Name: ___________________________________

Probability Basics
(Suggestion: complete this section before class)

Experiment 1.

An experiment consists of randomly selecting one letter from the word COMPUTER.

1. Find the set of all possible outcomes, the **sample space**, for this experiment.

$S = \{$________________________________$\}$

2. **Events** are *subsets* of the sample space S.
Find the following events using the sample space generated in **1.**

a. Let V be the event that a vowel is selected. $V = \{$____________$\}$

b. Let W be the event that a consonant is selected. $W = \{$__________________$\}$

c. Let X be the event that a "T" is selected. $X = \{$________$\}$

d. Let Y be the event that a "U" is selected. $Y = \{$________$\}$

3. Notice that some of the events above contained exactly one outcome and some contained more than one outcome. A **simple event** is an event that consists of exactly one sample point or outcome.

a. How many total *simple* events are there for this experiment? __________

List all of these simple events. ____

b. Which of the events in **2.** are simple events? V W X Y (Circle all that are simple.)

c. Can events X and Y occur at the same time? ________

d. What is the event $X \cap Y$? ________

e. Are X and Y mutually exclusive events? ________

All simple events are mutually exclusive.

A **uniform sample space** is a sample space in which *all* outcomes are equally likely.

4. **a.** Fill in the remaining information in the table below regarding the experiment in **1**.

Outcome	C	O	M	P	U	T	E	R
Frequency	1	1						
Probability (Relative Frequency)	$\frac{1}{8}$							

b. Is this a uniform sample space? __________

Why or why not? ____

c. What is the sum of the numbers in the second row? ____________

Experiment 2.

a. Fill in the remaining information in the table below regarding the experiment of choosing one letter from the word TELEVISION.

Outcome	T	E	L	V	I	S	O	N
Frequency	1	2						
Probability (Relative Frequency)	$\frac{1}{10}$							

b. Is this a uniform sample space? __________

Why or why not? ____

c. What is the sum of the numbers in the second row? ____________

The tables above are examples of **probability distributions.**

A probability distribution table has the following properties:

- **The outcomes in the table make up the entire sample space.**
- **The outcomes in the table cannot occur at the same time (the corresponding simple events are all mutually exclusive).**
- **The sum of all the probabilities is equal to 1.**

Once you know the probability of each outcome, to find the probabilities of larger events, add up the probabilities of each outcome in the event.

Going back to Experiment 1.
An experiment consists of randomly selecting one letter from the word COMPUTER.
Recall X is the event that a "T" is selected and Y is the event that a "U" is selected.

a. What is the *event* $X \cup Y$? ____________

b. $P(X \cup Y) =$ _________

c. $P(X) + P(Y) =$ ________+ ________ = ________

d. Does $P(X \cup Y) = P(X) + P(Y)$? ________

e. Since X and Y are mutually exclusive events, $P(X \cap Y) =$ ________

The set of all elements in the Universal set that belong to *both* sets A AND B is called the *intersection* of sets *A* and B, denoted $A \cap B$.

If two sets, *C* and *D*, have no elements in common, then the sets *C* and *D* are said to be *disjoint* sets and $A \cap B = \varnothing$.

If *E* and *F* are mutually exclusive events, $P(E \cup F) = P(E) + P(F)$.

Experiment 3.

An experiment consists of randomly selecting a letter from the word MATHEMATICS.

1. **a.** How many letters are there in this word? __________
b. Are the letters M and C equally likely to be selected? Why or why not? ___________________
____

2. Find the set of all possible outcomes in a *non-uniform* sample space S for this experiment.
$S = \{$___________________________________$\}$

Hint: In order to create a *non-uniform* sample space for this experiment, write each *different/distinct* letter as a separate outcome, as in **Experiment 2**.

3. **a.** Is the probability of selecting the first M in the word equal to the probability of selecting the second M in the word? __________
b. Is the probability of selecting the first M in the word equal to the probability of selecting the C? __________

4. Find the set of all possible outcomes in a *uniform* sample space S for this experiment.
$S = \{$__________________________________$\}$

Hint: In order to create a *uniform* sample space for this experiment, write every letter (repeated or not) as a separate outcome.

5. What is the probability of randomly choosing …

a. …the letter H? _________

b. …the letter M or the letter C? _________

c. …the letter A? _________

d. …a letter that is not A? _________

Note: Your answers from **c.** and **d.** above should sum to one, since these events are complements of one another and they combine to form the entire sample space.

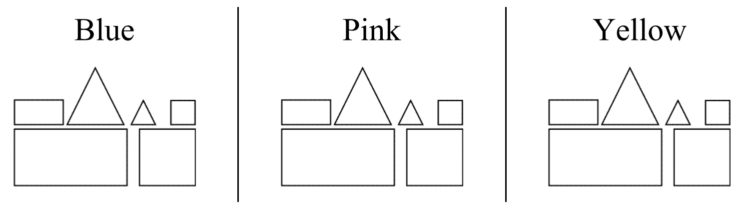
> **The set of all elements of the Universal set that are NOT objects of a given set *A*, is called the c*omplement* of *A*, denoted** A^C
>
> **For any event *E*,** $P(E) + P(E^C) = 1$**, and thus** $P(E^C) = 1 - P(E)$ **and** $P(E) = 1 - P(E^C)$**.**

Experiment 4.

In a class, the probability of choosing a female student at random is 0.38. What is the probability of choosing a male student at random from this class? __________

Sets & Venn Diagrams
(Suggestion: complete this section during class)

EXAMPLE1. For this activity, we will consider the following set of shapes which will be known as the elements of the Universal set (*U*) for our discussion: a large and small rectangle, triangle, and square, each in three different colors.

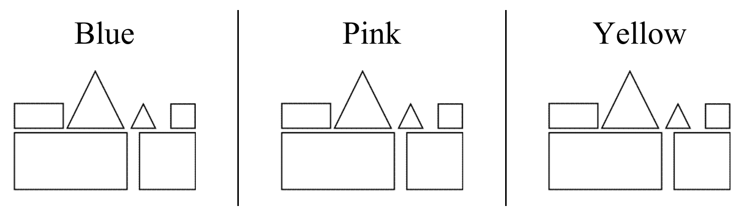


We will define the following sets:

$P = \{ x \in U \mid x \text{ is a pink shape} \}$ $B = \{ x \in U \mid x \text{ is a blue shape} \}$
$T = \{ x \in U \mid x \text{ is a triangular shaped piece} \}$ $S = \{ x \in U \mid x \text{ is a small shape} \}$
$L = \{ x \in U \mid x \text{ is a large shape} \}$

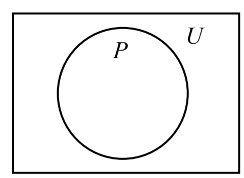
Part I

1. **a.** Put an "x" in all the shapes that are elements in the set *P*.



b. What is true about *all* the shapes **not** in set *P*?

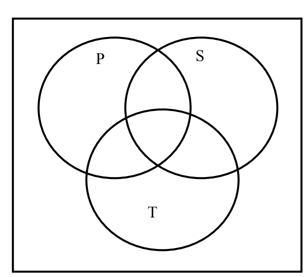
____



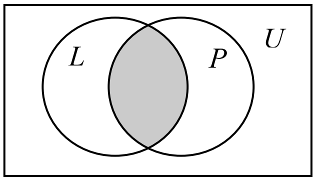
c. Shade the portion of the Venn diagram to the right that represents the set P^C.

2. **a.** What is true about all the shapes in the set T^C? ________________________________

b. Shade the portion of the Venn diagram to the right that represents the set T^C.

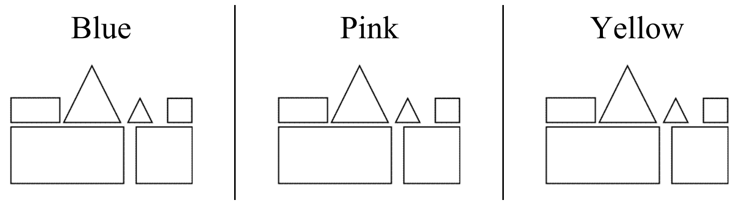


3. Consider the two-circle Venn diagram below with sets L and P as originally defined.



a. What is true about all the shapes that reside in the shaded region?
The shapes are all BOTH ___________ AND___________.

b. Place an "x" in all the shapes below that reside in this region.



The set in **3.** is the overlapping portion of sets L and P and is denoted $L \cap P$. When shading an intersection on a Venn diagram, shade the overlapping sections of the sets, since the intersection of sets is the set of elements they have in common.

4. Using the sets defined at the beginning of the activity, name two disjoint sets in the Universal set.

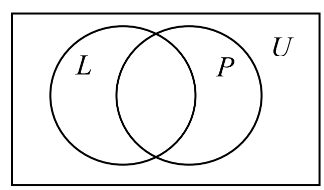
5. **a.** What is true about the shapes in the set $P \cap L^C$?

i. The shapes in set P are ______________________________.

ii. The shapes in set L^C are ______________________________.

Thus, the shapes in $P \cap L^C$ are BOTH ________________

AND _______________________.

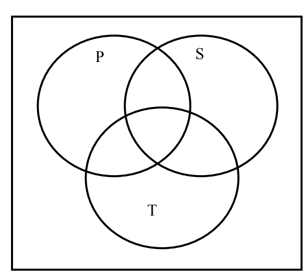


b. Shade the portion of the Venn diagram to the right that represents the set $P \cap L^C$. *Hint:* Where in the Venn diagram do these sets overlap – where are there shapes that are pink but are not large?

6. **a.** What is true about all the shapes in the set $S^C \cap T^C$?

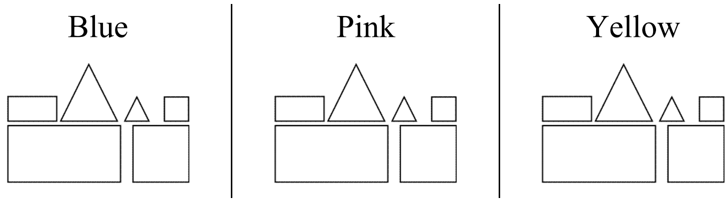
They are all BOTH _________________________

AND ____________________________.

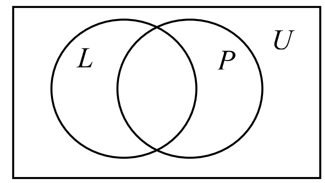


b. Shade the portion of the Venn diagram to the right that represents the set $S^C \cap T^C$.

7. **a.** Put an “x” in all the shapes below that are large.
b. Put an “x” in all the shapes below that are pink.
c. Put an “x” in all the shapes below that are both large and pink.
Note: You may place an extra “x” if a shape already contains an “x”.



d. Shade the portion of the Venn diagram to the right that represents the regions where all these shapes (ones that are large OR pink OR both) reside. *Hint:* Color the portions of the Venn diagram in the same order as you placed the x’s in the shapes above.



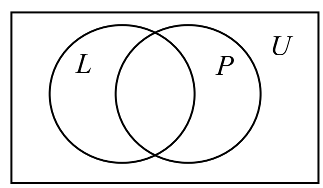
The set of all elements in the Universal set that belong to either set *A* OR to set *B* OR to both sets is called the *union* of sets *A* and *B*, denoted $A \cup B$.

The set shaded in **7d.** is the union of sets L and P and is denoted by $L \cup P$.
When shading a union of sets, shade all parts of any sets contained in the union.

8. **a.** What is true about the elements in the set $L \cup P^C$? The elements are either

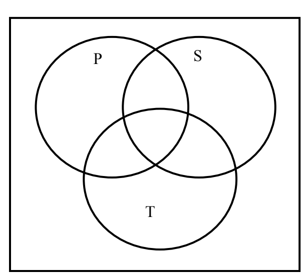
_______________ (in set L) OR __________________ (in set P^C) OR both (in set $L \cap P^C$).

b. Shade the portion of the Venn diagram to the right that represents the set $L \cup P^C$.



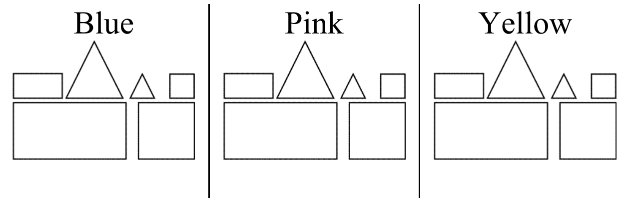
c. Why is part of set P colored above? Briefly explain.
Hint: Think about what is true of all the shapes in this region.

9. Shade the portion of the Venn diagram to the right that represents the set $S^C \cup T^C$.



10. Put an "x" in all the shapes that are in the following sets. Below each, describe the sets in words.

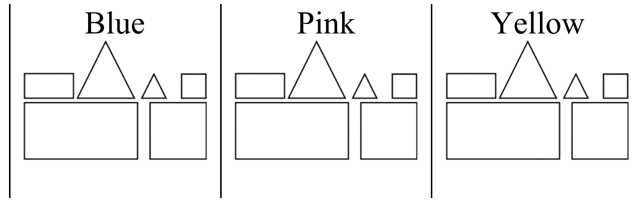
a. $S^C \cap B^C$



The set of all shapes that are both not small

AND ______________________________.

b. $S^C \cup B^C$

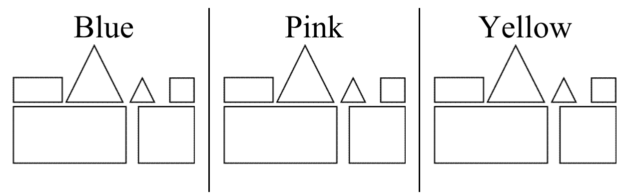


The set of all shapes that are not small

OR ________________________________

OR both.

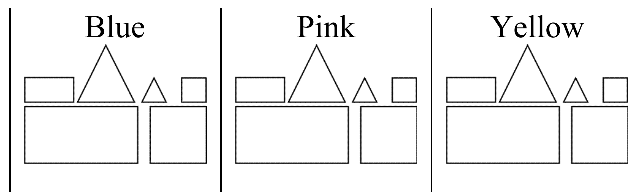
c. $(S \cap B)^C$



The set of all shapes that are _______________

____

d. $(S \cup B)^C$



The set of all shapes that are _______________

____

Two sets are equal if and only if they contain exactly the same elements.

e. Are any of the sets in **a.**– **d.** above equal?

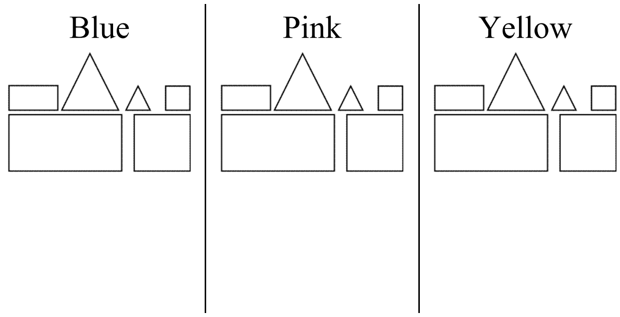
DeMorgan's Laws: For two sets A and B,

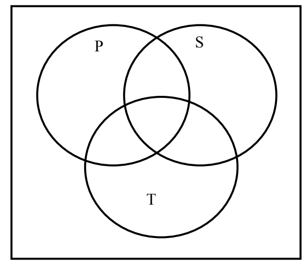
$$(A \cup B)^C = A^C \cap B^C$$
$$(A \cap B)^C = A^C \cup B^C$$

11. **a.** Use set notation (any of the following that are needed: $P, S, T, \cap, \cup$, etc.) to describe the set of *shapes that are small and pink, but not triangles*. Set Notation: ______________________

b. Put an "x" in all the elements in this set.

c. Shade the portion of the Venn Diagram below that represents this region.

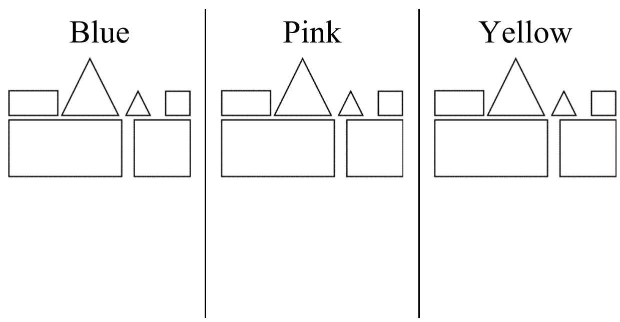


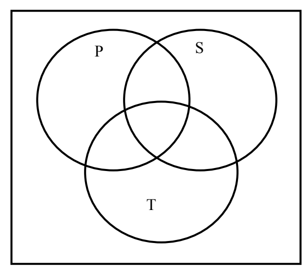


12. **a.** Use set notation (any of the following that are needed: $P, S, T, \cap, \cup$, etc.) to describe the set of *shapes that are not pink or not triangular or not small*. Set Notation: ______________________

b. Put an "x" in all the elements in this set.

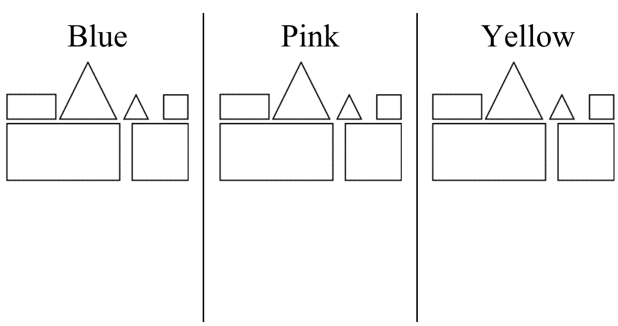
c. Shade the portion of the Venn Diagram below that represents this region.

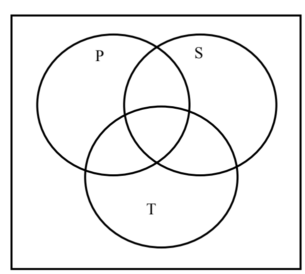




13. **a.** Put an "x" in all the elements in the set $\left(S \cap T^C\right) \cup P$.

b. Shade the portion of the Venn diagram that represents this set.





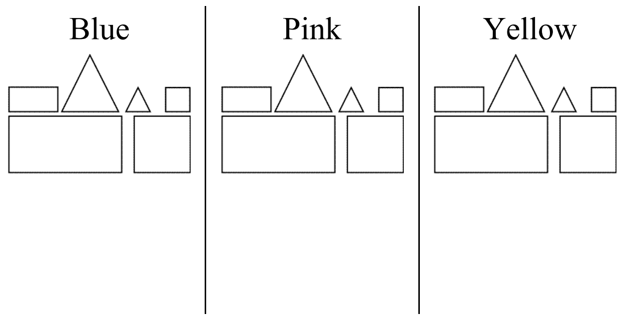
c. Describe this set in words: ____

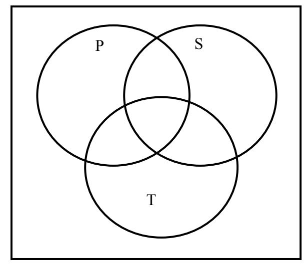
____

____

14. **a.** Put an "x" in all the elements in the set $\left(T \cup S\right) \cap P^C$.

b. Shade the portion of the Venn diagram that represents this set.





c. Describe this set in words: ____

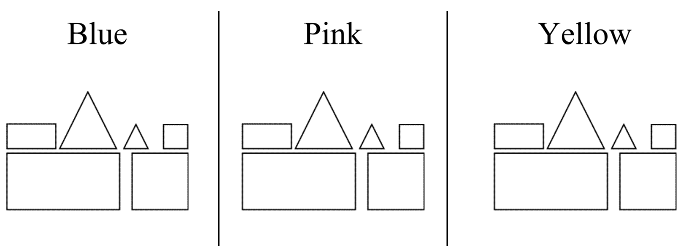
____

____

Part II

Let's now look at the *number* of elements in particular sets.

Recall our Universal set of shapes and defined sets:



$P = \{x \in U \mid x \text{ is a pink shape}\}$
$B = \{ x \in U \mid x \text{ is a blue shape}\}$
$T = \{ x \in U \mid x \text{ is a triangular shaped piece}\}$
$L = \{ x \in U \mid x \text{ is a large shape}\}$
$S = \{ x \in U \mid x \text{ is a small shape}\}$

1. Count the number of elements in set S. We write this as $n(S) =$ __________.

2. **a.** Find $n(L)$. __________ **b.** Find $n(L \cap S)$. ________ **c.** Find $n(L \cup S)$. __________

3. True or False? $n(L \cup S) = n(L) + n(S)$

4. What is true about sets L and S? L and S are ______________________ sets.
Hint: Refer to **Part I, 4.**

5. **a.** Find $n(B)$. __________ **b.** Find $n(B \cap S)$. ________ **c.** Find $n(B \cup S)$. __________

6. True or False? $n(B \cup S) = n(B) + n(S)$

7. Why or why not? ____

In general, for *any* two sets *A* and *B*, $n(A \cup B) =$ ________ + ________ − ________

If *A* and *B* are *disjoint* sets, then this becomes $n(A \cup B) =$ ________ + ________

8. Find the following:

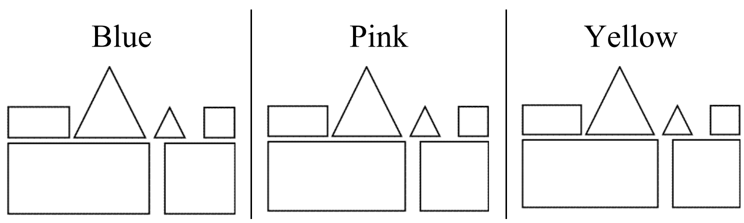
a. $n(T) =$ ________ **b.** $n(T^C) =$ ________ **c.** $n(U) =$ ________

Note: $n(T) + n(T^C) = n(U)$ since T and T^C are disjoint sets and combine to form U.

d. $n(P) =$ ________ **e.** $n(L \cap P) =$ ________

f. What is the value of $n(P) - n(L \cap P)$, and what does this number represent? ________
Hint: Refer to **Part I, 5b.**

__Part III__ Consider the set of shapes shown below. For each color, there is a small triangle, a big triangle, a small rectangle, a big rectangle, a small square, and a big square.



1. Suppose an experiment consists of selecting **one** of these shapes at random.
a. How many possible outcomes are there? ________

b. How many simple events are associated with this experiment? ________

2. Suppose B is the event that a blue shape is selected.
a. How many outcomes are in the event B? ________

b. What is the probability of B? $P(B) =$ ________

3. If M is the event that a small shape is selected,
a. How many outcomes are in the event M? ________

b. What is $P(M)$? ________

4. Consider the event $B \cap M$.
a. What kind of shape has been selected in order for this event to occur?
The shape is both ________ **and** ________.

b. How many outcomes are in the event $B \cap M$? ________

c. What is $P(B \cap M)$? ________

5. Consider the event $B \cup M$.

a. What kind of shape has been selected in order for this event to occur?
The shape is ___________ **or** ____________ **or** both.

b. How many outcomes are in $B \cup M$? ________

c. What is $P(B \cup M)$? ________

d. What is $P(B) + P(M)$? ________+ _________ = _________

e. Does $P(B \cup M) = P(B) + P(M)$? _________
Why or why not? *Hint:* Refer to your answer to **4c**.

____

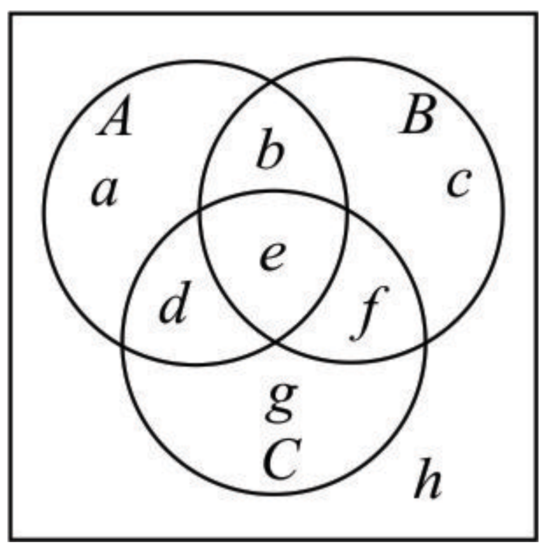
____

6. If E and F are any two events in a sample space S, how can the terms $P(E)$, $P(F)$, $P(E \cup F)$ and $P(E \cap F)$ be related in a formula? *Hint:* This is called the UNION RULE for probability.

EXAMPLE 2. **In the provided Venn diagram, set *A* are the students who like apples, set *B* are the students who like bananas, and set *C* is the set of students who like cantaloupe.**

A survey was conducted of 98 freshmen on their preference of fruit.
It found:

- 19 like only apples and cantaloupe
- 46 do not like apples
- 10 like bananas and cantaloupe
- 7 like only bananas
- 14 do not like any of these three fruits
- 4 like apples, bananas and cantaloupe
- 24 like bananas but not cantaloupe



Fill in the number of students in each set.

a = _________ b = _________ c = _________

d = _________ e = _________ f = _________

g = _________ h = _________