Homework Challenge #3

ECON 260A

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Set up

Let x_{it} be the stock of an invasive species in patch i at the beginning of time period t, and h_{it} is the control in patch i during time period t. The timing is as follows: The stock is observed in each patch, some level of control is undertaken in each patch, the remaining stock grows, and then moves across space. Movement from patch i to patch j is given by the constant D_{ij}. So the equation of motion is:

$$x_{it+1} = \sum_{j=1}^{N} D_{ji} g(e_{jt})$$

where e_{jt} is the residual stock in patch j and N is the number of patches. If the stock at the beginning of the period in patch i is x_i and the control is h_i (leaving residual stock e_i), then the total control cost during that period in patch i is $\int_{e_i}^{x_i} \theta_i c(s)\, ds$, where the downward-sloping function $\theta_i c(s)$ is the marginal control cost when the stock is s (the parameter θ_i is a constant). After control takes place, but before growth and spread occur, the residual stock imposes a patch-specific marginal damage of k_i, so the total damage in patch i during period t is given by $k_i e_i$.

Dynamics and the steady state of this system with myopic landowner

From the problem set up, we know that "*After control takes place, but before growth and spread occur, the residual stock imposes a patch-specific marginal damage of k_i, so the total damage in patch i during period t is given by $k_i e_i$*". Myopic landowners would not care about the future growth, dispersal, and damages of an invasive species. Instead, they would only care about damages and control costs in this time step.

Total costs for owner of property i at time t are given by:

$$\phi(x_{it}, e_{it}) = \int_{e_{it}}^{x_{it}} \theta c(s)\, ds + k e_{it}$$

Our current "damage function" assumes a constant marginal impact from remaining stock size. Therefore, our total costs can be shown by the graphical representation of the problem in Figure 1.

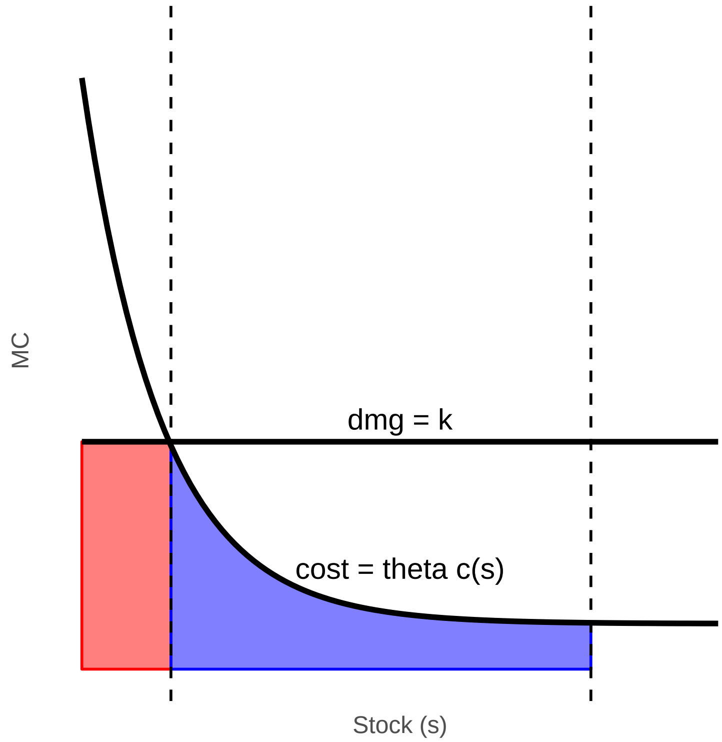

Figure 1: Illustration of marginal costs of control and damage. The area under each curve show the total cost of control (blue) and damage (red).

The myopic landowner would minimize present costs of control and damage, without accounting for the growth and dispersal of the stock and the future damaged that would come with it. Therefore, the myopic landowner seeks to minimize total costs (shaded area in Fig. 1) by chosing an optimal h:

$$
\begin{aligned}
&\min_e \phi(x_i, e_i) \\
&\min_e \left[\int_{e_i}^{x_i} \theta_i c(s)\, ds + k_i e_i \right] \\
&\min_e \left[\int_{e_i}^{x_i} \theta_i c(s)\, ds + k_i (x_i - h_i) \right]
\end{aligned}
$$

Here we would take first-order conditions and find the optimal h. But by visually inspecting Figure 1, we know that each individual landowner would control up to a point where marginal control costs are equal to marginal damage costs. With this in mind, and knowing that $e_{it} = x_{it} - h_{it}$, we drop the t subindeces to turn this into a simple problem, and denote e^* as the one-period equilibrium stock size left after harvesting and for which marginal costs of control are equal to the marginal costs of damages:

$$
\begin{aligned}
\theta_i c(e^*) &= k_i \\
c(e^*) &= \frac{k_i}{\theta_i}
\end{aligned}
$$

If we had a functional form for $c(s)$ we would substitute e^* into it and then just solve for e^* in the above. Intuitevely $h^* = x - e^*$: that is, the optimal h^* would be whatever brings the stock from x down to e^*.

Evidently, this approach does not account for the growth and dispersal of the remaining stock, as well as the future damages of the immigrating stock left from other landowner's suboptimal management.

Central planner determines control $\forall\, t$ and i

The central planner, on the other hand, would account for present reductions in damages as well as future ones. Therefore, the planner must balance between present costs of managing and future costs of damage.

Period t dynamic programming equation

The social planer would seek to minimize the sum of present value of total costs by optimally selecting the spatial and temporal control. I denote vectors as $\bar{x_t}$, so that $\bar{x_t} = x_{1t}, x_{2t}, ..., x_{N-1t}, x_{Nt}$. Then, the period-$t$ dynamic programming equation is given by:

$$V_t(\bar{x}_t) = \min_{e_{it}} \sum_{i=1}^{N} \phi(x_{it}, e_{it}) + \delta V_{t+1}(\bar{x}_{it+1})$$

Subject to the equation of motion given by:

$$\mathrm{s.t.\ x_{it+1}} = \sum_{\mathrm{j=1}}^{\mathrm{N}} \mathrm{D_{ji} g(e_{jt})}$$

My state variable is x_{it}, and my control is $e_{i,t}$. In this formulation, the planner doesn't explicitly choose how much to harvest, but rather how much to leave in every parcel i at every time period t.

Fixed-time, no salvage value

With a fixed-time problem and assuming no salvage value, the problem relies only in the present costs (from control and damages) but does not account for the dispersal, growth, and future damages because $\delta V_{t+1}(\bar{x}_{it+1}) = 0$. Therefore, the DPE becomes:

$$V_t(\bar{x}_t) = \min_{e_{it}} \sum_{i=1}^{N} \phi(x_{it}, e_{it}) + 0$$

- $\delta V_{t+1}(\bar{x}_{it+1}) = 0$ because the world ends after T, and there is no value of entering period $T+1$ with $\bar{x}_{it+1}$.
- The policy function would be as identified in the first exercise ($c(e^*) = \frac{k_i}{\theta_i}$), where we would solve for e^*.

Backward induction

The period-T solution is shown above, so we start at period $T - 1$

The DPE is given by:

$$V_t(\bar{x}_t) = \min_{e_{it}} \sum_{i=1}^{N} \phi(x_{it}, e_{it}) + \delta V_{t+1}(\bar{x}_{it+1})$$

Dependence of control in i on $x, k, \phi D and D$