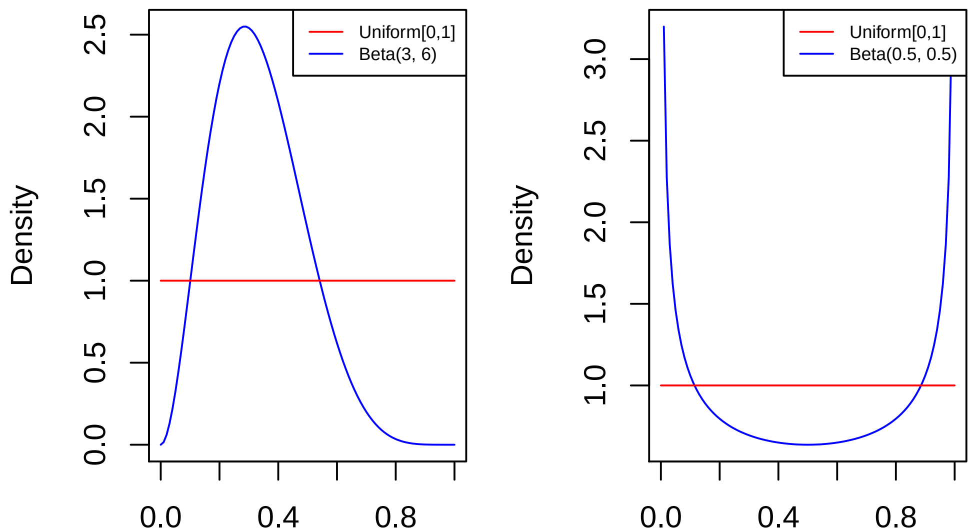
Acceptance-Rejection Sampling

Acceptance-rejection sampling is a common strategy to generate samples from a distribution with a density f - the **target** density - based on the ability to generate random draws from another distribution with density g - the **proposal** density - that satisfies:

$$\sum_x \frac{f(x)}{g(x)} \leqslant M < \infty$$

such that $1 < M < \infty$ for every x in the support of X (every x for which $f(x)$ is defined.



Algorithm:

1. Generate a draw $U \sim Uniform(0, 1)$
2. Generate a draw $X \sim g$
3. If $U \leqslant \frac{f(x)}{Mg(x)}$ then accept X as a draw from f.
4. Repeat until the required number of draws have been generated.

One can show that the rate of acceptance resulting from this algorithm is $\frac{1}{M}$:

$$P(accept) = \frac{1}{M}$$

And that the density of the distribution of the Xs that were kept is f:

$$\begin{aligned}
P(X \mid X \text{ is accepted}) &= \frac{P(X, X \text{ is accepted})}{P(X \text{ is accepted})} \\
&= \frac{P(X \text{ is accepted} \mid X) P(\text{drawing } X)}{P(X \text{ is accepted})} \\
&= P\left(U \leqslant \frac{f(X)}{Mg(X)}\right) \frac{P(\text{drawing } X)}{P(X \text{ is accepted})} \\
&= P\left(U \leqslant \frac{f(X)}{Mg(X)}\right) \frac{g(X)}{1/M} \\
&= \frac{f(X)}{Mg(X)} \cdot M \cdot g(X) \\
&= f(X)
\end{aligned}$$