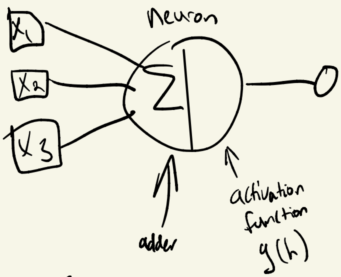
Artificial Intelligence

3-24-2023

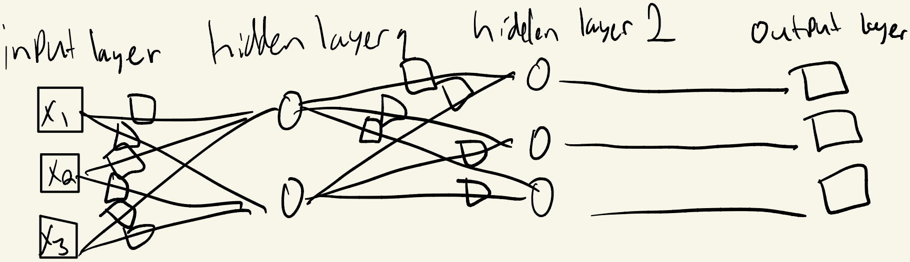
Neural Networks



$h = \sum x_i w_i$

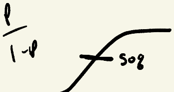
Linear Activation function: $g(x) = x$

Single neuron neural network → Same as Linear Regression



Logistic Regression

* Classification Algorithm
* Log-odds ratio } $\frac{p}{1-p}$
* Logit



$$\log_e\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$

$$\boxed{\frac{\partial}{\partial w}\left[g()\right]}$$

$$\cancel{(1-p)}\frac{p}{\cancel{1-p}} = e^{\vec{x}\vec{\beta}}(1-p)$$

$$p = e^{x\beta} - p(e^{x\beta})$$

$$p + p(e^{x\beta}) = e^{x\beta} = \boxed{p = \frac{e^{x\beta}}{1+e^{x\beta}}}$$

$$\boxed{\frac{e^{-x\beta}}{e^{-x\beta}} = 1}$$

$$\frac{1}{(e^{x\beta}+1)e^{-x\beta}}$$

$$\boxed{p = \frac{e^{x\beta}}{e^{x\beta}+1} = \frac{1}{1+e^{-x\beta}}}$$

easier to Derive

Take log-odds Ratio

Calculate the following Derivative:

$$\frac{d}{dx}\left[\frac{1}{1+e^{-x}}\right] = \frac{d}{dx}\left[(1+e^{-x})^{-1}\right]$$

$$= (-1)(1+e^{-x})^{-2} \cdot (0\ 1$$

$$\boxed{\text{Derivative } \frac{e^{-x}}{(1+e^{-x})^2}}$$

$$\boxed{\text{Idea: } +1-1=0}$$

$$\frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$$

$$= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}+1-1}{1+e^{-x}}$$

$$\frac{1}{1+e^{-x}} \cdot \left[\frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}}\right]$$

$$= \frac{1}{1+e^{-x}} \cdot \left[1 - \frac{1}{1+e^{-x}}\right]$$

$g(x)$ = logistic / sigmoid activation function

$$\boxed{g'(x) = g(x) \cdot (1 - g(x))}$$

derivative of activation function.