ECON 260A: A Spatial Renewable Resource Model

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1 Introduction

The distinguishing feature of *natural resource* economics is that it deals with the allocation of natural resources over time. For non-renewable resources, dynamics play a role because the extraction in one period reduces the overall stock, which limits the opportunity to extract in the future. For renewable resources, the growth of the stock often depends on the residual stock from the previous period. These motivate the need to account for the dynamics of natural resource growth and exploitation, which has been the focus of natural resource economics for over 50 years.

But a perhaps equally important dimension of natural resource use has gone practically unnoticed. The many natural resources are distributed, and move, across space. This is particularly true for renewable resources. Examples include:

- Fish (they swim or larvae are transported by ocean currents)
- Trees (their seeds are dispersed)
- Invasive species (are transported with trade, other economic activity, or by wind)
- Infectious diseases (are transported by human movement or other means)
- Game (migrate)

There are many issues one confronts when attempting to incorporate space into the "standard" renewable resource economics model. Some of these issues are noted below:

- Discrete vs. continuous space: For a variety of reasons, most models are in discrete space
- Implicit vs. explicit space: This is tricky and deserves more discussion.

- Endogenous or exogenous resource dynamics over space (does the stock of the resource drive movement of the resource)
- Economic payoffs across space, and aggregation of those payoffs.
- Game theoretical, optimization, or open access
- How to handle boundaries?
- Incorporation of spatial data.

In these notes, I will pose and solve a spatial generalization of the discrete-time renewable resource economics problem that was posed a few weeks ago.

2 Model

2.1 Simplest Model

Suppose the entire spatial domain is divided into two resource "patches" labeled 1 and 2. The resource stock in patch i at the beginning of time t is given by x_{it}, harvest is h_{it} and residual stock is $y_{it} \equiv x_{it} - h_{it}$. Resource growth in patch i is $f(y_{it})$, and the resource then migrates. The fraction of the resource stock that migrates from patch i to patch j is D_{ij}, so migration is completely characterized by a 2x2 matrix D where the rows sum to 1. The profit from harvesting in patch i is $p_i h_i$.

Suppose a sole owner seeks to perfectly exploit this spatial system to maximize the net present value of joint profits across both patches over an infinite horizon, so the objective is:

$$\max_{\{y_{1t}, y_{2t}\}} \sum_{t=1}^{\infty} \delta^t p_1(x_{1t} - y_{1t}) + p_2(x_{2t} - y_{2t}) \tag{1}$$

subject to:

$$x_{it+1} = \sum_{j=1}^{2} f(y_{jt}) D_{ji} \tag{2}$$

We can write this problem as a dynamic programming equation, as follows:

$$V_t(\mathbf{x}_t) = \max_{\mathbf{y}_t} \sum_{i=1}^{2} p_i(x_{it} - y_{it}) + \delta V_{t+1}(\mathbf{x}_{t+1}) \tag{3}$$

Which can be solved via backward induction. I will show the first two inductive steps and leave it as an exercise to prove that the same policy function holds for every subsequent inductive step. Suppose this were a finite horizon problem. In period T, the DPE would be:

$$V_T(\mathbf{x}_T) = \max_{\mathbf{y}_T} \sum_{i=1}^{2} p_i(x_{iT} - y_{iT}) \tag{4}$$

which has solution $y^*_{iT} = 0$, and thus $V_T(\mathbf{x}_T) = \sum_{i=1}^2 p_i(x_{iT})$. Stepping back one period, we have:

$$\begin{aligned}
V_{T-1}(\mathbf{x}_t) &= \max_{\mathbf{y}_{T-1}} \sum_{i=1}^{2} p_i(x_{iT-1} - y_{iT-1}) + \delta V_T(\mathbf{x}_T) && (5)\\
&= \max_{\mathbf{y}_{T-1}} \sum_{i=1}^{2} p_i(x_{iT-1} - y_{iT-1}) + \delta \sum_{i=1}^{2} p_i x_{iT} && (6)\\
&= \max_{\mathbf{y}_{T-1}} \sum_{i=1}^{2} p_i(x_{iT-1} - y_{iT-1}) + \delta \sum_{i=1}^{2} p_i \sum_{j=1}^{2} f(y_{jT-1}) D_{ji} && (7)\\
& && (8)
\end{aligned}$$

This has first order conditions:

$$\begin{aligned}
f'(y_1) &= \frac{p_1}{\delta(p_1 D_{11} + p_2 D_{12})} && (9)\\
f'(y_2) &= \frac{p_2}{\delta(p_1 D_{21} + p_2 D_{22})} && (10)
\end{aligned}$$

Or, more compactly:

$$f'(y_i) = \frac{p_i}{\delta \sum_{j=1}^2 p_j D_{ij}} \tag{11}$$

The interpretation is as follows: Patch i should be extracted down to the point where the benefit of one more unit of harvest (p_i) just equals the opportunity cost of that extraction. The opportunity cost is what that marginal unit of resource would have earned us in the future, which is the discounted growth multiplied by the value of that growth once it disperses. I leave it to the students to verify that Equation 11 holds for every period prior to T. The comparative statics are relatively easy after invoking the implicit function theorem. For example, the effect of a higher own price is found by totally differentiating:

$$dy_i[f''] - dp_i[\frac{\delta \sum_j p_j D_{ij} - \delta p_i D_{ii}}{(\cdot)^2}] \tag{12}$$

Which implies:

$$Sign(\frac{dy_i}{dp_i}) = Sign(\frac{\delta p_j D_{ij}}{f''}) < 0 \tag{13}$$

So the larger is own price, the smaller is the desired residual stock in that patch. The policy function given in Equation 11 has a few interesting special cases.

2.1.1 No spatial interactions

Without spatial interactions, $D_{12} = D_{21} = 0$ and $D_{11} = D_{22} = 1$. In that case, Equation 11 reduces to:

$$f'(y_i) = \frac{p_i}{\delta p_i D_{ii}} = \frac{1}{\delta} \tag{14}$$

which is the golden rule of growth for an a-spatial renewable resource. In this case, each patch should be harvested in the economically efficient manner, without regard for what is happening in the other patch.

2.1.2 Equal prices

If prices are equal across space, so $p_1 = p_2 = p$, Equation 11 reduces to:

$$f'(y_i) = \frac{p}{p\delta \sum_{j=1}^{2} D_{ij}} = \frac{1}{\delta} \tag{15}$$

So it is *as if* there is no space. This result holds because the 10th unit of resource left over in patch i is just as valuable as the 10th unit of resource left over in patch j - they both grow the same and disperse somewhere where the marginal value of harvest is identical.

2.2 Extensions

Numerous other extensions are possible from this model. I will note a few of them here:

- N resource patches: This seemingly complex problem has a simple solution, which is analogous to Equation 11. The solution is:

$$f'(y_i) = \frac{p_i}{\delta \sum_{j=1}^{N} p_j D_{ij}} \tag{16}$$

- Heterogeneous resource productivity. Suppose, for example, that some areas of ocean are more productive than others, so production in patch i is $f_i(y_i)$. Again, the solution is a straightforward extension:

$$f_i'(y_i) = \frac{p_i}{\delta \sum_{j=1}^{N} p_j D_{ij}} \tag{17}$$

- Including harvest costs. If harvests cost is linear in harvest, then this can be subsumed directly into the price. If harvest costs are non-linear in harvest, then this becomes a difficult problem that has no simple solution. If marginal harvest cost is nonlinear (and decreasing) in residual stock, so the marginal cost in patch i is $c_i(y_i)$, then this can be accommodated, as follows:

$$f_i'(y_i) = \frac{p_i - c_i(y_i)}{\delta \sum_{j=1}^{N}(p_j - c_j(x_j))D_{ij}} \tag{18}$$

- Game theoretic interactions. Suppose instead of a sole owner (who can simultaneously account for all patches), we have a set of N decentralized decision makers who compete in a game. Returning to the case of N agents without harvest costs, this can be accommodated as follows:

  $$f'(y_i) = \frac{1}{\delta D_{ii}} \tag{19}$$

  which suggests that agent i ignores the effect she will have on the other property owners.
- Stochasticity. We will deal with this in a separate lecture.
- Goods vs. bads. Suppose we were dealing with an invasive species or other pest. Then the stock of the resource would cause damage, and it would be costly to harvest it, so the problem becomes one of cost minimization. This can also be handled, but I will not do so here.