ISQS-7339: Homework 1

Problem 1

The WorldLight Company produces two light fixtures (products 1 and 2) that require both metal frame parts and electrical components. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of $1, and each unit of product 2, up to 60 units, gives a profit of $2. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.

a) (5 points) Formulate a linear programming model for this problem to maximize profit.
b) (5 points) Use the graphical method to solve this model. Show your work.
c) (5 points) Use Python to solve this problem and compare your answer with part b. Also, report the sensitivity analysis parameters such as slack and shadow prices for all constraints.
d) (5 points) What constraints are binding? Interpret the shadow price value of each constraint.

Problem 2

Consider the following problem, where the value of c_1 has not yet been ascertained.

$$\begin{aligned} &\textit{Max} && Z = c_1 x_1 + x_2 \\ &\textit{S.t.} && \\ & && x_1 + x_2 \leq 6 \\ & && x_1 + 2x_2 \leq 10 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

a) (5 points) Use graphical analysis to determine the optimal solution(s) for $(x1, x2)$ for the various possible values of c_1. Try 3 different values (for instance it can be 0.5, 1, and 2). Have you seen any change on the optimal solution of x_1, x_2 and objective value? Explain.
b) (5 points) Suppose $c_1 = 2$ (so $Z = 2x_1 + x_2$), use simplex algorithm to solve this model. Show the entire process.

Problem 3

Implement the following LP Minimization model in Python.

$$Min \quad Z = 4x_1 + 3x_2 + 5x\_3$$

S.t.

$$\begin{aligned} 2x_1 \quad + x_3 &\geq 8 \\ x_2 + x_3 &\geq 6 \\ 6x_1 + 8x_2 \quad &\geq 48 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

a) (5 points) Report the optimal solution for decision variables and the objective value.
b) (5 points) Report and interpret the reduced cost[1] value of each decision variable.

Problem 4

A trust officer at the Blacksburg National Bank needs to determine how to invest $500,000 in the following collection of bonds to maximize the annual return.

Bond	Annual Return	Maturity	Risk	Tax-Free
A	10.0%	Long	High	Yes
B	7.50%	Short	Low	Yes
C	9.00%	Long	Low	No
D	9.00%	Long	High	Yes
E	9.00%	Short	High	No

The officer wants to invest at least 50% of the money in short-term issues and no more than 50% in high-risk issues. Also, at least 30% of the funds should go into tax-free investments.

a) (5 points) Formulate an LP model and implement it in Python.
b) (5 points) What is the optimal solution?
c) (5 points) What constraints are binding? Interpret the shadow prices and slack values.
d) (5 points) Interpret the reduced cost value of each decision variable.

Problem 5

The CitruSun Corporation ships frozen orange juice concentrate from processing plants in Eustis and Clermont to distributors in Miami, Orlando, and Tallahassee. Each plant can produce 20 tons of concentrate each week. The company has just received orders of 10 tons from Miami for the coming week, 15 tons for Orlando, and 10tons for Tallahassee. The cost per ton for supplying each of the distributors from each of the processing plants is shown in the following table.

	Miami	Orlando	Tallahassee
Eustis	$260	$200	$290
Clermont	$220	$240	$330

The company wants to determine the minimum costly plan for filling their orders for the coming week.

a) (10 points) Formulate an Integer LP model for this problem.

b) (5 points) Implement the model in Python and solve it. What is the optimal solution?

Problem 6

A young couple, Eve and Steven, want to divide their main household tasks (shopping, cooking, dishwashing, and laundering) between them so that each has two tasks but the total time they spend on household duties is kept to a minimum. Their efficiencies on these tasks differ, where the time each would need to perform the task is given by the following table:

	Time needed per week			
	Shopping	**Cooking**	**Dishwashing**	**Laundry**
Eve	4.5 hours	7.5 hours	3.0 hours	3.0 hours
Steven	5.5 hours	7.2 hours	4.5 hours	3.2 hours

a) (5 points) Formulate a BIP (Binary Integer Programming) model for this problem. (Hint: obviously, you need to define 8 binary decision variables.)

b) (5 points) Use Python to solve this problem.

Problem 7

The Toys-R-4-U Company has developed two new toys for possible inclusion in its product line for the upcoming Christmas season. Setting up the production facilities to begin production would cost $45,000 for toy 1 and $75,000 for toy 2. Once these costs are covered, the toys would generate a unit profit of $10 for toy 1 and $15 for toy 2.

The company has two factories that are capable of producing these toys. However, to avoid doubling the start-up costs, just one factory would be used, where the choice would be based on maximizing profit. For administrative reasons, the same factory would be used for both new toys if both are produced.

Toy 1 can be produced at the rate of 50 per hour (or 1/50 hours for each toy) in factory 1 and 40 per hour (1/40 hours for each toy) in factory 2. Toy 2 can be produced at the rate of 40 per hour (1/40 hours for each toy) in factory 1 and 25 per hour in factory 2. Factories 1 and 2, respectively, have 500 hours and 700 hours of production time available before Christmas that could be used to produce these toys.

It is not known whether these two toys would be continued after Christmas. Therefore, the problem is to determine how many units (if any) of each new toy should be produced before Christmas to maximize the total profit.

a) (10 points) Formulate a MIP (Mixed Integer Programming) model for this problem.
b) (5 points) Use Python to solve this problem.