Joseph Dowling.

Exercise 1.a)

Exercise 3

We have that $(fg)' = fg' + f'g$ so $\int_\gamma (fg)' = \int_\gamma fg' + \int_\gamma f'g$ and then by the fundamental theorem we have

$$\int_\gamma (fg)' = f(b)g(b) - f(a)g(a)$$

Combining our two equations we get

$$\int_\gamma fg' + \int_\gamma f'g = f(b)g(b) - f(a)g(a)$$

Then rearanging

$$\int_\gamma fg' = f(b)g(b) - f(a)g(a) - \int_\gamma f'g$$

Exercise 5

We have $L(\gamma) = \int_0^{2\pi} |\gamma'(t)|dt = \int_0^{2\pi} |ie^{it}|dt = 2\pi$. And we also have

$$\begin{aligned}\sup_t |\frac{\sin(\gamma(t))}{\gamma(t)^2}| &= \sup_t |\frac{e^{i\gamma(t)} - e^{-i\gamma(t)}}{2i\gamma(t)^2}| \\ &= \sup_t \frac{|e^{i\gamma(t)} - e^{-i\gamma(t)}|}{|2i\gamma(t)^2|} \\ &\leq \sup_t \frac{|e^{i\gamma(t)}| + |e^{-i\gamma(t)}|}{|2i\gamma(t)^2|} \\ &= \sup_t \frac{1}{|\gamma(t)|^2} = 1?\end{aligned}$$

Then by the estimation lemma we get

$$|\int_\gamma \frac{\sin(z)}{z^2} dz| \leq 2\pi e$$

Exercise 6)

b)

c)