DS-GA 1007 | Lecture 3

Programming for Data Science

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DS-GA 1007 Curriculum

Programming for Data Science:

- Introduction to Programming in Python
- Best Practice Programming and Software Engineering
- **Program Efficiency**
- Interacting with Programs
- Array Manipulation for Scientific Computing
- Data Visualization
- Advanced Data Objects ($\times 4$)
- Environments for Collaborative Programming
- Industrial Applications

Program Efficiency

Program Efficiency

Last week:

- Modularization and Abstraction
- Functions and Objects in Python
- Testing and Debugging Programs

Today:

- **Program Run Time and Algorithmic Complexity**
- **Examples of Iterative and Recursive Algorithms**
- **Examples of Search and Sort Algorithms**

Program Run Time and Algorithmic Complexity

What is Program Efficiency?

How to compare the efficiency of two different programs?

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- How to measure the efficiency of an algorithm independent of machine or specific implemention?
- How to reason about an algorithm to predict the amount of time it will need to solve a problem of a particular size?

What is Program Efficiency?

How to compare the efficiency of two different programs?

- How to measure the efficiency of an algorithm independent of machine or specific implemention?
- How to reason about an algorithm to predict the amount of time it will need to solve a problem of a particular size?
- How to relate choices in algorithm design to time efficiency?

What is Program Efficiency?

How to compare the efficiency of two different programs?

- How to measure the efficiency of an algorithm independent of machine or specific implemention?
- How to reason about an algorithm to predict the amount of time it will need to solve a problem of a particular size?
- How to relate choices in algorithm design to time efficiency?

Example of algorithm to compute n^2:

```
def square(n):
    n2 = 0
    for i in range(n):
        for j in range(n):
            n2 += 1
    return n2
```

Isn't there a more efficient way?

Measures of a program's efficiency

Some options:

1. Measure the run time with a timer
2. Count the number of operations
3. Measure an order of growth as function of input size

Measure the run time of a program

Measure clock time on test data

- ✓ Run time varies between different algorithms
- × Run time varies between implementations
- × Run time varies between computers
- × Run time for large inputs not predictable based on small inputs

Example:

```
import time
t0 = time.time()
fact = 1
for i in range(1,1000):
   fact *= i
dt = time.time() - t0
print("Run time: {} seconds".format(dt))
```

Example of run time measurement

Measure run time with %timeit*

```
%timeit L = [n**2 for n in range(1000)]
```

Output:

```
148 µs ± 64 ns per loop (mean ± std dev of 7 runs, 10,000 loops each)
```

```
%timeit L = [square(n) for n in range(1000)]
```

Output:

```
7.34 s ± 60.5 ms per loop (mean ± std. dev. of 7 runs, 1 loop each)
```

*Details on Python magic commands:
jakevdp.github.io/PythonDataScienceHandbook/01.03-magic-commands.html

Counting Operations in a Program

Count number of operations as function of input size:

- ✓ #ops varies between different algorithms
- × #ops varies between implementations
- ✓ #ops does not vary between computers
- ✓ #ops for large inputs is predictable based on small inputs
- × No clear definition of which operations to count

Example:

```
def factorial(n):
    fact = 1                   1 ops
    for i in range(1,n+1):     1 ops
        fact *= i              2 ops
    return fact                => 1 + 3n ops
```

Generalization: Order of Growth

Count number of *key* operations in term of input size

- **#ops**: Measures an algorithm's run time in term of input size but does not measure scalability to arbitrarily large input size

- **Asymptotic Growth**: We need measure scalability to arbitrarily large input size, independent of specific implementation

- **Focus on Dominant Terms**: We need measure an invariant order of magnitude based on largest factors/bottlenecks

- **Lower/Upper Bound**: We can measure the average, best or worst case over all possible inputs of a given size

O() as Order of Growth of a Program

Upper Bound Asymptotic Growth relative to input size:

- ✓ $O()$ varies between algorithms, not implementations
- ✓ $O()$ measures rate of growth of run time as input size grows
- ✓ $O()$ is a tight upper bound on order of magnitude growth

Example:

```
def factorial(n):
    fact = 1                  1 ops
    for i in range(1,n+1):    1 ops
        fact *= i             2 ops
    return fact               => 1 + 3n ops
```

Ignore additive constants $=> 3n$ and multiplicative constants $=> n$

Worst case asymptotic complexity $= O(n)$

Analyzing Complexity of a Program

Order of Growth of a Program O():

- **Rule 1: Focus on dominant terms inside statements**: Drop additive and multiplicative constants

Examples:

- $1 + 3n$ has worst case complexity of $O(n)$
- $2 + 2n + n^2$ has worst case complexity of $O(n^2)$
- $10000 + 1000000n + n^2$ has worst case complexity of $O(n^2)$
- $n^5 + 5^n$ has worst case complexity of $O(5^n)$

Analyzing Complexity of a Program

Order of Growth of a Program O():

- **Rule 2: Use the law of addition for sequential statements**: Just *add up* orders of growth between consecutive operations $O(f(n) + g(n)) = O(f(n)) + O(g(n))$

Example:

```
for i in range(n):
   print(i)
for j in range(n*n):
   print(j)
```

- The first *for* loop has worst case complexity of $O(n)$
- The second *for* loop has worst case complexity of $O(n^2)$
- Worst case complexity $=> O(n) + O(n^2) = O(n^2)$

Analyzing Complexity of a Program

Order of Growth of a Program O():

- **Rule 3: Use the law of multiplication for nested statements**: Multiply orders of growth between nested loops because an inner loop is repeated for each outer loop iteration
  $O(f(n) \times g(n)) = O(f(n)) \times O(g(n))$

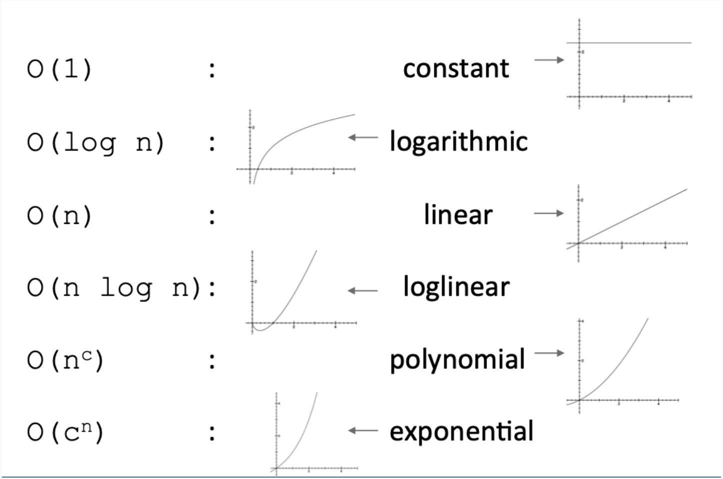
Example:

```
for i in range(n):
    for j in range(n):
        print(i,j)
```

- The first *for* loop has worst case complexity of $O(n)$
- The second *for* loop has worst case complexity of $O(n)$
- Worst case complexity $=> O(n) \times O(n) = O(n^2)$

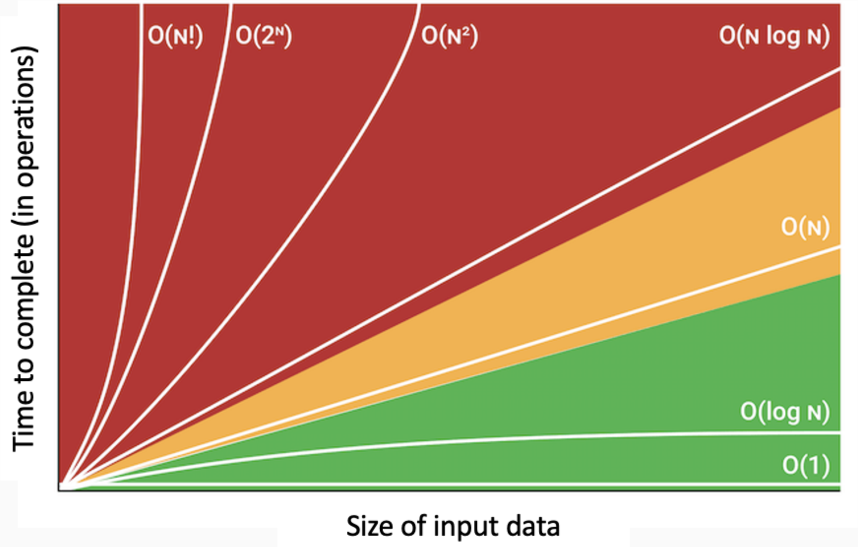
Map of Algorithmic Complexities

O() relates choices in algorithm design to time efficiency. The goal of the programmer is to write algorithms that go up in the map



O(1) :	constant
O(log n) :	logarithmic
O(n) :	linear
O(n log n):	loglinear
$O(n^c)$:	polynomial
$O(c^n)$:	exponential

Map of Algorithmic Complexities



Examples of Algorithm Complexity Analysis

Algorithm Complexity Analysis: x^2

Example of iterative nested loop to calculate x^2:

```
def square(x):
    x2 = 0
    for i in range(x):
        for j in range(x):
            x2 += 1
    return x2
```

Algorithm Complexity: $O(n^2)$

Algorithm Complexity Analysis: x²

Example of recursive function to calculate x^2

$$
\begin{aligned}
(x-1)^2 &= x^2 - 2x + 1 \\
<=> \quad x^2 &= (x-1)^2 + 2x - 1
\end{aligned}
$$

```
def square(x):
    if x == 0:
        return x
    else:
        return (square(x-1) + 2*x - 1)
```

Algorithm Complexity: $O(n)$

Algorithm Complexity Analysis: !x

Example of iterative loop to calculate !x:

```
def factorial(x):
    fact = 1
    for i in range(1,x+1):
        fact *= i
    return fact
```

Algorithm Complexity: $O(n)$

Algorithm Complexity Analysis: !x

Example of recursive function to calculate $!x$

```
def factorial(x):
    if x <= 1:
        return 1
    else:
        fact = x * factorial(x-1)
        return fact
```

Algorithm Complexity: $O(n)$

Search and Sort Algorithms

Linear Search of Element in List

Linear Search: Unsorted List

```
def search(L,e):
    found = False
    for i in range(len(L)):
        if L[i] == e:
            found = True
    return found
```

Algorithm Complexity: $O(n)$

(due to worst case scenarios)

Linear Search of Element in List

Linear Search: Sorted List

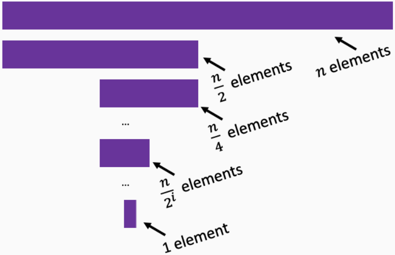
```
def search(L, e):
    for i in range(len(L)):
        if L[i] == e:
            return True
        if L[i] > e:
            return False
    return False
```

Algorithm Complexity: $O(n)$

(due to worst case scenarios)

Recursive Search of Element in List

Bisection Search



Search complete when
$\frac{n}{2^i} = 1 \ <=> \ i = log\, n$

Complexity: $O(log\, n)$

- ▶ Pick index that divides list in half, test if $L[m] == e$. If not, test if $L[m]$ is larger or smaller than e. Then, depending on the answer, search left or right half of L

- ▶ Bisection is a divide-and-conquer algorithm: Break original problem in smaller versions of the problem (smaller lists)

Recursive Search of Element in List

Bisection Search: Sorted List*

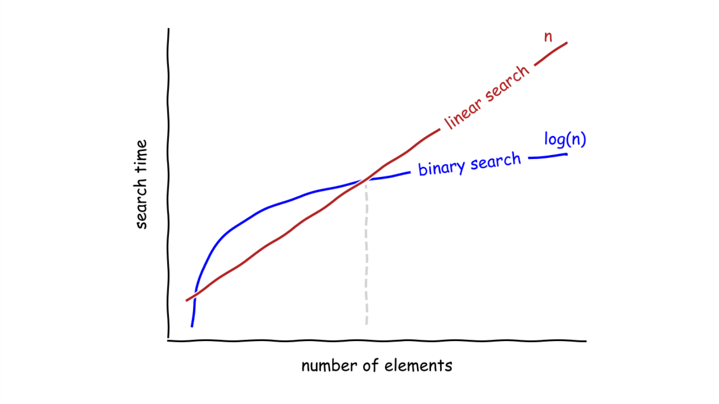
```
def bisection_search(L,e,low,high):
    if high == low:
        return L[low] == e
    mid = (low + high)//2
    if L[mid] == e:
        return True
    elif L[mid] > e:
        if low == mid:  # Nothing left to search
            return False
        else:
            return bisection_search(L,e,low,mid - 1)
    else:
        return bisection_search(L,e,mid + 1,high)
```

Algorithm Complexity: $O(\log n)$

**Sorting required*

Recursive Search of Element in List

Bisection Search is Faster for Large Lists



Sorting Algorithms

Is it Better to Sort before Searching?

- **Never True for Single Search**: Sorting a collection of elements requires to look at each one at least once

- **Amortize Cost**: Sort becomes beneficial for multiple searches: sort list once, then do many searches

- **Algorithmic Complexity of *k* searches**:
  $O(\text{sort}) + k \times O(log\, n) < k \times O(n)$?
  $=>$ When k is large, run time for sort may become irrelevant relative to run time for search

Sorting Algorithm: MonkeySort

MonkeySort: Randomly shuffle, repeat until sorted

```
def monkey_sort(L):
    while not is_sorted(L):
        random.shuffle(L)
```

Algorithm Complexity:

- Best case: $O(n)$
- Worst case: $O(\infty)$

Sorting Algorithm: SelectionSort

SelectionSort: For each index *i* in list, loop from *i* to end of list, find lowest element, swap with element at index *i*

```
def selection_sort(L):
    for i in range(len(L)):
        min_i = i
            for j in range(i+1, len(L)):
                if L[j] < L[min_i]:
                    min_i = j
        L[i], L[min_i] = L[min_i], L[i]
```

Algorithm Complexity: $O(n^2)$

Sorting Algorithm: BubbleSort

BubbleSort: Pass through pairs of elements, compare elements in each pair, swap so that smallest first, repeat

```python
def bubble_sort(L):
  swap = True
  while swap:
    swap = False
    for i in range(len(L)): # Pass through pairs
      if L[i] > L[i+1]: # Compare elements
        swap = True
        L[i], L[i+1] = L[i+1], L[i]
```

At each pass, largest unsorted element gets bubbled up to the end, so at most *n* passes in while loop $=>$ **Algorithm Complexity**: $O(n^2)$

Sorting Algorithm: MergeSort

MergeSort: Recursively split into half sublists, then sort sublists while merging them back together

```
def merge_sort(L):
    if len(L) < 2:
        return L[:]
    else:
        middle = len(L)//2
        left = merge_sort(L[:middle])
        right = merge_sort(L[middle:])
        return merge(left, right)
```

Fastest version (optional for this course)

MergeSort: Version less easy to read than on previous slide, but fastest (doesn’t copy sublists)

```
def merge_sort(L,l,h): # Start with l=1, h=len(L)
   if h == l:
      return L[l-1]
   else:
      mid = (l + h)//2
      left = merge_sort(L,l,mid)
      right = merge_sort(L,mid+1,h)
      return merge(left, right)
```

Algorithm Complexity: $O(\log n) \times$?

Merge Sorted Sublists

```python
def merge(left, right):
    result = []; i,j = 0,0
    while i < len(left) and j < len(right):
        if left[i] < right[j]:
            result.append(left[i])
            i += 1
        else:
            result.append(right[j])
            j += 1
    while (i < len(left)):
        result.append(left[i])
        i += 1
    while (j < len(right)):
        result.append(right[j])
        j += 1
    return result
```

Algorithm Complexity: $O(n)$

Sorting Algorithm: MergeSort

Divide and Conquer: $O(n\ log\ n)$

- **Divide using *merge_sort()***: $O(log\ n)$
  Divide lists into halves until each sublist contains only 1 element (which by definition are sorted) creates log(n) recursion levels $=>$ $O(log\ n)$ operations

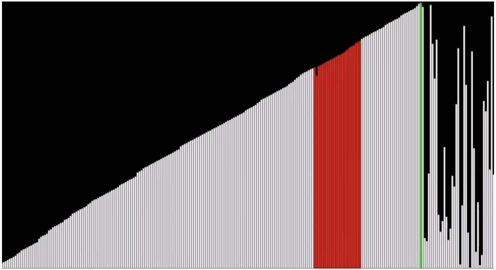
- **Conquer using *merge()***: $O(n)$
  At each recursion level, **sorting** pairs of sorted sublists is linear in number of elements (which is $2^i \times \frac{n}{2^i} = n$) because smallest elements are always the first elements. Copying (**merging**) each element takes exactly n operations $=>$ $O(n) + n$ operations

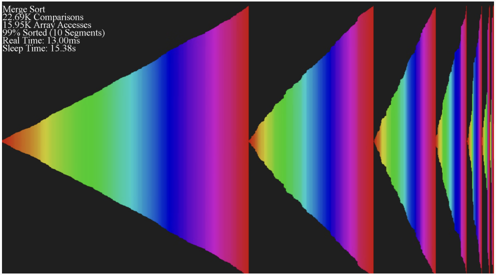
Sorting Algorithms: In a Nutshell

We looked at:

1. **Monkey sort:** $O(\infty)$
   Relies on randomness, potentially unbounded
2. **Selection sort:** $O(n^2)$
   Guarantees first i elements are sorted
3. **Bubble sort:** $O(n^2)$
   Guarantees last i elements are sorted & entire list gets *overall* more sorted at every step
4. **Merge sort:** $O(n \log n)$
   The fastest a sort can be...

Sorting Algorithms: In a Movie





```
Merge Sort
22.69K Comparisons
15.95K Array Accesses
99% Sorted (10 Segments)
Real Time: 13.00ms
Sleep Time: 15.38s
```

Thank you!