

Exercise 1.b)

$$\begin{aligned}|z + w|^2 &= |z|^2 + 2zw + |w|^2 \\ &\leq |z|^2 + |w|^2 \\ &\leq (|z| + |w|)^2 \\ |z + w| &\leq |z| + |w|\end{aligned}$$

Exercise 4

- a) Suppose $z \in \mathbb{C}, z \neq 0$, then $f(z) = \frac{z^4}{|z|^2} = \frac{z^4}{\bar{z}z}$. Hence applying lemma 2.22 we have

$$\frac{\partial}{\partial \bar{z}} f(z) = \frac{-z^3}{\bar{z}^2} \neq 0 \text{ for } z \neq 0$$

Now suppose $z = 0 \in \mathbb{C}$, let $h \in \mathbb{C}$, then

$$\lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{\frac{h^3}{\bar{h}}}{h} = \lim_{h \rightarrow 0} \frac{h^2}{\bar{h}} = 0$$

Hence f is complex differentiable at $z = 0$.

- b) From our function $f(z) = 2xy + i(x + \frac{2}{3}y^3)$ we have $u(x, y) = 2xy, v(x, y) = x + \frac{2}{3}y^3$ which are both trivially continuous as they are polynomials. So we have

$$u_x = 2y, u_y = 2x, v_x = 1, v_y = 2y^2$$

Hence for

$$\begin{aligned}u_x &= 2y = v_y = 2y^2 \\ u_y &= 2x = -v_x = -1\end{aligned}$$

The only solutions are $z = 0, z = -\frac{1}{2}$.

Exercise 7,a)

Exercise 9

Exercise 10

Let $f(z) = z^3, z = x + iy, x, y \in \mathbb{R}$. Then

$$f(z) = (x + iy)^3 = x^3 + 3x^2iy - 3xy^2 - iy^3$$

and so we have that

$$\operatorname{Re}(f) = x^3 - 3xy^2$$