Take additional textbook notes!

Ch1- Textbook Notes

8/31/16

Chapter 1

Section 1: The Distance and Midpoint Formulas

- Coordinate of the point - single real # assigned to point on the real # line
- x-axis - horizontal line
- y-axis - vertical line
- Origin - point of intersection between the x and y-axis
- this is a rectangular or Cartesian coordinate system
- plane formed by x and y-axis is the xy-plane, where the x and y axis are the coordinate axes
- Point can be located use an ordered pair (x, y), called the coordinates of P
- x value is the x-coordinate or abscissa
- y value is the y-coordinate or ordinate
- coordinate axes are divided into 4 quadrants
- distance formula
  - ↳ The distance formula provides a straightforward method for computing the distance between 2 points
  - ↳ The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$ is
    $$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
- midpoint of a line segment
  - ↳ The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is
    $$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Section 2: Graphs of Equations in Two Variable; Intercepts; Symmetry

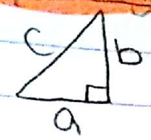
- an equation in two variable is a statement in which two expressions involving x and y are equal
- The points at which a graph crosses or touches an axis is called an intercept
- x-coordinate at which the graph crosses or touches the x-axis is an x-intercept

Make summary notes for each Chapter or Section!

8/31/16

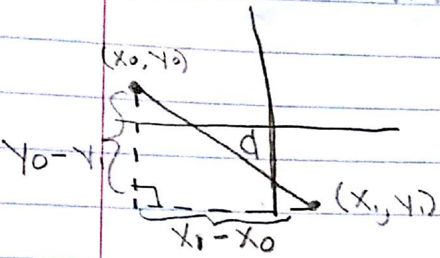
Chapter 1+2 Review

- **Pythagorean Theorem** (for right triangles)



$$a^2+b^2=c^2$$

- **Distance formula** for the distance between two points in the (x,y) plane



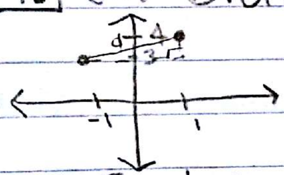
Distance formula

$(y_0-y_1)^2+(x_1-x_0)^2=d^2$

$d^2=(x_1-x_0)^2+(y_1-y_0)^2$

distance between (x_0,y_0) and (x_1,y_1) is $=d=\sqrt{(x_1-x_0)^2+(y_1-y_0)^2}$

1.) (a) Draw the points $(-1,3)$ and $(1,4)$



(b) Find the distance between $(-1,3)$ and $(1,4)$

$(x_1,y_1)=(-1,3)$, $(x_0,y_0)=(1,4)$

$$\begin{aligned} d &= \sqrt{(x_1-x_0)^2+(y_1-y_0)^2} \\ &= \sqrt{(-1-1)^2+(3-4)^2} \\ &= \sqrt{(-2)^2+(-1)^2} \\ &= \sqrt{4+1} \\ &= \boxed{\sqrt{5}} \end{aligned}$$

- **Equation of a circle**:

↳ All points (x,y) on a circle of radius r, where the circle is centered at (x_0,y_0), satisfy the following equation:

$$(x-x_0)^2+(y-y_0)^2=r^2$$

Make question sheet emphasizing hard homework problems!

[Appendix A Ch. 1 · 2] Review

8/31/16

$(x^a)^b = x^{ab}$ $\quad \left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

Questions from HW#1

7.) Similar Q. – Law of Exponents

$$\left(\frac{x^8 y^{-3}}{y^{-2} x^3}\right)^{-2} = \left(\frac{x^8 y^2}{x^3 y^3}\right)^{-2} = \left(\frac{x^5}{y}\right)^{-2} = \left(\frac{y}{x^5}\right)^2 =$$

$$\frac{y^2}{(x^5)^2} = \frac{y^2}{x^{10}}$$

Complete the Square

$$y = (x^2 - 5x) + \frac{25}{4} - \frac{25}{4} + 6$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-5}{2}\right)^2 = \frac{25}{4}$$

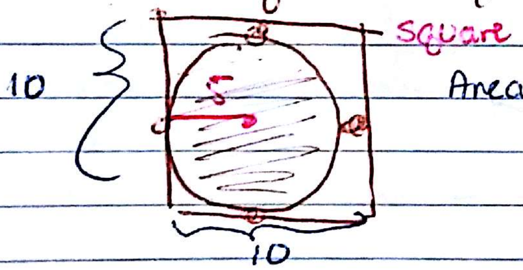
$$y = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 6$$

$$y = \left(x - \frac{5}{2}\right)^2 - \frac{1}{4}$$

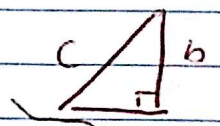
36.) $x^2 + bx + \underline{\ ?\ } - \underline{\ ?\ }$

This should be $\left(\frac{b}{2}\right)^2$

29.) Find the area of shaded region



Area of circle $= \pi r^2 = \pi (5)^2 = 25\pi$

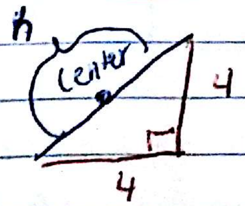


30.)

Area of circle $= \pi r^2 = \pi (2\sqrt{2})^2$

$= \pi \cdot 2^2 \cdot \sqrt{2}^2$ $\quad a^2 + b^2 = c^2$

$= \pi \cdot 4 \cdot 2 = \boxed{8\pi}$



Right triangles satisfy the Pythagorean Theorem.

$4^2 + 4^2 = h^2$ Solve for h:

$32 = h^2$

$\sqrt{32} = h$ $\quad \sqrt{16 \cdot 2} = h$

$4\sqrt{2} = h$ length of diagonal

Supply and Demand Functions

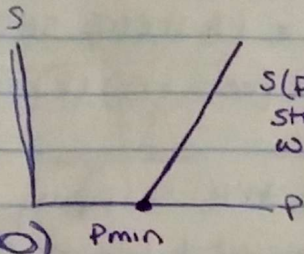
9/7/16

S(P) is a supply function

S = supply (# of items supplied to market)

P = price

Domain of S(P) function is $[P_{min}, \infty)$



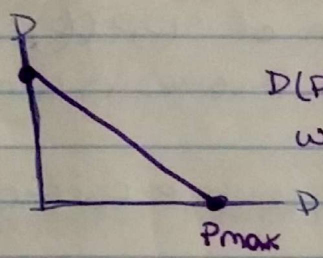
S(P) is a straight line w/ positive slope

D(P) is the demand function

D = demand (# items consumers want to buy)

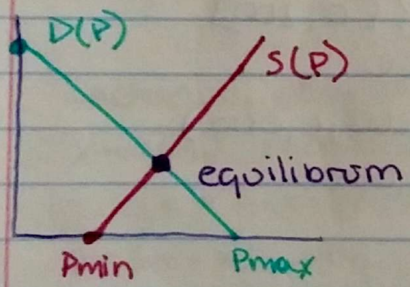
P = price

Domain of function is $[0, P_{max}]$



D(P) is a straight line w/ negative slope

Equilibrium price is the demand equals supply



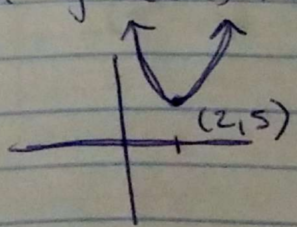
Use color for emphasis!

Review

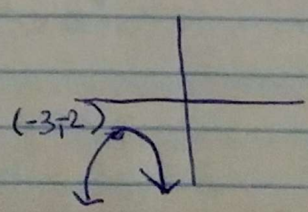
Completing the square on a quadratic polynomial ~~equation~~ allows you to graph it by shifting and finding the vertex of the polynomial which represents either max or min

Quadratic polynomial w/ square completed looks like the following: $y = a(x - x_0)^2 + y_0$

Ex $y = (x-2)^2 + 5$



Ex $y = -(x+3)^2 - 2$

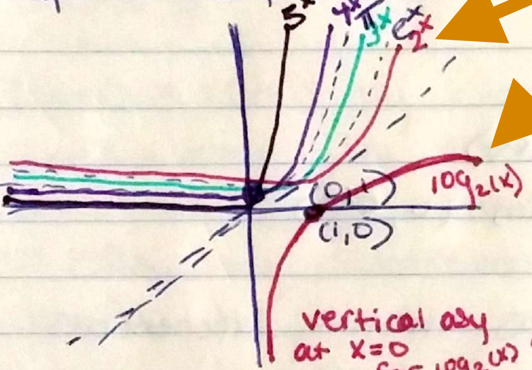


Exponential Functions

x	2^x	3^x	4^x	5^x
-3	1/8	1/27	1/64	1/125
-2	1/4	1/9	1/16	1/25
-1	1/2	1/3	1/4	1/5
0	1	1	1	1
1	2	3	4	5
2	4	9	16	25
3	8	27	64	125

Color coding makes related parts stand out!

Graph of exponential functions

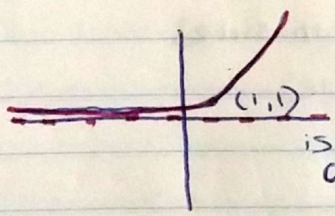


$e \approx 2.71$

$\pi \approx 3.14$

y=0 is a horiz asy.

consider the function $f(x) = 2^{x-1}$ Graph it.



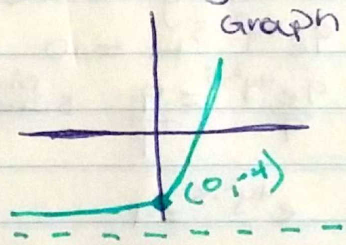
y=0 is still horiz asy.

a. Domain is $(-\infty, \infty)$

Range is $(0, \infty)$

consider the function $f(x) = 3^x - 5$

Graph



y=-5 is horiz. asy

a. Domain $(-\infty, \infty)$

Range is $(-5, \infty)$

Logarithms are inverse of exponentials.

$y = 2^x$

Take $\log_2$ of both sides

$\log_2(y) = \log_2(2^x)$ * inverses so they cancel out

$\log_2(y) = x$

Make formula sheets!

- Mid Point Formula: $m = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Area of a circle: $A = \pi r^2$
- Circumferance of a circle: $C = 2\pi r$
- Equation of a circle: $(x - x_0)^2 + (y - y_0)^2 = r^2$
  $(x_0, y_0) = $ center
- Area of a triangle: $A = 1/2\, b \cdot h$
- Pythagorean Theorem: $a^2 + b^2 = c^2$ * For right Δ's
- Distance Formula: $d^2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
- Surface Area of sphere: $4\pi r^2$
- Volume of a sphere: $V = 4/3\, \pi r^3$
- Surface Area of rectangle: $S = 2LH + 2(LW) + 2(WH)$
- Volume of a rectangle: $V = L \cdot W \cdot H$
- Differences of 2 squares: $x^2 - a^2 = (x-a)(x+a)$
- Point slope formula: $y - y_1 = m(x - x_1)$
- Parallel lines: have = slopes
- Perpendicular lines: $m_1 \cdot m_2 = -1$
- Slope of vertical line: undefined
- Slope of horozontal line: zero
- vertex: $-b/2a$
- $\left(\frac{b}{2}\right)^2$ - complete the square

Before each exam make a study sheet summarizing everything!

Exam 1 Study Sheet

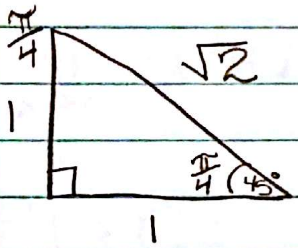
Trigonometry

SOH CAH TOA

$sec = \frac{H}{A}$

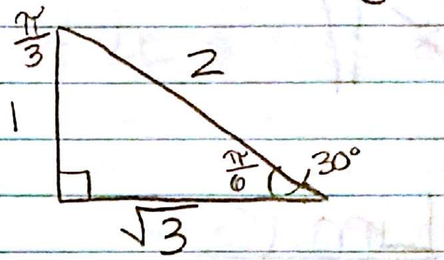
Pythagorean Theorm: $a^2 + b^2 = c^2$

45°-45°-Right



$1^2 + 1^2 = \sqrt{2}$

30°-60°-Right



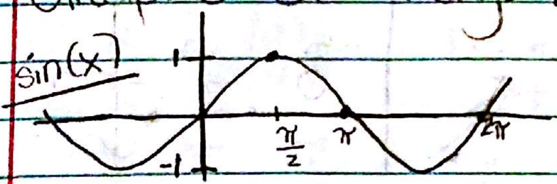
Trig. Identities: (Pythagorean identities)

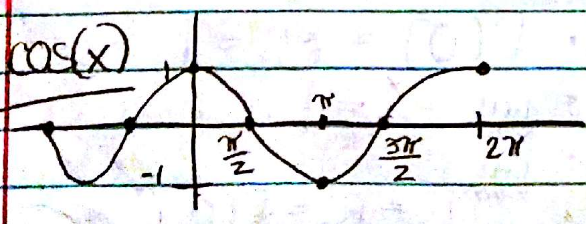
$\sin^2\theta + \cos^2\theta = 1$

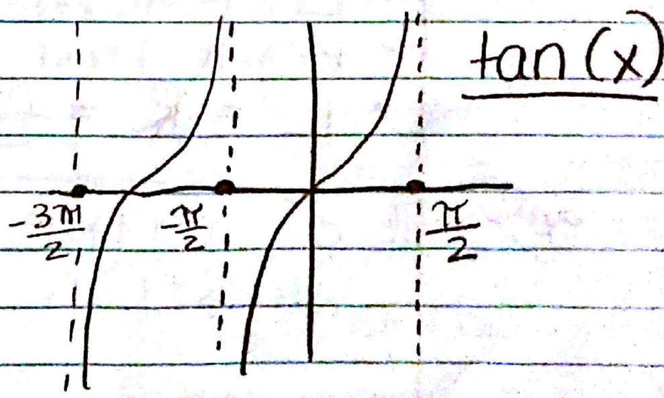
$1 + \cot^2\theta = \csc^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

Graphs of Trig. Functions:

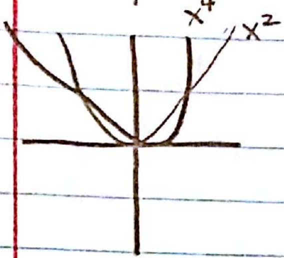


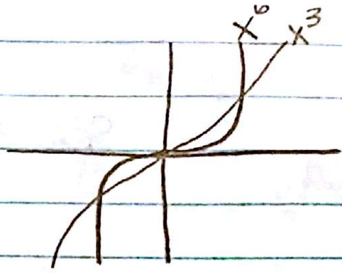




Graphs to know!

even degree





odd degree

Shifting

$x^2 - 5$ shift ↓

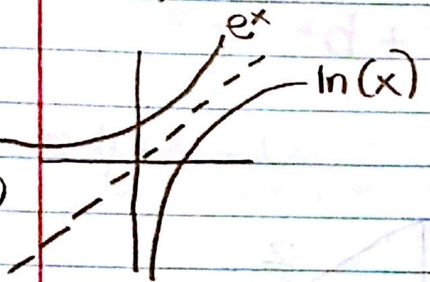
$x^2 + 5$ " ↑

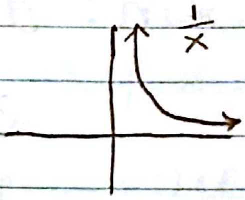
$(x-5)^2$ " →

$(x+5)^2$ " ←

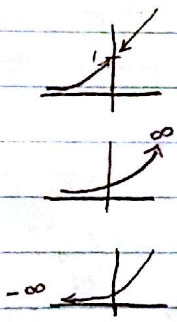
$e^0 = 1$

$\ln(1) = 0$





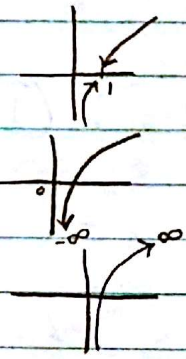
Limits



$\lim_{x\to 0} e^x = 1$

$\lim_{x\to \infty} e^x = \infty$

$\lim_{x\to -\infty} e^x = 0$



$\lim_{x\to 1} \ln(x) = 0$

$\lim_{x\to 0^+} \ln(x) = -\infty$

$\lim_{x\to \infty} \ln(x) = \infty$

Continuity Checklist

- $f(a)$ exists
- Left limit exists
- Right limit exists
- $L = R = f(a)$

$\frac{5}{0^+} \to +\infty$

$\frac{5}{0^-} \to -\infty$

$\frac{-5}{0^+} \to -\infty$

$\frac{-5}{0^-} \to \infty$

(divided by numbers approaching zero)

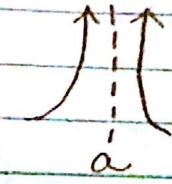
Example

Is e^x continuos @ $x = 0$?

- $f(0) = e^0 = 1$ exists✓
- $\lim_{x\to 0^-} e^x = 1$ exists✓
- $\lim_{x\to 0^+} e^x = 1$ exists✓
- $L = R = f(a)$ ✓

Asymtotes

- A vertical asymtote exists if



$\lim\limits_{x \to a} f(x) = \infty$ (not a finite #)

- A horizontal asymtote exists if

$\lim\limits_{x \to \infty} f(x) = L$ (must be a finite #)

Slopes

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$ = average rate of change on the interval $[x, x]$

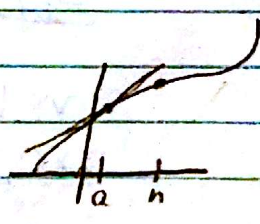
secant line - crosses graph twice $= \frac{\Delta y}{\Delta x}$

equation
$y = mx + b$
OR
$y - f(x) = m_{tan}(x-a)$

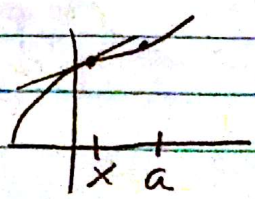
tangent line - touches graph once =
a.k.a the derivative of $f(x)$

Limit Definition of Derivatives

$f'(a) =$ #1: $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$



#2: $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

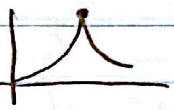


- $f(x)$ ± slopes
- $f'(x)$ ± heights

$f(x)$ is Differentiable if:

$$\lim_{x \to a^+} \frac{f(x)-f(a)}{x-a} = \lim_{x \to a^-} \frac{f(x)-f(a)}{x-a}$$

(slope of tangent line from the left) = (slope of tangent line from the right)



kink/cusp continuos, ~ differentiable
if $f(x) \sim C \to f(x) \sim D$ / if $D \to C$

Derivatives:

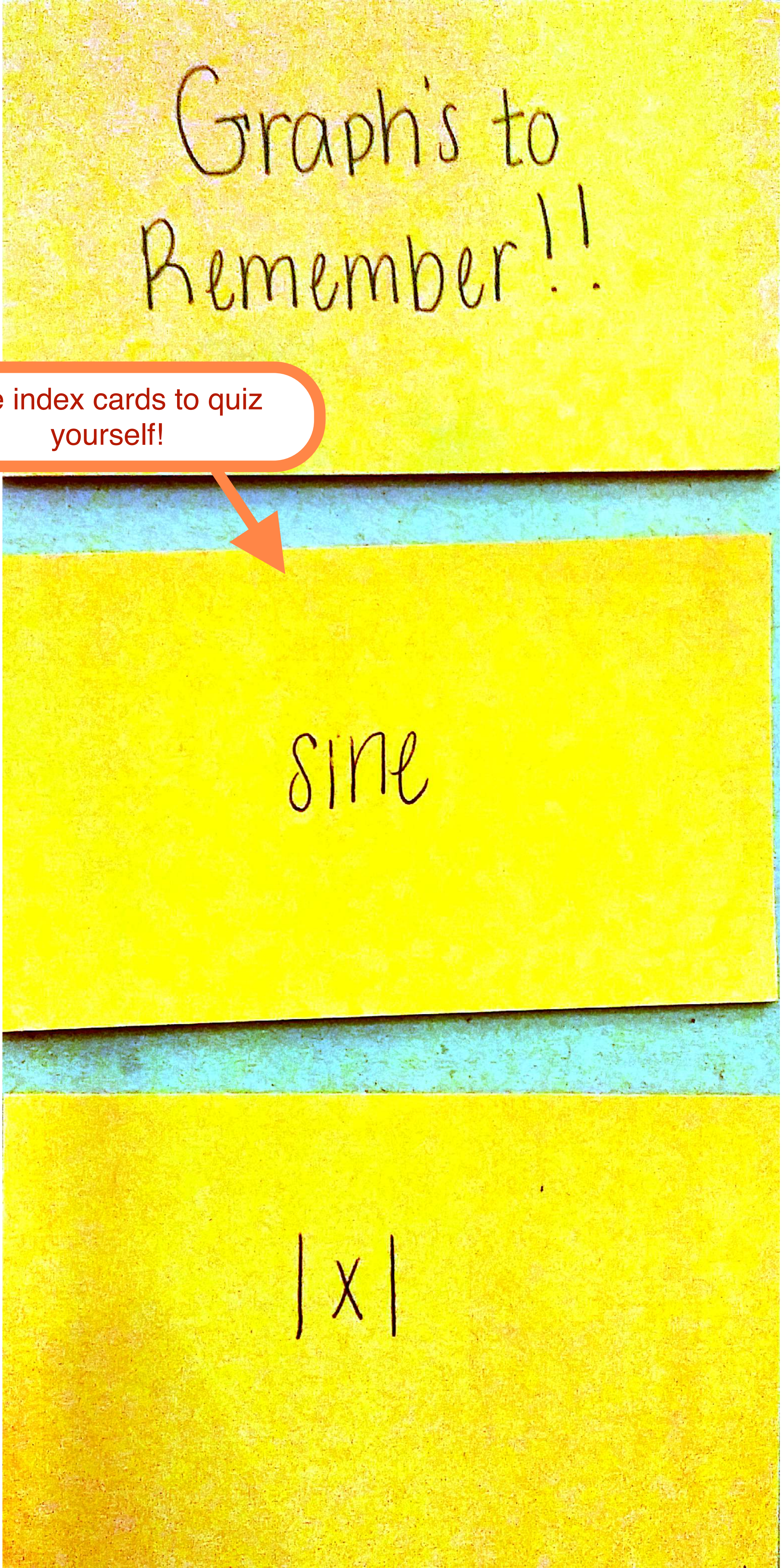
	$f(x)$	$f'(x)$
Power Rule	x^p	px^{p-1}
Constant	c	0
Linearity	$c \cdot f(x)$	$c \cdot f'(x)$
	$f(x)+g(x)$	$f'(x)+g'(x)$
	e^x	e^x

Physics Terminology

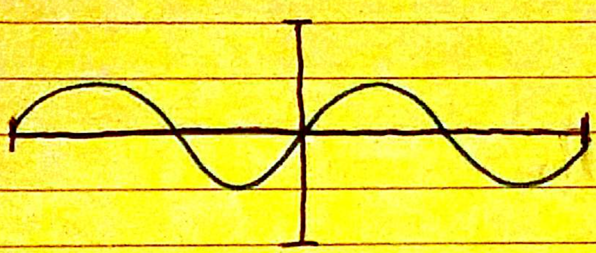
velocity = $f'(t)$
acceleration = $f''(t)$

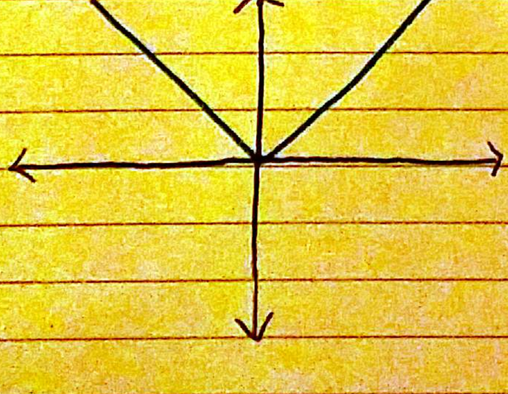
object stationary = $v(t)=0$ (highest point also)
object hit ground = $f(t)=0$

moving Right = $v(t)>0$
moving left = $v(t)<0$


Graph's to Remember!!
Use index cards to quiz yourself!
sine
|x|

Graph's to Remember!!





* e^x & $\ln(x)$ are inverses of each other

$$\lim_{x\to\infty} x^2 = \infty$$

$$\lim_{x\to\infty} x^4 = \infty$$

$$\lim_{x\to-\infty} x^3 = -\infty$$

$$\lim_{x\to\infty} \underbrace{-8}_{neg} \times \underbrace{x^{10}}_{pos} = -\infty$$

neg x pos = $-\infty$

$$\lim_{x\to\infty} \ln(x) = \infty$$

$$\lim_{x\to\infty} e^x = \infty$$

$$\lim_{x\to\infty} \cos(x) = \text{DNE}$$

$$\lim_{x\to 0} \sin(x) = 0$$

Use tabs to stay organized!

Study Sheet!

Study Sheet!

Test 9/28!

Calculus & Analytical Geometry
Chapters 2 & 3

equations to remember

$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ "Version 1"

$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ "Version 2"

$m = \frac{y_2 - y_1}{x_2 - x_1} \longrightarrow$ Slope

Shorthand Derivative Rules

$f(x)$	$f'(x)$
x^p	px^{p-1}
c	0
$c \cdot f(x)$	$c \cdot f'(x)$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$
a^x	$\ln(a) \cdot a^x$
$\log_a(x)$	$\frac{1}{\ln(a)\, x}$
$f(x)+g(x)$	$f'(x)+g'(x)$

Definition of Limits: $\lim_{x\to a} f(x) = L \to f(x)$ gets closer & closer to L as x gets closer & closer to a from both sides of a

Depends on value of f near a, not f(a)!

- An infinite limit occurs when function values ↑ or ↓ w/out bound near a point
- Limit at infinity occurs when independent variable x ↑ or ↓ w/out bound.

* if both the numerator & denominator are nonzero → just plug in value

* if denominator = 0 then lim approaches infinity

* numerator dominates → ∞, denominator dominates → 0

* if same highest powers in both num. & denom. then $\lim_{x\to\infty}$ = ratio of coefficients

Vertical Asymptotes: if $\lim_{x\to a^{\pm}} f(x) = \infty \longrightarrow$ must be dividing by 0

Horizontal Asymptotes: if $\lim_{x\to\infty} f(x) = L$

Slope: on a line, no matter what 2 pairs of points you pick, $\frac{\Delta y}{\Delta x}$ always gives you the same value of the slope m.

Slope in between 2 points, $\frac{y_1 - y_0}{x_1 - x_0}$ is called Average Rate of Change.

Slope of the tangent line at $x=a$ is called the Derivative at $x=a$

- f(x) is differentiable at $x=a$ if Version 1 & 2 limits exist

* if f(x) is NOT continuous at $x=a$, then f(x) is NOT differentiable

* if f(x) is differentiable at $x=a$, then f(x) is continuous at $x=a$.

- $f(x) \to f'(x)$ graphs: slope on f(x) = height on f'(x)

* positive slope on f(x) → positive height on f'(x)

* negative slope on f(x) → negative height on f'(x)

Physics terminology: $a(t) = v'(t) = p''(t)$

- $p'(t)$ represents $\frac{\Delta p}{\Delta t} \to$ velocity
- $p''(t) = \frac{d}{dt}(p'(t)) \to$ acceleration

Algebra Review

- $\ln(a \cdot b) = \ln(a) + \ln(b)$
- $\ln(\frac{a}{b}) = \ln(a) - \ln(b)$
- $\ln(a^b) = b\ln(a)$
- $\ln(e^x) = x$ — $e^0 = 1$
- $e^{\ln(x)} = x$ — $\ln(1) = 0$

Summary!

f(x) differentiable at x = # → Does slope match up with slope?

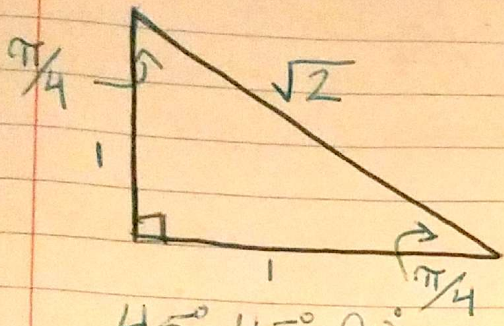
f(x) continuous at x = # → Does height match up with height?

- f(x) is differentiable at $x=a$ if $\underbrace{\lim_{x\to a^-} \frac{f(x)-f(a)}{x-a}}_{\text{slope of tan from left}} = \underbrace{\lim_{x\to a^+} \frac{f(x)-f(a)}{x-a}}_{\text{slope of tan from right}}$

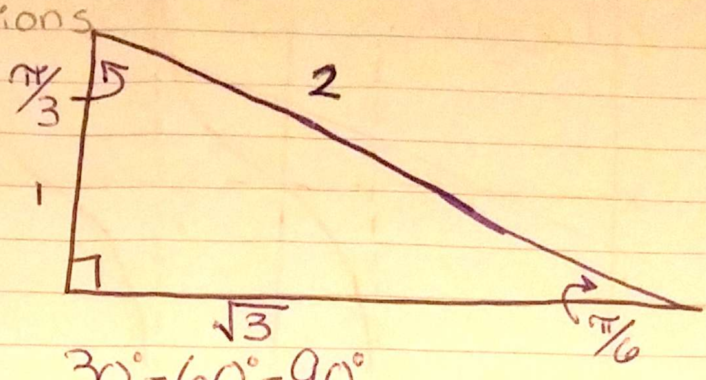
$\underbrace{\text{if } f(x) \neq \lim_{x\to a} f(x)}_{\text{Discontinuous}}$

Make study sheets reviewing the basics!

- More Values of Trig Functions



45°-45°-90°



30°-60°-90°

- Things to keep in mind:
  - Sin → csc
  - Cos → sec
  - Tan → cot
  - SOH-CAH-TOA

- $\sin = O/H$ • $\cos = A/H$ • $\tan = O/A$ • $\csc = H/O$ • $\sec = H/A$ • $\cot = A/H$

	0° 0	30° π/6	45° π/4	60° π/3	90° π/2
Sin(θ)	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos(θ)	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan(θ)	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	DNE
Cot(θ)	DNE	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0
Sec(θ)	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	DNE
csc(θ)	DNE	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1

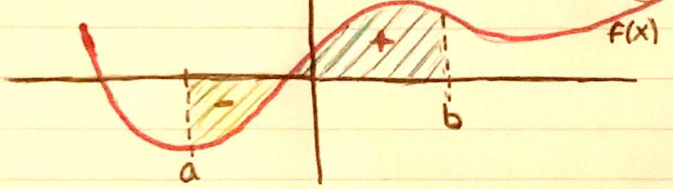
- Terminology
  - Definite Intergal = the net area between the x-axis and the graph of f(x) from x=a to x=b

$$\int_a^b f(x)\,dx$$

  - Indefinite Intergal = Simply antiderivatives of f(x) with respect to x.

$$\int f(x)\,dx = F(x)+C$$

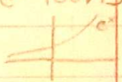
- Graph of Definite Intergal



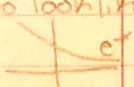
Color coding makes it easy to understand at a glance!

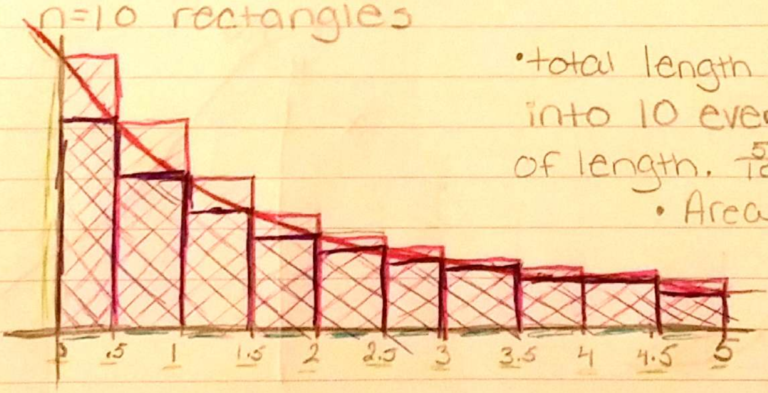
- Example
  - Estimate $\int_0^5 e^{-x}dx$ using ~~left~~ right handed Riemann Sum with n=10 rectangles

remember: e^x looks like



and to graph e^{-x} you flip it to look like





- total length is 5 and is divided into 10 evenly spaced intervals of length. $\frac{5}{10} = .5$
  - Area = width × height of the rectangle

Notice: when doing a left handed Riemann Sum all of the left handed endpoints got used

$$\int_0^5 e^{-x}dx \approx (0.5)(e^0) + (0.5)(e^{-0.5}) + (0.5)(e^{-1}) + (0.5)(e^{-1.5}) + (0.5)(e^{-2}) + (0.5)(e^{-2.5}) + (0.5)(e^{-3}) + (0.5)(e^{-3.5}) + (0.5)(e^{-4}) + (0.5)(e^{-4.5}) \approx 1.26$$

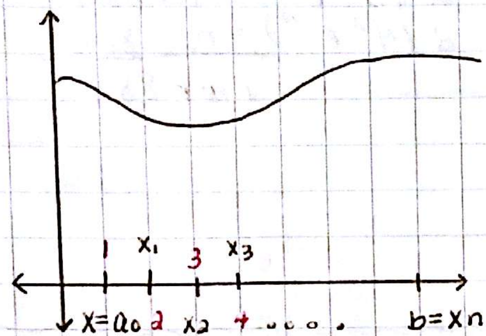
Overestimate of the value of definite intergal $\int_0^5 e^{-x}dx$

$$\int_0^5 e^{-x}dx \approx (0.5)(e^{-0.5}) + (0.5)(e^{-1}) + (0.5)(e^{-1.5}) + (0.5)(e^{-2}) + (0.5)(e^{-2.5}) + (0.5)(e^{-3}) + (0.5)(e^{-3.5}) + (0.5)(e^{-4}) + (0.5)(e^{-4.5}) + (0.5)(e^{-5}) \approx 0.77$$

Underestimate of the value of definite intergal $\int_0^5 e^{-x}dx$

Definite Integrals $\int_a^b f(x)\,dx$ can be written as a limit as $n \to \infty$
(# of rectangles $\to \infty$)

$$\int_a^b f(x)\,dx \approx \underbrace{\Delta x}_{\text{width of each rectangle}}$$



(n = # of rectangles)

Right Hand

$$\int_a^b f(x)\,dx \approx \Delta x f(x_1) + \Delta x f(x_2) + \Delta x f(x_3) \ldots + \Delta x (x_n)$$

Left Hand

$$\int_a^b f(x)\,dx \approx \Delta x f(x_0) + \Delta x f(x_1) + \Delta x f(x)_2 + \ldots \Delta x f(x_{n-1})$$

Sigma Notation

Σ "Sigma add"

Ⓡ $\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} \Delta x\, f(x_i)$; Ⓛ $\int_a^b f(x)\,dx \approx \sum_{i=0}^{n-1} \Delta x\, f(x)_i$

In order to get the exact value for the definite integral take the limit as $n \to \infty$

Ⓡ $\int_a^b (f(x)\,dx) = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\Delta x}_{w} \underbrace{f(x_i)}_{h}$

Ⓛ $\int_a^b (f(x)\,dx) = \lim_{n\to\infty} \sum_{i=0}^{n-1} \Delta x\, f(x_i)$

Make notes about the meaning of symbols!