Assignment No.9 (non-regular languages, Push Down Automata)

Name(s) ....

Please write only on one side of your papers

1. (18 points) Find a CFG for each of the following non-regular languages and trace the given word to determine whether it is accepted or rejected by the CFG

   a. $L=\{a^n b^p c^m | p=2m+n, m=0,1,2,\ldots$ and $n=1,2,3,\ldots\ldots\}$

      Trace $w1=a^2 b^4 c$

   b. $L=\{a^p b^n c^m | p=2n+3m; n=0,1,2,3,\ldots$ and $m=1,2,3,\ldots\ldots\}$

      Trace $w1=a^6 b^3 c^3$

   c. $L=\{a^n b^m | m>=n; n=1,2,3\ldots\ldots\}$

      Trace $w=a^3 b^5$

2. (6 points)
   A. Construct a PDA for $L=\{a^n b^m c^r | m=2r+n, r=1,2,\ldots n=0,1,2,\ldots\}$
   B. Write a CFG for L

3. (6 points)
   A. Construct PDA for $L=\{a^n b^m | n>m, m=1,2,\ldots\}$
   B. Write a CFG for L

4. (6 points) Given regular language $L=ab^* + b^*$. Construct
   a. FA to accept L
   b. PDA to accept L

(1a)

$$L = \{ a^n b^p c^m \mid \begin{matrix} p = 2m+n \\ m = 0,1,\dots \\ n = 1,2,\dots \end{matrix} \}$$

$$a^n b^{2m+n} c^m$$

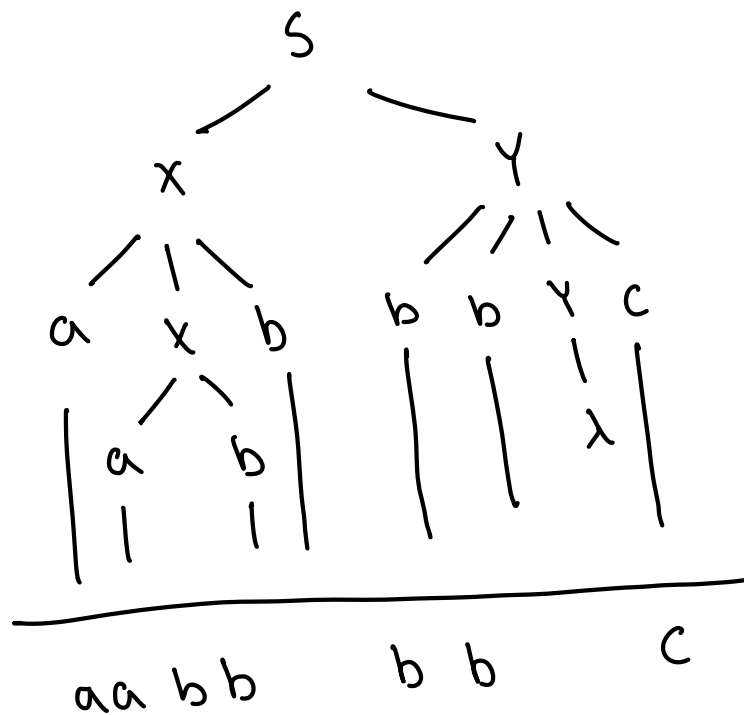$$\underbrace{a^n b^n}_{X} \underbrace{b^{2m} c^m}_{Y}$$

cfg: $S \to XY$

$X \to abX \mid ab$

$Y \to bbYc \mid \lambda$

Trace: $w1 = a^2 b^4 c$

$$a^2 b^2 b^2 c$$

aa bb   b b   c

$a^2 b^4 c$ accepted

(1b) $L = \{a^p b^n c^m \mid \begin{array}{l} p = 2n + 3m \\ n = 0, 1, \ldots \\ m = 1, 2, \ldots \end{array}\}$

$$a^{2n+3m} \quad b^n \quad c^m$$

$$a^{3m} \quad a^{2n} b^n \quad c^m$$

$$\underbrace{\phantom{a^{2n}b^n}}_{Y}$$

$$\underbrace{\phantom{a^{3m} a^{2n}b^n c^m}}_{S}$$

Cfg: $S \to aaaSc \mid aaaYc$

$\quad\quad Y \to aaYb \mid \lambda$

Trace: $w1 = a^6 b^3 c^3$
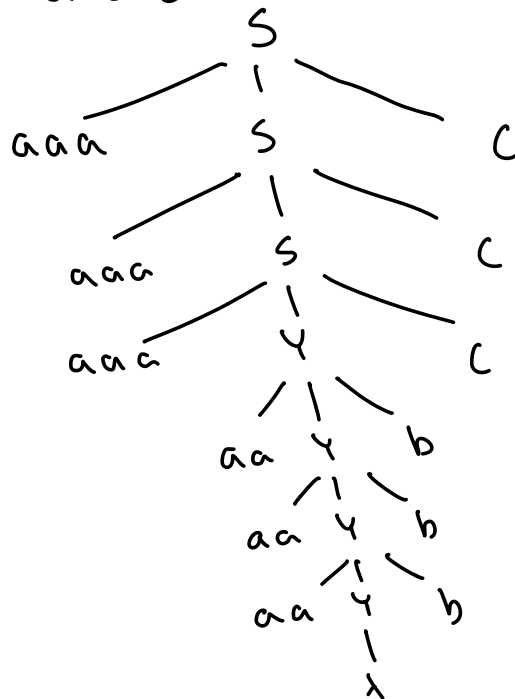

```
aaa aaa  aaa aa  aa aa bbb ccc
```

$\boxed{a^6 b^3 c^3 \quad \text{rejected}}$

(1c) $L = \{a^n b^m \mid \begin{matrix} m > n \\ n = 1, 2, \dots \end{matrix}\}$

$a^n b^m$

cfg: $S \to aSb \mid aXb$

$X \to bX \mid b$

Trace: $w = a^3 b^5$

```
              S
       ___ __|___ ___
      a      S       b
          _ _|_ _
         a    S    b
            _/|\_
           a  X  b
             _/ \_
            b    X
                 |
                 b
_____
   a  a  a  b   b  b  b  b
```
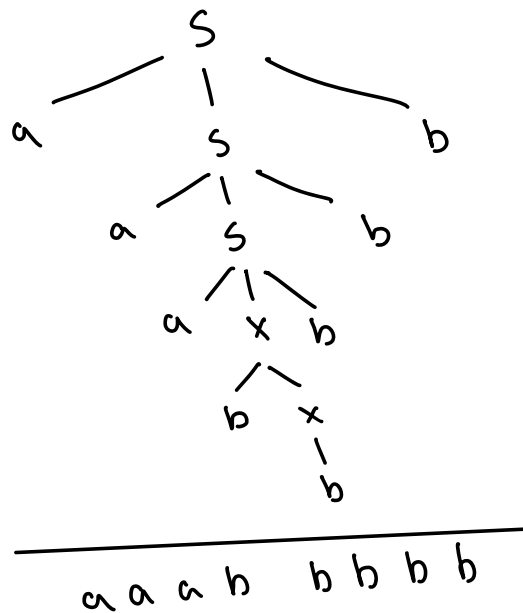
$\boxed{a^3 b^5 \quad \text{accepted}}$

(2a)

$$L = \{ a^n b^m c^r \mid \begin{matrix} m = 2r+n \\ r = 1,2,\cdots \\ n = 0,1,\cdots \end{matrix} \}$$

```
                start
                  |
                  v
   +----------> (read) --b--> (pop) --a--> (read)
   |              |             |         ^   \
 push a <--a-----/            Δ|          |    b
   ^                           v          +----/
   |                           |
   |                       push b
   |                           |
   |                           v
   +---------b----------> (read) --c--> (pop) --b--> (pop) --b--> (read) --Δ--> (pop)
                          <-c---                             <---c---          |
                                                                              Δ|
                                                                               v
                                                                             acc
```

(2b)
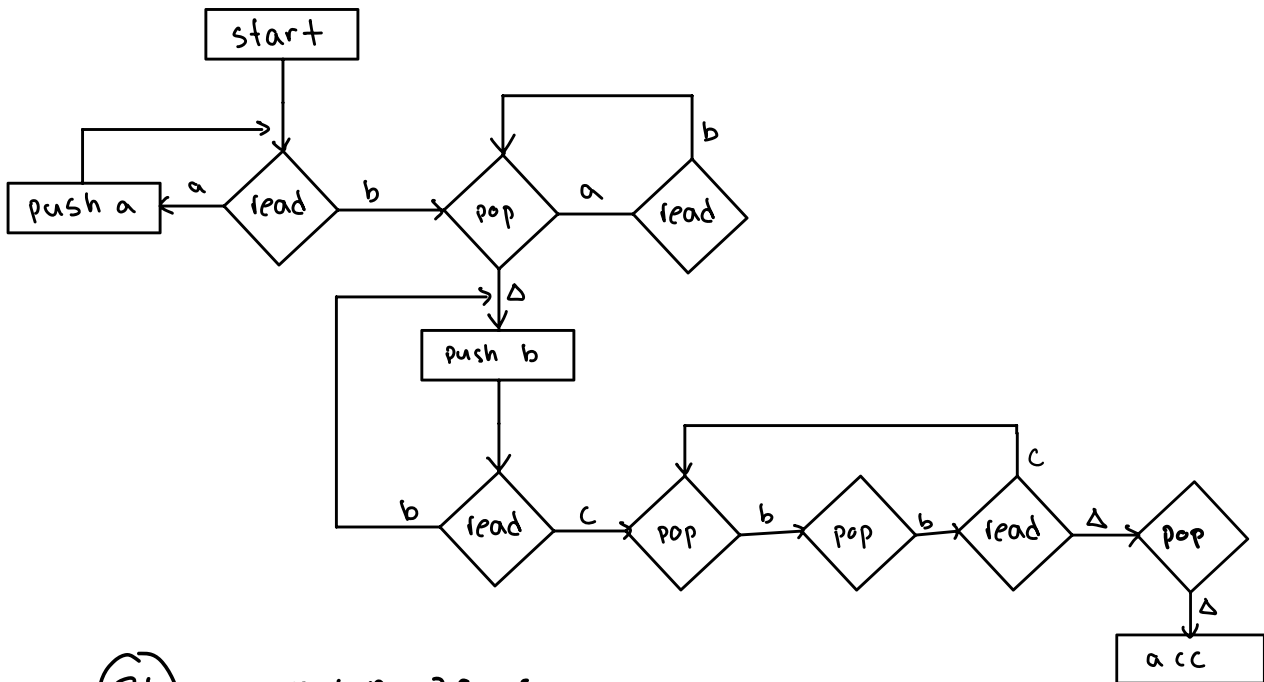
$$\underbrace{a^n\, b^n}_{X}\ \underbrace{b^{2r}\, c^r}_{Y}$$

cfg: $S \rightarrow XY$

$X \rightarrow aXb \mid \lambda$

$Y \rightarrow bbYc \mid bbc$

(3a) $L = \{ a^n b^m \mid \begin{array}{l} n > m \\ m = 1, 2, \ldots \end{array} \}$

```
                          ┌────────┐
                          │ start  │
                          └────────┘
                               │
                               ▼
        ┌────────┐    a   ◇ read ◇   b    ◇ pop ◇   a   ◇ read ◇   Δ   ◇ pop ◇
        │ push a │◄──────                      ◄──b──                      │
        └────────┘                                                        │ a
           │                                                              ▼ ◄─┐
           └──────────────►                                          ◇ pop ◇─┘ a
                                                                          │ Δ
                                                                          ▼
                                                                     ┌────────┐
                                                                     │  acc   │
                                                                     └────────┘
```

(3b) $a^k \underbrace{\quad}_{A} a^m b^m \underbrace{\quad}_{X}$ $k > 0$
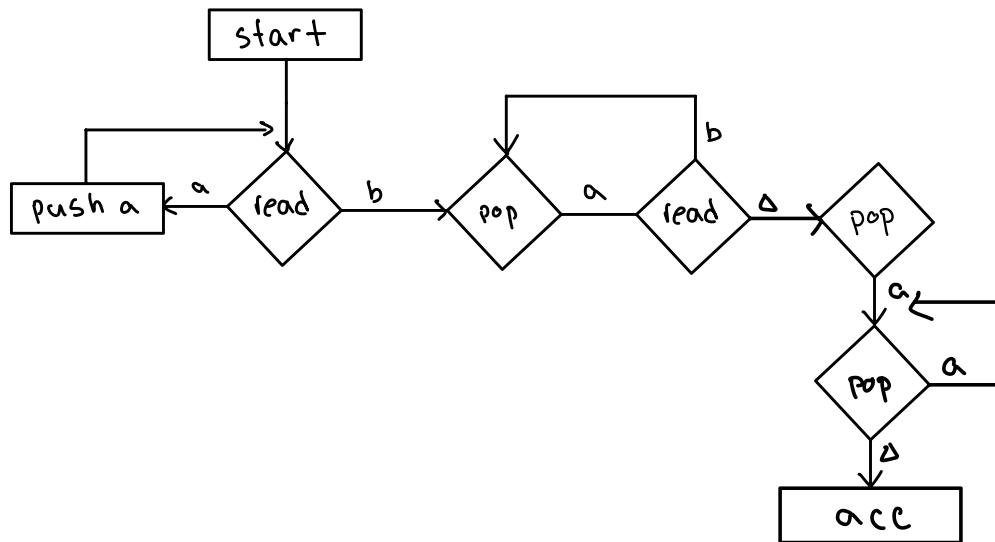
$S \rightarrow AX$

$A \rightarrow aA \mid a$

$X \rightarrow aXb \mid ab$

(4a) $L = ab^* + b^*$

(4b)