Limits

MATH 1151

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1 What is a Limit?

1.1 Limits as a Physics Problem

Limits were actually created by Physicists to solve the problem of finding out the speed of something at a certain time. The problem being that if we use our equation for speed at a certain time, we get a division by zero. So **limits** are used to look near the point but not at the point.

1.2 Notation

$\lim_{x \to 1} f(x) = 1$ the $x \to 1$ shows the variable and value we're looking at. $f(x)$ is the function we are looking at and the $= 1$ is the value the function is approaching.

1.3 One-Sided Limits

A function may actually behave differently on different sides of a point, so we need a way to analyze each side seperately.

- **Left-Sided Limits** or limits that are approaching a value slightly *less* than a are represented by $\lim_{x \to a^-} f(x)$
- **Right-Sided Limits** or limits that are approaching a value slightly *more* than a are represented by $\lim_{x \to a^+} f(x)$

One-Sided Limits implictly prove the existance of **Two-Sided Limits**. These are actually what a normal **Limit** is, **Two-Sided Limits** only exist if both **One-Sided Limits** exist and they are equal to eachother.

2 Continuity

Graphs that are **Continuous** are simple and easy to use. However, graphs that have **Discontinuity** can be tricky to deal with. We need to develop a language to deal with **Continuity**.

The definition of **continuity** is $\lim_{x \to a} f(a)$. If this equation is true, then $f(x)$ is **continuous** at a. This definition is really making three statements:

- $f(a)$ is defined, That is, a is in the domain of f
- $\lim_{x \to a} f(x)$ exits
- $\lim_{x \to a} f(x) = f(a)$

2.1 One-Sided Continuity

Since our definition of **Continuity** is built on **Limits** and limits have sides, this means that there is also **One-Sided Continuity**.

A function f is **Left Continuous** at a point a if $\lim_{x \to a^-} f(x) = f(a)$ and a function f is **Right Continuous** at a point a if $\lim_{x \to x^+} f(x) = f(x)$.

- If a function is **Continuous** at $x = a$, then it is also **Left-Continuous** and **Right-Continuous**.
- If a function is **Left-Continuous** and **Right-Continuous** at $x = a$, then is is also **Continuous** at $x = a$

2.2 Intervals of Continuity

Our previous definitions of **Continuity** have been based on points. However, we can also judge is a function is **Continous** on an interval.

$f(x)$ is continuous over the interval (a, b) if it is continuous at every point in the interval.

- When looking at **Intervals of Continuity**, it is very important to check endpoints.

3 Limit Laws