Michael Fu
Julian Dai

Project – III

mfu16, Linsanity8
jkdai, mono

1 Integer Constraint Model

We first analyze the constraint model for the true (i.e. without linear relaxation) supply chain problem. Let N_f be the maximum number of facilities, N_c be the number of customers, and N_v be the maximum number of vehicles per facility.

Variables: $\mathbf{M} \in \{0,1\}^{N_c \times N_f \times N_v}$

Constraints:

- Each customer is served by exactly one facility and vehicle: $\sum_{f,v} M_{c,f,v} = 1 \; \forall \; c$
- Each customers demand must be met in full: $\sum_{f,v} M_{c,f,v} = 1 \; \forall \; c$
- No facility may exceed its capacity: $\sum_{c,v} M_{c,f,v} \cdot Demand_c \leq Capacity_f \; \forall \; f$
- No vehicle may exceed its allowed driving distance: $\sum_{c,f} M_{c,f,v} \cdot Distance_{cf} \leq MaxDistance \; \forall \; v$

Objective: Minimize cost: $f \cdot C_f + v \cdot C_v + \sum_{c,f,v} M_{c,f,v} \cdot allocCost_{c,f}$

2 Linear Relaxation

To solve the linear relaxation of this problem, we allow facilities and vehicles to serve fractional customers. Formally:

Variables: $\mathbf{M} \in [0,1]^{N_c \times N_f}, \mathbf{V} \in [0, maxVehiclePerFacility]^{N_f}$

Constraints:

- Each customers demand must be met in full: $\sum_{f,v} M_{c,f} = 1 \; \forall \; c$
- No facility may exceed its capacity: $\sum_{c} M_{c,f} \cdot Demand_c \leq Capacity_f \; \forall \; f$
- No vehicle may exceed its allowed driving distance: $\sum_{c} M_{c,f} \cdot Distance_{cf} \leq \mathbf{V}_v \cdot MaxDistance \; \forall \; f$
- The distance between a facility and customer can't exceed the max distance: $M_{c,f} \cdot Distance_{cf} \leq MaxDistance \; \forall \; c, f$

Objective: Minimize cost: $f \cdot C_f + v \cdot C_v + \sum_{c,f,v} M_{c,f,v} \cdot allocCost_{c,f}$

3 Evaluation

To have some confidence in our results, we wrote tests for each of the constraints in our linear programming model.

Additionally, we constructed the integer programming model as described in the first section to sanity check the outputs of our linear programming relaxation. Unsurprisingly, the outcomes of the integer programming solution show that the linear relaxation consistently underestimates the true cost, but at least roughly follows the trends of the integer programming model, e.g. linear model estimates a more expensive value $\iff$ the integer model estimates a more expensive value.

4 Time Spent

We spent approximately 8 hours on this project.