A math problem at the VMEC magnetic axis

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Background

The W7-X stellarator is optimized to have very small equilibrium currents (Phirsh-Schlüter and Bootstrap) and thus the distortion to the flux surfaces are expected to be small as the plasma pressure (beta) increases. Because of the minimization of the currents, magnetic diagnostics for equilibrium reconstruction need to resolve very small signal levels. The synthetic diagnostic responses from the V3FIT/VMEC/V3RFUN solutions exhibit a sensitivity to a residual numerical non-physical VMEC on-axis current density which is present even in vacuum cases. This numerical artifact in the VMEC calculation leads to a large numerical error / offset in the synthetic diagnostic response on the same order as, or larger than, the actual expected plasma response. This numerical artifact manifests as a rotating dipole current density near the magnetic axis. The numerical error can be minimized at least two methods. The first method is to increase the total number of radial surfaces (while trying to minimize the achievable FTOL). The second method is to modify the radial grid spacing with the 'APHI' VMEC parameter. The error can be reduced, but not eliminated, and comes with the cost of increased computational resources (time and memory).

References

- **VMEC**: "Steepest-descent moment method for three-dimensional magnetohydrodynamic equilibria", Hirshman, S.P. and Whitson, J.C., Phys. Fluids, **26** (1983).
- **V3FIT**: "V3FIT: a code for three-dimensional equilibrium reconstruction" J. D. Hanson, S. P. Hirshman, S. F. Knowlton, L. L. Lao, E. A. Lazarus, and J. M. Shields, Nuclear Fusion, **40** (2009).

V3FIT: An iterative 3-D MHD equilibrium reconstruction

Initial guesses for
1. **Pressure profile**
2. **Current profile**
3. **Enclosed toroidal flux**

Coil set description
Stellarator symmetric
Coil currents are corrected to account for E-M deflections

MHD Calculation
VMEC
Free-boundary
MPOL = 12
NTOR = 12
NTHETA = 36
NZETA = 36
NS = 401
FTOL = 1e-14

Synthetic Diagnostics and Constraints, $S_i^M(\boldsymbol{p})$ and σ_i^S
Magnetic
No diamagnetic or compensation flux loops
Saddle Flux Loops
Rogowski coils (complete)
Rogowski coils (segmented)

Last closed flux surface
Contact with the divertor is a (weighted) target for the position of last closed flux surface

Experimental Data, S_i^O
Magnetic, 5% Arb Uncertainty

Evaluate

$$\chi(\boldsymbol{p})^2 = \sum_i W_i \left(\frac{S_i^O - S_i^M(\boldsymbol{p})}{\sigma_i^S}\right)^2$$

Check
1. $\chi(\boldsymbol{p}) < \chi_{target}$
2. Maximum iteration count reached
3. Some other exit condition reached?

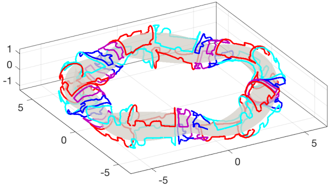
No

Adjust profiles, toroidal flux
Iterative step chosen based on SVD analysis of Jacobian (finite differences).

Yes

Reconstruction complete.
'Best fit' Pressure profile, current profile, and enclosed flux
Post-processing

Magnetic diagnostics are located behind the heat-shield, close to the plasma.



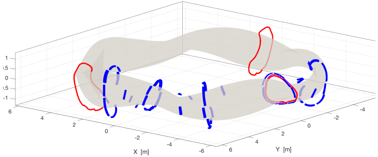


Figure 1: Top: 8 types of saddle coils for each of the 5 field periods. The plasma LCFS is shown in grey. Bottom: Segmented Rogowski coils (in blue) measure $\int \mathbf{B} \cdot dl$ and provide good poloidal coverage over 2 field periods.

Experimental Uncertainties (Arb.Units, $\sim$ Wb, T)
Saddle Loop Type 1: 4e-5
Saddle Loop Type 2: 5e-5
Saddle Loop Type 3: 7e-5
Saddle Loop Type 4: 2.5e-5
Segmented Rogowski Coils: 1e-5
Rogowski Coils: 5e-5

Computational Grids

MGRID - Magnetic 'lookup' table for fast vacuum field calculations

- 80 toroidal planes/Field Period
- 1 cm grid size in R-, Z- directions

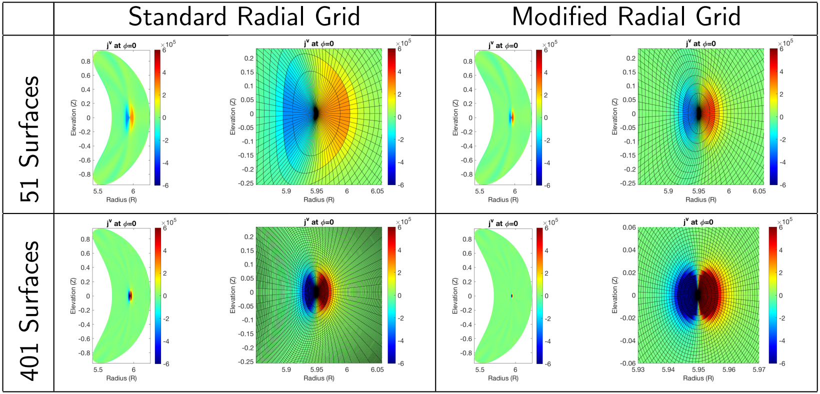
VMEC

- NTOR = 12 (Toroidal modes in spectrum -12 .. 12)
- MPOL = 12 (Poloidal modes in spectrum 0 .. 11)
- NS = 401 (Radial 'grid points')
- FTOL = 1e-14 (Residual force balance)
- NTHETA = 36 (Internal VMEC grid points - poloidal)
- NZETA = 80 (Internal VMEC grid points - toroidal)
- APHI = 0.05, 1, -0.05 (radial grid spacing, default: APHI = 1)

Effects of *APHI* on VMEC output

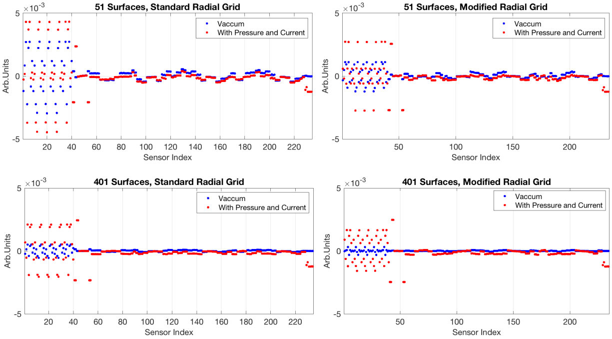
- The computational grid is linear in 's'
- APHI: $\Phi_N \equiv \phi/\phi_{LCFS} = a(0)\cdot s + a(1)\cdot s^2 + a(2)\cdot s^3 + ...$
  - Default: $\Phi_N = \phi/\phi_{LCFS} = 1\cdot s$
  - Modified: $\Phi_N = \phi/\phi_{LCFS} = 0.05\cdot s + 1\cdot s^2 - 0.05\cdot s^3 + ...$
- Current densities are modified (warning: check the usage of 2π)
- $jcuru = \frac{1}{2\pi} d(I_{pol})/ds = \frac{1}{2\pi} d(I_{pol})/d\Phi_N * d\Phi_N/ds$ and $jcurv = \frac{1}{2\pi} d(I_{tor})/ds = \frac{1}{2\pi} d(I_{tor})/d\Phi_N * d\Phi_N/ds$
- $d(I_{pol})/d\Phi_N = 2\pi\cdot jcuru/(d\Phi_N/ds)$ and $d(I_{tor})/d\Phi_N = 2\pi\cdot jcurv/(d\Phi_N/ds)$
  - Default: $d\Phi_N/ds = 1$
  - Modified: $d\Phi_N/ds = 0.05 + 2\cdot s - 0.15\cdot s^2$
  - General: $d\Phi_N/ds = a(0) + 2a(1)\cdot s + 3a(2)\cdot s^2 + ...$
- $I_{tor} = \int d\Phi_N(dI_{tor}/d\Phi_N) = 2\pi \int d\Phi_N(jcurv/(d\Phi_N/ds)) = 2\pi \int ds(jcurv)$
- $r_{eff} = Aminor * \sqrt{\Phi_N}$; $Area = pi * r_{eff}^2 = \pi * Aminor^2 * \Phi_N$; $\Phi_N = (\phi/\phi_{lcfs}) = Area/(\pi * Aminor^2)$
- How best to display radial profile?
  - $d(I_{tor})/d(Area) = (dI_{tor}/ds)\cdot(ds/d\Phi_N)\cdot(d\Phi_N/dArea) = 2\pi\cdot(jcurv)\cdot(ds/d\Phi_N)/(\pi * Aminor^2) = 2(jcurv)/(Aminor^2 * d\Phi_N/ds)$
  - $d(I_{tor})/d(r_{eff}) = (d(I_{tor})/d(Area))\cdot(d(Area)/d(r_{eff}) = 2(jcurv)/(Aminor^2 * d\Phi_N/ds)\cdot 2\pi r_{eff}$
  - $I_{tor} = \int_0^{A_0} dr_{eff}(4\pi r_{eff}(jcurv)/(Aminor^2 * d\Phi_N/ds)$

A residual dipole current density exists near the magnetic axis - Rotates poloidally 1/field period



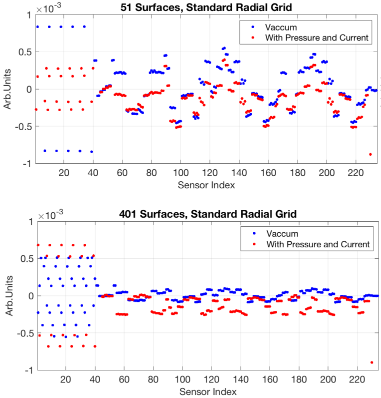
- Free-boundary Vacuum VMEC solution ($PRES\_SCALE = 0, ITOR = 0$)
- Increasing the number of surfaces and adjusting the grid spacing both reduce the residual on-axis current density and vacuum magnetic 'fingerprint'.
- Note: The plot limits are reduced for the 4th panel

Synthetic Magnetic Sensor Response is sensitive to the residual numerical current density - At vacuum and with finite beta, current.



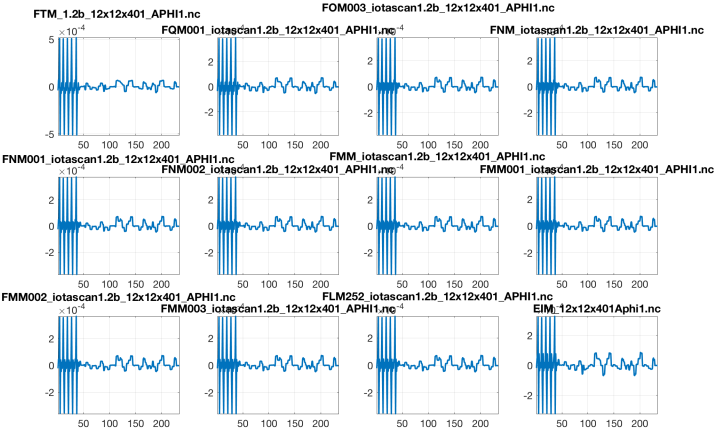
- Finite beta solution example has $\beta_{vol.ave.} = 0.2\%$, $\beta_{peak} = 0.4\%$ and $I_{tor} = 2kA$.
- Each diagnostic signal is adjusted/offset by an amount equal to its residual synthetic response.
- The adjusted signal is used as the target value for the reconstructions.

Synthetic Magnetic Sensor Response detects residual numerical current density - Even with finite beta, current



- Saddle coils of Type 3 (cyan) are the most sensitive.
- High resolution required for magnetic diagnostics. Other diagnostic systems (spectroscopy) can use much lower VMEC grid resolutions for reconstruction.

Synthetic Magnetic Response changes with vacuum field Response Arrary v. Configuration (Iota Scan)



The numerical artifact rotates with toroidal angle.

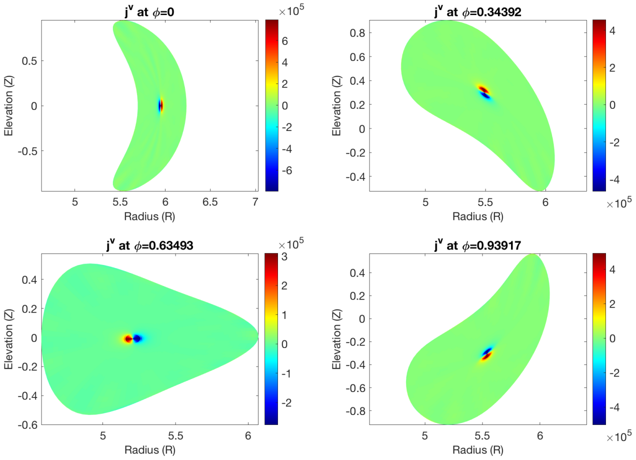


Figure 2: Dipole rotation of residual on-axis current density