Bayesian Statistics

Lecture 16

Multivariate statistics

Psychology 613 – Spring 2022

Overview

What is meant by “Bayesian”?

Bayes’ Rule

Bayes Factor

Applications of Bayes’ rule in statistics

What is “Bayesian”?

This term is used very loosely in the field.

There is a specific definition, but mostly people use ”Bayesian” to refer to any kind of analysis where:

estimates of probabilities of something (e.g., a hypothesis being true) are updated based on data.

What is “Bayesian”?

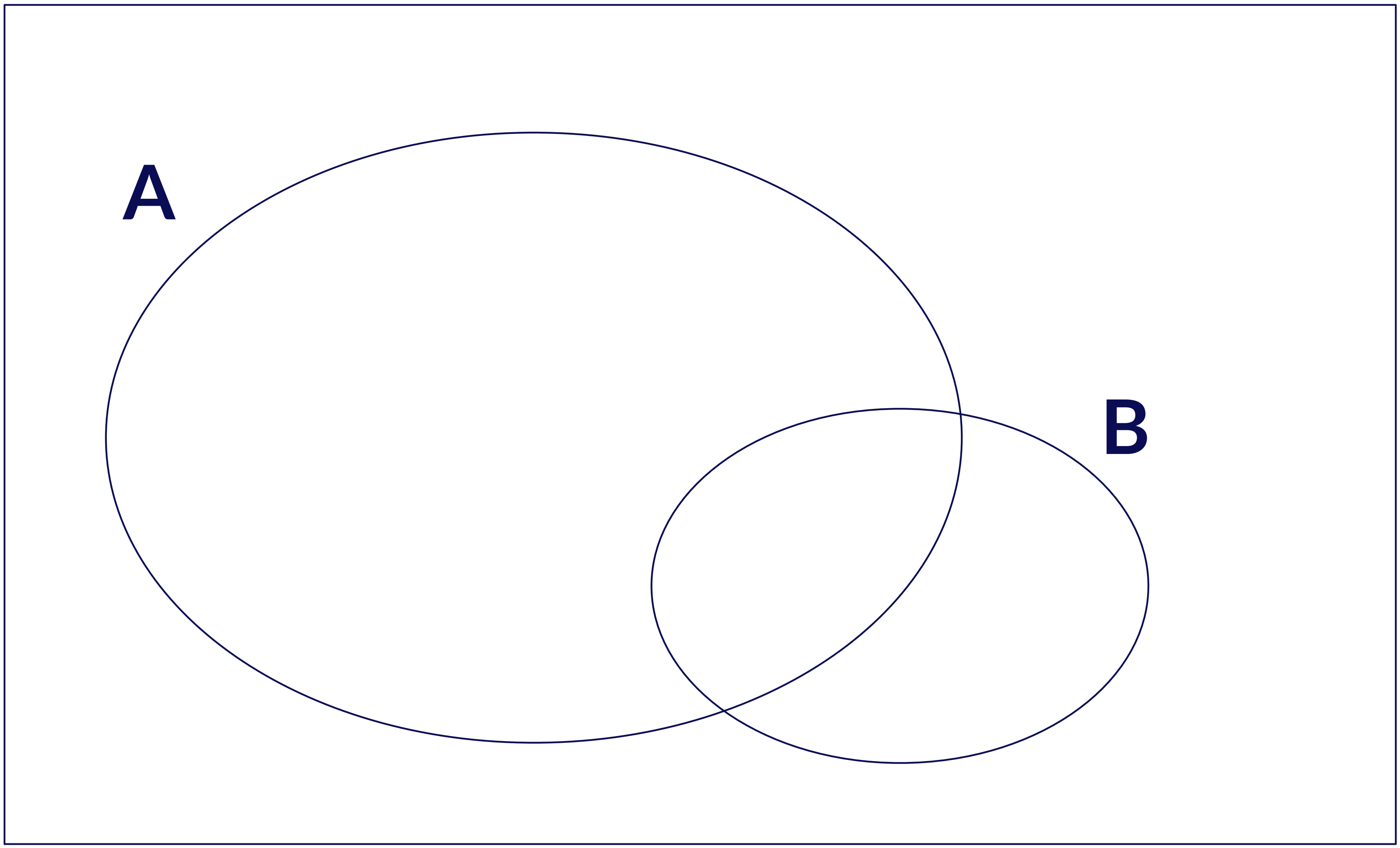
Intuition:

It’s a way of quantifying how much a probability *changes* with data.

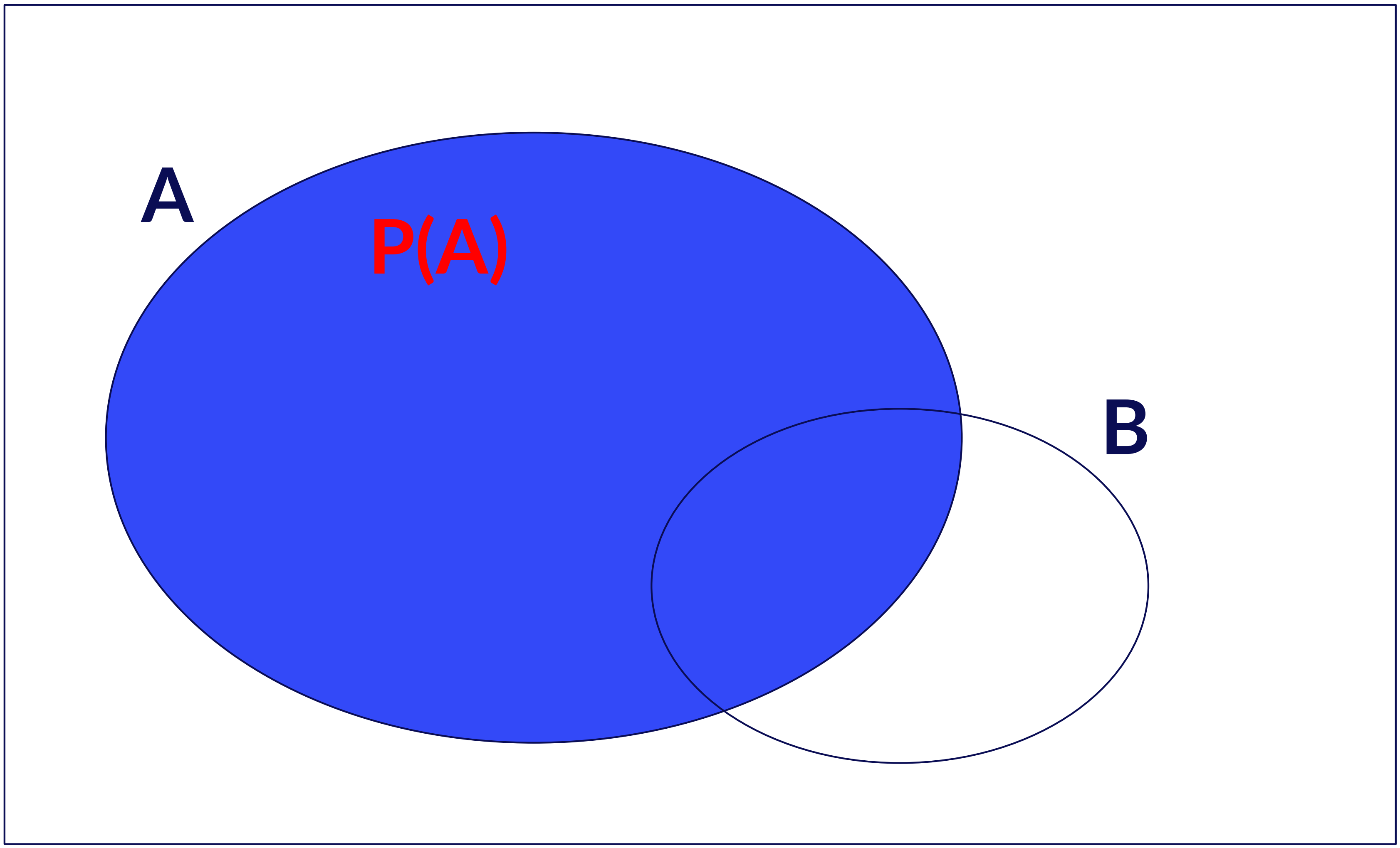
“Prior”: How likely is something?

“Posterior”: How likely is the thing *with data*?

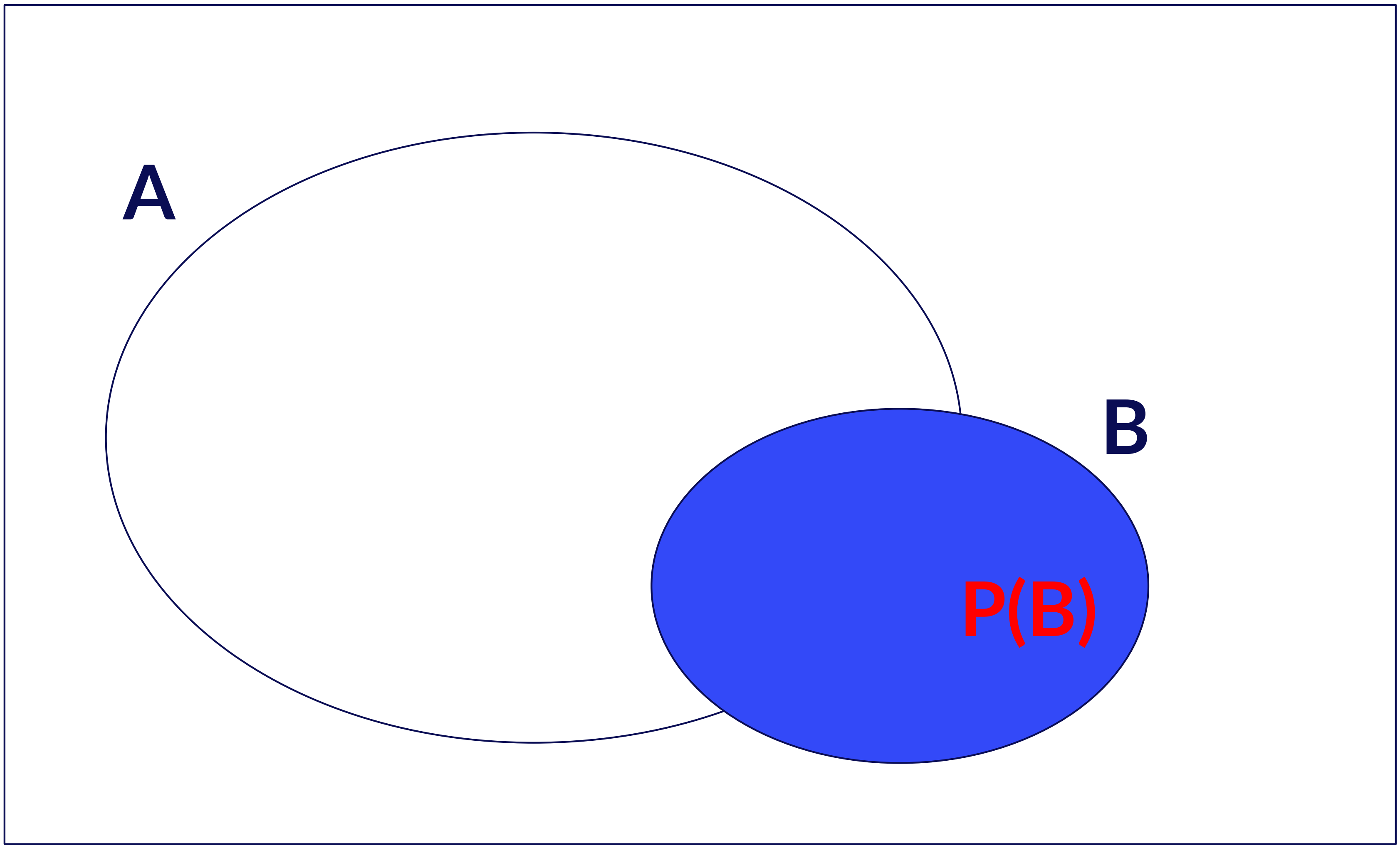
Bayes' Rule



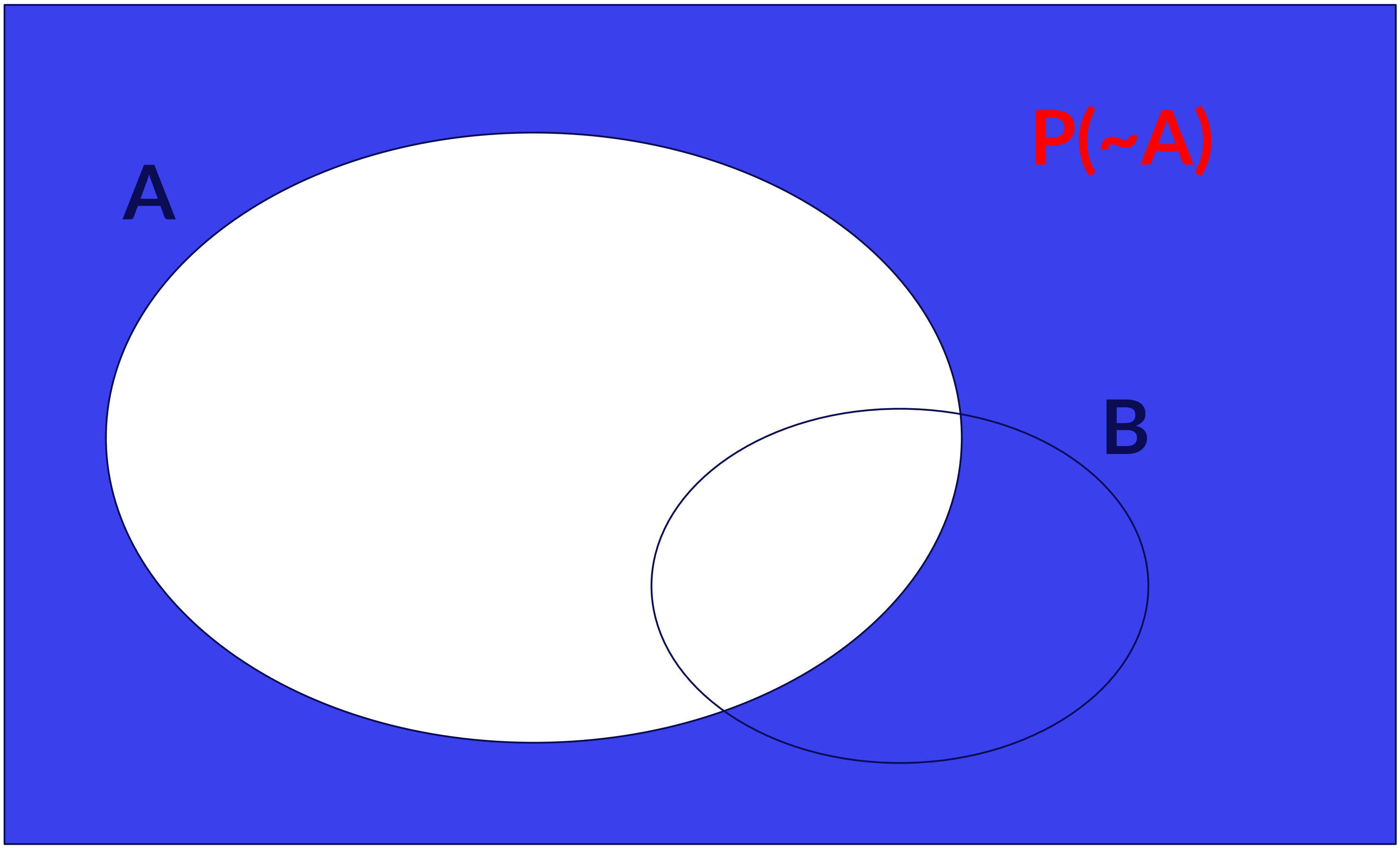
Bayes' Rule



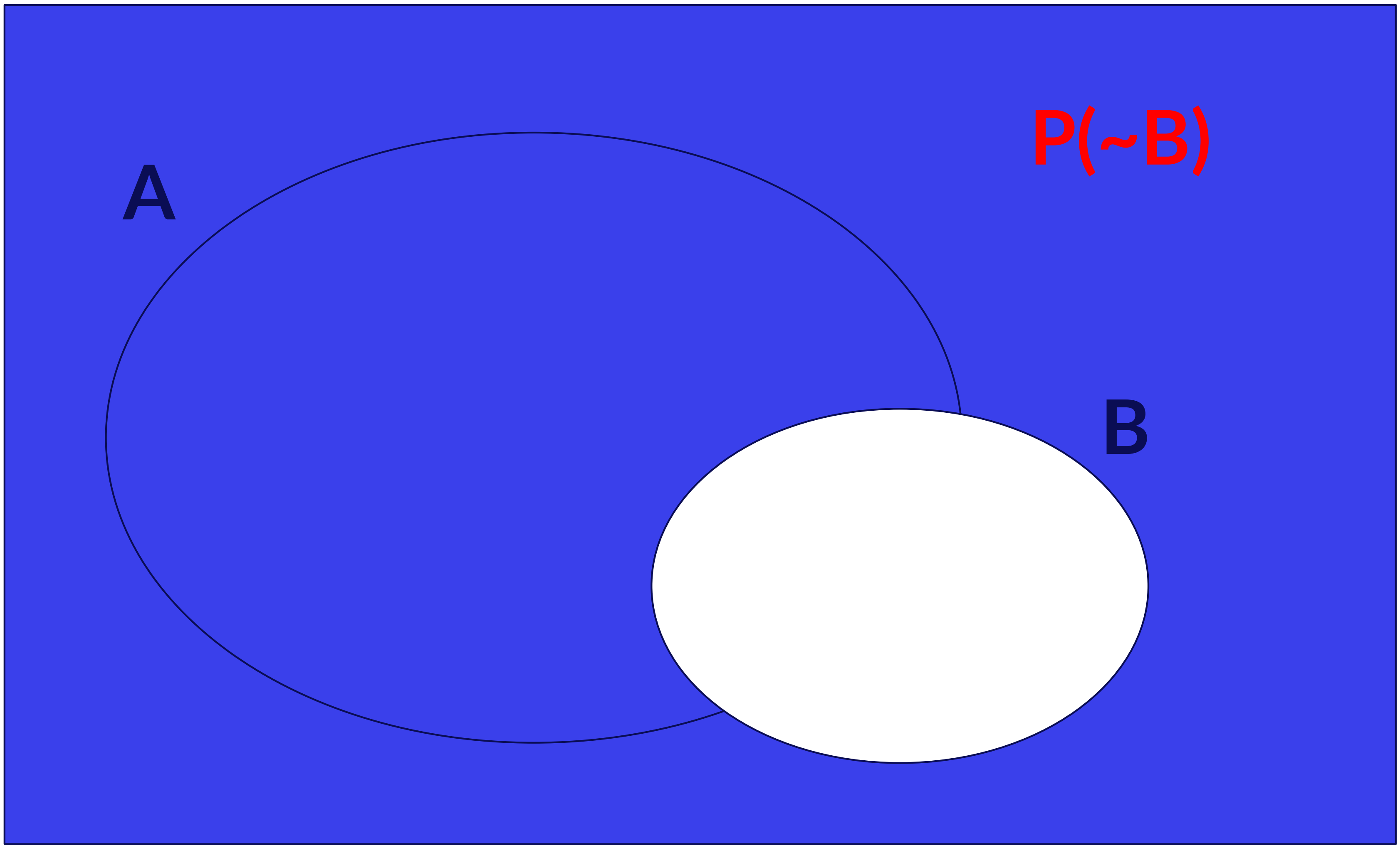
Bayes' Rule



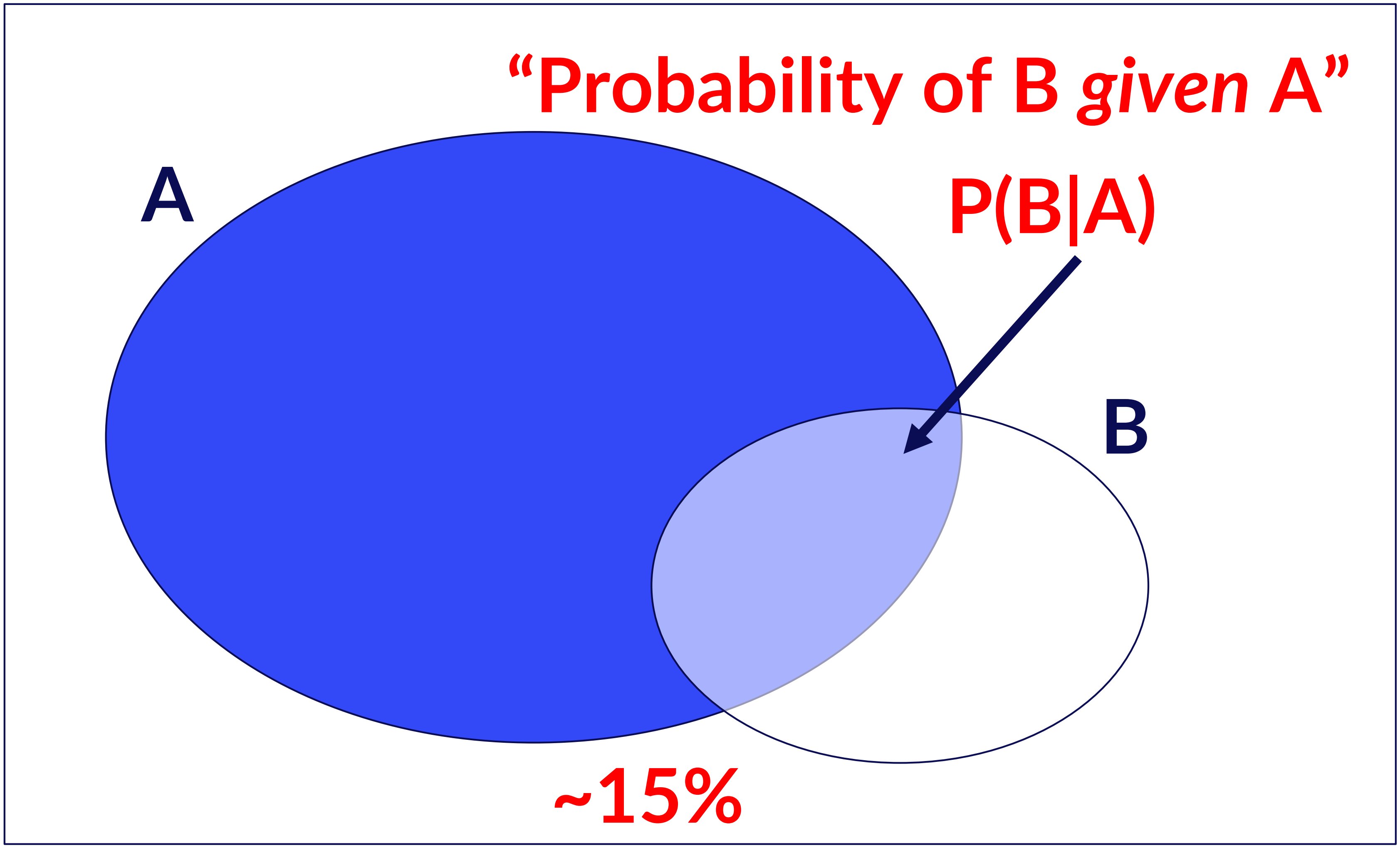
Bayes' Rule



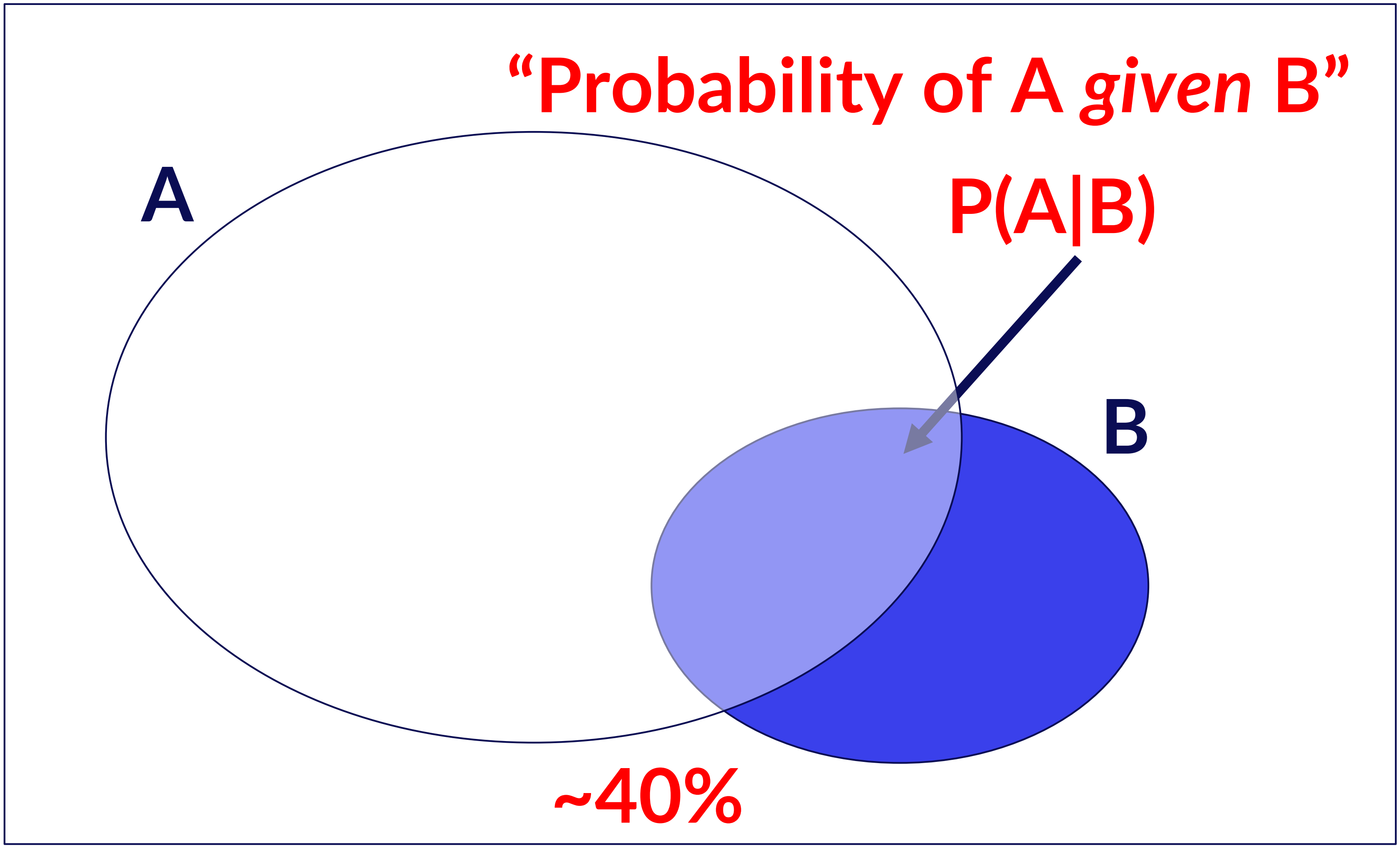
Bayes' Rule



Bayes' Rule



Bayes' Rule



Bayes' Rule

So how would we calculate the probability of being in the shaded area *overall*?

To be in the shaded area, two things need to happen:

A & B

Bayes' Rule

We can get A & B two ways:

P(B) & P(A | B) ← How often does "B" happen and, once that happens, also A?

P(A) & P(B | A) ← How often does "A" happen and, once that happens, also B?

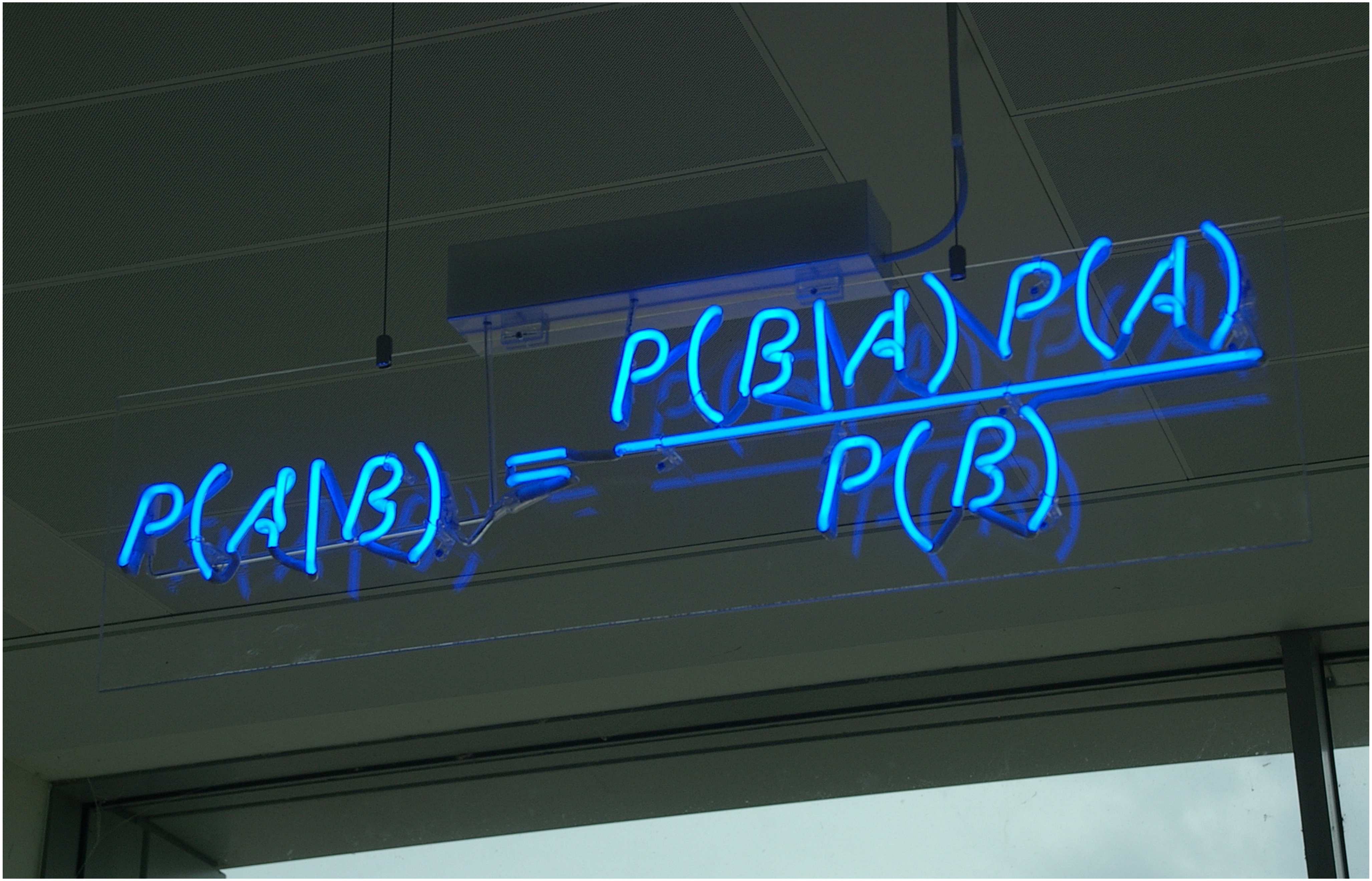
Bayes' Rule

Insight alert!

$$P(B) * P(A \mid B) = P(A) * P(B \mid A)$$

So:

$$\mathbf{P(A \mid B) = \frac{P(A) * P(B \mid A)}{P(B)}}$$


$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

Likelihood
(of B, when A is true)

Prior on A

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

"Posterior"
Probability of A
being true, if B is true

"Marginal"
Probability of B
being true

Bayes' Rule

When my kids play together, there is ~50% chance at least one of them ends up in tears.

They play together 20% of the time.

They cry about 12% of the time.

What is the chance they were playing if I hear one or more of them crying?

Bayes' Rule

$$P(\text{play} \mid \text{tears}) = \frac{P(\text{tears} \mid \text{play}) * P(\text{play})}{P(\text{tears})}$$

$$= \frac{0.5 \; * \; 0.2}{0.12} = 83\%$$

[Note that this also tells you that P(~play | tears) = 17%]

Bayes' Rule

Note that there is an implication of an effect size here. Before the tears, I believed there was a 20% chance they were playing – the prior.

Once I hear the crying, I now believe there was an 83% chance they were playing – the posterior. *I updated my beliefs!*

Bayes' Rule

Useful for:

- Genetics, such as P(risk | gene)
- Disease modeling, such as P(disease | symptom)
- Finding pirate treasure, with P(sink | scenario)
- Testing hypotheses, with P(H | data)

Bayesian Hypothesis Testing

Hypothesis	Tears	No tears	Marginal	
Kids are playing	0.1	0.1	0.2	P(Play)
Kids are not playing			0.8	P(~Play)
			1	
	P(Tears)	P(~Tears)		

$$P(\text{Tears} \mid \sim\text{play}) = \frac{P(\sim\text{play} \mid \text{tears}) * P(\text{tears})}{P(\sim\text{play})} = \frac{0.17 * 0.12}{0.8}$$

Bayesian Hypothesis Testing

Hypothesis	Tears	No tears	Marginal	
Kids are playing	0.1	0.1	0.2	**P(Play)**
Kids are not playing	0.02		0.8	**P(~Play)**
	0.12		1	
	P(Tears)	**P(~Tears)**		

Bayesian Hypothesis Testing

Hypothesis	Tears	No tears	Marginal	
Kids are playing	0.1	0.1	0.2	**P(Play)**
Kids are not playing	0.02	0.78	0.8	**P(~Play)**
	0.12		1	
	P(Tears)	**P(~Tears)**		

Bayesian Hypothesis Testing

Hypothesis	Tears	No tears	Marginal	
Kids are playing	0.1	0.1	0.2	**P(Play)**
Kids are not playing	0.02	0.78	0.8	**P(~Play)**
	0.12	0.88	1	
	P(Tears)	**P(~Tears)**		

H0: Kids are not playing. What is P(H0 | data)?

$$\frac{P(\text{data} \mid H0)*P(H0)}{P(\text{data})} = \frac{.025 * .8}{.12} = 0.17$$

This is the "posterior probability of the null"

Bayes Factor

It's an *odds ratio* of how likely the data are under the two hypotheses:

$$\frac{P(\text{Data} \mid H1)}{P(\text{Data} \mid H0)}$$

Bayes Factor

How likely are the data under one hypothesis compared to the other? How likely is crying given H1 vs. H0?

$$\frac{P(\text{Tears} \mid \text{Playing})}{P(\text{Tears} \mid {\sim}\text{Playing})} = \frac{0.50}{0.025} = 20$$

Bayes Factor

Posterior odds = Bayes Factor * Prior Odds

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$$\frac{P(H1 \mid Data)}{P(H0 \mid Data)} = BF * \frac{P(H1)}{P(H0)}$$

Bayes Factor

Posterior odds = Bayes Factor * Prior Odds

$$\frac{P(H1 \mid Data)}{P(H0 \mid Data)} = BF * \frac{P(H1)}{P(H0)}$$

$$\frac{0.83}{0.17} = 20 * \frac{0.2}{0.8}$$

Bayes Factor

Posterior odds = Bayes Factor * Prior Odds

$$\frac{P(H1 \mid Data)}{P(H0 \mid Data)} = BF * \frac{P(H1)}{P(H0)}$$

The Bayes Factor reflects *how much you update your prior beliefs once you know the data!*

Bayes Factor

Bayes factor	Interpretation
1 - 3	Negligible evidence
3 - 20	Positive evidence
20 - 150	Strong evidence
$>$150	Very strong evidence

Kass & Raftery, 1995; Navarro, 2018

Bayes Factor

"A Bayes Factor of 20 provides positive to strong evidence that my children were playing given that at least one of them is crying"

Bayesian Hypothesis Testing

Let's work with a simple two-groups t-test equivalent. We can solve P(H0 | data) with:

$$P(H0 \mid Data) = \frac{P(Data \mid H0) * P(H0)}{P(Data)}$$

So, we need the "likelihood", P(data | H0), and the prior, P(H0)

Bayesian Hypothesis Testing

Essentially, we can use the product of probabilities (assuming a normal distribution) to get P(Data | H0):

$P(x \mid H0) = pnorm(x, mean_{H0}, sd_{H0})$

Then multiply all the individual probabilities to get a (tiny) joint likelihood.

Bayesian Hypothesis Testing

Use the “BayesFactor” package in R.

Various options for simulation-based estimation of the Bayes factor and the posterior distribution.

See RMD for this lecture.

Why Bayes?

Is it better? Maybe. Is it more informative? Definitely.

Interpretation of p-value is convoluted and not what most people want.

Interpretation of Bayesian posteriors are *exactly* what most people want.