Assignment 5

Dimensionless form and types of cycles

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Convert a model of bacterial growth in a chemostat to dimensionless form

$$
\begin{aligned}
\frac{dN}{dt} &= \left(\frac{K_{max}C}{K_N + C}\right)N - \frac{FN}{V} \\
\frac{dC}{dt} &= -\alpha\left(\frac{K_{max}C}{K_N + C}\right)N - \frac{FC}{V} + \frac{FC_0}{V}
\end{aligned}
$$

First we need to define variables that split our variables (bacteria, nutrients, time) into a scalar and its dimensions:

- N^*, C^*, and t^* are the scalars
- $\hat{N}$, $\hat{C}$, and $\hat{t}$ are the time-independent units such that $N = N^* \times \hat{N}$, $C = C^* \times \hat{C}$, and $t = t^* \times \hat{t}$ represent our measurements.

Therefore, we can re-write Eq as:

$$
\begin{aligned}
\frac{d(N^*\hat{N})}{d(t^*\hat{t})} &= \left(\frac{K_{max}(C^*\hat{C})}{K_N + (C^*\hat{C})}\right)(N^*\hat{N}) - \frac{F(N^*\hat{N})}{V} \\
\frac{d(C^*\hat{C})}{d(t^*\hat{t})} &= -\alpha\left(\frac{K_{max}(C^*\hat{C})}{K_N + (C^*\hat{C})}\right)(N^*\hat{N}) - \frac{F(C^*\hat{C})}{V} + \frac{FC_0}{V}
\end{aligned}
$$

We can now multiply both sides by and $\hat{t}$, and divide by $\hat{N}$ and $\hat{C}$ obtaining:

$$
\begin{aligned}
\frac{dN^*}{dt^*} =& \hat{t}K_{max}\left(\frac{C^*}{\frac{K_N}{\hat{C}} + C^*}\right)N^* - \hat{t}\frac{F}{V}N^* \\
\frac{d(C^*\hat{C})}{d(t^*\hat{t})} =& \frac{-\alpha\hat{t}K_{max}\hat{N}}{\hat{C}}\left(\frac{C^*}{\frac{K_N}{\hat{C}} + C^*}\right)N^* - \hat{t}\frac{F}{V}C^* + \hat{t}\frac{FC_0}{V\hat{C}}
\end{aligned}
$$

Since V is in units of volume, and F represents a flow (*i.e.* volume over time), we can say that $\hat{t} = \frac{V}{F}$. This is also convenient to later re-write the system. By further setting $\hat{C} = K_N$ and $\hat{N} = \frac{\hat{C}}{\alpha\hat{t}K_{max}} = \frac{\hat{K_N}}{\alpha\hat{t}K_{max}}$ we can re-write the system as:

$$
\begin{aligned}
\frac{dN}{dt} &= \left(\frac{K_{max}C}{K_N + C}\right)N - \frac{FN}{V} \\
\frac{dC}{dt} &= -\alpha\left(\frac{K_{max}C}{K_N + C}\right)N - \frac{FC}{V} + \frac{FC_0}{V}
\end{aligned}
$$

Numerical simulation of a spring with no dampening

The model is described by:

$$\frac{dx}{dt} = y \frac{dy}{dt} = -Kx$$

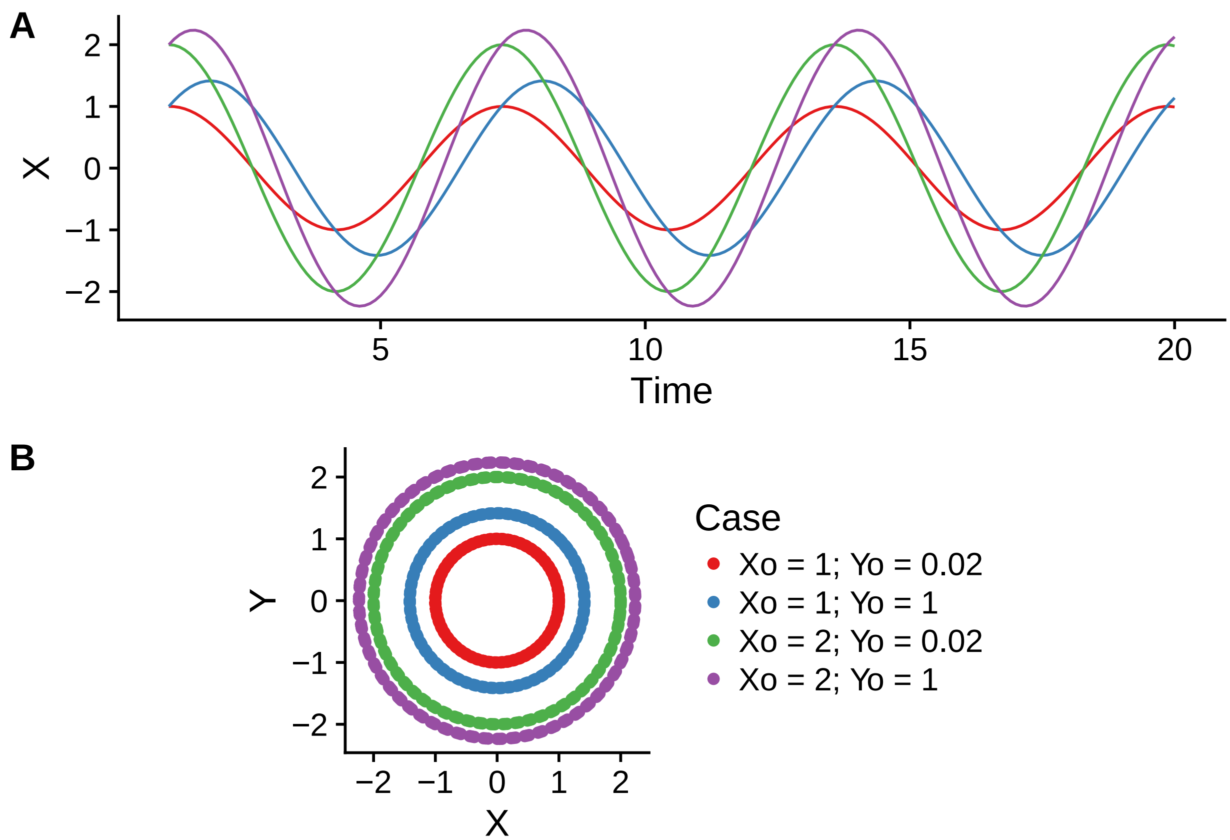

Figure 1: Different scenarios for a simple harmonic motion of a spring. A shows the distance (X) through time, and B shows the relationship between distance (X) and speed (Y). Red dots indicate initial conditions, blue dots indicate equilibrium.

Numerical solutions for the non-linear Lotka-Volterra predator-prey model

$$\begin{aligned} \frac{dN}{dt} &= rN - aNP \\ \frac{dP}{dt} &= caNP - mP \end{aligned} \tag{1}$$

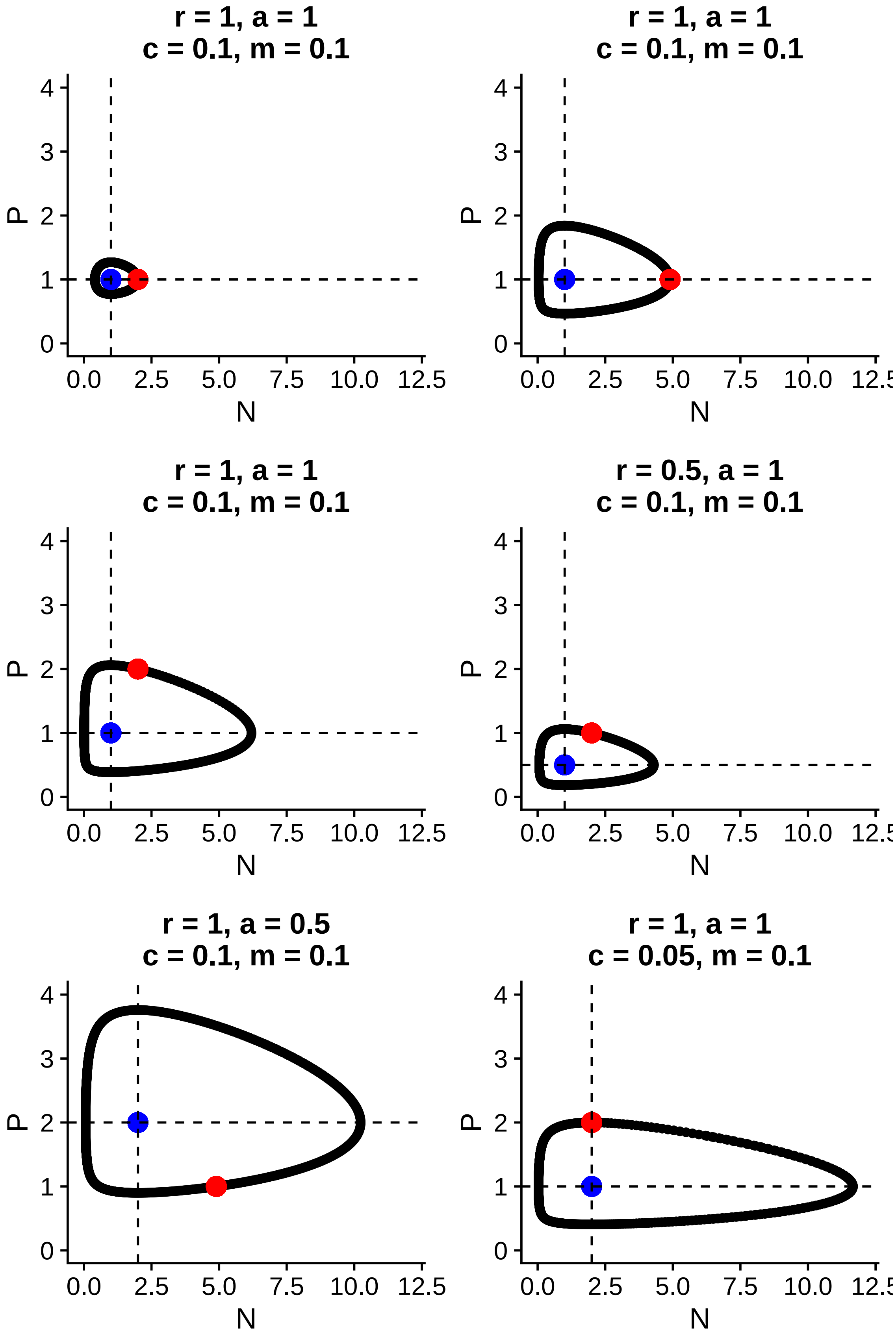

Figure 2: Combination of parameters and initial conditions for a Lotka-Volterra system with no density dependance and a Type-I functional response.