Pitot tube Icing Analysis Report
ENME331-0201 (Group 3)

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1. Introduction.

Aircraft icing is a serious problem in modern avionics. The presence of ice on the wing of an aircraft interrupts the smooth airflow on the wings, increasing drag, and decreasing lift. These effects of aircraft icing have caused a range of issues for pilots and technicians. The most grave of which can potentially cause catastrophic failure of the aircraft in question. In addition to ice accumulation on normal surfaces, ice can also clog the ports of instruments used to control and regulate the aircraft. Specifically in this report, we are taking into consideration an aircraft's pitot tube.

A Pitot tube is a device used to measure the stagnation pressure against the static pressure of the air flow about the tube. This information is subsequently used to find the velocity of the aircraft with respect to the airfield. Naturally, if this tube is clogged, it cannot acquire information about the pressure to find this velocity.

We are concerned with a simplified model of this phenomenon specific to aircraft icing. In particular, we are interested in modeling ice-accumulated clogging inside the pitot tube of an aircraft. At the altitude of a normal aircraft, water droplets are supercooled, instantly becoming ice upon impact against a solid surface (add reference). Therefore, a good model to measure ice accumulation would be one where supercooled, spherical ice particles are dispersed uniformly in an airflow, moving at a normal, constant velocity to the stagnation port of the pitot tube. Until reaching the pitot tube, upon which the particles' pathlines are perturbed due to the drag created from the stagnation point of the pitot tube.

Yet aircraft pitot tubes are subjected to a wide variety of droplet sizes, each with different pathlines and abilities to accumulate inside the pitot tube. Therefore, within our simplified model, we are primarily concerned with the effect that the diameter of the idealized spherical water droplet has on pitot tube clogging. More specifically, the flux of water droplets through the pitot tube's entrance. Flux can be correlated with clogging because, as each water droplet enters the pitot tube, they can be considered ice accumulation. This is a reasonable assumption assuming the water droplets can touch a solid surface to phase change into ice before they leave the pitot tube.

The way in which we will measure the variations in the "flux" is not directly through the rate of mass per unit area but by the ratio of droplets expected to cross the pitot tube entrance in a freestream with the actual amount that cross. We will call this term the "collection efficiency". We will define the collection efficiency as follows: the circular water droplet collection region (far from the pitot tube, where its effects are not seen), divided by the projected area of the static port (See figure 1). How and why the Collection efficiency is not always 100% will be discussed later in the report.

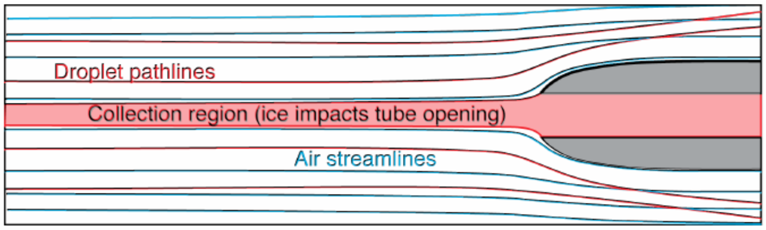


Figure 1: Water droplet pathlines collected into a pitot tube

In order to accomplish this, we use Dimensional Analysis techniques (The Buckingham Pi theory) in order to accomplish several different goals. First, attempt to scale an experiment down to a point where it would be feasible to perform in a laboratory wind tunnel (this will not actually be done by us, we simply want to assess feasibility). Replicating actual conditions for pitot tube icing is extremely difficult, so we will scale them down to those able to be studied in a laboratory setting yet will offer the same collection efficiency as actual conditions. Our second goal is to non-dimensionalize the equations of motions to help determine the most critical pi terms, in order to scale our model effectively. After all the scaling is done, we conduct a theoretical test of the scaling terms vs. the prototype terms using matlab in order to justify the scaled version of our experiment.

The results of the matlab computations will be what we use to conclude the effect of the particle's size on collection efficiency, as well as the feasibility for the hypothetical scaled model experiment.

Prototype system:

a) pitot tube dimensions: Outer diameter = .01m, Inner diameter = .004m
b) Airspeed: 250 m/s
c) Air density: .4 kg/m^3
d) Air viscosity: 1.4x10^-5
e) Ice particle effective spherical diameter range: 2-80 µm

Scaled system:

a) Wind tunnel: the test section has a cross section of .3x.3m and a length of .6m
b) Airspeed = 50 m/s
c) ρair= 1.2 kg/m^3

2. Discussion of Physical Phenomena

In order to find the collection efficiency, before even trying to non-dimensionalize the problem, we need to consider the physical system we are analyzing, and all of the variables that will likely come into play. Observing the system, we see three main components: the pitot tube, which has its prototype geometry set by the problem statement; the particle, which has Values of diameter predefined in the prototype information; and the flowing air, which has its velocity field set by a Matlab code provided to the group. Upon looking more in depth at each component, we can pull out several different variables that will effect the trajectory of the particle (altering the collection efficiency).

- The pitot tube has one main variable (since we are assuming we always know the velocity field, we don't need to provide more specific material properties and geometry), namely the diameter of the inner radius of the tube. We do not have to list the outer diameter, because it will be scaled according to the prototype values.
- The Particle has several values which we will want to consider. First it has a diameter (which is our main independent variable). Next, in order to calculate how the particle will react to an applied force, we will need to know its mass. The diameter is already known, all that is needed to find its mass is the density of the particle material. The Particle (which we assume starts out at the same velocity as the airfield) having a higher density, will not react as quickly as the air does with respect to the pitot tube, and will have its velocity deviate from the air. Because we are in a gravitational field, gravity will likely be important when we analyze how the particle moves.
- The air flow, as previously stated, has its velocity as a main variable. In order to determine the drag on the particle, we will also need to know the viscosity and density of the air as well.

Simply stated, the presence of the pitot tube will create a higher pressure around it as the air flow collides with it. The air flow will move around the tube, due to this pressure gradient. The changing fluid velocity will put a drag force on the particles in the field, and will cause the particles to deviate from their path which originally is normal to the stagnation port opening. This deviation will cause the collection efficiency to drop from 100%, to some value which we mean to find.

3. Dimensional Analysis

3.1 Variables

Collection efficiency is the dependent variable for this question, it is defined as:

$$Collection\ efficient = \frac{frontal\ height}{pitot\ tube\ inner\ diameter} \times 100\%$$

Frontal height can be affected by particle trajectory, therefore preliminary selection of independent variables should include the ones discussed in the previous section that could possibly change the water droplet trajectory (Figure 2) and they are listed as followed:

Table 1. Preliminary selection of variables

Parameters	Symbol	Dimension	Rationale	Pi-term group
Dependent variables				
Collection efficiency	η	N/A		η
Independent variables: geometry				
Particle diameter	d(particle)	M	Particle velocity	*
Pitot tube inner diameter	d(tube)	M	Collection efficiency	$\frac{d(tube)}{d(particle)}$
Independent variables: material property				
Viscosity of air	μ(air)	M*T^(-1)*L^(-1)	Particle inertia	$\frac{\mu(air)}{\rho(air)v(air)d(particle)}$
Velocity of air	v (air)	L*T^(-1)	Particle inertia	*
Density of air	ρ(air)	M*L^(-3)	Air characterizatio n	*
Velocity of particle	v(particle)	M*L(-1)	Particle trajectory description	$\frac{v(partilce)}{v(air)}$
Density of particle	ρ (particle)	M*L^(-3)	Collection efficiency	$\frac{\rho(particle)}{\rho(air)}$
Independent variables: external force				
Time of flying	t	T	Time*velocity =trajectory	$\frac{T * v(air)}{d(particle)}$
Gravity acceleration	g	L*T^(-2)	Affect particle trajectory	$\frac{g * d(particle)}{v_{(air)}^2}$

Denotation: M=mass, L=length, T= time, *repeating variables.

3.2 Pi-Terms

The pi-term generation was generated with Buckingham's pi theorem. Three basic dimensions: mass, length, and time were all involved in this problem, therefore we have three repeating variables d(droplet), v(air) and ρ(air). They were used as repeating variables because they have less complicated dimensions and they are independent. Using one of the variables, μ(air) as an example, we show here the procedure to generate dimensionless pi-terms using the non-repeating variables:

1.) Variables:

$$\mu(\text{air}) ==> \frac{\mu(\text{air})}{\rho(air)} ==> \frac{\mu(\text{air})}{\rho(air)v(air)} ==> \frac{\mu(\text{air})}{\rho(air)v(air)d(particle)}$$

2.) Corresponding Dimensions for the Variables above:

$$M * T^{-1} * L^{-1} ==> \frac{M * T^{-1} * L^{-1}}{M * L^{-3}} ==> \frac{T^{-1} * L^{2}}{L * T^{-1}} ==> \frac{L}{L} ==> 1$$

In this process, the dimensions of the original non-repeating variable was changed into a pi term by multiplying by a power (depends on the given non-repeating variable) of the repeating variables until the term is dimensionless, and becomes a "pi-term".

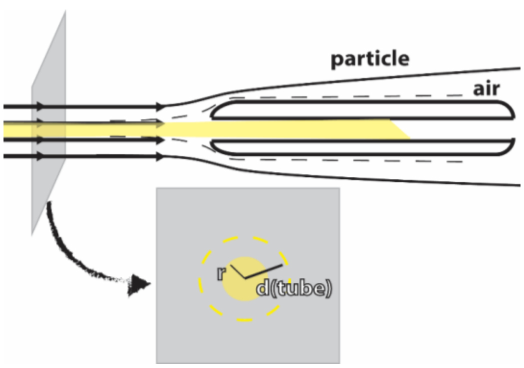


Figure 1. Illustration of water particle and air streamline and collection efficiency.

1.3 List of pi-terms

$(1)\ \frac{\rho(particle)}{\rho(air)}$ $(2)\ \frac{\mu(air)}{\rho(air) * v(air) * d(particle)}$ $(3)\ \frac{d(tube)}{d(particle)}$

$(4)\ \frac{v(particle)}{v(air)}$ $(5)\ \frac{g * d(particle)}{v_{(air)}^{2}}$ $(6)\ \frac{t * v(air)}{d(particle)}$

$(7)\eta$

3.3 Significance of the Pi terms

Pi-term (1) $\frac{\rho(particle)}{\rho(air)}$:

This describes the relationship between the density of the Particle and the density of air. If this Pi term deviates from being >>1, its behavior will change in the vertical direction. While this pi term is >>1, buoyant forces are negligible on the droplet, and it will accelerate downward with the expected [acceleration = 9.8-(negligible buoyant force)/m+outside forces/m]. As this term approaches 1, the droplet will have neutral buoyancy, and will be suspended in air as the buoyant force will counter gravitational acceleration: [9.8= buoyant force/m]. Lastly, as the term approaches 0, the buoyant force will overcome the gravitational acceleration and the particle will want to accelerate upwards.

Pi-term (2) $\frac{\mu(air)}{\rho(air)*v(air)*d(particle)}$:

This term is commonly denoted Reynolds number, however in our derivation of it, we found an inverted version of it (Re^-1). This term as it is presented relates the viscous forces on an object in a fluid flow, seen as the viscosity on the top of the fluid which is being traveled through (in this case, air), and the inertial forces on the object through the fluid, which is seen on the bottom, relating the density, velocity and diameter of the droplet. As this figure is presented, if it's value >>1, then the viscous forces will be observed to have a greater effect on the particles motion than the particles own inertial forces. Conversely, if the value is <<1, then the particles motion will have a more observable effect due to the inertial forces upon it.

Pi-term (3) $\frac{d(tube)}{d(particle)}$:

Easy enough to observe, it's apparent that as this term approaches one from infinity, and likewise approaches zero from one, the experiment becomes less and less physically meaningful, as it become more and more like taking a squirt gun and shooting the nozzle of the pitot tube. Indeed, in order for this experiment to find the meaningful data it seeks, it needs to have this term be >>1. Then the particles interaction with the pitot tube becomes similar to the way water particles will affect an actual pitot tube on an aircraft flying in the atmosphere.

Pi-term (4) $\frac{v(paritlce)}{v(air)}$:

This term changes as the particle flies through the flow field close to the pitot tube. Naturally, as the velocity of the air increases, the drag force on the particle will increase proportionally to it. The equation of drag that we used uses a difference of velocity rather than a ratio, however, this value still has some meaning with respect to the drag force apparent on the particle.

Pi-term (5) $\frac{g * d(particle)}{v_{(air)}^2}$:

This more obscure pi term relates the gravitational acceleration times the diameter of the droplet to the velocity of the air squared. This particular term provided a puzzling question, whether or not gravity could be ignored in our model. We ended up running two scenarios, one where gravity was taken into consideration, and the other where gravity was not (See table 3). The results of the two different approaches are discussed in the review section.

Pi-term (6) $\frac{\mathrm{t * v(air)}}{\mathrm{d(particle)}}$:

This term provides another scaling relationship between the time, the velocity of the air, and the diameter of the water droplet, our group couldn't think of any physical significance outside of this point.

Pi-term (7) η:

This is our primary dependent variable, described above in the document as Collection efficiency, it is what we are looking to find, while varying the diameter of the particle for our experiment.

4. Equation of motion for representative components of the system

4.1 Free-body diagram and equation of motion

To determine relevant pi-terms, we analyzed, and developed a force balance equation for a single droplet in the flow field. The free body diagram is shown in Figure 2. The particle is affected by a drag force and gravity. Drag forces can be expressed in x and y direction ($F_{drag,x}$ and $F_{drag,y}$; these are replaced below by the Stokes drag equation) and gravity only affects the droplet in the y direction, which gives us two equations of motion (equation (1) and (3)), the mass was express as a function of density and diameter due to the spherical droplet assumption. Velocity of particle was substituted by dx/dt and dy/dt for the respective dimensions, and acceleration was substituted by d(dx/dt)/dt and d(dy/dt)/dt (equation (2) and (4)).

> x direction:

$$\frac{dv(particle,x)}{dt} = \frac{18\mu}{\rho_p d^2(particle)}(v(air,x) - v(particle,x)) \quad (1)$$

$$\frac{d^2x}{dt^2} = \frac{18\mu}{\rho_p d^2(particle)}(v(air,x) - \frac{dx}{dt}) \quad (2)$$

> y direction:

$$\frac{dv(particle,y)}{dt} = \frac{18\mu}{\rho_p d^2(particle)}(v(air,y) - v(particle,y)) + g \quad (3)$$

$$\frac{d^2y}{dt^2} = \frac{18\mu}{\rho_p d^2(particle)}\left(v(air,y) - \frac{dy}{dt}\right) + g \quad (4)$$

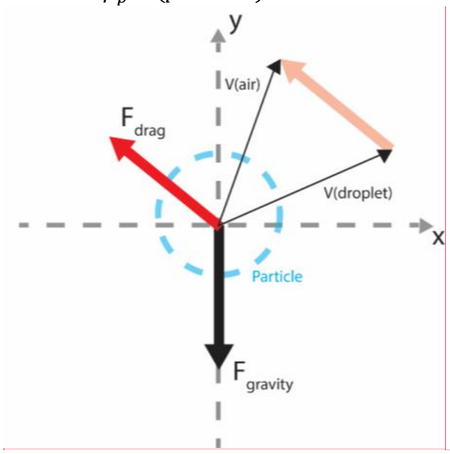


Figure 2. Free-body diagram of a water droplet in motion, drag force was shown as faded red arrow to illustrate that the direction is the same as the difference between v(air) and v(particle).

Commented [MW1]: V(particle)

4.2 Non-dimensionalization of equations of motion

To non-dimensionalize the equation, we first need to use the variables shown in table 1 to non-dimensionalize x, y and t. After this process they will turn into x*, y* and t* without dimension. Then we use these variables to calculate dimensionless derivatives.

$$x^* = \frac{x}{d(particle)} \qquad y^* = \frac{y}{d(particle)} \qquad t^* = \frac{t}{(\frac{d(particle)}{v(air)})}$$

1.) x direction:

$$\frac{dx^*}{dt^*} = \frac{d(particle)}{d(particle)*v(air,x)} * \frac{dx}{dt} \qquad \frac{d^2x^*}{dt^{*2}} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2(particle)}{d(particle)*v^2(air,x)} * \frac{d^2x}{dt^2}$$

2.) y direction

$$\frac{dy^*}{dt^*} = \frac{d(particle)}{d(particle)*v(air,y)} * \frac{dy}{dt} \qquad \frac{d^2y^*}{dt^{*2}} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d^2(particle)}{d(particle)*v^2(air,y)} * \frac{d^2y}{dt^2}$$

Equation (2) and (4) can be expressed using these dimensionless derivatives, they turn into:

$$(5) \qquad \frac{d^2x^*}{dt^{*2}} = \frac{18\mu}{\rho_p d^2(particle)}\left(v(air,x) - v(air,x) * \frac{dx^*}{dt^*}\right) * \frac{d^2(droplet)}{d(particle) * v^2(air,x)}$$

$$(6) \qquad \frac{d^2y^*}{dt^{*2}} = \frac{18\mu}{\rho_p d^2(particle)}\left(v(air,y) - v(air,y) * \frac{dy^*}{dt^*}\right) * \frac{d^2(droplet)}{d(particle) * v^2(air,y)} + g$$

Reorganize the equations gives:

$$(7) \qquad \frac{d^2x^*}{dt^{*2}} = \frac{18\mu}{\rho_p * d(particle) * v(air)} - \frac{dx^*}{dt^*} * \frac{18\mu}{\rho_p * d(particle) * v(air)} = \frac{18}{Re}(1 - \frac{dx^*}{dt^*})$$

$$(8) \qquad \frac{d^2y^*}{dt^{*2}} = \frac{18\mu}{\rho_p * d(particle) * v(air)} - \frac{dy^*}{dt^*} * \frac{18\mu}{\rho_p * d(particle) * v(air)} + \frac{g * d^2(particle)}{v^2(air)} = \frac{18}{Re}\left(1 - \frac{dy^*}{dt^*}\right) + \frac{g * d(particle)}{v^2(air)}$$

Representative pi-terms are the coefficients of the derivatives or constants, which are the Reynold's number and $\frac{g*d(particle)}{v^2(air)}$ (marked red in equation (7) and equation (8)).

5. Similarity Requirements and model scale

To determine the parameters in the scaling model, we need to compare the Pi-terms of scaling model and those of prototype. The parameters of prototype and scaling model is shown in table 2, the unknown parameters for the scaling model are left blank.

To start, we first determined the operating temperature of scaling model, which affects the viscosity of air. The tentative temperature is 293.15 K (20^{o}C), at this temperature the viscosity of air is 1.82 *10e-5 (kg/m*s). Using Pi-term (2) and setting the velocity of air as 50 m/s, the d(droplet) was calculated to be 4.33-173 µm. Then using Pi-term (3) the inner and outer diameter of pitot tube were calculated to be 8.68 mm and 21.7 mm. Using Pi-term (1), the density of particle was calculated to be 3000 kg/m^3, the potential materials applicable are glass, fluorite and aluminum, which have density around 3000 kg/m^3.[1](Table 3).

Table 2. Parameters known for prototype and scaling model.

Parameter	Prototype	Scaling model
d(droplet)	2-80µm	?
d(tube)	0.0004m (inner) 0.001m (outer)	?
µ(air)	1.4*10e-5(kg/m*s)	?
v(air)	250 m/s	50 m/s (max)
v(droplet)	250 m/s	50 m/s (max)
ρ (particle)	0-0.4g/m^3	?
ρ(air)	0.4 kg/m^3	1.2 kg/m^3

Table 3. Parameters for prototype and scaling model.

Parameter	Prototype	Scaling model (Pi term 5 not included)	Scaling model (Pi term 5 included)
d(droplet)	2-80	4.33-173.3 µm	1.1-43.96 µm
d(tube)	0.0004m (inner) 0.001m (outer)	0.00868m (inner) 0.0217m (outer)	0.0022m (inner) 0.0055m (outer)
µ(air)	1.4*10e-5(kg/m*s)	1.82*10e-5(kg/m*s)	1.82*10e-5(kg/m*s)
v(air)	250 m/s	50 m/s	185.3277 m/s (max)
v(droplet)	250 m/s	50 m/s	185.3277 m/s (max)
ρ (particle)	1000 kg/m^3	3000 kg/m^3	3000 kg/m^3
ρ(air)	0.4 kg/m^3	1.2 kg/m^3	1.2 kg/m^3

Commented [MW2]: The method may be not right, we should rescale g, not v, since Vmax is 50m/s, we can not set it to 185.3277. I think what we do is we need to rescale g, and calculate the collection efficiency. Since there is not much difference after rescale, we know that gravity could be neglected since it does not make a difference and the 5 pi-term is not useful

Commented [RM3R2]:

Figure 3 describes the process to determine unknown parameters in scaling model using balance Pi-term and known parameter in prototype and scaling model.

[1] W.E. Forsythe, Smithsonian Physical Tables, 9th revised, The Smithsonian Institution, 1956, p.292.

Known

$$\frac{\mu(air)}{\rho(air) * v(air) * d(particle)} = \frac{\mu(air)}{\rho(air) * v(air) * d(droplet)}$$

Known Known

Known

Fixed Can be determined

Figure 3. Determining parameter for scaling model using Pi-terms. Left side of the equation is the Pi-term for scaling mode, and on the right hand side, for prototype.

**It is important that the pitot tube size calculated using the Pi-terms would not cause the Venturi effect [where the decreased area due to the pitot tubes cross section would cause a pressure drop and velocity spike, creating errors] (Figure 4). in the wind tunnel, therefore we examine the deviation of wind speed caused by pitot tube using equation (9) where A represents cross section area of the pitot tube or wind tunnel. The deviation was calculated to be 0.41%, which provides adequate accuracy.

$$(9) \qquad Deviation = \frac{A(tube)}{A(channel)} * 100\%$$

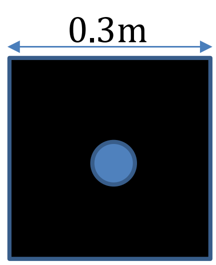


Figure 4. Blockage of wind tunnel (black) by pitot tube (blue)

6. Analysis of results

We've found through the Matlab simulation that the size of the droplet does have a significant effect on the collection efficiency, as can be noted by the graphs below in figure 4

Commented [MW4]: I think the right collection efficiency from Kiger is around 78%-100%?(not sure)

Prototype Collection efficiency   Scaled Collection efficiency

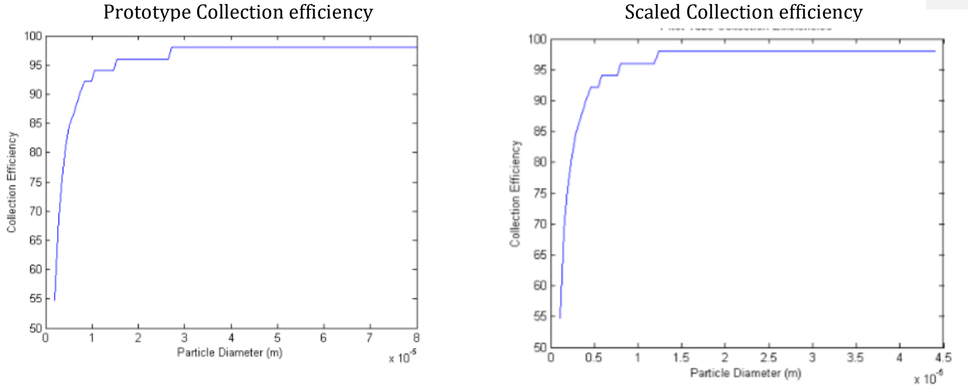


Figure 4 (left): Prototype collection efficiency (%) vs Particle Diameter (m) (right): Scaled Collection efficiency (same dimensions). The pi term for gravity was used to scale the iterations used to produce these graphs.

The Scaled Collection efficiency matches the Prototype collection efficiency to within .0014% averaged for each tested particle value, which makes sense, as the Scaled collection efficiency follows the same governing equations as the code used to calculate the prototype values, only with its own values scaled down using pi-terms.

One issue which we encountered was that when we neglected gravity, we had to scale the initial velocity of the air up to a value which the wind tunnel could not handle, to work around this issue, we considered removing Pi term #5, which related the velocity to gravity. This permitted us to achieve velocities within the range permitted by the wind tunnel design, the results of which produced similar graphs to the ones above, except that the average deviation from the prototype Collection efficiency was .0006 in this case (See Figure 5 next page).

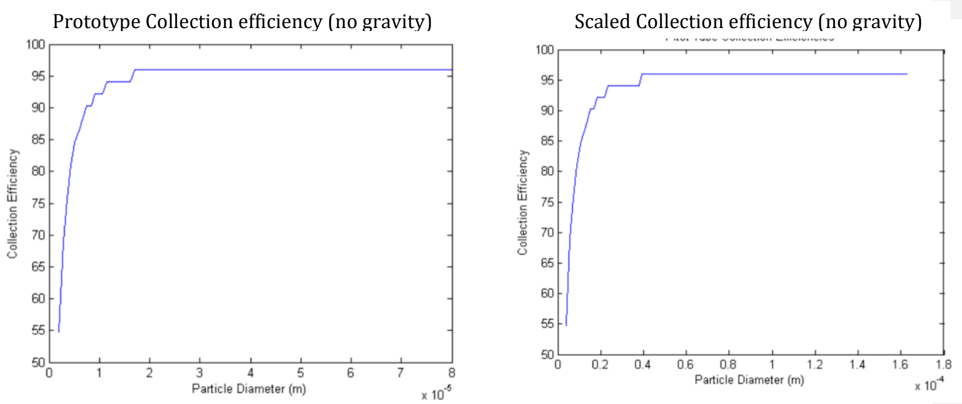


Figure 5 (left): Prototype collection efficiency without the effects of gravity (%) vs Particle Diameter (m) (right): Scaled Collection efficiency without the effects of gravity (same dimensions).

We can also see the effects that the particle's diameter had on the trajectories of the particles. Comparing the two extremes, we see that a larger particle will be more resistant to the change in its velocity (see figures 6a and 6b)

Commented [MW5]: Use the particle diameter from 4.3-173 to get the right graph

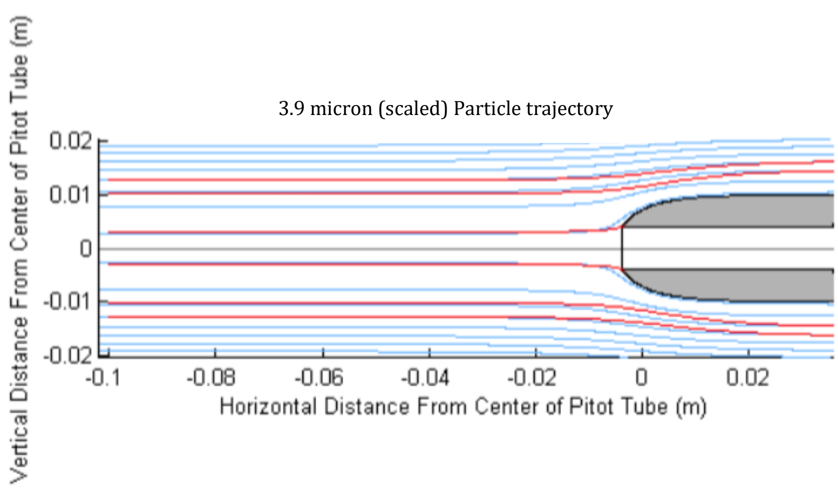


Figure 6: The smallest scaled value for the particle diameter size produces pathlines that curve noticeably around the pitot tube.

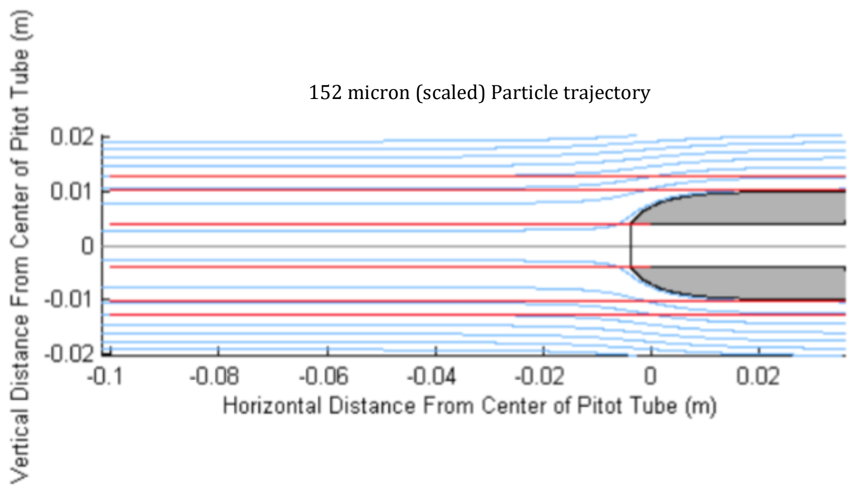
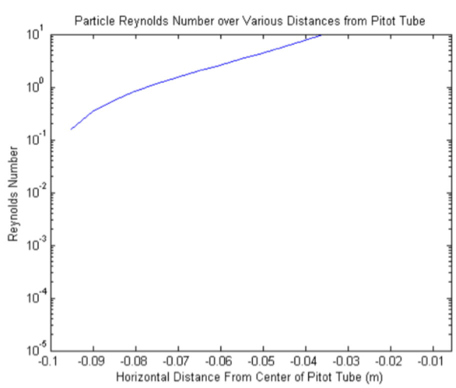
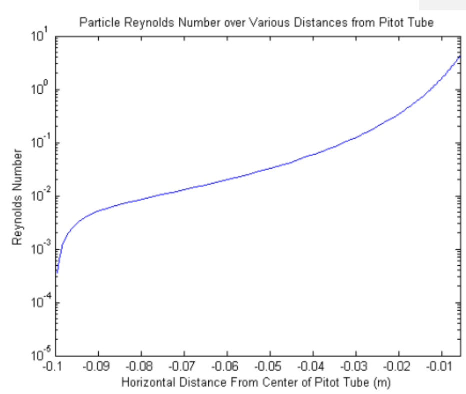


Figure 6b: The largest scaled value for the particle diameter size. The pathlines barely react to the change in the flow field at all when they reach this size.

Taking these results into consideration, we can observe how the Reynolds number changes for the two extreme cases in our experiment.



152 micron (Scaled, no gravity)



3.9 micron (Scaled, no gravity)

Figure 7: Reynolds number vs distance from pitot tube opening

7. Conclusion

We can conclude from our analysis several things. First, that when modeled in such close proximity to the pitot tube opening, the Pi-term used to relate gravity can be safely ignored, as it has a minor effect on the collection efficiency. When we take gravity out of consideration, we become able to perform the scaled test in the provided wind tunnel environment, within its specifications. This is important primarily because the cost implications of needing to use a larger facility, or worse, to have to design and test an actual prototype in the setting with which it is meant to operate are steep. Ignoring gravity, and using a different material for the particle, we are able to scale the system down to fit inside our provided tunnel. Another perk of this particular scale, is that we can use solid particles instead of liquid ones, that can help us ensure the assumptions of spherical particles, and lead to more accurate experimental values. However, the accuracy of using a scaled model without gravity is slightly decreased, as seen in the decrease in collection efficiency for both the prototype and model situation. So when looking for data, one must choose whether they want to spend more money and have more accurate results, or if the collection efficiency can have a margin of error reasonable enough to warrant saving the money.

The simulation results came out in such a way that the model scaled trial mirrors the prototype in their respective scales, both with and without the gravity scaling. Again, the issues lies in that while the scale models almost perfectly match the prototypes, the prototypes do not match perfectly when gravity is taken out of the picture. However, while the numbers do not perfectly match up between including and not including the gravity pi-term, the shapes of the graphs, and how they react are very similar. So if one were to simply study the effect the droplet size has on the collection efficiency, the gravityless model would suffice, while if one were to try to find exact quantities of ice flux into an actual prototype, we would recommend using the more accurate methods of finding that quantity.