Position Control

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Section: 06DB

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I. Procedure

To test the position control of the SRV02 motor using a PD controller, a simulation was first set up to verify the subsequent experiment. The simulation first used a direct derivative to calculate the derivative of the position for the PD controller. The first simulation used a step input for the reference/input value. Running the provided Simulink file returned the results of the ideal simulated response. This simulation was also run using a low-pass filter instead of direct derivative to mimic the subsequent experiment which only uses a low-pass filter. For the low-pass filter case, the simulation was also run using a ramp input (or triangular wave input) to determine the steady state error.

To perform the experiment with the SRV02 motor using a PD controller, the hardware was set up and connected to the provided Simulink file. The derivative (for the PD controller) was configured to use a low-pass filter as it is a bad idea to use the direct derivative of a noisy signal for controllers. First, a step input (or square wave) was given to the controller which controlled the SRV02 motor. Secondly, a ramp input (or triangular wave) was given to the controller which controlled the SRV02 motor.

To reduce the steady state error of the response to a ramp input, an integral controller was introduced. The integral controller should reduce the steady state error to zero due to the integral of a steady state error accumulating error. This causes a larger controlling force the longer the error is present eventually bringing it to zero.

There are many ways to view the controller, but looking at it as a black box with an input of controller gains, reference value, and measured value, and output of control voltage, there are 5 independent variables and one dependent variable. The independent variables are the 3 gain values: proportional, derivative, and integral gain, and 2 signals: reference signal and measured signal. The one dependent variable is the control voltage. The relation between the independent and dependent variables are as follows: the proportional gain causes the control voltage to vary proportionally with the error signal, the derivative gain causes the control voltage to vary proportionally with the derivative of the error signal, and the integral gain causes the control voltage to vary proportionally with the integral of the error signal. The three control voltage signals are then summed and the result is the final control voltage signal given to the SRV02 motor.

For this controller to reduce the error signal to zero, some assumptions were made such as: No noise is present in the measured signal. With noise, the error signal will never be zero because it will never be steady. The motor does not become saturated. If the ramp input requires the motor to spin at 100 rpm, but the max rate the motor can spin in 50 rpm, no controller would be able to reduce the steady state error. Friction is also negligible. With too much friction, the controller may not be able to supply the required torque to reach such speeds.

To reduce the steady-state error, a simulation was first run to verify the subsequent experiment. The integral gain was set to 38.9 V/(rad*s) and the derivative was calculated using a low-pass filter. The experiment was then performed where the same gains and low-pass filter was used, but now the measured signal was from the hardware instead of the simulation. The simulation and experimental plots were compared for validation.

II. Results

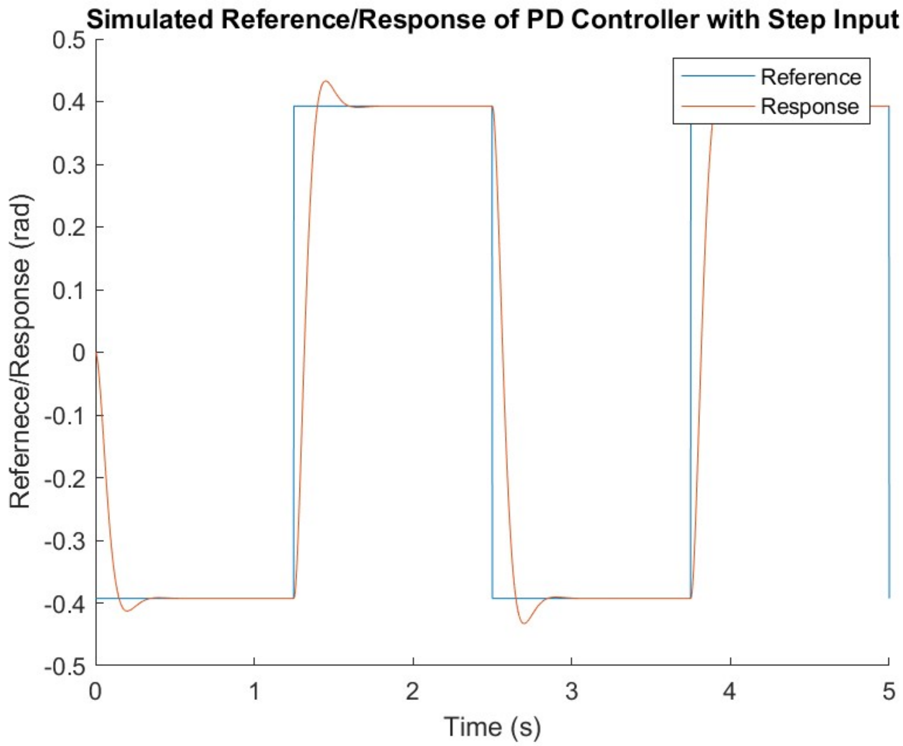

Figure 1: Simulated Response of PD controller with Step Input

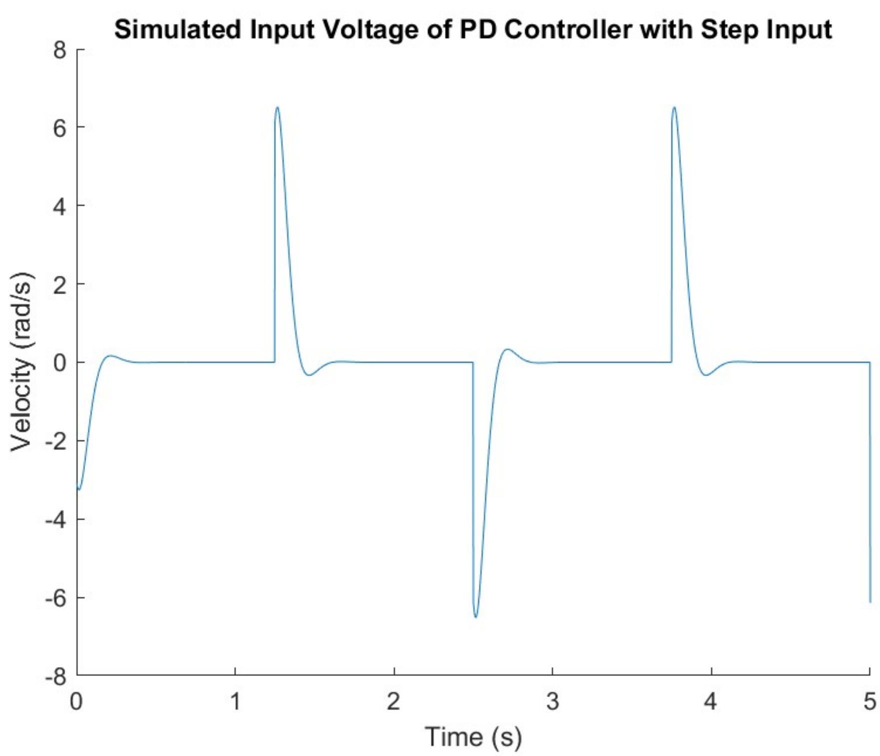

Figure 2: Simulated Input Voltage of PD Controller with Step Input

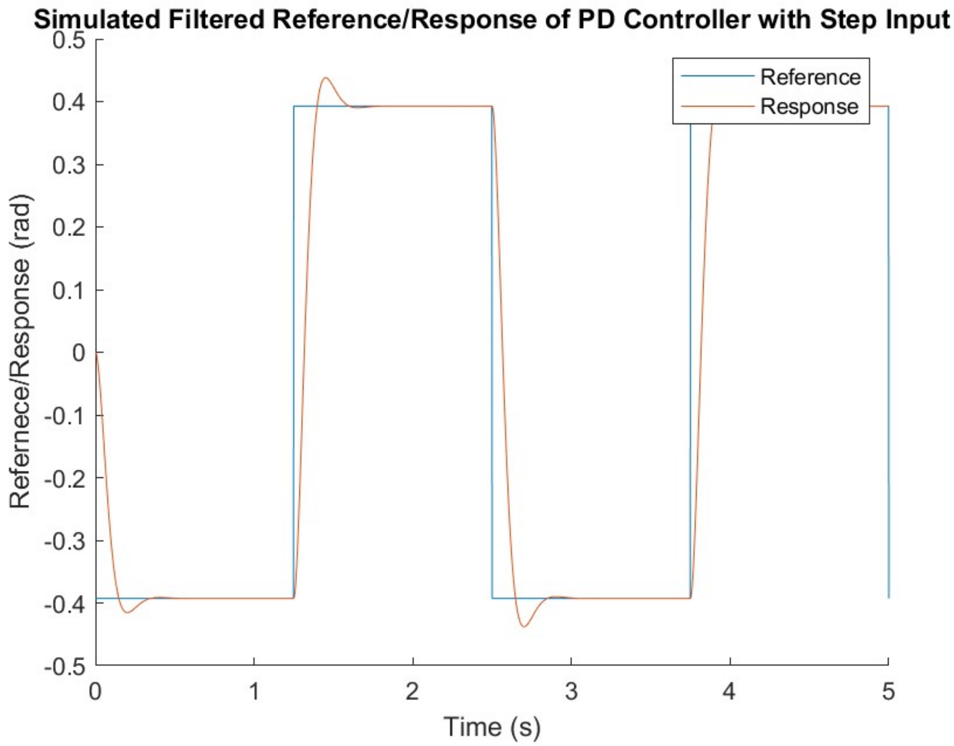

Figure 3: Simulated Filtered Response of PD Controller with Step Input

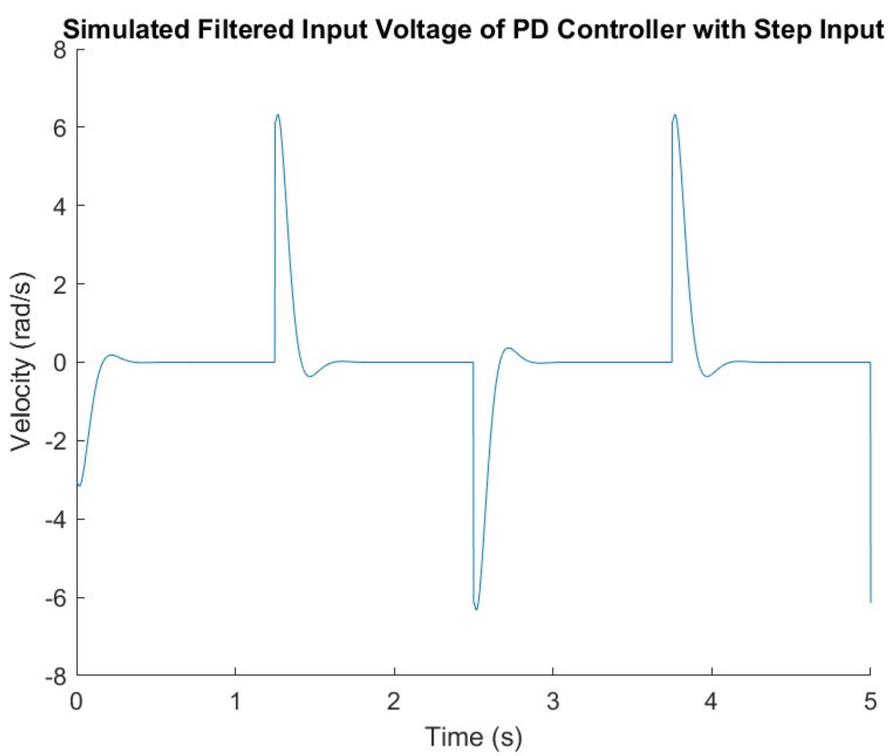

Figure 4: Simulated Filtered Input Voltage of PD Controller with Step Input

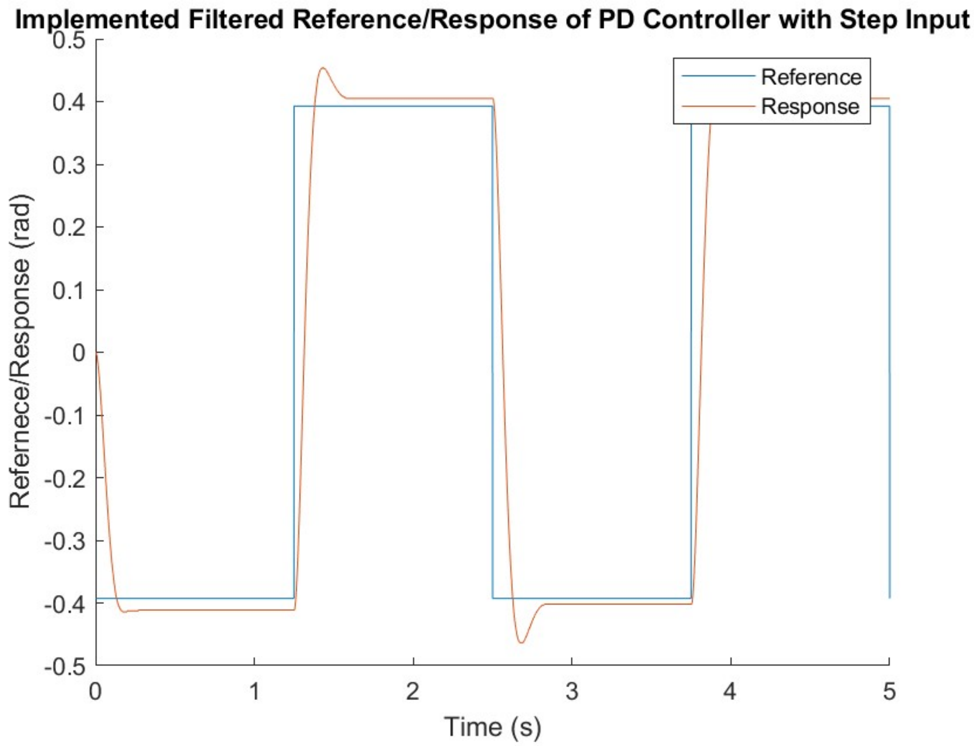

Figure 5: Implemented Filtered Response of PD Controller with Step Input

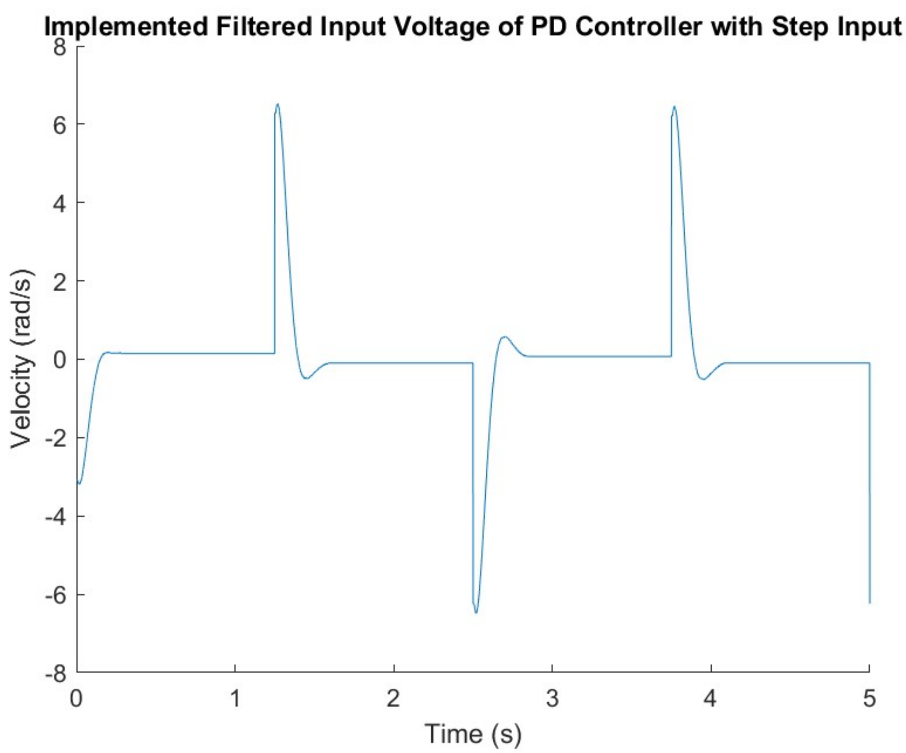

Figure 6: Implemented Filtered Input Voltage of PD Controller with Step Input

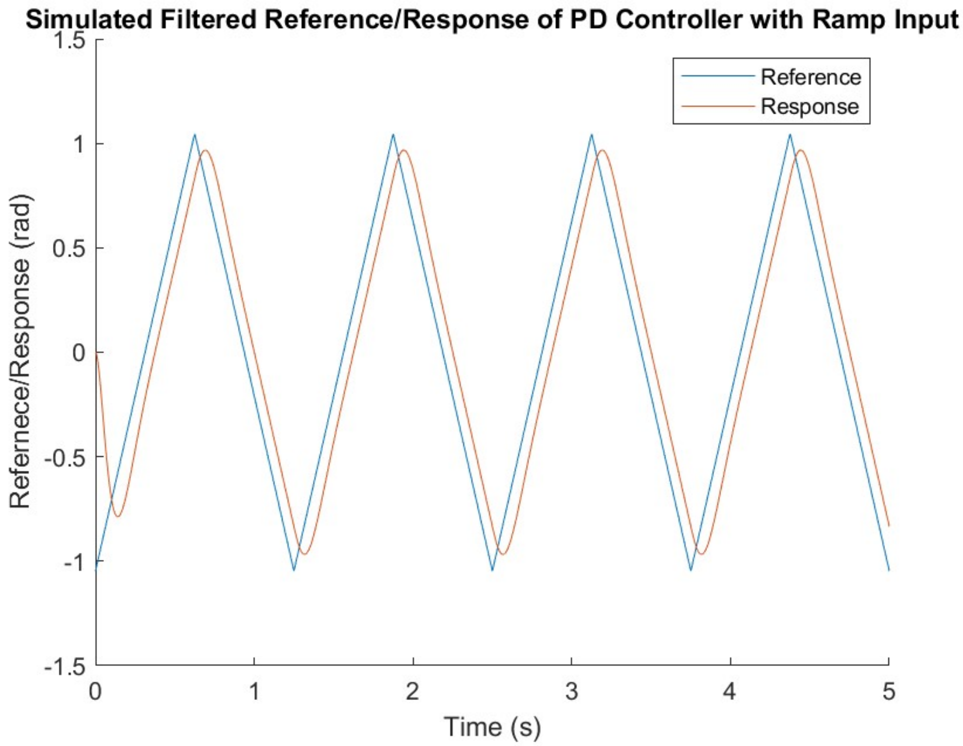

Figure 7: Simulated Filtered Response of PD Controller with Ramp Input

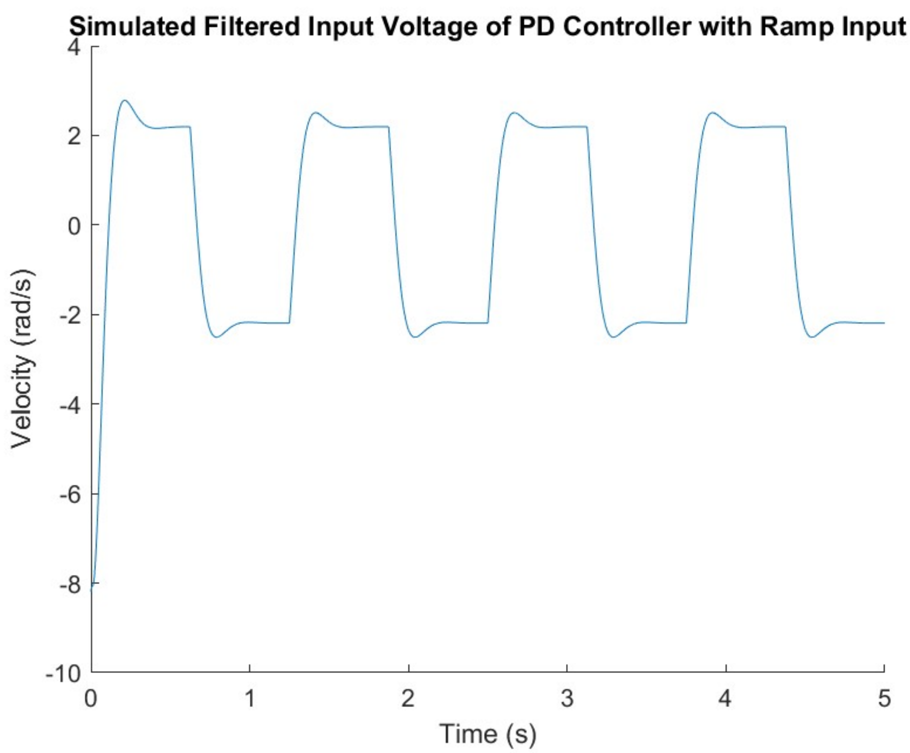

Figure 8: Simulated Filtered Input Voltage of PD Controller with Ramp Input

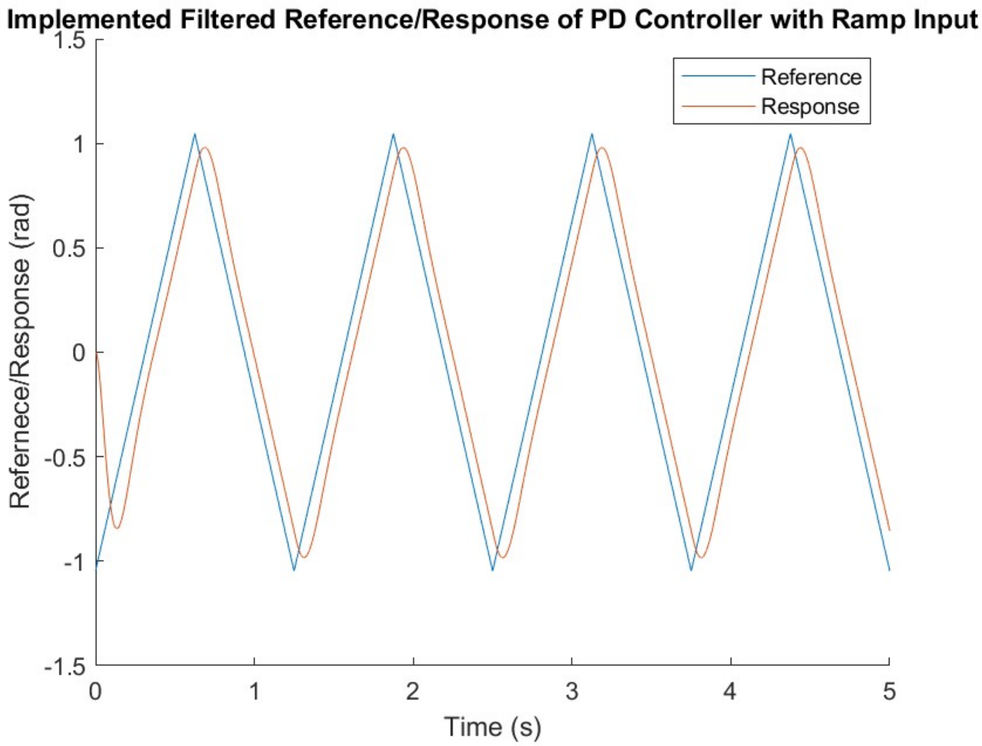

Figure 9: Implemented Filtered Response of PD Controller with Ramp Input

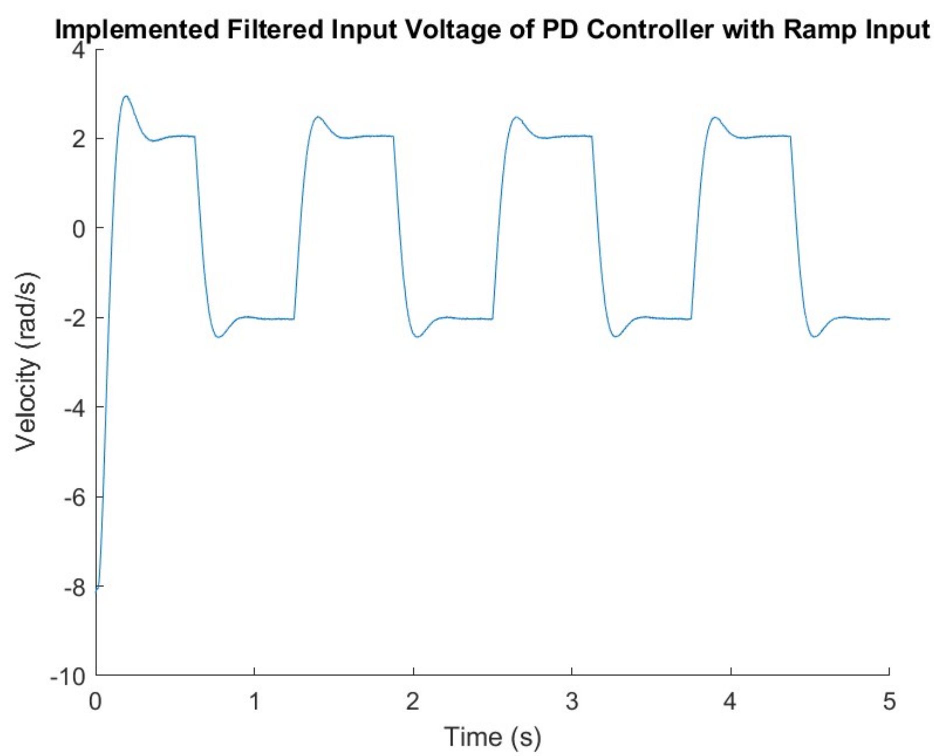

Figure 10: Implemented Filtered Input Voltage of PD Controller with Ramp Input

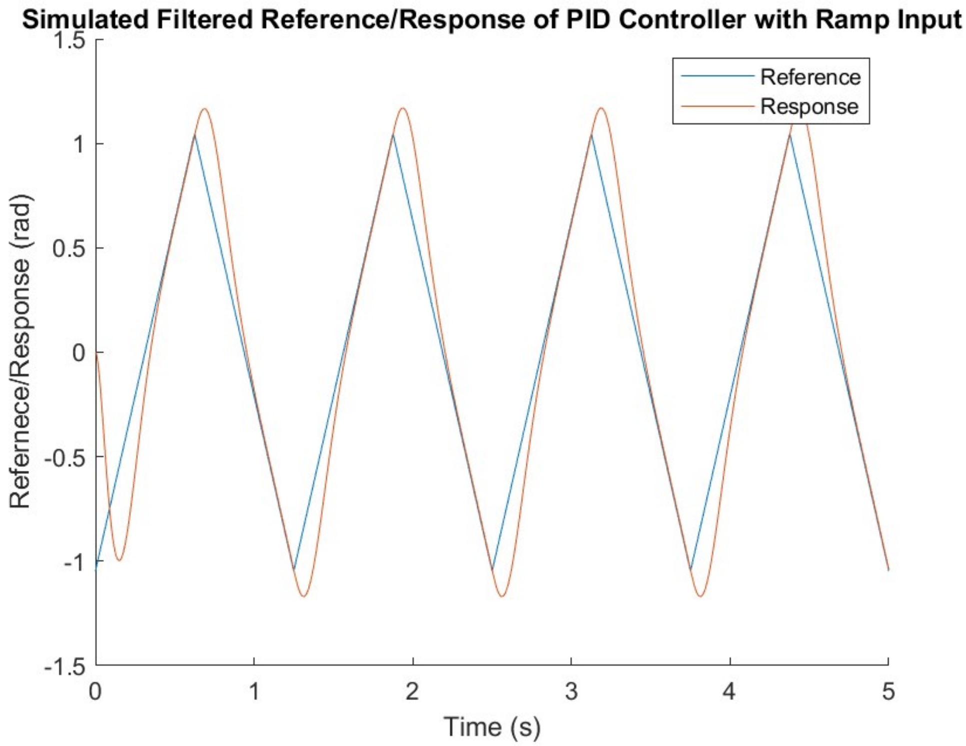

Figure 11: Simulated Filtered Response of PID Controller with Ramp Input

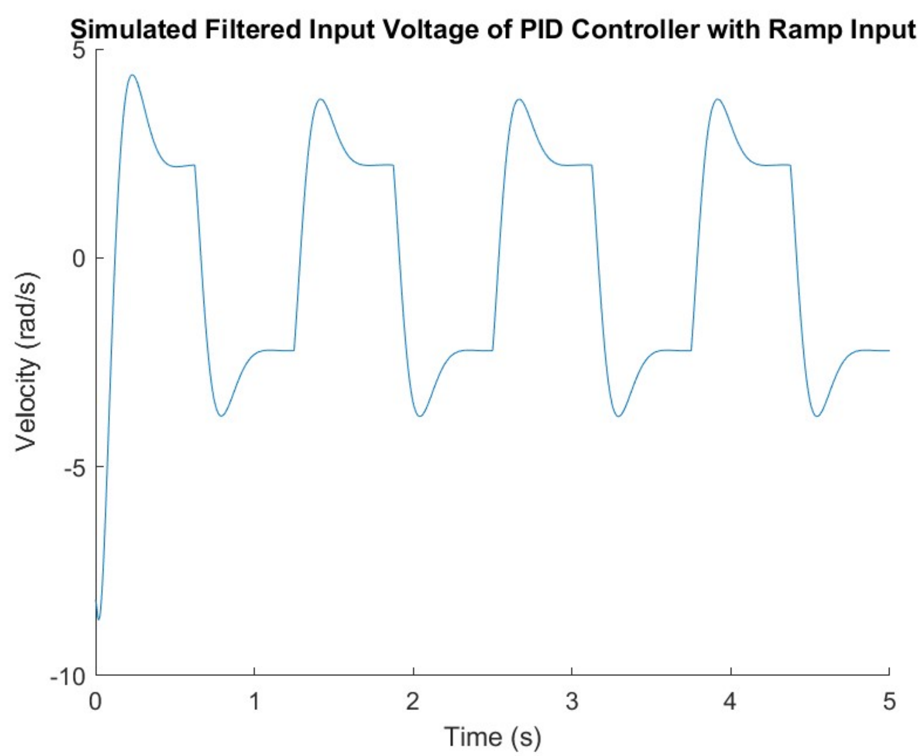

Figure 12: Simulated Filtered Input Voltage of PID Controller with Ramp Input

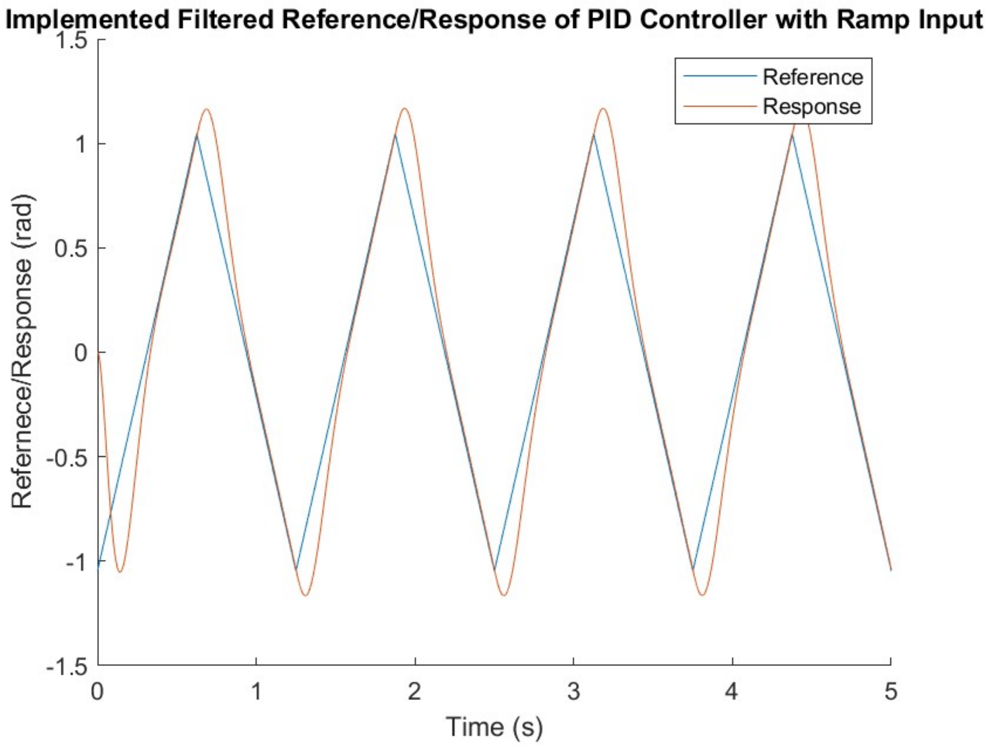

Figure 13: Implemented Filtered Response of PID Controller with Ramp Input

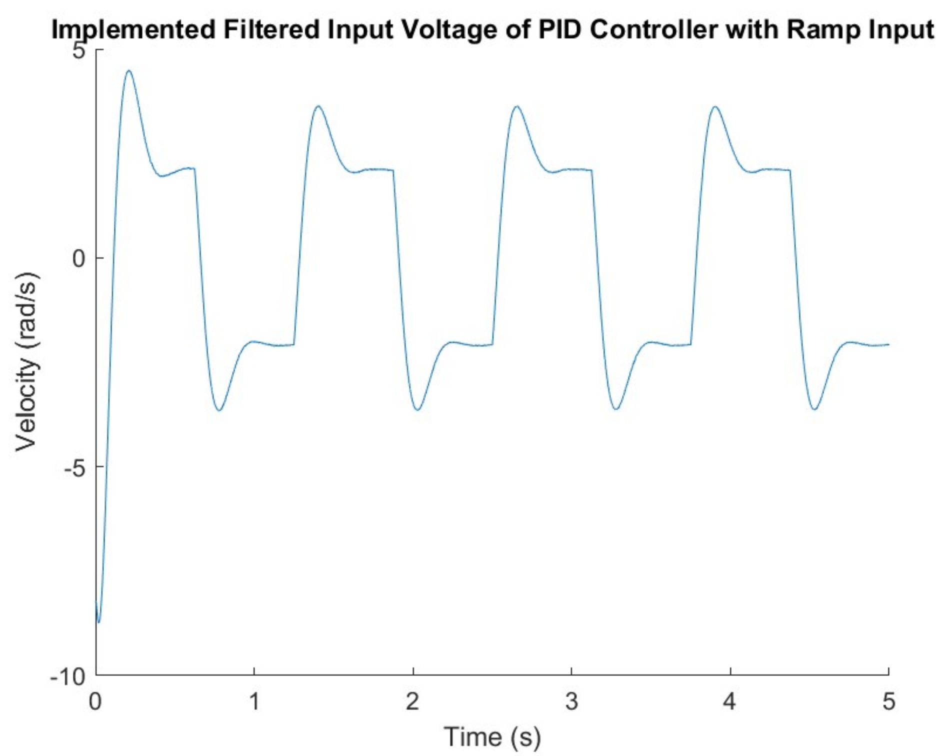

Figure 14: Implemented Filtered Input Voltage of PID Controller with Ramp Input

Table 1: Summary of results for the SRV02 Position Control Lab

Section/ Question	Description	Symbol	Value	Unit
Q 4	Pre-Lab: Model Parameters Open-Loop Steady-State Gain Open-Loop Time Constant	 K τ	 1.53 0.0254	 rad/(V*s) s
Q 4	Pre-Lab: PV Gain Design Proportional gain Velocity gain	 k_p k_v	 7.82 -0.157	 V/rad V*s/rad
Q 5	Pre-Lab: Control Gain Limits Maximum proportional gain	 $k_{p,max}$	 12.7	 V/rad
Q 6	Pre-Lab: Ramp Steady-State Error Steady-state error using PV	 e_{ss}	 0.214	 rad
Q 7	Pre-Lab: Integral Gain Design Integral gain	 k_i	 38.9	 V/(rad*s)
3.3.1.1	Step Response Simulation Peak time Percent overshoot Steady-state error	 t_p PO e_{ss}	 0.204 5.10 0	 s % rad
3.3.1.1	Filtered Step Response Using PD Peak time Percent overshoot Steady-state error	 t_p PO e_{ss}	 0.204 5.10 0	 s % rad
3.3.1.2	Step Response Implementation Peak time Percent overshoot Steady-state error	 t_p PO e_{ss}	 0.191 7.617 0.0123	 s % rad
3.3.2.1	Ramp Response Simulation with PV Steady-state error	 e_{ss}	 -0.213	 rad
3.3.2.2	Ramp Response Implementation with PV Steady-state error	 e_{ss}	 -0.193	 rad
3.3.3	Ramp Response Simulation with no steady-state error Steady-state error	 e_{ss}	 -0.007	 rad
3.3.3	Ramp Response Implementation with no steady-state error Steady-state error	 e_{ss}	 -0.009	 rad

The following are calculations needed to obtain values for Q 6 and Q7 in Table 1:

$$e_{ss} = \lim_{s\to 0} s * E(s) = \lim_{s\to 0} s * \frac{2.36 * s(\tau s + 1 + K * k_v)}{s(\tau s^2 + s + K * k_p + K * k_v s)} = 0.214\, rad \tag{1}$$

$$V(t) = e_{ss}(k_p + k_i t_i) \tag{2}$$

$$k_i = \frac{V_m(t) - k_p e_{ss}}{t_i e_{ss}} = 38.9\, V/(rad * s) \tag{3}$$

III. Analysis

For the analysis, the following equations are used:

$$t_p = t(y_{min}) - t(y_{max}) \tag{4}$$

$$PO = \frac{100 * (\Delta y_{ref} - \Delta y_{measured})}{\Delta y_{ref}} \tag{5}$$

$$e_{ss} = y(t_{ss})_{ref} - y(t_{ss})_{measured} \tag{6}$$

For the simulated step input case with PD controller and direct derivative, the following can be obtained using equations 4, 5 and 6 respectively:

Table 2: Summary of results for the simulated step input with direct derivative.

Variable	Criteria	Pass/Fail
$t_p = 0.204$ s	$t_p \leq 0.2$ s	Approximate
$PO = 5.10\%$	$PO \leq 5\%$	Approximate
$e_{ss} = 0$ rad	$e_{ss} = 0$ rad	Pass

Small errors can be observed, likely due to numerical simulation methods. For the simulated step input case with PD controller and low-pass filter derivative, the following can be obtained using equations 4, 5, and 6 respectively:

Table 3: Summary of results for the simulated step input with low-pass filter derivative.

Variable	Criteria	Pass/Fail
$t_p = 0.204$ s	$t_p \leq 0.2$ s	Approximate
$PO = 5.10\%$	$PO \leq 5\%$	Approximate
$e_{ss} = 0$ rad	$e_{ss} = 0$ rad	Pass

Implementation of a low pass filter caused no effect on the simulation response. The criteria, for the most part, is met within a tolerance.

For the implementation of the PD controller with a low-pass derivative, the following can be obtained using equations 4, 5, and 6, respectively:

Table 4: Summary of results for the implemented step input with low-pass filter derivative.

Variable	Criteria	Pass/Fail
$t_p = 0.191$ s	$t_p \leq 0.2$ s	Pass
$PO = 7.617\%$	$PO \leq 5$	Fail
$e_{ss} = 0.0123$ rad	$e_{ss} = 0$ rad	Fail

The implementation of the PD controller with a low-pass derivative appears to have too much disturbance/not accounted for terms that were not in the dynamics of the problem. Most criteria failed to pass. The reason for the steady state error is likely due to static friction the gearbox cannot overcome.

For the ramp input simulation, the steady state error (using equation 6) comes out to be -0.213 rad. This is almost identical to the value calculated in the prelab of -0.213 rad. This difference likely comes from the numerical solver (ode 1).

For the ramp input implementation, the steady state error came out to be -0.193 rad. This is an interesting result as previous encounters with friction would hint at his steady state error being larger than the simulated error. One possible explanation for this result would be added inertia of the system causing the servo to overshoot the expected value.

For the ramp input with no steady state error, the simulated steady state error was -0.007 rad which is approximately 0. The implementation had a very similar steady state error of -0.009 rad.

IV. Conclusions

After analyzing the response to step and ramp inputs for position control of a servo motor, it has been found that a PD controller is theoretically enough to reduce the steady state error of a step input, and an integral controller is required to reduce the steady state error of a ramp input. For the implemented results, it appears that a PD controller does not properly control the system as well as expected. It would also appear that implementing a low pass filter causes minimal deviation from the desired results. One can conclude that using a PD controller for the step input would also reduce the steady state error for the implemented step input case. Based on the input control voltage plots for all of these cases, it can also be concluded that the 10 V maximum operating voltage was never exceeded.