Question 2

Part A

$$\hat{u} = prediction\ error$$

$$\begin{aligned} \hat{u}_i &= y_i - \hat{y}_i && (1)\\ &= y_i - \left(\hat{\beta}_o + \hat{\beta}_1 x_i\right) && (2)\\ &= y_i - \hat{\beta}_o - \hat{\beta}_1 x_i && (3)\\ & && (4) \end{aligned}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\begin{aligned} \hat{u}_i &= y_i - \left(\bar{y} - \hat{\beta}_1 \bar{x}\right) - \hat{\beta}_1 x_i && (5)\\ &= (y_i - \bar{y}) - \hat{\beta}_1 (u_i - \bar{x}) && (6)\\ & && (7) \end{aligned}$$

$$\begin{aligned} \sum_{i=1}^{n} \hat{u}_i &= \sum_{i=1}^{n} \left[(y_i - \bar{y}) - \hat{\beta}_1 (u_i - \bar{x})\right] && (8)\\ &= \sum_{i=1}^{n} (y_i - \bar{y}) - \sum_{i=1}^{n} \hat{\beta}_1 (u_i - \bar{x}) && (9)\\ &= \sum_{i=1}^{n} (y_i - \bar{y}) - \hat{\beta}_1 \sum_{i=1}^{n} (u_i - \bar{x}) && (10)\\ &= 0 && (11)\\ & && (12) \end{aligned}$$

Part B

$$SSR\ (b_0, b_1) = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2$$

$$\begin{aligned} \left.\frac{\partial SSR(b_0, b_1)}{\partial (b_0)}\right|_{\hat{\beta}_0, \hat{\beta}_1} &= -2 \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i) = 0 && (13)\\ \left.\frac{\partial SSR(b_0, b_1)}{\partial (b_i)}\right|_{\hat{\beta}_0, \hat{\beta}_1} &= -2 \sum_{i=1}^{n} x_i (y_i - b_0 - b_1 x_i) = 0 && (14)\\ & && (15) \end{aligned}$$

where $\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=0}^{n} ((x_i - \bar{x})^2}$ and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ are the OLS estimators

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{16}$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{17}$$

$$\tag{18}$$

$\bar{\beta}_0$ is the predicted value of y_i when $x_i = 0$

$$\sum_{i=0}^{n} \hat{u}_i = 0$$

where $u_i = y_i - \hat{y}_i,\ i = 1,\ 2,\ \ldots\ , n$
$u_i = y_i - \hat{\beta}_0 - \hat{\beta}_0, x_i,\ i = 1,\ 2,\ \ldots\ , n$