

## Question 2

### Part A

$$\begin{aligned}
 \hat{u}_i &= \text{residuals} \\
 \hat{u}_i &= y_i - \hat{y}_i, \quad i = 1, 2, \dots, n \\
 &= y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i) \\
 &= y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i
 \end{aligned}$$

from Assignment eq(7)  
by definition  $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

$$\begin{aligned}
 \sum_{i=1}^n \hat{u}_i &= \sum_{i=1}^n y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i) \\
 &= \sum_{i=1}^n y_i - \sum_{i=1}^n (\hat{\beta}_0 + \hat{\beta}_1 x_i) \\
 &= \sum_{i=1}^n y_i - \sum_{i=1}^n \hat{\beta}_0 - \sum_{i=1}^n \hat{\beta}_1 x_i \\
 &= \sum_{i=1}^n y_i - \sum_{i=1}^n \hat{\beta}_1 x_i - n \hat{\beta}_0 \\
 &\quad \text{rearranging terms and summing } \hat{\beta}_0 \\
 &= \frac{n}{n} \sum_{i=1}^n y_i - \frac{n}{n} \sum_{i=1}^n \hat{\beta}_1 x_i - n \hat{\beta}_0 \\
 &= n \bar{y} - n \hat{\beta}_1 \bar{x} - n \hat{\beta}_0 \\
 &= n \bar{y} - n \hat{\beta}_1 \bar{x} - n(\bar{y} - \hat{\beta}_1 \bar{x}) \\
 &= n \bar{y} - n \hat{\beta}_1 \bar{x} - n \bar{y} + n \hat{\beta}_1 \bar{x} \\
 &= n \bar{y} - n \bar{y} - n \hat{\beta}_1 \bar{x} + n \hat{\beta}_1 \bar{x} \\
 &= 0
 \end{aligned}$$

times first and second sum by  $\frac{n}{n}$   
substituting from Assignment eq(6)

### Part B

$$\begin{aligned}
 SSR(b_0, b_1) &= \sum_{i=1}^n (y_i - b_0 - b_1 x_i)^2 \\
 \frac{\partial SSR(b_0, b_1)}{\partial b_0} \Big|_{\hat{\beta}_0, \hat{\beta}_1} &= -2 \sum_{i=1}^n (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0 \\
 \frac{\partial SSR(b_0, b_1)}{\partial b_i} \Big|_{\hat{\beta}_0, \hat{\beta}_1} &= -2 \sum_{i=1}^n x_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0 \quad \text{call this (1)}
 \end{aligned}$$

$$\begin{aligned}
 \hat{u}_i &= y_i - \hat{y}_i, \quad i = 1, 2, \dots, n \\
 &= y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i) \\
 &= y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i
 \end{aligned}$$

from Assignment eq(7)  
by definition  $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$   
call this (2)

$$-2 \sum_{i=1}^n x_i (\hat{u}_i) = 0 \quad \text{substituting (2) into (1)}$$

$$\sum_{i=1}^n x_i \hat{u}_i = 0 \quad \text{call this (3)}$$

Now if  $\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \\ (n \times 1) \end{pmatrix}$  and  $\hat{\mathbf{u}} = \begin{pmatrix} \hat{u}_1 \\ \vdots \\ \hat{u}_n \\ (n \times 1) \end{pmatrix}$  then the dot product is

$$\begin{aligned} \mathbf{x}' \hat{\mathbf{u}} &= (x_1 \quad \dots \quad x_n) \begin{pmatrix} \hat{u}_1 \\ \vdots \\ \hat{u}_n \end{pmatrix} \\ &= x_1 \hat{u}_1 + \dots + x_n \hat{u}_n \quad \text{just the linear combinations of column vector } \mathbf{u} \\ &= \sum_{i=1}^n x_i \hat{u}_i \quad \text{where the scalars are the components of } \mathbf{x} \\ &= 0 \quad \text{from (3)} \end{aligned}$$