

1 Econ 260A. Fall 2018. Homework Challenge 2

1. An exhaustible resource is costlessly extracted in continuous time by competitive firms (assume all the other assumptions of the Hotelling model apply). The market inverse demand curve is $p(y) = a - by$ where $p(t)$ and $y(t)$ denote the price and quantity in time t and a and b are finite parameters. The initial stock of the resource is given by $x(0) = x_0$, the interest rate is r , and the time horizon is $0 \leq t \leq T$.

(a) Use optimal control theory to characterize the dynamic market equilibrium. Clearly state all of the first-order and transversality conditions that must hold.

(b) Use your results from part (a) to derive an implicit function for T in terms of the parameters of the problem (a, b, x_0, r) . Show how T changes with marginal changes in each of the parameters and discuss the economic intuition for each comparative static result.