# Sigmoid Proof

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$$\log_e\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_p x_p$$

$$\log_e\left(\frac{p}{1-p}\right) = x\beta$$

$$\cancel{e^{\log_e}}\left(\frac{p}{1-p}\right) = e^{x\beta}$$

$$\left(\frac{p}{1-p}\right) = e^{x\beta}$$

$$\cancel{(1-p)}\left(\frac{p}{\cancel{1-p}}\right) = e^{x\beta}(1-p)$$

$$p = e^{x\beta}(1-p)$$

$$p = e^{x\beta} - pe^{x\beta}$$

$$p + pe^{x\beta} = e^{x\beta}$$

$$p(1+e^{x\beta}) = e^{x\beta}$$

$$\frac{p(1+e^{x\beta})}{1+e^{x\beta}} = \frac{e^{x\beta}}{1+e^{x\beta}}$$

$$\frac{p\cancel{(1+e^{x\beta})}}{\cancel{1+e^{x\beta}}} = \frac{e^{x\beta}}{1+e^{x\beta}}$$

$$p = \frac{e^{x\beta}}{1+e^{x\beta}}$$

$$\frac{e^{-x\beta}}{e^{-x\beta}} \cdot \frac{e^{x\beta}}{1+e^{x\beta}} = p$$

$$p = \frac{1}{1+e^{-x\beta}}$$

$$\frac{d}{dx}\left[\frac{1}{1+e^{-x\beta}}\right]$$

$$\frac{d}{dx}\left[(1+e^{x})^{-1}\right]$$

$$= (-1)(1+e^{x})^{-2} \cdot (-e^{-x})$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{x}}$$

$$= \frac{1}{1+e^{-x}} \cdot \left[\frac{\cancel{1+e^{-x}}}{\cancel{1+e^{-x}}} - \frac{1}{1-e^{-x}}\right]$$

$$= \frac{1}{1+e^{-x}} \cdot \left[1 - \frac{1}{1-e^{-x}}\right]$$