Question 2

Part A

$$
\begin{aligned}
\hat{u}_i &= residuals \\
\hat{u}_i &= y_i - \hat{y}_i,\ \ i = 1, 2, ...n && \text{from Assignment eq(7)} \\
&= y_i - \left(\hat{\beta}_o + \hat{\beta}_1 x_i\right) && \text{by definition } \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \\
&= y_i - \hat{\beta}_o - \hat{\beta}_1 x_i
\end{aligned}
$$

$$
\begin{aligned}
\sum_{i=1}^{n} \hat{u}_i &= \sum_{i=1}^{n} y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i) \\
&= \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} (\hat{\beta}_0 + \hat{\beta}_1 x_i) \\
&= \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \hat{\beta}_0 - \sum_{i=1}^{n} \hat{\beta}_1 x_i \\
&= \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \hat{\beta}_1 x_i - n\hat{\beta}_0 && \text{rearranging terms and summing } \hat{\beta}_0 \\
&= \frac{n}{n} \sum_{i=1}^{n} y_i - \frac{n}{n} \sum_{i=1}^{n} \hat{\beta}_1 x_i - n\hat{\beta}_0 && \text{times first and second sum by } \frac{n}{n} \\
&= n\bar{y} - n\hat{\beta}_i \bar{x} - n\hat{\beta}_0 \\
&= n\bar{y} - n\hat{\beta}_i \bar{x} - n(\bar{y} - \hat{\beta}_i \bar{x}) && \text{substituting from Assignment eq(6)} \\
&= n\bar{y} - n\hat{\beta}_i \bar{x} - n\bar{y} + n\hat{\beta}_i \bar{x} \\
&= n\bar{y} - n\bar{y} - n\hat{\beta}_i \bar{x} + n\hat{\beta}_i \bar{x} \\
&= 0
\end{aligned}
$$

Part B

$$
\begin{aligned}
SSR\ \ (b_0, b_1) &= \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2 \\
\left.\frac{\partial SSR\,(b_0, b_1)}{\partial b_0}\right|_{\hat{\beta}_0, \hat{\beta}_1} &= -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0 \\
\left.\frac{\partial SSR\,(b_0, b_1)}{\partial b_i}\right|_{\hat{\beta}_0, \hat{\beta}_1} &= -2 \sum_{i=1}^{n} x_i (y_i - b_0 - b_1 x_i) = 0 \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
\hat{u}_i &= y_i - \hat{y}_i,\ \ i = 1, 2, ...n && \text{from Assignment eq(7)} \\
&= y_i - \left(\hat{\beta}_o + \hat{\beta}_1 x_i\right) && \text{by definition } \hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \\
&= y_i - \hat{\beta}_o - \hat{\beta}_1 x_i && (2)
\end{aligned}
$$

$$
\begin{aligned}
-2\sum_{i=1}^{n} x_i(\hat{u}_i) &= 0 && \text{substituting (2) into (1)} \\
\sum_{i=1}^{n} x_i\hat{u}_i &= 0 && (3)
\end{aligned}
$$

Now if $\underset{(n\times 1)}{\boldsymbol{x}} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ and $\underset{(u\times 1)}{\hat{\boldsymbol{u}}} = \begin{pmatrix} \hat{u}_1 \\ \vdots \\ \hat{u}_n \end{pmatrix}$ then the dot product is

$$
\begin{aligned}
\boldsymbol{x}'\hat{\boldsymbol{u}} &= \begin{pmatrix} \hat{x}_1 & ... & \hat{x}_n \end{pmatrix} \begin{pmatrix} \hat{u}_1 \\ \vdots \\ \hat{u}_n \end{pmatrix} \\
&= x_1\hat{u}_1 + \ ... \ + x_n\hat{u}_n && \text{just the linear combinations of column vector } \boldsymbol{u} \\
& && \text{where the scalars are the components of } \boldsymbol{x} \\
&= \sum_{i=1}^{n} x_i\hat{u}_i \\
&= 0 && \text{from (3)}
\end{aligned}
$$