

# Linear Perturbation Series Documentation

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## 1 Exact Solution Of 2x2 Case

For  $E_1, E_2, a \in \mathbb{R}$  let

$$H := \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

and

$$V := \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix}.$$

Define  $T : \mathbb{C} \rightarrow \mathbb{C}^{2 \times 2}$  by  $T x = H + x \cdot V$ .

The two eigenvalues  $E_{\pm}$  of  $T x$  (the roots of the characteristic polynomial) can be found as

$$E_{\pm}(x) = \frac{E_1 + E_2}{2} \pm \frac{|E_1 - E_2|}{2} \sqrt{1 + \left( \frac{2ax}{E_1 - E_2} \right)^2}. \quad (1)$$

Now the binomial series says that for  $x \in \mathbb{C}$  with  $|x| < 1$ :

$$(1 + x)^{1/2} = \sum_{k=0}^{\infty} \binom{1/2}{k} x^k, \quad (2)$$

where the radius of convergence of the power series is 1. Therefore

$$\begin{aligned} E_{\pm}(x) &= \frac{E_1 + E_2}{2} \pm \frac{|E_1 - E_2|}{2} \sum_{k=0}^{\infty} \binom{1/2}{k} \left( \frac{2ax}{E_1 - E_2} \right)^{2k} \\ &= \end{aligned} \quad (3)$$

where the radius of convergence is

$$\frac{|E_1 - E_2|}{2a}. \quad (4)$$