Taller 05

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Departamento de Matemáticas

Estadística 2

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Estimadores de Máxima Verosimilitud

1. De Thijseen (2016), leer y sintetizar la Sección 6.2 (*Maximum likelihood estimators*, p. 77).

2. (Hogg et al. 2015, 6.4-7) Let $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$. Let $X_1, \ldots, X_n$ be a random sample of size n from this distribution.

   a. Sketch the probability density function of X for (i) $\theta = 1/2$, (ii) $\theta = 1$, and (iii) $\theta = 2$.

   b. Show that
   $$\hat{\theta}_{\text{MLE}} = -\frac{n}{\log\left(\prod_{i=1}^n X_i\right)}$$
   is the maximum likelihood estimator of θ.

   c. For each of the following three sets of 10 observations from the given distribution, calculate the values of the maximum likelihood estimate and the method-of- moments estimate of θ. See the data sets in Hogg et al. (2015, p. 265).

3. (Hogg et al. 2015, 6.4-8) Let
$$f(x;\theta) = \frac{1}{\theta}\, x^{(1-\theta)/\theta}, \qquad 0 < x < 1, \qquad 0 < \theta < \infty.$$

(a) Show that the maximum likelihood estimator of θ is

$$\hat{\theta}_{\text{MLE}} = -\frac{1}{n}\sum_{i=1}^{n} \log X_i.$$

(b) Show that $\mathbb{E}\left[\hat{\theta}_{\text{MLE}}\right] = \theta$ and thus that $\hat{\theta}_{\text{MLE}}$ is an unbiased estimator of θ.

Distribución Exponencial

(Hogg et al. 2015, example 6.4-1) Let $X_1, \ldots, X_n$ be a random sample from the Exponential distribution with probability density function

$$f(x;\theta) = \lambda\, e^{-\lambda\, x}, \qquad 0 < x < \infty, \qquad \lambda > 0.$$

Provide a general expression for the maximum likelihood estimator of λ.

Distribución Poisson

(Rice 2007, Sec. 8.5, example A) Let $X_1, \ldots, X_n$ be a random sample from the Poisson distribution with probability mass function

$$f(x;\lambda) = \mathbb{P}\mathrm{r}\left[X = x\right] = \frac{e^{-\lambda}\,\lambda^x}{x!}, \qquad x \in \{0, 1, 2, \ldots\}, \qquad \lambda > 0.$$

Provide a general expression for the maximum likelihood estimator of λ.

Distribución Normal

(Thijseen 2016, example 6.2) Let $X_1, \ldots, X_n$ be a random sample from the Normal distribution with mean μ and variance σ^2, where both μ and σ^2 are unknown. Recall that the probability density function of the Normal distribution is

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2}\,\frac{(x-\mu)^2}{\sigma^2}}, \quad -\infty < x < \infty, \;\; -\infty < \mu < \infty, \;\; 0 < \sigma^2 < \infty.$$

Provide a general expression for the maximum likelihood estimators of μ and σ^2.

Distribución Geométrica

(Rice 2007, Chap. 8, problem 7) Let $X_1, \ldots, X_n$ be a random sample from the Geometrix distribution with probability mass function

$$f(x; \theta|) = \mathbb{P}\mathrm{r}\left[X = x\right] = \theta(1-\theta)^{x-1}, \qquad x \in \{1, 2, \ldots\}, \qquad 0 < \theta < 1.$$

Provide a general expression for the maximum likelihood estimator of θ.