - (y^*_{t_i} - y^*_{t_{i-1}}) = Q_0 x^*(Q_0) - y^*(Q_0) + \int_0^T |x^*(Q_t)| dQ_t - (Q_T x^*(Q_T) - y^*(Q_T))$$

$$\equiv V_0 + \int_0^T |x^*(Q_t)| dQ_t - V_T$$

For the third term we get the following limit:

$$\lim_{N\to\infty}\sum_{i=1}^{N} \frac{1}{2\tilde{x}'_{t_{i-1}}}\left(x^*_{t_{i-1}} - x^*_{t_i}\right)^2 = \int_0^T \frac{1}{2\tilde{x}^{*\prime}(Q_t)}(d(x^*(Q_t)))^2$$

It follows that

$$ARB_T = V_0 + \int_0^T |x^*(Q_t)| dQ_t - \int_0^T C(dx^*(Q_t)) - V_t = R_t - V_T = LVR_T$$

By Ito's formula, as $dt^2 = 0$, $dtdB_t = 0$ and $dB_t^2 = dt$, we obtain

$$d(x^*(Q_t))^2 = \sigma^2 Q_t^2 (x^{*\prime}(Q_t))^2 dt.$$

Thus the cost term is

$$\int_0^T \frac{\sigma^2 Q_t^2}{2\tilde{x}^{*\prime}(Q_t)}(x^{*\prime}(Q_t))^2 dt$$

and we obtain the adjusted formula for the arbitrage gain:

$$ARB_T = V_0 - V_T + \int_0^T |x^*(Q_t)| dQ_t - \int_0^T \frac{\sigma^2 Q_t^2}{2\tilde{x}^{*\prime}(Q_t)}(x^{*\prime}(Q_t))^2 dt.$$

By Ito's lemma and using the Envelope theorem:

$$V_T - V_0 = \int_0^T V'(Q_t) dQ_t + \int_0^T \tfrac{1}{2} V''(Q_t)\sigma^2 Q_t^2 dt = \int_0^T x^*(Q_t) dQ_t + \int_0^T \tfrac{1}{2} x^{*\prime}(Q_t)\sigma^2 Q_t^2 dt$$

We get the following adjustment of the LVR formula:

Proposition 1. *For the bounded liquidity case, with liquidity $\tilde{x}^*(Q)$ of the risky asset in the more liquid market, LVR takes the form,*

$$LVR_T = \int_0^T \ell(\sigma, Q_t) dt$$

where

$$\ell(\sigma, Q) := \tfrac{\sigma^2 Q^2}{2}\left(1 - \tfrac{|x^{*\prime}(Q)|}{|\tilde{x}^{*\prime}(Q)|}\right)|x^{*\prime}(Q)|.$$

Corollary 1. *In case the two market use CPMMs, the formula simplifies to*

$$LVR_T = \int_0^T \ell(\sigma, Q_t, r_t)dt$$

with

$$\ell(\sigma, Q, r) := \tfrac{\sigma^2 Q^2}{2}(1 - \tfrac{1}{r})|x^{*\prime}(Q)|,$$

where

$$r_t \geq 1$$

is the relative liquidity in the two markets.

Interpretation and Discussion

What does Proposition 1 tell us? Let's first look at the DEX-DEX case and normalize LVR by unit of liquidity provided. For the case of two CPMMs the LVR is:

$$\frac{\ell}{V} = \tfrac{\sigma^2}{8}(1 - \tfrac{1}{r}).$$

Assuming that both DEXes use the same liquidity curve and that fee income per unit of liquidity provided is comparable between the two DEXes, the returns to liquidity provision are higher on the more liquid DEX. Thus, in the long run we would expect liquidity to concentrate in one DEX. Of course this is an artifact of the particular liquidity curve. The liquidity effect on LPing is implicitly a slippage effect: A relatively larger fraction of the liquidity is provided around the equilibrium price in the less liquid market, such that the effect of adverse selection is more severe. But that is just the case if we make a market by committing to the constant product curve. Ideally, the less liquid market would quote less aggressively.

Proposition 1 tells us, moreover, that there are gains to be made from active market making. This is, of course, not surprising at all qualitatively, but we get a quantitative estimation of this effect. What determines the relative LVR in the two exchanges is the relative marginal liquidity provided in the two exchanges at equilibrium price:

$$\frac{|x^{*\prime}(Q)|}{|\tilde{x}^{*\prime}(Q)|}.$$

If LPs actively manage liquidity by trying to predict the evolution of equilibrium exchange rates and react to it by adjusting supply, they are naturally better off compared to an LP that commits to a supply curve x^* and that independently of how much capital they can deploy.

2 Extensions & Open Research Questions

Other Cost Functions

The previous results are dependent on the shape of the cost function. As discussed previously, for high volume tickers on Binance, linear marginal cost is a bad approximation and we should work with the following model instead:

$$C'(\Delta x) = \begin{cases} c, & \Delta x \leq \alpha, \\ \frac{\Delta x - \alpha}{|x'(Q)|} + c, & \Delta x > \alpha, \end{cases}$$

for constants $\alpha > 0$ and $c > 0$. At this point our model assumption of a diffusion process becomes crucial and we would obtain quite different results otherwise. This is because for the GBM model, almost surely all arbitrage trades are "small" i.e. they are below the α threshold so that we can work with a constant marginal cost assumption. If we would literally follow the logic of the LVR calculation, instead of quadratic variation, we would need to look at the absolute variation for the cost,

$$\lim_{N\to\infty} \sum_{i=1}^{N} c \left| x^*_{t_{i-1}} - x^*_{t_i} \right|$$

which diverges. Hence, the result does not extend. Of course, arbitraging not being worthwhile directly contradicts reality. The problem is that immediately arbitraging is not optimal in the model. So we need a different model, which is a possible direction of future research.

Finite Slot Times

The LVR framework has been extended to the case of fees, where arbitrageurs are restricted to trade on chain at discrete (possibly random) slot times (Milionis et al., 2024; Nezlobin and Tassy, 2025). It would be relatively straightforward to extend their analysis by introducing a slippage cost while maintaining the no-spread assumption for the more liquid market. However, this is arguably not a reasonable model: ideally we would have a model that would have fees/spreads in both markets between which the arbitrageur arbitrages. Such extensions, seems however non-trivial to obtain. This is, at least partially, because there is no notion of a single equilibrium exchange rate any longer. Rather there is a corridor of exchange rates where there is no profitable arbitrage. Coming up with a tractable model and characterizing arbitrageur's profit in the model is an interesting open question.

Empirical Research

Realistic estimates of arbitrageur as well as market maker P&L and cost of liquidity are arguably largely missing from the DeFi literature. Existing literature seems to systematically overestimate the profits of arbitrageurs. Thus, it would be interesting to construct a reasonable multi-factor model of cost for these actors and to estimate it from public data.

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