The Rising Sea: Vakil

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Exercises

Chapter 1

Just Enough Category Theory to be Dangerous

1.1 Motivation

1.2 Categories and Functors

Exercise 1.2.1. A category in which each morphism is an isomorphism is called a groupoid.

(a) A perverse definition of a group is: is a groupoid with one object. Make sense of this.

(b) Describe a groupoid that is not a group.

Exercise 1.2.2. If A is an object in the category $\mathcal{C}$, show that the invertible elements of $\mathrm{Mor}(A, A)$ from a group. What are the automorphism groups of a set X and a vector space V.

Exercise 1.2.3. Let $(\cdot)^{\vee\vee} : f.d.Vec_k \to f.d.Vec_k$ be the double dual functor from the category of finite dimensional vector spaces over k to itself. Show that $(\cdot)^{\vee\vee}$ is naturally isomorphic to the identity functor on $f.d.Vec_k$.

Exercise 1.2.4. Let $\mathcal{V}$ be the category whose objects are the k-vector spaces k^n for $n \geqslant 0$ and whose morphisms are linear transformations. Show that $\mathcal{V} \to f.d.Vec_k$ gives an equivalence of categories by describing an "inverse" functor.

1.3 Universal Properties Determine an Object up to Isomorphism

1.4 Limits and Colimits

1.5 Adjoints

1.6 An Introduction to Abelian Categories