1 Nonrenewable Resources Lecture

1.1 Background

In the standard renewable resource problem, $\dot{x} = F(x(t) - y(t)$. For a nonrenewable resource, $F(x(t)) = 0$, so we can write $\dot{x}(t) = -y(t)$, and denote the starting stock as $x(0) = x_0$. As with renewable resources, the central question in the economics literature is how to optimally exploit the resource to maximize its discounted value. Since the stock is fixed, and ignoring the uninteresting case in which the resource is too costly to extract (implying $y(t) = 0$ for all t), we must consider a finite horizon problem because at some point the stock will be exhausted if the optimal $y(t) > 0$.

Distinctions are often made between nonrenewable and exhaustible resources and resources and reserves. Resources correspond to the total physical quantity of materials (minerals, oil, etc.) in the universe, while reserves are a subset of the resources that are known about and economical to extract. If reserves are equal to resources, this can be considered an exhaustible resource. Nonrenewable resources refers to the case where reserves are not equal to resources. In this case, the stock of reserves can be expanded through exploration or technological change. An example is ocean-bed minerals. There are large stocks of minerals under the ocean floor. The problem is they are under the ocean floor and uneconomical to extract at current prices. We will focus on the simpler model of exhaustible resource use in which the initial stock is fixed.

1.2 Hotelling

The seminal paper on exhaustible resource use is by Hotelling (1931). He considered the case of 1) a competitive industry, 2) a homogenous resource, 3) costless extraction, 4) a common alternative rate of return r, 5) perfect foresight and common knowledge, and proposed what is known now as Hotelling's rule: in equilibrium, prices for the resource should grow at the rate of interest. Specifically, $\frac{\dot{p}(t)}{p(t)} = r$.

Consider decision by a single firm to extract a unit of the resource today or tomorrow. If today, the resource is sold for p_t and invested in the alternative asset earning interest r and yielding a future value $FV_{t+1} = p_t(1+r)$. If tomorrow, $FV_{t+1} = p_{t+1}$. If $p_t(1+r) > p_{t+1}$, then what happens? As long as this is true, then all of the resource will be extracted, driving down $p_t(1+r)$ until $p_t(1+r) = p_{t+1}$. Conversely, if $p_t(1+r) < p_{t+1}$, nobody extracts today, driving up current price until $p_t(1+r) = p_{t+1}$. The equilibrium in which nobody wants to change their behavior thus occurs at $p_t(1+r) = p_{t+1}$.

In continuous time, the Hotelling rule is $\frac{\dot{p}(t)}{p(t)} = r$, which can solved for $p(t)$: $\int \frac{\dot{p}(t)}{p(t)} dt = \int r dt$, $lnp(t) = rt + c$, $p(t) = e^{rt+c}$, $p(0) = e^c$, $p(t) = p(0)e^{rt}$.

This is the intuition. This result can be derived more formally by solving the social planner's problem of maximizing consumer plus producer surplus, which we know from the welfare theorems is the outcome achieved by a competitive market. Although, in this case, extraction is costless so the problem is to maximize the total discounted benefit of consumption measured, at each instant in time, as the area under the inverse demand curve $p(s)$. The benefit flow is $U(y(t)) = \int_0^{y(t)} p(s)ds$, and the planner problem is:

$$\max \int_0^T U(y(t))e^{-rt}dt \quad \text{s.t.} \quad \dot{x}(t) = -y(t), \quad x(0) = x_0 \tag{1}$$

The current value Hamiltonian is $\tilde{H} = U(y(t)) - \mu(t)y(t)$ and the maximum principle gives: $\tilde{H}_y = U'(y(t)) - \mu(t) = 0$, $\dot{\mu}(t) - r\mu(t) = -\tilde{H}_x = 0$, $\dot{x}(t) = \tilde{H}_\mu = -y(t)$. What about transversality conditions? This is a free-time problem requiring $\tilde{H}(T) = 0$. The free-state condition can be written $\mu(T)x(T) = 0$. Earlier, we wrote the condition as $F'(x(T)) = \lambda(T)$. In the present problem, there's no salvage value so $F' = 0$, but as you'll see in a moment we are unlikely to satisfy $\mu(T) = 0$. The general way to write the condition is $\mu(T)x(T) = 0$, which could be satisfied by $x(T) = 0$. The general free-state condition says that either the resource is used up or it isn't worth anything.

Applying Leibnitz rule, we can write $\tilde{H}_y = p(y(t)) - \mu(t) = p(t) - \mu(t) = 0$ and the adjoint equation implies $\frac{\dot{\mu}(t)}{\mu(t)} = r$. Combining these results, we get Hotelling's rule $\frac{\dot{p}(t)}{p(t)} = r$, or $p(t) = p(0)e^{rt}$. The optimal control $y(t)$ is selected in such a way that the price grows at the rate of interest. The multiplier $\mu(t)$ is the (current value) shadow price of the stock, or the value to the optimal program of one more unit of the resource in time t. According to the maximum condition, on the margin the value of the resource should equal its market price. If this wasn't the case, there would be a way of increasing the total value of the program. For example, if $p(t) > \mu(t)$, then selling an additional unit in time t is worth more than holding it in the ground until next period.

Now, let's consider what happens in time T. The free-state condition says $\mu(T)x(T) = 0$. How will this be satisfied? By ensuring $x(T) = 0$. Why? Because $\mu(T) > 0$. We know this because $\mu(t)$ equals price, $p(t) > 0$ for all t, and $\mu(t)$ must grow at the rate of interest and, thus, must get larger over time. To show that price must be positive, we have to rule out the possibility that the optimal program is to exhaust all of the resource in time 0 by choosing a large value of $y(0)$ that makes $p(0) = 0$. The free-time condition $\tilde{H}(T) = 0$ isn't satisfied in this case because the area under the demand curve would be positive and thus the Hamiltonian couldn't be zero because $\mu(0)y(0) = p(0)y(0) = 0$.

Let's consider the free-time condition. If there is a finite choke price, where will we be on the inverse demand curve in period T when the last unit is sold? The free-time condition, $\tilde{H}(T) = U(y(T)) - \mu(T)y(T) = 0$, requires that $y(T) = 0$. The area under a downward sloping demand curve must be larger than $\mu(T)y(T) = p(T)y(T)$ for any positive $y(T)$. Intuition: why would it never be optimal to supply the last unit at a price below the choke price? Because you could improve on the program but holding back one unit and supplying it in the next period at the choke price. What if the price goes to infinity as quantity goes to zero? You would have to satisfy the transversality condition in the limit, $\lim_{T\to\infty} \tilde{H}(T) = 0$, because otherwise you could improve on the program. Another possibility is that there's a backstop technology that kicks in at a high enough price. This is considered by Heal, G. 1976. The Relationship between Price and Extraction Cost for a Resource with a Backstop Technology. Bell Journal of Economics 7(2):371-78.

To summarize, the main features of the optimal program are that prices grow at the rate of interest and the initial price is chosen in such a way that the resource is exhausted at the precise moment when demand goes to zero. That is, $p(t) = p(0)e^{rt}$, $\int_0^T y(t)dt = x_0$, $y(T) = p^{-1}(p(0)e^{rT}) = 0$.

1.3 Issues and Extensions

Competitive industry: If there are N firms, not necessarily identical, the same result can be obtained if the firms accurately predict the time path of prices (i.e., have rational expectations).

Adding extraction costs: If we add a constant marginal cost of extraction we get $p(t) - c = \mu(t)$ and $\frac{(p(t)\dot{-}c)}{(p(t)-c)} = r$. Or, $\frac{\dot{p}(t)}{p(t)} = r(1 - \frac{c}{p(t)})$. How do costs change the optimal extraction path? Prices increase more slowly, implying the initial price must be higher in order to reach the choke price. And, the exhaustion date must be later because otherwise there would be some of the resource left (i.e., the two price paths have to cross because otherwise the quantity with marginal costs is always less). Intuition? Extract more slowly to push costs off to the future. Alternatively, if we make marginal costs a declining function of stock, $c'(x) < 0$, then $\frac{\dot{p}(t)}{p(t)} = r(1 - \frac{c(x)}{p(t)})$. In this case, as x goes to zero, $c(x)$ will get large and the rate of price growth will go to zero. This implies an ever flatter price path and a more distant exhaustion date. Both factors push costs off to the future.

Scarcity: a perennial question is whether nonrenewable resources are becoming scarcer? The relevant scarcity measure is $\mu(t)$, the shadow price. This is the value of having an additional unit of the resource in the ground and $\mu(t)$ rises over time as the resource becomes scarcer. Note that the market price may not be a good indicator of scarcity because it is difficult to disetangle prices from extraction costs. If c is falling due to technological change, then price can actually fall while $\mu(t)$ is going up. Thus, $p(t)$ can fall as the resource get more scarce. Unfortunately, we can readily observe $p(t)$ but we don't directly observe $\mu(t)$ (except in the case of timber).

Oil: We will look at the Anderson et al. paper next time, which suggests that Hotelling's framework does not apply to oil extraction. Instead, the maximum production rate from any well is physically constrained by the pressure available in the underground oil reservoir, and this pressure declines toward zero as more and more oil is extracted. Using detailed data on well-level production and drilling from Texas, the authors show that oil production from existing wells exhibits essentially zero response to price shocks, contradicting a basic prediction of Hotelling's standard model. Instead, production declines steadily toward zero, consistent with a model in which firms always produce their wells at their maximum flow rate. In contrast, they show that the rate of drilling of new wells responds substantially to oil price shocks, as does the cost of renting drilling rigs.