CMSC 170: Planning

Hidden Markov Models

Katherine Loren M. Tan
Institute of Computer Science
University of the Philippines Los Baños

LEARNING OUTCOMES

At the end of the session, the students should be able to:

- understand the Markov chain;
- understand the hidden Markov chain;
- implement the hidden Markov model;
- use hidden Markov chain to solve problems.

Markov Chain

It is a stochastic model used to model systems.

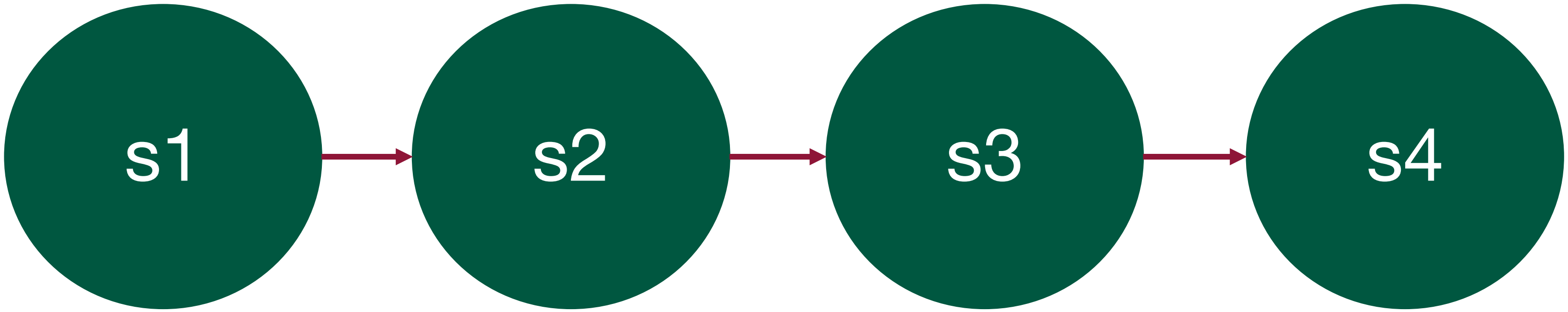
Markov Chain

It assumes that the next state is solely dependent on the current state.

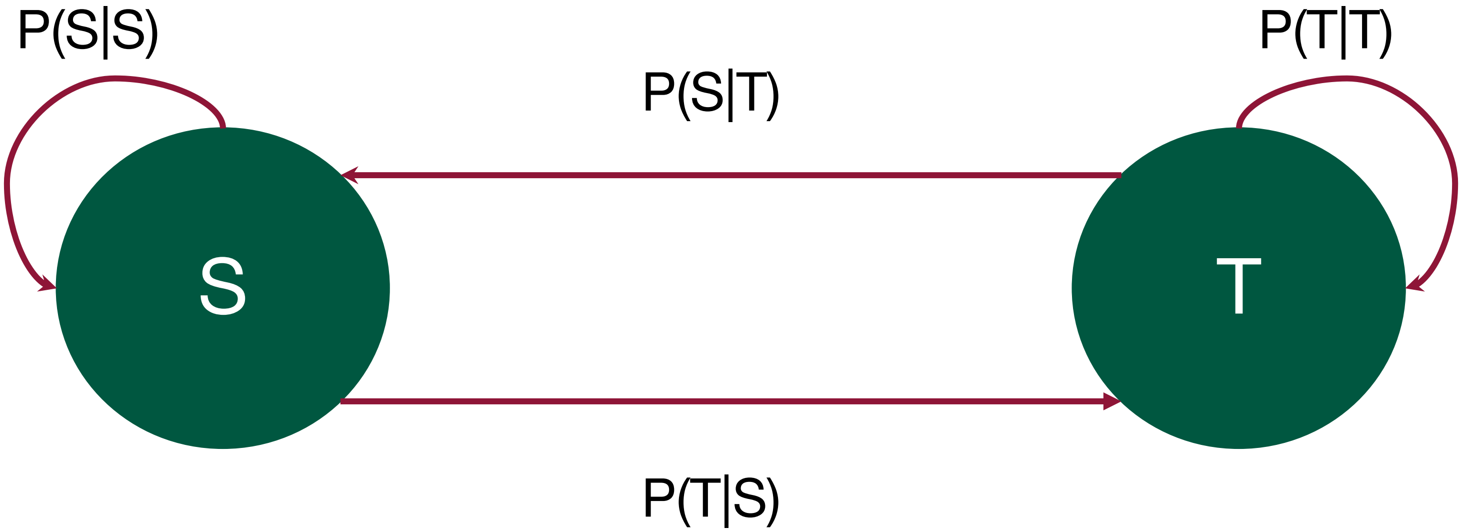
Markov Chain

It is independent of any past and future states.

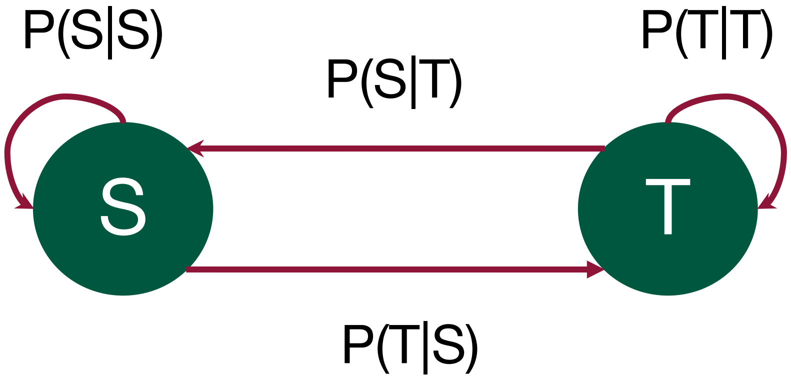
EXAMPLE OF A MARKOV CHAIN



MARKOV CHAIN



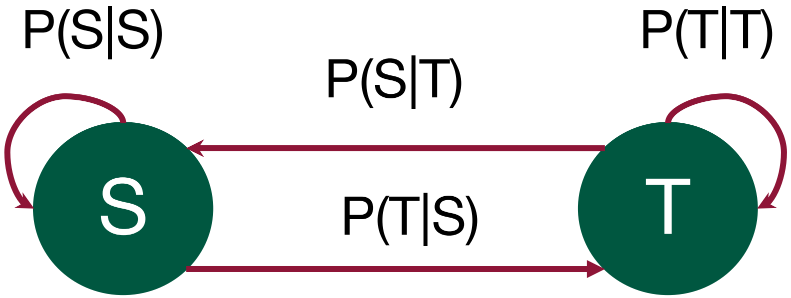
Solving for the Transition Probabilities



Sequence:

STSSTSTSSSSTT

Solving for the Transition Probabilities

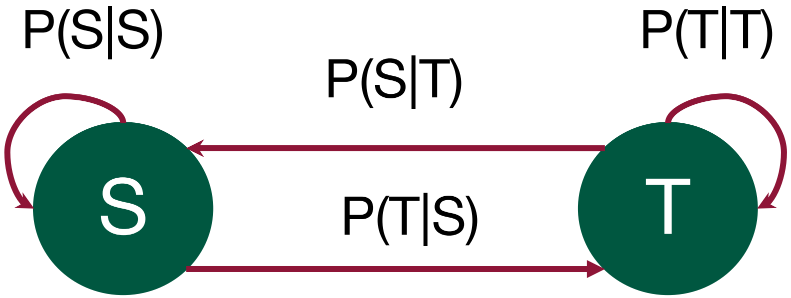


Sequence:

STSSTSTSSSTT

Probability	Value	Explanation
$P(S_0)$	1	The sequence starts with an S.
P(S|S)	3/7	Out of 7 occurences of S with a next state, 3 were followed by S.

Solving for the Transition Probabilities

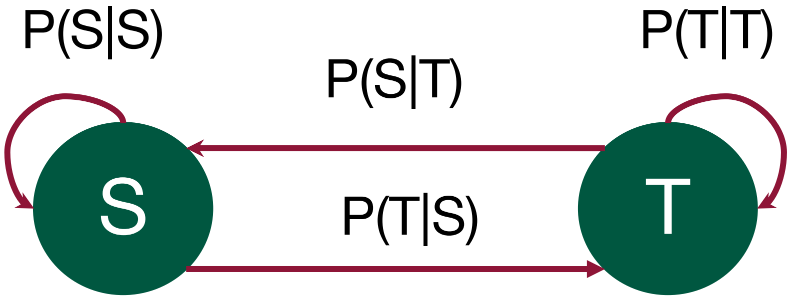


Sequence:

STSSTSTSSSTT

Probability	Value	Explanation
P(T|S)	4/7	Out of 7 occurences of S with a next state, 4 were followed by a T.
P(T|T)	1/4	Out of 4 occurences of T with a next state, 1 was folled by a T.

Solving for the Transition Probabilities



Sequence:

STSSTSTSSSTT

Probability	**Value**	**Explanation**
P(S|T)	3/4	Out of 4 occurences of T with a next state, 3 were followed by an S.

TOTAL PROBABILITY

Once the first state $P(S_0)$ is computed. It can be used to predict the probability value of the future states.

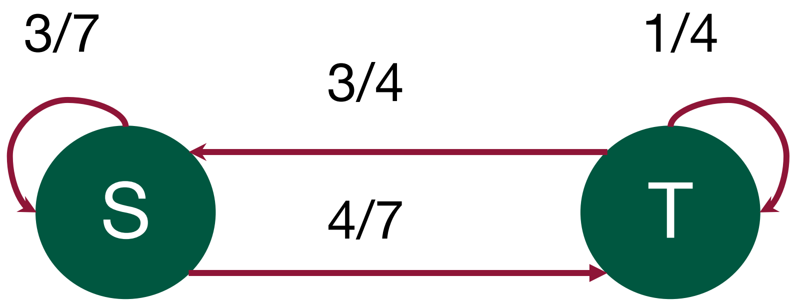
TOTAL PROBABILITY

$$P(S_n) = P(S_n|S_{n-1})P(S_{n-1}) + P(S_n|T_{n-1})P(T_{n-1})$$

Or

$$P(T_n) = P(T_n|S_{n-1})P(S_{n-1}) + P(T_n|T_{n-1})P(T_{n-1})$$

Solving for the Transition Probabilities

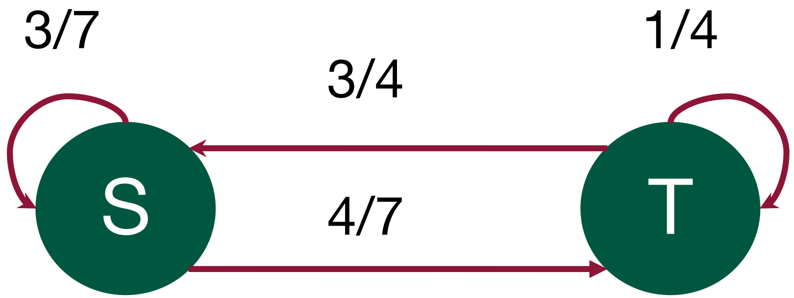


Sequence:

STSSTSTSSSTT

$$P(T_1) = P(T_1|S_0)P(S_0) + P(T_1|T_0)P(T_0)$$
$$P(T_1) = 4/7 \times 1 + ¼ \times 0$$
$$P(T_1) = 0.5714$$

Solving for the Transition Probabilities



Sequence:

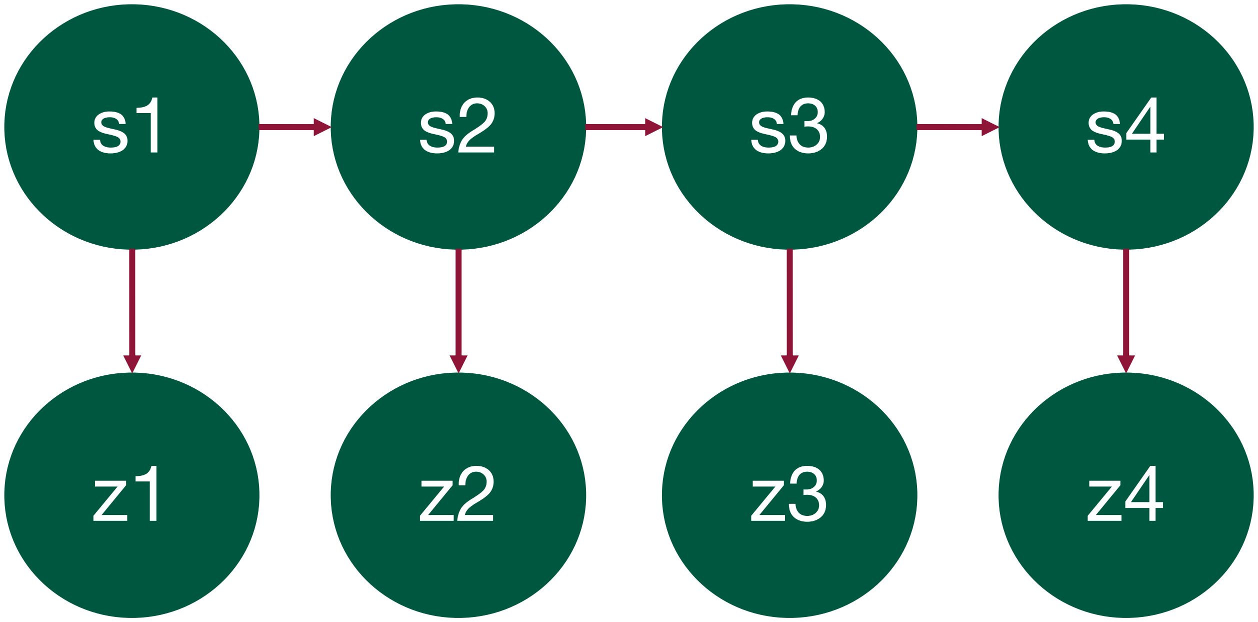
STSSTSTSSSTT

$$P(T_2) = P(T_2|S_1)P(S_1) + P(T_2|T_1)P(T_1)$$
$$P(T_2) = 4/7 * (1-0.5714) + ¼ * 0.5714$$
$$P(T_2) = 0.3877$$

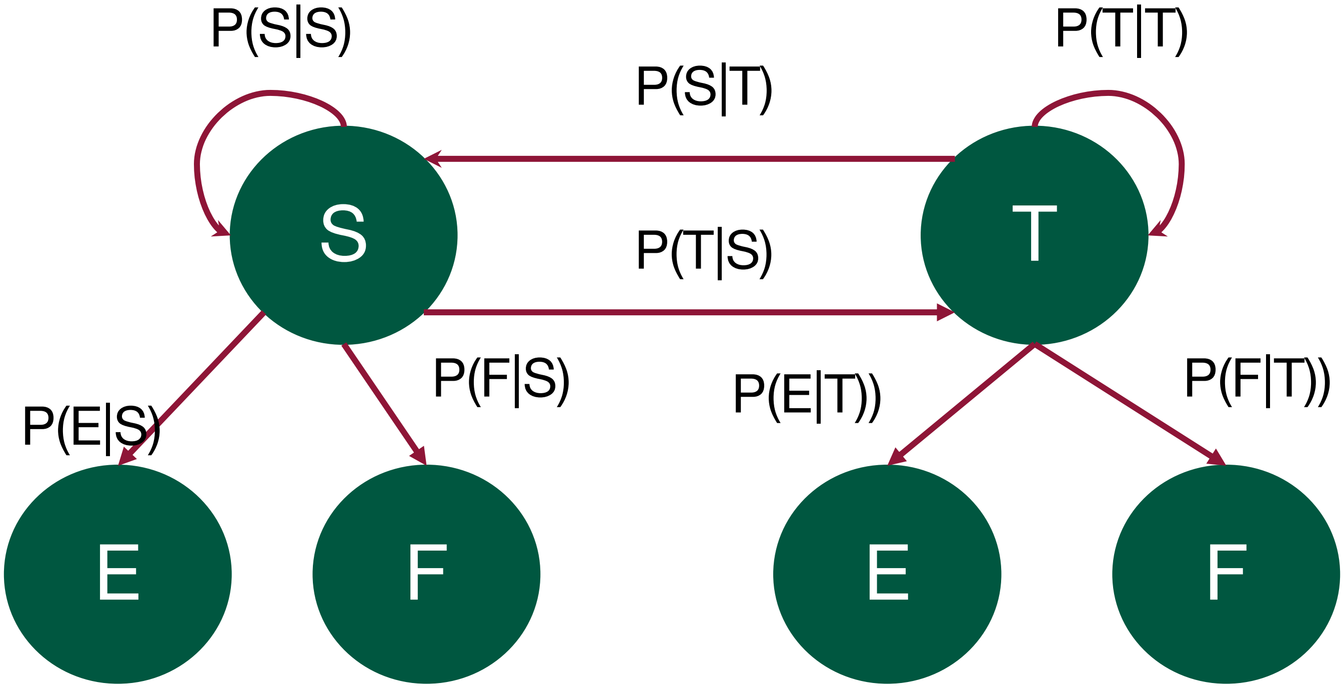
Hidden Markov Model

It is a statistical model where a system is represented as a simple Bayesian model called the Markov chain.

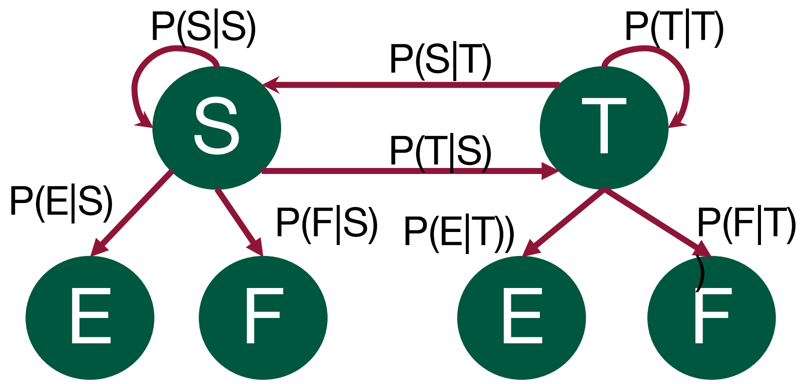
EXAMPLE OF A MARKOV CHAIN



Using Measurement Values

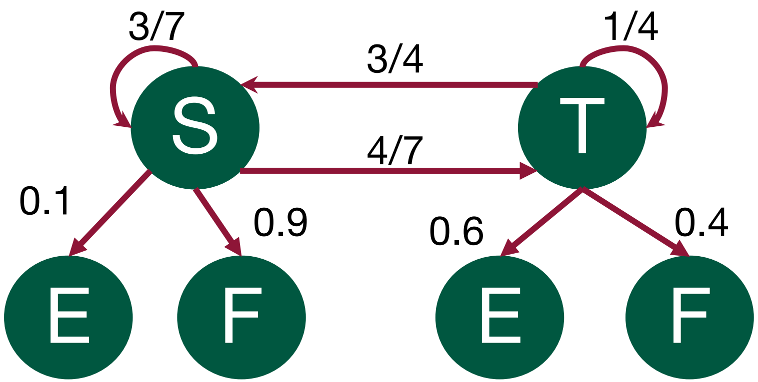


Using Measurement Values



- E and F are the possible measurement values of the possible states S and T.
- We can solve the probability of the current state in the Markov chain using the observable measurements values that have a dependence on each state of the markov chain.

Using Measurement Values



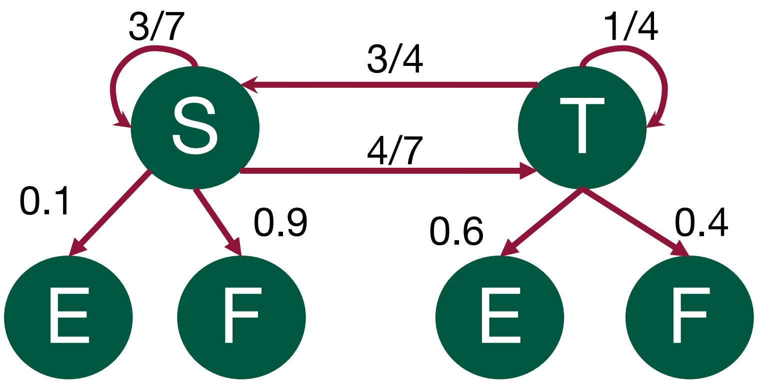
Sequence:

STSSTSTSSSTT

We can compute for $P(S_1|E_1)$ using the Bayes Rule:

$$P(S_1|E_1) = \frac{P(E_1|S_1)P(S_1)}{P(E_1)}$$

Using Measurement Values



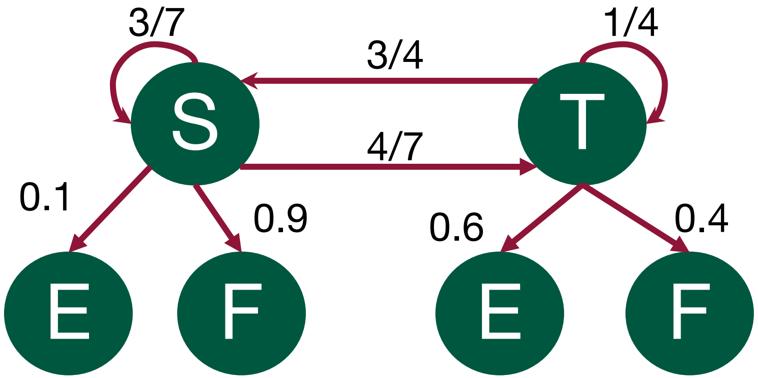
Sequence:

STSSTSTSSSTT

$$P(S_1|E_1) = \frac{P(E_1|S_1)P(S_1)}{P(E_1)}$$

Compute $P(S_1)$ using total probability

Using Measurement Values



Sequence:

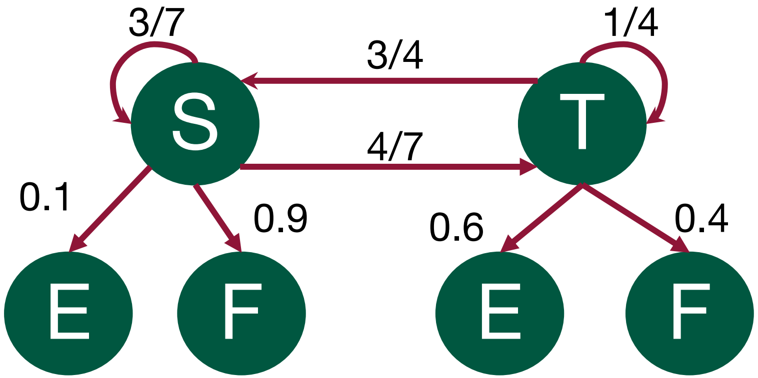
STSSTSTSSSTT

$$P(S_1) = P(S_1|S_0)P(S_0) + P(S_1|T_o)P(T_0)$$

$$P(S_1) = 3/7 \times 1 + ¾ \times 0$$

$$P(S_1) = \mathbf{0.4285}$$

Using Measurement Values



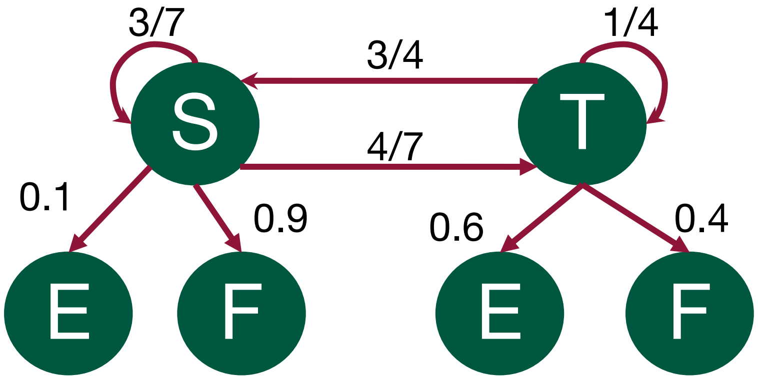
Sequence:

STSSTSTSSSTT

$$P(S_1|E_1) = \frac{P(E_1|S_1)P(S_1)}{P(E_1)}$$

Compute $P(E_1)$ using total probability

Using Measurement Values



Sequence:

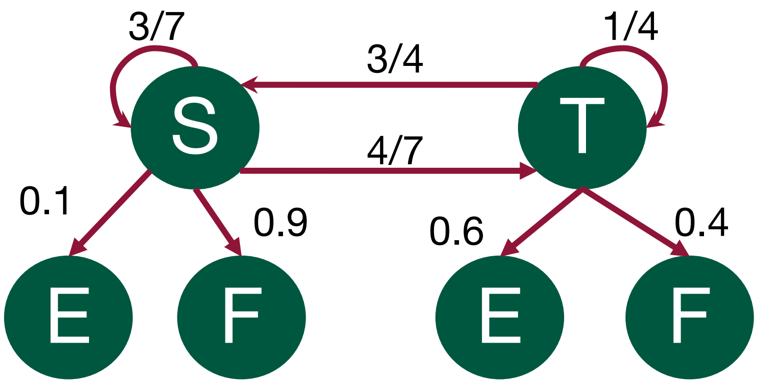
STSSTSTSSSTT

$$P(E_1) = P(E_1|S_1)P(S_1) + P(E_1|T_1)P(T_1)$$

$$P(E_1) = 0.1 \times 0.4285 + 0.6 \times 0.5714$$

$$P(E_1) = \mathbf{0.3856}$$

Using Measurement Values



Sequence:

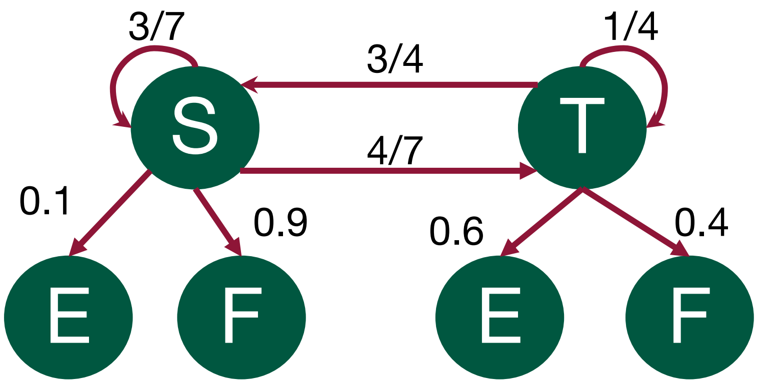
STSSTSTSSSTT

$$P(S_1|E_1) = \frac{P(E_1|S_1)P(S_1)}{P(E_1)}$$

$$P(S_1|E_1) = \frac{0.1 * 0.4285}{0.3856}$$

$$P(S_1|E_1) = \mathbf{0.1111}$$

Using Measurement Values



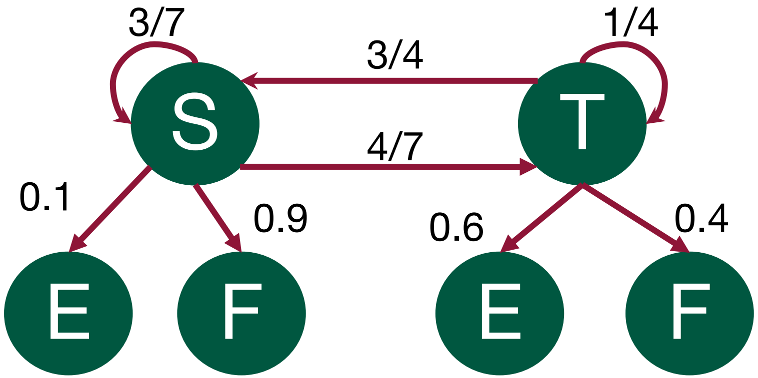
Sequence:

STSSTSTSSSTT

$$P(S_3|E_3) = \frac{P(E_3|S_3)P(S_3)}{P(E_3)}$$

$$P(S_3|E_3) = \ ?$$

Using Measurement Values



Sequence:

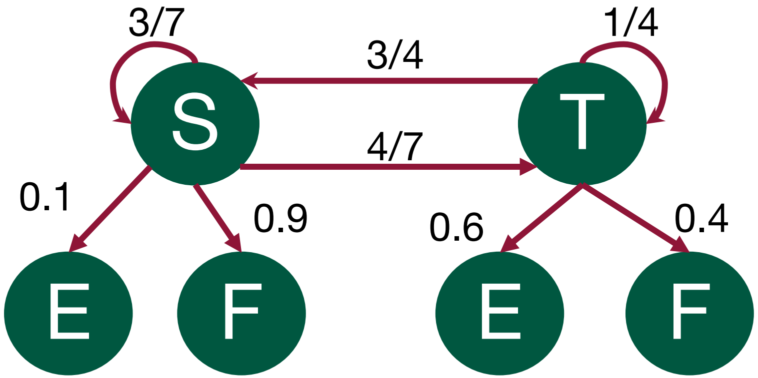
STSSTSTSSSTT

$$P(S_3|E_3) = \frac{P(E_3|S_3)P(S_3)}{P(E_3)}$$

$$P(S_3) = P(S_3|S_2)P(S_2) + P(S_3|T_2)P(T_3)$$

$$P(S_2) = ?$$

Using Measurement Values



Sequence:

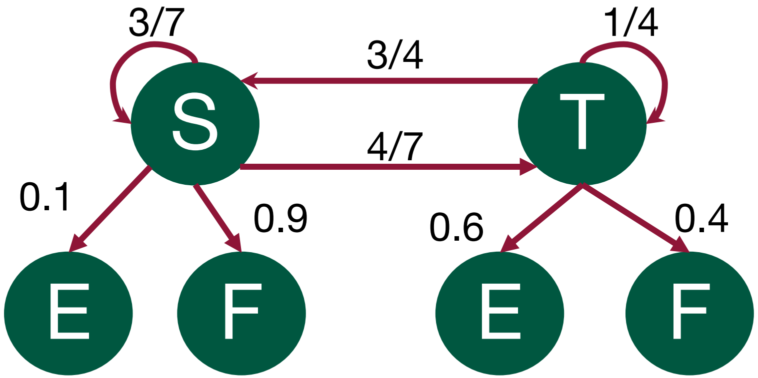
STSSTSTSSSTT

$$P(S_2) = P(S_2|S_1)P(S_1) + P(S_2|T_1)P(T_1)$$

$$P(S_2) = 3/7 \times 0.4285 + \tfrac{3}{4} \times 0.5715$$

$$P(S_2) = \mathbf{0.6122}$$

Using Measurement Values



Sequence:

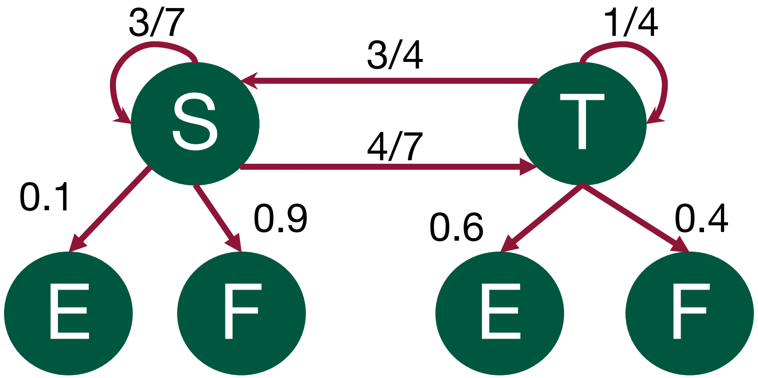
STSSTSTSSSTT

$$P(S_3) = P(S_3|S_2)P(S_2) + P(S_3|T_2)P(T_3)$$

$$P(S_3) = 3/7 \times 0.6122 + ¾ \times 0.3878$$

$$P(S_3) = \mathbf{0.5531}$$

Using Measurement Values



Sequence:

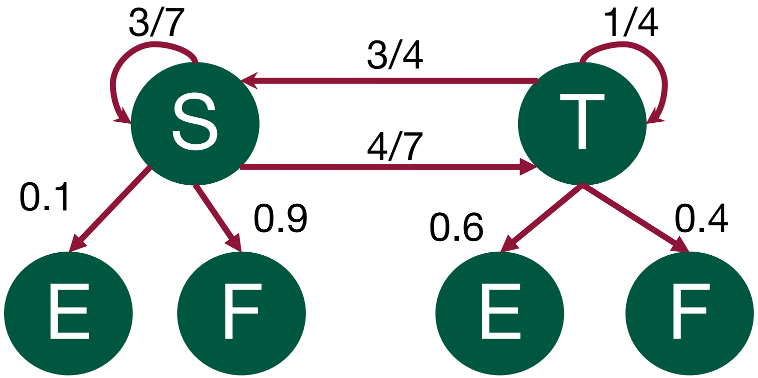
STSSTSTSSSTT

$$P(S_3|E_3) = \frac{P(E_3|S_3)P(S_3)}{P(E_3)}$$

$$P(E_3) = P(E_3|S_3)P(S_3) + P(E_3|T_3)P(T_3)$$

$$P(E_3) = ?$$

Using Measurement Values



Sequence:

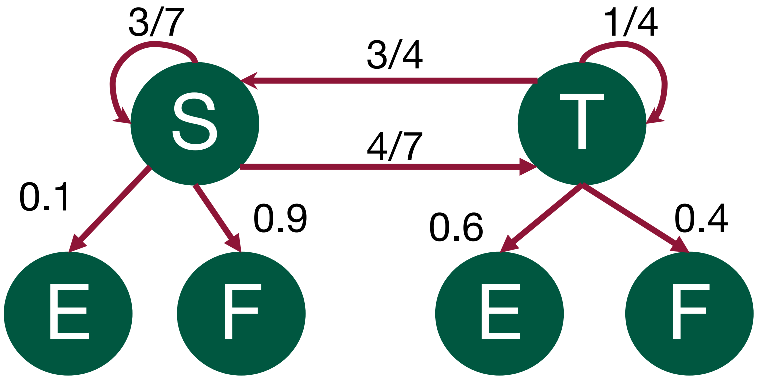
STSSTSTSSSTT

$$P(E_3) = P(E_3|S_3)P(S_3) + P(E_3|T_3)P(T_3)$$

$$P(E_3) = 0.1 * 0.5531 + 0.6 * 0.4469$$

$$P(E_3) = \mathbf{0.3234}$$

Using Measurement Values



Sequence:

STSSTSTSSSTT

$$P(S_3|E_3) = \frac{P(E_3|S_3)P(S_3)}{P(E_3)}$$

$$P(S_3|E_3) = \frac{0.1 * 0.5531}{0.3234}$$

$$P(S_3|E_3) = \mathbf{0.1710}$$

For questions and inquiries, you can email me at

kmtan4@up.edu.ph

EXERCISE on Hidden Markov Model

An input file named **hmm.in** has the following format:

2 no. of string considered

STSSTSSSTT string sequence 1

TSSSSSTTSS string sequence 2

S T the possible values for each state in the Markov chain (MC)

E F possible observable measurement values for each state in the MC

P(E|S) P(F|S) pair values for P(E|S) and P(F|S), respectively

P(E|T) P(F|T) pair values for P(E|T) and P(F|T), respectively

3 no of cases to be considered for the strings

S1 given E1 compute for P(S1|E1)

T3 given F3 compute for P(T3|F3)

S2 given F2 compute for P(S2|F2)

EXERCISE on Hidden Markov Model

The output file named **hmm.out** has the following format:

STSSTSSSTT

S1 given E1 //compute for P(S1|E1)

T3 given F3 // compute for P(T3|F3)

S2 given F2 // compute for P(S2|F2)

STSSSSTTSS

S1 given E1 //compute for P(S1|E1)

T3 given F3 // compute for P(T3|F3)

S2 given F2 // compute for P(S2|F2)

EXERCISE on Hidden Markov Model

SOME REMINDERS:

- Naming convention for exercise: surname_hmm
- Python or Java can only be used for the exercise.
- Do not used built-in libraries for Hidden Markov Models.
- Do not forget to put a journal in your ReadMe file in Github.
- Lastly, **Honor and Excellence.**