Mixed Integer Programming - Applications

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If only some of the variables are required to have integer values, this model is referred to as mixed integer programming (MIP).

Application 1: Making Choices When the Decision Variables Are Continuous

The Research and Development Division of the GOOD PRODUCTS COMPANY has developed three possible new products. However, to avoid undue diversification of the company's product line, management has imposed the following restriction:

Restriction 1: From the three possible new products, at most two should be chosen to be produced. Each of these products can be produced in either of two plants. For administrative reasons, management has imposed a second restriction in this regard.

Restriction 2: Just one of the two plants should be chosen to be the sole producer of the new products.

The production cost per unit of each product would be essentially the same in the two plants. However, because of differences in their production facilities, the number of hours of production time needed per unit of each product might differ between the two plants. These data are given in Table bellow, along with other relevant information, including marketing estimates of the number of units of each product that could be sold per week if it is produced. The objective is to choose the products, the plant, and the production rates of the chosen products so as to maximize total profit.

	Production Time Used for Each Unit Produced			Production Time Available per Week
	Product 1	Product 2	Product 3	
Plant 1	3 hours	4 hours	2 hours	30 hours
Plant 2	4 hours	6 hours	2 hours	40 hours
Unit profit	5	7	3	(thousands of dollars)
Sales potential	7	5	9	(units per week)

Application 2: A Customer-Assignment Model for Supply Chain Optimization

Supply chain optimization is one of the broadest applications of integer optimization and is used extensively today as companies seek to reduce logistics costs and improve customer service in tough economic environments.

Suppose that a company has numerous potential locations for distribution centers that will ship products to many customers and wants to redesign its supply chain by selecting a fixed number of distribution centers. In an effort to provide exceptional customer service, some companies have a single-sourcing policy-that is, every customer can be supplied from only one distribution center. The problem is to determine how to assign customers to the distribution centers so as to minimize the total cost of shipping to the customers.

Example: Paul & Giovanni Foods distributes supplies to restaurants in five major cities: Houston, Las Vegas, New Orleans, Chicago, and San Francisco. In a study to reconfigure their supply chain, they have identified four possible locations for distribution centers: Los Angeles, Denver, Pensacola, and Cincinnati. The costs of supplying each customer city from each possible distribution center are shown next:

Sourcing Costs	Houston	Las Vegas	New Orleans	Chicago	San Francisco
Los Angeles	$40,000	$11,000	$75,000	$70,000	$60,000
Denver	$72,000	$77,000	$120,000	$30,000	$75,000
Pensacola	$24,000	$44,000	$45,000	$80,000	$90,000
Cincinnati	$32,000	$55,000	$90,000	$20,000	$105,000

Four possible locations for distribution centers have been identified, <u>but they only want to assign 2 distribution centers.</u> P&G Foods wishes to determine the best supply chain configuration to minimize cost.

Practice: K&L Designs

Part 1: K&L Designs is a home-based company that makes hand-painted jewelry boxes for teenage girls. Forecasts of sales for the next year are 150 in the autumn, 400 in the winter, and 50 in the spring. Plain jewelry boxes are purchased from a supplier for $20. The cost of capital is estimated to be 24% per year (or 6% per quarter); thus, the holding cost per item is 0.06($20) = $1.20 per quarter. The company hires art students part-time to craft designs during the autumn, and they earn $5.50 per hour. Because of the high demand for part-time help during the winter holiday season, labor rates are higher in the winter, and workers earn $7.00 per hour. In the spring, labor is more difficult to keep, and the owner must pay $6.25 per hour to retain qualified help. Each jewelry box takes 2 hours to complete. How should production be planned over the three quarters to minimize the combined production and inventory-holding costs?

Part 2: Consider the multiperiod production-inventory-planning model for K&L Designs that we developed in part 1. Suppose that the company must rent some equipment, which costs $65 for 3 months. The equipment can be rented or returned each quarter, so if nothing is produced in a quarter, it makes no sense to incur the rental cost.