Coordinate mapping for pan/tilt Lidar

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Conversion from polar coordinates and lidar servo structure to cartesian coordinates is done with translation/rotation matrix multiplication.

These are the structure conversions, using positive x as "forward", y as "right" and z as "upwards" axies - a "left handed" axis setup, and starting with measurements lidar distance d, pan angle α and tilt angle β:

- Rotation arm attach point is the origin.
- The pan servo and attachment arm gives us rotation alpha around z-axis
- The tilt servo rotates the device by β around the y-axis.
- Distance from Lidar is the measurement position relative to Lidar $X = \begin{bmatrix} d & 0 & 0 & 1 \end{bmatrix}^T$

Pan rotation matrix:

$$M_{pan} = \begin{bmatrix} cos(\alpha) & -sin(\alpha) & 0 & 0 \\ sin(\alpha) & cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Tilt rotation matrix:

$$M_{tilt} = \begin{bmatrix} cos(\beta) & 0 & -sin(\beta) & 0 \\ 0 & 1 & 0 & 0 \\ sin(\beta) & 0 & cos(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Due to bracket attachments, there is also *translation* terms in addition to rotation. In the early stages we could assume they are zero, or significantly less than measured distances and therefore irrelevant.

The transformation matrices above look like this with translation terms:

Pan transformation matrix:

$$M_{pan} = \begin{bmatrix} cos(\alpha) & -sin(\alpha) & 0 & p_x \\ sin(\alpha) & cos(\alpha) & 0 & p_y \\ 0 & 0 & 1 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Tilt transformation matrix:

$$M_{tilt-t} = \begin{bmatrix} cos(\beta) & 0 & -sin(\beta) & t_x \\ 0 & 1 & 0 & t_y \\ sin(\beta) & 0 & cos(\beta) & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

In my arrangement, d_y term is zero in both cases - there is no sideways translation.

The applied transformation is in entirety:

$$X' = M_{pan} * M_{tilt} * X \tag{5}$$

Without translation terms:

$$X' = \begin{bmatrix} cos(\alpha) * cos(\beta) & -sin(\alpha) & cos(\alpha) * -sin(\beta) & 0 \\ sin(\alpha) * cos(\beta) & cos(\alpha) & sin(\alpha) * -sin(\beta) & 0 \\ sin(\beta) & 0 & cos(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} d \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{6}$$

$$X' = \begin{bmatrix} d'_x \\ d'_y \\ d'_z \\ 1 \end{bmatrix} = \begin{bmatrix} d * cos(\alpha) * cos(\beta) \\ d * sin(\alpha) * cos(\beta) \\ d * sin(\beta) \\ 1 \end{bmatrix} \tag{7}$$

With translation terms (remember that $t_y = p_y = 0$) the combined translation matrix is:

$$X' = \begin{bmatrix} cos(\alpha) * cos(\beta) & -sin(\alpha) & cos(\alpha) * -sin(\beta) & cos(\alpha) * t_x + p_x \\ sin(\alpha) * cos(\beta) & cos(\alpha) & sin(\alpha) * cos(\beta) & sin(\alpha) * t_x + p_y \\ sin(\beta) & 0 & cos(\beta) & t_z + p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} * X \tag{8}$$

Transformed coordinates are in this case:

$$X' = \begin{bmatrix} d * cos(\alpha) * cos(\beta) + cos(\alpha) * t_x + p_x \\ d * sin(\alpha) * cos(\beta) + sin(\alpha) * t_x \\ d * sin(\beta) + t_z + p_z \\ 1 \end{bmatrix} \tag{9}$$