Ito simulations

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Set up

The inverse demand function is given by

$$
\begin{aligned}
p(Q) &= \alpha + \beta q \\
\alpha &= 15 + \epsilon \\
\beta &= -0.03 \\
\epsilon &\sim \mathcal{N}(0, 2)
\end{aligned}
$$

Simulate data

I simulate 50 demand curves for two time steps. Each time step has a different marginal market price (an increase of 3 units), but the "kink" is the same for both time steps.

So, the red and blue dots represent two parallel worlds. One in which consumers respond to marginal prices (red dots) and one in which consumers respond to average prices (blue dots).

Bunching test

The first test Ito does is a bunching test. We'll do this for each "world", so there will be two bunching tests, and I'll do it just for one year (it shouldn't matter what year it is). The takeaway is that we should see no buncing of the blue dots, but we should see it for the red dots.

Estimation

The bunching test shows that in the red world (marginal) there is bunching around the kink point. There is no bunching around the kink point in the blue world (average). Let's move on to estimate the effect of changes in p to changes in Q. For each demand curve, we know the equilibrium point in year T = 1 and T = 2, and we know the equilibrium poiunts for each world (marginal vs average). Therefore, we calculate changes in comsumption and changes in prices for each individual and world.

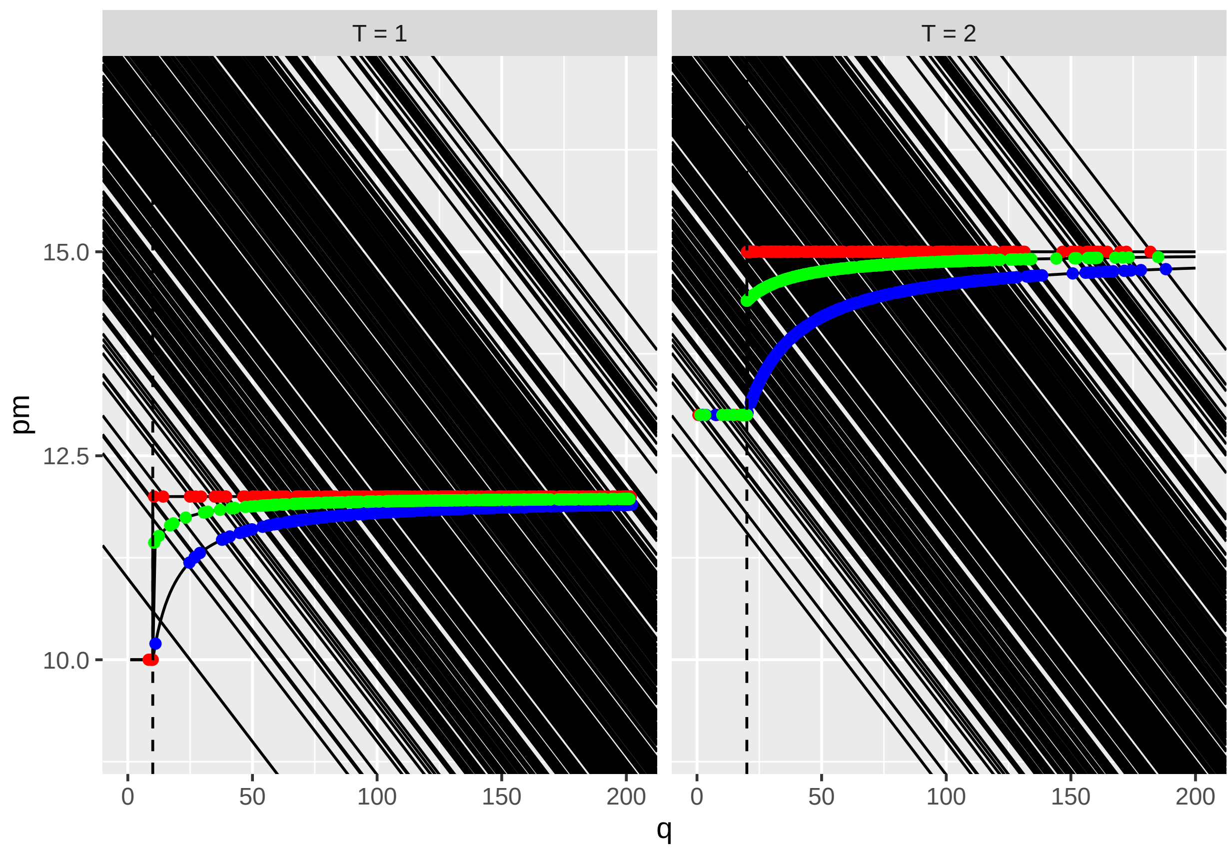


Figure 1: Suply and demand curves. Points mark the equilibrium points for each demand curve when consumers respond to marginal (red) or average (blue) prices.

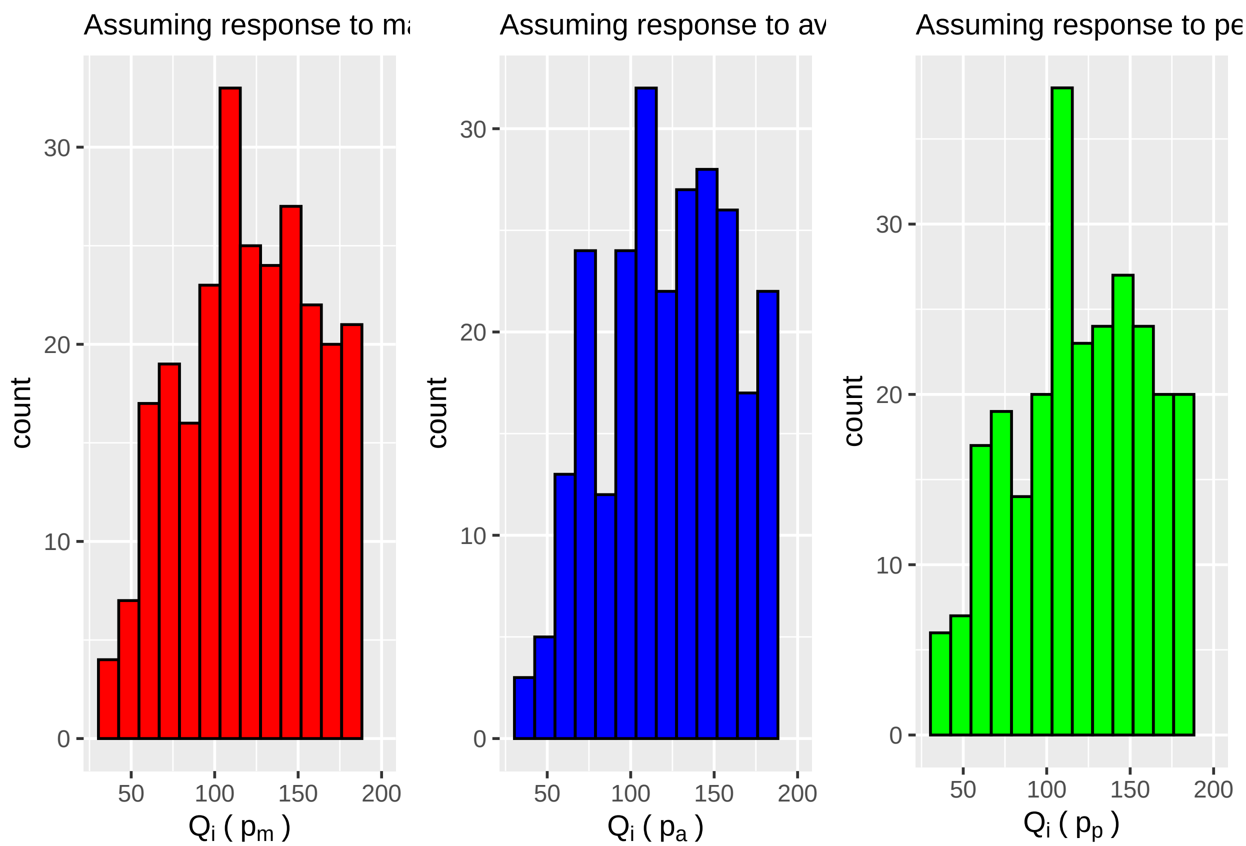
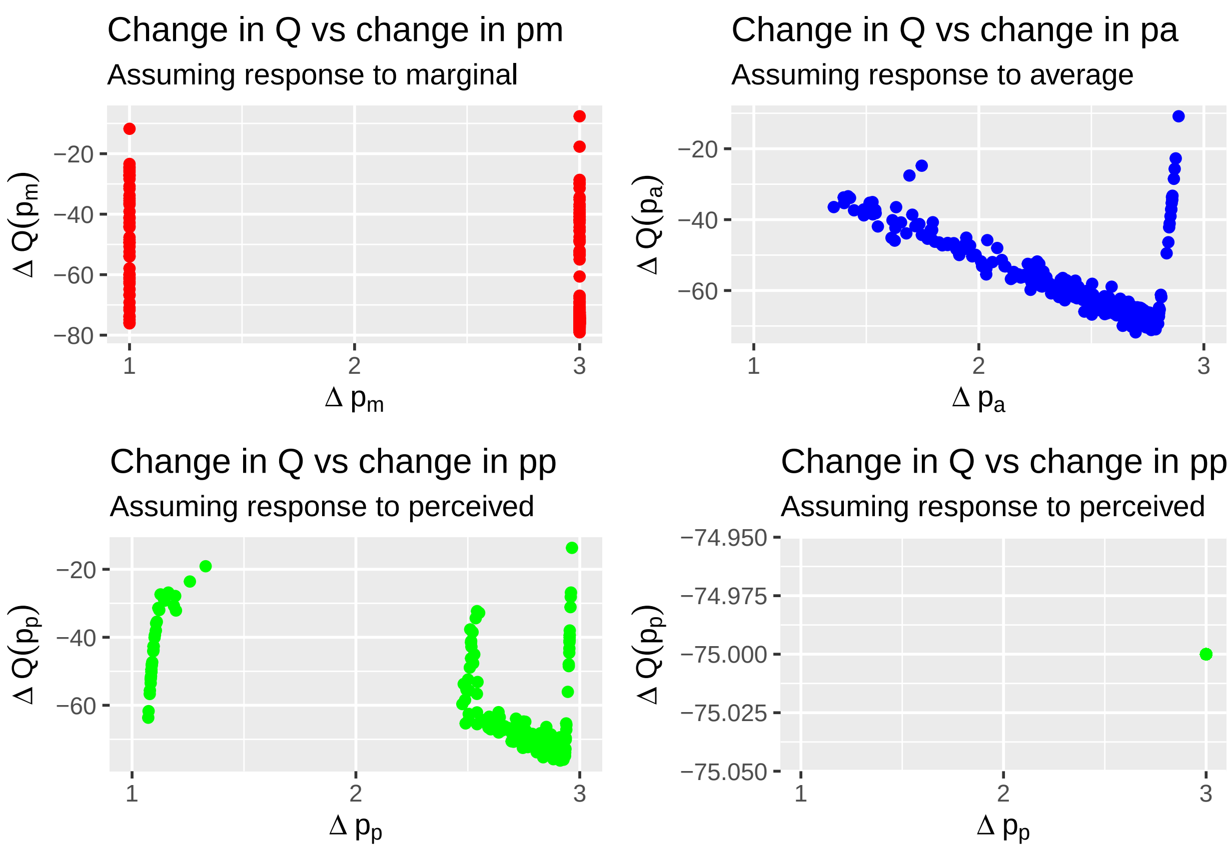


Figure 2: Bunching analysis for observed consumptions for a marginal world (red) and averag eworld (blue).



Since we have calculated the differences, that takes care of "individual-level fixed effects". It also adds the assumption that the curve should have an origin at (0, 0), so we estimate the following three models for each world:

$$\Delta Q_i = \beta_1 \Delta p_m + \beta_2 \Delta p_a + \mu$$

And the corresponding encompassing tests of

$$\Delta Q_i = \beta_1 p_m + \mu$$
$$\Delta Q_i = \beta_2 p_a + \mu$$

Again, we'll fit these three models twice. First on a world where ΔQ_i is calculated from consumers responding to marginal prices, and one where consumers respond to average prices (The figure above).

Table 1: Outcome variable is the difference in consumption observed when individuals respond to marginal prices

	Model 1	Model 2	Model 3
Δp_m	-23.922*** (0.348)		-2.136* (1.200)
Δp_a		-27.392*** (0.268)	-25.033*** (1.352)
Num.Obs.	297	297	297
R2	0.941	0.972	0.973

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Outcome variable is the difference in consumption observed when individuals respond to marginal prices

	Model 1	Model 2	Model 3
Δp_m	-20.891*** (0.294)		-1.023 (0.912)
Δp_a		-23.960*** (0.203)	-22.829*** (1.027)
Num.Obs.	297	297	297
R2	-0.550	0.418	0.421
Std. errors	Standard	Standard	Standard

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Results

From the set up, we know that the real slope is $\beta = 0.3$ (so, we want to recover 1 / beta). We see that, when consumers respond to marginal prices only (Table 1), the first model (pm as a dependent variable only) correctly recovers the estimand. The second model, with only pa, recovers a produces a biased estimate, likely due to spurious correlation. In the full model, there is no value of knowing pa, once pm is known.

When consumers respond to average prices only, the opposite occurs. The first model, with just pm as a predictor, produces a biased estimate, The second model, with just pa, recovers the slope. In the full model we see there is no value of learning pm, once pa is known.

Table 3: Outcome variable is the difference in consumption observed when individuals respond to marginal prices

	Model 1	Model 2	Model 3
Δp_m	-22.925*** (0.324)		-1.584 (1.043)
Δp_a		-26.272*** (0.233)	-24.522*** (1.175)
Num.Obs.	297	297	297
R2	-0.322	0.462	0.466
Std. errors	Standard	Standard	Standard

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$