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Chemical Engineering and Processing 36 (1997) 219–229

Chemical Engineering and Processing

Studies of sedimentation in settlers with packing

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Received 29 June 1996; accepted 20 October 1996

Abstract

This paper deals with packed-type (multicurrent) settlers, which — compared to conventional ones — display a number of advantages. Two experimental stands, viz. with tubular elements and plate packing, have been constructed. Eight kinds of suspensions were tested. The results of investigations of tubular and plate settlers constituted a basis for the determination of sedimentation effectiveness being a function of dimensionless groups. Moreover, the experimental results have served to verify models and calculational methods, quoted in the literature. © 1997 Elsevier Science S.A.

Keywords: Sedimentation; Packed-type settlers; Tubular settlers; Plate settlers; Settlers' effectiveness

1. Introduction

The sedimentation process takes place as a result of the action of the gravitational force field upon the solid particles in such a way that additional investment costs are not required, the latter factor constituting a considerable advantage of this process. This process occurs in apparatuses known as settlers. As their successors packed-type settlers may be considered. Compared with conventional settlers, the former display a number of advantages, viz. they are inexpensive, highly effective and may operate at high flow rates of a suspension. The packed-type (multicurrent) settlers (Fig. 1) are used in the technologies of raw materials preparation, in utilisation-type technologies, and in the purification and treatment of water for industrial or municipal purposes. Moreover, they may find application in industrial processes, replacing traditionally used filters or sedimentation centrifuges with high energy consumption and high investment costs.

This versatility of application of packed-type settlers and their economical significance have stimulated numerous research tasks and practical applications aimed at the improvement of the operation of existing settlers as well as the design of new ones.

2. Theoretical principles

Initial studies aimed at the design of settlers — with plate — or tubular packing, based on experimental studies, concerned the sedimentation of selected suspensions. The results of these investigations, usually presented graphically or in the form of qualitative recommendations, had a limited scope of application.

The 1960s brought with them the development of theoretical research, leading to the determination of analytical relationships, enabling calculation of the length of tubes or plates. The starting point in these considerations was the assumption of a definite flow profile of liquid, a method of settling of particles and determination of the trajectory of their motion.

The simplest model (Table 1), proposed by Culp [1,2], is based on a rectangular velocity distribution and assumes that the trajectory of particles' motion is rectilinear. The condition for the separation of a solid particle from suspension is the intersection of the tube surface by the trajectory of particle motion in the outlet section.

Yao [3] assumed in his studies laminar liquid flow in a duct. He has determined the conditions which have to be fulfilled by the trajectory of a particle, independently of the shape of the duct, in order to stop a particle in it. Yao has introduced in his equations the parameter S, the value of which depends on conduit shape, as well as the ratio of settling velocity and liquid flow velocity

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PII S0255-2701(96)04190-6

which becomes stabilised for systems with relative length exceeding 40.

Successive mathematical models of the sedimentation process, taking into consideration the liquid flow velocity profile, confine themselves to the assumption of fully developed laminar flow in spite of the fact that any other case of laminar flow development may occur here, from the rectangular profile to the characteristic parabolic distribution.

Olszewski and Suchanecka [4], based on Yao's formula and results of testing the plate packing, have elaborated an experimental equation for overall sedimentation effectiveness.

The group of models assuming laminar liquid flow in a tube also include the model presented by McMichael [5]. For inclined tubes the trajectory of solid particles in a three-dimensional coordinate system has been determined; also the existence of two tube regions, in which the solid particles countermove, has been proved.

The solid particle trajectory in the model of Haba, Nosowicz and Pasiński [6] has been described by means of the Hazen number; also, the critical diameter of a particle, required for calculation of the overall sedimentation effectiveness, has been determined.

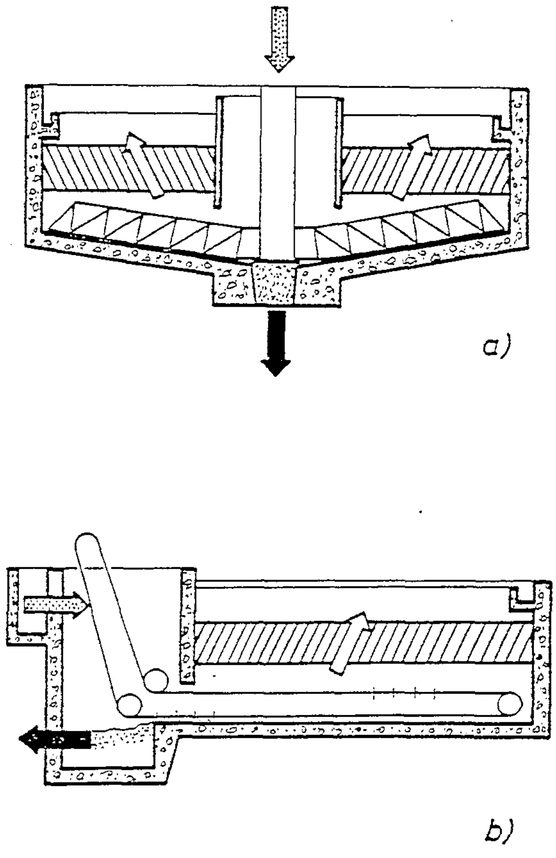

Fig. 1. Countercurrent settlers with packing: (a) circular settler; (b) rectangular settler.

Willis' model [7], deduced on an experimental basis, constitutes an approximation, which, despite the assumption of a non-typical trajectory (vertical line) of a solid particle and a very simple mathematical form, finds a full confirmation in the design of settlers with tubular packing.

In Czajkowski's model [8] it is assumed that the radius of a particle is a random variable obeying the logarithmic–normal distribution, and the overall sedimentation effectiveness depends on the characteristic parameter p.

Niedźwiedzki [9] has worked out an equation of the trajectory of a critical diameter particle; also, the range of optimal solutions, considering the construction of packing and its operation, has been determined.

The sedimentation model, suggested by Kowalski [10], is based on the transformation of the probability density function of particle size distribution to the probability density function of particle velocity distribution.

A different approach is presented in the model given in [11], derived by dimensional analysis, in which dimensionless groups occur, taking into account the dynamic, physical and geometrical characteristics of a system.

3. Investigation of packed-type settlers

The analysis of a sedimentation process occurring in packed-type settlers, comprising technological applications and calculational models, enables one to state that the design methods, found in the literature, are not of a general character. The restrictions refer to the physico-chemical features, to hydraulic parameters, or — finally — to the constructional solutions of settlers.

In such a situation the idea of the verification of settlers is justified; it involves:

- Evaluation of tested materials by the analysis of the particle size distribution function and particle density.
- Estimation of the effect of settler's construction (representative solutions: plate and tubular model). These models constitute a basis of modified packings, being at the same time a subject of solutions of a main group of mathematical models.
- Estimation of the effect of hydraulic parameters.

The accomplishment of such a program required the construction of experimental stands, taking into account the possibility of variation of basic parameters of the sedimentation process, as well as constructional parameters with the allowance for the principles of rational scale-up.

Table 1
Models for settlers with tubular- or plate packing

No	Author	Equations	Remarks
1	Culp, Hansen, Richardson [1,2]	$\frac{w_p - w_{0g}\sin\alpha}{l} = \frac{w_{0g}\cos\alpha}{D}$	Tubular packing
2	Yao [3]	$S = \frac{w_0}{w_p}(\sin\alpha + L\cos\alpha)$	Tubular and plate packing
3	Olszewski, Suchanecka [4]	$\eta_{ov} = 1 - 0.7\left[\frac{1}{w_p}(\sin\alpha + L\cos\alpha)\right]^{-0.45}$	Plate packing (counterflow) w_p (mm/s)
4	McMichael [5]	$L = \frac{4w_p}{3w_{0g}\cos\alpha}\left(1 - \frac{w_{0g}\sin\alpha}{2w_p}\right)^{3/2}$	Tubular packing
5	Haba, Nosowicz, Pasiński [6]	$\eta_{ov} = \int_{d_{min}}^{d_g} f(d)\eta(d)\,\mathrm{d}d + \int_{d_g}^{d_{max}} f(d)\,\mathrm{d}d$ $d_g = 3\sqrt{\frac{2\mu_c}{(\rho_s - \rho_c)g}\frac{w_p h}{\left(1 - \frac{2h}{\sin 2\alpha}\right)\cos\alpha}}$	Plate packing (cocurrent flow)
6	Willis [7]	$L = \frac{\gamma KV}{F_c\sin\alpha\cos\alpha}$	Tubular packing
7	Czajkowski [8]	$\eta_{ov} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{p}\exp\left(-\frac{1}{2}L^2\right)\mathrm{d}L$ where $p = \frac{\ln\left\{\frac{A(\rho_s - \rho_c)g\cos\alpha}{\eta_{ov}}(L + \mathrm{tg}\,\alpha)\right\} + 2(m_r + 3\sigma_r^2)}{2\sigma_r}$	Tubular and plate packing A-geometric parameter
8	Niedźwiedzki [9]	$\eta_i = \frac{1}{h}\frac{d_i^2 k\cos\alpha}{w_p - d_i^2 k\sin\alpha}$ $k = \frac{(\rho_s - \rho_c)g}{18\mu_c}$	Plate packing
9	Kowalski [10]	$\eta_{ov} = \int_{w_{0\,min}}^{w_{0g}} \eta(w_0)f(w_0)\,\mathrm{d}w_0 + \int_{w_{0g}}^{w_{0\,max}} f(w_0)\,\mathrm{d}w_0$ $f(w_0) = \frac{1}{\sqrt{2\pi}\,\sigma_w w_0}\exp\left[-\frac{1}{2}\left(\frac{\ln w_0 - m_w}{\sigma_w}\right)^2\right]$ $m_w = 2m_d + \ln\left[\frac{(\rho_s - \rho_c)g}{18\mu_c}\right]$ $\sigma_w = 2\sigma_d$	Tubular and plate packing
10	Hehlmann [11]	$\mathrm{Mo}^* = x_1\left(\frac{\rho_s - \rho_c}{\rho_c}\right)^{x_2}\mathrm{Ga}^{x_3}\left(\frac{w_0}{w_p}\right)^{x_4}\left(\frac{l}{h}\right)^{x_5}\left(\frac{l_s}{h}\right)^{x_6}\left(\frac{n}{n_0}\right)^{x_7}$	Tubular and plate packing

3.1. Stand for the study of sedimentation in a tubular settler

The purpose of constructing such a stand was the determination of sedimentation effectiveness for selected suspensions in a tubular element representing the operation of a tubular settler.

The scheme of the experimental unit is presented in Fig. 2. This stand comprises: suspension tank 1, circulation pump 2, tubular settler with the sedimentation tube 3, suspension head 4 with supply of suspension and venting, clear liquid head 5 with venting. Both heads have drains — for sludge and clarified liquid — to tank 1. In the pipeline system pipe connectors and clamp fittings 6, of 'Akwar' type, have been employed. The sampling of test suspension from the circulation system takes place by means of a concentric probe installed in the T-connection 7. The control of flow rates occurs by means of ball valves Z1–6.

The circulation tank, stabilising the experimental conditions, has a capacity of 60 l, and the pump 2, operating in a sink system, has a delivery amounting to 4.5 m^3/h. In the settler the polypropylene and organic glass tubes, with internal diameters, $D = 0.023$ and 0.030 m, lengths $l = 0.950$, 1.200 and 2.200 m, and inclination angles $\alpha = 30$, 45 and 60°, have been employed.

The height of the settler in its extreme position is equal to ca. 3.0 m. The measurement of flow rates (FIR-01,02) has been executed by means of an electromagnetic meter of EMF 51-12 type with an EKT-7 transducer and SR 106-71 E 850 000 recorder; the measurement range of the set amounted to (0.88 ± 17.7) m^3/h with an error of $\pm 1.6\%$. The flow rate measurements (FI-03) have been accomplished by means of a non-invasive 'Digital Doppler Flowmeter' of HMF type with a range (0.3 ± 6.0) m/s and reading error of $\pm 0.2\%$. The temperature

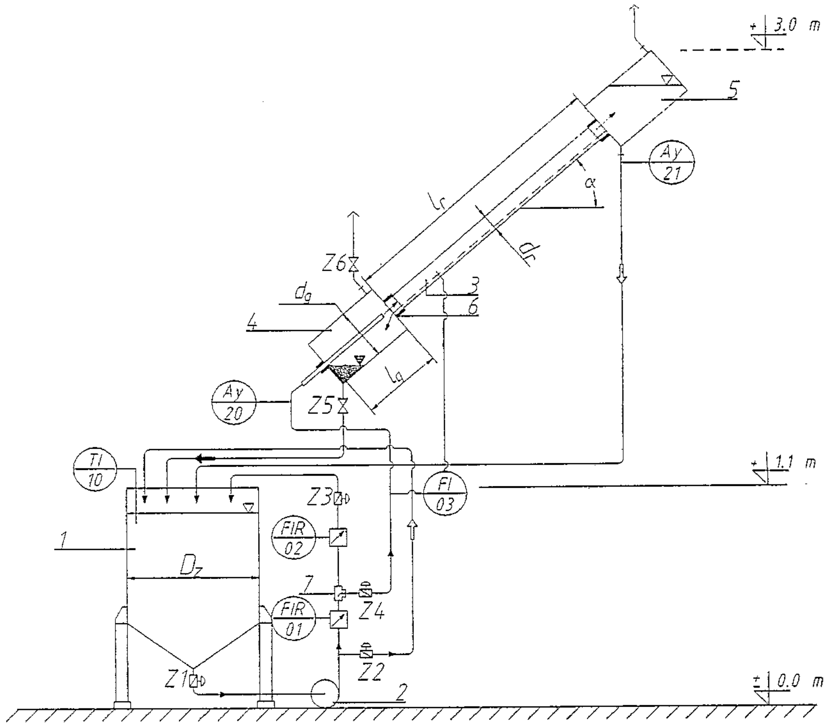

Fig. 2. Scheme of a stand for the investigation of sedimentation in a tubular settler.

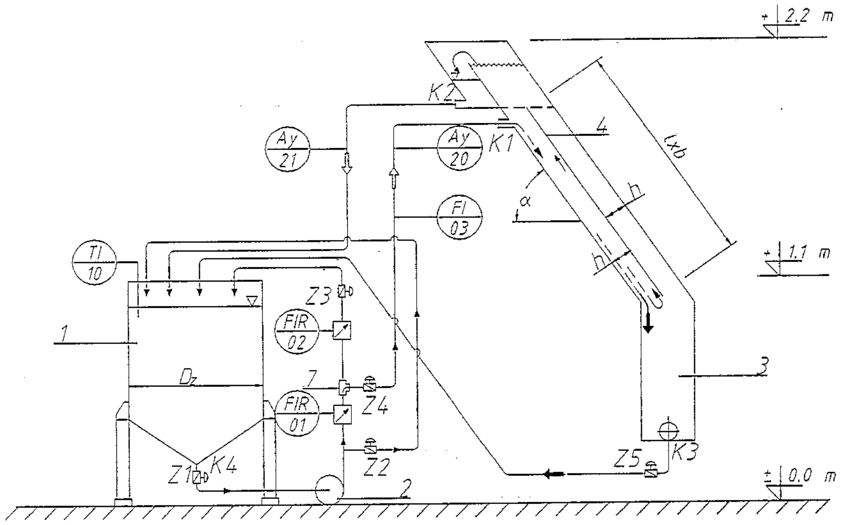

Fig. 3. Scheme of a stand for the investigation of sedimentation in a plate (lamella-type) settler.

(T 1–10) has been measured by means of a standard digital thermocouple of class 1.0.

The appropriate experiments have been conducted in 14 series (runs), the variable parameters being:

- the type of suspension (material, particle size distribution);
- process parameters: suspension flow rate, initial concentration;
- geometric parameters: tube diameter D, tube length l, inclination angle α.

After stabilisation of the flow velocity, 100 ml samples (Ay-20 and 21) were taken at appropriate time intervals. Next, the sediment from bottom head 4 was drawn off and a successive flow velocity w_p underwent stabilisation.

All samples were filtered using filter paper, dried and weighed beforehand. Afterwards the filter paper was dried in a laboratory dryer for ca. 12 h at 110°C and weighed again. In this manner sediment masses at a constant sample volume for each measurement were obtained. Then, the overall process effectiveness for individual measurements was calculated using:

$$\eta_{ov} = \frac{\Delta m_p - \Delta m_k}{\Delta m_p} = \frac{c_p - c_k}{c_p} \tag{1}$$

After completion of a run, the stand was prepared for a new experiment.

3.2. *Stand for the study of sedimentation in a plate-type settler*

A scheme of the setup is presented in Fig. 3. This stand is constructed as a large laboratory unit, making it possible to scale-up. The unit is provided with a system of suspension preparation, as in the previous case.

The settler 3 is fed through two pipe connectors K1, introduced to lateral chambers. Each of them has individual deaeration. The suspension flows in the direction of the settling chamber, in which the flow direction changes upwards through plate packing 4.

The clarified liquid flows, through an overflow, to the outlet chamber, and next, by means of stub connector K2, to the circulation tank 1. The sediment flows gravitationally from the plates to the bottom of the chamber and, after each measurement, is removed periodically through the pipe connector K3. The settler is fixed by joints, and so various inclination angles α may be attributed to the plate system. The experimental procedure is identical to the case of the tubular settler; hence it does not require a separate description.

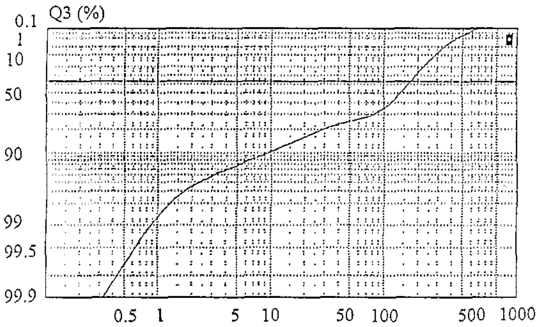

Fig. 4. Particle size distribution for a selected sample of quartz sand (item 1, Table 2).

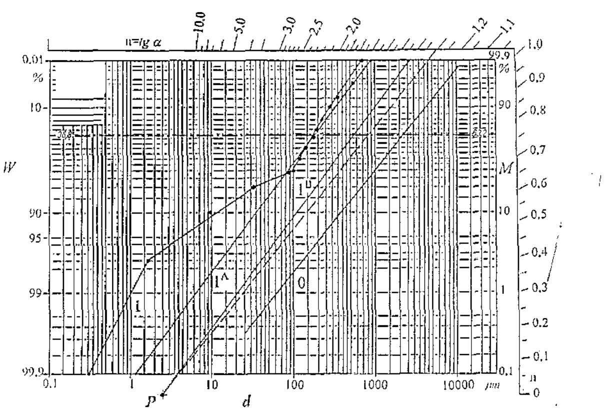

Fig. 5. Transformation of a particle size distribution represented by a broken line to a rectilinear equivalent RRSB distribution for the quartz sand (item 1, Table 2).

The technical characteristics of the plate settler are as follows:

Height of settler	$H = 2.2$ m
Settling chamber:	
distance between plates	$h = 7.1 \times 10^{-5}$, 3.25×10^{-5}, 1.575×10^{-5} m
width	$b = 0.8$ m
length	$l = 1.0$ m
Number of plates	$z = 1, 2, 4$
Inclination angle	$\alpha = 30, 45, 60°$
Working capacity	$V_n = 0.16$ m^3
Plate material	Textolite

3.3. *Characteristics of solid materials used*

The particle characteristics were varied by grinding the basic sample. The particle size distribution analysis was performed by means of the laser particle sizer 'Fritsch-Analysette 22'. An exemplary printout, for quartz sand 1, is presented in Fig. 4. This printout corresponds to the RRSB model. The read-out: $d_{50} = 141.93$ μm, $d' = 174.62$ μm.

In case of some solids used, the particle size distributions, obtained as above, are represented by a broken line. Thus, it has been accepted that the assumption of RRSB distribution is appropriate; in the description of the latter, however, one should employ the equivalent value of exponent n. It has has suggested to calculate this exponent additively, according to the relationship:

$$n = \sum n_i r_i \tag{2}$$

$$r_i = \frac{\Delta M(d)}{100} \tag{3}$$

The particle-size distribution in the RRSB system has the following characteristics: $M(d) = 50\%$, $d_{50} = 141.93$ μm, and $n = 1.400$; for a quartz sand sample (Fig. 5) this represents a straight line.

The density of solid particles has been determined by means of a pycnometer with the application of methyl alcohol. The experimental and calculational procedure corresponds to the Polish Standard PN-81/C-04307 'Determination of true density, apparent density and porosity'. The results are listed in Table 2.

Table 2
Results of analyses of solid particles used in experiments

No.	Material	Density, ρ_s (kg/m^3)	Particle diameter (µm)		Exponent, n
			d'	d_{50}	
1	Quartz sand 1	2761.0	184.40	141.93	1.400
2	Quartz sand 2	2761.0	31.53	23.65	1.274
3	Quartz sand 3	2761.0	98.77	82.72	2.066
4	Slag dust	2390.0	39.88	29.52	1.218
5	Ash	3222.3	16.24	12.51	1.405
6	Talc ('micro')	2080.0	159.29	133.30	2.057
7	Talc (technical)	1981.3	122.71	89.31	1.154
8	Coal dust	1352.9	55.31	43.53	1.530

3.4. Investigations of sedimentation in the tubular settler

The experiments have been performed in the unit presented in Fig. 1. The specification and characteristics of individual runs (aqueous suspensions being used):

- Quartz sand 1, settling velocity $w_0 = 1.745 \times 10^{-2}$ m/s.
- Quartz sand 2, settling velocity $w_0 = 5.35 \times 10^{-4}$ m/s.
- Coal dust, settling velocity $w_0 = 3.38 \times 10^{-4}$ m/s.
- Parameters of the settler — tube diameter $D =$ 0.023, 0.030 m; length $l = 0.950$, 1.200, 2.200 m.
- Angle of inclination of the tube $\alpha = 30$, 45, 60°.

The experimental results comprise 14 runs; the selected series are presented in Table 3.

3.5. Investigation of sedimentation in the plate-packed (lamellar) settler

The experiments were carried out in a large laboratory-scale unit (Fig. 3). The methodology was similar to the case of the tubular settler.

The specification and characteristics of individual runs (aqueous suspensions being used) were as follows:

- Quartz sand 3, settling velocity $w_0 = 6.708 \times 10^{-3}$ m/s.
- Slag dust, settling velocity $w_0 = 9.43 \times 10^{-4}$ m/s.
- Dust settling velocity $w_0 = 1.90 \times 10^{-4}$ m/s.
- Talc ('micro'), settling velocity $w_0 = 1.017 \times 10^{-2}$ m/s.
- Talc (technical), settling velocity $w_0 = 4.272 \times 10^{-3}$ m/s.
- Settler's parameters: number of plates $z = 1$, 2, 4; angle of inclination $\alpha = 30$, 45, 60°.

The experiments were performed in 13 runs. Table 4 presents the results of selected experiments and calculated values of overall sedimentation effectiveness. The experiments carried out constituted the basis of a comparative analysis of selected models and served also to determine the correlation equations according to the present model of sedimentation for the tubular and plate system.

4. Elaboration of results

The results of investigations have been worked out in two particular bearings. The first of them concerned the regression calculations of our own equation based on the formula for overall sedimentation effectiveness:

$$\eta_{ov} = \frac{c_p - c_k}{c_p} = 1 - \exp(-\mathrm{Mo}^*) \tag{4}$$

$$\mathrm{Mo}^* = k_s / w_p \tag{5}$$

where the sedimentation number Mo* is defined as (k_s is the coefficient characterizing the sedimentation kinetics).

The sedimentation number Mo* is described by an equation obtained from the analysis of the phenomenon of particle settling during suspension flow through an inclined tubular or plate system:

$$\mathrm{Mo}^* = x_1 \mathrm{Ar}^{x_2} \left(\frac{w_0}{w_p}\right)^{x_3} \left(\frac{l}{D}\right)^{x_4} (\Pi_\alpha)^{x_5} \left(\frac{n}{n_0}\right)^{x_6} \tag{6}$$

where $\Pi_\alpha = \mathrm{tg}\,\alpha$.

The second bearing concerned an analytical–graphical comparison of selected process models on the grounds of a consistent data basis, obtained in our own research programme, including also Binder's data basis [12,13], the latter comprising aqueous and glycerol solutions.

4.1. Settler with tubular packing

The experimental results have been worked out on the basis of 65 data points; the corresponding correlation equation has the form:

Table 3
Results of selected measurements of overall sedimentation effectiveness in a tubular settler

No.	$d \times 10^6$ (m)	$D \times 10^3$ (m)	l (m)	α (°)	t (°C)	$w_p \times 10^3$ (m/s)	$m_{0p} \times 10^4$ (g)	$m_{1p} \times 10^4$ (g)	$\Delta m_p \times 10^4$ (g)	$m_{0k} \times 10^4$ (g)	$m_{1k} \times 10^4$ (g)	$\Delta m_k \times 10^4$ (g)	η_{ov}
Kind of suspension: coal dust in water													
Solid density: $\rho_s = 1352.9$ kg/m^3													
1	43.53	23	0.95	45	17	11.1	3957	6015	2058	3976	4736	760	0.6307
2	43.53	23	0.95	45	17	16.3	3911	6061	2150	3787	4673	886	0.5879
3	43.53	23	0.95	45	17	48.6	3831	6255	2424	3758	5106	1348	0.4439
4	43.53	23	0.95	45	17	63.9	3691	6292	2601	3780	5494	1714	0.3410
5	43.53	30	2.20	45	17	8.0	3921	5198	1277	3743	4095	352	0.7244
6	43.53	30	2.20	45	17	16.4	3895	5643	1748	3766	4399	633	0.6379
King of suspension: quartz sand in water													
Solid density: $\rho_s = 2761$ kg/m^3													
7	141.93	30	2.20	30	20	35.2	3963	6110	2147	3711	4103	392	0.8174
8	141.93	30	2.20	30	20	48.2	3841	6634	2793	4169	4754	585	0.7905
9	141.93	30	2.20	30	20	54.2	3777	6351	2574	3828	4441	613	0.7618
10	141.93	30	2.20	30	20	64.7	3908	7791	3883	3711	4597	886	0.7718
11	141.93	30	2.20	45	20	32.8	3965	6041	2076	3740	4122	382	0.8160
12	141.93	30	2.20	45	20	65.5	4122	8785	4663	3947	5176	1229	0.7364

Table 4
Results of selected measurements of overall sedimentation effectiveness in a plate type settler

No.	$d \times 10^6$ (m)	$h \times 10^3$ (m)	l (m)	α (°)	t (°C)	$w_p \times 10^3$ (m/s)	$m_{0p} \times 10^4$ (g)	$m_{1p} \times 10^4$ (g)	$\Delta m_p \times 10^4$ (g)	$m_{0k} \times 10^4$ (g)	$m_{1k} \times 10^4$ (g)	$\Delta m_k \times 10^4$ (g)	η_{ov}
Kind of suspension: quartz sand (3) in water													
Solid density: $\rho_s = 2761.0$ kg/m^3													
1	82.75	71.00	1	30	21	3.291	4874	7758	2884	4835	4990	155	0.9462
2	82.75	71.00	1	60	21	0.572	4243	11843	7600	4085	4121	36	0.9952
3	82.75	32.50	1	30	24	4.351	4729	9278	4549	4553	4685	132	0.9710
4	82.75	32.50	1	30	24	4.050	4757	6800	2043	4825	4844	19	0.9907
5	82.75	32.50	1	30	24	3.651	4662	5579	917	4600	4607	7	0.9924
6	82.75	32.50	1	30	24	3.307	4884	5389	505	4770	4775	5	0.9901
Kind of suspension: slag dust in water													
Solid density: $\rho_s = 3290.0$ kg/m^3													
7	29.52	15.75	1	45	15	7.811	4052	12561	8509	4079	4753	674	0.9207
8	29.52	15.75	1	45	15	5.658	4130	11437	7307	4177	4758	581	0.9205
9	29.52	15.75	1	45	15	3.666	4016	11086	7070	4132	4477	345	0.9512
10	29.52	15.75	1	60	15	7.632	3931	10131	6200	4047	4953	906	0.8539
11	29.52	15.75	1	60	15	5.634	4097	9380	5283	4091	4710	619	0.8828
12	29.52	15.75	1	60	15	3.564	3943	9524	5581	4032	4319	287	0.9486

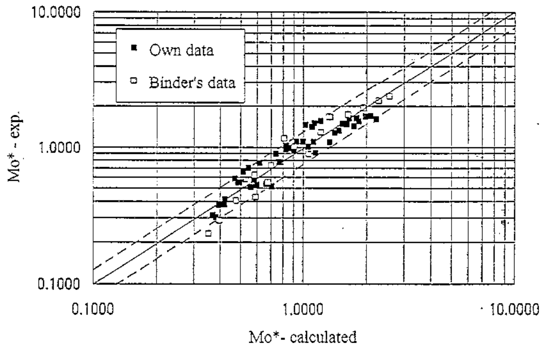

Fig. 6. Comparison of Mo* numbers (experimental values vs. the calculated ones—authors' model) for a settler with tubular packing.

$$\mathrm{Mo}^* = 0.2872\mathrm{Ar}^{-0.1360}\left(\frac{w_0}{w_p}\right)^{0.4116}\left(\frac{l}{D}\right)^{0.6333}(\mathrm{tg}\,\alpha)^{-0.1571}\left(\frac{n}{n_0}\right)^{0.0532} \quad (7)$$

The confidence level assumed was equal to $p = 95\%$. The relative error amounted to $\pm 23.5\%$; the equation holds for the following range of dimensionless groups:

$$\mathrm{Ar} = 0.0047\text{–}48.86; \qquad \frac{w_0}{w_p} = 0.0039\text{–}1.71;$$

$$\frac{l}{d} = 25.64\text{–}73.33; \qquad \mathrm{tg}\,\alpha = 0.57\text{–}1.73;$$

$$\frac{n}{n_0} = 1.0\text{–}9.8$$

the standard value of n_0 being equal to 1.25.

The regression coefficients appearing in Eq. (7) determine the effect of individual dimensionless groups. The Archimedes number characterises the effect of inertial and gravitational forces as well as the forces of buoyancy and resistance of the medium, exerted on a particle undergoing settling in the suspension stream. The velocity number (w_0/w_p) constitutes a simplified ratio of Reynolds number for a settling particle and suspension stream. An increase in this quantity results in an increase in the value of Mo*.

The relative tube length number (l/D) is a geometrical (constructional) parameter. Its increase causes a rise in the value of Mo* and may be obtained either by augmentation of tube length l or by reduction of tube diameter D. The increase of angle α causes a reduction in the value Mo*. This effect is evident and results from the rise in the settling path of a particle.

The particle size distribution number affects to a small extent the Mo* value. As mentioned before, a standard value $n_0 = 1.25$, corresponding to average distributions for quartz sand, has been assumed. The augmentation of sedimentation effectiveness will occur with an increase in n (decrease of polydispersity). For a monodisperse system $n = \infty$.

The comparative analysis of individual methods, discussed in the Introduction, has been accomplished on the grounds of obtained data and our own model of the process. The aim of this analysis is the estimation of usefulness of individual methods in design procedures. Fig. 6 presents a comparison of sedimentation number Mo* — obtained from experiments and calculated according to Eq. (7). This figure also includes the experimental data of Binder. The inclusion of this set is of some importance, since it concerns, amongst others, the settling in glycerol solutions at a high value of the

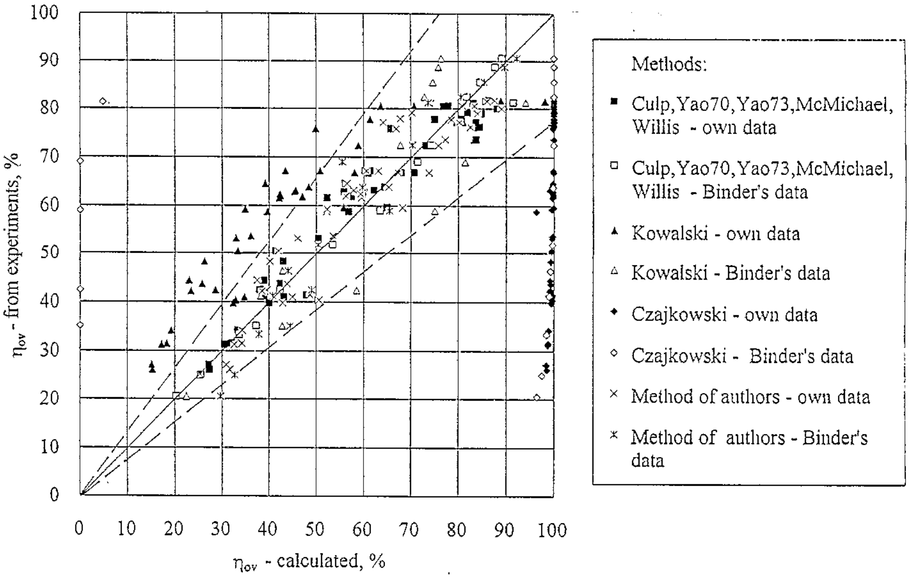

Fig. 7. Comparison of overall sedimentation effectiveness — experimental values vs. those calculated according to individual methods for a settler with tubular packing.

n/n_0 parameter, amounting to 9.8. The analysis of this diagram enables one to state that Eq. (7) correlates well both our data and that of Binder, which indicates that the proposed method is of a general nature. This thesis is supported by the results shown in Fig. 7. In this graph a scatter zone of $\pm 23.5\%$, corresponding to the relative error of the correlation Eq. (7), has been assumed.

The analysis of data shown makes it possible to formulate the following conclusions:

1. The methods of Culp [1], Yao [3], McMichael [5] and Willis [7] may be considered as satisfactory with respect to our data and that of Binder, the accuracy limits being within the scatter zone. Hence, it may be stated that they are suitable for the design. Some inconvenience in their application is linked with their complex analytical form.
2. Kowalski's method shows an excessive deviation with respect to our data. The calculated values are ca. twice as small as the experimental ones. This method displays better results with respect to Binder's data, comprising, however, quasimonodisperse suspensions with a value of $n = 12.25$. Thus, the method discussed has a limited application.
3. Czajkowski's method yields specific results, strongly deviating from the other ones. The distribution of our data and of Binder's data, both referring to aqueous suspensions, is, however, consistent. At the same time an essential deviation of Binder's data concerning glycerol solutions (close to the ordinate) is noticed. This is not the case with our model, which correlates the whole set of data in the standard scatter zone.
4. The method described in this paper holds well for the whole set of data in the assumed scatter zone.

Summing up, it may be assumed that our method and the methods of Culp, Yao, McMichael and Willis are comparable from the point of view of calculational accuracy. Hence these methods may be applied in a similar range of dimensionless groups and types of suspensions. The choice of method may be influenced in individual cases by general optimisation criteria.

4.2. Settler with plate packing (lamellar settler)

The experimental results together with those obtained from individual methods discussed in the Introduction comprised 272 data points. Treatment of these data by the least-squares method led to the following correlation equation:

$$\mathrm{Mo}^* = 0.3700\mathrm{Ar}^{-0.1269}\left(\frac{w_0}{w_p}\right)^{0.5403}\left(\frac{l}{h}\right)^{0.6911}(\mathrm{tg}\,\alpha)^{-0.4710}\left(\frac{n}{n_0}\right)^{0.0571} \tag{8}$$

The confidence level assumed was $p = 95\%$. The relative error of Eq. (8) amounted to $\pm 23.2\%$; the equation is valid for the following range of variability of individual groups:

$$Ar = 1.89 \times 10^{-6}–49.4; \qquad \frac{w_0}{w_p} = 0.01–10.3;$$

$$\frac{l}{h} = 14–63.5; \qquad \mathrm{tg}\,\alpha = 0.57–1.73; \qquad \frac{n}{n_0} = 0.92–1.64$$

the standard value of n_0 being equal to 1.25.

The regression coefficients appearing in Eq. (8) are close to the values for the tubular settler (Eq. (7)), hence the physical interpretation remains analogous, which at the same time emphasises the general character of the suggested model.

Eq. (8), worked out for a lamellar settler, enables one to calculate the overall sedimentation effectiveness after the model in Eq. (5). Thus, it is possible to carry out a comparative analysis of individual methods. The results of such an analysis, referring to the sedimentation number Mo*, are shown in Fig. 8. This comparison, comprising 272 data points, indicates a good agreement between the methods in question. Fig. 9 presents the overall sedimentation efficiency, a parameter determining a final appreciation of the settler. It can be seen clearly that the methods of Czajkowski [8], Olszewski [4] and Kowalski [10] are characterised by considerable errors, in particular for the latter method in a smaller effectiveness range. So, it may be stated that, similarly as in the case of a settler with tubular packing, our method displays a satisfactory accuracy for practical purposes. An essential advantage of this method is a simple design algorithm, making it possible to optimise a selected parameter of the settler.

5. Recapitulation and conclusions

Two representative solutions of packed-type settlers, viz. of tubular type and of plate type have been tested. In the experiments aqueous suspensions of 8 kinds of solid particles have been applied, of importance for the treatment of waste water and technological liquids. The experiments comprised 14 runs for a tubular settler and 13 runs for a plate settler, altogether 337 data points. This basis has been supplemented by the results of Binder's investigations, comprising, amongst others, quasimonodisperse glycerol suspensions ($n = 12.24$). The results obtained constituted a basis for elaboration of our own process model and verification of the models of other authors. On the grounds of the presented verification studies a number of conclusions of a general nature, indicating the achieved theoretical and practical results, may be formulated.

1. The solids used comprised particles with very different parameter values: viz., density ρ_s (1352.9–3223.3) kg/m^3; characteristic diameter of RRSB distribution d_{50} (12.51–141.93) µm; power exponent of the cumulative distribution function n (1.218–2.066).

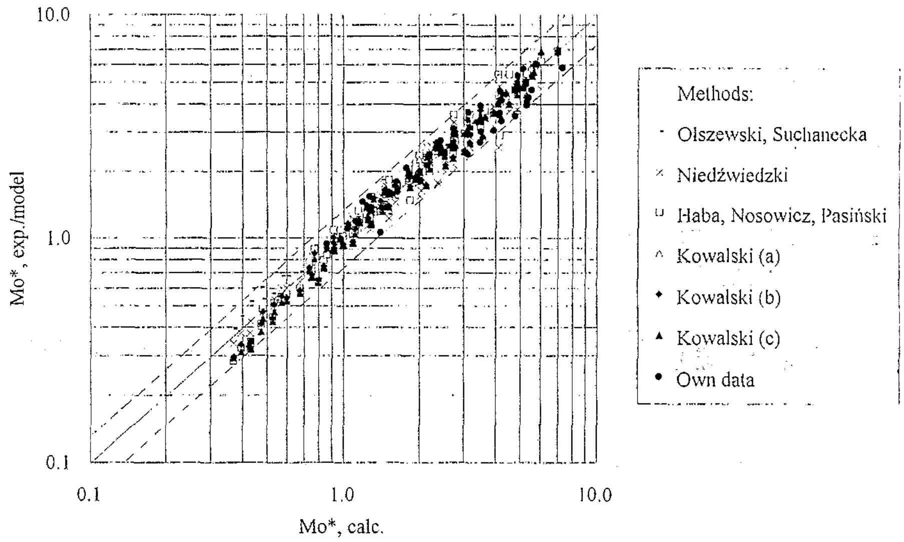

Fig. 8. Comparison of Mo* values calculated from Eq. (8) vs. the experimental ones and those calculated from individual models for a settler with plate packing.

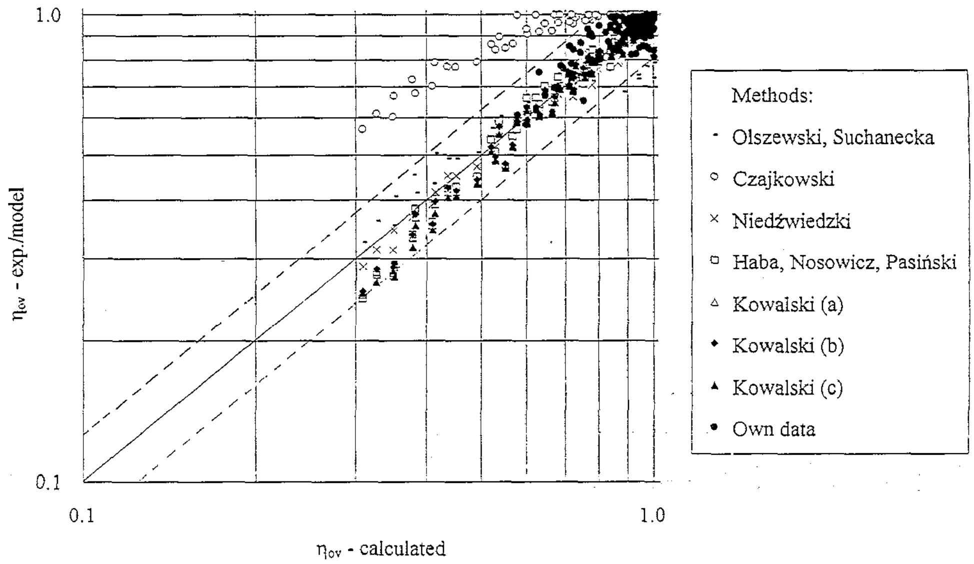

Fig. 9. Comparison of η_{ov} values calculated from Eq. (8) vs. the experimental ones and those calculated from individual models for a settler with plate packing.

2. In the research program used a uniform method of estimation of key parameters has been applied, characterising the solid material undergoing sedimentation, viz.:
   - particle size distribution has been determined by means of laser analysis and use has been made of the RRSB model, enabling one to determine the diameters d and d_{50} as well as the power exponent n;
   - for distributions represented by a broken line the procedure of transformation of a distribution leading to a rectilinear equivalent distribution has been worked out and verified, which made possible a common elaboration of the whole basis of experimental data.
3. A process model, based on the overall sedimentation efficiency in the continuous system Eq. (5) and on the correlation equations, Eqs. (7) and (8), has been presented in this paper. These relationships display the similar effect of individual dimensionless groups upon the sedimentation number Mo*, the latter indicating the intensity of the suspension's clarification process. Thus, a thesis may be formulated that the proposed model is a complete one and, at the same time, characteristic of a universal method.
4. The method elaborated, as well as selected methods of other authors, has been subjected to verification, on the basis of experimental data available. The results of a critical analysis have been graphically interpreted, enabling an estimation of the suitability

of individual methods, in particular with respect to calculational accuracy.

As a result of this analysis it has been found that certain methods (e.g. those of Czajkowski, Kowalski and Olszewski) are limited to selected types of suspensions, yielding, in general cases, considerable calculational deviations. The method described here, as well as those of Culp, Yao, McMichael and Willis, are of a comparable accuracy, differing, however, considerably in time consumption, the authors' method proving to be most advantageous.

Appendix A. Nomenclature

Ar	$= d^3\rho_c g(\rho_s-\rho_c)/(\mu_c^2\rho_c)$, Archimedes number
b	width of settling chamber, m
c	concentration of suspension, kg/m^3
D	internal diameter of sedimentation tube, m
d	particle diameter, m
d'	particle diameter corresponding to a value of $M(d)=63.2\%$, m
F_f	facial area of a tubular module, m^2
g	gravitational acceleration, m/s^2
Ga	$= d^3\rho_c g/\mu_c^2$, Galilei number
h	distance between plates, m
H	height of settler, m
K	ratio of the cross-sectional area perpendicular to the tubes to the total internal areas of all tubes
l	length of sedimentation tube (or plate), m
l_r	relative length (l/D or l/h)
m	mean value
m_0	mass of the measuring vessel (tare)
m_1	mass of the measuring vessel and of solids (gross)
Δm	mass of solids, g
Mo*	sedimentation number
$\Delta M(d)$	percent mass difference corresponding to the straight-line segments of particle size classes of distribution line 1 according to Fig. 5, %
n	exponent of the RRSB distribution
n_i	tangent of the angle of inclination of i-th straight-line segment, read directly on the angular scale of the RRSB distribution
n_0	standard value equal to 1.25
r_i	volumetric fraction of particle size class, represented by the straight-line segment
S	Yao parameter (Table 1)
t	suspension temperature, °C
V	total flow rate of suspension, m^3/s
w_p	suspension flow velocity between plates (or in a sedimentation tube), m/s
w_0	settling velocity, m/s
z	number of plates
α	inclination angle of a sedimentation system, °
γ	reciprocal of the settling velocity, s/m
η	sedimentation effectiveness
μ	dynamic viscosity, Pa·s
ρ	density, kg/m^3
σ	standard deviation

Subscripts

c	liquid
d	diameter
g	critical value
i	ith fraction of solids
k	outlet
max	maximal value
min	minimal value
ov	overall
p	inlet
r	radius
s	solid
w	velocity
50	median value

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