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Exercise 1

I) $log(z-1)$: Has a branch cut of $[1,\infty)$.

II) $log(z+2)$: Has a branch cut of $[-2,\infty)$.

III) $log[(z-1)(z+2)]$: Has one branch cut at $[-2,1]$.

Exercise 2

I) $D=\{z\in\mathbb{C}|Im(z)>0\}$: Has an antiderivative of $F:=Log(z)$, differentiable in D, (as $D\subseteq\mathbb{C}\backslash[0,+\infty)$), as defined in the notes.

II) $D=A_{r,R}(0)$: f has no antiderivative as we know $\int_{S_p^+(0)}1/z=2\pi i\neq 0,\forall p$.

III) $D=\mathbb{C}\backslash[0,+\infty)$: Has an antiderivative of $F:=Log(z)$, differentiable in D as defined in the notes.

Exercise 3

I) Let $f(z)=1,\forall z$, suppose $|a|<r$. Then we have by cauchys integral formula

$$\int_{S_r^+(0)}\frac{f(z)}{z-a}dz=f(z)2\pi i=2\pi i$$

For $|a|=r$, then we can find some $R>r$ such that $S_r^+(0)\sim S_R^+(0)$, then by the deformation theorem and above

$$\int_{S_r^+(0)}\frac{1}{z-a}dz=\int_{S_R^+(0)}\frac{1}{z-a}dz=2\pi i$$

For $|a|>r$ Then $\int_{S_r^+(0)}\frac{1}{z-a}dz=0$ by cauchys theorem. As $S_r^+(0)$ is simply connected and f is differentiable in $S_r^+(0)$.

II) Let $f(z)=\sin(z)/(z+2)$, f holomorphic for $\mathbb{C}\backslash\{-2\}$. So by cauchys integral formula

$$\int_{S_1^+(0)}\frac{f(z)}{z-0}dz=2\pi i f(0)=0$$

Exercise 4

We know from the definition of the homotopy map, $\gamma_0 \sim \gamma_1 \sim \gamma_s \forall s \in [0,1]$. So for any $s \in [0,1]$, we have that, by the deformation theorem, $\int_{\gamma_s} f = \int_{\gamma_0} f = \int_{\gamma_1} f = c$ for some constant $c \in \mathbb{C}$. Then,

$$\frac{d}{ds} I(s) = \frac{d}{ds} c = 0$$