

Homework Challenge #3

ECON 260A

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2018-11-07

Set up

Let x_{it} be the stock of an invasive species in patch i at the beginning of time period t , and h_{it} is the control in patch i during time period t . The timing is as follows: The stock is observed in each patch, some level of control is undertaken in each patch, the remaining stock grows, and then moves across space. Movement from patch i to patch j is given by the constant D_{ij} . So the equation of motion is:

$$x_{it+1} = \sum_{j=1}^N D_{ji} g(e_{jt})$$

where e_{jt} is the residual stock in patch j and N is the number of patches. If the stock at the beginning of the period in patch i is x_i and the control is h_i (leaving residual stock e_i), then the total control cost during that period in patch i is $\int_{e_i}^{x_i} \theta_i c(s) ds$, where the downward-sloping function $\theta_i c(s)$ is the marginal control cost when the stock is s (the parameter θ_i is a constant). After control takes place, but before growth and spread occur, the residual stock imposes a patch-specific marginal damage of k_i , so the total damage in patch i during period t is given by $k_i e_i$.

Dynamics and the steady state of this system with myopic landowner

From the problem set up, we know that “After control takes place, but before growth and spread occur, the residual stock imposes a patch-specific marginal damage of k_i , so the total damage in patch i during period t is given by $k_i e_i$ ”. Myopic landowners would not care about the future growth, dispersal, and damages of an invasive species. Instead, they would only care about damages and control costs in this time step

Total costs for owner of property i at time t are given by:

$$\int_{e_{it}}^{x_{it}} \theta c(s) ds + k e_{it}$$

Our current “damage function” assumes a constant marginal impact from remaining stock size. Therefore, our total costs can be shown by the graphical representation of the problem:

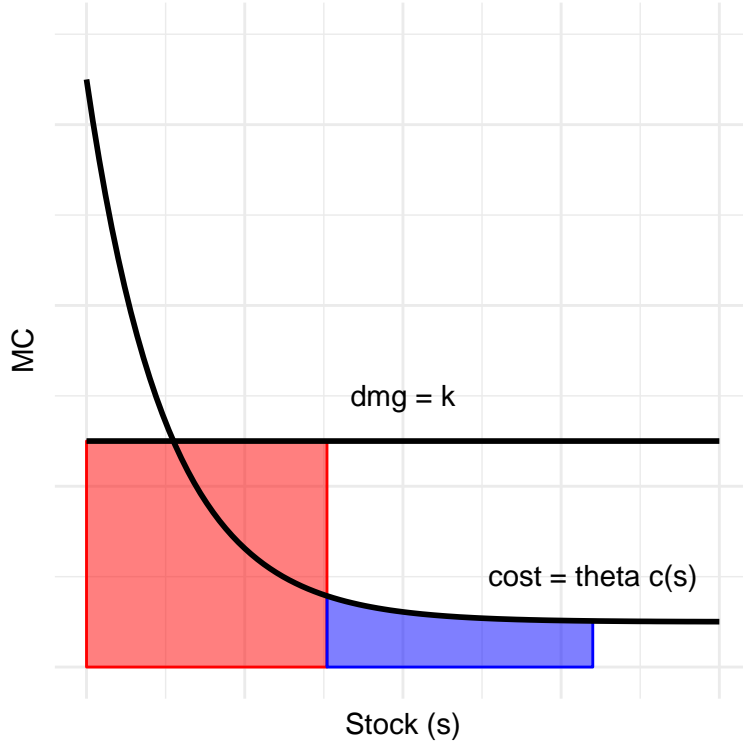


Figure 1: Illustration of marginal costs of control and damage. The area under each curve show the total cost of control (blue) and damage (red).

Each individual landowner would then control up to a point where control costs are equal to damage costs. With this in mind, and knowing that $e_{it} = x_{it} - h_{it}$, the above becomes:

$$\begin{aligned}
 \int_{e_{it}}^{x_{it}} \theta c(s) ds &= k e_{it} \\
 \int_e^x \theta c(s) ds &= k(x - h) \\
 \int_e^x \theta c(s) ds &= kx - kh \\
 kh &= kx - \int_e^x \theta c(s) ds \\
 h &= x - \frac{\int_e^x \theta c(s) ds}{k}
 \end{aligned}$$

Evidently, this approach does not account for the growth and dispersal of the remaining stock, as well as the future damages of the immigrating stock left from other landowner's suboptimal management.

Central planner determines control $\forall t$ and i

The central planner, on the other hand, would account for present reductions in damages as well as future ones. Therefore, the planner must balance between present costs of managing and future costs of damage.

Period t dynamic programming equation

$$\min_{h_t} \sum_{t=0}^{\infty} \left[\sum_{i=1}^N \left(\int_{e_i}^{x_i} \theta_i c(s) ds + k_i e_i \right) + V_{t+1}(x_{it+1}) \right]$$

$$\text{s.t. } x_{it+1} = \sum_{j=1}^N D_{ji} g(e_{jt})$$

Fixed-time, no salvage value

Backward induction

Dependence of control in i on $x, k, \phi D$ and D