MATH 135: Introduction to the Theory of Sets

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Chapter 1

Introduction

1.1 August 25

1.1.1 Introduction

Foundations of Mathematics: language, axioms, formal proofs

- We focus on the axioms in set theory
- We use ZFC (Zermelo-Fraenkel + Choice)
- There is only one primitive notion : $\in$
- Within the ZFC universe, everything is a set

Course Outline:

- Basic axioms
- Operations, relations, functions
- $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
- carindals
- AC
- ordinals

1.1.2 Basics

Principle of Extensionality: Two sets A, B are the same $\leftrightarrow$ they have the same elements $\forall x(x \in A \leftrightarrow x \in B)$
Example 1.1.1. $2, 3, 5 = \{5, 2, 4\} = \{2, 5, 2, 3, 3, 2\}$

Definition 1.1.2. There is a set with no elements, denoted $\varnothing$

- $\varnothing \neq \{\varnothing\}$
- $A \subseteq B$: A is a subset of B $\leftrightarrow$ each element of A is in B (use $\subsetneq$ to denote proper subset)

- $\{2\} \subseteq \{2,3,5\}$ but $\{2\} \notin \{2,3,5\}$
- Power set opertaion: $\mathcal{P}(A) = \{B \mid B \subseteq A\}$

We can define a hierarchy:
$V_0 = \varnothing$, $V_1 = \mathcal{P}(\varnothing) = \{\varnothing\}$, $V_2 = \mathcal{P}\mathcal{P}(\varnothing) = \{\varnothing, \{\varnothing\}\}$
$V_3 = \mathcal{P}(V_2) = \{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$, $V_4, \ldots$
$V_\omega = \bigcup_{n \in \mathbb{N}} V_n$, $\mathcal{P}(V_\omega)$, $\mathcal{P}\mathcal{P}(V_\omega), \ldots, V_{\omega+\omega}, \ldots, V_{\omega+\omega+\cdots}, \ldots, V_{\omega\times\omega}, \ldots, V_{\omega^\omega}$