

# 1 Complex Numbers

Defined as  $\mathbb{C} = \{(x, y) : x, y \in \mathbb{R}\}$  subject to conditions, for  $(x_1, y_1), (x_2, y_2) \in \mathbb{C}$

- Addition (+):  
 $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$
- Multiplication ( $\cdot$ ):  
 $(x_1, y_1) \cdot (x_2, y_2) = (x_1x_2 - y_1y_2, x_2y_1 + x_1y_2)$

$(\mathbb{C}, +), (\mathbb{C}, \cdot)$  are abelian groups, with units  $(0, 0)$  and  $(1, 0)$  respectively

## 1.1 Lemma

$(\mathbb{C}, +, \cdot)$  is a field with multiplicative inverse

$$z \in \mathbb{C} \setminus \{0, 0\}, \quad z^{-1} = \left( \frac{x}{x^2 + y^2}, \frac{-y}{x^2 + y^2} \right)$$

such that  $z * z^{-1} = (1, 0)$

We will define as follows:

$$1 := (1, 0), \quad i := (0, 1), \quad 0 := (0, 0)$$

Allowing us to write complex numbers as