

Design of Experiments

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Contents

1	About	7
1.1	License and contributing	7
2	Experimenting	9
2.1	Why do we experiment? The causality problem	10
2.2	The elements of an experiment	10
2.3	Validity of an experiment	10
2.4	Running an experiment	10
2.5	To explore further	10
3	Analyzing an experiment	11
3.1	Compiling the experimental matrix	12
3.2	Analyzing the experiment	12
3.3	Identifying relevant contributions	12
3.4	Visualizing results, effects and interactions	12
3.5	Analyzing through models	12
3.6	Errors estimation	12
4	One factor experiments	13
4.1	Two-level experiments	13
4.2	Multiple-level experiments	13
4.3	To explore further	13

5	Full factorial designs	15
5.1	OFAT and their limitations	15
5.2	Full factorial designs	15
5.3	Degrees of freedom	15
5.4	Interactions and error	15
5.5	To explore further	15
6	Fractional and saturated factorial designs	17
6.1	Fractional factorial designs	18
6.2	Saturated factorial designs	18
6.3	Plackett-Burman designs	18
6.4	Taguchi's approach to orthogonal arrays	18
6.5	Follow-up	18
6.6	To explore further	18
7	Non orthogonal arrays	19
7.1	Modified Taguchi's orthogonal arrays	19
7.2	Optimal designs	19
7.3	Supersaturated designs	19
8	Taguchi Robust Design strategy	21
8.1	Noise and control factors	21
8.2	Inner and outer arrays designs	21
8.3	Two-step optimization technique	21
9	Surface response designs	23
9.1	Three-level full factorial designs	23
9.2	Box-Wilson central composite designs	23
9.3	Box-Behnken designs	23
9.4	Doehlert designs	23
9.5	Sequential design of experiments	23
9.6	To explore further	23

<i>CONTENTS</i>	5
10 Sequential optimization	25
10.1 EVOP	25
10.2 Simplex	25
11 Advanced and uncovered topics	27
11.1 Designs for nonlinear effects	27
11.2 Designs for systems with restrictions	27
11.3 Random levels	27
References	29

Chapter 1

About

This book has been developed by Jordi Cuadros (ORCID) and Vanessa Serrano as a reference material for a Master's course in Design of Experiments at IQS Universitat Ramon Llull (<http://www.iqs.url.edu>) in Barcelona, Spain. First version (always a draft) was published on September 2022.

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1.1 License and contributing

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Should you be interested in contributing to this work, suggest corrections or improvements, or just say thanks, feel free to reach the authors at jordi.cuadros@iqs.url.edu.

Chapter 2

Experimenting

Experimenting is a common task in any scientific endeavor. However, and this is not uncommon, being a common task does not warrant a full comprehension of it.

Let's refresh here some fundamental concepts.

2.1 Why do we experiment? The causality problem

2.2 The elements of an experiment

2.2.1 A basic schema for an experiment

2.2.2 Types of variables

2.2.3 Data types

2.3 Validity of an experiment

2.3.1 Internal validity

2.3.2 External validity

2.4 Running an experiment

2.4.1 Replication and repetition

2.4.2 Randomization

2.5 To explore further

2.5.1 Hierarchy of evidence

https://en.wikipedia.org/wiki/Hierarchy_of_evidence

Chapter 3

Analyzing an experiment

3.1 Compiling the experimental matrix

3.2 Analyzing the experiment

3.2.1 Ortogonality

3.2.2 Confounded variables

3.2.3 Optimality

3.2.4 Efficacy

3.2.5 Sphericity

3.3 Identifying relevant contributions

3.3.1 Pareto chart of the effects

3.3.2 Pareto chart of the contributions

3.3.3 Normal plot

3.3.4 Half-normal plot

3.4 Visualizing results, effects and interactions

3.4.1 Main effects plots

3.4.2 Interaction plots

3.5 Analyzing through models

3.5.1 Quantitative models

3.5.2 Models with nominal variables

Chapter 4

One factor experiments

4.1 Two-level experiments

4.2 Multiple-level experiments

4.2.1 Quantitative variables

4.2.2 Nominal levels

4.3 To explore further

4.3.1 Single-subject experiments

4.3.2 Salomon design

Chapter 5

Full factorial designs

5.1 OFAT and their limitations

5.2 Full factorial designs

5.2.1 Two-levels factorial designs

5.2.1.1 Standard coding of the experimental conditions

5.2.1.2 Yates order

5.2.2 Multiple-levels factorial designs

5.2.3 Mixed-levels factorial designs

5.3 Degrees of freedom

5.4 Interactions and error

5.5 To explore further

5.5.1 Latin and grecolatin square

5.5.2 Nested, split-plot, split-split and strip-strip designs

5.5.3 BIBD

Chapter 6

Fractional and saturated factorial designs

6.1 Fractional factorial designs

6.1.1 Two-levels fractional factorial design

6.1.2 Alias structure and design generator

6.1.3 Resolution of a design

6.2 Saturated factorial designs

6.3 Plackett-Burman designs

6.4 Taguchi's approach to orthogonal arrays

6.5 Follow-up

6.5.1 Follow-up designs

6.5.2 Confirmatory runs

6.6 To explore further

6.6.1 Adding center points

6.6.2 Definitive screening designs

Chapter 7

Non orthogonal arrays

7.1 Modified Taguchi's orthogonal arrays

7.1.1 Column upgrading/merging

7.1.2 Dummy treatment/level or column downgrading/degrading

7.2 Optimal designs

7.3 Supersaturated designs

Chapter 8

Taguchi Robust Design strategy

8.1 Noise and control factors

8.2 Inner and outer arrays designs

8.3 Two-step optimization technique

Chapter 9

Surface response designs

9.1 Three-level full factorial designs

9.2 Box-Wilson central composite designs

9.3 Box-Behnken designs

9.4 Doehlert designs

9.5 Sequential design of experiments

9.6 To explore further

9.6.1 Optimal designs for RSM

Chapter 10

Sequential optimization

10.1 EVOP

10.2 Simplex

Chapter 11

Advanced and uncovered topics

11.1 Designs for nonlinear effects

11.2 Designs for systems with restrictions

11.2.1 Mixtures designs

11.3 Random levels

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