PROBLEM SET # 3

Astro 512 – Spring 2019
Extragalactic Astronomy

In class we discussed how different types of stars dominate the light and the mass of a single stellar population. This problem is designed to quantitatively demonstrate the different roles that various stellar types play in determining the mass, luminosity, and ionizing luminosity of a single stellar population.

For all problems below, please plot your results for the full range of masses between $0.08\,\mathrm{M}_\odot$ and $120\,\mathrm{M}_\odot$. Please label your axes with physically meaningful words and astrophysical units (angstroms or microns, not centimeters; solar masses, not grams; Myr or Gyr, not seconds or years, etc), and choose limits and scalings (linear vs log) that make the behavior of the data clear.

To reduce the time you have to spend on the problem set, I have provided an iPython notebook (`main_sequence.ipynb`) on the course website that reads in the main sequence star properties from Ekers et al 2018 (from the table below, also included on the class website) and sets up some useful interpolating functions. The notebook also includes some useful functions to save you time.

Problem 1: Behavior of Main Sequence Stars as a Function of Mass

Much of the behavior of evolving stellar populations is set by the behavior of individual main sequence stars – their lifetimes, temperatures, and ionizing fluxes. In what follows, you will make some simplifying assumptions that should allow you to develop intuition for the behavior of main sequence stars as a function of stellar mass. The first major assumption is that you will assume stars' bolometric (i.e., total) luminosity L_{bol} is distributed in wavelength like a black-body with temperature T_{eff}, set the the temperature of the stars' photospheres.

a) Make a plot of main sequence stars' peak wavelength (in angstroms) as a function of their mass (in solar masses). You may wish to use the Wien displacement law, which relates the peak wavelength λ_{peak} of a black-body spectrum to its temperature T (i.e., $(\lambda_{peak}/\mu\mathrm{m}) = 2898\,(T/\mathrm{K})$).

b) For main sequence stars, at what effective temperature, stellar mass, and approximate spectral type does the peak move out of the visible and into the near-UV? Use the blue edge of the SDSS u-band filter as an approximate indicator as the start of the near-UV.

c) For main sequence stars, at what effective temperature, stellar mass, and approximate spectral type does the peak move out of the visible and into the near-IR? Use the red edge of the SDSS z-band filter as an approximate indicator as the start of the near-IR.

d) Photons are capable of ionizing Hydrogen out of the ground state when their wavelength is blueward of 912Å. Plot the *fraction* of ionizing flux (i.e., $f_{ion} = L_{ion}/L_{bol}$) for a main sequence star, as a function of the star's mass M. Plot the same quantity as a function of the star's effective temperature T_{eff}, as well.

e) Beyond what stellar mass, effective temperature, and approximate spectral type do stars begin emitting more than 10% of their flux at ionizing wavelengths?

f) Using your result from (d), plot the luminosity L_{bol} and the ionizing luminosity L_{ion} of main sequence stars as a function of mass.

g) What are the approximate power-law exponents α and α_{ion} for the dependences of luminosity and of ionizing luminosity on stellar mass, respectively (i.e., $L_{bol} \propto M^{\alpha}$, and $L_{ion} \propto M^{\alpha_{ion}}$), for stars $> 10\,\mathrm{M}_{\odot}$? (*Note: You can estimate the power-law exponent of $y = \beta\, x^{\alpha}$ as the slope on a log-log plot. You can either fit a line to $\log y = \alpha \log x + \log \beta$, or measure how many decades the y value changes ($\Delta \log y$) when the x value changes by one decade ($\Delta \log x = 1$). For example, if your x-value changes from 10^0 to 10^1, and your y-value changes from 10^2 to 10^5, the power-law exponent is 3 ($= (5-2)/(1-0)$), and $y \propto x^3$. When α is a large number, it indicates that y depends very strongly on x, such that small changes in x produce very large changes in y.*).

Problem 2: The Impact of the Initial mass function

In galaxies there are vastly more lower mass stars than higher mass stars, and the *net* behavior of an ensemble of stars can be very different than you calculated in the first problem.

We characterize the relative number of stars with the "stellar mass function". For the first part of this problem, you will assume that the mass function of the stellar population is the same as a Salpeter initial mass function (IMF):

$$\xi(M) = \frac{\xi_0}{\mathrm{M}_{\odot}} \left(\frac{M}{\mathrm{M}_{\odot}} \right)^{-2.35}$$

i.e. $\xi(M)dM$ is the number of stars which have masses between M and $M + dM$. The IMF normalization ξ_0 can be chosen so that either $\xi(M)$ is a probability distribution function for drawing a mass M (with $\int \xi(M)\,\mathrm{d}M = 1$), or that $\xi(M)$ describes the production of a stellar population with a total mass of $1\,\mathrm{M}_{\odot}$ (with $\int \xi(M)\,M\,\mathrm{d}M = 1\,\mathrm{M}_{\odot}$); the function for calculating $\xi(M)$ in the ipython notebook uses the former. Assume that no stars form with masses below $M_{min} = 0.08 M_{\odot}$ or above $M_{max} = 120 M_{\odot}$; if we had not made this assumption, then the number of stars would be infinite. More modern IMFs are not as steep at low masses, but to keep this problem more tractable it is fine to just assume the Salpeter IMF, which remains an adequate description of the relative numbers of stars with masses greater than $1\,\mathrm{M}_{\odot}$. Note that when using the IMF for calculations, it is often helpful to set up a calculation in terms of a "$dN(M) = \xi(M)dM$", which you can then multiply by a quantity that depends on M before integrating over M.

a) Write down the expression for the fraction of mass in stars below some mass M (i.e., $f_{mass}(M) = M_{tot}(< M)/M_{tot}(< \infty)$). You do not need to solve any integrals once you've set them up.

b) Write down the expression for the fraction of the luminosity in stars below some mass M (i.e., $f_{lum}(M) = L_{tot}(< M)/L_{tot}(< \infty)$), assuming you have some function $L_{bol}(M)$ that gives the bolometric luminosity of a main sequence stars with mass M. You do not need to solve any integrals once you've set them up.

c) Write down the expression for the fraction of the ionizing luminosity in stars below some mass M (i.e., $f_{lumion}(M) = L_{ion,tot}(< M)/L_{ion,tot}(< \infty)$), assuming you have (1) some function $L_{bol}(M)$ that gives the bolometric luminosity of a main sequence stars with mass M and (2) some function $f_{ion}(M)$ that gives the fraction of that bolometric luminosity that comes out at short enough wavelengths to ionize hydrogen, for a main sequence star with mass M. You do not need to solve any integrals once you've set them up.

d) Using the provided ipython notebook, and the functions you calculated in the first problem, plot the expressions for parts (a), (b), and (c) on a single plot. Include a horizontal line at 0.5; the mass at which this line crosses your curves is the stellar mass above which half the total mass, half the total luminosity, and half the total ionizing luminosity comes from.

e) By either interpolating your results or using a ruler on your plot, what are the masses above which half the total mass, half the total luminosity, and half the total ionizing luminosity comes from? What spectral type of star do those each correspond to? What effective temperature?

Problem 3: Timescale for Luminosity Evolution

During a burst of star formation, stars are born with a distribution of masses that follows the IMF. As time passes, however, stars evolve, and more massive stars disappear from the main sequence with time. The main sequence lifetime τ_{MS} as a function of mass can be approximated as

$$\log_{10}(\tau/\mathrm{yr}) = 10.09 - 3.139\log_{10}\left(\frac{M}{\mathrm{M}_\odot}\right) + 0.238238\log_{10}\left(\frac{M}{\mathrm{M}_\odot}\right)^2 + 0.26163378\log_{10}\left(\frac{M}{\mathrm{M}_\odot}\right)^3,$$

based on the Bertelli et al (2009) evolutionary tracks (see http://www.pas.rochester.edu/~emamajek/images/stellar_lifetimes.png). This relation was based on fitting to stars below $20\,\mathrm{M}_\odot$. Above $20\,\mathrm{M}_\odot$, it is preferable to use

$$\log_{10}(\tau/\mathrm{yr}) = 9.01 - 1.57\log_{10}\left(\frac{M}{\mathrm{M}_\odot}\right).$$

a) Based on your answers from part 2(e), and the code provided in the ipython notebook, how long until half of the ionizing luminosity is gone?

b) How long until half of the total luminosity is gone?

c) What fraction of the total initial luminosity is left after 10 Myr, 100 Myr, 1 Gyr, and 10 Gyr?

d) On a single panel, plot the *relative* luminosity contributions of stars of different stellar masses for each the ages in part (c). What stellar mass dominates the luminosity at each age? In other words, when you calculate the total luminosity by integrating/summing up the luminosity $dL(M)$ contributed by stars with masses between M and $M + dM$, which stellar mass produces the largest $dL(M)$ at each age?

e) Based on your answer for (d), what stellar spectral type do you expect to dominate the integrated spectrum at each age? (Note that this ignores any contributions from red evolved stars that have not yet become SNe or WDs, and thus you would expect these spectral features to be most dominant in the bluer parts of the spectrum)

Problem 4 (Extra Credit): The Impact of the IMF Slope

All of the results in Problem 2 and 3 depend on the slope as the IMF. For extra credit, derive how the total initial luminosity, total initial ionizing luminosity, and the half-life for the ionizing flux depend on the slope of the IMF (considering values from -1.5 to -3.5).

Appendix of Useful Data

Table 1: Properties of Main Sequence Stars from Eker et al 2018

Spt	$\log T_{eff}$	$B-V$ (mag)	$U-B$ (mag)	BC (mag)	M_V (mag)	T_{eff} (K)	M ($M_\odot$)	R ($R_\odot$)	$\log g$ (cgs)	M/L ($\odot$)	L/M (erg s^{-1} gr^{-1})
O2	4.720	-0.33	-1.22	-4.52	-6.44	52483	63.980	16.734	3.80	0.00003	57628
O3	4.672	-0.33	-1.21	-4.19	-5.62	46990	44.260	12.312	3.90	0.00007	28981
O4	4.636	-0.33	-1.20	-3.94	-5.13	43251	34.809	10.301	3.95	0.00010	18514
O5	4.610	-0.33	-1.19	-3.77	-4.80	40738	29.669	9.236	3.98	0.00014	13743
O6	4.583	-0.33	-1.18	-3.58	-4.50	38282	25.380	8.362	4.00	0.00019	10270
O7	4.554	-0.33	-1.17	-3.39	-4.20	35810	21.660	7.616	4.01	0.00025	7642
O8	4.531	-0.32	-1.15	-3.23	-3.99	33963	19.215	7.131	4.02	0.00032	6111
O9	4.508	-0.32	-1.13	-3.03	-3.83	32211	17.123	6.721	4.02	0.00039	4929
B0	4.470	-0.31	-1.08	-2.84	-3.45	29512	14.277	6.171	4.01	0.00055	3511
B1	4.400	-0.28	-0.98	-2.40	-2.93	25119	10.459	5.454	3.98	0.00098	1965
B2	4.325	-0.24	-0.87	-2.02	-2.35	21135	7.699	4.967	3.93	0.00174	1110
B3	4.265	-0.21	-0.75	-1.62	-1.68	18408	6.123	3.989	4.02	0.00373	518
B5	4.180	-0.17	-0.58	-1.22	-0.76	15136	4.516	3.214	4.08	0.00928	208
B6	4.145	-0.15	-0.50	-1.02	-0.44	13964	4.007	2.974	4.09	0.01327	146
B7	4.115	-0.13	-0.43	-0.85	-0.18	13032	3.625	2.797	4.10	0.01790	108
B8	4.080	-0.11	-0.35	-0.66	0.13	12023	3.234	2.617	4.11	0.02518	77
B9	4.028	-0.07	-0.19	-0.39	0.57	10666	2.743	2.394	4.12	0.04121	47
A0	3.995	-0.01	-0.01	-0.24	0.86	9886	2.478	2.274	4.12	0.05587	35
A1	3.974	0.02	0.03	-0.15	0.90	9419	2.325	2.362	4.06	0.05897	33
A2	3.958	0.05	0.06	-0.08	1.06	9078	2.216	2.292	4.06	0.06919	28
A3	3.942	0.08	0.08	-0.03	1.23	8750	2.113	2.226	4.07	0.08107	24
A5	3.915	0.15	0.10	0.00	1.57	8222	1.952	2.123	4.08	0.10557	18
A6	3.902	0.18	0.10	0.01	1.74	7980	1.879	2.077	4.08	0.11971	16
A7	3.889	0.21	0.09	0.02	1.91	7745	1.810	2.033	4.08	0.13563	14
A8	3.877	0.25	0.08	0.02	2.07	7534	1.749	1.994	4.08	0.15209	13
F0	3.855	0.31	0.05	0.01	2.37	7161	1.643	1.928	4.08	0.18726	10
F1	3.843	0.34	0.02	0.01	2.53	6966	1.588	1.893	4.08	0.20956	9.22
F2	3.832	0.37	0.00	0.00	2.69	6792	1.540	1.863	4.09	0.23219	8.33
F3	3.822	0.40	-0.01	0.00	2.82	6637	1.498	1.838	4.09	0.25448	7.60
F5	3.806	0.45	-0.02	-0.01	3.30	6397	1.354	1.588	4.17	0.35692	5.42
F6	3.800	0.48	-0.01	-0.02	3.49	6310	1.305	1.508	4.20	0.40325	4.79
F7	3.794	0.50	0.00	-0.02	3.65	6223	1.259	1.434	4.23	0.45450	4.25
F8	3.789	0.53	0.02	-0.03	3.80	6152	1.222	1.377	4.25	0.50125	3.86
G0	3.780	0.59	0.07	-0.04	4.06	6026	1.161	1.283	4.29	0.59553	3.25
G1	3.775	0.61	0.09	-0.04	4.19	5957	1.128	1.236	4.31	0.65401	2.96
G2	3.770	0.63	0.13	-0.05	4.33	5888	1.098	1.191	4.33	0.71723	2.70
G3	3.767	0.65	0.15	-0.06	4.42	5848	1.080	1.165	4.34	0.75756	2.55
G5	3.759	0.68	0.21	-0.07	4.64	5741	1.031	1.097	4.37	0.87824	2.20
G6	3.755	0.70	0.23	-0.08	4.72	5689	1.019	1.081	4.38	0.92834	2.08
G7	3.752	0.72	0.26	-0.09	4.78	5649	1.011	1.069	4.39	0.96765	2.00
G8	3.745	0.74	0.30	-0.10	4.92	5559	0.990	1.041	4.40	1.06553	1.81
K0	3.720	0.81	0.45	-0.18	5.45	5248	0.922	0.951	4.45	1.49639	1.29
K1	3.705	0.86	0.54	-0.24	5.77	5070	0.884	0.903	4.47	1.82840	1.06
K2	3.690	0.91	0.65	-0.32	6.11	4898	0.848	0.858	4.50	2.22864	0.867
K3	3.675	0.96	0.77	-0.41	6.46	4732	0.813	0.817	4.52	2.71007	0.713
K5	3.638	1.15	1.06	-0.65	7.32	4345	0.736	0.727	4.58	4.34779	0.445
M0	3.580	1.40	1.23	-1.18	9.07	3802	0.558	0.541	4.72	10.16323	0.190
M1	3.562	1.47	1.21	-1.39	9.60	3648	0.524	0.508	4.75	12.75337	0.152
M2	3.544	1.49	1.18	-1.64	10.15	3499	0.492	0.479	4.77	15.91749	0.121
M3	3.525	1.53	1.15	-2.02	10.85	3350	0.462	0.452	4.79	20.00404	0.097
M4	3.498	1.56	1.14	-2.55	12.29	3148	0.323	0.338	4.89	32.12216	0.060
M5	3.477	1.61	1.19	-3.05	13.37	2999	0.249	0.284	4.93	42.55068	0.045

Figure 1: Properties of Main Sequence Stars from Eker et al 2018