Econ 260A: Homework Challenge #1
Due October 18 in class

The stock of a renewable resource at the beginning of period t is x_t and the harvest is h_t, leaving residual stock $y_t \equiv x_t - h_t$. The price is p=1, assume initially that the cost of fishing is zero (so the period-t payoff is just h_t), and the discount factor is δ. The equation of motion is:

$$x_{t+1} = f(y_t) \tag{1}$$

Answer the following questions:

1. Write the period-t dynamic programming equation.

2. Use backward induction to characterize the optimal policy function and value function at any arbitrary period $t < T$ in a T-period problem.

3. How does the period-t policy function depend on p?

4. Now suppose the current period payoff is $\alpha h_t - \beta h_t^2$ (for $\alpha > 0$ and $\beta > 0$). *Guess* that the period-t dynamic programming equation is a linear function of the state. Then try to verify that the Bellman equation is satisfied (or is not satisfied). In other words, try to prove or disprove that the DPE is linear in the state.

5. Retaining the non-linear payoff function, now assume the stock evolves in a stochastic fashion: $x_{t+1} = z_t f(y_t)$, where $z_t = \{1 - \theta, 1 + \theta\}$ (i.e. it takes one of those two values, each with probability 0.5), and $f(y_t) = y_t + r y_t(1 - y_t/K)$. Write a computer program that solves this problem numerically. You can use the following parameters: $\delta = .9$, $\alpha = 20$, $\beta = .6$, $r = .3$, $K = 100$, $\theta = .3$. How does the converged optimal policy function depend on θ, r, δ, and α?

6. Suppose $x_0 = 15$. Using the infinite-horizon optimal policy function, simulate the optimized system forward for 20 years under baseline parameters above. Run 10 separate simulations and plot the trajectory of x_t and h_t over time for each simulation. (You should produce 2 plots, one for the 10 trajectories of x_t and one for the 10 trajectories of h_t).