Moderation & Interaction

Lecture 5

Multivariate statistics

Psychology 613 – Spring 2022

Moderation

Suppose:

X is an IV

M is a moderator

Y is a DV

Moderation is when the relationship between X and Y changes as a function of M.

In this case, X and M are said to *interact* in predicting Y.

Moderation: Example

Group	Control	Treatment 1	Treatment 2	MEAN
Bajorans	30	50	100	60
Cardassians	30	100	50	60
MEAN	30	75	75	

No main effect of species

Main effect of treatments vs. control

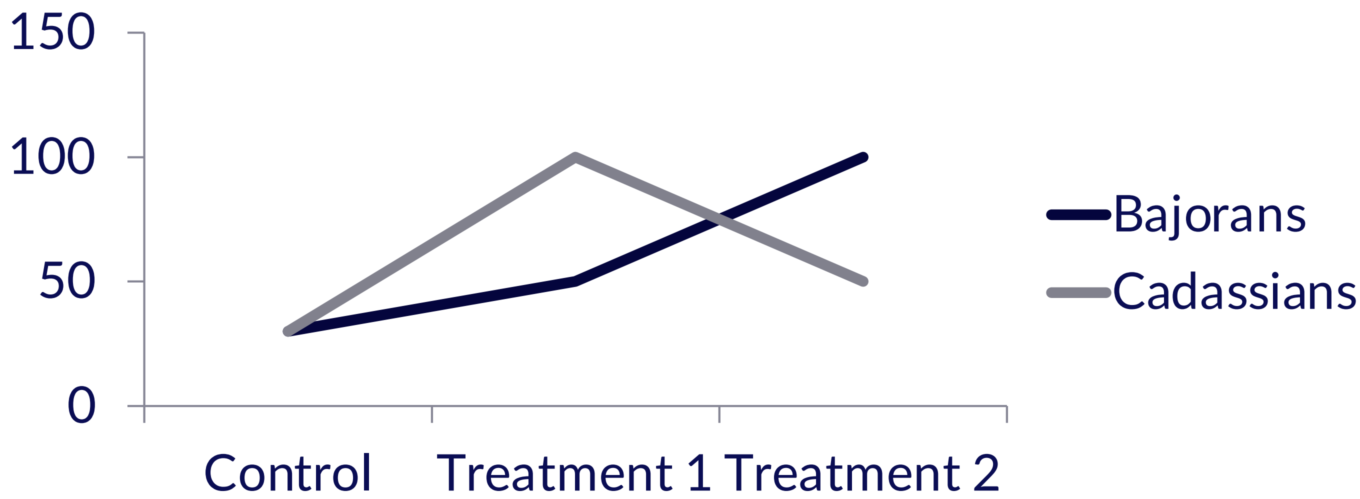
Huge interaction of species by treatments

→ The effect of treatment depends on species

→ Treatment and species *interact* in predicting outcome

→ Species *moderates* the effect of treatment

Moderation: Example



No main effect of species

Main effect of treatments vs. control

Huge interaction of species by treatments

→ The effect of treatment depends on species

→ Treatment and species *interact* in predicting outcome

→ Species *moderates* the effect of treatment

Moderation in regression

Typically, we assume simple additivity:

$$Y = b_0 + b_1 X1 + b_2 X2 + e$$

→ The effect of X1 (over and above X2) is the same regardless of the level of X2.

However, X1 may not operate the same way across the entire range of X2; particular combinations of X1 and X2 may produce specific effects (*interactions*).

The effect of one variable may depend on the level of the other.

Moderation in regression

Moderation model:

$$Y = b_0 + b_1X1 + b_2X2 + b_3(X1^*X2) + e$$

→ The unique effect of X1 may vary as a function of X2.

b_0: The expected value of Y when X1, X2, and X1X2 = 0.

b_1: The “main” effect of X1 controlling for X2 and X1X2.

b_2: The “main” effect of X2 controlling for X1 and X1X2.

b_3: The interaction (i.e., joint effect) of X1 and X2 controlling for X1 and X2.

Reflects how, say, the Y~X1 relationship changes with X2

Example!

nature International weekly journal of science

Home | News & Comment | Research | Careers & Jobs | Current Issue | Archive | Audio & Video | Fo

News & Comment > News > 2014 > April > Article

NATURE | NEWS

Male researchers stress out rodents

Rats and mice show increased stress levels when handled by men rather than women, potentially skewing study results.

Alla Katsnelson

28 April 2014

Sorge et al., 2014

Centering alters interpretation

Uncentered: $Y = b_0 + b_1X1 + b_2X2 + b_3X1{*}X2$

b_1: slope of Y on X1 when X2 = 0

b_2: slope of Y on X2 when X1 = 0

b_3: partialled interaction

Centered: $Y = b_0 + b_1X1_{cen} + b_2X2_{cen} + b_3X1_{cen}{*}X2_{cen}$

b_1 = slope of Y on X1 at average of X2

The average regression of Y on X1 across range of X2

b_2 = slope of Y on X2 at average of X1

The average regression of Y on X2 across range of X1

b_3 = partialled interaction

Center and interact

Var 1	*	Var 2		Interaction term (product)
$X1-m_{X1}$	*	$X2-m_{X2}$	→	$(X1-m_{X1})*(X2-m_{X2})$
D1	*	$X2-m_{X2}$	→	$(D1)*(X2-m_{X2})$
D1	*	D2	→	D1 * D2
D1,D2	*	$X2-m_{X2}$	→	$D1(X2-m_{X2})$, $D2(X2-m_{X2})$
D1,D2	*	D3,D4	→	D1D3,D1D4,D2D3,D2D4

Interpretation tips

In equations with higher-order terms (e.g., interactions), variability shared by higher- and lower-order terms is typically attributed to the lower-order term.

Interpret coefficients of lower-order terms at zeros for all other variables.

Interaction: Procedure

<u>Step 1</u>

The initial analysis focuses on determining whether or not the interaction is significant.

If there is a single interaction variable, examine the t-test for the associated b coefficient.

If multiple vectors code for the interaction (e.g., dummy variables), test simultaneously with R^2 change test from nested models.

Interaction: Procedure

Step 2a: Interaction is non-significant

- Non-significant interaction: the effects of each variable are constant across the range of the other
- Follow-up with tests of main effects, which collapse across values of the other variable.

OR Step 2b: Interaction is significant

- Significant interaction: the effect of one variable changes with the value of another
- Follow-up with tests of simple effects, which are conducted *at particular values* of the other variable.

Interaction: Procedure

Step 3a: No interaction

Examine effect of X1 on Y at:

X2 = mean

X2 = +1 SD

X2 = -1 SD

$Y = b_0 + b_1X1 + b_2X2$

Rearrange terms: $Y = (b_0 + b_2X2) + b_1X1$

→ *With no interaction, intercept changes with X2 but the slope of X1 on Y does not*

Interaction: Procedure

Step 3b: Significant interaction

Examine effect of X1 on Y at:

X2 = mean

X2 = +1 SD

X2 = -1 SD

$Y = b_0 + b_1X1 + b_2X2 + b_3X1X2$

Rearrange terms: $Y = (b_0 + b_2X2) + X1(b_1 + b_3X2)$

→ *With an interaction, intercept AND SLOPE of X1 on Y change with X2.*

Interaction: Example

- Predicting adolescents' sexual behavior from their sexual attitudes and agreeableness
  - Sexual attitudes: Sexual Opinion Survey (SOS)
  - Agreeableness: "a" from Big Five Inventory
  - Behavior: Sociosexual Orientation Inventory (SOI)

2. With how many different partners have you had sexual intercourse on *one and only one* occasion?

☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5-6	7-9	10-19	20 or more

CORRELATION MATRIX - RAW	Sexual behavior	Sexual attitudes	Agreeableness	Interaction
	soitot	sos	a	sosXa
soitot	1.0000000	0.4821981	-0.2235485	0.3185900
sos	0.4821981	1.0000000	-0.1987734	0.8325501
a	-0.2235485	-0.1987734	1.0000000	0.3564782
sosXa	0.3185900	0.8325501	0.3564782	1.0000000

CORRELATION MATRIX - Centered

	soitot	Zsos	Za	sosXa_cent
soitot	1.00000000	0.4821981	-0.22354850	-0.08306904
Zsos	0.48219809	1.0000000	-0.19877344	0.03556010
Za	-0.22354850	-0.1987734	1.00000000	0.08511869
sosXa_cent	-0.08306904	0.0355601	0.08511869	1.00000000

REGRESSION RESULTS - RAW

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -3.0269282  1.3295076  -2.277  0.02355 *
sos          0.0520891  0.0166726   3.124  0.00197 **
a            0.0294081  0.0306397   0.960  0.33797
sos:a       -0.0006718  0.0003893  -1.726  0.08548 .
---
```

REGRESSION RESULTS - Centered

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0318105  0.0527445  -0.603   0.5469
c_sos        0.0239538  0.0027222   8.799   <2e-16 ***
c_a         -0.0200553  0.0084629  -2.370   0.0185 *
sosXa_cent  -0.0006718  0.0003893  -1.726   0.0855 .
---
```

Fit the model: 2 MEs + 1 Int

```
Call:
lm(formula = soitot ~ c_sos + c_a + sosXa_cent)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.9874 -0.6363 -0.1924  0.4840  3.3481 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.0318105  0.0527445  -0.603   0.5469    
c_sos        0.0239538  0.0027222   8.799   <2e-16 ***
c_a         -0.0200553  0.0084629  -2.370   0.0185 *  
sosXa_cent  -0.0006718  0.0003893  -1.726   0.0855 .  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Visualization with simple slopes

- Create one variable with the range of sexual attitudes (20 -> 120, **centered**)
- Create three more variables based on the simple slope equations:

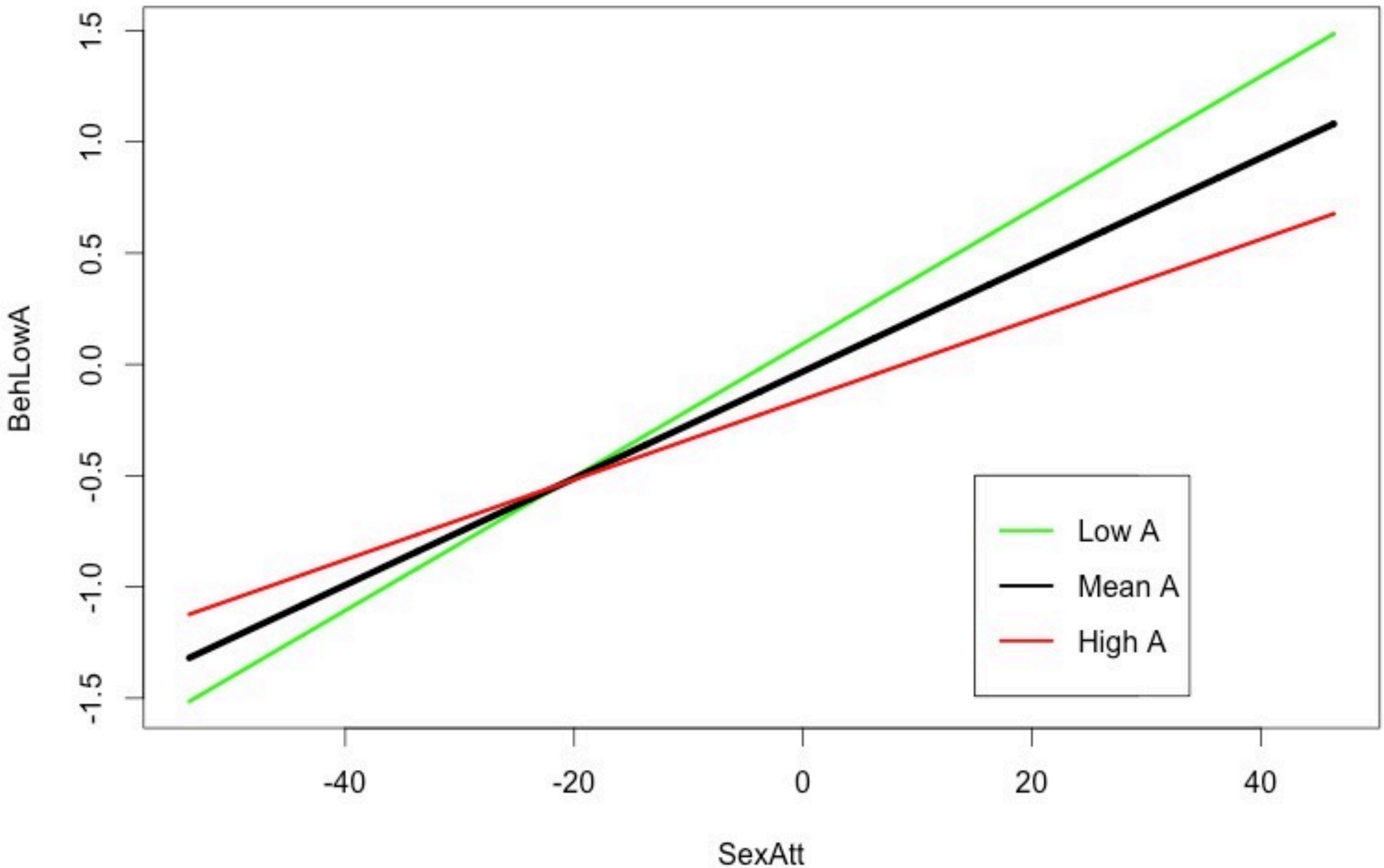
High “a”: BehHighA <- -.158 + .018*SexAtt

Mean “a”: BehMeanA <- -.032 + 0.024*SexAtt

Low “a”: BehLowA <- .094 + .03*SexAtt

Visualization with simple slopes

```
> SexAtt <- seq(20, 120, by=.1) – 73.6237
> BehHighA <-  -.158 + SexAtt*.018
> BehMeanA <-  -.032 + SexAtt*.024
> BehLowA <-  .094 + SexAtt*.03
> plot(SexAtt,BehLowA, type=“l”, col=“green”, lwd=3)
> lines(SexAtt,BehMeanA, type=“l”, col=“black”, lwd=5)
> lines(SexAtt,BehHighA, type=“l”, col=“red”, lwd=3)
> legend(15,-.5,c(“Low A”,”Mean A”, “High A”),
        lty=c(1,1,1),  lwd = c(3,5,3),
        col=c(“green”, ”black”,”red”))
```



BehLowA
SexAtt
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-40
-20
0
20
40
Low A
Mean A
High A

Invariance of interpretation

Note that the **interpretation of the slopes** is the same regardless of centering.

E.g., for uncentered params:

SexBeh = (-3.03 + .029a) + SexAtt(.052 -.001a)

At mean:

SexBeh = (-3.03 + .029(42)) + SexAtt(.052-.001(42))

SexBeh = -1.8 - SexAtt*.024

Centered

– Mean “a”: SexBeh <- -.032 + 0.024*SexAtt

Significance testing of simple slopes

Need variance/covariance matrix of regression coefficients (Add "/STATISTICS BCOV" to "REGRESSION" syntax)

$$SE_{b\_at\_X2} = \sqrt{SE_{b1}^2 + 2 * X2(\mathrm{cov}(b_1, b_3)) + X2^2 SE_{b3}^2}$$

$$t_{b\_at\_X2} = (b_1 + b_3 X2) / SE_{b\_at\_X2}$$

Distributed ~t with N-K-1 df

Significance testing of simple slopes

```
> vcov(full_model)
              (Intercept)         c_sos           c_a     c_sos:c_a
(Intercept)  2.781978e-03 -1.385954e-06 -7.547667e-06  3.687111e-06
c_sos       -1.385954e-06  7.410307e-06  4.668899e-06 -5.696186e-08
c_a         -7.547667e-06  4.668899e-06  7.162083e-05 -3.101022e-07
c_sos:c_a    3.687111e-06 -5.696186e-08 -3.101022e-07  1.515570e-07
```

Covariances

Variances (= SE^2)

Significance testing of simple slopes

b_1 = effect of attitudes
b_2 = effect of "a"
b_3 = interaction

Test high "a" slope:

$$SE_{b\_at\_X2} = \sqrt{SE_{b1}^2 + 2 * X2(\mathrm{cov}(b_1, b_3)) + X2^2 SE_{b3}^2}$$

$$SE_{b\ at\ X2} = \sqrt{.000007^2 + 2 * X2(-.00000005) + X2^2 * .0000002^2}$$

$$t_{b\_at\_X2} = (b_1 + b_3 X2) / SE_{b\_at\_X2}$$

$$t_{b\_at\_highA} = (.024 - .001 * 6.3) / .003 = 5.9$$

Continuous/categorical interactions

Compute interaction terms in the same way

Always center continuous predictors

No need to center categorical variables (since interpretation of “mean” of a categorical variable is meaningless)

Cont/Cat interaction: Example

The “salary” dataset on Canvas

Faculty salary data from the early 1980s

(though sadly unchanged)

Includes other variables of interest such as # of publications, gender, and citations.

Cont/Cat interaction: Example

Predict faculty salary based on # of publications, gender, and their interaction.

```
Call:
lm(formula = salary ~ c_pubs + female + c_pubs * female)

Residuals:
   Min     1Q Median     3Q    Max 
-12950  -6583    239   6003  21408 

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)         55649.48    1394.00  39.921  < 2e-16 ***
c_pubs                446.80      91.39   4.889 8.39e-06 ***
femaleFemale        -2794.01    2130.06  -1.312   0.1948    
c_pubs:femaleFemale  -350.19     163.36  -2.144   0.0363 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Cont/Cat interaction: Example

Salary = 55,650 + 446*pubs - 2794*female
… - 350*pubs*female

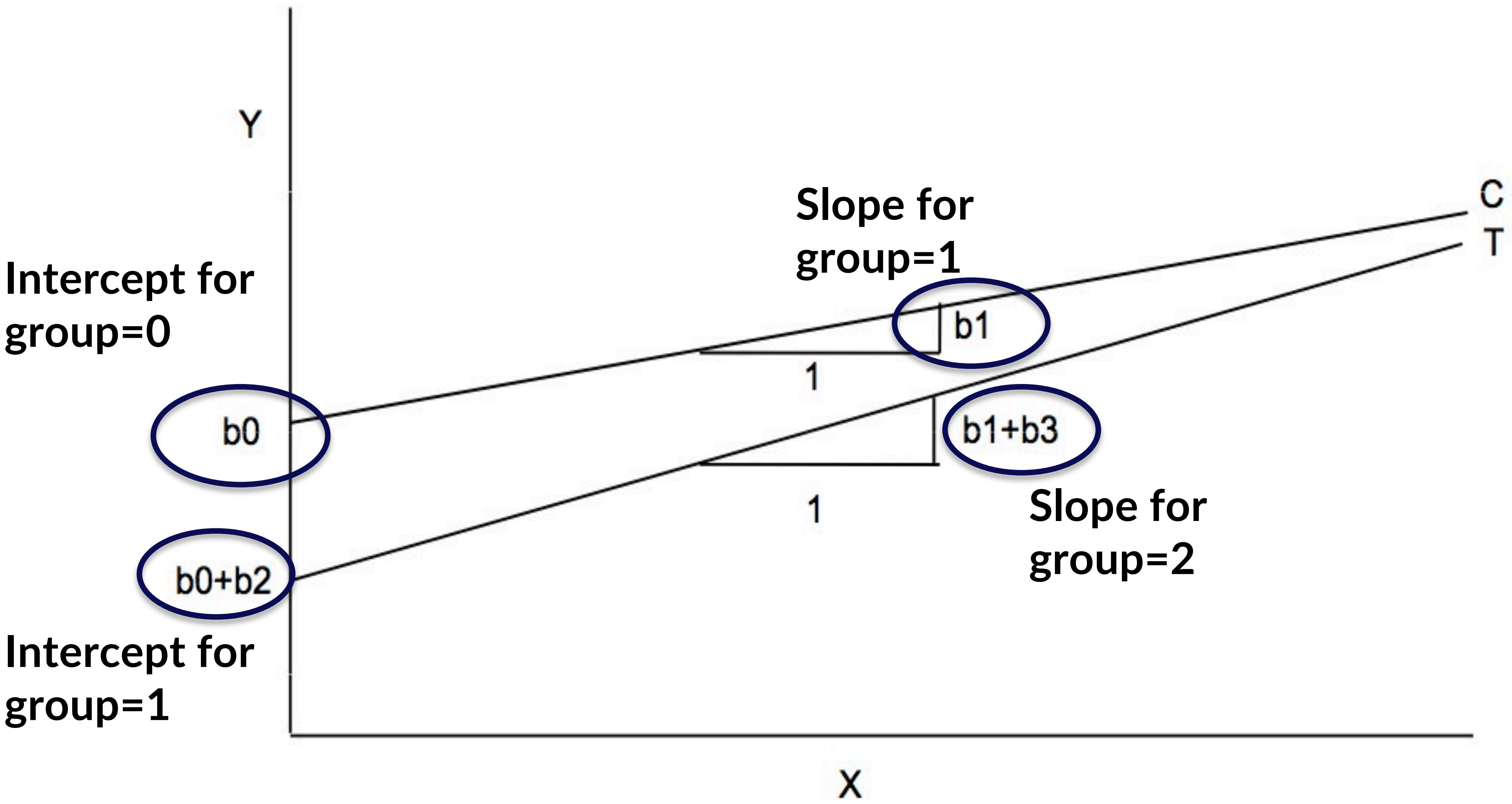
Male salary (female=0):

= 55,650 + 446*pubs

Female salary (female=1):

= (55,650-2794) + (446-350)*pubs

General Cont/Cat graph



Cont/Cat interaction: Example

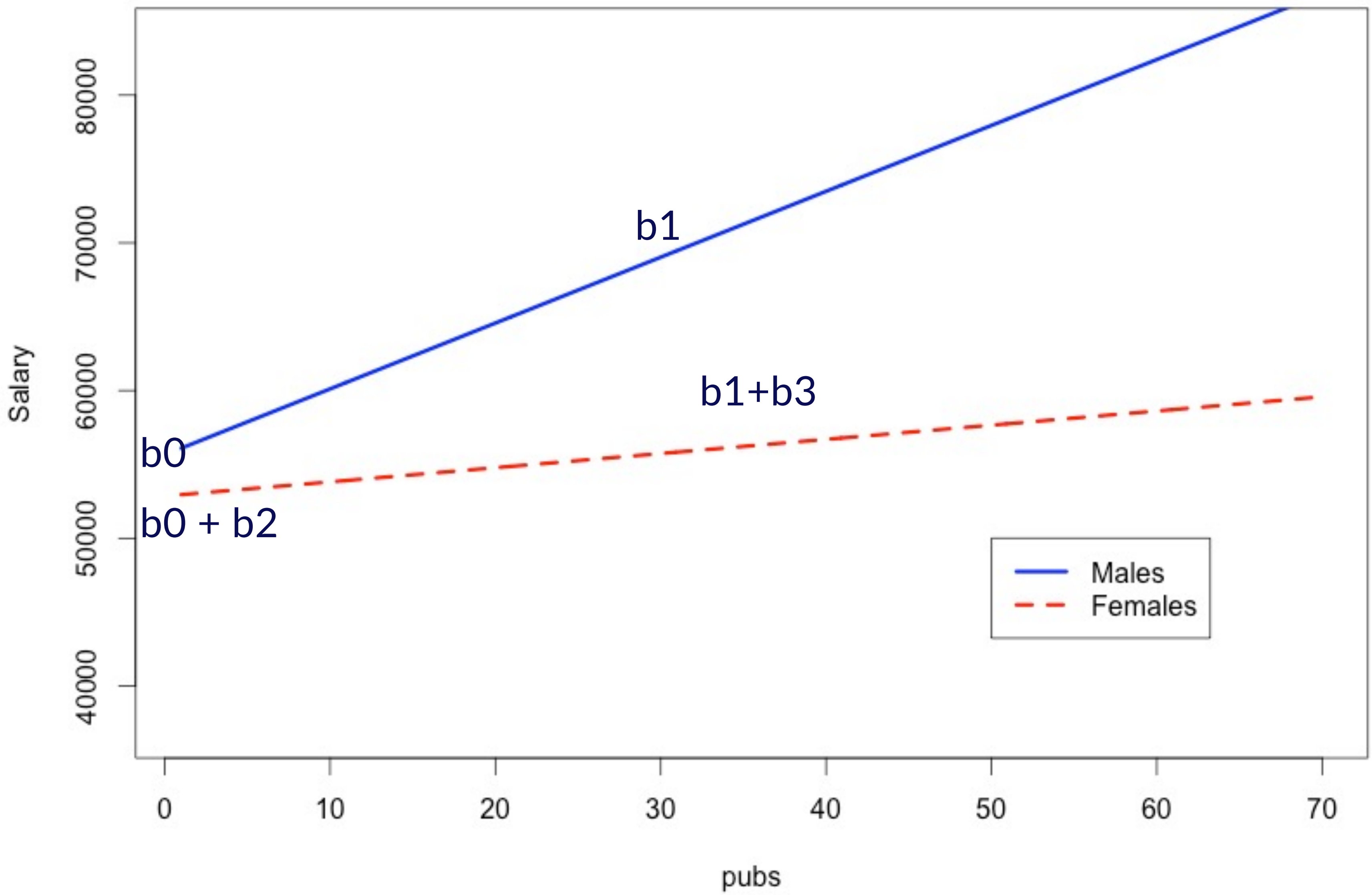
- b_1 represents the *difference* in intercepts between the groups
  - Significance test compares group intercepts
- b_3 represents the *difference* in slopes
  - Significance test compares group slopes

```
Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)           55649.48    1394.00  39.921  < 2e-16 ***
c_pubs                  446.80      91.39   4.889 8.39e-06 ***
femaleFemale          -2794.01    2130.06  -1.312   0.1948
c_pubs:femaleFemale    -350.19     163.36  -2.144   0.0363 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

b_1 test

b_3 test

In this case…


Salary
80000
70000
60000
50000
40000
0
10
20
30
40
50
60
70
pubs
Males
Females
b1
b1+b3
b0
b0 + b2

Cont/Cat Interaction: Example

Tests of simple effects (i.e., for each gender separately):

- **Could:** test simple slopes as above
- **Easier:** split sample, regress outcome on predictor (i.e., salary on pubs), then compute the F-test based on MS-predictor (from simple model) and MS-error (from larger model).

Cont/Cat Interaction: Example

R: Mdata = subset(dataset, female=0)

Males

```
Response: salary
          Df     Sum Sq    Mean Sq F value    Pr(>F)
c_pubs     1 1599322474 1599322474  22.442 3.993e-05 ***
Residuals 33 2351778342   71266010
---
```

Females

```
Response: salary
          Df     Sum Sq  Mean Sq F value Pr(>F)
c_pubs     1   34062834 34062834  0.5568 0.4625
Residuals 25 1529377297 61175092
```

Cont/Cat Interaction: Example

Full model

```
Response: salary
               Df     Sum Sq    Mean Sq F value    Pr(>F)    
c_pubs          1 1472195326 1472195326 22.0005 1.702e-05 ***
female          1   85752814   85752814  1.2815   0.26228    
c_pubs:female   1  307516045  307516045  4.5955   0.03626 *  
Residuals      58 3881155638   66916477                      
---
```

For males: $F(1,58) = 1.6b / 67m = 24$, $p<.001$

For females: $F(1,58) = 34m / 67m = 0.50$, p *ns*

Types of interactions

Synergistic (enhancing) interactions

- All three terms in same direction

Buffering interaction

- Predictors of opposite sign
- Risk and protective factors

Antagonistic interaction

- Predictors same sign, interaction opposite sign
- Total influence is less than sum of the parts
- e.g., Ability + Hard with – (ability*hard work)

Final notes on interactions

- There is no test of difference between simple slopes
- Only interpret interactions within the range of data
- Do not interpret beta weights with interaction in the equation (use simple slopes)
- All lower-order terms must be in the model to interpret an interaction