

CPSC 323

Assignment No.9 (non-regular languages, Push Down Automata)

Name(s)

Please write only on one side of your papers

1. (18 points) Find a CFG for each of the following non-regular languages and trace the given word to determine whether it is accepted or rejected by the CFG

a. $L = \{a^n b^p c^m \mid p = 2m + n, m = 0, 1, 2, \dots \text{ and } n = 1, 2, 3, \dots\}$

Trace $w_1 = a^2 b^4 c$

b. $L = \{a^p b^n c^m \mid p = 2n + 3m; n = 0, 1, 2, 3, \dots \text{ and } m = 1, 2, 3, \dots\}$

Trace $w_1 = a^6 b^3 c^3$

c. $L = \{a^n b^m \mid m \geq n; n = 1, 2, 3, \dots\}$

Trace $w = a^3 b^5$

2. (6 points)

A. Construct a PDA for $L = \{a^n b^m c^r \mid m = 2r + n, r = 1, 2, \dots, n = 0, 1, 2, \dots\}$

B. Write a CFG for L

3. (6 points)

A. Construct PDA for $L = \{a^n b^m \mid n > m, m = 1, 2, \dots\}$

B. Write a CFG for L

4. (6 points) Given regular language $L = ab^* + b^*$. Construct

a. FA to accept L

b. PDA to accept L

1a) $L = \{ a^n b^p c^m \mid \begin{matrix} p = 2m + n \\ m = 0, 1, \dots \\ n = 1, 2, \dots \end{matrix} \}$

$$a^n b^{2m+n} c^m$$

$$\underbrace{a^n b^n}_x \underbrace{b^{2m} c^m}_y$$

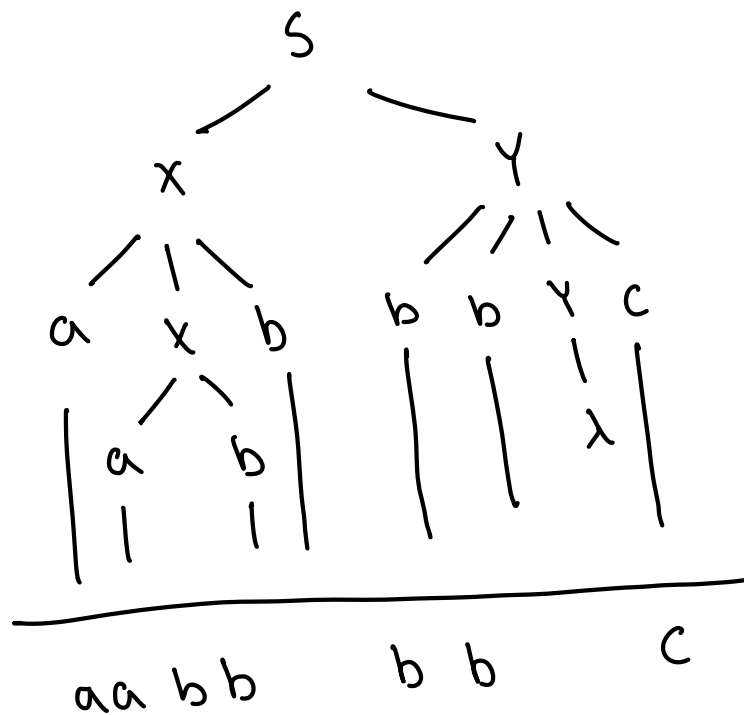
cfg: $S \rightarrow XY$

$$X \rightarrow abX \mid ab$$

$$Y \rightarrow bbYc \mid \lambda$$

Trace: $w_1 = a^2 b^4 c$

$$a^2 b^2 b^2 c$$



$a^2 b^4 c$ accepted

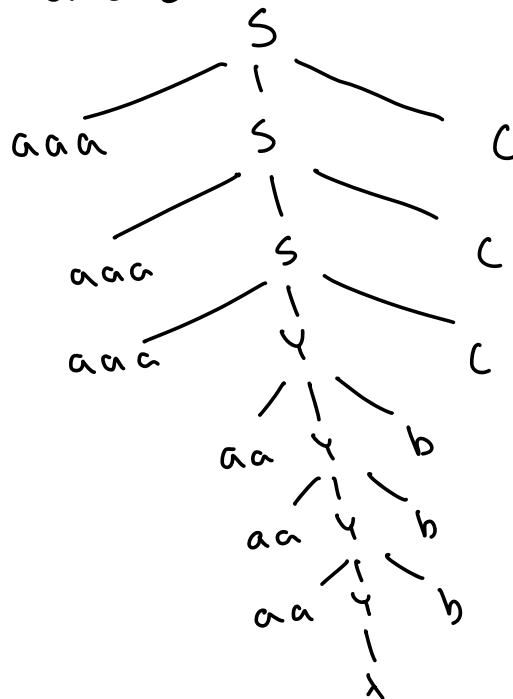
$$(1b) \quad L = \{ a^p b^n c^m \mid \begin{matrix} p = 2n + 3m \\ n = 0, 1, \dots \\ m = 1, 2, \dots \end{matrix} \}$$

$$\begin{array}{c} a^{2n+3m} b^n c^m \\ a^{3m} \underbrace{a^{2n} b^n}_Y c^m \\ \hline S \end{array}$$

$$\text{cfg: } S \rightarrow aacSc \mid aaaYc$$

$$Y \rightarrow aaYb \mid \lambda$$

$$\text{Trace: } w_1 = a^6 b^3 c^3$$



aaa aaa aaa aa aaaa bbb ccc

$a^6 b^3 c^3$ rejected

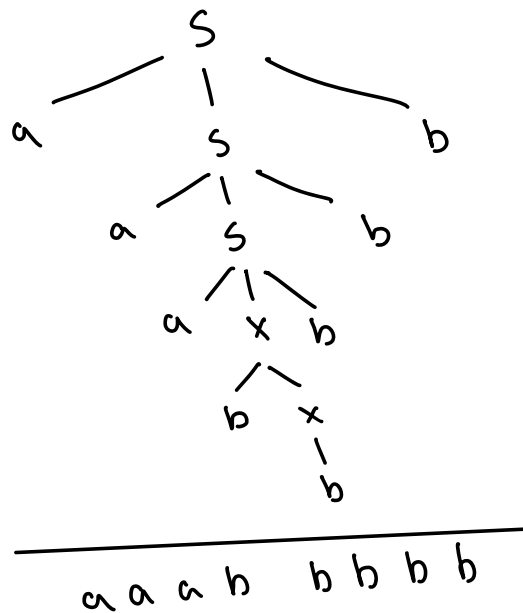
$$(1c) L = \{a^n b^m \mid m > n, n = 1, 2, \dots\}$$

$$a^n b^m$$

$$CFG: S \rightarrow a S b \mid a X b$$

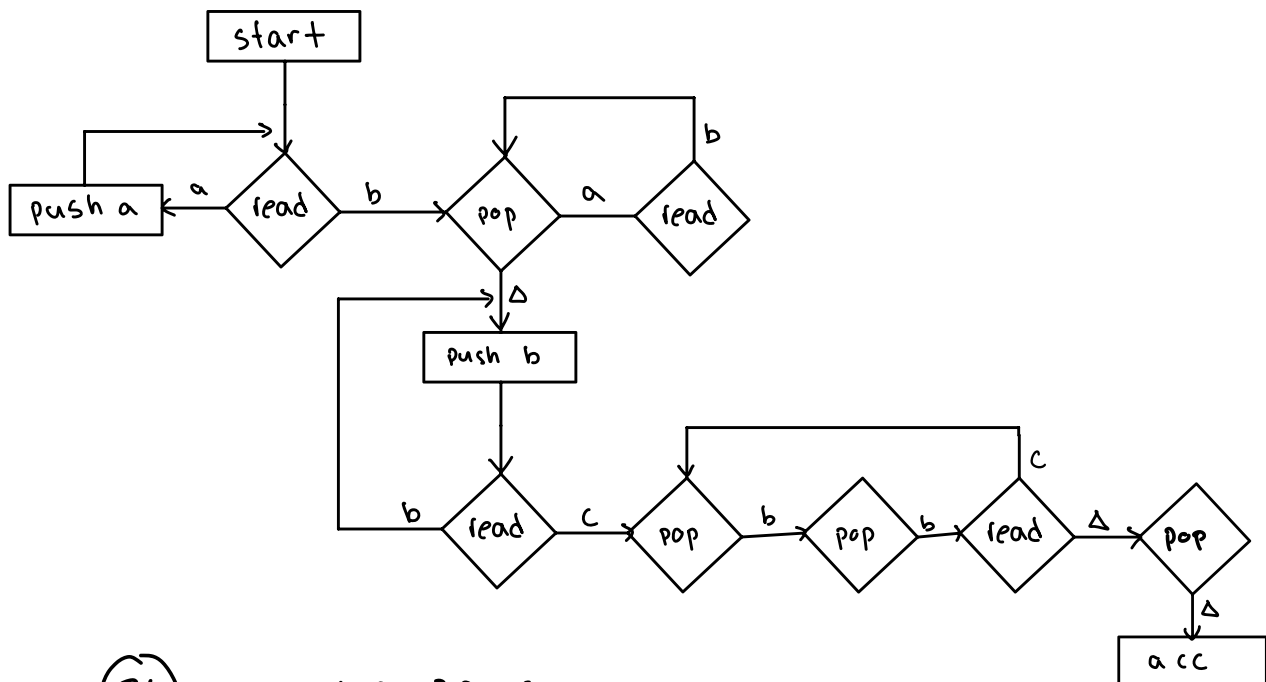
$$X \rightarrow b X \mid b$$

$$Trace: w = a^3 b^5$$



$a^3 b^5$ accepted

(2a) $L = \{a^n b^m c^r \mid \begin{matrix} m = 2r + n \\ r = 1, 2, \dots \\ n = 0, 1, \dots \end{matrix} \}$



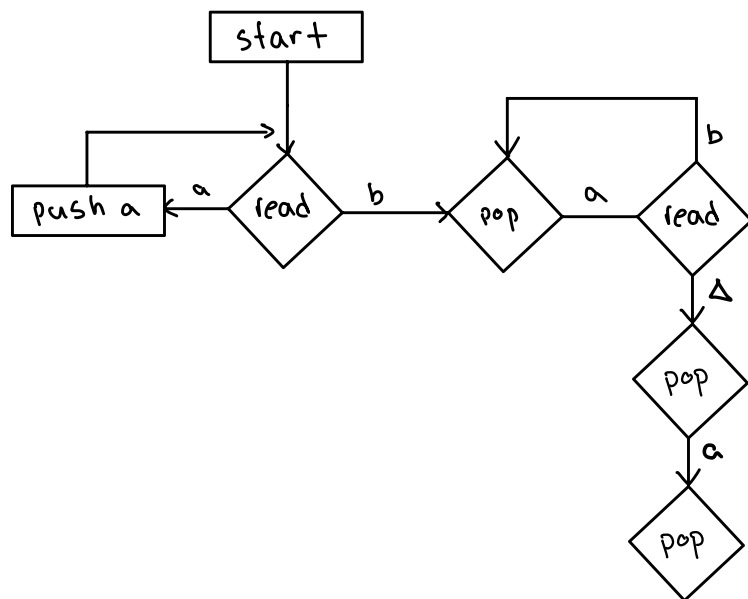
(2b) $a^n b^n b^{2r} c^r$

cfg: $S \rightarrow XY$

$X \rightarrow aXb \mid \lambda$

$Y \rightarrow bYc \mid bbc$

3a $L = \{ a^n b^m \mid n > m, m = 1, 2, \dots \}$



$a a a a a b b$

$\begin{matrix} a \\ a \\ a \\ a \\ a \end{matrix}$