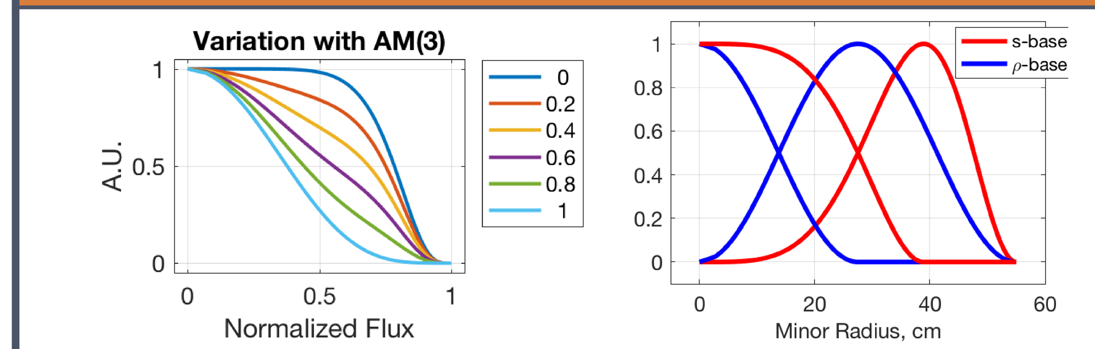
Abstract

The present status of the V3FIT code [1] for the free-boundary reconstruction of the magneto-hydrodynamic (MHD) equilibrium of W7-X plasmas with finite plasma pressure and toroidal current are presented. The reconstruction of the equilibrium is a vital tool for fusion experiments to model and interpret diagnostic signals. The iterative procedure involves solving the MHD equilibrium, calculating synthetic diagnostic signals and comparing the signals to those from the experiment. The parameters that describe the plasma position, boundary location and internal pressure and current profiles are adjusted until a 'best-match' is found between the synthetic signals and measured signals. Profile and parameter uncertainties are based on a best-linear estimate. The reconstruction output includes the shape and location of the plasma boundary, and profile information of the plasma parameters. This set of information is then used to interpret diagnostic signals and can be used for further analysis of the equilibrium state. Typically, the MHD solution is provided by VMEC [2], which only permits nested closed flux surface. The SIESTA code [3], which is already coupled with V3FIT, permits islands to be present in the free-boundary MHD solution.

Pressure and Current Profiles



Pressure profiles: Either a **'power series'** or a **'two-two-power'**:
$\frac{P(s)}{P_0} = \left((1 - AM_3)\left(1 - s^{AM_1}\right)^{AM_2} + AM_3\left(1 - s^{AM_4}\right)^{AM_5}\right)$

Current density: either a **'power series'** or ...

- Radial profile is specified by an analytic 'sum of cos^2' profiles with $AC_0 = N_{cssq}$ independent radial 'zones'.
- Current profile shape is normalized by the net toroidal current carried by the plasma. $CURTOR = \int_{s=0}^{s=1} ds 2\pi J(s)$.
- Radial coordinate can be $s = \Phi_{tor}/\Phi_{tor,LCFS}$ or $\rho = \sqrt{s}$.
- AC_1 and AC_2 specify the relative magnitude of the local current density near the axis and mid-radius, respectively.
- Right: Demonstrating the difference between the s-based and ρ-based profiles on a common radial grid for cos^2-basis functions for $N_{cssq} = 3$.
- In both cases, the edge current density is restricted to be $j_{tor}(LCFS) = 0$ by restricting the $AC_3 = 0$.

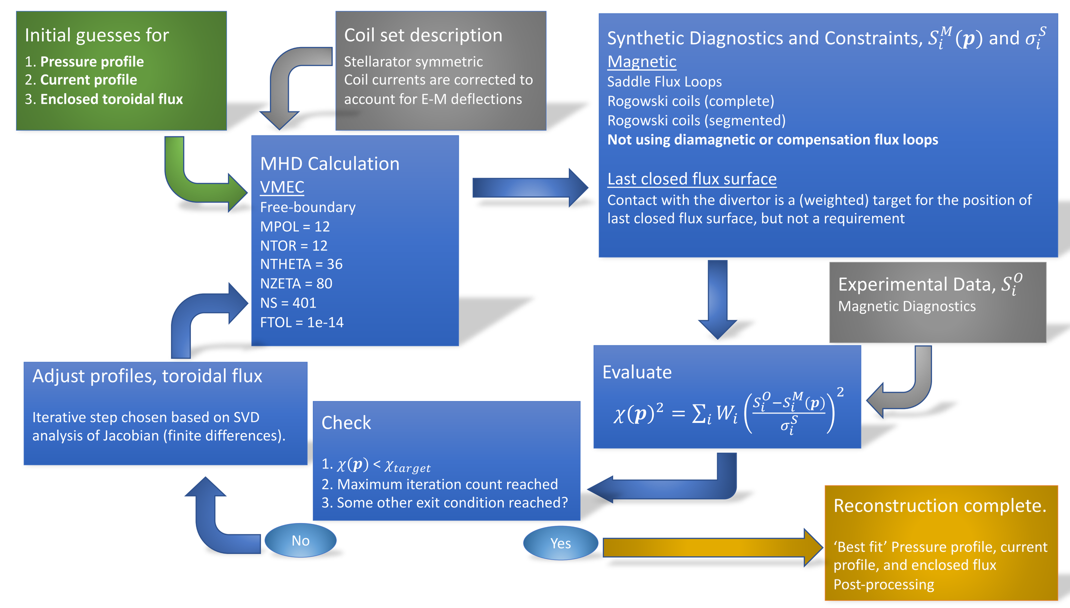
Uncertainty Propagation [4]

- S_i^O, σ_i: The observed (or measured) values and associated uncertainties
- $S_i^M(\mathbf{p})$: The synthetic model values
- $e_i = \frac{S_i^O - S_i^M(\mathbf{p})}{\sigma_i}$: Mismatch between measurement and model
- The optimizer minimizes the cost function, $\chi^2 = \sum_i W_i e_i^2$
- $(\mathbf{C_s})_{i,j} = \sigma_i \sigma_j \delta_{ij}$: Signal covariance matrix (uncorrelated)
- $\mathbf{C_p}$: Parameter covariance matrix, $C_p^{-1} = J^T \cdot C_s^{-1} \cdot J$; $J_{i,j} = \frac{\partial e_i}{\partial p_j}$
- $\mathbf{C_M}$: Model-derived covariance matrix, $C_M = K^T \cdot C_p \cdot K$
- The change in the model-derived parameter M_i with respect to changes in the parameter, p_j: $K_{i,j} = \frac{\partial M_i(\mathbf{p})}{\partial p_j}$

The parameter here is the total plasma pressure or toroidal current density at radial index $i \in [1, N_s]$:

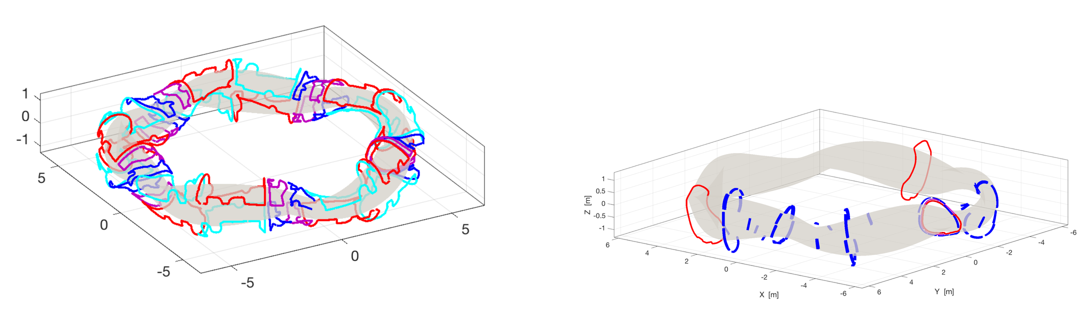
- $M_i = J_{tor}((i-1)/(N_s-1))$ or $M_i = P_{total}((i-1)/(N_s-1))$
- $\sigma_{M_i} = \sqrt{\mathbf{C_M(i,i)}}$

V3FIT Workflow



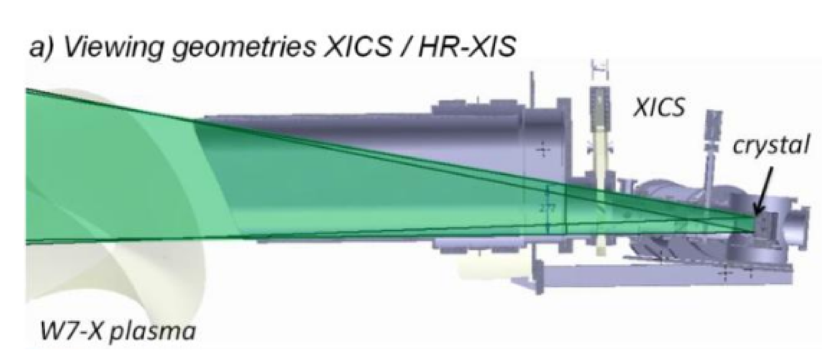
Diagnostics

Magnetics



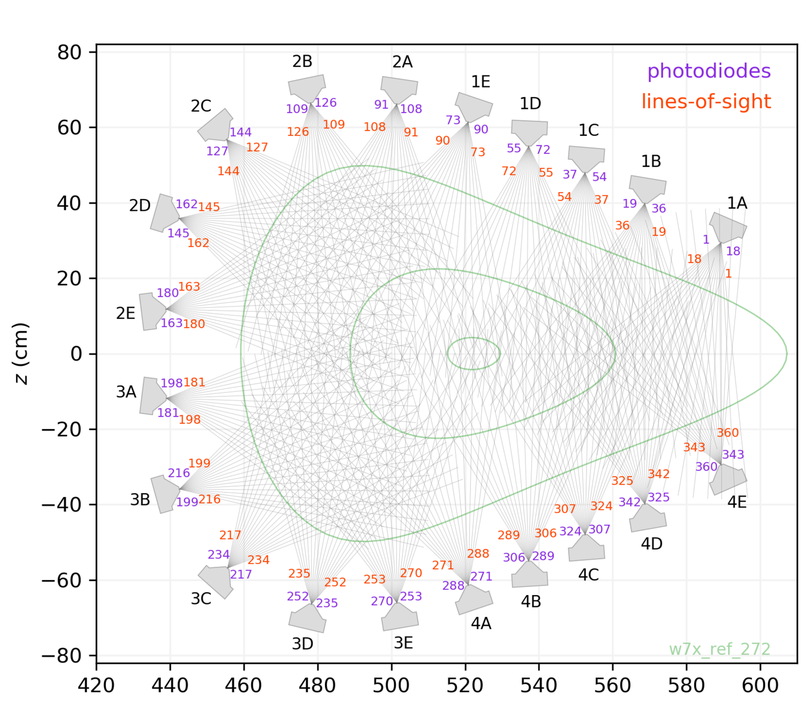
Left: 4 types of stellarator symmetric saddle coils for each of the 5 field periods. The plasma LCFS is shown in grey.
Right: Segmented Rogowski coils (in blue) measure $\int \mathbf{B} \cdot dl$ and provide good poloidal coverage over 2 field periods.

XICS [5, 6]



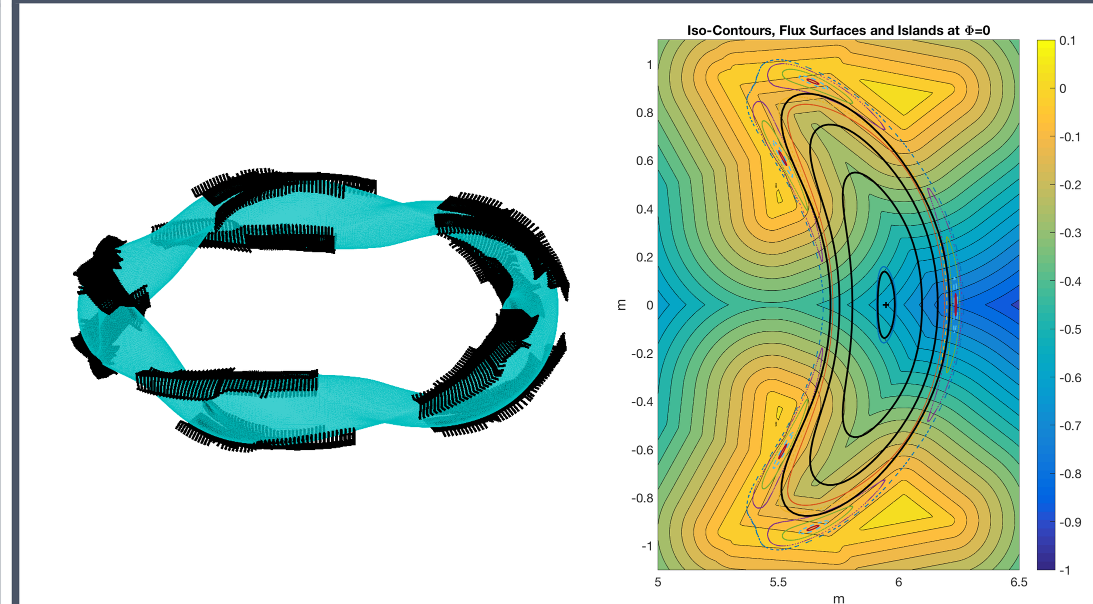
Two (2) X-ray Imaging Crystal Spectrometer systems provide multi-channel line-integrated measurements of impurity ion temperatures and emissivity profiles.

XMCTS [7]



- X-ray multichannel tomography system measures soft x-ray emission along 360 lines-of-sight
- Shafranov shift and MHD mode dynamics are detectable

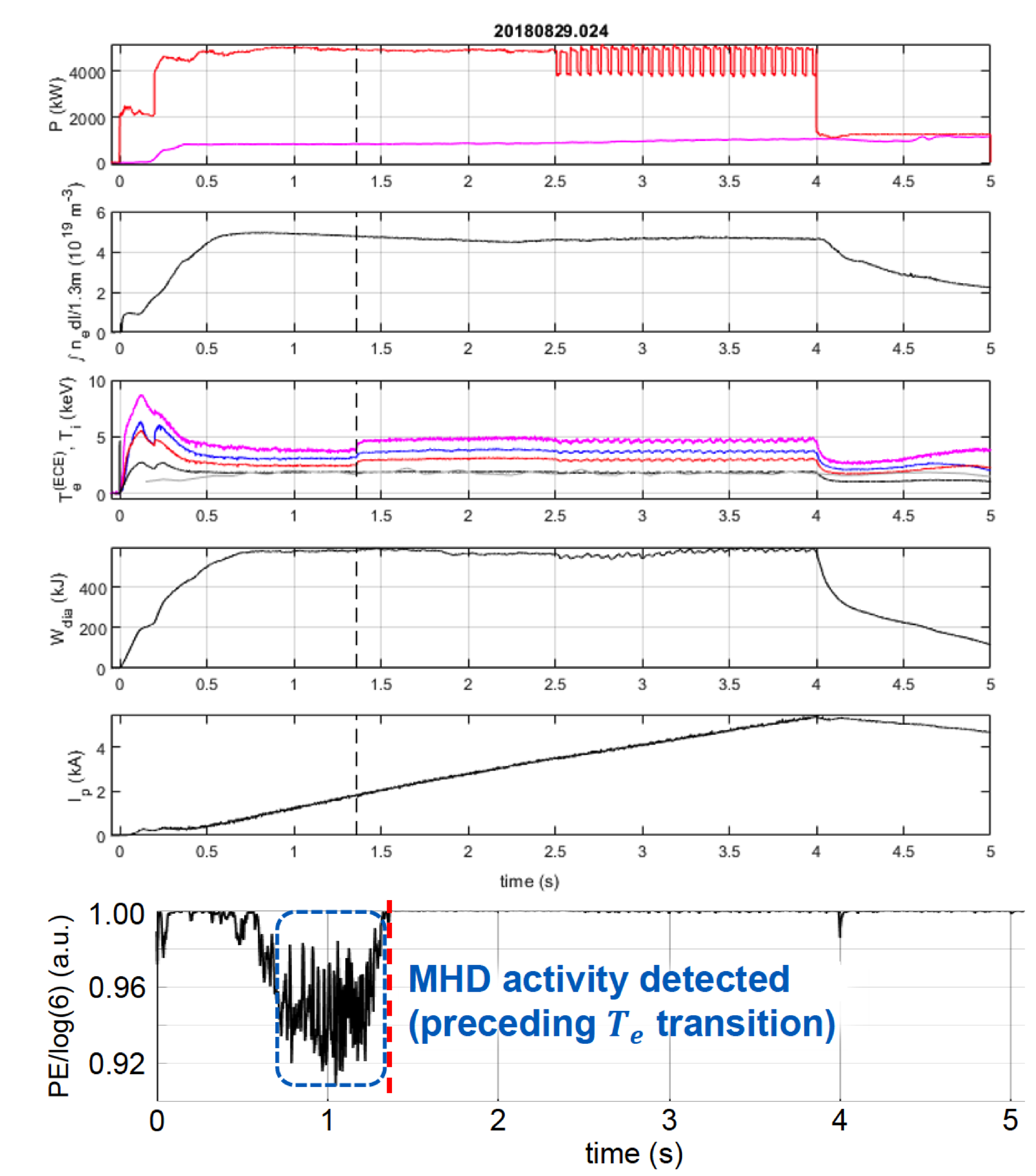
Edge Constraints



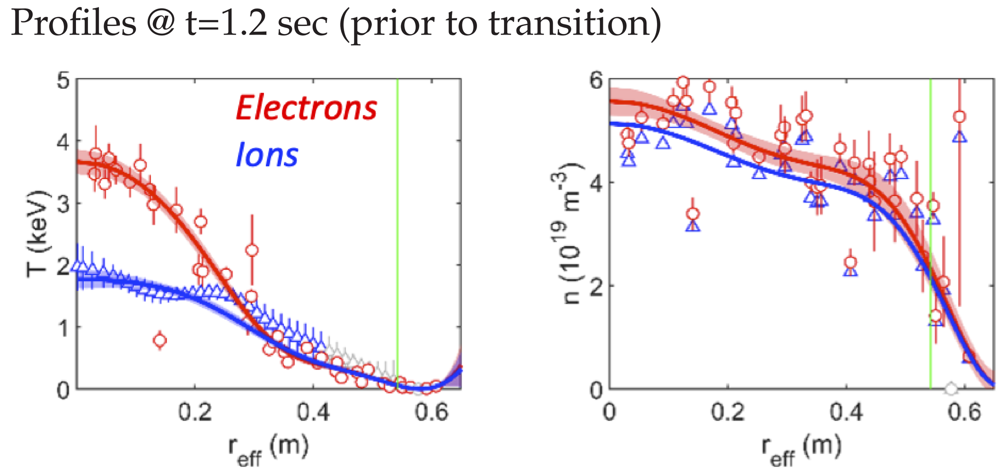
- Iso-surfaces quantify the distance from plasma to divertor
- Typical reconstructions target (but not constrain) the LCFS to touch the divertor

Spontaneous high-Te transitions [8]

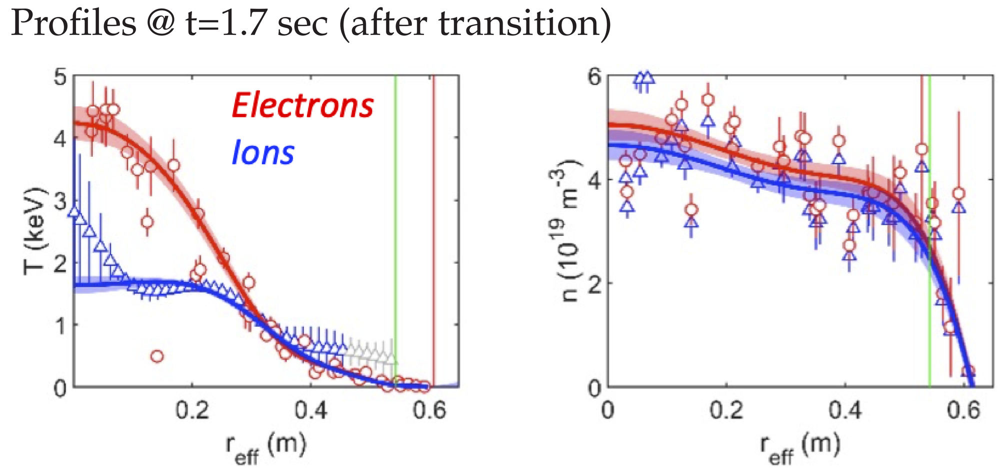
- Spontaneous transitions to high core electron temperature are seen in low-ι plasmas. See Reference [8] for more details.
- MHD activity prior to transition (Permutation energy) [9]



- Profiles @ t=1.2 sec (prior to transition)

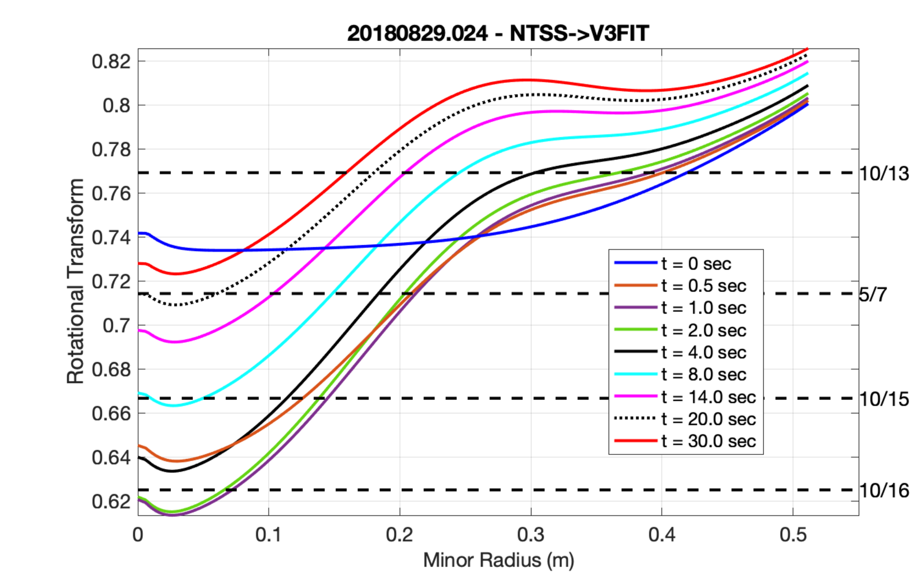


- Profiles @ t=1.7 sec (after transition)

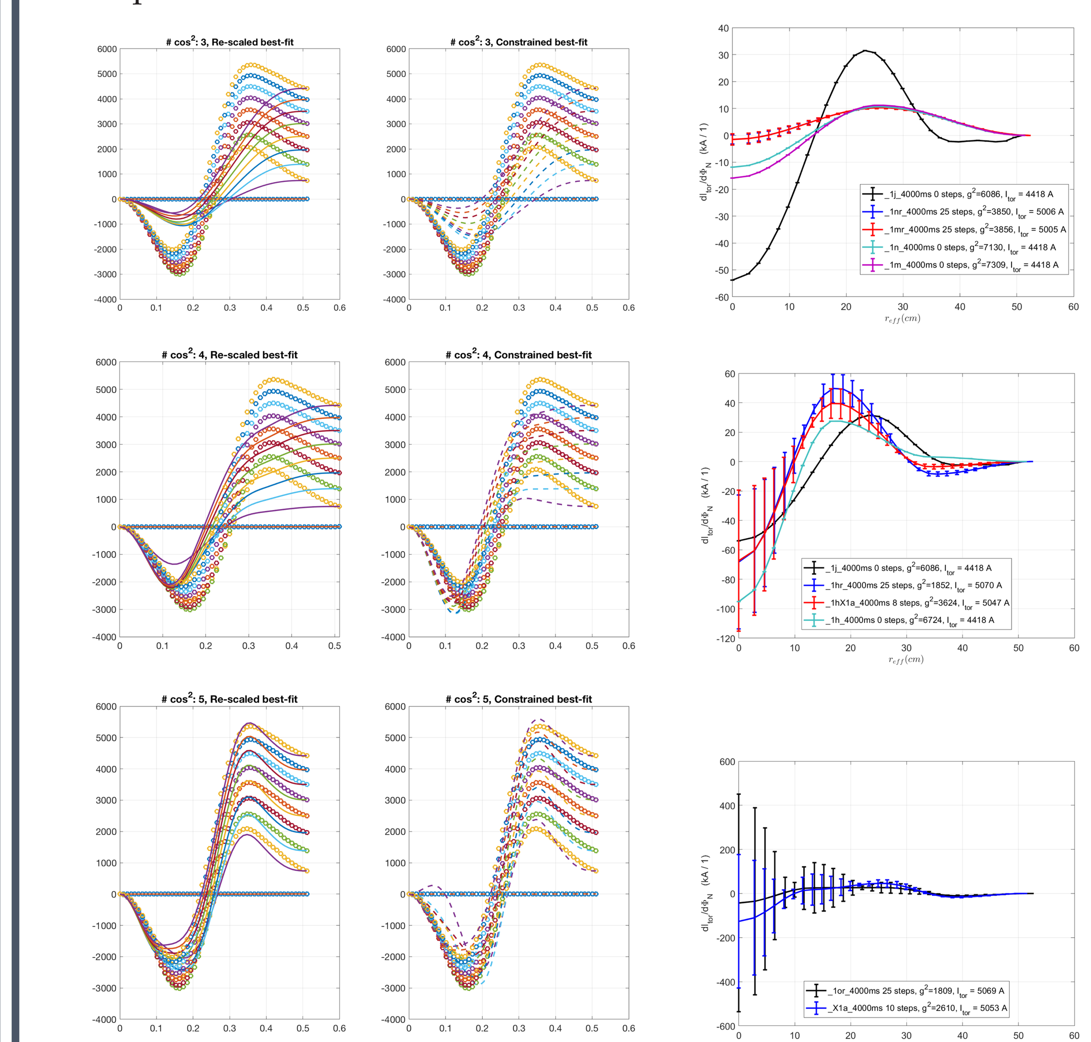


XP 20180829.024, so-called low-ι

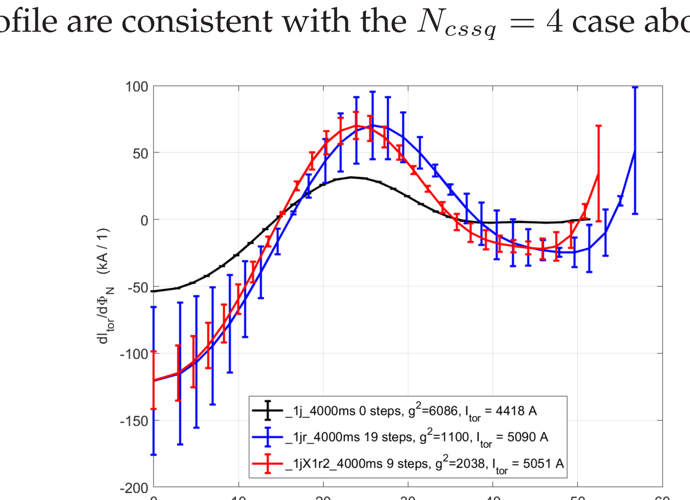
- NTSS [10] provides forward model predictions of the time-evolved bootstrap current.
- The transform passes through many low-order rational values



- Left: Enclosed current profiles ($\int dsJ$) and best-fit curves for 'sum of cos^2'-profiles with $N_{cssq} = (3, 4, 5)$ at t = (0, 500ms, 1000ms, ..., 4000ms). The curves were re-normalized to match the net current profile at each timeslice.
- Middle: As Left, with the net current constrained in the fit procedure.
- Right: Current *density* profiles based on best-fit curves and from reconstructions.
- If too few basis functions are used, the coverage is insufficient and will not provide accurate results



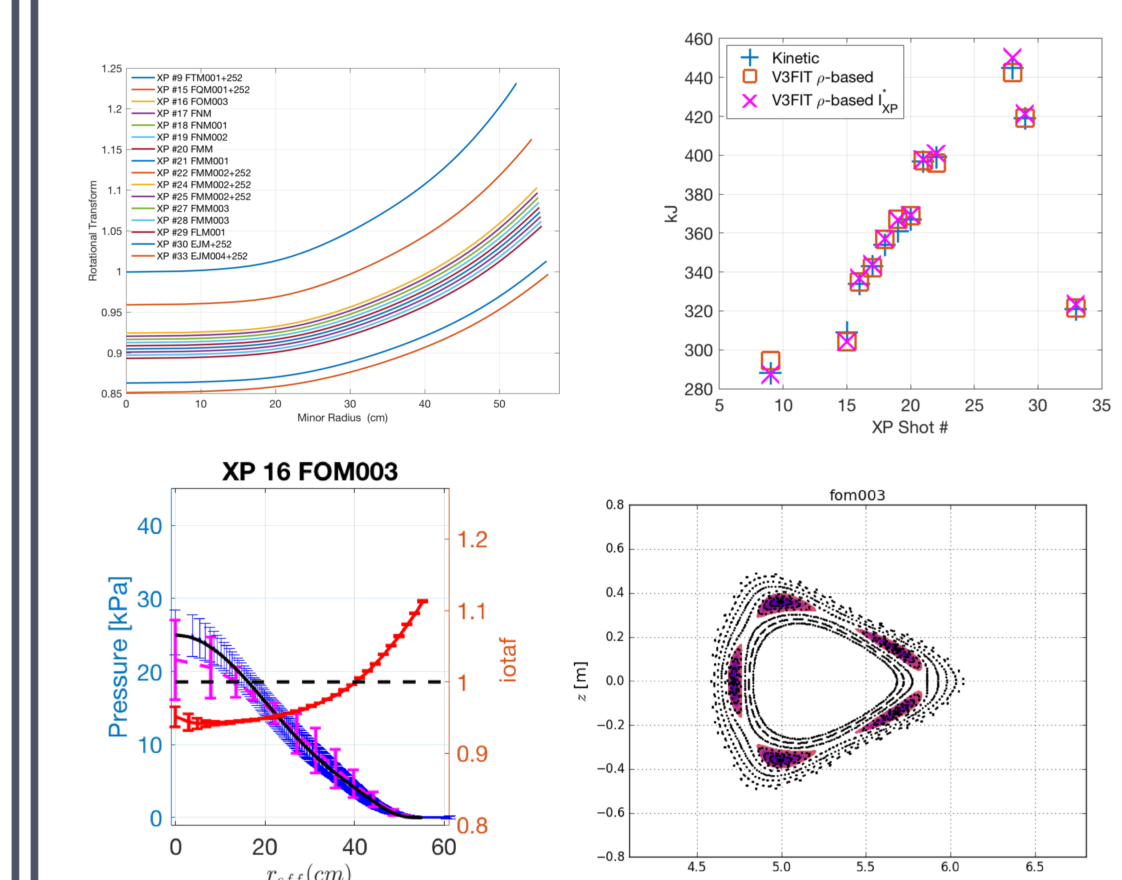
- Reconstructions were performed with 'Magnetics-only' and with 'Magnetics+XICS' (an 'X' in the legend).
- Including multi-channel spectroscopy is beneficial to constraining the current density profile. This is particularly important as the number of degrees of freedom increases.
- V3FIT results with power series and 8 degrees of freedom for the current profile are consistent with the $N_{cssq} = 4$ case above.



- A net-parallel current density may be present in/near the edge (5/6 island edge)
- Constraining the edge current density to '0' with 'sum-of-cos^2' may cause inaccurate near-axis results.

Effects of Islands

- The vacuum transform was adjusted while heating and fueling were consistent in "Iota Scan, Session 53"
- The total stored energy from reconstructions agrees well with kinetic measurements (Thomson+XICS).
- VMEC-based reconstructions exhibit little effect due to 5/5-island chain located near $r_{eff} \approx 40cm$.
- The relationship between the edge island structure and stored energy is not well understood.
- The ι-scan experiments are planned to be re-run with improved diagnostic coverage in the next campaign.
- Finite-beta free-boundary equilibrium analysis with SIESTA (forward modeling + reconstructions) is planned for this fall.



References

[1] J. D. Hanson et al. V3fit: a code for three-dimensional equilibrium reconstruction. *Nuclear Fusion*, 48, 2009.

[2] S.P. Hirshman and J.C. Whitson. Steepest-descent moment method for three-dimensional mhd equilibria. *Phys. Fluids*, 26:3553–3568, 1983.

[3] H. Peraza-Rodriguez et al. Extension of the siesta mhd equilibrium code to free-plasma-boundary problems. *Phys.Plasmas*, 24, 2017.

[4] M.R. Cianciosa et al. Uncertainty analysis in 3d equilibrium reconstruction. *Fusion Science and Technology*, 74, 2018.

[5] A. Langenberg et al. Forward modeling of x-ray imaging crystal spectrometers within the minerva bayesian analysis framework. *Fus.Sci.Techn.*, 69, 2016.

[6] N. Pablant et al. Investigation of the core radial electric field in wendelstein 7-x plasmas. *43rd EPS Conference*, 2016.

[7] C. Brandt et al. Installation of the soft x-ray multi-camera tomography system (xmcts) in the wendelstein 7-x stellarator. *Fusion Engineering and Design*, 123, 2017.

[8] J.F. Guerrero Arnaiz et al. Investigation of spontaneous transitions to high core-electron temperatures in w7-x low-iota plasmas. *23rd International Stellarator Heliotron Workshop*, 2022.

[9] J.F. Guerrero Arnaiz et al. Fast characterization of plasma states in w7-x with permutation entropy. *Plasma Phys. Control. Fusion*, 64, 2022.

[10] Y. Turkin et al. Neoclassical transport simulations for stellarators. *Physics of Plasmas*, 18, 2011.

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