

# PS207 Quantitative Causal Inference

## Experiments

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Special thanks to Chad Hazlett (UCLA) for select slides, used with permission

# Randomization Solves the Selection Problem

Recall the selection bias formula for diff. in means ( $\tilde{\tau}$ )

$$\begin{aligned}\tilde{\tau} &= \mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] \quad (\text{obs. diff. in means}) \\ &= \mathbb{E}[Y_{1i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 0] \\ &= \underbrace{\mathbb{E}[Y_{1i} - Y_{0i}|D_i = 1]}_{\tau_{ATT}} + \underbrace{\mathbb{E}[Y_{0i}|D_i = 1] - \mathbb{E}[Y_{0i}|D_i = 0]}_{\text{Bias}}\end{aligned}$$

How can we eliminate the bias term?

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How can we eliminate the bias term?

**Random assignment** of  $D_i$  will make the treated and untreated units identical on average, such that

$$\mathbb{E}[Y_{0i} | D_i = 1] = \mathbb{E}[Y_{0i} | D_i = 0]$$

This implies Bias = 0.

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- Large increase in the use of experiments in the social sciences: laboratory, survey, and field experiments
- *Abbreviated* list of examples (from Green 2008):
  - **Program evaluation**: development programs, education programs, SAT prep classes, weight loss programs, diversity training, deliberative polls, advertising campaigns, website designs...
  - **Public policy evaluation**: teacher pay, student incentives, class size, speed traps, vouchers, alternative sentencing, job training, health insurance subsidies, tax compliance, public housing
  - **Behavioral research**: persuasion, mobilization, education, income, interpersonal influence, conscientious health behaviors, media exposure, deliberation, discrimination
  - **Research on institutions**: transparency, corruption, electoral systems, information

# Outline

- 1 Identification
- 2 Hypothesis Testing
- 3 Randomization Inference
- 4 Threats to Validity
- 5 Reviewing What We've Covered So Far

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- What is an unbiased **estimator** for  $\beta$ ?  $\hat{\beta} = (\sum_{i=1}^n X_i X_i^\top)^{-1} (\sum_{i=1}^n X_i Y_i)$ .

# Classical Randomized Experiment

## Setup:

- Units:  $i = 1, \dots, N$
- Treatment:  $D_i \in \{0, 1\}$ , randomly assigned
- Potential outcomes:  $Y_{0i}, Y_{1i}$
- Observed outcome:  $Y_i = Y_{D_i i}$
- Number of treated/untreated units:  $N_1 = \sum_{i=1}^N D_i$  and  $N_0 = N - N_1$

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Randomization (simple or complete) implies:  $\{Y_{1i}, Y_{0i}\} \perp\!\!\!\perp D_i$

# Identification of Average Treatment Effect

Identification assumption (guaranteed by random assignment):

$$\{Y_{1i}, Y_{0i}\} \perp\!\!\!\perp D_i$$

Quantity of interest:

$$\tau_{ATE} \equiv \mathbb{E}[Y_{1i} - Y_{0i}] = \frac{1}{N} \sum_{i=1}^N (Y_{1i} - Y_{0i})$$

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$$\begin{aligned} \mathbb{E}[Y_i | D_i = 1] &= \mathbb{E}[D_i \cdot Y_{1i} + (1 - D_i) \cdot Y_{0i} | D_i = 1] \\ &= \mathbb{E}[Y_{1i} | D_i = 1] \end{aligned}$$

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So it follows that

$$\begin{aligned} \tau_{ATE} &= \mathbb{E}[Y_{1i}] - \mathbb{E}[Y_{0i}] = \underbrace{\mathbb{E}[Y_i | D_i = 1] - \mathbb{E}[Y_i | D_i = 0]}_{\text{observed difference in means}} \\ &= \frac{1}{N_1} \sum_{i=1}^N D_i Y_i - \frac{1}{N_0} \sum_{i=1}^N (1 - D_i) Y_i \end{aligned}$$

# SATE and PATE

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- Compare to the **Population Average Treatment Effect (PATE)**
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  - need to account for two sources of variation:
    - variation from the sampling process
    - variation from treatment assignment (randomness of  $D_i$ )
- Given additional uncertainty in PATE, you might (correctly) expect

$$\text{Var}(\widehat{PATE}) > \text{Var}(\widehat{SATE})$$

# Uncertainty for SATE and PATE

A first attempt at standard errors,

- Treat  $\bar{Y}_1$  as if computed from a random sampling of  $Y_{1i}$  in population
- Likewise, treat  $\bar{Y}_0$  as if from a (separate) sampling of the  $Y_{0i}$ 's
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Get variance of the difference in means:

$$\begin{aligned}\mathbb{V}(\bar{Y}_1 - \bar{Y}_0) &= \mathbb{V}(\bar{Y}_1) + \mathbb{V}(\bar{Y}_0) + 2cov(\bar{Y}_1, \bar{Y}_0) \\ &= \frac{\sigma_{Y_1}^2}{N_1} + \frac{\sigma_{Y_0}^2}{N_0} \\ SE_{ATE} &= \sqrt{\frac{\sigma_{Y_1}^2}{N_1} + \frac{\sigma_{Y_0}^2}{N_0}}\end{aligned}$$

where  $\sigma_{Y_1}^2$  and  $\sigma_{Y_0}^2$  are the variance of the  $Y_{1i}$  and  $Y_{0i}$  in the population

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This is what we typically use, and it is

- the correct SE for the PATE
- what you get from t-tests (unequal variance), regression with robust SE
- conservative for the SATE, where the sample fixed by  $D_i$  is random

# Standard Error for Sample ATE

In finite sample, sampling without replacement occurs; changes variance of group means, and induces (small) covariance

## Standard Error for Sample ATE

Given complete randomization of  $N$  units with  $N_1$  assigned to treatment and  $N_0 = N - N_1$  to control, the true standard error of the estimated sample ATE is given by

$$SE_{\widehat{ATE}} = \sqrt{\left(\frac{N - N_1}{N - 1}\right) \frac{Var[Y_{1i}]}{N_1} + \left(\frac{N - N_0}{N - 1}\right) \frac{Var[Y_{0i}]}{N_0} + \left(\frac{1}{N - 1}\right) 2Cov[Y_{1i}, Y_{0i}]}$$

with population variances and covariances  $Var[Y_{di}]$ ,  $Cov(Y_{1i}, Y_{0i})$ .

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Is this identifiable?

Standard error decreases if:

- $N$  grows
- $Var[Y_1]$ ,  $Var[Y_0]$  decrease
- $Cov[Y_1, Y_0]$  decreases



# Proof: $SE_{\widehat{ATE}} \leq \widehat{SE}_{\widehat{ATE}}$

Upper bound for standard error is when  $Cor[Y_1, Y_0] = 1$ :

$$Cor[Y_1, Y_0] = \frac{Cov[Y_1, Y_0]}{\sqrt{Var[Y_1]Var[Y_0]}} \leq 1 \iff Cov[Y_1, Y_0] \leq \sqrt{Var[Y_1]Var[Y_0]}$$

$$\begin{aligned} SE_{\widehat{ATE}} &= \sqrt{\left(\frac{N - N_1}{N - 1}\right) \frac{Var[Y_1]}{N_1} + \left(\frac{N - N_0}{N - 1}\right) \frac{Var[Y_0]}{N_0} + \left(\frac{1}{N - 1}\right) 2Cov[Y_1, Y_0]} \\ &= \sqrt{\frac{1}{N - 1} \left( \frac{N_0}{N_1} Var[Y_1] + \frac{N_1}{N_0} Var[Y_0] + 2Cov[Y_1, Y_0] \right)} \\ &\leq \sqrt{\frac{1}{N - 1} \left( \frac{N_0}{N_1} Var[Y_1] + \frac{N_1}{N_0} Var[Y_0] + 2\sqrt{Var[Y_1]Var[Y_0]} \right)} \\ &\leq \sqrt{\frac{1}{N - 1} \left( \frac{N_0}{N_1} Var[Y_1] + \frac{N_1}{N_0} Var[Y_0] + Var[Y_1] + Var[Y_0] \right)} \end{aligned}$$

Last step follows from the following inequality

$$\begin{aligned} (\sqrt{Var[Y_1]} - \sqrt{Var[Y_0]})^2 &\geq 0 \\ Var[Y_1] - 2\sqrt{Var[Y_1]Var[Y_0]} + Var[Y_0] &\geq 0 \iff Var[Y_1] + Var[Y_0] \geq 2\sqrt{Var[Y_1]Var[Y_0]} \end{aligned}$$

# Proof: $SE_{\widehat{ATE}} \leq \widehat{SE}_{\widehat{ATE}}$

$$\begin{aligned}
 SE_{\widehat{ATE}} &\leq \sqrt{\frac{1}{N-1} \left( \frac{N_0}{N_1} \text{Var}[Y_1] + \frac{N_1}{N_0} \text{Var}[Y_0] + \text{Var}[Y_1] + \text{Var}[Y_0] \right)} \\
 &\leq \sqrt{\frac{N_0^2 \text{Var}[Y_1] + N_1^2 \text{Var}[Y_0] + N_1 N_0 (\text{Var}[Y_1] + \text{Var}[Y_0])}{(N-1)N_1 N_0}} \\
 &\leq \sqrt{\frac{(N_0^2 + N_1 N_0) \text{Var}[Y_1] + (N_1^2 + N_1 N_0) \text{Var}[Y_0]}{(N-1)N_1 N_0}} \\
 &\leq \sqrt{\frac{(N_0 + N_1)N_0 \text{Var}[Y_1]}{(N-1)N_1 N_0} + \frac{(N_1 + N_0)N_1 \text{Var}[Y_0]}{(N-1)N_1 N_0}} \\
 &\leq \sqrt{\frac{N \text{Var}[Y_1]}{(N-1)N_1} + \frac{N \text{Var}[Y_0]}{(N-1)N_0}} \\
 &\leq \sqrt{\frac{N}{N-1} \left( \frac{\text{Var}[Y_1]}{N_1} + \frac{\text{Var}[Y_0]}{N_0} \right)} \\
 &\leq \sqrt{\frac{N}{N-1} \left( \frac{\widehat{\text{Var}}[Y_1]}{N_1} + \frac{\widehat{\text{Var}}[Y_0]}{N_0} \right)}
 \end{aligned}$$

So the estimator for the standard error is conservative.

# Seeing the Standard Error for Sample ATE

Want to know how ATE would differ under  $J$  randomizations of treatment, same

sample:  $SE_{\hat{\theta}} \equiv \sqrt{\frac{1}{J} \sum_1^J (\hat{\theta}_j - \bar{\hat{\theta}})^2}$

$i$	$Y_{1i}$	$Y_{0i}$	$Y_i$	$D_i$	$P(D_i = 1)$
1	3	0	3	1	2/4
2	1	1	1	1	2/4
3	2	0	0	0	2/4
4	2	1	1	0	2/4

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ATE estimates given all possible random assignments with two treated units:

Treated Units:	1 & 2	1 & 3	1 & 4	2 & 3	2 & 4	3 & 4
$\widehat{ATE}$ :	1.5	1.5	2	1	1.5	1.5

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Treated Units:	1 & 2	1 & 3	1 & 4	2 & 3	2 & 4	3 & 4
$\widehat{ATE}$ :	1.5	1.5	2	1	1.5	1.5

Average  $\widehat{ATE}$  is 1.5 and true standard error is  $SE_{\widehat{ATE}} =$

$$\sqrt{\frac{1}{6} [(1.5 - 1.5)^2 + (1.5 - 1.5)^2 + (2 - 1.5)^2 + (1 - 1.5)^2 + (1.5 - 1.5)^2 + (1.5 - 1.5)^2]} \approx .28$$

# Standard Error for Sample ATE

To convince yourself the “true” formula is correct, try it using the (partial unobservable) quantities in the prior table. You would get:

$$\begin{aligned}
 SE_{\widehat{ATE}} &= \sqrt{\left(\frac{N - N_1}{N - 1}\right) \frac{Var[Y_{1i}]}{N_1} + \left(\frac{N - N_0}{N - 1}\right) \frac{Var[Y_{0i}]}{N_0} + \left(\frac{1}{N - 1}\right) 2Cov[Y_{1i}, Y_{0i}]} \\
 &= \sqrt{\left(\frac{4 - 2}{4 - 1}\right) \frac{.25}{2} + \left(\frac{4 - 2}{4 - 1}\right) \frac{.5}{2} + \left(\frac{1}{4 - 1}\right) 2(-.25)} \\
 &\approx .28
 \end{aligned}$$

# Standard Error for Sample ATE

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 &\approx .28
 \end{aligned}$$

But of course both ways we got to this number involved unobservables.



# Standard Error for Sample ATE

How does this compare to what we'll actually compute, on average?

$i$	$Y_{1i}$	$Y_{0i}$	$Y_i$
1	3	0	3
2	1	1	1
3	2	0	0
4	2	1	1

$\widehat{SE}_{ATE}$  estimates given all possible assignments with two treated units:

Treated Units:	1 & 2	1 & 3	1 & 4	2 & 3	2 & 4	3 & 4
$\widehat{ATE}$ :	1.5	1.5	2	1	1.5	1.5
$\widehat{SE}_{ATE}$ :	1.11	.5	.71	.71	.5	.5

The average  $\widehat{SE}_{ATE}$  is  $\approx .67$  compared to true  $SE_{ATE} \approx .28$

# Outline

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# Example: Effect of Training on Earnings

- Treatment Group:
  - $N_1 = 7,487$
  - Estimated Average Earnings  $\bar{Y}_1$ : \$16,199
  - Estimated Sample Standard deviation  $\hat{\sigma}_{Y|D_i=1}$ : \$17,038
- Control Group :
  - $N_0 = 3,717$
  - Estimated Average Earnings  $\bar{Y}_0$ : \$15,040
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$$\widehat{SE}_{\widehat{\tau}_{ATE}} = \sqrt{\frac{\hat{\sigma}_{Y|D_i=1}^2}{N_1} + \frac{\hat{\sigma}_{Y|D_i=0}^2}{(N_0)}} = \sqrt{\frac{17,038^2}{7,487} + \frac{16,180^2}{3,717}} \approx \$330$$

- Is this consistent with a zero average treatment effect  $\alpha_{ATE} = 0$ ?

# Testing the Null Hypothesis of Zero Average Effect

Null hypothesis  $H_0: \tau_{ATE} = 0$ , the average potential outcomes in the population are the same for treatment and control:  $\mathbb{E}[Y_1] = \mathbb{E}[Y_0]$ .

- However, we observe a difference in mean earnings of  $\hat{\tau}_{ATE} = 1,159$
- What is the probability of observing a difference this large if the true average effect of the training were zero (i.e. the null hypothesis were true)?

# Testing the Null Hypothesis of Zero Average Effect

- Use a two-sample t-test with unequal variances:

$$t = \frac{\hat{\tau}}{\sqrt{\frac{\hat{\sigma}_{Y_i|D_i=1}^2}{N_1} + \frac{\hat{\sigma}_{Y_i|D_i=0}^2}{N_0}}} = \frac{\$1,159}{\sqrt{\frac{\$17,038^2}{7,487} + \frac{\$16,180^2}{3,717}}} \approx 3.5$$

- We know that  $t_N \xrightarrow{d} \mathcal{N}(0, 1)$
- And for a standard normal distribution, the probability of observing a value of  $t$  that is larger than  $|t| > 1.96$  is  $< .05$
- So obtaining a value as high as  $t = 3.5$  is very unlikely under the null hypothesis of a zero average effect
- We reject the null hypothesis  $H_0: \tau_0 = 0$  against the alternative  $H_1: \tau_0 \neq 0$  at asymptotic 5% significance level whenever  $|t| > 1.96$ .
- Inverting the test statistic we can construct a 95% confidence interval

$$\hat{\tau}_{ATE} \pm 1.96 \cdot \widehat{SE}_{ATE}$$

- What assumptions did we need along the way?

# Testing the Null Hypothesis of Zero Average Effect

```
> d <- read.dta("jtpa.dta")  
> t.test(earnings~assignmt, data=d, var.equal=FALSE)
```

Welch Two Sample t-test

```
data:  earnings by assignmt  
t = -3.5084, df = 7765.599, p-value = 0.0004533  
alternative hypothesis: true difference in means is not equal to 0  
95 percent confidence interval:  
 -1807.2427  -511.6239  
sample estimates:  
mean in group 0 mean in group 1  
    15040.50      16199.94
```



# Regression to Estimate the Average Treatment Effect

## Estimator (Regression)

*The ATE can be expressed as a regression equation:*

$$\begin{aligned}
 Y_i &= D_i Y_{1i} + (1 - D_i) Y_{0i} \\
 &= Y_{0i} + (Y_{1i} - Y_{0i}) D_i \\
 &= \underbrace{\bar{Y}_0}_{\alpha} + \underbrace{(\bar{Y}_1 - \bar{Y}_0)}_{\tau_{Reg}} D_i + \underbrace{\{(Y_{i0} - \bar{Y}_0) + D_i \cdot [(Y_{i1} - \bar{Y}_1) - (Y_{i0} - \bar{Y}_0)]\}}_{\varepsilon} \\
 &= \alpha + \tau_{Reg} D_i + \varepsilon_i
 \end{aligned}$$

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 \end{aligned}$$

- Does this assume constant treatment effects?
- Our SE estimator allows different variance for  $D = 1$  and  $D = 0$ .
  - Implies heteroskedasticity
  - Use “HC2” heteroskedasticity-robust variance:

$$\hat{\sigma}_{HC2}^2 = \frac{S_1^2}{N_1} + \frac{S_0^2}{N_0} = \hat{V}(\tilde{\tau})$$

# Regression to Estimate the Average Treatment Effect

```
> library(sandwich)
> library(lmtest)
>
> lout <- lm(earnings~assignmt,data=d)
> coeftest(lout,vcov = vcovHC(lout, type = "HC2"))
```

t test of coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	15040.50	265.38	56.6752	< 2.2e-16	***
assignmt	1159.43	330.46	3.5085	0.0004524	***
--					

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# Testing in Small Samples: Fisher's Exact Test

- Test of differences in means with large  $N$ :

$$H_0 : \mathbb{E}[Y_1] = \mathbb{E}[Y_0], \quad H_1 : \mathbb{E}[Y_1] \neq \mathbb{E}[Y_0] \text{ (weak null)}$$

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Let  $\Omega$  be the set of all possible ways to assign treatments.

Fisher's exact test procedure:

- 1 Calculate a statistic  $\hat{\theta}_{true}$  (e.g. difference in means) from original treatment assignment data
- 2 Obtain the null distribution of the statistic by calculating the same statistic  $\hat{\theta}(\omega)$  under the sharp null for every possible (or many)  $\omega$  in  $\Omega$  (or many)
- 3 Compare  $\hat{\theta}_{true}$  to the null distribution of  $\hat{\theta}(\omega)$ 's to see how “extreme” it is

# Testing in Small Samples: Fisher's Exact Test

$i$	$Y_{1i}$	$Y_{0i}$	$D_i$
1	3	?	1
2	1	?	1
3	?	0	0
4	?	1	0
$\widehat{\tau}_{ATE}$			1.5

What do we know given the sharp null  $H_0 : Y_1 = Y_0$ ?

# Testing in Small Samples: Fisher's Exact Test

$i$	$Y_{1i}$	$Y_{0i}$	$D_i$
1	3	3	1
2	1	1	1
3	0	0	0
4	1	1	0
$\hat{\tau}_{ATE}$			1.5
$\hat{\tau}(\omega)$			1.5

Given the full schedule of potential outcomes under the sharp null, we can compute the null distribution of  $ATE_{H_0}$  across all possible randomization.

# Testing in Small Samples: Fisher's Exact Test

$i$	$Y_{1i}$	$Y_{0i}$	$D_i$	$\omega_1$
1	3	3	1	1
2	1	1	1	0
3	0	0	0	1
4	1	1	0	0
$\hat{\tau}_{ATE}$			1.5	
$\hat{\tau}(\omega)$			1.5	0.5

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$i$	$Y_{1i}$	$Y_{0i}$	$D_i$	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$
1	3	3	1	1	1	0	0
2	1	1	1	0	0	1	1
3	0	0	0	1	0	1	0
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$\hat{\tau}_{ATE}$			1.5					
$\hat{\tau}(\omega)$			1.5	0.5	1.5	-1.5	-0.5	-1.5

So  $\Pr(\hat{\tau}(\omega) \geq \hat{\tau}_{ATE}) = 2/6 \approx .33$ .

Which assumptions are needed?

# Testing in Small Samples: Fisher's Exact Test

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Which assumptions are needed? None! Randomization as “reasoned basis for causal inference” (Fisher 1935)

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# Threats to Internal and External Validity

- **Internal validity**: can we estimate the treatment effect for our particular sample?
  - Fails when there are differences between treated and controls (other than the treatment itself) that affect the outcome
  - In other words, how sure is the randomization?

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  - Fails when there are differences between treated and controls (other than the treatment itself) that affect the outcome
  - In other words, how sure is the randomization?
- **External validity**: can we extrapolate our estimates to other populations?
  - Might  $Y_{1i}$  and  $Y_{0i}$  have looked different in a different part of the population?

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  - e.g. in JTPA: only 62% of women and 66% of men assigned for treatment actually enrolled in JTPA training (on the other side, compliance was almost perfect in the control group).
- Differential attrition
  - if probability of attrition depends on treatment status and potential outcomes, you break the randomization.
  - Two scenarios: (i) people attrite from treatment but you still get to measure outcome; (ii) people attrite and you can't measure anything.

# Example: Clingingsmith, Khwaja, Kremer



# Example: Natural Experiment in Pakistan

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  - 11% who lose lottery still attend the Hajj (via private tours).
- Since randomization is not controlled by researcher, balance checks and qualitative checks are crucial!

# Example: Natural Experiment in Pakistan

Two pieces of information to bolster the randomization assumption:

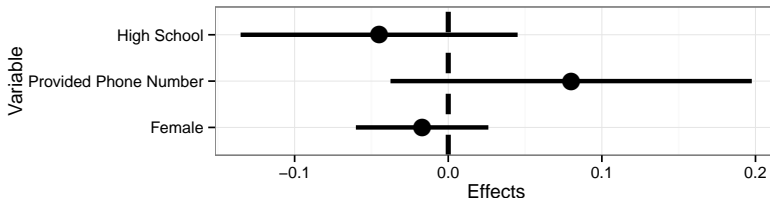
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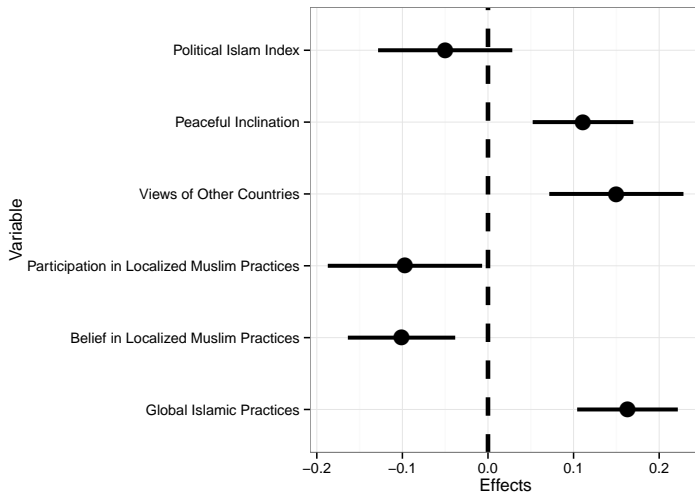
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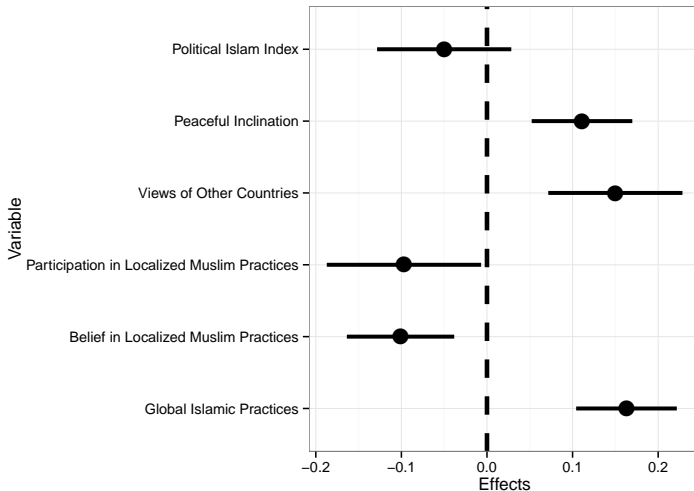
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- Balance tests:



# Effects of winning the Hajj lottery



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Would you say these are the “effects of going on the Hajj”?

# Most Common Threats to External Validity

- Non-representative sample
  - e.g. laboratory experiment using a convenience sample
  - units are randomly assigned, but not from the pop of interest
- Non-representative treatment
  - the treatment differs in actual implementations
  - e.g. survey experiment about the effect of media priming on voting
  - scale effects
  - actual treatments may be bundled

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Which one is more important?

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“One common view is that internal validity comes first. If you do not know the effects of the treatment on the units in your study, you are not well-positioned to infer the effects on units you did not study who live in circumstances you did not study.” (Rosenbaum 2010, p. 56)

- Randomization ensures internal validity
- External validity may be partially addressed by comparing the results of several internally valid studies conducted in different circumstances and at different times
- Note that the same external validity issues often apply in observation studies

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That said, it is a balancing act

Often a series of studies with lower internal validity gives us the inspiration and qualitative knowledge needed for an experiment



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# Review

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- What's Next: Advanced Experimental Design and Analysis
  - covariate adjustment
  - cluster randomization
  - block randomization

# Some Important Topics in Experimental Design and Analysis

A brief introduction to some common topics:

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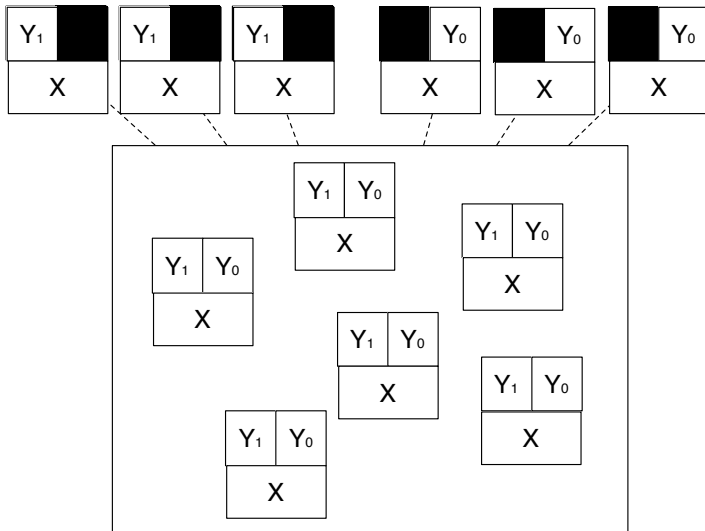
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# Some Important Topics in Experimental Design and Analysis

A brief introduction to some common topics:

- Covariate adjustment
- Blocking
- Ethical Considerations

# Covariates and Experiments



# Covariates

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# Covariates

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- Good idea to test for covariate balance on important covariates, using “balance tests” (eg. t-tests, F-tests, etc.)



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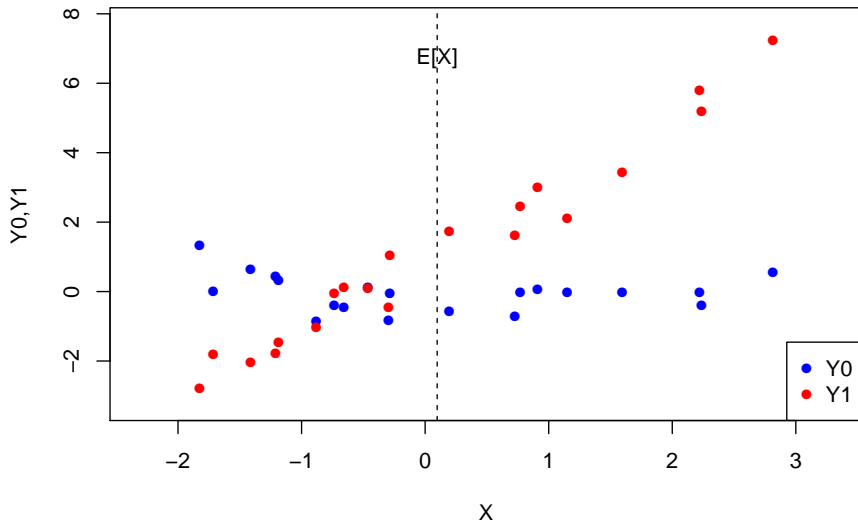
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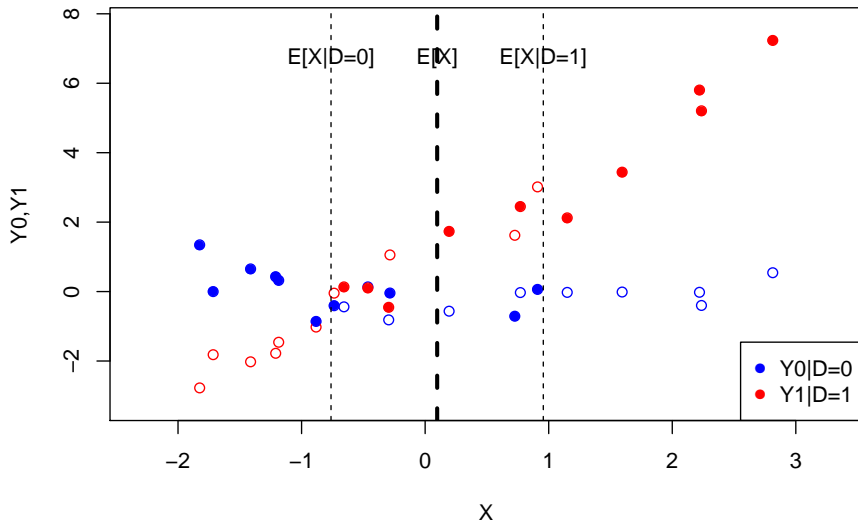
# Regression Adjustment For Experiments

True ATE = 1.169



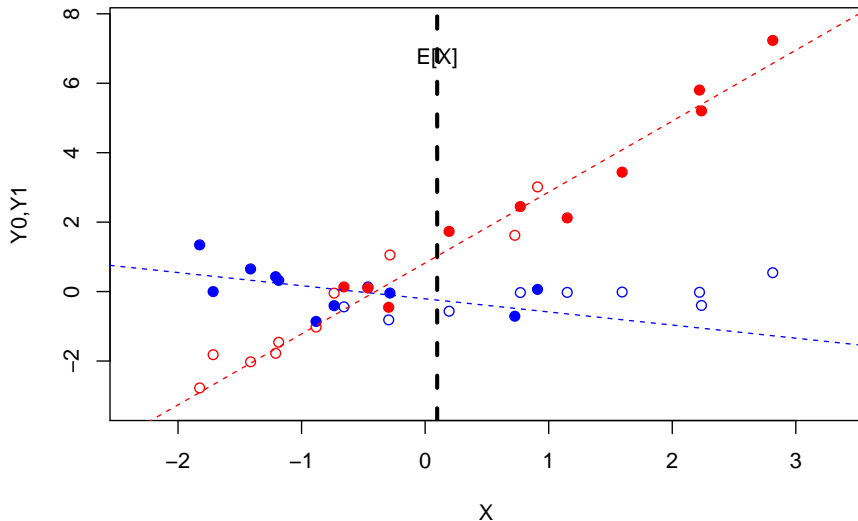
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**Est. ATE = 2.698**



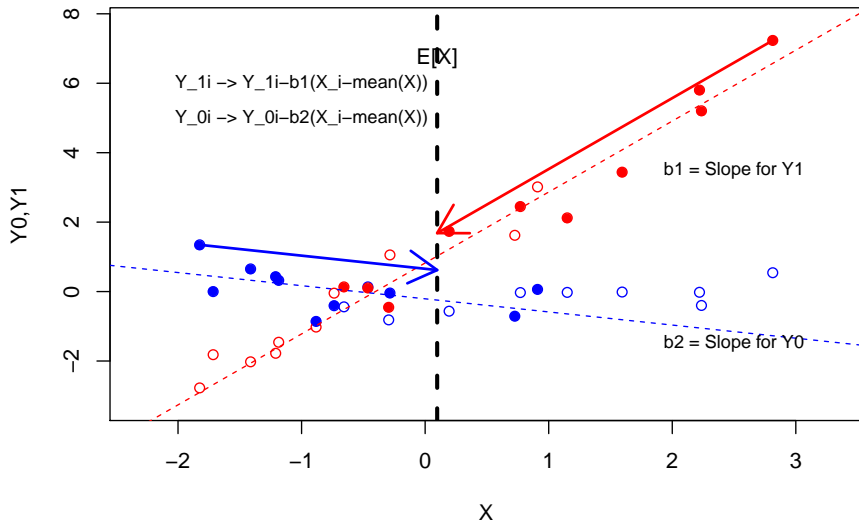
# Regression Adjustment For Experiments

Consider regression lines (interacting with D)



# Regression Adjustment For Experiments

Regression Adjusted ATE = 1.102



# Covariate Adjustment with Regression

Freedman (2008) shows that regression of the form:

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Lin (2013) replies, showing:

- in sufficient samples, these problems are minimal
- robust SEs give asymptotically consistent or conservative CIs
- if you interact covariates with treatment, it never hurts asymptotic precision, likely increases precision:

$$Y_i = \alpha + \tau_{interact} D_i + \beta_1 \cdot (X_i - \bar{X}) + \beta_2 \cdot D_i \cdot (X_i - \bar{X}) + \varepsilon_i$$

- interactions not needed in large sample, but might as well



# Why specify the interactive model this way?

Consider our model,

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- allowing the interaction is a good idea, though not yet standard practice

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Two facts:

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All together, this gives the Frisch-Waugh-Lovell (FWL) theorem:

$$\beta_k = \frac{Cov(\tilde{Y}_i, \tilde{X}_{ki})}{Var(\tilde{X}_{ki})}$$

Where,

- $\tilde{X}_{ki}$  is the residual from a regression of  $X_{ki}$  on all other covariates
- $\tilde{Y}_i$  may have all other  $X$  partialled out as well or you can use the original  $Y_i$ .

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- What does this suggest about the effect of adding covariates to the simple regression of  $Y$  on  $D$ ?



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- Since covariates are balanced by design, results typically not model dependent
- Best reason to be sceptical of adjustment: fishing.
  - always report unadjusted result first
  - best if adjustment strategy pre-specified

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  - Simple random assignment into two treated/control will place  $\{2, 2\}$  and  $\{8, 8\}$  together in the same group one-third of the time

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or more generally, if there are  $J$  strata or blocks, then

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or more generally

$$\text{Var}(\hat{\tau}_B) = \sum_{j=1}^J \left( \frac{N_j}{N} \right)^2 \text{Var}(\hat{\tau}_j)$$

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To correct for this, weight each observation by something that makes probabilities the same. Let  $p_{ij}$  be  $Pr(D_i = 1 \mid \text{block for unit } i)$ :

$$w_{ij} = \left( \frac{1}{p_{ij}} \right) D_i + \left( \frac{1}{1 - p_{ij}} \right) (1 - D_i)$$

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- Adding DOF adjustment, we end up asking whether  $\frac{SSR_{\hat{\varepsilon}^*}}{n-k-1} < \frac{SSR_{\hat{\varepsilon}}}{n-2}$

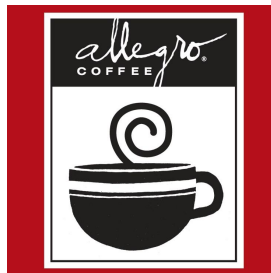
# Fair Trade Labeling Experiment (Hainmueller et al, 2012)

## Label Experiment

Treatment



Control



# Example: Fair Trade Labeling Experiment

## Matched Pairs: Phase 1



# Example: Fair Trade Labeling Experiment

```
> d <- read.dta("FTdata.dta")
> head(d)
```

	store	pair	FTweek	lnsalesd
1	1	1	1	3.20
2	4	1	0	2.77
3	6	2	1	4.18
4	9	2	0	4.04
5	21	3	1	4.30
6	24	3	0	3.93

# Example: Fair Trade Labeling Experiment

```
> cr.out <- lm(lnsalesd~FTweek,data=d)
> coeftest(cr.out,vcov = vcovHC(cr.out, type = "HC1"))
```

t test of coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	4.35000	0.16079	27.0537	<2e-16 ***
FTweek	0.12385	0.21424	0.5781	0.5686

--



# Example: Fair Trade Labeling Experiment

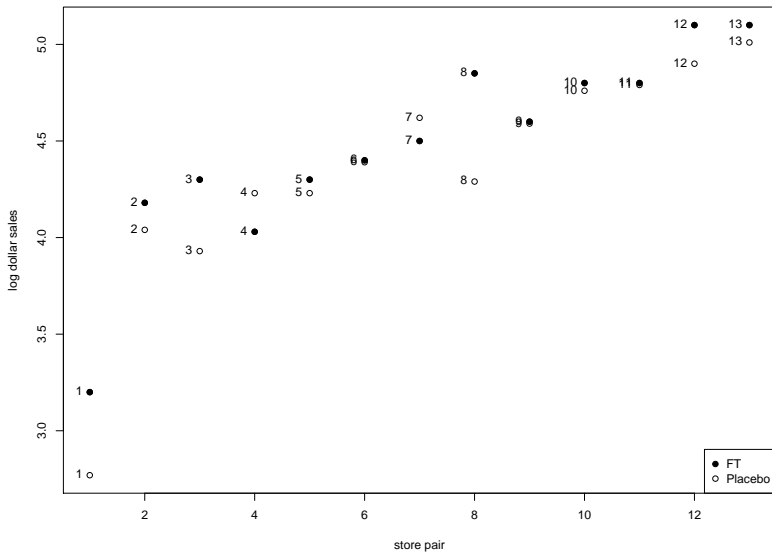
```
> br.out <- lm(lnsalesd~FTweek+as.factor(pair),data=d)
> coeftest(br.out,vcov = vcovHC(br.out, type = "HC1"))
```

t test of coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	2.923077	0.162144	18.0277	4.671e-10	***
FTweek	0.123846	0.060176	2.0581	0.0619840	
as.factor(pair)2	1.125000	0.159549	7.0511	1.335e-05	***
as.factor(pair)3	1.130000	0.204440	5.5273	0.0001304	***
as.factor(pair)4	1.145000	0.231925	4.9369	0.0003439	***
as.factor(pair)5	1.280000	0.161773	7.9123	4.208e-06	***
as.factor(pair)6	1.410000	0.169987	8.2948	2.591e-06	***
as.factor(pair)7	1.575000	0.203689	7.7324	5.317e-06	***
as.factor(pair)8	1.585000	0.277319	5.7154	9.675e-05	***
as.factor(pair)9	1.610000	0.169987	9.4713	6.420e-07	***
as.factor(pair)10	1.795000	0.165195	10.8660	1.450e-07	***
as.factor(pair)11	1.810000	0.169987	10.6479	1.810e-07	***
as.factor(pair)12	2.015000	0.164183	12.2729	3.763e-08	***
as.factor(pair)13	2.070000	0.160298	12.9134	2.127e-08	***

--

# Example: Fair Trade Labeling Experiment



# How much is explained by blocks?

```
> summary(lm(lnsalesd~as.factor(pair), data=d))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	2.9850	0.1212	24.621	2.72e-12	***
as.factor(pair)2	1.1250	0.1715	6.562	1.82e-05	***
as.factor(pair)3	1.1300	0.1715	6.591	1.74e-05	***
as.factor(pair)4	1.1450	0.1715	6.678	1.52e-05	***
as.factor(pair)5	1.2800	0.1715	7.466	4.73e-06	***
as.factor(pair)6	1.4100	0.1715	8.224	1.65e-06	***
as.factor(pair)7	1.5750	0.1715	9.186	4.77e-07	***
as.factor(pair)8	1.5850	0.1715	9.245	4.44e-07	***
as.factor(pair)9	1.6100	0.1715	9.390	3.71e-07	***
as.factor(pair)10	1.7950	0.1715	10.469	1.05e-07	***
as.factor(pair)11	1.8100	0.1715	10.557	9.56e-08	***
as.factor(pair)12	2.0150	0.1715	11.752	2.68e-08	***
as.factor(pair)13	2.0700	0.1715	12.073	1.94e-08	***

--

Residual standard error: 0.1715 on 13 degrees of freedom

Multiple R-squared: 0.9474, Adjusted R-squared: 0.8988

F-statistic: 19.5 on 12 and 13 DF, p-value: 2.356e-06

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How does blocking help?

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- Two constraints: data you can get, number of units to work with.
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## How to block?

- Stratification
- Pair or two pair-matching
- Check: `blockTools` library.

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- Failure to account for the method of randomization in these ways can result in incorrect test size.

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- Iron law of field experiments: keep it simple. Many ways to fail.
- Getting difficult to publish studies that only examine the efficacy of some intervention: look for substantively interesting treatment and outcome.

# Ethics and Experimentation

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*These changes included stronger language about the importance of protecting anonymity, random audits of community behavior, facilitation of anonymous reporting of violations of game protocol by participants, and a new opportunity to receive supplemental funds in a postproject lottery if no reports of harassment were received.*

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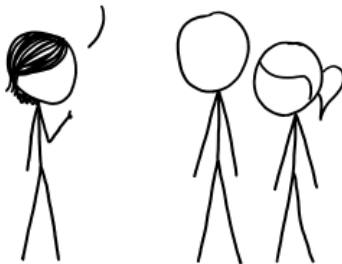
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- **Benefice:** Avoid knowingly doing harm. Does not mean that all risk can be eliminated, but possible risks must be balanced against overall benefits to society of the research.
  - Control group is often difficult: denying access to some benefit.
  - however, project benefits often finite anyway, so why not distribute them by randomization...but that won't always prevent possible envy problems
  - treatments that are occurring already in the wild pose minimal additional risk, but still may be criticized when used for research
- **Justice:** Benefits and risks should accrue to same group or individuals
  - evaluate interventions that are relevant to the subject population

FACEBOOK SHOULDN'T CHOOSE WHAT  
STUFF THEY SHOW US TO CONDUCT  
UNETHICAL PSYCHOLOGICAL RESEARCH.

THEY SHOULD ONLY MAKE THOSE  
DECISIONS BASED ON, UH...

HOWEVER THEY WERE  
DOING IT BEFORE.

WHICH WAS PROBABLY  
ETHICAL, RIGHT?



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  - can your experiment reveal an interesting effect?
  - will it have external validity
  - ethical concerns