# 1 Complex Numbers

Defined as $\mathbb{C} = \{(x,y) : x, y \in \mathbb{R}\}$ subject to conditions, for $(x_1, y_1), (x_2, y_2) \in \mathbb{C}$

- Addition $(+)$:
  $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

- Multiplication $(\cdot)$:
  $(x_1, y_1) + (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_2 y_1 + x_1 y_2)$

$(\mathbb{C}, +), (\mathbb{C}, \cdot)$ are albelian groups, with units $(0,0)$ and $(1,0)$ respectivly

## 1.1 Lemma

$(\mathbb{C}, +, \cdot)$ is a field with multiplicative inverse

$$z \in \mathbb{C} \backslash \{0, 0\}, \quad z^{-1} = (\frac{x}{x^2 + y^2}, \frac{-y}{x^2 + Y^2})$$

such that $z * z^{-1} = (1, 0)$

We will define as follows:

$$1 := (1, 0), \quad i := (0, 1), 0 := (0, 0)$$

Allowing us to write complex numbers as