OFFICE HOURS

Parsing λ-expressions

$((A\ B)\ C)\ D$

$(\text{add}\ e1)\ e2$

$\underbrace{((\lambda x.\lambda y.x+y)\ 2)}$ 3

$(\lambda y.\ 2+y)$ 3

$(\lambda f.\lambda x.\ f f x)\ \lambda x.\lambda y.x y$

$(\lambda f.(\lambda x.(f f) x))\ (\lambda x\ (\lambda y.x y))$

$(\lambda f.(\lambda x.f\ (f\ x)))\ (\lambda x.x)$

- $\lambda x.(\lambda y.\ A\ B)$
- $\lambda x.(\lambda y.\ A)\ B$

two different parse trees

Exercise Practice Test

$aaa \to$
$aa \to b$
$ab \to$
$bb \to a$

INVARIANT:
$(\#a + 2 \cdot \#b) \bmod 3$

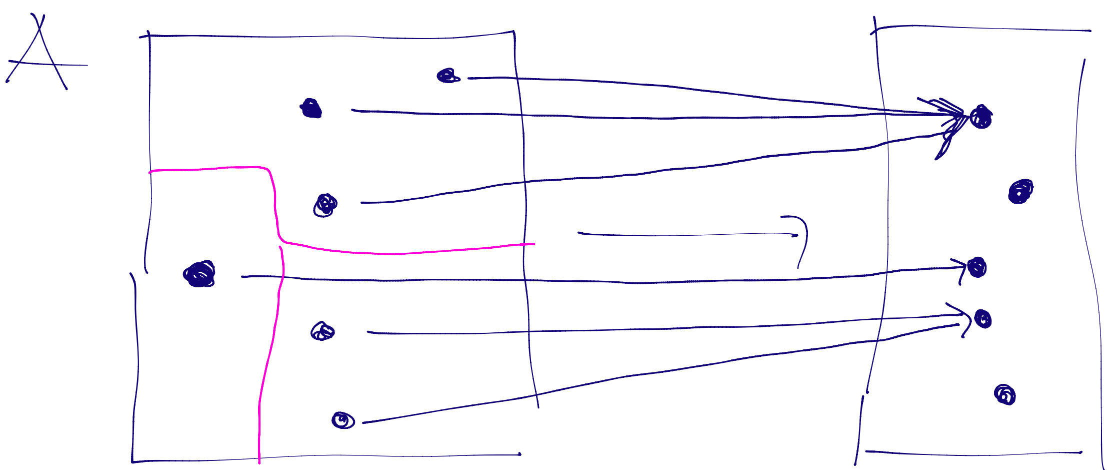
1) $\{ w \mid w \xrightarrow{*} a \} =$ "the set of all words such that $(\#a + 2 \cdot \#b) \bmod 3 = 1$"

2) $\{ w \mid w \xrightarrow{*} [\,] \} = \ldots$

3) $\{ w \mid w \xrightarrow{*} b \} = \ldots$

Discrete Math : Relations

$$f : \underset{\text{domain}}{A} \longrightarrow \underset{\text{co-domain}}{B}$$



the equivalence class of $a \in A$

is $\{a' \mid f(a) = f(a')\}$

every function partitions its domain into equ. classes

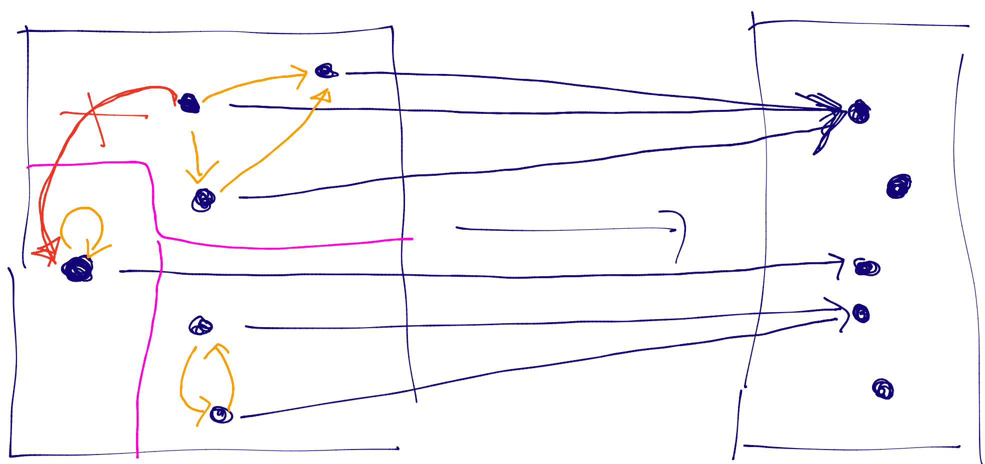
f could be an invariant

$(A, \to)$

f invariant

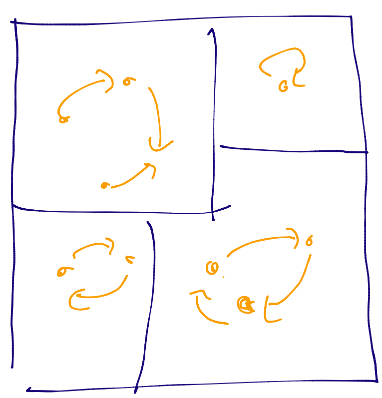
$\to$ $\boxed{\text{does not connect}}$ two different equ classes

A

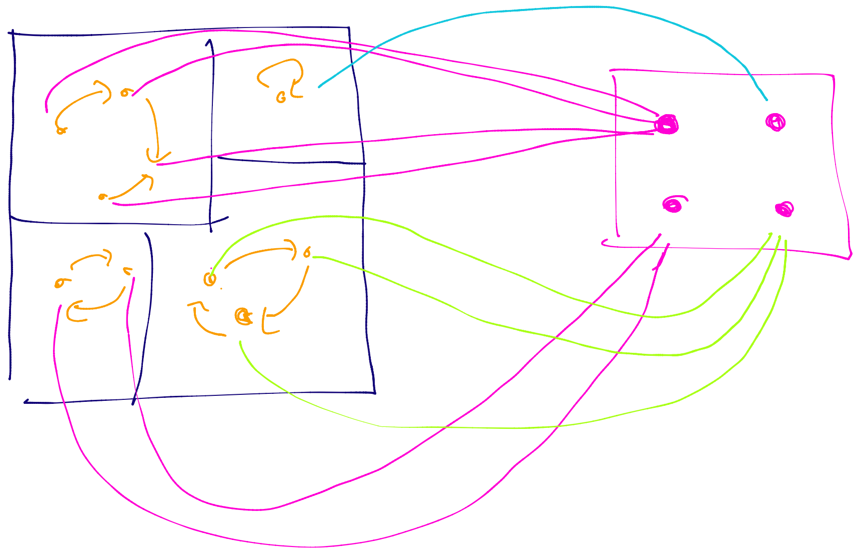


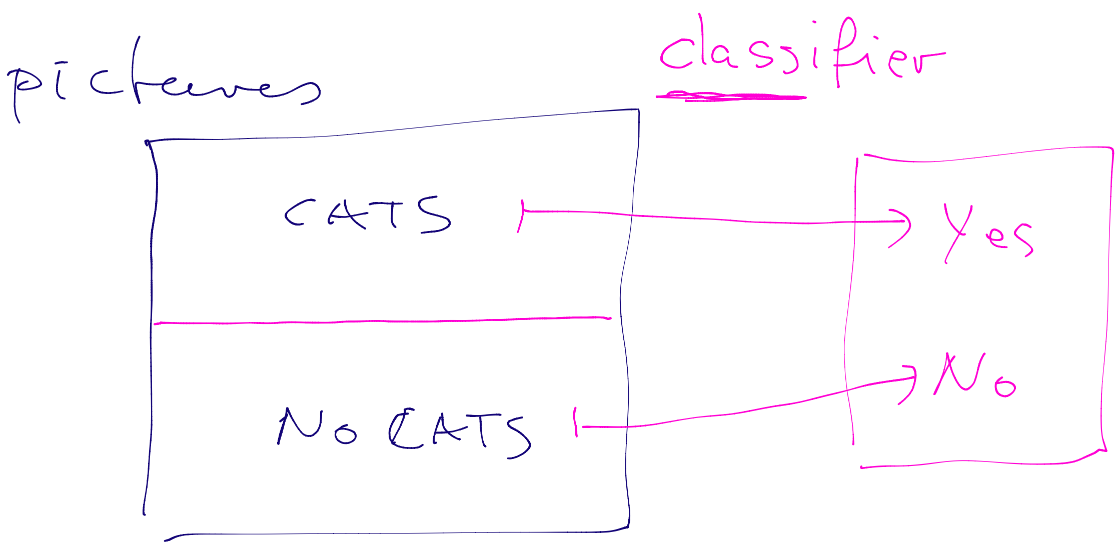
Given $(A, \to)$

the equ.-classes are already determined by $\to$



FIND INVARIANT MEANS TO FIND f such that:





Method of how to find an invariant

3 steps:

(1) idea

(2) articulate a guess

(3) prove that it's an invariant

Example:

aaa →

ab →

ac → b

bb → a

idea 1

mod 3

mod 3

three times - table

3	6	9	12	15	...

"sequence of threes"

---

ab → ba ba → ab

$\boxed{\text{order doesnt matter}}$

aabacb $\to$ 4a+2b

abaaba

aaaabb 4a+2b

concatenation plus

idea 2

ab $\to$

1+2 O

so far: 2 ideas ... next:

(2) articulate guess

$$(\#a + 2 \cdot \#b) \bmod 3$$

(3) prove

	$(\#a + 2\#b) \bmod 3$ before	after
aaa →	0	0
ab →	0	0
aa → b	2	2
bb → a	1	1

It is invariant because the "before" and "after" columns agree

summarize: $(\#a + 2 \cdot \#b) \bmod 3$ is a (complete) invariant

(SR2) $ab \to ba$ and $ba \to ab$

$aa \to$

$ba \to bbaa$

equ. class of $[\,]$? $\boxed{\#a \bmod 2 = 0 \text{ and no } b\text{'s}}$

equ. class of a ? $\boxed{\#a \bmod 2 = 1 \text{ and no } b\text{'s}}$

can ignore $ba \to bbaa$ ... why?

what happens if we have b's?

$$b = baa = bbaaa = bba$$
$$= bbbaa = bbb = bbbbb = b^7 = \dots$$

notation $\begin{cases} b = b^3 = b^7 = b^9 = \dots \\ \text{or} \\ b = 3b = 7b = 9b = \dots \end{cases}$

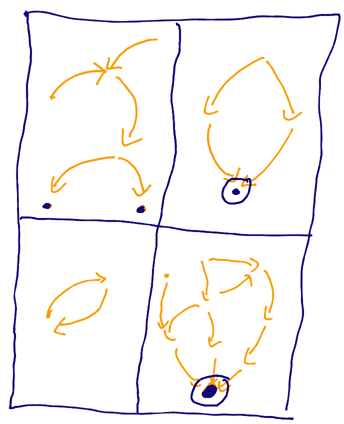
- idea ... we are still here
- guess ...
- proof ...

AFTERNOON LECTURE

Revision:

Unique NF

$A, \to$



$b = b^3 = b^5 = \cdots$

$b^2 = b^4 = b^6 = \cdots$

$ba = ?$

$bba = ?$

• normal forms

⊙ unique normal forms (unique for an equ. class)

An ARS has unique n.f.'s if all equ. classes have exactly one n.f.

(SR1) $aa \to$
$b \to$

$\boxed{\text{Does } w \text{ has an even \#a's ?}}$

$$aabbabb \to bbabb \xrightarrow{*} a$$

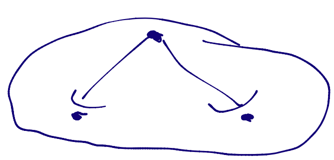
Because of UNF there are only 2 possible results a and [].

Generally speaking, if you have UNF and <u>termination</u>, then you also have <u>deterministic</u> algorithm.

termination	confluence
there are n.f.'s	n.f.'s are unique
	results are determined by the input

joinable $\Rightarrow$ equivalent

equivalent $\not\Rightarrow$ joinable

Exle 

(SR1) $\quad aa \to$

$\qquad\quad b \to$

For SR1, equivalent $\Rightarrow$ joinable

$$w,v \text{ equivalent} \iff \begin{cases} w,v \text{ have even } \#a \\ w,v \text{ have odd } \#a \end{cases}$$

If w,v have $\#a$ <u>even</u>

then w,v are joinable

Why? $\quad$ • eliminate all b's

• eliminate all aa's

then we get $[\,]$

in a picture:

$$\begin{tikzcd} w \arrow[dr, "*"] & & v \arrow[dl, "*"'] \\ & {[\,]} & \end{tikzcd}$$

LECTURE

Add O S to the interpreter?

$\boxed{\text{evalCBN (ENatO)} = ?}$

abstract syntax

let us first think in terms of concrete syntax

eval (S (S O)) = ?

eval (2+3) = 5

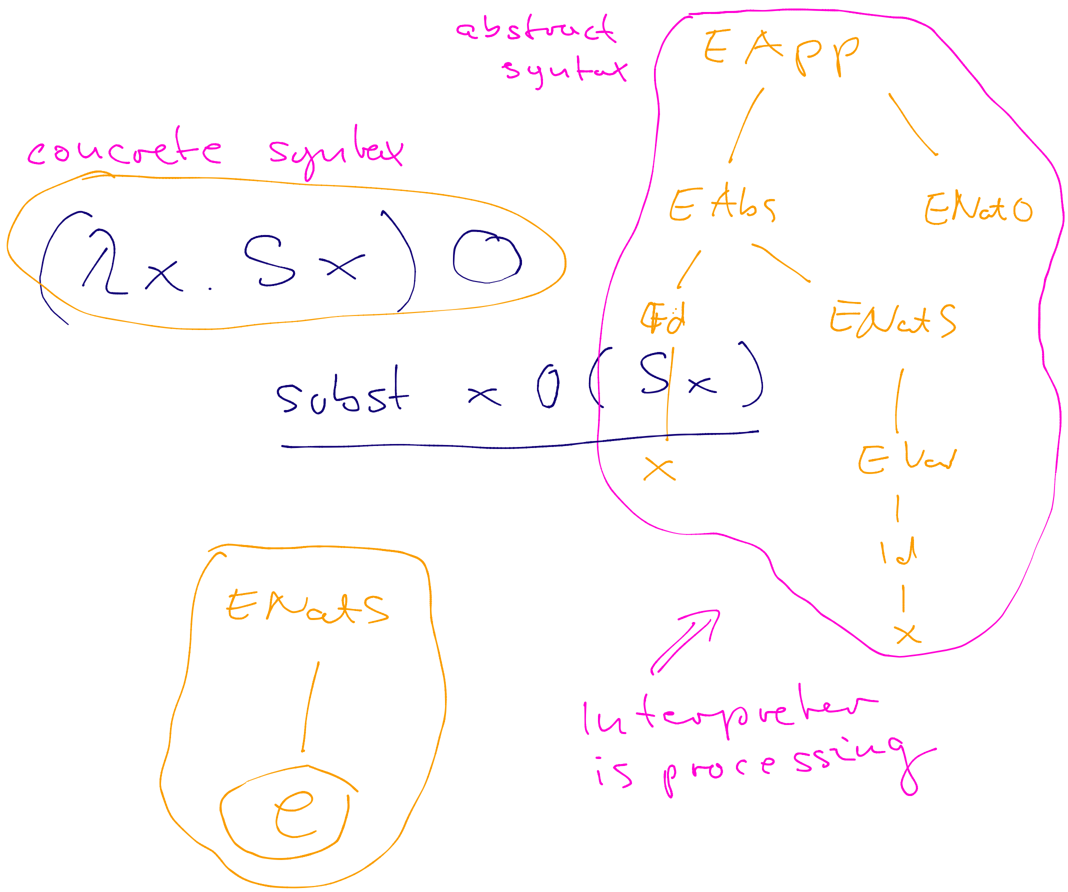
eval 2 = 2

eval (S (S O)) = S (S O)

eval O = O

$\boxed{\text{evalCBN (ENatO)} = \text{ENatO}}$

line 60 (compare with line 51)




```
subst i s (ENat S e) =
ENatS (subst i s e)
```

```
evalCBN (ENatS e) =
    ENat S (evalCBN e)
```

Homework

I give you λ-calc + Numbers

I ask you to add if-then-else