Ideal Learning Progression

$\star$ starred bullets are graded, all bullets are expected

• IN CLASS = work with peers, all resources allowed

★ HOMEWORK = individual work, all resources allowed, go to office hours, seek tutoring, get crucial help you need

• QUIZ YOURSELF = individual work, no notes, calculator ok, work hard on developing independence by seeking out extra textbook problems based on your personal weaknesses

★ PRE-TEST QUIZZES = individual work, no notes, calculator ok, work harder on developing independence

• EXAM REVIEW PROBLEMS = individual work, no notes, calculator ok, work hardest on developing independence by simulating exam scenario with **no resources, only a blank sheet of paper**

★ EXAMS = individual work, no notes, no calculator, **no resources, only a blank sheet of paper**

Calculus II (All Exams) Review Problems

always show all work

Below is NOT a list of exact exam problems! It's a list of topics and possibilities to jog your memory.
Exams are created by: modifying functions used, changing up algebra needed, using negatives instead of positives or vice versa, using other possible modifications that we covered in classes, on homeworks, on quizzes, etc..

WARNING: Although sections are listed below, they are NOT listed on the exam! Make sure you can do the problems without knowing what section it comes from!

SECTION 4.9

1. Find the following indefinite integrals.
   (a) $\displaystyle\int \frac{1}{1+9x^2}dx$
   (b) $\displaystyle\int \frac{1}{\sqrt{1-9x^2}}dx$
   (c) $\displaystyle\int \csc(9x)\left(\cot(9x)-\csc(9x)\right)dx$

SECTIONS 5.1-5.3

2. Consider the definite integral $\displaystyle\int_0^{\pi/2} \cos^4(x)dx$.
   (a) Estimate the integral using a right-handed Riemann sum with $n = 4$.
   (b) Estimate the integral using the Midpoint Rule with $n = 4$.
3. Consider the definite integral $\displaystyle\int_0^4 \frac{x^3-x}{x}dx$
   (a) Draw the area corresponding to the definite integral.
   (b) Estimate the definite integral using a left-handed Riemann sum with $n = 4$.
   (c) Find the exact value of the definite integral using the Fundamental Theorem of Calculus.
4. Consider the definite integral $\displaystyle\int_0^{\pi/4} \tan(x)dx$.
   (a) If you used a right-handed Riemann sum to estimate this definite integral, would it result in an overestimate or an underestimate of the actual value?
   (b) If you used a left-handed Riemann sum to estimate this definite integral, would it result in an overestimate or an underestimate of the actual value?

SECTION 5.2

5. Draw the region indicated by the definite integral, and use basic geometry to evaluate it.

(a) $\int_0^1 \sqrt{1-x^2}dx$

(b) $\int_0^3 (x-1)dx$

(c) $\int_{-3}^0 \left(1+\sqrt{9-x^2}\right) dx$

(d) $\int_0^5 |2x-4|\, dx$

(e) $\int_{-4}^4 f(x)dx$ where $f(x) = \begin{cases} x & \text{for } x \leq 0 \\ x+1 & \text{for } x > 0 \end{cases}$

SECTIONS 4.9 & 5.3

6. Find the following indefinite integrals.

(a) $\int \frac{\sqrt{x}+1}{x}dx$

(b) $\int (x^2-3)(x+4)dx$

(c) $\int \left(2\csc^2(4x) + \frac{e^{-4x}+x}{4}\right) dx$

(d) $\int \frac{3}{10+10x^2}dx$

(e) $\int \frac{\sqrt{9-9x^2}-3}{\sqrt{9-9x^2}}dx$

SECTION 5.4

7. Find the value of the definite integral $\int \frac{x^3\cos(x)}{x^6+3}dx$.

8. Find the value of the definite integrals $\int_{-\pi/3}^{\pi/3} \frac{x\sec(x)}{\sec^2(x)+1}dx$

9. Use the trig. identity $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$ in order to find $\int_0^{\pi/6} \cos^2(3x)dx$.

10. Find the average value of $f(x) = \sec^2(x)$ on the interval $\left[0, \frac{\pi}{4}\right]$.

11. Find the average value of $f(x) = x^2$ on the interval $[0,1]$.

SECTION 6.1

12. An object moves according to the acceleration function $a(t) = t+4$, has an initial velocity $v(0) = 5$ and the location at time $t = 1$ is $s(1) = 10$. Find the position function $s(t)$.

13. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)dt$ represent?

14. Water flows from a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute for $0 \leq t \leq 50$. Find the amount of water that has leaked out of the tank in the first ten minutes.

SECTION 5.5

15. Find the following indefinite integrals.

(a) $\int \frac{x}{1+9x^2}dx$

(b) $\int \frac{x}{\sqrt{1-9x^2}}dx$

(c) $\int \frac{5}{25+x^2}dx$

(d) $\int \frac{r^2}{r^3+1}dr$

(e) $\int \frac{1}{x\left(\ln(x)\right)^3}dx$

(f) $\int (1+\cos(t))^6 \sin(t)dt$

(g) $\int y^3(y^4+5)^6 dy$

(h) $\int x\cos(x^2)\sin(x^2)dx$

(i) $\int \frac{\sec^2(x)\tan(x)}{\sqrt{1+\sec^2(x)}}dx$

(j) $\int \frac{1}{2+6x+9x^2}dx$
Hint: Start by completing the square in the denominator, and then do a substitution.

16. Find the average value of $f(x) = \tan(x)$ on the interval $\left[0, \frac{\pi}{4}\right]$.

SECTION 6.2

17. Find the area of the region enclosed by the curves $y = x^2 - 2x$, $y = x + 4$.

18. Find the area of the region enclosed by the curves $x = 2y^2$, $x = 4 + y^2$.

19. Draw and find the area enclosed by $y = 0$, $y = \sqrt{x}$, $y = \sqrt{4-x}$.

SECTION 6.3

20. Find the volume of the solid obtained by rotating the region bounded by $y = 1$, $y = \sqrt{\sin(x)}$, $x = 0$ about the x-axis.s

21. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $x = 0$, $y = 8$ in the first quadrant about the y-axis.

SECTION 6.4

22. Use shells/cylinders to redo the previous problem.

23. Find the volume of the solid obtained by rotating the region bounded by $y = e^{x^2}$, $x = 0$, $y = e^9$ in the first quadrant about the y-axis. Use shells/cylinders.

SECTION 6.5

24. Find the length of the line $y = 3x + 2$ from $x = 1$ to $x = 5$ using (a) basic geometry, and then (b) the arclength formula. Be sure that you obtain the same answer for both parts (a) and (b).

SECTION 6.6

25. Find the surface area of the surface generated by rotating $f(x) = \frac{x^4}{8} + \frac{1}{4x^2}$ about the x-axis from $x = 1$ to $x = 2$.

SECTIONS 5.5, 7.2 - 7.3

26. Find the following indefinite integrals using the appropriate method.

(a) $\int \cos^7(t)\sin^3(t)dt$

(b) $\int \tan^7(w)\sec^4(w)dw$

(c) $\int (\pi x)^2 e^{4x}dx$

(d) $\int \frac{1}{2+6x+9x^2}dx$

(e) $\int e^{2x}\sin(x)dx$

(f) $\int s\sec^2(s)ds$

(g) $\int x^2\ln(x)dx$

(h) $\int x\sin^2(x)dx$

(i) $\int x^3\sin(x^2)dx$

SECTION 6.3 & 7.2

27. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{\frac{\pi}{2}}$, $y = \sqrt{\arcsin(x)}$, $x = 0$ about the x-axis.

28. Find the volume of the solid obtained by rotating the region bounded by $y = e^{x^2}$, $x = 0$, $y = e^9$ in the first quadrant about the y-axis.

SECTIONS 7.4 - 7.5

29. Find the following indefinite integrals using the appropriate method.

(a) $\displaystyle\int \frac{1}{x^2\sqrt{1-9x^2}}dx$

(b) $\displaystyle\int \frac{1}{1-9x^2}dx$

(c) $\displaystyle\int \frac{x^3}{\sqrt{1+9x^2}}dx$

(d) $\int \frac{5x^2+x+3}{x^3+x}dx$

(e) $\int \frac{x^3+4}{x^2+4}dx$
Note: PFD only works if the degree of the numerator is LESS than the degree of the denominator. For this problem begin by using polynomial division. Then use PFD on the resulting expression.

SECTION 7.8

30. Find the improper integrals.

(a) $\displaystyle\int_0^1 \frac{1}{x^3+x}dx$

(b) $\displaystyle\int_0^{\infty} te^{-5t}dt$

(c) $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}dx$

(d) $\displaystyle\int_e^{\infty} \frac{1}{x\,(\ln(x))^3}dx$

(e) $\displaystyle\int_{-1}^{1} \frac{e^x}{e^x-1}dx$

SECTIONS 8.3-8.4

31. Find whether each series converges or diverges. State clearly which test you are using and how you come to your conclusions.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n^2}+\frac{2}{n}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{5^n}{n^5}$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{e^n}{2^{n-1}}$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{3}{n^2+3n}$

(e) $\displaystyle\sum_{n=1}^{\infty}\frac{14n^2+2n+3}{11n^2+5}$

(f) $\displaystyle\sum_{n=0}^{\infty}\frac{1+2^n}{3^n}$

(g) $\displaystyle\sum_{n=0}^{\infty}(0.46)^{n-1}$

SECTIONS 8.4 - 8.6

32. Determine whether the following series converge or diverge. State clearly which test you are using and how you come to your conclusions.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1 + 2n + n^2}{n^2} \right)^{n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n) n^7}{8n^7 + 1}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n} + n^3}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{-5\ln(n)}{n^6}$

(f) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1}\ln(n)}{n}$

SECTION 9.1 - 9.3

33. Determine for which x values the series $\displaystyle\sum_{n=0}^{\infty} \frac{(2x-7)^{2n+1}}{3^n n!}$ converges.

34. Determine for which x values the series $\displaystyle\sum_{n=0}^{\infty} \frac{(2x-7)^{2n+1}}{3^n}$ converges.

35. Find the quadratic approximation for the function $f(x) = \ln(1-x)$ at $x = 0$ and use it to estimate $\ln(0.2)$.