

## Question 2

### Part A

$$\hat{u}_i = \text{residuals}$$

$$\hat{u}_i = y_i - \hat{y}_i, \quad i = 1, 2, \dots, n$$

$$= y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)$$

$$= y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$$

from Assignment eq(7)

by definition  $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

$$\sum_{i=1}^n \hat{u}_i = \sum_{i=1}^n y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)$$

$$= \sum_{i=1}^n y_i - \sum_{i=1}^n (\hat{\beta}_0 + \hat{\beta}_1 x_i)$$

$$= \sum_{i=1}^n y_i - \sum_{i=1}^n \hat{\beta}_0 - \sum_{i=1}^n \hat{\beta}_1 x_i$$

$$= \sum_{i=1}^n y_i - \sum_{i=1}^n \hat{\beta}_1 x_i - n\hat{\beta}_0$$

rearranging terms and summing  $\hat{\beta}_0$

$$= \frac{n}{n} \sum_{i=1}^n y_i - \frac{n}{n} \sum_{i=1}^n \hat{\beta}_1 x_i - n\hat{\beta}_0$$

times first and second sum by  $\frac{n}{n}$

$$= n\bar{y} - n\hat{\beta}_1 \bar{x} - n\hat{\beta}_0$$

$$= n\bar{y} - n\hat{\beta}_1 \bar{x} - n(\bar{y} - \hat{\beta}_1 \bar{x})$$

substituting from Assignment eq(6)

$$= n\bar{y} - n\hat{\beta}_1 \bar{x} - n\bar{y} + n\hat{\beta}_1 \bar{x}$$

$$= n\bar{y} - n\bar{y} - n\hat{\beta}_1 \bar{x} + n\hat{\beta}_1 \bar{x}$$

$$= 0$$

### Part B

$$SSR(b_0, b_1) = \sum_{i=1}^n (y_i - b_0 - b_1 x_i)^2$$

$$\left. \frac{\partial SSR(b_0, b_1)}{\partial b_0} \right|_{\hat{\beta}_0, \hat{\beta}_1} = -2 \sum_{i=1}^n (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$

$$\left. \frac{\partial SSR(b_0, b_1)}{\partial b_1} \right|_{\hat{\beta}_0, \hat{\beta}_1} = -2 \sum_{i=1}^n x_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$

call this (1)

$$\hat{u}_i = y_i - \hat{y}_i, \quad i = 1, 2, \dots, n$$

from Assignment eq(7)

$$= y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)$$

by definition  $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

$$= y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$$

call this (2)

$$-2 \sum_{i=1}^n x_i (\hat{u}_i) = 0 \quad \text{substituting (2) into (1)}$$

$$\sum_{i=1}^n x_i \hat{u}_i = 0 \quad \text{call this (3)}$$

Now if  $\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}_{(n \times 1)}$  and  $\hat{\mathbf{u}} = \begin{pmatrix} \hat{u}_1 \\ \vdots \\ \hat{u}_n \end{pmatrix}_{(n \times 1)}$  then the dot product is

$$\begin{aligned} \mathbf{x}' \hat{\mathbf{u}} &= (x_1 \quad \dots \quad x_n) \begin{pmatrix} \hat{u}_1 \\ \vdots \\ \hat{u}_n \end{pmatrix} \\ &= x_1 \hat{u}_1 + \dots + x_n \hat{u}_n \end{aligned}$$

just the linear combinations of column vector  $\mathbf{u}$   
where the scalars are the components of  $\mathbf{x}$

$$\begin{aligned} &= \sum_{i=1}^n x_i \hat{u}_i \\ &= 0 \end{aligned}$$

from (3)