Question 2

Part A

$$\hat{u}_i = residuals$$
$$\hat{u}_i = y_i - \hat{y}_i, \ \ i = 1, 2, ...n \qquad\qquad \text{from Assignment eq(7)}$$
$$= y_i - \left(\hat{\beta}_o + \hat{\beta}_1 x_i\right) \qquad\qquad \text{by definition } \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$
$$= y_i - \hat{\beta}_o - \hat{\beta}_1 x_i$$

$$\sum_{i=1}^{n} \hat{u}_i = \sum_{i=1}^{n} y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)$$
$$= \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} (\hat{\beta}_0 + \hat{\beta}_1 x_i)$$
$$= \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \hat{\beta}_0 - \sum_{i=1}^{n} \hat{\beta}_1 x_i$$
$$= \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \hat{\beta}_1 x_i - n\hat{\beta}_0 \qquad\qquad \text{rearranging terms and summing } \hat{\beta}_0$$
$$= \frac{n}{n} \sum_{i=1}^{n} y_i - \frac{n}{n} \sum_{i=1}^{n} \hat{\beta}_1 x_i - n\hat{\beta}_0 \qquad\qquad \text{times first and second sum by } \frac{n}{n}$$
$$= n\bar{y} - n\hat{\beta}_i\bar{x} - n\hat{\beta}_0$$
$$= n\bar{y} - n\hat{\beta}_i\bar{x} - n(\bar{y} - \hat{\beta}_i\bar{x}) \qquad\qquad \text{substituting from Assignment eq(6)}$$
$$= n\bar{y} - n\hat{\beta}_i\bar{x} - n\bar{y} + n\hat{\beta}_i\bar{x}$$
$$= n\bar{y} - n\bar{y} - n\hat{\beta}_i\bar{x} + n\hat{\beta}_i\bar{x}$$
$$= 0$$

Part B

$$SSR \ (b_0, b_1) = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2$$
$$\left.\frac{\partial SSR \ (b_0, b_1)}{\partial b_0}\right|_{\hat{\beta}_0, \hat{\beta}_1} = -2\sum_{i=1}^{n} \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i\right) = 0$$
$$\left.\frac{\partial SSR \ (b_0, b_1)}{\partial b_i}\right|_{\hat{\beta}_0, \hat{\beta}_1} = -2\sum_{i=1}^{n} x_i \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i\right) = 0 \qquad\qquad \text{Call this (1)}$$

$$\hat{u}_i = y_i - \hat{y}_i, \ \ i = 1, 2, ...n \qquad\qquad \text{from Assignment eq(7)}$$
$$= y_i - \left(\hat{\beta}_o + \hat{\beta}_1 x_i\right) \qquad\qquad \text{by definition } \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$
$$= y_i - \hat{\beta}_o - \hat{\beta}_1 x_i \qquad\qquad \text{Call this (2)}$$

$$-2\sum_{i=1}^{n} x_i(\hat{u}_i) = 0 \qquad\qquad \text{substituting (2) into (1)}$$

$$\sum_{i=1}^{n} x_i\hat{u}_i = 0 \qquad\qquad \text{Call this (3)}$$

Now if $\boldsymbol{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}_{(n\times 1)}$ and $\hat{\boldsymbol{u}} = \begin{pmatrix} \hat{u}_1 \\ \vdots \\ \hat{u}_n \end{pmatrix}_{(n\times 1)}$ then the dot product is

$$\boldsymbol{x'}\hat{\boldsymbol{u}} = \begin{pmatrix} x_1 & \dots & x_n \end{pmatrix} \begin{pmatrix} \hat{u}_1 \\ \vdots \\ \hat{u}_n \end{pmatrix}$$

$$= x_1\hat{u}_1 + \dots + x_n\hat{u}_n \qquad\qquad \text{just the linear combinations of column vector } \boldsymbol{u}$$
$$\text{where the scalars are the components of } \boldsymbol{x}$$

$$= \sum_{i=1}^{n} x_i\hat{u}_i$$

$$= 0 \qquad\qquad \text{from (3)}$$