

## Question 2

### Part A

$\hat{u} = \text{prediction error}$

$$\hat{u}_i = y_i - \hat{y}_i \quad (1)$$

$$= y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i) \quad (2)$$

$$= y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i \quad (3)$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{u}_i = y_i - (\bar{y} - \hat{\beta}_1 \bar{x}) - \hat{\beta}_1 x_i \quad (4)$$

$$= (y_i - \bar{y}) - \hat{\beta}_1 (x_i - \bar{x}) \quad (5)$$

$$\sum_{i=1}^n \hat{u}_i = \sum_{i=1}^n [(y_i - \bar{y}) - \hat{\beta}_1 (x_i - \bar{x})] \quad (6)$$

$$= \sum_{i=1}^n (y_i - \bar{y}) - \sum_{i=1}^n \hat{\beta}_1 (x_i - \bar{x}) \quad (7)$$

$$= \sum_{i=1}^n (y_i - \bar{y}) - \hat{\beta}_1 \sum_{i=1}^n (x_i - \bar{x}) \quad (8)$$

$$= 0 \quad (9)$$

### Part B

$$SSR(b_0, b_1) = \sum_{i=1}^n (y_i - b_0 - b_1 x_i)^2$$

$$\left. \frac{\partial SSR(b_0, b_1)}{\partial b_0} \right|_{\hat{\beta}_0, \hat{\beta}_1} = -2 \sum_{i=1}^n (y_i - b_0 - b_1 x_i) = 0 \quad (10)$$

$$\left. \frac{\partial SSR(b_0, b_1)}{\partial b_1} \right|_{\hat{\beta}_0, \hat{\beta}_1} = -2 \sum_{i=1}^n x_i (y_i - b_0 - b_1 x_i) = 0 \quad (11)$$

where  $\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$  and  $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$  are the OLS estimators

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i \quad (12)$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (13)$$

$\bar{\beta}_0$  is the predicted value of  $y_i$  when  $x_i = 0$

$$\sum_{i=0}^n \hat{u}_i = 0$$

where  $u_i = y_i - \hat{y}_i$ ,  $i = 1, 2, \dots, n$   
 $u_i = y_i - \hat{\beta}_0 - \hat{\beta}_0 x_i$ ,  $i = 1, 2, \dots, n$