1 Econ 260A. Fall 2018. Homework Challenge 4. Suggested Answers

1. Consider the following two-period investment opportunity. The cost of the investment is $I = 400$ and the revenues generated in year one are $V_1 = 200$. In year two, the investment will generate revenues of $V_2 = 600$ with probability p and $V_2 = 100$ with probability $1-p$. The investment is irreversible once made, and the value of V_2 is revealed at the start of year two. Assume, for now, that the discount factor δ is equal to one.

a. Derive an expression for the Dixit-Pindyck option value in terms of p. Display this graphically and interpret.

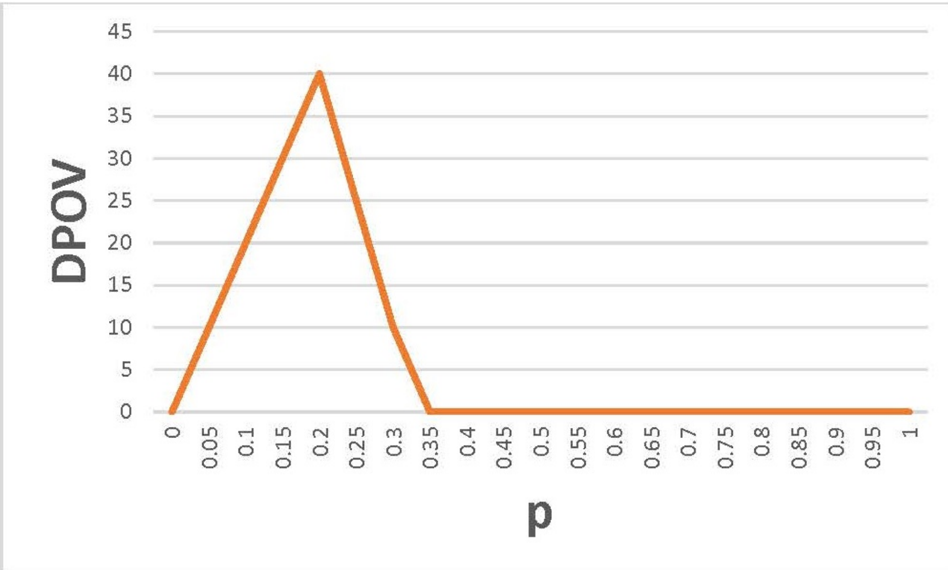
First, write $V^l(1) = V^n(1) = -400 + 200 + 600p + 100(1-p)$. This expression is positive if $p \geq 0.2$. Thus, we have:

$$V^l(1) = -100 + 500p \qquad \text{if } p \geq 0.2$$
$$V^l(1) = 0 \qquad \text{if } p \leq 0.2$$

Further, we have $V^l(0) = p(600-400) + (1-p)0 = 200p$. The first part of the DPOV expression equals $V^l(0)$ if $200p \geq 500p - 100$ or if $p \leq 1/3$. For the second part of the DPOV expression we need to compare $V^n(1)$ and $V^n(0)$. $V^n(0) = p(600-400)+(1-p)(100-400) = 500p - 300$. By inspection, it can never be greater than $V^n(1)$. Therefore, we can write the DPOV as:

$$DPOV = 200p \qquad \text{if } p \leq 0.2$$
$$DPOV = 100 - 300p \qquad \text{if } 0.2 \leq p \leq 1/3$$
$$DPOV = 0 \qquad \text{if } p \geq 1/3$$

Figure 1: Graph of DPOV against p



The figure shows that as the probability of the high payoff increases, the value of postponing the investment in order to learn about the payoff decreases. The reason is that the high payoff is becoming a "sure thing" and so there is no benefit to foregoing the investment today. The DPOV is also small for low values of p because the low payoff is close to a "sure thing" and so the value of learning about tomorrow's payoff is correspondingly low.

b. Suppose there is a spread in the distribution of year two revenues. Specifically, $V_2 = 600 + 100u$ with probability p and $V_2 = 100 - 100u$ with probability $1 - p$ where $0 \leq u \leq 1$. Derive an expression for the Dixit-Pindyck option value in terms of p and u. How does the option value change as u gets larger? Explain. How does the option value vary across $p - u$ space?

With the new expression for V_2, we have:

$$
\begin{aligned}
V^l(1) =& -100 + 500p + 200up - 100u \qquad \text{if } p \geq \frac{100 + 100u}{500 + 200u} \\
V^l(1) =& \, 0 \qquad \text{otherwise}
\end{aligned}
$$

Further, $V^l(0) = 200p + 100up$, which is weakly positive for all p and u. As in part a, $V^n(1) \geq V^n(0)$. Therefore, to determine the DPOV, we can compare $V^l(0)$ and $V^l(1)$. Assume the condition for $V^l(1) \geq 0$ holds. Then, $V^l(0) - V^l(1) = 100 - 300p - 100up + 100u$. This expression is positive as long as $p \leq \frac{100+100u}{300+100u}$. Note that this term is always greater than the cut-off value of p for $V^l(1)$ to be positive. Therefore,

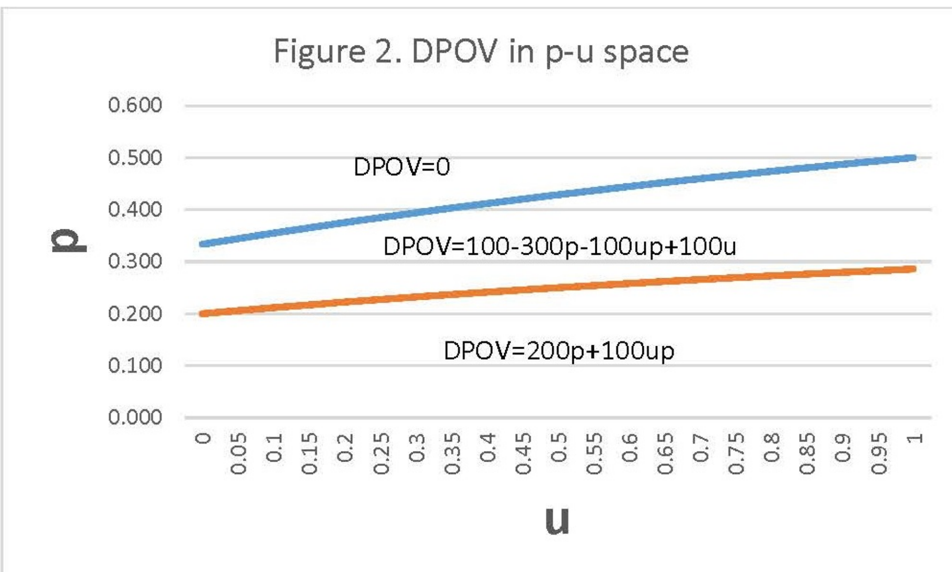
$$DPOV = 200p + 100up \qquad \text{if } p \leq \frac{100u + 100}{500 + 200u}$$

$$DPOV = 100 - 300p - 100up + 100u \qquad \text{if } \frac{100u + 100}{500 + 200u} \leq p \leq \frac{100 + 100u}{300 + 100u}$$

$$DPOV = 0 \qquad \text{if } p \geq \frac{100 + 100u}{300 + 100u}$$

The option value is increasing in u because a higher u raises the high payoff, which makes waiting for information more valuable. A higher u also decreases the low payoff, but since this outcome is avoided when the decision is delayed, only the upside risk matters. In the figure below, I've indicated the value of the DPOV for different values of p and u. The option value is zero for high values of p, for reasons discussed above. This region shrinks as u increases because the higher payoff increases the value of delaying the decision.

Figure 2: DPOV in p-u space



c. Now suppose that $\delta \leq 1$ and $u = 0$. Derive an expression for the option value in terms of p and δ. How does the Dixit-Pindyck option value change as δ gets larger? Explain. How does the option value vary across $p - \delta$ space?

This problem is just like in part a except that the period 2 payoffs are multiplied by δ. The DPOV becomes:

$$DPOV = \delta 200p \qquad \text{if } p \leq \frac{200 - 100\delta}{500\delta}$$

$$DPOV = -300\delta p + 200 - 100\delta \qquad \text{if } \frac{200 - 100\delta}{500\delta} \leq p \leq \frac{200 - 100\delta}{300\delta}$$

$$DPOV = 0 \qquad \text{if } p \geq \frac{200 - 100\delta}{300\delta}$$

A larger value of δ decreases the option value because discounting reduces the second period payoff, making it less valuable to learn about that value. In the figure below, the DPOV is zero for high values of p, as before. However, discounting counteracts the effect of the high probability of a high payoff, increasing the incentive to delay the investment.

Figure 3: DPOV in $\delta - u$ space