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Decision Trees

Most business decisions involve uncertainties of some kind. A drug company may decide to invest in research and development (R&D), for example, not knowing whether it will be able to obtain Food and Drug Administration (FDA) approval for its product. An automobile maker may choose to build a large factory to exploit scale economies without knowing whether the market will be large enough to allow the factory to work at full capacity. Although uncertainty can be daunting, you will see that by laying out the set of events that can occur in a systematic way and anticipating your actions following the resolution of uncertainty, you can make better decisions.

This note outlines a set of techniques to analyze decisions under uncertainty. You will be able to apply the techniques to classes of decisions that have three properties:

1. The *alternatives* are well-defined,
2. The critical *uncertainties* can be quantified,
3. The *objectives* are clear.

Not all decisions have these characteristics. However, many, and perhaps all, decisions can be simplified in such a way that these three properties hold. Because decisions with these three properties are straightforward to solve, you will find that good decision makers are distinguished by their ability to cast complex problems in a way that highlights these properties.

You will learn how to construct a graphical device called a *decision tree*. Decision trees serve two primary purposes. First, they tell you which alternatives to choose. Second, they identify the value of having those alternatives in the first place.

A Simple Business Problem

Allison Tate runs a small company that manufactures low-cost ergonomic chairs, sold via the Internet. Her firm has several popular models, each with annual sales of \$200,000 to \$450,000.

Her research staff has produced a prototype that improves on the top-of-the-line model in several ways. However, because of the number of innovations in the new chair, Tate does not expect to be able to produce it using her existing manufacturing facility. Tate knows that a new facility will cost \$300,000 and is unsure whether there will be sufficient demand for the chair to cover this large

Professors Robin Greenwood and Lucy White prepared this note as the basis for class discussion.

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investment. If the market is good, she thinks she might sell 8,000 chairs at a profit of \$100 each, generating a cash flow with present value of \$800,000.¹ On the other hand, if the market is poor, she thinks she might sell only 1,000 chairs, generating a cash flow with present value of only \$100,000. How should she make this decision?

Before we can determine how to make a decision where the outcome of the decision is uncertain, we need a way of determining the value of uncertain prospects. Uncertain gains and losses can be evaluated using a device known as a *probability tree*. In the next section we show how Tate can use a probability tree to evaluate the value of investment. Then, in the following section, we allow Tate to make a *sequence of decisions*. This is done using a device known as a *decision tree*.

Probability Trees

What Is the Value of Tate's Investment Opportunity?

If Tate invests in the factory to produce the new chairs, this will cost \$300,000. If the market is large, cash flows from sales will total \$800,000. If Tate is certain that the market will be large, then her decision will be easy: she should go ahead and invest, since the project is worth \$500,000 (\$800,000 minus the cost of investment of \$300,000).

Her decision is more difficult, however, since she is not certain whether the market will be large or small. If the market for the chairs is small, cash flows from sales will be only \$100,000. In this case, she will lose \$200,000 (\$100,000 minus the cost of investment of \$300,000). How should she think about the opportunity to invest now? She begins by laying out the alternative outcomes if she invests, shown in **Figure A** below. (Costs and benefits in all the figures are expressed in thousands.)

Figure A



How should she trade off the possibility of gaining \$800,000 versus gaining \$100,000 in relation to her certain cost of investment of \$300,000? Intuitively, Tate will be more inclined to make the investment the more likely it is that the market will be large. The likelihood of such an *event* occurring is expressed mathematically by its *probability*. If an event is impossible, we assign it probability of zero. At the other end of the spectrum, if an event is certain to occur, we assign it probability of one. Most real-world events are between these two extremes—and the probability that they occur is between zero and one, with more likely events being assigned higher probabilities.

In order to decide whether to invest in the factory, Tate needs to decide just how likely it is that the market will be large. Considering what she knows about the market, she reasons that the chairs

¹ To simplify matters, all cash flows in this note are discounted to the time of the investment decision.

are a new design and it is a little more likely that the market will be small than that it will be large; that is, the probability of a small market is more than half. On the other hand, there is still a decent chance that the market will be large. She decides that the probability of a large market is around 0.40. Since the sum of the probabilities of the two outcomes must equal one, the probability of a small market is therefore 0.60. Assessing a 40% probability of a large market means that she thinks that this is as likely to occur as a draw of a red ball from a jar containing 40 red and 60 black balls. Armed with this information, she can now construct a simple probability tree to represent the value of the investment opportunity:

Figure B



What is the expected value of her investment? Suppose we were to replay the scenario many times. Forty percent of the time Tate would expect to make \$800,000, and 60% of the time, she would expect to get \$100,000. However, she always incurs the cost of investment of \$300,000. On average she expects cash flows of

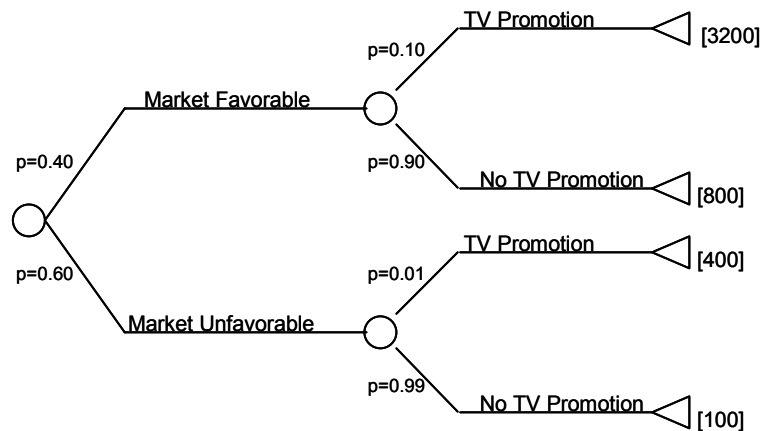
$0.40 \times 800,000$ Probability-weighted monetary value of good market outcome	+	$0.60 \times 100,000$ Probability-weighted monetary value of poor market outcome	–	$1.0 \times 300,000$ Certain cost of investment	=	$80,000$ Expected monetary value of investment, net of costs
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The equation above calculates the expected monetary value of the investment opportunity. *The expected monetary value of a probability tree is the probability-weighted average of the outcomes.* The expected monetary value of Tate's *investment* is the expected monetary value of the tree minus the cost of her investment. Tate's opportunity thus has an expected monetary value of \$80,000. To maximize the expected returns to her shareholders, Tate should choose to invest, since this offers a higher expected monetary value than not investing.

Evaluating More Complicated Probability Trees

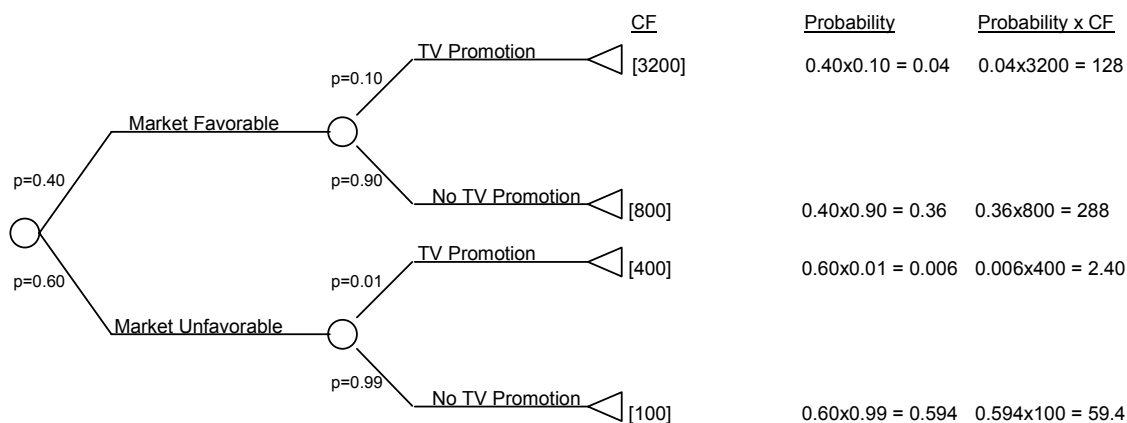
Other more complicated, risky prospects can be evaluated in the same way as the simple problem above. Suppose that there is a chance that the innovative and controversial chairs will be shown on national television. This will increase sales fourfold: that is, to \$3.2 million if the market is large and to \$400,000 if the market is small. Tate estimates that the chance of her product's being featured on TV is 10% if the market is large and only 1% if the market is small. We can extend the tree as follows:

Figure C



What is the value of this tree? As before, the expected monetary value of a probability tree is the probability-weighted average of the outcomes. To get the probability of an outcome, we multiply the probabilities along the branches of the tree. Thus, the probability that the market is favorable and there is a TV promotion is equal to $0.40 \times 0.10 = 0.04$. Similarly, the probability that the market is favorable and there is no TV promotion is equal to $0.40 \times 0.90 = 0.36$. We can do this calculation for each of the branches of the tree, as follows:

Figure D



If done correctly, the sum of the probabilities of the outcomes should equal 1. We can check that indeed $0.04 + 0.36 + 0.006 + 0.594 = 1$. To get the expected monetary value, we take the probability-weighted average of the four outcomes (summing the product of the outcomes with the probability of each outcome). This gives us $0.04 \times 3,200,000 + 0.36 \times 800,000 + 0.006 \times 400,000 + 0.594 \times 100,000 = 477,800$. Thus the value of this investment opportunity, net of costs, is $\$477,800 - \$300,000 = \$177,800$.

Decision Trees

In the probability tree we just analyzed, Tate did not have to make any decisions, as uncertainty was resolved. She just had to figure out the value of the investment opportunity by calculating the expected monetary value of the probability tree, and having figured out this value, she simply compared it to the cost of investment. How would we modify the analysis to take into account the fact that often we may be able to partially control events by making decisions as time goes on? To analyze a series of decisions where there is uncertainty, decision makers often use a device called a *decision tree*. Like a probability tree, a decision tree represents a chronological sequence of events presented in a simple graphical manner. A decision tree also allows for decisions along the way as uncertainty is resolved. We typically draw the decision tree from left to right, emphasizing the chronological nature of decisions.

Decision trees serve two primary goals. First, they help you decide which decision to make. At each decision node, you will be faced with several alternatives. Using a tree, you will be able to decide which of these alternatives is the right one to choose. Second, the decision tree identifies the value of any particular decision or set of options. For example, you may want to know not only whether you prefer one alternative to another, but by how much. At other times, you might also want to know how valuable the right to make a decision at a particular point is.

There are three important elements in the decision tree:

□ A decision is represented by a square, or *decision node*. This node could refer to the decision to invest or not invest, to purchase a piece of equipment or not to purchase, or how much to offer in an auction.

○ Uncertain events are represented with a circle, or *chance node*. Uncertain events may occur before or after, or both before and after, decisions.

◁ *Outcomes* are represented with a triangle. The decision process arrives at an outcome when all uncertainties have been resolved and there are no further decisions to be made; at this point the decision maker knows the payoff he will receive. Outcomes can occur at various stages of a complex decision. For example, if the decision is whether to continue or abandon, the selection of abandonment is an outcome, while the selection of continuance may lead to future decisions or uncertainties.

The three elements of a decision tree are connected with lines. Although we do not draw them, you can think of the lines as arrows that go from left to right, in the same direction as the chronology of decisions. The lines are often called “branches” because decisions may lead to future decisions, which lead to future decisions . . . and so on.

The order in which the shapes in the decision tree are connected is very important. A square that is followed by a circle, for example, means that the decision is made before the uncertainty is resolved. In contrast, a circle that is followed by a square denotes a decision that occurs after uncertainty is resolved.

A Simple Decision Problem: Tate's Ergonomic Chairs

Let us go back to our simple example to illustrate the principles involved in constructing and solving a decision tree. We saw earlier that Tate was faced with deciding whether to invest \$300,000 in a new manufacturing facility when she was unsure whether there would be sufficient demand for the chair it would produce to cover the large investment. She knows that if the market is large, she

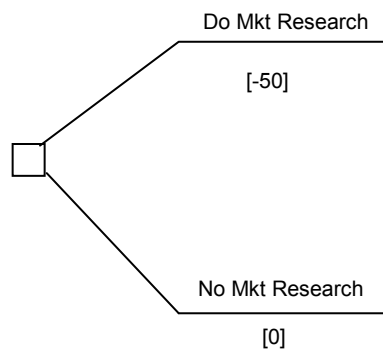
will sell 8,000 chairs at a profit of \$100 each, generating a cash flow with present value of \$800,000. On the other hand, if the market is small, she will sell only 1,000 chairs, generating a cash flow with a present value of only \$100,000.

A market research firm has submitted a proposal to evaluate the potential market for the chair. Tate is confident that this will resolve her uncertainty about whether the market for the chair will be large or small. But she does not know whether it is worth the \$50,000 that the firm is asking to carry out the survey. How should she decide whether to pay for the market research?

Designing the Tree

Step 1: Lay out the alternatives Decision trees are designed forwards, one branch at a time, starting with the first decision. Tate must begin by deciding whether to invest in market research or not. (There would be no point in investing in market research after making her investment decision, since the value of the research is in helping to make the investment decision.) Because we start with a decision and there are two alternatives, we draw a square with two lines coming out of it:

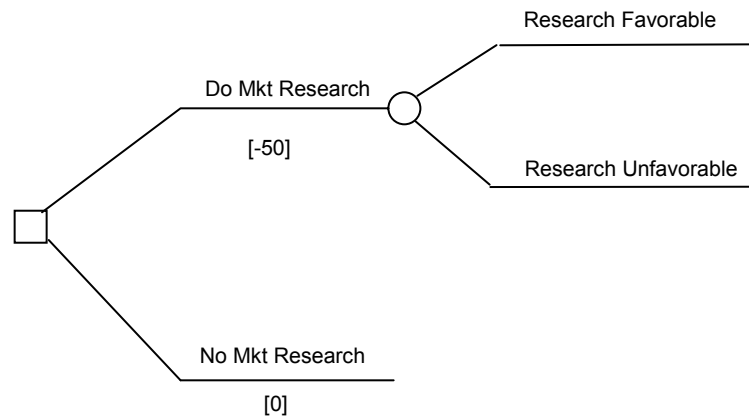
Figure E



We keep track of costs (and benefits) as they occur. Below the top branch, we note the cost of \$50,000. If Tate does no research, she incurs no costs. Thus we write a zero under the bottom branch.

Now what? We start with the top branch. The market research will identify the market opportunities to be either good or bad. Tate is unsure which of these events will occur.

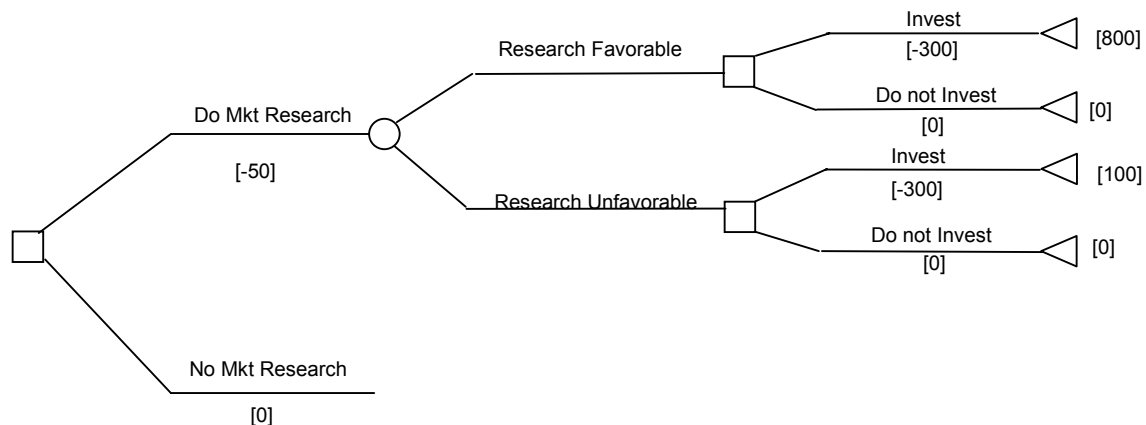
Figure F



Because the results of the market research are out of her control, we mark the resolution of uncertainty with a circle. Why are there no cash flows associated with either of these two branches? Even though the research results may influence her course of action, there are no direct cash flows associated with the results from the research survey.

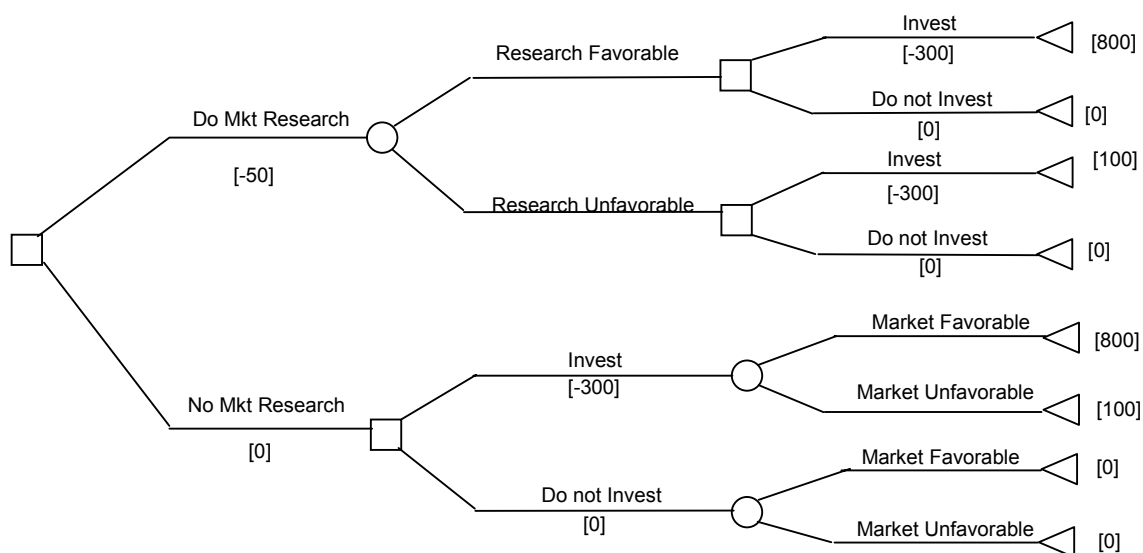
What happens next? Armed with the results of the research survey, Tate can purchase a new facility or abandon her efforts to produce the new chair. If the market is large, then if she purchases the new facility this will cost \$300,000 and she will receive \$800,000 in revenue subsequently. In contrast, if the market is small, then if she purchases the new facility this will cost \$300,000 and yield cash flows of \$100,000. In either case, if she abandons the project, her cash flows will be zero.

Figure G



Finally, we repeat these steps for the case for which Tate does not purchase the market research. Notice that this part of the tree contains no further decisions and is just like the probability tree we analyzed earlier. The difference between the upper and lower sections of the tree is that in the lower half Tate must make a decision *before* the uncertainty about the size of the market is resolved, whereas in the upper half she already knows the market size before she makes her investment decision. Therefore, the final decision tree is:

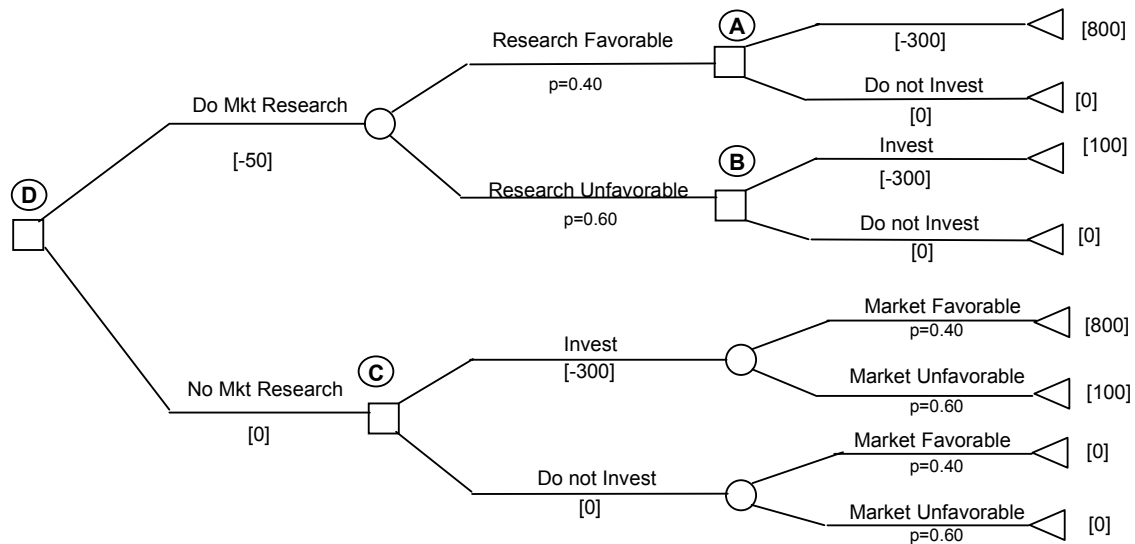
Figure H



Step 2: Quantify the uncertainties As with probability trees, Tate must estimate the probability that each of the branches of uncertain outcomes occurs. We have already done this for the lower half of the tree in the previous section. Recall that Tate believes that the probability that opportunities are good is 0.40 and the probability that opportunities are bad is 0.60.

If we look at the top half of the tree, there is only one chance node, with two outcomes, that corresponds to research saying that sales opportunities are good or bad. Tate believes that the research reports are 100% accurate. So what is the probability that the research says that the market will be large? The research will report a large market precisely when the market actually is large, which, according to Tate, happens with probability 0.40. Similarly, the probability of the research's reporting a small market is 0.60. We now have enough information to fill out the rest of the tree.

Figure I Tate's Decision Tree



For easy reference, we label the important nodes of the decision tree **A**, **B**, **C**, and **D**.

Step 3: Specify the objectives What should Tate's objective be when choosing among alternatives?

In our previous analysis, we assumed that Tate was aiming to make the decision offering the highest monetary value. This choice of objective may appear problematic, because it seems to assume that Tate is *risk neutral*, which is to say, she values a probability tree at its expected monetary value. Tate treated a 40% chance at \$800,000 and a 60% chance of \$100,000, an expected monetary value of \$380,000, as being equivalent to \$380,000 for sure. Most people, however, are *risk averse*, meaning that they would prefer to receive the expected value of a gamble to the gamble itself. In Tate's case, it means that she may prefer to invest \$300,000 and receive \$380,000 for sure than to invest and take her chances. To take a simpler case, most people would prefer to receive \$100 for sure than to take a 50% chance at receiving \$200.

Nevertheless, the maximization of expected value is a reasonable starting point for most business decisions. Underlying this decision rule are two important assumptions. First, we assume that the decision maker is trying to make the best decision on behalf of diversified investors. If each investor holds only a very small part of his portfolio in Tate's company, the risks that Tate's company takes have very little impact on the investor's overall wealth. Second, we assume that the uncertainties in the tree are not related to the riskiness of the investor's portfolio. In the context of our example, this means that the probability that the market for ergonomic chairs is favorable is not related to the overall performance of the stock market.²

Of course, these assumptions may sometimes be violated. If Tate is the sole owner of her company and most of her wealth is tied up in the company, then she may act in a risk-averse way, avoiding investments with a positive expected monetary value if they are very risky. Risk aversion can be incorporated into our analysis by having the decision maker specify a *utility* for each final outcome (a

² These intuitions are formalized by the Capital-Asset Pricing Model, or CAPM.

measure of how happy or unhappy she would feel about her enhanced or diminished wealth level at that outcome) and then allowing her to maximize *expected utility* rather than expected value.

Maximization of expected monetary value is a much simpler decision rule, however, and is a good first approximation in most cases where the risks the decision maker faces are not too large. Therefore, in this note, we will continue to assume Tate values a set of uncertain alternatives at its expected monetary value. This means that, when faced with two or more alternatives, she always prefers the one with the highest expected monetary value, no matter what the risks. *At a decision node, we always choose the branch with the highest expected monetary value.*

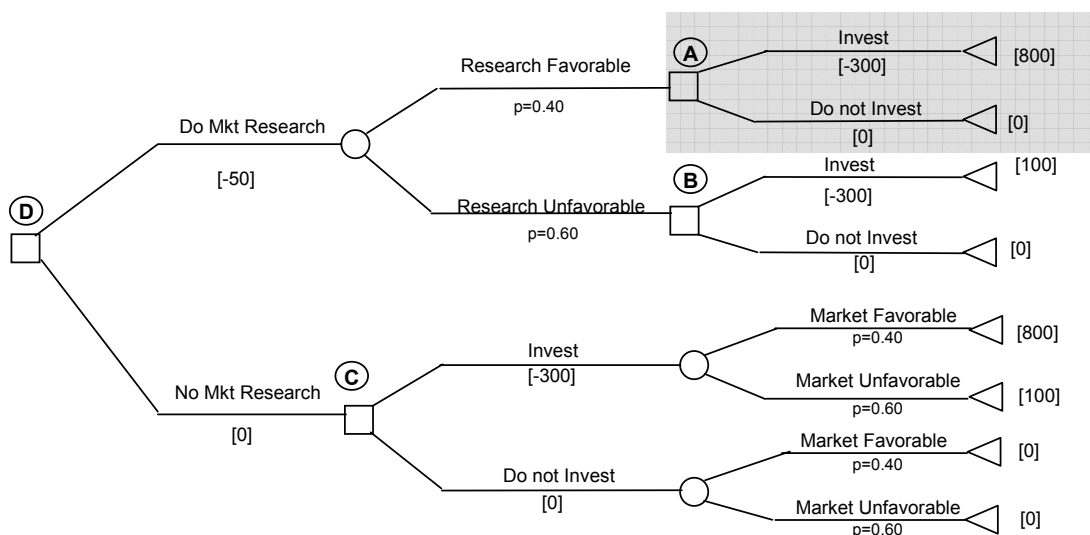
Solving the Tree

Tate is now ready to solve the decision tree. To solve any complex decision problem, we break the decision tree into a series of smaller decision problems. Looking back on the decision tree in **Figure I**, we count four decision nodes: **A**, **B**, **C**, and **D**. Tate must decide what to do at each.

Where do we start? The chronology of the tree makes it easy. We start on the right-hand side of the tree and work backwards. Why do we start at the end? While it may seem counterintuitive to work backwards, in fact we often do this in making day-to-day decisions. Suppose you are considering whether it is worth the extra time and effort of going into town to eat, versus eating at your local restaurant. In making this decision, you might think first about what you would eat if you went to town, versus what you would eat if you ate locally. In other words, you would imagine that you had already gone into town and consider how you would act next. Then, having solved this problem, you would work backwards to make your first decision, comparing how much more you would enjoy your selected meal in town to your selected local meal considering the extra time and effort of going into town.

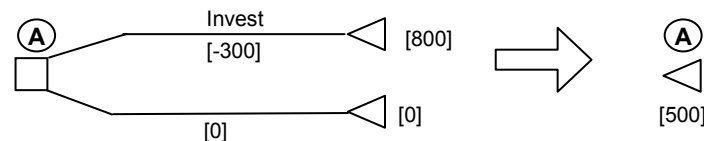
In a similar way, Tate starts by imagining that she has already invested in market research and discovered that the market is favorable. That is to say, she first supposes that she is in position **A** in the tree below. What would she do then?

Figure J



If Tate found herself at **A**, she would have to choose between investing and not investing. By virtue of being in this position, she knows that the market is good and that investing would yield a cash flow of \$500,000 compared with not investing, which would yield zero. Therefore, she knows that if she were to reach point **A**, she would always choose to invest.

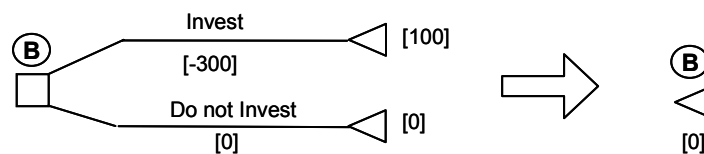
Figure K



Because she knows what she would do if she were at **A**, she knows that the value of being at that point is equal to the value when she makes the best decision available there: \$500,000.

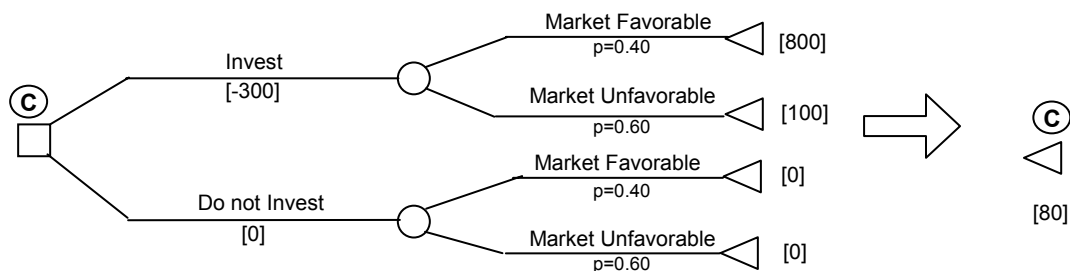
Following the same technique, she thinks about what she would do if she invested in market research and found that it was unfavorable, at point **B**. Comparing the certain loss of 200 (-300 plus 100) if she invests with 0 if she does not invest, she chooses not to invest.

Figure L



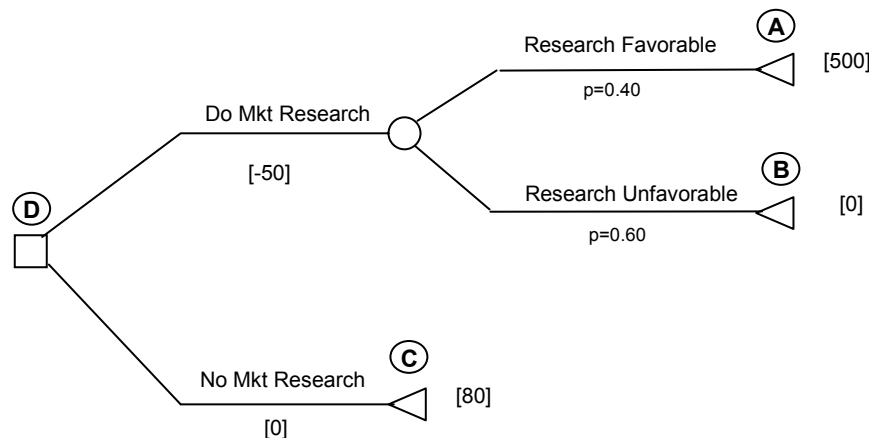
Finally, she thinks about what she would do if she were at point **C**. Here, her decision is more complicated because she does not know for sure what the consequences of her actions will be. She compares the expected monetary value of each of the branches of the tree and chooses the best one. Since the expected monetary value of investing ($0.40 \times 800,000 + 0.60 \times 100,000 - 300,000 = 80,000$) exceeds the expected monetary value of not investing ($0.40 \times 0 + 0.60 \times 0 - 0 = 0$), she would choose to invest. As we saw in our analysis of probability trees earlier, the value of this decision is the expected monetary value of the best decision, or \$80,000. Tate's value of being at **C** is \$80,000.

Figure M



At this point, we know what Tate would do at points **A**, **B**, and **C**. We can condense the tree into a smaller set of decisions as follows:

Figure N



With our simplified tree, we can now ask what Tate should do at **D**. She compares the expected monetary value of the first path (doing market research) with the expected monetary value of the second path (not doing market research). In the top path, she gets 500,000 with a probability of 0.40 and 0 with a probability of 0.60 but incurs a certain cost of 50,000. The expected cash flow after choosing to commission the market research is therefore $500,000 \times 0.4 + 0 \times 0.6 = 200,000$, less the cost of the research itself, 50,000, yielding a net expected cash flow of \$150,000. In the bottom path, she has an expected cash flow of \$80,000. Therefore, she should pay for the market research, as it increases her expected cash flows by \$70,000, from \$80,000 to \$150,000.

Figure O summarizes the steps involved in solving Tate's decision problem.

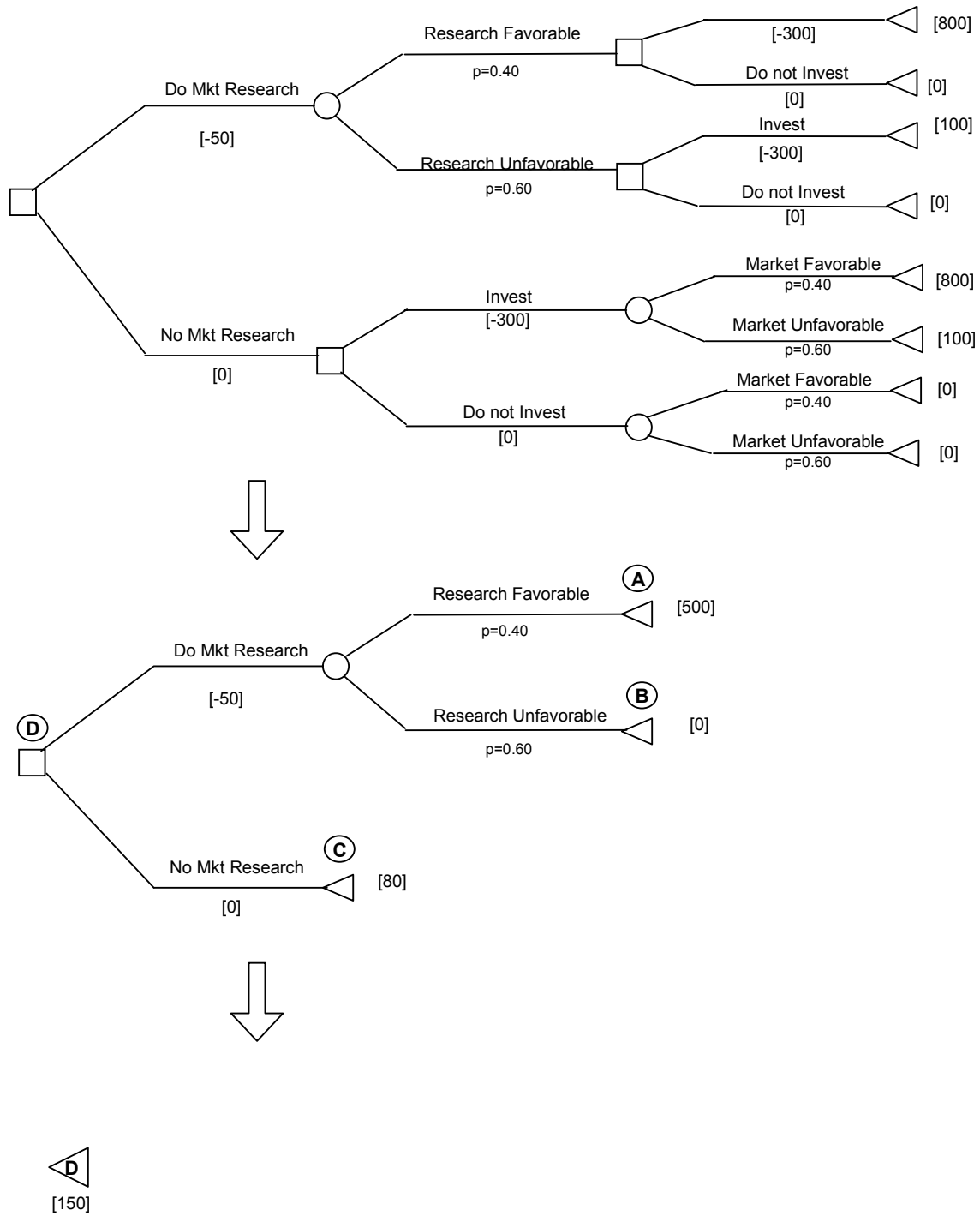
Sensitivity Analysis

Having written down and solved the decision tree for her business problem, Tate is ready to proceed. However, she may want to know how her decision would change if there were an intermediate stage of the market, in which cash flows from sales were \$400,000 rather than limited to the \$100,000 and \$800,000 extremes. Alternatively, she may want to know whether she would still want to purchase the information if the price were raised by \$30,000. Finally, she may want to analyze how her decision would change if the probabilities of the market's being high or low were different.

To answer these questions, Tate would redo the decision tree for each of these scenarios. In each case, she would analyze her actions and the value of those actions. Why would she go to so much trouble? The reason is that she may want to identify what the crucial assumptions are on which her decision hinges.

In many business decisions, *sensitivity analysis* of this kind can be extremely important. Remember that the model is only as good as its inputs. Understanding which of the inputs are crucial to the decision can help you clarify where the uncertainties are and ultimately make better decisions.

Figure O Solving the Tree Backwards



Summary

Solving complex decision problems is easy with a few simple tools. In this note, we solved a decision problem in which (1) the alternatives were well-defined, (2) the critical uncertainties were quantified, and (3) the objectives were clear. To solve the problem, we built the decision tree in three steps. First, we laid out the alternatives, paying close attention to the chronology of events and understanding the sources of uncertainty. Second, we quantified the uncertainties. This meant that we assigned a probability to each of the branches of the event nodes. Third, we said that the decision maker's objective was to choose alternatives based on their expected monetary value.

Once we set up the tree, solving it was simple (see **Figure O** for a summary). Starting at the end of the tree, we folded back the event nodes and decision nodes. Event nodes were folded back by replacing them with their expected monetary value. Decision nodes were folded back by calculating the expected monetary value of each alternative and selecting the best one. Finally, we were left with a single node that gave us the value of the project. In the process, we also learned which decisions we would make at each of the decision nodes.