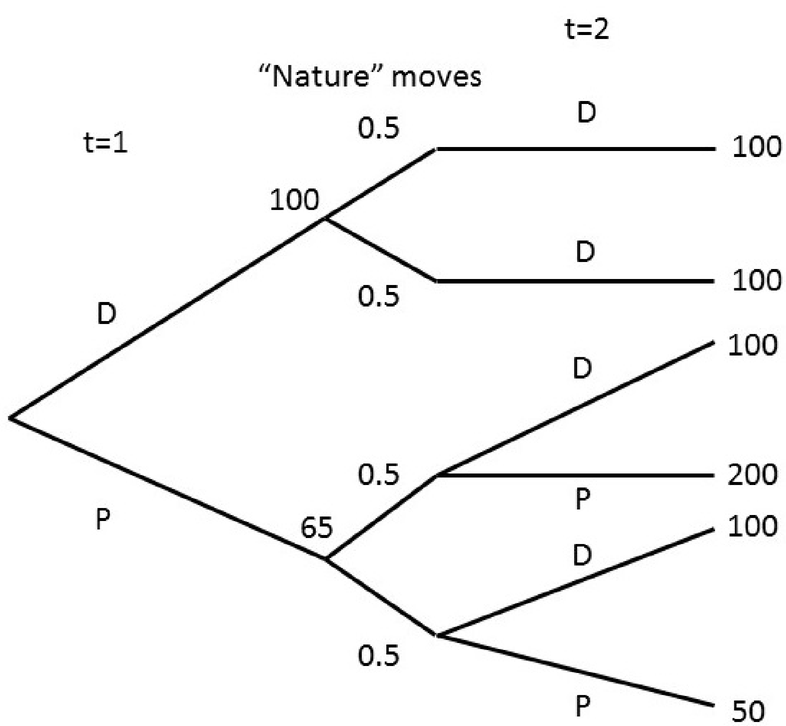
1 Option Values

1.1 Simple example

Consider the decision to develop or preserve a plot of land in period 1. The payoff to development is 100 at the end of period 1 and 2 and development is irreversible, such that if it is undertaken in period 1, the choice is development in period 2. Preservation is not irreversible, so that if it is chosen in period 1, preservation or development can be chosen in period 2. The period 1 payoff to preservation is known with certainty and equal to 65. However, there is uncertainty about the period 2 payoff to preservation. With probability 0.5 it is equal to 200 and with probability 0.5 it is equal to 50. The timing of payoffs is represented in the graph below. The decision maker selects develop (D) or preserve (P) in periods 1 and 2 and "Nature" moves between period 1 and period 2 to determine the value of period 2 preservation.

Figure 1: Develop v. preservation decision



One decision rule would be to compute the NPV of development and the expected net present value of preservation and choose the larger. Specifically, if a once-and-for-all decision is made in period 1, then $ENPV_D = 100+100 = 200$ and $ENPV_P = 65+0.5*200+0.5*50 = 190$, which says that development should be chosen in period 1. But this rule ignores the possibility of learning about the value of preservation. That is, if preservation is chosen in period 1, then the decision-maker gets to observe the move by "Nature": whether the value of preservation goes up to 200 or goes down to 50. If the former, then preservation in period

2 continues to be the best choice, but if the latter, development in period 2 is a better choice. So, an alternative rule is based on anticipatory learning, in which case preservation in period 1 has an expected value of $ENPV_P = 65 + 0.5 * 200 + 0.5 * 100 = 215$. The value of development in period 1 is still 200 because the choice is irreversible: learning about the value of preservation doesn't help you if you've chosen irreversible development in period 1. Under learning, we see that the expected value of preservation in period 1 is greater than the value of development in period 1. The notion of option value, which we will develop formally in a moment, recognizes that there is a value to delaying irreversible decisions in order to learn about uncertain payoffs.

1.2 Formal model set-up, part 1

Following Traeger (2014 Resource and Energy Economics), we consider a discrete decision to preserve or develop a plot of land. This is a two-period problem where $x_i = 0$ indicates preservation in period $i = 1, 2$ and $x_i = 1$ indicates development. The payoff from preservation and development is given by:

$$v(x_1, x_2, \tilde{\theta}) = u_1(x_1) + u_2(x_1, x_2, \tilde{\theta}) \tag{1}$$

where $\tilde{\theta}$ is a random variable that affects the period 2 payoff. In general, x_1 can affect the period 2 payoff, which can be thought of like a "stock effect". In addition, there is no explicit discounting term, but it could be implicit in the u_2 function.

1.3 NPV rule

Given this set-up, let's consider one type of decision rule, referred to as as the NPV (net present value) rule. V will be used to denote the expected present value of the plot and the term following V in parentheses will indicate the period 1 decision. The option value models we will consider are focused on the current period (period 1) decision about whether to preserve or develop. Using this notation, we have:

$$V^n(0) = u_1(0) + Eu_2(0, 0, \tilde{\theta}) \tag{2}$$
$$V^n(1) = u_1(1) + Eu_2(1, 1, \tilde{\theta}) \tag{3}$$

The NPV rule says that development should be chosen in period 1 if $NPV = V^n(1) - V^n(0) \geq 0$ and, otherwise, preservation should be chosen. In the above expressions, n refers to a "naive" decision maker. Why? What is missing from the rule is 1) the possibility of making different choices in period 1 and 2 (e.g., preserve, then develop), 2) any consideration of conditioning the period 2 decision on a revealed value of $\tilde{\theta}$, and 3) any consideration of the irreversibility of the development decision.

1.4 Set-up, part 2

We augment the set-up of the problem. First, we assume that development is irreversible. Thus, $x_1 = 1$ implies $x_2 = 1$, but if $x_1 = 0$, x_2 can be either 0 or 1. Irreversibility has been an implicit feature of the natural resource economics models we have examined. For example, we have assumed that once fish are harvested, trees are cut, or minerals are mined, the decision cannot be undone and the resource must be sold at the prevailing price. In other words, we are assuming a cost of undoing the decision or storing the resource that is prohibitive. Second, we are precise about the informational timing of model. We assume that $\tilde{\theta}$ is revealed after period 1, but before the period 2 decision must be made.

1.5 The optimal rule

To find the optimal solution to the preservation/development problem we should apply Bellman's Principle of Optimality. Formally, we solve:

$$v(x_1^*, x_2^*, \tilde{\theta}) = \max_{x_1 \in \{0,1\}} u_1(x_1) + E \max_{x_2 \in \{x_1,1\}} u_2(x_1, x_2, \tilde{\theta}) \tag{4}$$

Note that the expectations operator is outside of the max function in period 2. The decision-maker is able to observe $\tilde{\theta}$ before making the period 2 decision and, thus, anticipates being in period 2 and maximizing u_2 conditional on the revealed value of $\tilde{\theta}$. However, from the perspective of period 1, the solution to this maximization problem is only known in expectation. The expected present value terms are given by:

$$\begin{aligned} V^l(0) &= u_1(0) + E \max_{x_2 \in \{0,1\}} u_2(0, x_2, \tilde{\theta}) \\ V^l(1) &= u_1(1) + E u_2(1, 1, \tilde{\theta}) \end{aligned}$$

where "l" denotes learning. That is, the decision-maker anticipates that she will learn about the value of $\tilde{\theta}$ and accounts for this when making the period 1 decision. The optimal rule can be stated: develop in period 1 if $V^l(1) - V^l(0) \geq 0$ and, otherwise, preserve.

1.6 Postponement

A third possibility is that the decision-maker will choose x_2 optimally once in period 2 (i.e., by conditioning on the revealed value of $\tilde{\theta}$) but not account for this in making the period 1 choice. This rule is also called an open-loop control. In particular, the decision-maker considers postponing the period 1 decision given information available in period 1 but does not anticipate that new information will be forthcoming in the next period. The expected present value terms in this case are given by:

$$V^p(0) = u_1(0) + \max_{x_2 \in \{0,1\}} Eu_2(0, x_2, \tilde{\theta})$$
$$V^p(1) = u_1(1) + Eu_2(1, 1, \tilde{\theta})$$

where "p" denotes postponement. Now the expectations operator is inside the max function because the decision-maker is not accounting for the fact that the period 2 decision will be made after $\tilde{\theta}$ is revealed.

1.7 Option values

With the above notation, we can now define two types of option values. The first is the one discussed by Arrow and Fisher (1974 QJE), Henry (1974 AER), and Hanemann (1989 JEEM). These authors point out that the NPV rule is incorrect and will tend to lead to too much development. There is an additional value associated with preservation that is not captured in the NPV rule. They define a "quasi-option value" as the difference between the net value of postponement under anticipated learning and the net value of postponement without learning. Specifically,

$$QOV = (V^l(0) - V^l(1)) - (V^p(0) - V^p(1)) = V^l(0) - V^p(0) \tag{5}$$

The above expression simplifies because $V^l(1) = V^p(1) = V^n(1)$. It can be shown that $QOV \geq 0$. Conditional on preservation in period 1, the QOV gives the additional value from being able to learn about $\tilde{\theta}$ before making the period 2 decision. We can also write the full value of optimal decision-making relative to the naive approach as $V^l(0) - V^n(0)$, or:

$$V^l(0) - V^n(0) = (V^l(0) - V^p(0)) + (V^p(0) - V^n(0)) \tag{6}$$

where the first term is the QOV and the second term is the "simple option value" (SOV). The key result on QOV provided by Traeger is:

Result 1. The optimal decision-maker who anticipates learning is strictly better off developing in period 1 if and only if $NPV > QOV + SOV \geq 0$.

One can see that the NPV rule is incorrect. For developing in period 1 to be optimal, the NPV must be larger than the options (QOV and SOV) that are extinguished when development is chosen.

In addition to the QOV, there is a related concept called the Dixit and Pindyck option value (DPOV). A good reference is their 1994 book titled "Investment Under Uncertainty." This notion of option value is based on the real options literature in finance. In the context of our problem, the DPOV is the value of the option to postpone development. Specifically,

$$DPOV = \max\{V^l(1), V^l(0)\} - \max\{V^n(1), V^n(0)\} \tag{7}$$

The DPOV is interpreted as the net value of postponement conditional on learning. This value can be zero if development is optimal in period 1. In general, $DPOV \geq 0$. The key result provided by Traeger on DPOV is:

Result 2. The optimal decision-maker who anticipates learning is weakly better off developing in period 1 if $NPV > 0$ and $DPOV = 0$ and is strictly better off postponing if $DPOV > 0$.

Developing in period one is optimal as long as the net value to waiting for the forthcoming information about $\tilde{\theta}$ is zero.