You Should Know ... Basic Operations (+,–, ×, ÷)

> Do not memorize without understanding!
> *Understand first*, THEN *memorize!*

FACTS

Properties of Addition:

- Associative (parentheses don't matter):
  $A + B + C = (A + B) + C = A + (B + C)$
- Commutative (order doesn't matter):
  $A + B = B + A$
- Identity is 0 (anything plus zero is itself):
  $A + 0 = A$

Properties of Subtraction:

- Multiplying times a negative, and subtracting, ARE EQUIVALENT (they are the same thing!):
  $A + B \cdot (-1) = A - B$
- Not Associative (parentheses do matter!):
  $(A - B) - C = A - B - C$
  $A - (B - C) = A - B + C$
- Not Commutative (order does matter!):
  $A - B \neq B - A$
- Multiplying times a negative, and switching the order of subtraction, ARE EQUIVALENT:
  $A - B = -(B - A)$

Properties of Multiplication:

- Associative (parentheses don't matter):
  $A \cdot B \cdot C = (A \cdot B) \cdot C = A \cdot (B \cdot C)$
- Commutative (order doesn't matter):
  $A \cdot B = B \cdot A$
- Identity is 1 (anything times one is itself):
  $A \cdot 1 = A$
- Multiplication distributes over addition:
  $A \cdot (B + C) = A \cdot B + A \cdot C$
- Multiply two sums using FOIL method:
  $(A + B) \cdot (C + D) = A \cdot C + A \cdot D + B \cdot C + B \cdot D$
- Squaring means multiply times itself:
  $$(A + B)^2 = (A + B) \cdot (A + B) = A^2 + A \cdot B + B \cdot A + B^2 = A^2 + 2AB + B^2$$
- Difference of squares factors like so:
  $A^2 - B^2 = (A - B)(A + B)$

Properties of Division:

- Dividing by a negative, and multiplying by a negative ARE EQUIVALENT:
  $$B \cdot (-1) = \frac{B}{-1} = -B$$
- More next page (see Fractions).

Equality Notation:

- $A = B = C$ means all combos are equal:
  $A = B$ and $A = C$ and $B = C$.

Solutions are posted at www.jdambroise.com/studyaids

TRUE OR FALSE

1. $-(x - 12) = -x + 12$ T
   ↕ same
2. $-(x - 12) = 12 - x$ T
3. $-(x + 12) = -x + 12$ F (need to distribute minus)
4. $(x + 12)^2 = x^2 + 144$ F (missing cross term)
5. $-(x + 12)^2 = -x^2 - 144$ F (missing cross term)
6. $-(x + 12)^2 = -(x^2 + 24x - 144)$ F (+144)
7. $-(x + 12)^2 = -x^2 - 24x + 144$ F (−144)
8. $xy + 16x^2 = xy(1 + 16x)$ F (no y in second term)
9. $xy + 16x^2y = xy(1 + 16x)$ T
10. $x + z - y = -y + x + z$ T
11. $(xz + 8y) + x = xz + 8y + x$ T
12. $(-4)y + x = x - 4y$ T
13. $(-4)(-y)xx = 4x^2y$ T
14. $yxx = 2xy$ F ($xx = x^2$ not $2x$)
15. $y + y - 3y = y^2 - 3y$ F ($y+y=2y$ not y^2)
16. $4x^2 + 8x - x - 2 = (4x - 1)(x + 2)$ T
17. $4x^2 + 7x - 2 = (4x - 1)(x + 2)$ T
18. $2x^2 + 8x = (2x + 1)8x$ F (no 8 in first term)
19. $(2 - x)^2 = (x - 2)^2$ T
20. $2xy^2 = 4x^2y^2$ F (the 2 isn't squared)
21. $(2xy)^2 = 4x^2y^2$ T
22. $(2x - y)^2 = 4x^2 - y^2$ F (missing cross term)
23. $(2x - y)^2 = 4x^2 - 4xy + y^2$ T
24. $4x^2 - y^2 = (2x - y)(2x + y)$ T
25. $(\sqrt{2} - y)^2 = 2 - \sqrt{2}y - y^2$ F ((1) $+y^2$ (2) cross term missing 2)
26. $(\sqrt{2} - y)^2 = 2 - 2\sqrt{2}y - y^2$ F ($+y^2$)
27. $\frac{1}{-xy} = \frac{-1}{xy} = -\left(\frac{1}{xy}\right) = -\frac{1}{yx}$ T

You Should Know ... Fractions

Do not memorize without understanding! *Understand first,* THEN *memorize!*

FACTS

Dealing with Constants: Any constant can be made into a fraction by inserting "over one".

$$A = \frac{A}{1}$$

Multiplying: *Multiply numerators, multiply denominators.*

$$\frac{A}{B}\cdot\frac{C}{D} = \frac{A\cdot C}{B\cdot D}$$

Therefore...

- A constant C multiplied goes in the numerator:

$$C\left(\frac{A}{B}\right) = \frac{C}{1}\cdot\frac{A}{B} = \frac{A\cdot C}{B}$$

Dividing: *Flip the bottom fraction, then multiply.*

$$\frac{\left(\frac{A}{B}\right)}{\left(\frac{C}{D}\right)} = \left(\frac{A}{B}\right)\cdot\left(\frac{D}{C}\right) = \frac{A\cdot D}{B\cdot C}$$

Therefore...

- A constant C divided goes in the denominator:

$$\frac{\left(\frac{A}{B}\right)}{C} = \frac{\left(\frac{A}{B}\right)}{\left(\frac{C}{1}\right)} = \left(\frac{A}{B}\right)\cdot\left(\frac{1}{C}\right) = \frac{A}{BC}$$

Adding: *Need a common denominator!*

- **#1:** These fractions have a common denominator:

$$\frac{A}{B}+\frac{C}{B} = \frac{A+C}{B}$$

- **#2:** These fractions have different denominators:

$$\frac{A}{B}+\frac{C}{D} = \frac{A}{B}\left(\frac{D}{D}\right)+\frac{C}{D}\left(\frac{B}{B}\right) = \frac{A\cdot D}{B\cdot D}+\frac{C\cdot B}{B\cdot D} = \frac{AD+BC}{BD}$$

TRUE OR FALSE

1. $2\cdot\frac{8}{7} = \frac{16}{14}$ F (constant 2 should go in numerator)
2. $2\cdot\frac{8}{7} = \frac{8}{14}$ F (same ↗)
3. $2\cdot\frac{8}{7} = \frac{16}{7}$ T
4. $\left(\frac{103}{21}\right)/3 = \frac{103}{7}$ F (constant 3 divided goes in denom.
5. $\left(\frac{103}{21}\right)/3 = \frac{103}{63}$ T
6. $\left(\frac{4}{3}\right)\cdot 3 = 4$ T
7. $\left(\frac{4}{3}\right)/3 = \frac{4}{1} = 4$ F (constant 3 divided goes in denom.)
8. $\left(\frac{4}{3}\right)/3 = \frac{4}{9}$ T
9. $\left(\frac{4}{3}\right)+3 = \frac{7}{6}$ F (need to get common denom. By adding)
10. $\left(\frac{4}{3}\right)+3 = \frac{4}{3}+\frac{3}{1} = \frac{7}{4}$ F (same ↗)
11. $\left(\frac{4}{3}\right)+3 = \frac{4}{3}+\frac{3}{1}\cdot\frac{3}{3} = \frac{13}{3}$ T
12. $\frac{3}{3x-12y+6} = \frac{1}{x-12y+6}$ F (cannot cancel 3 only in one term)
13. $\frac{y}{x}+\frac{3}{(x-1)} = \frac{y}{x}\cdot\frac{(x-1)}{(x-1)}+\frac{3}{(x-1)}\cdot\frac{y}{x} = \frac{xy-y+3y}{x(x-1)}$ F (This should be x

EXAMPLES

Example A. Correctly add the fractions:

$$\frac{y}{x}+\frac{3}{(x-1)} = \frac{y}{x}\cdot\frac{(x-1)}{(x-1)}+\frac{3}{(x-1)}\cdot\frac{x}{x}$$

$$= \frac{yx-y+3x}{x(x-1)}$$

Example B. Correctly factor and/or cancel:

$$\frac{3}{3x-12y+6} = \frac{1}{x-4y+2}$$

Example C. Choose among the following words/phrases in order to complete the sentences below: "every single term", or "numerator", or "denominator", or "common denominator".

- When adding fractions, you must have a common denom..
- When multiplying fractions multiply the numerator of the first fraction with the numerator of the second fraction, and multiply the denom. of the first fraction with the denom. of the second fraction.
- If a constant C is multiplied times a fraction, then the constant will be multiplied into the numerator of the fraction.
- If a fraction is divided by a constant C, then the constant will be multiplied into the denom. of the fraction.
- If a number appears in every single term of a fraction then you can cancel it in every single term.

You Should Know … Exponents

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FACTS

To multiply/divide with common base x:
add/subtract exponents.

$$x^a \cdot x^b = x^{a+b} \qquad \text{and} \qquad \frac{x^a}{x^b} = x^{a-b}$$

To multiply/divide with common exponent a:
use parentheses.

$$x^a \cdot y^a = (xy)^a \qquad \text{and} \qquad \frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

Raising an exponent to an exponent: multiply exponents.

$$(x^a)^b = x^{a \cdot b}$$

Roots are fractional exponents:

- $\sqrt{x} = x^{1/2}$
- $\sqrt[3]{x} = x^{1/3}$
- $\sqrt[4]{x} = x^{1/4}$
- $\sqrt[5]{x} = x^{1/5}$
- … etc.

Move exponents between denominator and numerator by negating the exponent:

- $\frac{1}{x^{26}} = x^{-26}$
- $\frac{1}{x^{-26}} = x^{-(-26)} = x^{26}$
- $yx^{-5} = \frac{y}{x^5}$
- $\frac{y^{-6}}{x^3} = \frac{1}{y^6x^3}$
- $\frac{1}{\sqrt[3]{x}} = x^{-1/3}$
- … etc.

Combine fraction rules and exponents rules carefully:

- $x^{4/5} = x^{\frac{1}{5} \cdot 4} = \left(\sqrt[5]{x}\right)^4 = OR = \sqrt[5]{x^4}$
- $x^{-8/3} = x^{-\frac{1}{3} \cdot 8} = \frac{1}{\left(\sqrt[3]{x}\right)^8} = OR = \frac{1}{\sqrt[3]{x^8}}$
- … etc.

Anything to the zero power is one: $x^0 = 1$

If no exponent then assume exponent one: $x = x^1$

Examples where you **cannot simplify**:

- cannot combine different powers added
  $x^5 + x^9 + 1 \quad ((\neq x^{14} + 1))$
- cannot separate terms added in denominator
  $\frac{1}{x^2+x^4+7} \quad ((\neq x^{-2} + x^{-4} + \frac{1}{7}))$
- cannot distribute roots over addition
  $\sqrt{x+y} \quad ((\neq \sqrt{x} + \sqrt{y}))$

Solutions are posted at www.jdambroise.com/studyaids

TRUE OR FALSE

1. $x \cdot \sqrt[4]{x^{10}} = x \cdot x^{5/2} = x^{7/2}$ T
2. $\frac{x^3}{x^5+x^9}$ cannot be simplified F cancel x^3 $= \frac{1}{x^2+x^6}$
3. $\frac{x^3}{x^5+x^9+1}$ cannot be simplified T
4. $\frac{x^{13}}{4x^5} = \frac{1}{4}x^8$ T
5. $\frac{x^{13}y^7}{9\,y^{20}} = \frac{1}{9}\left(\frac{x}{y}\right)^{13}$ T
6. $\left(\frac{\sqrt{x}}{y+x}\right)^{10} = \frac{x^5}{y^{10}+x^{10}}$ F ($(y+x)^{10}$ is not equal to $y^{10}+x^{10}$)
7. $\left(\frac{\sqrt{x}}{y+x}\right)^{10} = \frac{x^5}{(y+x)^{10}}$ T

EXAMPLES

Example A. Fully simplify:
Hint: Recall the rules for multiplying fractions!

$$\frac{x^5\cancel{\sqrt{x}}}{9\,y^{-2}} \cdot \frac{3x}{\cancel{\sqrt{x}}} = \frac{1}{3} \cdot x^6 \cdot y^2$$

Example B. Fully simplify:

$$\sqrt[3]{\frac{54\,x}{9\,x^{-2}} + \frac{27\,x^{7/2}}{\sqrt{x}}} = \sqrt[3]{6x^3 + 27x^3}$$

$$= \sqrt[3]{33\;x^3}$$

$$= \sqrt[3]{33} \cdot x$$

33 is not a perfect cube so this number isn't an exact integer so best to leave it in exact form

Example C. Fully simplify:

$$\sqrt[3]{\frac{54\,x}{9\,x^{-2}} + \frac{27\,x^8}{x}} = \sqrt[3]{6x^3 + 27x^7}$$

$$= \sqrt[3]{x^3 \cdot (6+27x^4)}$$

$$= x \cdot \sqrt[3]{3(2+9x^4)}$$

No perfect cube under root, so no more simplifying

You Should Know ... LINES

Do not memorize without understanding! *Understand first,* THEN *memorize!*

GENERAL CONCEPTS

Zeros / x-intercepts
- An x-value satisfying $f(x) = 0$
- Set $y = 0$ and solve for x.

y-intercepts
- The y-value of $y = f(0)$
- Set $x = 0$ and solve for y.

LINES

Equations
- Slope-y-intercept format: $y = mx + b$
- Point-slope format: $y = m(x - x_0) + y_0$

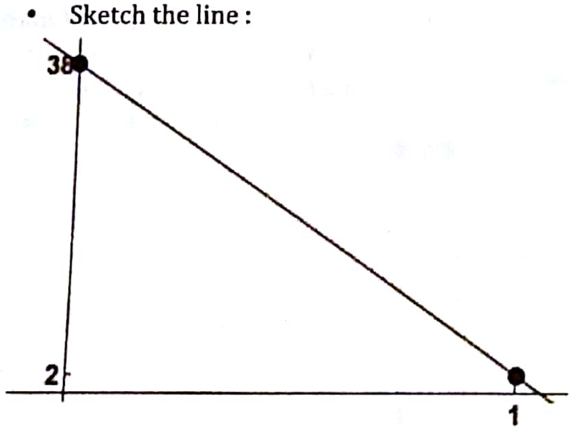
Symbols
- $m = \frac{\Delta y}{\Delta x}$ is the slope of the line
- (x_0, y_0) is a point on the line
- $(0, b)$ is the y-intercept point

Special Cases
- For *positive slope* $(m > 0)$ the line goes UP
- For *negative slope* $(m < 0)$ the line goes DOWN
- $y = C$ is a *horizontal line* with slope $m = 0$
- $x = C$ is a *vertical line* with infinite slope
- Lines with the same slope are *parallel*
- Lines with the same slope and y-intercept *overlap*
- Lines with slopes m and $-\frac{1}{m}$ are *perpendicular*

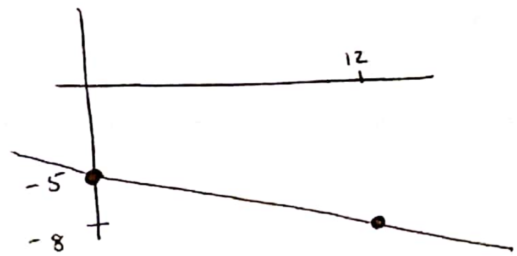
How to Draw a *Rough* Sketch *Quickly & Efficiently*
- Find two points on the line.
- Connect the two points with a line.
- Double check the slope roughly goes the correct way. ***DO NOT draw every single tick mark!***

EXAMPLES

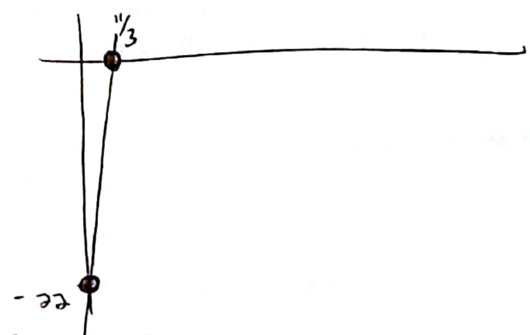
Demo Example. $y = -36(x - 1) + 2$
- $(1, 2)$ is a point on the line
- $(0, 38)$ is the y-intercept point
- The slope is $m = -36 = \frac{-36}{1}$
- If x goes up by 1 then y goes down by 36.
- Sketch the line :



Example A. $y = -\frac{1}{4}(x - 12) - 8$
- (12 , -8) is a point on the line
- (0 , -5) is the y-intercept point
- If x goes up by 4 , y goes down by 1 .
- Sketch the line :



Example B. $y = 6x - 22$
- (0 , -22) is the y-intercept point
- (11/3 , 0) is the x-intercept point
- If x goes up by 1 , y goes up by 6 .
- Sketch the line:



Example C. Complete the sentences.
- For slope-intercept format it is easiest to find the X -intercept and the Y -intercept.
- For point-slope format it is easiest to find the point (x_0 , y_0) and the Y -intercept.
- $y = \frac{1}{3}x - 7$ is perpendicular to $y =$ -3 x+8.
- X = 5 is a vertical line
- y = -37 is a horizontal line
- The line that goes through $(-1, 6)$ and is parallel to $y = -x + 4$ is $y =$ $-1 \cdot (x+1) + 6$.
- The line that goes through $(-1, 6)$ and is perpendicular $y = -x + 4$ is $y =$ $1 \cdot (x+1) + 6$.

You Should Know ... QUADRATICS

Do not memorize without understanding! *Understand first,* THEN *memorize!*

GENERAL CONCEPTS

Shifting rules (assuming $c > 0$)

- $f(x - c)$ shifts $f(x)$ c-units right
- $f(x + c)$ shifts $f(x)$ c-units left
- $f(x) + c$ shifts $f(x)$ c-units up
- $f(x) - c$ shifts $f(x)$ c-units down

Reflecting rules

- $f(-x)$ flips $f(x)$ left-to-right, about the y-axis
- $-f(x)$ flips $f(x)$ up-down, about the x-axis

A QUADRATIC IS A PARABOLA

Equations

- Standard format: $f(x) = ax^2 + bx + c$
- Completed Square format: $y = a(x - x_0)^2 + y_0$

Symbols

- (x_0, y_0) is the vertex of the parabola
- $(0, c)$ is the y-intercept of the parabola
- $x_0 = -\frac{b}{2a}$ is the x-value of the vertex
- $y_0 = f(x_0)$ is the y-value of the vertex

Special Cases

- For $a > 0$ the parabola curves UP
- For $a < 0$ the parabola curves DOWN
- To find the x-intercepts (a.k.a. zeros) set $y = 0$ and solve for x: $0 = ax^2 + bx + c$
  - If a, b, c are nice, factor, set each factor to zero, then solve for x.
  - If a, b, c are not nice, use the quadratic formula to solve for x:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

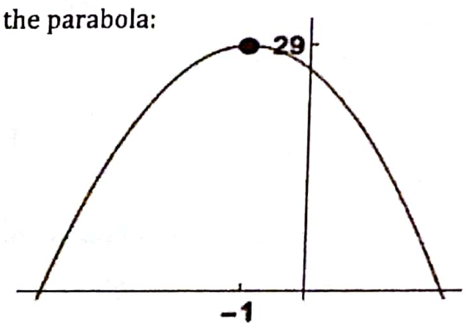
How to Draw a *Rough* Sketch *Quickly & Efficiently*

- Determine the vertex (x_0, y_0).
- Decide if the parabola curves UP or DOWN.
- Quickly sketch. ***DO NOT draw every tick mark!***

EXAMPLES

Demo Example. $y = -3(x + 1)^2 + 29$

- $(-1, 29)$ is the vertex
- Since $a = -3 < 0$ the parabola curves DOWN
- Sketch the parabola:



Demo Example Continued. $y = -3(x + 1)^2 + 29$

- Put the quadratic in standard form (expand).

$y = -3(x^2 + 2x + 1) + 29$

$= -3x^2 - 6x + 26$

$a =$ -3, $b =$ -6, $c =$ 26

- Find the zeros. Express your answer exactly.

$0 = -3x^2 - 6x + 26$

$x = \dfrac{6 \pm \sqrt{36 - 4\cdot(-3)(26)}}{-6}$

$= \dfrac{6 \pm \sqrt{348}}{-6} = \dfrac{6 \pm \sqrt{4 \cdot 87}}{-6}$

$= -1 \pm \frac{1}{3}\sqrt{87}$

Example A. $y = 6x^2 - 5x + 1$

Express all answers exactly.

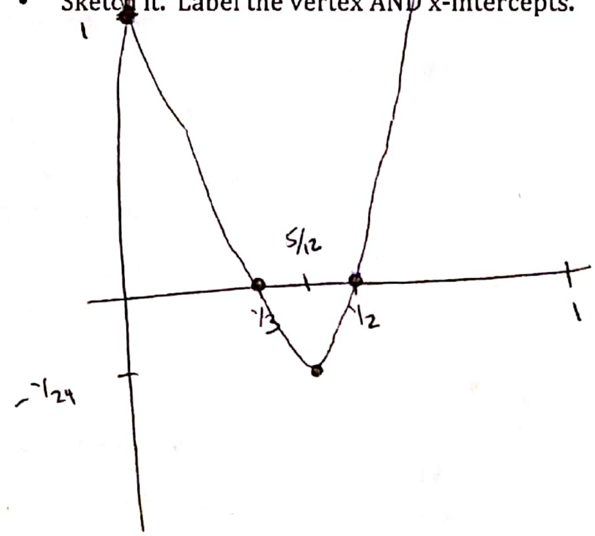
- $a =$ 6, $b =$ -5, $c =$ 1
- ($\frac{5}{12}$, $-\frac{1}{24}$) is the vertex
- Find the zeros.

$0 = 6x^2 - 5x + 1$

$0 = (2x - 1)(3x - 1)$

$x = 1/2,\ 1/3$

- Sketch it. Label the vertex AND x-intercepts.



You Should Know ... OTHER BASIC FUNCTIONS

Do not memorize without understanding!
Understand first, THEN *memorize!*

GENERAL CONCEPTS

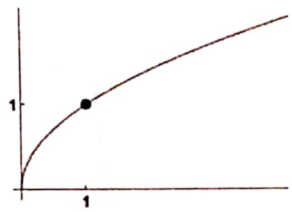
Properties of Functions

- x-intercepts, y-intercepts, shifting and reflecting rules (see previous pages)
- Domain is the set of *allowed* x-values
- Range is the set of all *output* y-values

OTHER BASIC FUNCTIONS

Any Even Root
$y = x^{1/n}$
n is even integer

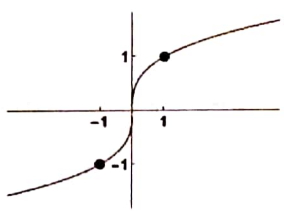
Domain $[0, \infty)$
Range $[0, \infty)$



- Example: Square root $y = \sqrt{x} = x^{1/2}$.
- Example: Sixth root $y = \sqrt[6]{x} = x^{1/6}$

Any Odd Root
$y = x^{1/n}$
n is odd integer

Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$

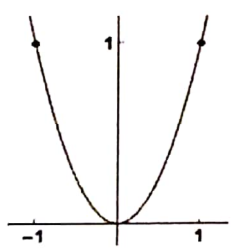


- Example: Cube root $y = \sqrt[3]{x} = x^{1/3}$.
- Example: Seventh root $y = \sqrt[7]{x} = x^{1/7}$

Any Even Power Monomial
$y = x^p$
p is even integer

Domain $(-\infty, \infty)$
Range $[0, \infty)$

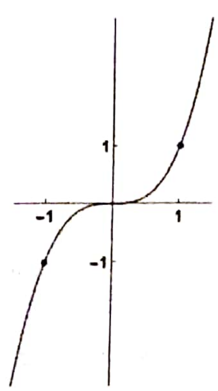
Examples: $y = x^2, y = x^{14}$



Odd Power Monomial
$y = x^p$
p is odd integer

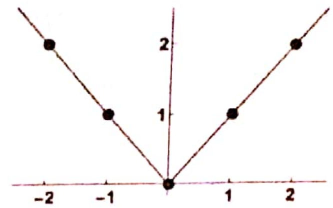
Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$

Examples: $y = x^3, y = x^{19}$



Absolute Value
$y = |x|$

Domain $(-\infty, \infty)$
Range $[0, \infty)$



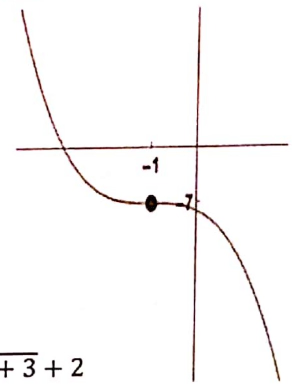
Writing absolute value in piecewise format:

$$|x| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$$

EXAMPLES

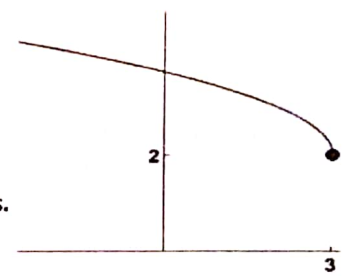
Demo Example. $y = -(x+1)^3 - 7$
The function is
an odd power,
flipped upside down
(due to minus sign in front),
shifted 1 unit left
and 7 units down.

Domain $(-\infty, \infty)$
Range $(-\infty, \infty)$



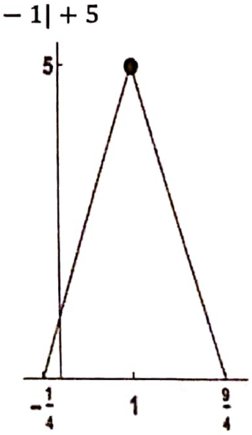
Demo Example. $y = \sqrt{-x+3} + 2$
To see shift/reflection
must factor out minus
$y = \sqrt{-(x-3)} + 2$
therefore square root
is shifted right 3 units,
up 2 units, and reflected
left-to-right due to minus.

Domain $(-\infty, 3]$
Range $[2, \infty)$



Demo Example. $y = -4|x-1| + 5$
This is a vee-shape
with slope 4
flipped upside down
(due to minus)
then shifted right 1 unit
and up 5 units.

Domain $(-\infty, 5]$
Range $(-\infty, \infty)$



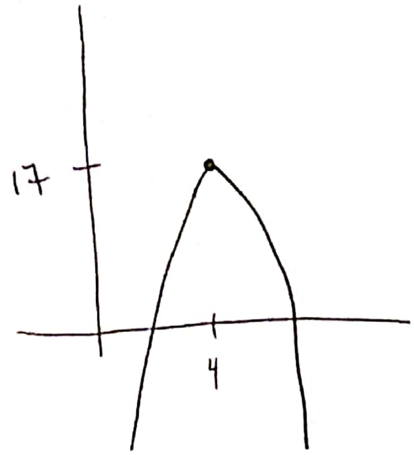
Write in piecewise format:

$$-4|x-1| + 5 = \begin{cases} -4(x-1)+5, & if\ x \geq 1 \\ 4(x-1)+5, & if\ x < 1 \end{cases}$$

Example A. Draw rough sketch, and state domain/range.

$$y = -2(x-4)^8 + 17$$

Note: doesn't say label intercepts.



Domain $(-\infty, \infty)$

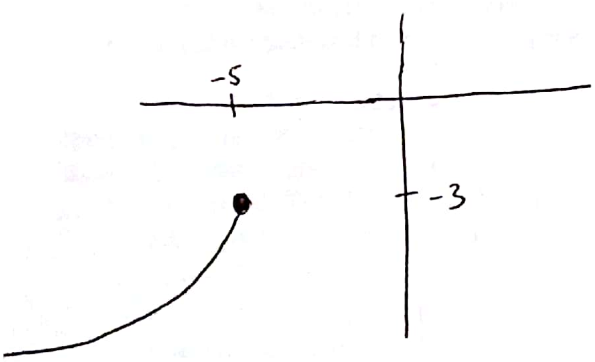
Range $(-\infty, 17]$

Example B. Draw rough sketch, and state domain/range.

$$y = -2\sqrt[6]{-x-5} - 3$$

$y = -2\sqrt[6]{-(x+5)} - 3$ ← shift down

↑ flip up down; ↑ flip left/right; ↑ shift left



Domain $(-\infty, -5]$

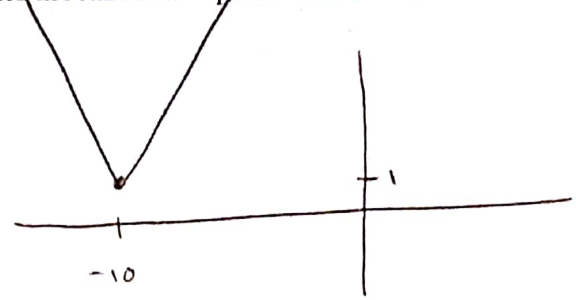
Range $(-\infty, -3]$

Note: Doesn't say label intercepts.

Example C. Draw rough sketch, and state domain/range.

$$y = 2|x+10| + 1$$

Write the function in piecewise format too.



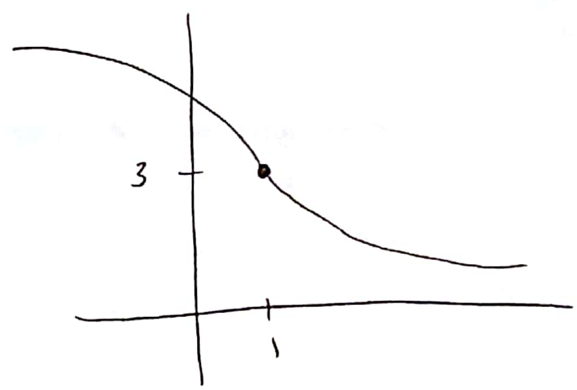
Domain $(-\infty, \infty)$

Range $[1, \infty)$

$$y = \begin{cases} 2(x+10)+1 & \text{if } x \geq -10 \\ -2(x+10)+1 & \text{if } x < -10 \end{cases}$$

Example D. Draw rough sketch, and state domain/range.

$$y = -20(x-1)^{1/5} + 3$$



Domain $(-\infty, \infty)$

Range $(-\infty, \infty)$

You Should Know ... CIRCLES

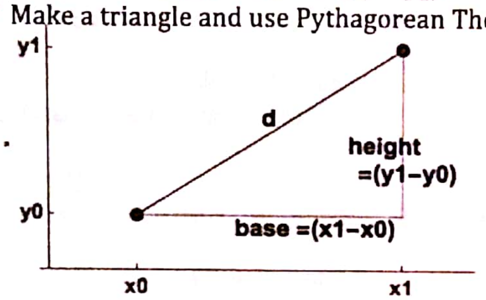
Do not memorize without understanding! *Understand first,* THEN *memorize!*

GENERAL CONCEPTS

Distance d between (x_0, y_0) and (x_1, y_1) is ...

$$d = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}$$

Why? Make a triangle and use Pythagorean Theorem:



CIRCLES

Equations

- Standard format: $(x - x_0)^2 + (y - y_0)^2 = r^2$
- General format: $x^2 + Ax + y^2 + By + C = 0$

Symbols

- r is the radius of the circle
- (x_0, y_0) is the center of the circle
- A, B, C are constants

Special Cases

- $x^2 + y^2 = r^2$, a circle radius r centered at origin
- $y = \sqrt{r^2 - x^2}$ is the top half of a circle of radius r centered at origin
- $y = -\sqrt{r^2 - x^2}$ is the bottom half of a circle of radius r centered at origin

How to Draw a *Rough* Sketch *Quickly & Efficiently*

- Identify center and radius.
- From the center of the circle go up, down, left, and right the radius-length and mark 4 dots.
- Roughly connect the dots creating the circle.

EXAMPLES

Demo Example. Write circle $x^2 - 2x + y^2 + 10y = 0$ in standard format. (How? COMPLETE THE SQUARE).

- Use the VERTEX FORMULA from previous page!
  The x-part is $G(x) = x^2 - 2x$.
  $x_0 = -\frac{b}{2a} = -\frac{-2}{2 \cdot 1} = 1$,
  $y_0 = G(1) = 1 - 2 = -1$.
  Therefore $G(x) = (x-1)^2 - 1$.
- Now complete the square for the y-part.
  $H(y) = y^2 + 10y$.
  $x_0 = -\frac{b}{2a} = -\frac{10}{2 \cdot 1} = -5$
  $y_0 = H(-5) = 25 - 50 = -25$.
  Therefore $H(y) = (y+5)^2 - 25$.

Put It Together: $(x-1)^2 - 1 + (y+5)^2 - 25 = 0$

Therefore ...

Standard Format is: $(x-1)^2 + (y+5)^2 = 26$.

Solutions are posted at www.jdambroise.com/studyaids

Demo Example Continued. Therefore ...

- The center is (1 , −5).
- The radius is $r = \sqrt{26}$

Example A. Write the circle $x^2 + 2x + y^2 - 3y = 0$ in standard format. State the center and radius. Then draw and label the circle.

x-part

$x^2 + 2x = (x+1)^2 - 1$
because
$x_0 = \frac{-2}{2} = -1$
$y_0 = -1$

y-part

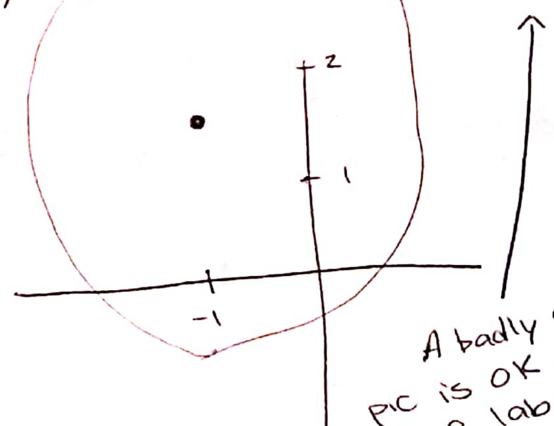
$y^2 - 3y = (y - \frac{3}{2})^2 - \frac{9}{4}$
because
$x_0 = \frac{+3}{2}$
$y_0 = \frac{9}{4} - \frac{9}{2} = \frac{-9}{4}$

Therefore eqn. of circle is...

$(x+1)^2 - 1 + (y - 3/2)^2 - \frac{9}{4} = 0$

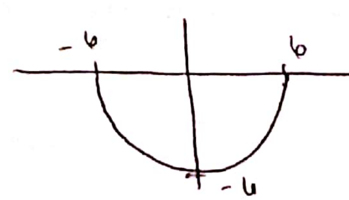
$\Rightarrow (x+1)^2 + (y - 3/2)^2 = \frac{13}{4}$

$\Rightarrow$ Center $(-1, 3/2)$ Radius $\frac{\sqrt{13}}{2}$



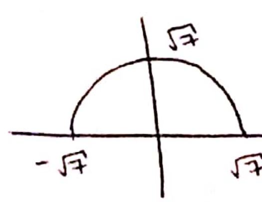
A badly drawn pic is OK as long as the labels compensate. :)

Example B. Draw a rough sketch of $y = -\sqrt{36 - x^2}$.



Bottom half circle of radius 6 centered at origin

Example C. Draw a rough sketch of $y = \sqrt{7 - x^2}$.



Top half circle radius √7 origin center

You Should Know ... TRIGONOMETRY

Do not memorize without understanding! *Understand first,* THEN *memorize!*

RADIANS V. DEGREES

Facts: $\pi = 180^o$ and $2\pi = 360^o$.

Example A.

$\pi/3 =$ 60°, $\pi/4 =$ 45°, $\pi/6 =$ 30°

SOHCAHTOA

Example B. What does it mean?

$\sin(\theta) = \frac{opp}{hyp}$, $\cos(\theta) = \frac{adj}{hyp}$, $\tan(\theta) = \frac{opp}{adj}$

OTHER FACTS

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \quad \text{and} \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

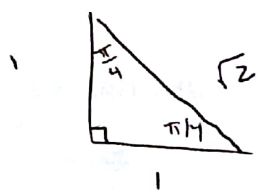
Example C. Complete the other functions:

$\csc(\theta) = \frac{1}{\sin(\theta)}$ and $\sec(\theta) = \frac{1}{\cos(\theta)}$

SPECIAL TRIANGLE: 45°-45°-right

Example D. Draw the triangle.

Hint: The sides are 1, 1, $\sqrt{2}$.

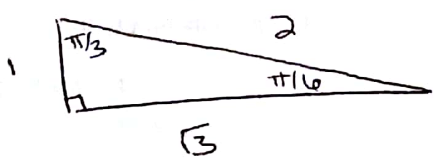


SPECIAL TRIANGLE: 30°-60°-right

Example E. Draw the triangle.

Hint1: The sides are 1, $\sqrt{3}$, 2.

Hint2: The smallest side is opposite the smallest angle.



SPECIAL VALUES

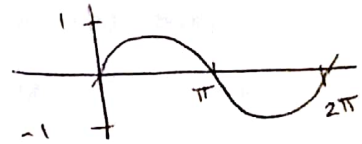
Example F.

- $\sec(\pi/4) =$ $\sqrt{2}$
- $\cot(\pi/6) =$ $\sqrt{3}$
- $\csc(\pi/3) =$ $\frac{2}{\sqrt{3}}$
- $\tan(\pi/4) =$ 1

SINE FUNCTION FACTS

- $\sin(0) = 0$
- Sine has period 2π.
- Sine is odd: $\sin(-x) = -\sin(x)$.

Example G1. Draw at least one cycle of the function. Label at least two points on each of the x,y axes.



Example G2. State the domain and the range.

Domain $(-\infty, \infty)$

Range $[-1, 1]$

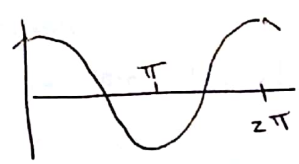
Example G3. Use the graph to find these special values.

- $\sin(\pi/2) =$ 1
- $\sin(\pi) =$ 0
- $\sin(3\pi/2) =$ -1
- $\sin(2\pi) =$ 0

COSINE FUNCTION FACTS

- $\cos(0) = 1$
- Cosine has period 2π.
- Cosine is even: $\cos(-x) = \cos(x)$.

Example H1. Draw at least one cycle of the function. Label at least two points on each of the x,y axes.



Example H2. State the domain and the range.

Domain $(-\infty, \infty)$

Range $[-1, 1]$

Example H3. Use the graph to find these special values.

- $\cos(\pi/2) =$ 0
- $\cos(\pi) =$ -1
- $\cos(3\pi/2) =$ 0
- $\cos(2\pi) =$ 1

MORE SPECIAL VALUES

Example I. *Hint: Recall that dividing by zero is undefined.*

- $\sec(\pi) =$ -1
- $\cot(\pi/2) =$ 0
- $\csc(3\pi/2) =$ -1
- $\cot(2\pi) =$ undefined

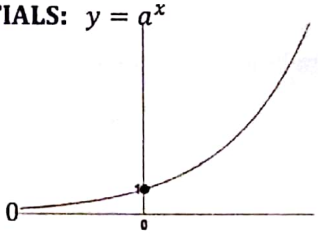
You Should Know ... EXP & LOG FUNCTIONS

Do not memorize without understanding! *Understand first,* THEN *memorize!*

EXPONENTIALS: $y = a^x$

With base a > 1
the graph is increasing.

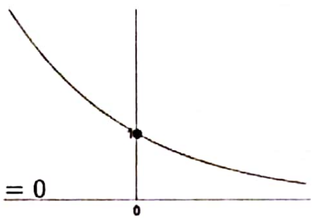
Domain $(-\infty, \infty)$
Range $(0, \infty)$
Horizontal Asymptote: y = 0



- Examples: 2^x, e^x, 3^x, π^x, 4^x, 5^x, etc.

With base 0 < a < 1
the graph is decreasing.

Domain $(-\infty, \infty)$
Range $(0, \infty)$
Horizontal Asymptote: y = 0



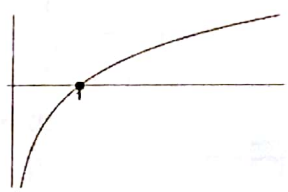
- Examples: $\left(\frac{1}{2}\right)^x$, $\left(\frac{1}{3}\right)^x$, $\left(\frac{1}{4}\right)^x$, etc.

Additional Facts
- $a^0 = 1$ therefore all graphs above go thru (0,1).

LOGARITHMS: $y = log_a(x)$

With base a > 1
the graph is increasing.

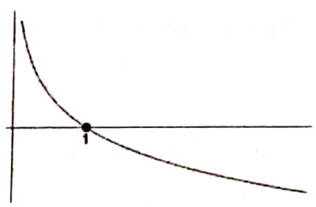
Domain $(0, \infty)$
Range $(-\infty, \infty)$
Vertical Asymptote: x = 0



- Examples: $log_2(x)$, $log_3(x)$, $log_\pi(x)$, etc.
- Note: $\ln(x)$ means $log_e(x)$
- Note: $\log(x)$ means $log_{10}(x)$.

With base 0 < a < 1
the graph is decreasing.

Domain $(0, \infty)$
Range $(-\infty, \infty)$
Vertical Asymptote: x = 0



- Examples: $log_{1/2}(x)$, $log_{1/3}(x)$, $log_{1/\pi}(x)$, etc.

Additional Facts
- $log_a(1) = 0$ therefore all logs above go thru (1,0).
- $log_a(a) = 1$
- $log_a(xy) = log_a(x) + log_a(y)$
- $log_a\left(\frac{x}{y}\right) = log_a(x) - log_a(y)$
- $log_a(x^y) = y \cdot log_a(x)$

EXP and LOG ARE INVERSES

Fact: $log_a(a^x) = x$ (log composed with exp cancels)
Fact: $a^{log_a(x)} = x$ (exp composed with log cancels)

TRUE OR FALSE

1. $ln(8 + x) = ln(8) + ln(x)$ F (log isn't linear)
2. $log_2(8x^{10}) = 10 \cdot log_2(8x)$ F (8 isn't raised to 10 power)
3. $log_2(8x^{10}) = log_2(8) + 10 \cdot log_2(x)$ T
4. $log\left(\frac{8-x^3}{y^{10}}\right) = log(8 - x^3) - 10 \cdot log(y)$ T
5. $log_3(3^{y-4}) = y - 4$ T

EXAMPLES

Example A. Expand using log rules: $log\left(\frac{7yx^2}{4+y^2}\right) =$

$\log(7) + \log(y) + 2\log(x) - \log(4+y^2)$

Example B. Condense into a single log expression:
$y \cdot log(8) + 10 \cdot log(x - y) - 3log(z) =$

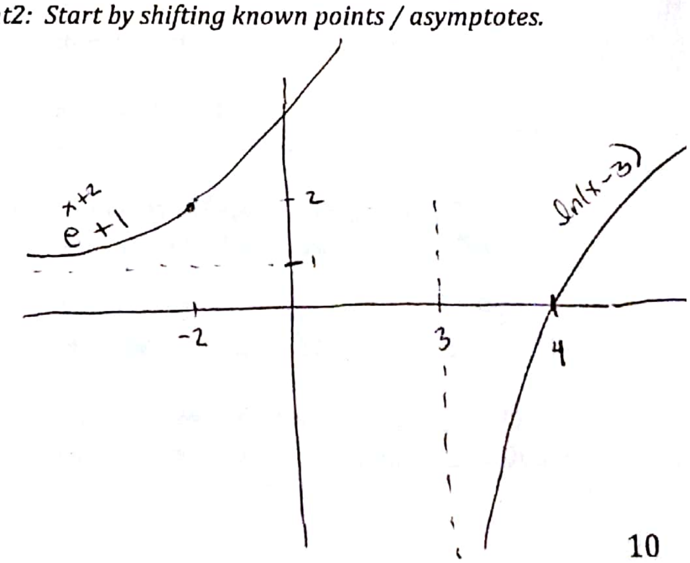
$$\log\left(\frac{8^y \cdot (x-y)^{10}}{z^3}\right)$$

Example C. Simplify:
$2 \cdot log(10^{3x-5}) + 10 \cdot ln(e) - 3log_{1/2}\left(\frac{1}{2^x}\right) =$

$2(3x-5) + 10 - 3x$

$= 6x - 3x = 3x$

Example D. Draw $y = e^{x+2} + 1$ and $y = \ln(x - 3)$ on the same set of axes. *Hint1: Remember shift rules. Hint2: Start by shifting known points / asymptotes.*



You Should Know ... DERIVATIVES

> Do not memorize without understanding!
> *Understand first,* THEN *memorize!*

RULES

$\frac{d}{dx}(cf(x)) = cf'(x)$ Multiplied constants are "pulled out"

$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$ Linearity

$\frac{d}{dx}(f(x) \cdot g(x)) = f(x)g'(x) + g(x)f'(x)$ Product Rule

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$ Quotient Rule

$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$ Chain Rule

$\frac{d}{dx}(f(ax)) = af'(ax)$ "Times-a rule" (a is a constant)

FACTS

$\frac{d}{dx}(c) = 0$ Derivative of a constant is zero

$\frac{d}{dx}(cx) = c$ Derivative of a linear function is constant

$\frac{d}{dx}(x^p) = px^{p-1}$ Power rule

$\frac{d}{dx}(e^x) = e^x$ Derivative of e^x is itself

$\frac{d}{dx}(ln(x)) = \frac{1}{x}$ Derivative of ln (x) is $\frac{1}{x}$

$\frac{d}{dx}(a^x) = a^x \cdot \ln(a)$ General exponential derivative rule

$\frac{d}{dx}(log_a(x)) = \frac{1}{\ln(a) \cdot x}$ General logarithm derivative rule

Derivatives of trig functions:

$\frac{d}{dx}(sin(x)) = cos(x)$ $\qquad$ $\frac{d}{dx}(cos(x)) = -sin(x)$

$\frac{d}{dx}(sec(x)) = sec(x)tan(x)$ $\qquad$ $\frac{d}{dx}(tan(x)) = sec^2(x)$

$\frac{d}{dx}(csc(x)) = -csc(x)cot(x)$ $\qquad$ $\frac{d}{dx}(cot(x)) = -csc^2(x)$

Derivatives of inverse trig functions:

$\frac{d}{dx}(Arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}(Arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(Arctan(x)) = \frac{1}{1+x^2}$ $\qquad$ $\frac{d}{dx}(Arccot(x)) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(Arcsec(x)) = \frac{1}{|x|\sqrt{x^2-1}}$ $\qquad$ $\frac{d}{dx}(Arccsc(x)) = \frac{-1}{|x|\sqrt{x^2-1}}$

EXAMPLES

Demo Example. *Note: Wise algebra can make it easier!*

$$\frac{d}{dx}\left(-cos(4x) + \pi^2 + \frac{sec^3(x)tan(x) + 1}{tan(x)}\right)$$

$$= \frac{d}{dx}(-cos(4x)) + \frac{d}{dx}(\pi^2) + \frac{d}{dx}(sec^3(x)) + \frac{d}{dx}(cot(x))$$

$$= 4sin(4x) + 0 + 3(sec^2(x)) \cdot sec(x)tan(x) - csc^2(x)$$

$$= 4sin(4x) + 3sec^3(x)tan(x) - csc^2(x)$$

Example A. Find the derivative:

$$\frac{d}{dx}\left(e^{-3x} + \frac{1}{x} + ln\left(3x^3 + \frac{1}{2}x^4\right) + \frac{\pi-2}{e}\right) =$$

$$-3e^{-3x} - \frac{1}{x^2} + \frac{(9x^2 + 2x^3)}{(3x^3 + \frac{1}{2}x^4)} + 0$$

Example B. Find the derivative. *Hint: Simplify first, and DO NOT use the quotient rule!*

$$\frac{d}{dx}\left(\frac{cos(2-\pi x)}{cot(2-\pi x)}\right) = \frac{d}{dx}\left(\sin(2-\pi x)\right)$$

$$= -\pi \cos(2-\pi x)$$

Example C. Find the derivative.

$$\frac{d}{dx}(log_3(5x-2) + Arcsin(5^x)) =$$

$$\frac{5}{\ln(3)\cdot(5x-2)} + \frac{5^x \cdot \ln(5)}{\sqrt{1-(5^x)^2}}$$

Example D. Suppose an object is moving with velocity

$$v(t) = \pi t^2 tan(t^3)$$

Find the acceleration.

$$a(t) = v'(t) = 2\pi t \cdot \tan(t^3) + \pi t^2 \cdot \sec^2(t^3) \cdot 3t^2$$

Example C. Choose among the following words/phrases in order to complete the sentences below: "positive", or "negative", or "moving left", or "moving right", or "increasing", or "decreasing".

- If $f'(x)$ is positive
  then $f(x)$ is increasing
- If $f'(x)$ is negative
  then $f(x)$ is decreasing
- If velocity $v(t)$ is positive
  then the object is moving right
- If velocity $v(t)$ is negative
  then the object is moving left
- If acceleration $a(t)$ is positive
  then the velocity $v(t)$ is increasing
- If acceleration $a(t)$ is negative
  then the velocity $v(t)$ is decreasing

Solutions are posted at www.jdambroise.com/studyaids

You Should Know ... ANTIDERIVATIVES

Do not memorize without understanding!
Understand first, THEN *memorize!*

NOTATION/TERMINOLOGY

- If $\frac{d}{dx}(f(x)) = f'(x)$ then $\int f'(x)dx = f(x) + C$.
  - Therefore $f(x)$ is antiderivative of $f'(x)$.
- If $\frac{d}{dx}(F(x)) = f(x)$ then $\int f(x)dx = F(x) + C$.
  - Therefore $F(x)$ is antiderivative of $f(x)$.

RULES

$\int cf(x)dx = c\int f(x)dx$ Multiplied constants "pull out"
$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$ Linearity
$\int f(ax)dx = \frac{1}{a}F(ax) + C$ "The $\frac{1}{a}$-rule" (a is a constant)
$\int f(g(x))g'(x)dx = F(g(x)) + C$ Substitution($u = g(x)$)
$\int u \cdot dv = uv - \int v \cdot du$ Integration by parts (IBP)

EXAMPLES with Linearity & $\frac{1}{a}$- Rule

Demo Example. *Note: Wise algebra is essential!*

$$\int \left(csc^2(4x) + \pi^2 + \frac{x^3+1}{5x}\right)dx$$
$$= \int (csc^2(4x))\,dx + \int \pi^2 dx + \frac{1}{5}\int x^2 dx + \frac{1}{5}\int \frac{1}{x}dx$$
$$= -\frac{1}{4}\cot(4x) + \pi^2 x + \frac{1}{15}x^3 + \frac{1}{5}\ln|x| + C$$

Example A. Find the antiderivative:
$\int \left(e^{-3x} + \frac{\sec^2(x)\tan(x)+8x^5\sec(x)}{\sec(x)} + \frac{\pi-2}{e}\right)dx=$

$$= \int e^{-3x}dx + \int sec(x)\tan(x)dx + \int 8x^5 dx + \int \frac{\pi-2}{e}dx$$
$$= \frac{-1}{3}e^{-3x} + \sec(x) + \frac{4}{3}x^6 + \left(\frac{\pi-2}{e}\right)x + C$$

EXAMPLES: Substitution

Demo Example.
$\int xsec^2(4x^2)\,dx$ Set $u = 4x^2$ therefore $du = 8xdx$.
Therefore $\int xsec^2(4x^2)\,dx = \frac{1}{8}\int sec^2(u)du$
$= \frac{1}{8}\tan(u) + C = \frac{1}{8}tan(4x^2)$+C

Example B. Find the antiderivative:
$\int \sin(3x)e^{\cos(3x)}dx=$ $\int -\frac{1}{3}e^u du$

$u=\cos(3x)$
$-\frac{1}{3}du = \sin(3x)dx$

$= \frac{-1}{3}e^{\cos(3x)} + C$

Example C. Find the antiderivative:
$\int \frac{1}{x+x\cdot\ln^2(x)}dx= \int \frac{1}{(1+\ln^2(x))}\cdot\frac{1}{x}dx$

$u=\ln(x)$
$du=\frac{1}{x}dx$

$= \int \frac{1}{1+u^2}du$

$= \text{Arctan}(\ln(x)) + C$

EXAMPLES: IBP

Demo Example.
$\int xsec^2(x)\,dx$ Set $u = x$ and $dv = sec^2(x)dx$.
Therefore $du = dx$ and $v = \tan(x)$.
Using the IBP formula:
$\int xsec^2(x)\,dx = xtan(x) - \int \tan(x)\,dx$
Now integrate tangent by using its definition
$= xtan(x) - \int \frac{\sin(x)}{\cos(x)}dx$
Use substitution with $u = \cos(x)$. *Write it out!!*
Final answer: $xtan(x) + \ln|\cos(x)| + C$.

Example D. Find the antiderivative:
$\int 2xe^{-x}dx=$ $\rightarrow -2xe^{-x} + 2\int e^{-x}dx$

$u=2x$ $dv=e^{-x}dx$
$du=2dx$ $v=-e^{-x}$

$= -2xe^{-x} - 2e^{-x} + C$

$= -2e^{-x}(x+1) + C$

Example E. Find the antiderivative (use IBP twice):
$\int x^2\cos(6x)dx=$

$u=x^2$ $dv=\cos(6x)dx$
$du=2xdx$ $v=\frac{1}{6}\sin(6x)$

$= \frac{1}{6}x^2\sin(6x) - \frac{1}{3}\int x\sin(6x)\,dx$

$u=x$ $dv=\sin(6x)dx$
$du=dx$ $v=-\frac{1}{6}\cos(6x)$

$= \frac{1}{6}x^2\sin(6x) - \frac{1}{3}\left(-\frac{1}{6}x\cos(6x) + \frac{1}{6}\int\cos(6x)dx\right)$

$= \frac{1}{6}x^2\sin(6x) + \frac{1}{18}x\cos(6x)$

$- \frac{1}{108}\sin(6x) + C$