Foundations of AI Part I

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Consider the following algorithm:

$$\sum_i \sum_j (x_i - \bar{x_j})^2$$

These symbols represent certain functions in relation to the overall algorithm. The summation function is useful for adding whole units.

$$\int \int (x_i - \bar{y_j})^2 dx\ dy$$

We have different tools to add things that don't have a definite whole shape. We can use integrals to add them. This distinction is known as being *discrete* and *continuous*.

Discrete data	Continuous data
Summation	Integral

Parts of a Function

Functions consist of a name, domain, and range.

$$name\ :\ domain \rightarrow range\ (codomain)$$

$$f\ :\ \mathbb{N} \rightarrow \mathbb{N}$$

Consider the following function:

```
def f(x):
        if x>10;
                return 1
        else
                return 2
```

The possible outputs for this function is 1 and 2. The domain is any integer and the range is 1 and 2. The range is a subset of the codomain.

Let's break down this function.

$$g(-30, 132)$$

The name of this function is g. g accepts two parameters that are negative of positive integers.

$$g\ :\ \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{N}$$

Function g takes the cartesian product of its parameters.

Consider the function:

$$f\ :\ \{0, 1\}^5 \rightarrow \{pos, neg\}$$

Name	f
Domain	{0,1}^5
Range	Positive and Negative Integer

According to this notation, a valid input would be:

$$f(0,0,1,0,1)$$

Derivatives

There are several notations for derivatives.

$$\frac{d}{dx},\ f'x,\ y'$$

Programmatically, these represent function assignments.

```
double y = f(x);
```

We can use the $\frac{d}{dx}$ notation to avoid confusion when we relate to coding.

A higher order function accepts a function and outputs a function.

$$\frac{d}{dx}[f(x)]$$

Consider:

$$y = f(x)$$

Represented by:

$$\frac{d}{x}[y] = \frac{d[y]}{dx} = \frac{dy}{dx}$$

Parts Of A Derivative

$$\frac{d}{dy}[f(x)]$$

y represents the variable of interest. x represents an input variable. The variable of interest determines how to derive an answer with respect to that variable in a given function.

Applications of Derivatives

Derivatives are useful when finding minima and maxima of a function.