PS 207: Problem Set 1

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Due Thursday 11 April at 2:00pm

Instructions:

- Responses should be typeset in LaTeX, or through Rstudio/Rmarkdown or similar if possible. If you have no background in these tools, that is okay for now but you should start learning it.
- A basic tutorial for Rmarkdown can be completed here: `http://rmarkdown.rstudio.com/lesson-1.html` or here: `https://www.youtube.com/watch?v=-apyD5f9nwg`
- For an introduction to typesetting in LaTeX, you may find the following tutorial helpful https://www.latex-tutorial.com/tutorials/
- Many people find it easier to write their first LaTeXdocuments using `https://www.overleaf.com`. Overleaf will typeset your document in real-time so that you can learn the ropes. Overleaf also has various templates that you can browse through to understand how to typeset different types of documents. It's all free to use.
- I know this may be a bit challenging at first - but learning LaTeXand Rmarkdown will be extremely rewarding for you moving forward. I have intentionally not included any R coding in this problem set, to give you some time to familiarize yourself with these typesetting programs.

Problem 1: Thinking about Potential Outcomes

a) Consider binary treatment $D_i \in \{0, 1\}$, observed outcome variable Y_i. What is the meaning of Y_{1i} and Y_{0i}? Describe both in words, and choose an example to illustrate. (2pt)

b) What is the difference between Y_{1i} and "Y_i for a unit that actually received the treatment"? Explain the difference using the example you began in section *a*. (2pt)

c) Define the average treatment effect (ATE) and average treatment effect among the treated (ATT) using potential outcomes notation. Describe in words what each quantity means. (2pt)

d) When will the ATT and the ATE be the same? Prove it. (2pt)

Problem 2. Potential Outcomes with Interference

a) You decide to conduct a get-out-the-vote experiment. For your treatment, you encourage a randomly assigned set of voters to put up yard signs supporting a particular candidate. Your hypothesis is that people who are asked to publicly declare their support for a candidate are more likely to remember to vote. Is this experiment likely to violate the SUTVA assumption? Why or why not?

b) Now suppose you decide to conduct a different get-out-the-vote experiment. With this experiment, you send a randomly assigned set of voters a mailer. These mailers tell each treated individual whether their neighbors voted or not in the last election. Is this experiment likely to violate the SUTVA assumption? Why or why not?

c) Imagine you have the following study population. For this population, calculate the ATE, ATT, and ATC. Then calculate the ATE for the subgroup of odd-numbered cases.

i	D_i	Y_{1i}	Y_{0i}
1	0	6	5
2	1	-1	3
3	1	2	2
4	0	5	2
5	1	2	-3

d) Assume some arbitrary treatment. Now suppose that a small boy's treatment status depends on the treatment status of his sister, his mother and his father. Write out the full set of potential outcomes for this boy. How many such potential outcomes are there? How many τ_i (e.g. unit-level treatment effects) can be defined for this situation?

e) It turns out that the boy and his sister get treated, by construction, jointly. Otherwise the same conditions apply. Now how many potential outcomes exist for the boy? How many tau_i are defined for this new situation?

Problem 3. Potential Outcomes and the Difference in Means Estimand

In this question we will explore how the difference in means estimator differs from causal quantities of interest such as the ATT, ATC, and ATE. Assume we have potential outcomes Y_{1i}, Y_{0i}, treatment assignment D_i, and observed outcome Y_i. Note that the difference in means *estimator* is what we use on a given observed sample; the difference in means *estimand* is the expectation of this.

a) Show that the difference in means *estimand* (the expectation of the difference in means estimator) can be decomposed into the ATT and a bias term. Derive the decomposition, showing your work – do not simply state it. Also describe clearly and concisely the meaning of the bias term. (4pt)

b) Show how the difference in means estimand can be decomposed into the average treatment effect on the controls (ATC) and a bias term. Again derive the decomposition and describe clearly the meaning of the bias term. (4pt)

c) * Harder * Now show how the difference in mean estimands can be decomposed into the ATE and a bias term. Again describe the meaning of the bias term. (Hint: you may want to start by showing how the ATE relates to the ATT and ATC, using the law of iterated expectations - in other words, interpret the ATE as a weighted average of the ATT and ATC). (4pt)