Taller 03

Universidad Externado de Colombia

Departamento de Matemáticas

Estadística 2

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Inferencia estadística

(Thijseen 2016, problem 3.2) In the following scenarios, indicate (i) the population, (ii) the variable, (iii) the parameter, and (iv) the statistic.

1. A researcher wishes to know the average duration of unemployment among recent university graduates. She bases her conclusions on the average of a random sample of 210 unemployed recent university graduates.

2. A marketing officer wants to know the proportion of Vogue readers who would buy a new mascara. She bases her conclusions on the sample proportion of a random sample of 89 Vogue readers agreeing with the statement *I will use this new mascara.*

3. A quality control officer is concerned about the variability (as measured by the variance) of the diameter of a hinge used in the tail sections of Airbus aircraft of the A320 type. She bases her conclusion on the sample variance of a random sample of 121 hinges obtained from the sole manufacturer of these hinges.

Simulación

Se dice que una variable aleatoria X tiene distribución Gamma con parámetros $\alpha > 0$ (*shape parameter*) y $\beta > 0$ (*rate parameter*), lo cual se escribe $X \sim \mathsf{Gam}(\alpha, \beta)$, si la función de densidad de X es

$$f(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \, x^{\alpha-1} \, e^{-\beta x}$$

donde $\Gamma(x)$ es la función Gamma. Ver Sosa et al. (2012, Sección 10.5, p. 306) para más detalles acerca de la distribución Gamma.

Considere una población con distribución Gamma con $\alpha = 3$ y $\beta = 5$.

1. Escribir y graficar la función de densidad $f(x; \alpha, \beta)$ de la población.
2. ¿Cuál es el valor esperado (media) μ de la población? ¿Cuál es la varianza σ^2 de la población?
3. Teniendo en cuenta los resultados del numeral anterior, encontrar el valor esperado de la media muestral $\mu_{\bar{X}} = \mathbb{E}\left[\bar{X}\right]$ y la varianza de la media muestral $\sigma^2_{\bar{X}} = \mathbb{V}\text{ar}\left[\bar{X}\right]$.
4. Simular $M = 100,000$ muestras de tamaño $n = 30$ de la población y para cada una de ella calcular el promedio muestral $\bar{x}$.
5. Calcular el promedio de la variable promedio y compararlo con el valor teórico $\mu_{\bar{X}}$ correspondiente.
6. Calcular la varianza de la variable promedio y compararlo con el valor teórico $\sigma^2_{\bar{X}}$ correspondiente.
7. De acuerdo con el Teorema del Límite Central, ¿cuál es la distribución probabilística de la media muestral $\bar{X}$?
8. En una misma figura, graficar la población (en color gris) junto con la distribución teórica de la media muestral (en color rojo) y un histograma de la distribución empírica de la media muestral (en color negro).

Distribución muestral de la proporción

1. De Berenson et al. (2012), leer y sintetizar la Sección 7.4 (*Sampling Distribution of the Mean*, p. 258).

2. De Anderson et al. (2011), leer y sintetizar la Sección 7.6 (*Sampling Distribution of* $\bar{p}$, p. 289).

3. (Keller 2014, example 9.2) In the last election, a state representative received 52% of the votes cast. One year after the election, the representative organized a survey that asked a random sample of 300 people whether they would vote for him in the next election. If we assume that his popularity has not changed, what is the probability that more than half of the sample would vote for him?

4. (Berenson et al. 2014, problem 7.29) Companies often make flextime scheduling available to help recruit and keep female employees who have children. Other workers sometimes view these flextime schedules as unfair. An article in USA Today indicates that 25% of male employees state that they have to pick up the slack for moms working flextime schedules. (Data extracted from D. Jones, Poll Finds Resentment of Flextime, www.usatoday. com, May 11, 2007.) Suppose you select a random sample of 100 male employees working for companies offering flextime.

   a. What is the probability that 25% or fewer male employees will indicate that they have to pick up the slack for moms working flextime?

   b. What is the probability that 20% or fewer male employees will indicate that they have to pick up the slack for moms working flextime?

   c. If a random sample of 500 is taken, how does this change your answers to (a) and (b)?

5. (Anderson 2011, Sec. 7.6, problem 41) The Food Marketing Institute shows 17% of households spend more than $100 per week on groceries. Assume the population proportion is $p = 0.17$ and a simple random sample of $n = 800$ households will be selected from the population.

   a. Show the sampling distribution of $\bar{p}$, the sample proportion of households spending more than $100 per week on groceries.

b. What is the probability that the sample proportion will be within $\pm.02$ of the population proportion?

c. Answer part (b) for a sample of 1600 households.