

Frank E. Grubbs and Glenn Beck, "Extension of Sample Sizes and Percentage Points for Significance Tests of Outlying Observations", *Technometrics*, 14(4), 847-854 (1972).

TABLE I

Table of Critical Values for T (One-Sided Test of T_1 or T_n) when the Standard Deviation is Calculated from the Same Sample

Number of Observations n	Upper .1% Significance Level	Upper .5% Significance Level	Upper 1% Significance Level	Upper 2.5% Significance Level	Upper 5% Significance Level	Upper 10% Significance Level
3	1.155	1.155	1.155	1.155	1.153	1.148
4	1.499	1.496	1.492	1.481	1.463	1.425
5	1.780	1.764	1.749	1.715	1.672	1.602
6	2.011	1.973	1.944	1.887	1.822	1.729
7	2.201	2.139	2.097	2.020	1.938	1.828
8	2.358	2.274	2.221	2.126	2.032	1.909
9	2.492	2.387	2.323	2.215	2.110	1.977
10	2.606	2.482	2.410	2.290	2.176	2.036
11	2.705	2.564	2.485	2.355	2.234	2.088
12	2.791	2.636	2.550	2.412	2.285	2.134
13	2.867	2.699	2.607	2.462	2.331	2.175
14	2.935	2.755	2.659	2.507	2.371	2.213
15	2.997	2.806	2.705	2.549	2.409	2.247
16	3.052	2.852	2.747	2.585	2.443	2.279
17	3.103	2.894	2.785	2.620	2.475	2.309
18	3.149	2.932	2.821	2.651	2.504	2.335
19	3.191	2.968	2.854	2.681	2.532	2.361
20	3.230	3.001	2.884	2.709	2.557	2.385
21	3.266	3.031	2.912	2.733	2.580	2.408
22	3.300	3.060	2.939	2.758	2.603	2.429
23	3.332	3.087	2.963	2.781	2.624	2.448
24	3.362	3.112	2.987	2.802	2.644	2.467
25	3.389	3.135	3.009	2.822	2.663	2.486
26	3.415	3.157	3.029	2.841	2.681	2.502
27	3.440	3.178	3.049	2.859	2.698	2.519
28	3.464	3.199	3.068	2.876	2.714	2.534
29	3.486	3.218	3.085	2.893	2.730	2.549
30	3.507	3.236	3.103	2.908	2.745	2.563
31	3.528	3.253	3.119	2.924	2.759	2.577
32	3.546	3.270	3.135	2.938	2.773	2.591
33	3.565	3.286	3.150	2.952	2.786	2.604
34	3.582	3.301	3.164	2.965	2.799	2.616
35	3.599	3.316	3.178	2.979	2.811	2.628
36	3.616	3.330	3.191	2.991	2.823	2.639
37	3.631	3.343	3.204	3.003	2.835	2.650
38	3.646	3.356	3.216	3.014	2.846	2.661
39	3.660	3.369	3.228	3.025	2.857	2.671
40	3.673	3.381	3.240	3.036	2.866	2.682
41	3.687	3.393	3.251	3.046	2.877	2.692
42	3.700	3.404	3.261	3.057	2.887	2.700
43	3.712	3.415	3.271	3.067	2.896	2.710
44	3.724	3.425	3.282	3.075	2.905	2.719
45	3.736	3.435	3.292	3.085	2.914	2.727
46	3.747	3.445	3.302	3.094	2.923	2.736
47	3.757	3.455	3.310	3.103	2.931	2.744
48	3.768	3.464	3.319	3.111	2.940	2.753
49	3.779	3.474	3.329	3.120	2.948	2.760
50	3.789	3.483	3.336	3.128	2.956	2.768

TABLE I (Continued)

Table of Critical Values for T (One-Sided Test of T_1 or T_n) when the Standard Deviation is Calculated from the Same Sample

Number of Observations n	Upper .1% Significance Level	Upper .5% Significance Level	Upper 1% Significance Level	Upper 2.5% Significance Level	Upper 5% Significance Level	Upper 10% Significance Level
51	3.798	3.491	3.345	3.136	2.964	2.775
52	3.808	3.500	3.353	3.143	2.971	2.783
53	3.816	3.507	3.361	3.151	2.978	2.790
54	3.825	3.516	3.368	3.158	2.986	2.798
55	3.834	3.524	3.376	3.166	2.992	2.804
56	3.842	3.531	3.383	3.172	3.000	2.811
57	3.851	3.539	3.391	3.180	3.006	2.818
58	3.858	3.546	3.397	3.186	3.013	2.824
59	3.867	3.553	3.405	3.193	3.019	2.831
60	3.874	3.560	3.411	3.199	3.025	2.837
61	3.882	3.566	3.418	3.205	3.032	2.842
62	3.889	3.573	3.424	3.212	3.037	2.849
63	3.896	3.579	3.430	3.218	3.044	2.854
64	3.903	3.586	3.437	3.224	3.049	2.860
65	3.910	3.592	3.442	3.230	3.055	2.866
66	3.917	3.598	3.449	3.235	3.061	2.871
67	3.923	3.605	3.454	3.241	3.066	2.877
68	3.930	3.610	3.460	3.246	3.071	2.883
69	3.936	3.617	3.466	3.252	3.076	2.888
70	3.942	3.622	3.471	3.257	3.082	2.893
71	3.948	3.627	3.476	3.262	3.087	2.897
72	3.954	3.633	3.482	3.267	3.092	2.903
73	3.960	3.638	3.487	3.272	3.098	2.908
74	3.965	3.643	3.492	3.278	3.102	2.912
75	3.971	3.648	3.496	3.282	3.107	2.917
76	3.977	3.654	3.502	3.287	3.111	2.922
77	3.982	3.658	3.507	3.291	3.117	2.927
78	3.987	3.663	3.511	3.297	3.121	2.931
79	3.992	3.669	3.516	3.301	3.125	2.935
80	3.998	3.673	3.521	3.305	3.130	2.940
81	4.002	3.677	3.525	3.309	3.134	2.945
82	4.007	3.682	3.529	3.315	3.139	2.949
83	4.012	3.687	3.534	3.319	3.143	2.953
84	4.017	3.691	3.539	3.323	3.147	2.957
85	4.021	3.695	3.543	3.327	3.151	2.961
86	4.026	3.699	3.547	3.331	3.155	2.966
87	4.031	3.704	3.551	3.335	3.160	2.970
88	4.035	3.708	3.555	3.339	3.163	2.973
89	4.039	3.712	3.559	3.343	3.167	2.977
90	4.044	3.716	3.563	3.347	3.171	2.981
91	4.049	3.720	3.567	3.350	3.174	2.984
92	4.053	3.725	3.570	3.355	3.179	2.989
93	4.057	3.728	3.575	3.358	3.182	2.993
94	4.060	3.732	3.579	3.362	3.186	2.996
95	4.064	3.736	3.582	3.365	3.189	3.000
96	4.069	3.739	3.586	3.369	3.193	3.003
97	4.073	3.744	3.589	3.372	3.196	3.006
98	4.076	3.747	3.593	3.377	3.201	3.011
99	4.080	3.750	3.597	3.380	3.204	3.014
100	4.084	3.754	3.600	3.383	3.207	3.017

TABLE I (*Continued*)

Table of Critical Values for T (One-Sided Test of T_1 or T_n) when the Standard Deviation is Calculated from the Same Sample

Number of Observations n	Upper .1% Significance Level	Upper .5% Significance Level	Upper 1% Significance Level	Upper 2.5% Significance Level	Upper 5% Significance Level	Upper 10% Significance Level
101	4.088	3.757	3.603	3.386	3.210	3.021
102	4.092	3.760	3.607	3.390	3.214	3.024
103	4.095	3.765	3.610	3.393	3.217	3.027
104	4.098	3.768	3.614	3.397	3.220	3.030
105	4.102	3.771	3.617	3.400	3.224	3.033
106	4.105	3.774	3.620	3.403	3.227	3.037
107	4.109	3.777	3.623	3.406	3.230	3.040
108	4.112	3.780	3.626	3.409	3.233	3.043
109	4.116	3.784	3.629	3.412	3.236	3.046
110	4.119	3.787	3.632	3.415	3.239	3.049
111	4.122	3.790	3.636	3.418	3.242	3.052
112	4.125	3.793	3.639	3.422	3.245	3.055
113	4.129	3.796	3.642	3.424	3.248	3.058
114	4.132	3.799	3.645	3.427	3.251	3.061
115	4.135	3.802	3.647	3.430	3.254	3.064
116	4.138	3.805	3.650	3.433	3.257	3.067
117	4.141	3.808	3.653	3.435	3.259	3.070
118	4.144	3.811	3.656	3.438	3.262	3.073
119	4.146	3.814	3.659	3.441	3.265	3.075
120	4.150	3.817	3.662	3.444	3.267	3.078
121	4.153	3.819	3.665	3.447	3.270	3.081
122	4.156	3.822	3.667	3.450	3.274	3.083
123	4.159	3.824	3.670	3.452	3.276	3.086
124	4.161	3.827	3.672	3.455	3.279	3.089
125	4.164	3.831	3.675	3.457	3.281	3.092
126	4.166	3.833	3.677	3.460	3.284	3.095
127	4.169	3.836	3.680	3.462	3.286	3.097
128	4.173	3.838	3.683	3.465	3.289	3.100
129	4.175	3.840	3.686	3.467	3.291	3.102
130	4.178	3.843	3.688	3.470	3.294	3.104
131	4.180	3.845	3.690	3.473	3.296	3.107
132	4.183	3.848	3.693	3.475	3.298	3.109
133	4.185	3.850	3.695	3.478	3.302	3.112
134	4.188	3.853	3.697	3.480	3.304	3.114
135	4.190	3.856	3.700	3.482	3.306	3.116
136	4.193	3.858	3.702	3.484	3.309	3.119
137	4.196	3.860	3.704	3.487	3.311	3.122
138	4.198	3.863	3.707	3.489	3.313	3.124
139	4.200	3.865	3.710	3.491	3.315	3.126
140	4.203	3.867	3.712	3.493	3.318	3.129
141	4.205	3.869	3.714	3.497	3.320	3.131
142	4.207	3.871	3.716	3.499	3.322	3.133
143	4.209	3.874	3.719	3.501	3.324	3.135
144	4.212	3.876	3.721	3.503	3.326	3.138
145	4.214	3.879	3.723	3.505	3.328	3.140
146	4.216	3.881	3.725	3.507	3.331	3.142
147	4.219	3.883	3.727	3.509	3.334	3.144

TABLE II

Table of Critical Values for $S_{n-1,n}^2/S_0^2$ or $S_{1,2}^2/S_0^2$ for Simultaneously Testing the Two Largest or Two Smallest Observations

Number of Observations n	Lower .1% Significance Level	Lower .5% Significance Level	Lower 1% Significance Level	Lower 2.5% Significance Level	Lower 5% Significance Level	Lower 10% Significance Level
4	.0000	.0000	.0000	.0002	.0008	.0031
5	.0003	.0018	.0035	.0090	.0183	.0376
6	.0039	.0116	.0186	.0349	.0564	.0920
7	.0135	.0308	.0440	.0708	.1020	.1479
8	.0290	.0563	.0750	.1101	.1478	.1994
9	.0489	.0851	.1082	.1492	.1909	.2454
10	.0714	.1150	.1414	.1864	.2305	.2863
11	.0953	.1448	.1736	.2213	.2667	.3227
12	.1198	.1738	.2043	.2537	.2996	.3552
13	.1441	.2016	.2333	.2836	.3295	.3843
14	.1680	.2280	.2605	.3112	.3568	.4106
15	.1912	.2530	.2859	.3367	.3818	.4345
16	.2136	.2767	.3098	.3603	.4048	.4562
17	.2350	.2990	.3321	.3822	.4259	.4761
18	.2556	.3200	.3530	.4025	.4455	.4944
19	.2752	.3398	.3725	.4214	.4636	.5113
20	.2939	.3585	.3909	.4391	.4804	.5270
21	.3118	.3761	.4082	.4556	.4961	.5415
22	.3288	.3927	.4245	.4711	.5107	.5550
23	.3450	.4085	.4398	.4857	.5244	.5677
24	.3605	.4234	.4543	.4994	.5373	.5795
25	.3752	.4376	.4680	.5123	.5495	.5906
26	.3893	.4510	.4810	.5245	.5609	.6011
27	.4027	.4638	.4933	.5360	.5717	.6110
28	.4156	.4759	.5050	.5470	.5819	.6203
29	.4279	.4875	.5162	.5574	.5916	.6292
30	.4397	.4985	.5268	.5672	.6008	.6375
31	.4510	.5091	.5369	.5766	.6095	.6455
32	.4618	.5192	.5465	.5856	.6178	.6530
33	.4722	.5288	.5557	.5941	.6257	.6602
34	.4821	.5381	.5646	.6023	.6333	.6671
35	.4917	.5469	.5730	.6101	.6405	.6737
36	.5009	.5554	.5811	.6175	.6474	.6800
37	.5098	.5636	.5889	.6247	.6541	.6860
38	.5184	.5714	.5963	.6316	.6604	.6917
39	.5266	.5789	.6035	.6382	.6665	.6972
40	.5345	.5862	.6104	.6445	.6724	.7025
41	.5422	.5932	.6170	.6506	.6780	.7076
42	.5496	.5999	.6234	.6565	.6834	.7125
43	.5568	.6064	.6296	.6621	.6886	.7172
44	.5637	.6127	.6355	.6676	.6936	.7218
45	.5704	.6188	.6412	.6728	.6985	.7261
46	.5768	.6246	.6468	.6779	.7032	.7304
47	.5831	.6303	.6521	.6828	.7077	.7345
48	.5892	.6358	.6573	.6876	.7120	.7384
49	.5951	.6411	.6623	.6921	.7163	.7422
50	.6008	.6462	.6672	.6966	.7203	.7459

TABLE II (*Continued*)

Table of Critical Values for $S_{n-1,n}^2/S_0^2$ or $S_{1,2}^2/S_0^2$ for Simultaneously Testing the Two Largest or Two Smallest Observations

Number of Observations n	Lower .1% Significance Level	Lower .5% Significance Level	Lower 1% Significance Level	Lower 2.5% Significance Level	Lower 5% Significance Level	Lower 10% Significance Level
51	.6063	.6512	.6719	.7009	.7243	.7495
52	.6117	.6560	.6765	.7051	.7281	.7529
53	.6169	.6607	.6809	.7091	.7319	.7563
54	.6220	.6653	.6852	.7130	.7355	.7595
55	.6269	.6697	.6894	.7168	.7390	.7627
56	.6317	.6740	.6934	.7205	.7424	.7658
57	.6364	.6782	.6974	.7241	.7456	.7687
58	.6410	.6823	.7012	.7276	.7489	.7716
59	.6454	.6862	.7049	.7310	.7520	.7744
60	.6497	.6901	.7086	.7343	.7550	.7772
61	.6539	.6938	.7121	.7375	.7580	.7798
62	.6580	.6975	.7155	.7406	.7608	.7824
63	.6620	.7010	.7189	.7437	.7636	.7850
64	.6658	.7045	.7221	.7467	.7664	.7874
65	.6696	.7079	.7253	.7496	.7690	.7898
66	.6733	.7112	.7284	.7524	.7716	.7921
67	.6770	.7144	.7314	.7551	.7741	.7944
68	.6805	.7175	.7344	.7578	.7766	.7966
69	.6839	.7206	.7373	.7604	.7790	.7988
70	.6873	.7236	.7401	.7630	.7813	.8009
71	.6906	.7265	.7429	.7655	.7836	.8030
72	.6938	.7294	.7455	.7679	.7859	.8050
73	.6970	.7322	.7482	.7703	.7881	.8070
74	.7000	.7349	.7507	.7727	.7902	.8089
75	.7031	.7376	.7532	.7749	.7923	.8108
76	.7060	.7402	.7557	.7772	.7944	.8127
77	.7089	.7427	.7581	.7794	.7964	.8145
78	.7117	.7453	.7605	.7815	.7983	.8162
79	.7145	.7477	.7628	.7836	.8002	.8180
80	.7172	.7501	.7650	.7856	.8021	.8197
81	.7199	.7525	.7672	.7876	.8040	.8213
82	.7225	.7548	.7694	.7896	.8058	.8230
83	.7250	.7570	.7715	.7915	.8075	.8245
84	.7275	.7592	.7736	.7934	.8093	.8261
85	.7300	.7614	.7756	.7953	.8109	.8276
86	.7324	.7635	.7776	.7971	.8126	.8291
87	.7348	.7656	.7796	.7989	.8142	.8306
88	.7371	.7677	.7815	.8006	.8158	.8321
89	.7394	.7697	.7834	.8023	.8174	.8335
90	.7416	.7717	.7853	.8040	.8190	.8349
91	.7438	.7736	.7871	.8057	.8205	.8362
92	.7459	.7755	.7889	.8073	.8220	.8376
93	.7481	.7774	.7906	.8089	.8234	.8389
94	.7501	.7792	.7923	.8104	.8248	.8402
95	.7522	.7810	.7940	.8120	.8263	.8414
96	.7542	.7828	.7957	.8135	.8276	.8427
97	.7562	.7845	.7973	.8149	.8290	.8439
98	.7581	.7862	.7989	.8164	.8303	.8451

TABLE II (Continued)

Table of Critical Values for $S_{n-1,n}^2/S_0^2$ or $S_{1,2}^2/S_0^2$ for Simultaneously Testing the Two Largest or Two Smallest Observations

Number of Observations n	Lower .1% Significance Level	Lower .5% Significance Level	Lower 1% Significance Level	Lower 2.5% Significance Level	Lower 5% Significance Level	Lower 10% Significance Level
51	.6063	.6512	.6719	.7009	.7243	.7495
52	.6117	.6560	.6765	.7051	.7281	.7529
53	.6169	.6607	.6809	.7091	.7319	.7563
54	.6220	.6653	.6852	.7130	.7355	.7595
55	.6269	.6697	.6894	.7168	.7390	.7627
56	.6317	.6740	.6934	.7205	.7424	.7658
57	.6364	.6782	.6974	.7241	.7456	.7687
58	.6410	.6823	.7012	.7276	.7489	.7716
59	.6454	.6862	.7049	.7310	.7520	.7744
60	.6497	.6901	.7086	.7343	.7550	.7772
61	.6539	.6938	.7121	.7375	.7580	.7798
62	.6580	.6975	.7155	.7406	.7608	.7824
63	.6620	.7010	.7189	.7437	.7636	.7850
64	.6658	.7045	.7221	.7467	.7664	.7874
65	.6696	.7079	.7253	.7496	.7690	.7898
66	.6733	.7112	.7284	.7524	.7716	.7921
67	.6770	.7144	.7314	.7551	.7741	.7944
68	.6805	.7175	.7344	.7578	.7766	.7966
69	.6839	.7206	.7373	.7604	.7790	.7988
70	.6873	.7236	.7401	.7630	.7813	.8009
71	.6906	.7265	.7429	.7655	.7836	.8030
72	.6938	.7294	.7455	.7679	.7859	.8050
73	.6970	.7322	.7482	.7703	.7881	.8070
74	.7000	.7349	.7507	.7727	.7902	.8089
75	.7031	.7376	.7532	.7749	.7923	.8108
76	.7060	.7402	.7557	.7772	.7944	.8127
77	.7089	.7427	.7581	.7794	.7964	.8145
78	.7117	.7453	.7605	.7815	.7983	.8162
79	.7145	.7477	.7628	.7836	.8002	.8180
80	.7172	.7501	.7650	.7856	.8021	.8197
81	.7199	.7525	.7672	.7876	.8040	.8213
82	.7225	.7548	.7694	.7896	.8058	.8230
83	.7250	.7570	.7715	.7915	.8075	.8245
84	.7275	.7592	.7736	.7934	.8093	.8261
85	.7300	.7614	.7756	.7953	.8109	.8276
86	.7324	.7635	.7776	.7971	.8126	.8291
87	.7348	.7656	.7796	.7989	.8142	.8306
88	.7371	.7677	.7815	.8006	.8158	.8321
89	.7394	.7697	.7834	.8023	.8174	.8335
90	.7416	.7717	.7853	.8040	.8190	.8349
91	.7438	.7736	.7871	.8057	.8205	.8362
92	.7459	.7755	.7889	.8073	.8220	.8376
93	.7481	.7774	.7906	.8089	.8234	.8389
94	.7501	.7792	.7923	.8104	.8248	.8402
95	.7522	.7810	.7940	.8120	.8263	.8414
96	.7542	.7828	.7957	.8135	.8276	.8427
97	.7562	.7845	.7973	.8149	.8290	.8439
98	.7581	.7862	.7989	.8164	.8303	.8451

Table B19. Lower percentage points of the Grubbs' test for one outlier in each tail, $S_{1,n}^2/S^2$ (based on simulation of 20,000 samples)

n	percentage points				
	0.5%	1%	2.5%	5%	10%
10	0.096	0.117	0.158	0.197	0.246
15	0.222	0.254	0.300	0.345	0.394
20	0.328	0.363	0.407	0.444	0.488
25	0.416	0.444	0.485	0.519	0.557
30	0.481	0.503	0.541	0.573	0.607
35	0.532	0.554	0.589	0.615	0.646
40	0.566	0.586	0.621	0.647	0.675
45	0.602	0.623	0.653	0.677	0.703
50	0.631	0.648	0.676	0.698	0.723
60	0.671	0.692	0.715	0.736	0.757
70	0.709	0.724	0.747	0.765	0.784
80	0.738	0.753	0.772	0.788	0.804
90	0.758	0.774	0.791	0.805	0.820
100	0.777	0.791	0.808	0.821	0.835
250	0.895	0.900	0.907	0.913	0.919
500	0.941	0.944	0.948	0.951	0.954