Exercise 1

i)

i)

Exercise 6

Exercise 10

i) Want to show $f^{-1}(\emptyset) = \emptyset$
Suppose to the contrary that $\exists x \in f^{-1}(\emptyset)$. This would imply $f(x) \in \emptyset$ which is a contradiction. Hence $f^{-1}(\emptyset)$ has no elements and thus $f^{-1}(\emptyset) = \emptyset$.

ii)

iii)

iv)

$$\begin{aligned} x \in f^{-1}(\cup_\alpha E_\alpha) &\iff f(x) \in \cup_\alpha E_\alpha \\ &\iff f(x) \in E_{\alpha_1} \quad \text{for some} \alpha_1 \\ &\iff x \in f^{-1}(E_{\alpha_1}) \\ &\iff x \in \cup_\alpha f^{-1}(E_\alpha) \end{aligned}$$

v)

$$\begin{aligned} x \in f^{-1}(\cap_\alpha E_\alpha) &\iff f(x) \in \cap_\alpha E_\alpha \\ &\iff f(x) \in E_\alpha \quad \forall \alpha \\ &\iff x \in f^{-1}(E_\alpha) \\ &\iff x \in \cap_\alpha f^{-1}(E_\alpha) \end{aligned}$$