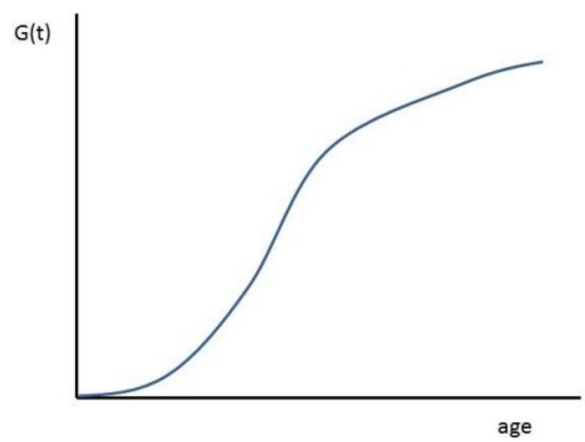
1 Forestry Lecture

1.1 The standard rotation model

The standard renewable resources model does not provide a very good description of forests. With many forests, harvesting occurs periodically with many years separating harvests. Also, the age of trees is an important consideration. The age-old problem in forestry is when to harvest an even-aged stand of trees.

If we start with bare ground and define the starting period to be $t = 0$, then age and calendar time are both t. The value of the stand at age (in time) t is $V(t) = P(t)G(t)$ where $P(t)$ is the price in time t and $G(t)$ is the volume of the stand (e.g., cubic feet of timber) at age t. The graph of $G(t)$ looks like:

Figure 1: Timber volume as a function of age



The cost of planting the stand is c. Then, if the stand is grown for one rotation, the problem is:

$$\max_t P(t)G(t)e^{-rt} - c$$

Solving gives:

$$\frac{P'(t)}{P(t)} + \frac{G'(t)}{G(t)} = r$$

which says the trees should be harvested at the age (t_1) when the growth rate in their value (price and volume increases) equal the interest rate. Similar to the fishery case, you should grow the stand as long as its value is increasing at a higher rate than the alternative rate of return.

The second order condition requires the growth rate in $V(t)$ to be declining at the maximum. Assume price growth is constant. Then, we need

$$\frac{d}{dt}\left(\frac{G'(t)}{G(t)}\right)|_{t=t^*} < 0$$

From the graph above, you can see that beyond the inflection point this condition holds. $\frac{G'}{G}$ usually rises to a maximum before falling.

Now, suppose that the land is to be replanted, another stand is to be grown and harvested, and so on indefinitely. The rotation in this case is called the Faustmann rotation (t^*) after the 19th century German forester who studied this problem (this is precursor to capital theory that came later with Ramsey). As we will see, $t^* \neq t_1$ (in general). The reason has to do with Bellman's Principle of Optimality. Define the present value of the infinite sequence of harvests as:

$$\begin{aligned} J(t) &= PG(t)e^{-rt} - c + [PG(t)e^{-rt} - c]e^{-rt} + [PG(t)e^{-rt} - c]e^{-r2t} + ... \\ &= PG(t)e^{-rt} - c + J(t)e^{-rt} \end{aligned}$$

This assumes that every rotation has the same length. Why would this be true? Because immediately following a rotation, the problem facing the manager is always the same. This problem makes more sense if price is constant, as above (otherwise, we need price to go back to its original level at the start of each rotation). If we write this as a dynamic programming problem, we would have:

$$J(t) = \max_t [PG(t)e^{-rt} - c] + e^{-rt}J(t)$$

The single rotation problem ignores the second term, namely, the present discounted value of future rotations, which is affected by the choice of t. Bellman's Principle of Optimality requires that an action be optimal with respect to the state that results from the action. Rearranging the expression for $J(t)$ from above gives:

$$J(t) = \frac{PG(t)e^{-rt} - c}{1 - e^{-rt}}$$

We now solve:

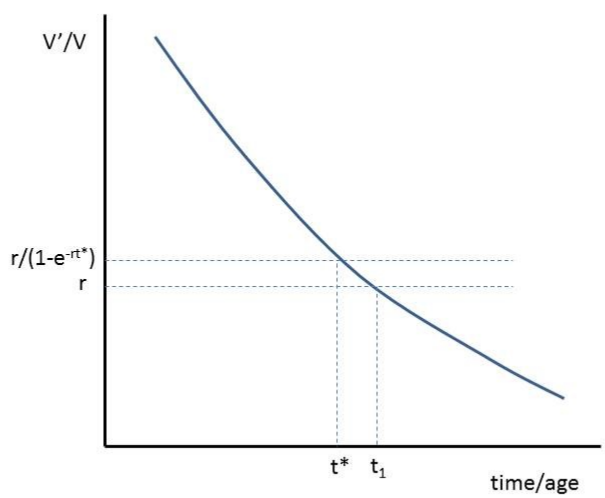
$$\max_t J(t) = \frac{PG(t)e^{-rt} - c}{1 - e^{-rt}}$$

which yields:

$$\frac{V'(t^*)}{V(t^*) - c} = \frac{r}{1 - e^{-rt^*}}$$
$$\frac{G'(t^*)}{G(t^*)} = \frac{r}{1 - e^{-rt^*}} \left(1 - \frac{c}{PG(t^*)}\right)$$

With zero costs, $t^* < t_1$; otherwise, the result is ambiguous – delaying the rotation delays the planting costs but also postpones the harvests of all subsequent rotations. For the $c = 0$ case, the graph looks like:

Figure 2: The Faustmann rotation



1.2 The Hartman model

Hartman (1976 Economic Inquiry) examined the optimal rotation when a standing forest has values (e.g., for wildlife, carbon, etc.). Suppose the flow of these services in monetary terms are $A(t)$ when the stand is age t, where $A'(0) > 0, A''(t) < 0$. This assumption allows for the case in which the amenity flow is monotonically increasing, as well as the possibility that it rises then falls. How would we write the present discounted value of amenity benefits over an infinite number of rotations?

$$PV_A = \frac{\int_0^t A(s)e^{-rs}ds}{1 - e^{-rt}}$$

The maximization problem with timber included becomes:

$$\max_t J(t) = \frac{PG(t)e^{-rt} + \int_0^t A(s)e^{-rs}ds - c}{1 - e^{-rt}}$$

Suppose that $A'(t) > 0$ for all t. How should the rotation length change relative to the timber only case? It will always be longer because:

$$\frac{dPV_A}{dt}|_{t=t^*} > 0$$

This implies that J(t) is increasing at t^* and, thus, the rotation should be extended. It is possible that t^* should be infinite or if $A'(t) < 0$ for some $t > 0$, the rotation could be shorter than the timber only rotation.

1.3 Fire Risk

Reed (1984 Journal of Environmental Economics and Management) considered how the optimal rotation length should be modified when there is fire risk. William Reed is a mathematician from UBC who works on dynamic optimization problems and has made some really nice contributions to the natural resource economics literature. Reed assumes that fires occur in a Poisson process at rate λ. That is, if λ is the average number of fires that occur each year, the probability that exactly k fires ($k = 1, 2, ...$) occur is:

$$f(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

λt is the rate of fires occurring over a period of t years. Thus, the probability that $k = 0$ fires occur over t years is given by:

$$f(0; \lambda) = e^{-\lambda t}$$

and the cumulative probability that more than zero fires occur is $F(t) = 1 - e^{-\lambda t}$. Let $X_1, X_2, ...$ denote the times *between* successive destructions either by fire ($t < T$) or logging ($t > T$) where T is the age at which the stand is harvested. The distribution of these random variables X_n is given by:

$$\begin{aligned} F_X(t) &= 1 - e^{-\lambda t} \qquad t < T \\ &= 1 \qquad t \geq T \end{aligned}$$

The net payoffs from the stand are:

$$\begin{aligned} Y_n &= -c_2 \qquad X_n < T \\ &= V(T) - c_1 \qquad X_n = T \end{aligned}$$

where $V(t) = pG(t)$. The expected present value from an infinite sequences of rotations is given by:

$$J = E\left(\sum_{n=1}^{\infty} e^{-r(X_1+X_2+...+X_n)}Y_n\right)$$

where the term in parentheses is the total number of years between today and the nth stand destruction. The independence of stand destructions is implicit in the Poisson formulation, and allows the expression above to be written as:

$$J(t) = \frac{(\lambda + r)(V(T) - c_1)e^{-(\lambda+r)T}}{r(1 - e^{-(\lambda+r)T})} + \frac{\lambda}{r}c_2$$

This expression is maximized with respect to T. Then, we get an expression similar to what we had before:

$$\frac{V'(T^*)}{V(T^*) - c_1} = \frac{\lambda + r}{1 - e^{-(\lambda+r)T^*}}$$

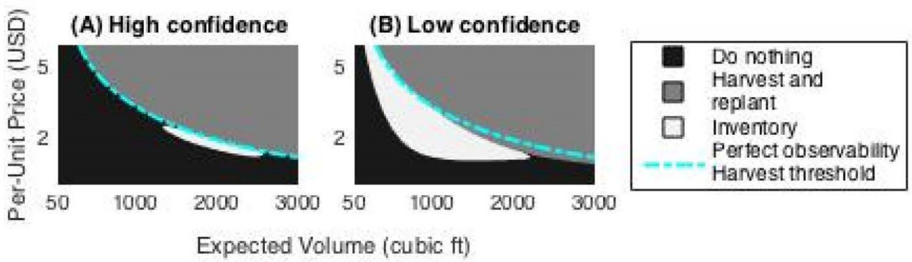
It can be shown that the T^* that solves this expression is less than the T^* that solves the Faustmann problem in the absence of fire risk. With the risk of destruction by fire, it is optimal to shorten the rotation in order to guard against this loss. In effect, you use a higher discount rate, $\lambda + r$, which makes you more impatient for timber harvest revenues, than in the case with no fire risk.

1.4 Timber inventories

It is typically assumed in forestry models (and natural resource models more generally), that the state variable is perfectly observable. That is, in the current period the manager knows with certainty the amount of the stock. This is never the case. With forests, managers have to undertake costly inventories (called timber cruises) to obtain an estimate of the stock volume. I'm working with Matt Sloggy (former PhD student at Oregon State) and David Kling (assistant professor at Oregon State) on a paper called, "Measure Twice Cut Once: Optimal Inventory and Harvest under Volume Uncertainty and Price Volatility." Our question is: how is the optimal timing of timber harvests affected by imperfect observability of the stock? In our model, timber volume and price evolve stochastically. The timber volume is unobservable but the manager can get an estimate of the volume by conducting an inventory. Prices are stochastic but perfectly observable in the current period. In every period, the manager observes price, forms beliefs about the volume, and decides whether to harvest, do an inventory, or do neither. There are three state variables in the model: 1) price, 2) belief mean, 3) belief confidence (coefficient of variation). The following graph shows the policy function for two difference levels of confidence:

Figure 3: Measure Twice, Cut Once

Figure 1: Policy functions for the with-inventory (WI) case, holding confidence constant at a high level (A) and a low level(B). The cv values associated with the low and high levels are 0.11 and 0.52, respectively. The perfect observability threshold is overlaid for comparison.



We parameterize our model using volume, price, and cost data for timber stands in the southern United States and evaluate scenarios with and without inventories and under alternative informational assumptions. We find that the option to conduct inventories raises the present value of timber stands and nearly reaches the value when timber volumes are perfectly observable. A key insight from our results is that price stochasticity has an important influence on inventory decisions. When prices follow a stationary autoregressive process, inventories are most likely to occur (specifically, the range of expected timber volumes is greatest) at prices near the mean of price process. In this case, the cost to delaying harvest in order to conduct an inventory is low because only small changes in price are expected. In contrast, when prices are high, the cost to delaying harvest is also high because prices are expected to fall, and so inventories are less common. Finally, at low prices, the manager is better off waiting to see if prices rise before committing to either a harvest or an inventory. This asymmetry in inventory decisions is magnified by the degree of uncertainty over the timber volume estimate. When there is a high degree of confidence in the estimate, inventories are only conducted at prices near the mean. However, as confidence decreases, it becomes optimal to conduct inventories at ever greater prices. With low confidence, an inventory may indicate a large timber volume that can be harvested before prices fall further.

An additional insight is that state uncertainty can enlarge the region of the state space where harvesting is optimal. We find that when confidence in the timber volume estimate is low, it can be optimal to harvest at some price and expected volume combinations when delaying harvest would be optimal if the volume were known with certainty. The counterintuitive finding that uncertainty increases the likelihood of an irreversible harvesting decision is related to the truncation of timber volumes at zero. For a given expected timber volume, a higher degree of uncertainty increases the probability of large timber volumes at which it is highly profitable to harvest. Truncation of timber volumes at zero limits the potential losses from low volumes.