Econ 260A: Homework Challenge #3
Due November 13 in class

This homework challenge is about the optimal control of a mobile invasive species. Let x_{it} be the stock of an invasive species in patch i at the beginning of time period t, and h_{it} is the control in patch i during time period t. The timing is as follows: The stock is observed in each patch, some level of control is undertaken in each patch, the remaining stock grows, and the moves across space. Movement from patch i to patch j is given by the constant D_{ij}. So the equation of motion is

$$x_{it+1} = \sum_{j=1}^{N} D_{ji} g(e_{jt}) \tag{1}$$

where e_{jt} is the residual stock in patch j and N is the number of patches. If the stock at the beginning of the period in patch i is x_i and the control is h_i (leaving residual stock e_i), then the total control cost during that period in patch i is $\int_{e_i}^{x_i} \theta_i c(s) ds$, where the downward-sloping function $\theta_i c(s)$ is the marginal control cost when the stock is s (the parameter θ_i is a constant). After control takes place, but before growth and spread occur, the residual stock imposes a patch-specific marginal damage of k_i, so the total damage in patch i during period t is given by $k_i e_i$.

Answer the following questions:

1. Suppose each patch is owned by a separate landowner, and that all landowners were myopic. Describe the dynamics and the steady state of this system.

2. For the remainder of this assignment, assume that a central planner can determine the level of control in each patch in each time period.

   (a) Write down the period t dynamic programming equation. Identify the state and control variables and the equation of motion.

   (b) Assume a T-period time horizon, with no salvage value. What is the period T dynamic programming equation? Derive the period T value function and policy function.

   (c) Work backward through time to derive the period t value function and policy function.

   (d) In an infinite horizon problem, how does the level of control in patch i depend on: x_i, k_i, θ_i, D_{ii}, D_{ij}. Try to interpret each of these comparative statics.